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A Model for Ranking Efficient Decision Making Units in the Situation of Nondiscretionary Factors
(Case Study: Islamic Azad University)

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Abstract

In various applications of Data Envelopment Analysis (DEA), there may exist two common problems. One problem is that there may exist nondiscretionary factors that are beyond the control of a Decision Making Units (DMU)'s management. Another problem is the lack of discrimination in DEA scores, i.e., efficient DMUs can not be ranked. The objective of this paper is to present a model for ranking efficient DMUs in the situation of nondiscretionary factors. A case study demonstrates the application of the proposed method.

Key words: Data envelopment analysis; Ranking; Nondiscretionary factors; Discretionary factors; Decision making units.

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1. Introduction

Data Envelopment Analysis (DEA[1]) proposed by Charnes & et al, (1978, 429-444) and developed by Banker & et al, (1984, 1078-1092) is an approach for evaluating the efficiencies of Decision Making Units (DMUs). Outcome of input-oriented DEA models is an efficiency score equal to one to efficient DMUs and less than one to inefficient DMUs. So, for inefficient DMUs a ranking is given but efficient DMUs can not be ranked. One problem that has been discussed frequently in the DMUs ranking literature, has been the lack of discrimination in DEA applications, in particular when there are insufficient DMUs or the number of inputs and outputs is too high relative to the number of DMU's. The research on ranking efficient DMU's can be divided into the following streams (Adler & et al, 2002, 249-265):

In the first stream, the research was pioneered by Sexton & et al, (1989, 73-105). In their research, the ranking of DMUs was based on a cross-efficiency. In the second stream, the ranking of DEA-efficient DMUs is based on benchmarking, an approach initially developed by Torgersen & et al, (1996, 379-396). They concluded that a DMU was highly ranked if it was chosen as a reference by many other inefficient DMUs. In the third stream, researchers like Fridman & Sinuany-Stern (1997, 629-637), who initiated the research in this direction, used multivariate statistical tools such as canonical correlation analysis and discriminate analysis to rank both efficient and inefficient DMU's. To increase efficiency discrimination among DMUs, in some of the papers on DEA, there have been explained that, if correlation coefficient between each pair of input (output) vectors are strong and positive, one of the input (output) vectors could be omitted (Charnes & et al, 1988, 1-26), (Dielman, 1991, 281-283), (Farzipoor Saen & et al, 2005, 503-521), (Kao & et al, 1993, 73-83), (Kleinbaum & et al, 1998, 248), (Seifert & Zhu, 1998, 279-296), (Smith, 1997, 233-252), (Sueyoshi, 1991, 463-477), (Thanassoulis, 2000, 1-13), (Jenkins & Anderson, 2003, 51-61). The last, yet the most popular, research stream in ranking DMUs is called super-efficiency. An advantage of super-efficiency method is the capability to rank both efficient and inefficient DMUs. Here is presented a concise review:

The research in this area was first developed by Andersen & Petersen (1993, 1261-1264). They proposed the idea of modifying the envelopment Linear Programming (LP) formulation so that the corresponding column of the DMUs being scored is removed from the coefficient matrix. Thrall (1996, 109-138) pointed out that the model developed by Andersen and Petersen (AP) may result in instability when some inputs are close to zero. Then, to avoid this problem, Mehrabian & et al, (1999, 261-266), (MAJ) and Slack-Based Measure (SBM) (Tone, 2002, 32-41) models were proposed.

Jahanshahloo & et al, (2004, 215-224) presented a method for ranking extreme efficient decision making units in DEA models with constant and variable returns to scale. They exploited the leave-one-out idea and l_1-norm. Jahanshahloo & et al, (2005, 371-379) using Monte Carlo method developed a method which is able to rank all efficient (extreme and non-extreme) DMUs.

Jahanshahloo & et al, (2005, 265-281) introduced a method for ranking of DMUs using Common Set of Weights (CSW). Jahanshahloo & Afzalinejad (2006, 248-260) suggested a ranking method which basically differs from previous methods. In this ranking method, DMUs are compared against a full-inefficient frontier. This method can be used to rank all DMUs to get analytic information about the system, and also to rank only efficient DMUs to discriminate between them.

