RESPONSE OF PILE FOUNDATIONS DUE TO LATERAL FORCE AND SOIL MOVEMENTS

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ABSTRACT

This research has investigated the response of pile foundations subjected to lateral force applied directly to pile head and loadings arising from lateral soil movements of the surrounding ground.

The behaviour of pile foundations subjected to lateral soil movements was studied through physical modelling with a specially designed testing apparatus. Laboratory experiments have been undertaken on a single pile embedded in progressive moving sand. A triangular loading block was used in the model tests to induce a progressive soil movement profile. Apart from eight general tests, sixteen tests were conducted on a single pile to examine the effects of the distance between the source where soil movements were induced and the pile location, the magnitude of axial load applied at pile head, the variation of loading block angle, varying combination of sliding and stable layer depths, and pile diameter on the responses of piles. The results of previously conducted pile tests with a uniform soil movement profile were compared with those of the current tests to examine the effect of soil movement profiles on the pile behaviour. Simple solutions were proposed for predicting the pile responses. They provided good estimate of the development of maximum bending moment and maximum shear force in the piles with soil movement. Importantly, the maximum bending moments induced by the soil movements were found to be linearly related to the maximum shear forces (sliding thrust), independent of the magnitude and depth of soil movement and soil movement profiles.

Experiments have also been conducted on pile groups in progressive moving sand,
including various pile group configurations and spacing. Both free-head and capped-head fixity conditions have been considered. The findings show that the resistances of the piles to lateral soil movements significantly rely on their locations in a group, especially for piles arranged in a line parallel to the soil movement direction. The results of the pile group tests were compared with those of the single pile tests. Group factors were defined in terms of maximum bending moment and modulus of subgrade reaction to quantify the impact of group effect. The simple solutions developed were extended for predicting the response of individual piles in a group with soil movement.

The static and cyclic responses of laterally loaded piles in cohesionless soils have been investigated as well. Guideline for estimating the design parameters for laterally loaded rigid piles in cohesionless soils were provided from extensive back calculation of measured responses of fifty-one pile tests. The elastic-plastic solutions presented by Guo (2008) were used in the back calculation. Simple expressions were presented for estimating the parameters used in the solutions. The reliability of the back calculation, the effects of the ratio of loading eccentricity to pile embedded length on the nonlinear pile response and lateral load capacity were investigated. Additionally, the apparatus was modified to apply cyclic lateral loading, with which a series of model tests were conducted on piles in dry sand under static and cyclic loadings. Analyses of the test results show that the cyclic load level has a greater impact on the pile behaviour than the number of cycles. It is noted that the gradient of the limiting force profile will decrease and the modulus of subgrade reaction will increase, after a number of unloading and reloading cycles. The induced maximum bending moment can be estimated from the applied lateral load, eccentricity of the load, and the depth at which the maximum bending moment occurs.
STATEMENT OF ORIGINALITY

This work has not previously been submitted for a degree or diploma in any university.

To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

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Hongyu Qin
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<td>$K_i$</td>
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<td></td>
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$K_p$ coefficient of passive earth pressure

$K_r$ flexibility factor

$L, l$ embedded length of pile

$L_c, l_c$ critical length of pile

$L_{m}$ sliding layer depth

$L_{s}$ stable layer depth

$m$ a constant

$M_0$ moment at ground line

$M_{m}, M_{max}$ maximum bending moment

$M_p$ plastic moment of pile

$M_u$ ultimate moment capacity

$M(z)$ bending moment in a pile at a depth $z$

$n$ a constant

$N_g$ a non-dimensional factor for calculating the $A_r$

$N_m$ fictitious tension for a stretched membranes used to tie together the springs around the pile shaft

$N_p$ a non-dimensional factor for calculating the $p_u$

$p$ soil reaction (force per unit length)

$p_u$ ultimate soil resistance per unit length

$p(z)$ soil pressure along a pile at a depth $z$

$q_c$ cone tip resistance

$q_u$ Uniaxial Compressive Strength (UCS) of intact rock

$r_0$ outer radius of a cylindrical pile

$R_L$ sliding depth ratio ($= L_{m}/L$)