Amirteimoori & et al, (2005, 122-135) described a new DEA ranking approach that uses l_2-norm. Jahanshahloo & et al, (2006, 1054-1059) showed that the technique used for rendering MAJ model unit-invariant causes the ranking to change when some inputs of some inefficient DMUs change, without causing any change in the new Production Possibility Set (PPS). They modified MAJ model so that this problem will not occur.

1. Since the DEA models have become common knowledge, the readers are directed to the references.

In order to obtain a complete ranking of efficient DMUs when the two assumptions are relaxed, Chen (2004, 213-219) proposed a modified super-efficiency DEA model to overcome the infeasibility problem and to correctly capture the super-efficiency represented by the input saving or the output surplus. Chen (Ibid, 213-219) suggested using both input-oriented and output-oriented super-efficiency models to characterize the super-efficiency when infeasibility occurs.

Jahanshahloo & et al, (2004, 209-219) proposed a method based on using gradient line, for ranking DMUs. Wang & Luo (2006, 902-915) developed two DEA models, one is based on virtual Ideal Decision Making Unit (IDMU) and the other is based on virtual Anti-ideal Decision Making Unit (ADMU). The former evaluates DMUs using the best possible relative efficiency and can be used to identify DEA efficient units; while the latter evaluates DMU's using the worst possible relative efficiency and can be used to identify those DEA inefficient units. The two distinctive efficiencies are integrated using a relative closeness index, which provides the overall performance assessment of each DMU and can thus be used as the basis of comparing and ranking the DMUs.

Saati & el al. (2001, 47-59) suggested a modification for MAJ model and proved that the modified version is always feasible and the ranking lies in (0,1]. Unlike the previous models, this model is both input and output oriented, simultaneously.

On the other hand, in any realistic situation there may exist exogenously fixed or nondiscretionary inputs or outputs that are beyond the control of a DMU's management. Here, a literature review is presented briefly.

Banker and Morey (1986, 513-521) provided the first of these DEA models by including the nondiscretionary variables in the model and defining efficiency relative to the discretionary inputs only (in the input oriented version). Currently, Banker and Morey's model can be considered as a standard model for the inclusion of nondiscretionary inputs or nondiscretionary outputs in DEA. The most important advantage of Banker and Morey's model lies in the simultaneous introduction in the same DEA programme of all the variables relevant to the research.

Ruggiero (1996, 553-565) provided an alternative model by relaxing the convexity constraint with respect to the nondiscretionary variables and used simulation analysis to highlight the advantages of his model over the Banker and Morey model. A weakness of Ruggiero's model was the tendency to identify DMUs as efficient by default as the number of exogenous factors increased. Ruggiero (1998, 461-469) extended this model to allow multiple nondiscretionary inputs. This model requires three stages and uses regression analysis to develop an aggregate index capturing the influence of the nondiscretionary factors.

Maital & Vaninsky (2001, 206-212) showed that supplying additional resources in order to achieve full efficiency is equivalent to expansion of the input/output space with additional factors that are nondiscretionary. Resource-intensive and resource-extensive efficiencies are introduced and defined.

To evaluate relative efficiency of 14 police precincts, Sun (2002, 51-71) utilized Banker and Morey's model. Muñiz (2002, 625-643) provided an alternative three-stage model to control for the effect that non-controllable inputs have no production. Like the first stage in Ruggiero (1998, 461-469), this model considers only discretionary inputs and output in the first stage. This model, however, focuses not only on the radial measure of efficiency but also the remaining slacks that remain after equi-proportionate projection to the overall frontier. Moreover, by using only DEA methods, it has the advantage that there is no need to assume any functional form in any of the stages.

Ruggiero (2004, 250-257) analyzed the effects of correlation between efficiency and nondiscretionary factors. He showed that if the true technical efficiency is negatively correlated with the nondiscretionary inputs, the existing DEA efficiency estimates will be biased upward. So, to effectively handle the problem, a corrected model was introduced.