$s_u$ undrained shear strength of the cohesive soil

$\bar{s}_u$ average undrained shear strength over pile embedded length

$S_{b}$ distance between the pile location and shear box boundary

$S_{b}$ Pile spacing in the direction parallel to the direction of soil movement

$S_{v}$ Pile spacing in the direction perpendicular to the direction of soil movement

$T_0$ Lateral load capacity at a defined (tip yield or yield at rotation point) state

$T_{max}$ maximum shear force

$T_t$ lateral load for active piles

$T_{tmax}$ maximum lateral load for cyclic loading

$T_{tmin}$ minimum lateral load for cyclic loading
Tu
T(z)
ul
u0
ui
u(z)
u*
w_e
w_f
w_i
x_p
y
y0
y_s
z
z0
z1
zm, z_max
z_r

Greek

α
α_0
∆y
φ'_s
γ
γ'
γ_s
γ'_s
ν_s
σ'_v
θ
ω

Tu
ultimate lateral load
T(z)
shear force in a pile at a depth z
ul
pile displacement at groundline for active piles
u0
pile displacement at groundline for active piles
ui
pile head displacement for active piles
u(z)
pile displacement along a pile at a depth z
u*
local threshold u above which pile soil relative slip is initiated
w_e
effective frame movement
w_f
frame movement measured at the top of the shear box
w_i
initial frame movement
x_p
slip depth from elastic state to plastic state
y
pile deflection
y0
pile deflection at groundline due to soil movement
y_s
free-field soil movement
z
depth measured from groundline
z0
slip depth initiated from groundline
z1
Slip depth initiated from pile base
zm, z_max
depth of the maximum bending moment
z_r
depth of rotation point

Greek

α
a constant
α_0
equivalent length to account for ground line limiting force
∆y
relative displacement between free field soil movement and pile deflection
φ'_s
effective internal friction angle of soil
γ
load transfer factor (a non-dimensional parameter)
γ_s
unit weight of soil
γ'_s
effective unit weight of soil
ν_s
Poisson’s ratio of soil
σ'_v
vertical effective stress
θ
angle of triangular loading blocks
ω
rotational angle of pile
Principal subscripts / superscripts

ave average
e effective
f frame movement
max maximum
min minimum
u undrained

Principal Abbreviations

LFP Limiting Force Profile
RS tests using Rectangular loading block at a Shallow depth
RD tests using Rectangular loading block at a Deep depth
TS tests using Triangular loading block at a Shallow depth
TD tests using Triangular loading block at a Deep depth
YRP Yield at Rotation Point
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LIST OF PUBLICATIONS

The following publications were produced to disseminate the results of the work undertaken by the author during the course of this PhD study.

Journal Publications


Conference Publications


1 INTRODUCTION

1.1 BACKGROUND

Pile foundations are often subjected to lateral static and cyclic loading generated by wind, waves, currents, traffic, and so on. Earthquakes, landslides and human activities, such as tunnelling and deep excavation in the vicinity of piles, might cause irreversible soil movements. These soil movements can generate lateral thrusts on the piles supporting both onshore and offshore structures, in a variety of circumstances. Typical examples include:

- piles supporting bridge abutments constructed on soft clay undergoing lateral soil movements resulting from an adjacent surcharge loading;
- piles adjacent to embankments;
- piles near tunnelling and excavation;
- piles used for slope stabilization;
- pile foundations for offshore platforms subjected to submarine slides; and
- liquefaction-induced lateral spreading on pile foundations during earthquakes.

The lateral forces acting on piles from soil movement are of significant concern in the design of pile foundations, especially since they may result in additional internal forces and excessive deflection on the piles. In extreme cases, they might damage the piles and compromise the serviceability and stability of the supported structures.

A number of methods have been developed for analyzing laterally loaded piles. Broadly, they can be classified into one or more of the following categories: analytical (closed-form) solutions (Hetenyi, 1946; Guo and Lee, 2001); the load transfer approach ($p \sim y$ curves) (Matlock and Reese, 1960; Reese and Van Impe, 2001);
boundary element methods (Poulos and Davis, 1980); and finite difference and finite element methods (Randolph, 1981). Recent analyses have tended to concentrate on numerical methods, in particular, the three-dimensional finite element methods. The importance of incorporating interface elements to simulate possible slippage and separation between the pile and soil, and capturing the soil nonlinearity using advanced constitutive models, has been widely recognized (Yang and Jeremic, 2002, 2003; Comodromos and Pitilakis, 2005). While advanced numerical analyses may be appropriate and necessary for detailed design, they are generally not suited to a rapid assessment in the initial stage due to high computational cost, time constraints and demand of skilled analysts. Therefore, most engineering practitioners are still inclined to use reasonably simplified analytical or semi-analytical approaches.

Predicting the behaviour of piles subjected to lateral loading arising from horizontal soil movement is a more complicated issue since free soil movement, in the absence of piles, can not be easily and accurately estimated. Great efforts have been made to clarify the responses of piles under different situations through physical modelling, and analytical and numerical analysis. Among these are studies on:

- piles supporting bridge abutments or piles adjacent to an embankment (Springman, 1989; Stewart et al., 1994; Bransby and Springman, 1997; Goh et al., 1997; Ellis and Springman, 2001);
- single piles and pile groups subjected to excavation-induced soil movements in sand (Finno et al., 1991, 2005; Leung et al., 2000, 2003) and clay (Ong et al.,
lateral spreading and its effects on pile foundations through centrifuge experiments (Boulanger et al., 2003; Dobry et al., 2003); and

• full-scale blasting tests (Ashford et al., 2006; Juirnarongrit and Ashford et al., 2006).

Recently, the performance of pile foundations during past earthquakes has raised numerous concerns regarding the failure mechanism of vertically loaded piles in liquefiable soils (Bhattacharya and Madabhushi, 2008; Knappett and Madabhushi, 2009a, 2009b). While extensive investigations have improved our understanding of the performance of piles undergoing lateral soil movements, many issues are still not clear, such as:

• mechanism of mobilisation of soil pressure on the piles due to lateral soil movement;
• estimation of lateral forces caused by soil movement;
• response of single piles and pile groups in progressive moving soil;
• effects of various soil movement profiles and soil sliding depths; and
• effects of combined loading of vertical load and lateral soil movements.

More experimental modelling and analytical analysis are required to develop robust, yet simple solutions for predicting the nonlinear response of piles to soil movement.

1.2 OBJECTIVES

The proposed experimental work in this thesis was part of an Australian Research Council (ARC) Discovery Project (Grant no. DP0209027): The response beams
subjected to axial load and lateral soil movement. In this project, an experimental programme on piles embedded in soils undergoing axial load and lateral soil movement was outlined and aims to:

- establish guidelines for estimating axial pile capacity subjected to lateral soil movement;
- identify possible modes of soil movement and pile failure;
- establish guidelines for the selection of failure modes to be used in the analyses based on pile flexibility, boundary condition, soil movement profiles;
- establish appropriate guidelines for modulus of subgrade reaction of piles in moving soil;
- provide difference in the subgrade reaction due to lateral loading and soil movement; and
- verify the predicted soil movement and pile response from relevant analytical analysis.

Under the ARC project support, Guo and Ghee (2004) developed a new experimental apparatus for studying the responses of piles subjected to lateral soil movement and axial load. Using the apparatus, Ghee (2010) has carried out model tests on single piles and pile groups subjected to uniform lateral soil movement induced by a rectangular loading block. The effect of pile diameter, axial load level, and soil sliding layer depth on the behaviour of single piles due to lateral soil movement has been investigated. The effect of pile group configuration, pile spacing and pile head fixity conditions on the behaviour of pile groups subjected to soil movement has also been explored. He has undertaken preliminary analysis of the test results. With these tests, he has clarified the behaviour of piles undergoing uniform lateral soil movement.
Nevertheless, the aims of the ARC project have not been fulfilled. The author is obligated to conduct further experimental investigation.

In line with the ARC project, this thesis aims to study the response of vertical piles to lateral forces and soil movements. The specific aims are:

1. to study the behaviour of single piles in progressively moving sand with focus on the impact of distance from the free soil movement source and pile location, variation in axial load level, variation of sliding depth, soil movement profiles, magnitude of soil movement on the responses of piles.