Syrjänen (2004, 20-33) discussed the interpretation of nondiscretionary factors and suggested a generalized model for incorporating different types of inputs and outputs in DEA. He first compared two approaches to the inclusion of continuous nondiscretionary factors in a one-stage model, one introduced by Banker and Morey (1986, 513–521) and the other by Lovell (1994, 175-248) and Ruggiero (1996, 553-565). The approaches were compared theoretically, on the basis of axiomatic systems, and empirically, on the basis of examples and simulated data. To utilize the properties of the different models simultaneously, a generalized DEA model allowing for this possibility was presented.

Paradi & Schaffnit (2004, 719-735) evaluated the performance of the commercial branches of a large Canadian bank using DEA. They introduced nondiscretionary factors to reflect specific aspects of the environment a branch is operating in, such as risk and economic growth rate of the region. Farzipoor Saen (2005, 1440-1447) presented a methodology for treating nondiscretionary in SBM formulation. Muñiz & et al. (2006, 1173-1183) compared the performance of the various models that allow for nondiscretionary factors. This extends (1998, 461-469) by including the alternative models developed by Muñiz (2002, 625-643).

To the best of author's knowledge, there is not any reference that presents the model for ranking efficient DMUs in the situation of nondiscretionary factors. The objective of this paper is to propose a model which ranks efficient DMUs in the situation of nondiscretionary factors.

This paper proceeds as follows. In Section 2, proposed model which ranks efficient DMUs in the situation of nondiscretionary factors is presented. Case study and concluding remarks are discussed in Sections 3 and 4 respectively.

2. Proposed model

Suppose that there is a set of n DMUs, $\{DMU_j;\ j = 1, 2, \dots, n\}$, which produce multiple outputs y_{rj} ($r = 1, 2, \dots, s$), by utilizing multiple inputs x_{ij} ($i = 1, 2, \dots, m$). In particular, DMU_p consumes x_{ip} (i=1, …, m), the amount of input i, to produce y_{rp} (r=1, …, s), the amount of output r. Omitting the column corresponding to DMU_p, the DMU under consideration, the ranking model is obtained as follows (Saati & et al. 2001, 47-59):

$$
\begin{array}{llll}
\min & w_p + 1 & & \\
s.t. & \sum_{j=1, j\neq p}^{n} \lambda_j x_{ij} \le x_{ip} + w_p \mathbf{1}, & i = 1,\dots,m & (1)\\
 & \sum_{j=1, j\neq p}^{n} \lambda_j y_{rj} \ge y_{rp} - w_p \mathbf{1}, & r = 1,\dots,s & \\
 & \lambda_j \ge 0 \qquad j = 1,\dots,n & &
\end{array}
$$

where w_p is a free variable and **1** is a vector of ones. Model (1) by decreasing inputs and increasing outputs of the DMU under consideration by equal sizes, project it on the frontier. Simultaneous changes in input and output are equal in size because otherwise due to giving different preferences to them, the problem becomes a multi objective programming one and hence yielding a complex situation.

Since the inputs and outputs are not homogeneous and scale of objective function in model (1) is depended on the units of measurement of input and output data, unit independence is obtained by normalization, e.g. dividing each input and output to the largest of them as one of the techniques for normalization.

In the meantime, with respect to aforementioned advantages of Banker and Morey's model, Banker and Morey's output-oriented model as a standard model for the inclusion of nondiscretionary outputs is presented as follows (Banker & Morey, 1986, 513–521):

Suppose that the output variables may be partitioned into subsets of discretionary (D) and nondiscretionary (N) variables. Thus,