2. to investigate the response of pile groups to lateral soil movement for various pile spacing, configuration and pile head fixity conditions;

3. to develop simple solutions for preliminary evaluation of the internal forces and deformations in piles imposed by horizontal loading arising from soil movements;

4. to verify the relationship between the maximum bending moment and lateral load, which is used in solutions for passive piles and active piles; and

5. to obtain the design parameters for laterally loaded rigid piles in sand and to identify any difference between active and passive piles.

The logic behind the proposed study is shown in Figure 1.1. To fulfil the objectives, literatures on laterally loaded piles and piles subjected to lateral soil movements have been reviewed. Model tests have been conducted to investigate the behaviour of single
piles (aim 1) and pile groups (aim 2) subjected to soil movement. The results from pile group tests were compared with those of the single pile tests to study the pile-soil-pile interactions and coupling effects in pile groups subjected to lateral soil movements. Based on the test results, Guo and Qin (2010) proposed simple solutions. A conceptual model was proposed in this these to explain the use of equivalent load in Guo and Qin (2010) solutions (aim 3).

The expressions in Guo and Qin (2010) solutions were drawn from the theory for a laterally loaded rigid pile presented by Scott (1981). The maximum shear force was used as an equivalent load in the analysis of the results of the model tests on piles subjected to soil movement. Broms (1964b), Scott (1981), and Guo (2008) have demonstrated theoretically that the maximum bending moment linearly depends on the lateral load for rigid piles in sand. This has never been verified experimentally. Therefore, static and cyclic laterally loaded pile tests (aim 4) were also conducted in the same sand using the same apparatus as the tests on piles subjected to lateral soil movement to gain the relationship between the maximum bending moment and applied lateral load.

The existing methods for passive piles are based on modulus for active piles (Chen and Poulos, 1997; Cai and Ugai, 2003). However, the difference in the subgrade reaction between lateral loading and soil movement was not clear. Therefore, analyses were conducted for 51 laterally loaded free-head rigid piles in sand. The modulus of subgrade reaction was back calculated from the measured responses of piles using the solutions presented by Guo (2008). The results were then compared with those deduced from the passive pile tests. This study will promote a better understanding of
the responses of pile foundation due to lateral loading and soil movement (aim 5).

1.3 ORGANISATION OF THE THESIS

This thesis comprises nine chapters, the outline for which is described below:

Chapter 1 outlines the background, objectives and organisation of the thesis.

Chapter 2 presents a review of the literature relevant to this research, covering the behaviour of piles under active loading at their heads and passive lateral loading arising from horizontal soil movement.

Chapter 3 describes the test apparatus, experimental procedures and programs for a single pile and pile groups subjected to lateral soil movement. Thereafter, the methods for the determination of pile responses from measured data were explained.

Chapter 4 firstly presents the experimental results of eight general tests on a single pile in progressive moving sand, induced by a triangular loading block. Then the variation of the distance between the free-field soil movement source and the pile location, axial load applied at the pile head, soil movement depth, and angle of the loading block on the performance of the pile were examined. Note that these test results allow Guo and Qin (2010) to have developed simple solutions to estimate the evolution of the maximum bending moment and the maximum shear force with the soil movement. Herein in this thesis, a conceptual model was presented to explain the use of equivalent load in analysing the model tests. Modulus of subgrade reaction was
deduced for piles subjected to lateral soil movement. The limitations and practical values of Guo and Qin’s (2010) solutions were discussed. Limitation of existing solutions and model tests were also highlighted.

Chapter 5 examines the effect of lateral soil movement profiles on vertically loaded single piles. The concept of effective frame movement was clarified by two groups of six tests using a rectangular and a triangular block, respectively. Subsequently, the results of eight previously conducted tests using a rectangular loading block were compared with another eight tests using the triangular loading block to examine the effect of soil movement profiles on the responses of single piles. The modulus of subgrade reaction was further deduced and the influence of test parameters on the modulus was examined. The relationship between the maximum bending moment and maximum shear force was further validated by in situ tests, model tests and centrifuge tests. The influence of the scale effect and variation of soil movement profiles on this relationship was highlighted.

Chapter 6 presents the experimental results of twenty-one tests on pile groups in progressive moving sand, including two piles in a row, two piles in a line and four piles in a square with free head and capped-head fixity conditions. These tests were analysed using simple solutions developed by Guo and Qin (2010). These allow group factors to be obtained to quantify the group interaction. The current group factors were compared with the previous experimental and numerical analysis results.

Chapter 7 establishes guidelines for estimating the design parameters for laterally loaded rigid piles in cohesionless soils. The measured responses of 51 free-headed rigid
piles were analysed, from which the parameters describing the limiting force and modulus of subgrade reaction were deduced. The reliability of back estimation, the effect of the ratio of loading eccentricity to pile embedment depth on the nonlinear pile response and lateral capacity were also explored. The difference in the modulus of subgrade reaction for piles subjected to lateral loading and soil movement was investigated.

Chapter 8 presents the test results of laterally loaded piles in sand under static and cyclic loading. The theoretical relationship between the maximum bending moment and the lateral load used in chapters 4 to 6 was examined experimentally. The test apparatus in chapter three was modified to apply cyclic lateral load. A series of model tests were conducted on a pile under static and cyclic loading. The effect of the cyclic loading on the pile responses via design parameters was also examined.

Chapter 9 summarises the major findings from this research, and highlights areas requiring further research.
Figure 1.1 Methodology and structure of the thesis
2 LITERATURE REVIEW

2.1 INTRODUCTION

In practice, laterally loaded piles may be classified as active piles or passive piles, with regard to the loading transferring direction between the piles and the surrounding soils (De Beer, 1977). An active pile is principally loaded at its top, with the lateral load being transferred to the soil; such as piles acting as foundations for transmission towers, advertisement posts and offshore structures. A passive pile usually sustains lateral thrusts along its shaft arising from horizontal movement of the surrounding soil; such as piles in a moving slope or landslides.

A variety of design and analysis methods have been developed for both active and passive piles. These methods range from relatively simplistic approaches that calculate the ultimate lateral capacity to relatively sophisticated methods involving advanced numerical analyses that estimate the pile deflections. Methods are reviewed separately for active and passive piles within this chapter. The findings of previous researchers are used to compare and examine the general background literature on laterally loaded piles.

First, the approaches for analysing active piles are reviewed. They are subdivided into five sections: (1) pile flexibility and critical length; (2) failure modes; (3) limiting force or ultimate soil resistance profiles; (4) ultimate lateral capacity; and (5) load-deflection calculation of laterally loaded piles.

The subsequent section deals with design and approaches for analysing passive piles.
Due to the large amount of information on piles undergoing lateral soil movements, only soil movements due to landslides or slope instability are considered. Similar to the presentation for active piles, the following content will be reviewed: (1) failure modes and ultimate lateral capacity of slope-stabilising piles; (2) ultimate soil resistance for passive loading; (3) analysis methods for passive piles; (4) piled-slope stability analysis; and (5) laboratory and field tests on piles related to the current experimental study.

A summary of the major findings of the literature review is then presented with the areas of concerned and needing further research being highlighted.

2.2 ACTIVE PILES

2.2.1 Flexibility and critical length

The behaviour of a laterally loaded pile embedded in a soil having a Young’s modulus, $E_s$ and Poisson’s ratio, $\nu_s$, depends on its flexibility. The flexibility can be assessed in terms of the dimensionless length (Broms, 1964a, 1964b), flexibility factor, $K_r$ (Poulos, 1971) or pile soil relative stiffness, $E_p/G^*$ (Randolph, 1981; Carter and Kulhawy, 1992; Guo and Lee, 2001). Piles subjected to lateral load may be classified as rigid or flexible piles using these factors, according to criteria derived from analyses, based on the subgrade reaction approach and the elastic continuum theory. Importantly, Randolph (1981) proposed an equivalent modulus, $G^*$, to incorporate the effect of variation in Poisson’s ratio on the deformation of a laterally loaded pile, given by

$$G^* = G_s(1 + 3\nu_s / 4)$$  \hspace{1cm} (2.1)
where $G_s$ is the shear modulus of soil. He suggested that a pile would behave as if it were infinitely long from the results of the finite element analysis when