$O = \{1,2,\ldots,s\} = O_D \cup O_N$, $O_D \cap O_N = \Phi$

When a DMU is under evaluation, there is

$$
\begin{array}{lll}
\max & \eta + \varepsilon\left(\sum_{i=1}^{m} t_i^- + \sum_{r \in D} t_r^+\right) & \\
s.t. & x_{io} = \sum_{j=1}^{n} x_{ij}\mu_j + t_i^-, & i = 1,\ldots,m \\
 & \eta y_{ro} = \sum_{j=1}^{n} y_{rj}\mu_j - t_r^+, & r \in D \\
 & y_{ro} = \sum_{j=1}^{n} y_{rj}\mu_j - t_r^+, & r \in N \\
 & t_r^+ = 0, & r \in N \\
 & \mu_j \geq 0, & j = 1,\ldots,n \\
 & t_i^- \geq 0, & i = 1,\ldots,m \\
 & t_r^+ \geq 0, & r = 1,\ldots,s
\end{array}
\qquad (2)
$$

where $\mu = \{\mu_j\}$ is vector of DMU loadings, determining "best practice" for the DMU being evaluated. The variable t_r^+ is shortfall amount of output r and t_i^- is excess amount of input i.

The higher the value of objective function of model (2), the less efficient the DMU.

By combining ideas of models (1) and (2), the proposed model is presented as follows:

$$
\begin{aligned}
\max \quad & \eta - w_o - 1 + \varepsilon\left(\sum_{i=1}^{m} t_i^- + \sum_{r \in D} t_r^+\right) \\
s.t. \quad & \sum_{j=1}^{n} x_{ij}\mu_j - w_o 1 + t_i^- = x_{io}, && i = 1,\dots,m \\
& \sum_{j=1}^{n} y_{rj}\mu_j + w_o 1 - t_r^+ = \eta y_{ro}, && r \in D \\
& \sum_{j=1}^{n} y_{rj}\mu_j + w_o 1 - t_r^+ = y_{ro}, && r \in N \\
& t_r^+ = 0, && r \in N \\
& \mu_j \geq 0, && j = 1,\dots,n \\
& t_i^- \geq 0, && i = 1,\dots,m \\
& t_r^+ \geq 0, && r = 1,\dots,s
\end{aligned}
\tag{3}
$$

The higher the value of objective function of model (3), the less efficient the DMU. Model (3) ranks efficient DMUs in the situation of nondiscretionary factors.

3. Case study

To apply the proposed model (3), research deputies of Islamic Azad University (IAU) in Tehran county was selected. In IAU, there are 5 research deputies (DMUs). Two inputs and 2 outputs are considered. Table 1 shows the data for inputs and outputs. Number of patents, for some considerations, is a nondiscretionary factor. Table 2 shows the normalized data set.

Table 1. Inputs and outputs (1999-2000)

DMU	Inputs		Outputs	
	Number of instructors	Number of assistant professors and higher	Number of patents	Number of published books
Karaj	161	99	1	6
Central Tehran	645	278	2	26
Shahr-e-rey	122	46	1	3
South Tehran	780	420	1	1
North Tehran	383	249	1	10

Table 2. Normalized data set

DMU	Inputs		Outputs	
	Number of instructors	Number of assistant professors and higher	Number of patents	Number of published books
Karaj	.206	.236	.5	.231
Central Tehran	.827	.662	1	1
Shahr-e-rey	.156	.11	.5	.115
South Tehran	1	1	.5	.038
North Tehran	.491	.593	.5	.385

In Table 3, the efficiency results by using model (2), have been displayed. Research deputies of Karaj, Central Tehran, and Shahr-e-rey are efficient.

Table 3. Efficiency results

DMU	Efficiency
Karaj	1
Central Tehran	1
Shahr-e-rey	1
South Tehran	13.17
North Tehran	1.3

To rank efficient DMUs, model (3) has been used. In Table 4, the ranking results by using model (3), have been displayed. The research deputies have been ranked in increasing order of their objective values. As Table 4 shows, Central Tehran is the most efficient research deputy.

Table 4. Final solution

DMU rank	Research deputy	Objective Value
1	Central Tehran	0
2	Karaj	1.052
3	Shahr-e-rey	3.207

4. Concluding remarks

Ranking efficient DMUs in the situation of nondiscretionary factors is one of the most applied topics. This paper proposed an innovative method for ranking efficient DMUs in the situation of nondiscretionary factors.

The problem considered in this study is at initial stage of investigation and many further researches can be done based on the results of this paper. Some of them are as follows:

Similar research can be repeated for the cases of imprecise data, stochastic data, and generally, ranking efficient DMUs under uncertainty when some factors are nondiscretionary.

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