$$\frac{l}{d} \geq \left( \frac{E_p}{G_s} \right)^{2/7} \tag{2.2}$$

Carter and Kulhawy (1992) reported that a pile would behave as a rigid body when

$$\frac{l}{d} \leq 0.05 \left( \frac{E_p}{G_s} \right)^{1/2} \tag{2.3}$$

where $l$ is the length of the pile, $d$ is the outer diameter or width of the pile, and $E_p$ is the effective Young’s Modulus of the pile, calculated by

$$E_p = \frac{E_p l_p}{\pi d^4 / 64} \tag{2.4}$$

where $E_p$ is the Young’s Modulus of the pile, $I_p$ is moment of inertial of pile, and $E_p l_p$ is the bending rigidity of the actual pile.

A rigid pile displaces as a whole body, whereas a ‘critical length’, $l_c$, exists for a flexible pile beyond which the pile-head response is essentially unaffected by any additional pile length. Poulos and Hull (1989) pointed out that the existence of a critical length has some practical consequences. Firstly, only increasing the pile length beyond the critical length cannot eliminate excess pile deflection, so other measures are required; such as increasing the diameter or stiffness (changing the Young’s modulus of the pile). Secondly, the pile deformation and bending moments are limited within the critical length and are confined to the upper part of the piles, generally less than $(8 \sim 10)d$ (Randolph, 1981; Duncan et al., 1994; Poulos et al., 2001; Guo, 2006). Therefore, the assessment of the soil profile and the soil parameters in this range, below the ground surface, is critical to achieving an accurate analysis and design of the laterally loaded piles.
Expressions for the estimation of the critical length are presented in Table 2.1. These formulas were derived from the results of analysis using the subgrade reaction approach and the elastic continuum theory, discussed in section 2.2.5, with an assumed soil profile. The profiles considered have been a homogeneous stratum having constant stiffness, $E_s$ or modulus of subgrade reaction, $k$, and a non-homogeneous stratum whose stiffness increases linearly with depth. In the current thesis, Guo and Lee’s (2001) formula will be used to determine the critical length of a pile.

### 2.2.2 Failure modes

Broms (1964a) identified that the failure mode of a laterally loaded pile relies on the pile length, on the stiffness of the pile section, and on the load-deformation characteristics of the soil. An unrestrained or free-headed pile, subjected to lateral load, fails either by the development of a plastic hinge at some depth below the ground surface (pile failure) shown in Figure 2.1(a), or by rotation as a rigid body with the kicking out of the pile toe (soil failure) shown in Figure 2.1(b). For a restrained or fixed-headed pile, with the pile cap providing perfect restraint from rotation, the pile may fail in three ways, depending on the pile length as illustrated in Figure 2.2:

1. Translation of the pile with its cap and no hinges developed (Figure 2.2(a));

2. Formation of a plastic hinge at the top end of the pile with subsequent rotation of the pile around a depth below the ground (Figure 2.2(b));

3. Formation of two plastic hinges, with the first one at the top of the pile, due to a negative bending moment, and then the second at some distance down the pile, due to
a positive bending moment, which are approximately equal in magnitude (Figure 2.2(c)).

It should be noted that the degree of pile head fixity or rotational restraint has a significant influence on the bending moments and deflections of laterally loaded piles. Figure 2.3 shows that the piles may rotate with respect to the cap, while the cap itself may also rotate for partial pile head fixity. Duncan et al. (2005) performed a parametric study to evaluate the effects of partial pile head fixity on bending moments and deflections in laterally loaded piles. Their results show that the smallest values of maximum bending moment at any location in the pile were approximately 60% of those obtained for the fully fixed head condition, which always occurs at 50% fixity. Additionally, pile head deflections decreased with increasing pile head fixity, while deflections for the fixed-head condition were 25% of those for the free-head condition (Guo and Lee, 2001).

2.2.3 Limiting force profile

The ultimate lateral resistance that soils can provide against the piles is a key quantity in the analysis of laterally loaded piles, not only for the calculation of the ultimate lateral capacity, but also for the construction of the p ~ y curves, in which the ultimate soil resistance is incorporated at large pile deflection (Matlock, 1970; Reese et al., 1974; Murff and Hamilton, 1993). In the literature, several terms are used to describe this quantity, such as soil reaction (Broms, 1964a, 1964b; Poulos and Davis, 1980), limiting pressure (Randolph and Houlsby, 1984; Fleming et al., 2009) and limiting force per unit length (Guo, 2006, 2008, 2009a), which makes it very confusing. The ultimate soil resistance and limiting force are used interchangeably in this thesis.
The ultimate soil resistance or limiting force is the sum of the passive soil resistance acting on the face of pile in the direction of the soil movement, and sliding resistance on the side of the pile, less any force due to active earth pressure on the rear face of the pile (Guo, 2006). Variation in the limiting force per unit length with depth is referred to as the limiting force profile (LFP). Murff and Hamilton (1993) stated that, because of the three-dimensional and non-linear behaviour of a laterally loaded pile, establishing the limiting force profile using analytical methods is undesirable. Several methods have been adopted for the construction of limiting force profiles for different types of soils, including:

- force equilibrium on a passive soil wedge (Reese, 1958, 1962; Reese et al., 1974; Gabr and Borden, 1990);
- upper bound method of plasticity (Murff and Hamilton, 1993; Stewart, 1999);
- Cavity expansion theory (Yu, 2000);
- ‘strain wedge’ mode of soil failure (Ashour and Norris, 2000); and
- a generic expression derived using large amounts of field test data and rigorous closed-form solutions (Guo, 2006).

The following section reviews the limiting force profiles for two different types of soils: undrained and drained soils.

**2.2.3.1 Limiting force profile for cohesive soils**

The ultimate soil resistance that may be mobilised at any depth by a laterally loaded pile can be estimated by considering the failure mechanisms of the surrounding soil. Near ground level, soil moves upwards in the direction of least resistance and a passive wedge develops (see Figure 2.4(a)); while at a greater depth, the soil moves...
horizontally around the pile since the potential upward movement is suppressed by the overburden weight of the soil, as shown in Figure 2.4(b). By considering the failure mechanisms, two analytical approaches: limit equilibrium analysis and limit analysis based on plasticity theory, were used to determine the ultimate soil resistance in cohesive soil. These approaches are discussed separately for shallow and greater depths.

The ultimate soil resistance per unit length, \( p_u \), may be expressed by

\[
 p_u = N_p s_u d
\]

(2.5)

where \( s_u \) is the average undrained shear strength of the cohesive soil. \( N_p \) is a non-dimensional factor and is usually expressed as a function of depth. It should be noted that \( p_u, N_p, \) and \( s_u \) are all functions of depth.

(a) at a shallow depth \( (z \leq 3d) \)

Reese (1958) considered a wedge type of soil failure in front of a pile and derived an expression for the ultimate resistance at a shallow depth from limit equilibrium analyses. The factor \( N_p \) can be estimated by

\[
 N_p = 2 + \frac{\gamma_p' z}{s_u} + J \frac{z}{d}
\]

(2.6)

where \( \gamma_p' \) is the average effective unit weight, \( z \) is the depth from the ground surface, and \( J = 2.83 (\approx 2\sqrt{2}) \) is a non-dimensional factor.

This expression has been used for stiff clay in the presence of free water by Reese et al. (1975). Matlock (1970) suggested that the value of \( J \) may be taken as 0.5 for a soft clay and 0.25 for a medium clay. He also replaced the first term of 2 by 3 in equation (2.6). This term represents the resistance near the ground surface. Randolph and
Houlsby (1984) postulated that the value of factor $N_p$ at the ground surface ($z = 0$) was approximately 2–3, allowing for side resistance. They also suggested taking $J = 1.5$. The ultimate resistance given by equation (2.6) is applicable up to a depth of 3 diameters (Broms, 1964a; Poulos and Davis, 1980; Randolph and Houlsby, 1984).

(b) at a greater depth ($z \geq 3$)

The value of $N_p$ at greater depths is generally obtained by means of limit analysis in plasticity theory.

Broms (1964a) stated that the value of $N_p$ varies from 8.28, for a smooth square section, to 12.56, for a rough flat plate, depending on the pile section shape and the roughness of the pile surface. For a routine design and analysis, Broms took a constant value of 9.

Charts were provided by Poulos and Davis (1980) for their estimation of $N_p$ in terms of the ratio of the pile adhesion to cohesion and aspect ratio of a rhomb moving parallel to its diagonal, giving 8.28 for a smooth square pile and 11.14 for a perfectly rough pile.

Randolph and Houlsby (1984) presented upper bound and lower bound plasticity solutions for calculating the limiting pressure on a circular pile translating laterally in cohesive soil. The soil was assumed to be a homogeneous, isotropic and rigid perfectly plastic medium deforming at a constant volume. The value of $N_p$ varies in a range of 9.14 for a perfectly smooth pile to 11.94 for a perfectly rough pile. Martin and Randolph (2006) revisited the upper bound analysis. Their improved results were
obtained by devising a combined failure mechanism. The maximum discrepancy with respect to the closed-form lower bound solution was reduced to 0.65%. Klar (2008) also examined the upper bound solution and assumed the velocity field associated with the failure mechanism to be proportional to the displacement field from the elastic solution for a laterally loaded rigid circular disc, in a semi-infinite elastic medium under plane strain conditions, as presented by Baguelin et al. (1977). Although the assumed failure mechanism was continuous and without discontinuity, the solution did not enhance the results presented by Martin and Randolph (2006).

The values of $N_p$ at the ground surface and greater depth obtained from the different methods are summarised in Table 2.2. A limiting force profile (LFP) for cohesive soil may be constructed, considering the variation of ultimate resistance with depth. Two widely used LFP for cohesive soils are described herein.

Broms (1964a) ignored the soil resistance from the ground surface to a depth of $1.5d$ and assumed the ultimate resistance as being equal to $9s_d$ ($N_p = 9$). The limiting force profile is shown in Figure 2.5(a).

Fleming et al. (2009) suggested a limiting force profile with the value of $N_p$ increasing from 2 at the ground surface to 9 at a depth of $3d$, and then remaining constant below this depth, as shown in Figure 2.5(b).

2.2.3.2 Limiting force profile for cohesionless soils

Several methods have been suggested to estimate the ultimate soil resistance of laterally loaded piles in cohesionless soil (Brinch Hansen, 1961; Broms, 1964b; Reese,
These methods were developed using the earth pressure theory, with some modifications which considered the three-dimensional effect of the pile–soil interaction, and were calibrated with the results of full scale field, small scale model or centrifuge tests. Compared with the approaches for estimating the ultimate soil resistance in cohesive soils, these methods are less theoretically based. The expressions for calculating the ultimate resistance using these methods are summarised in Table 2.3. Accordingly, the various limiting force profiles are plotted in Figure 2.6. The primary difference among these limiting force profiles lies in the gradient of the average ultimate resistance across the width of the pile. Fleming et al. (2009) took the ultimate soil resistance at any depth along the pile as

$$p_u = N_p \gamma \cdot z d$$  \hspace{1cm} (2.7)

The $N_p$ is generally correlated to earth pressure coefficients. By choosing $N_p$ as $3K_p$ and $K_p^2$, the limiting force profile becomes those suggested by Broms (1964b) and Barton (1982), respectively. Moreover, the $N_p$ can be back-calculated from the measured pile response using the closed-form elastic-plastic solutions (Guo, 2008). This analysis will be presented in Chapter Seven.

Importantly, Guo (2008) commented that a number of the proposed soil pressure profiles for rigid piles (Petrasovits and Award, 1972; Meyerhof et al., 1981; Prasad and Chari, 1999) are in essence, on-pile force profiles, rather than limiting force profiles, since they may correspond to different states of the pile–soil interaction. Various expressions have been developed, based on these kinds of soil pressure profiles, to estimate the ultimate lateral capacity. Eight expressions are summarised in Table 2.4. These expressions generally offer different predicted ultimate capacities as
demonstrated from the comparison between the measured and predicted capacities for both the laboratory and full scale field tests (Prasad and Chari, 1999; Haldar et al., 2000). Haldar et al. (2000) compared the measured ultimate moments of six full-scale field tests with those predicted from the theoretical methods proposed by Brinch Hansen (1961), Broms (1964b), Petrasovits and Awad (1972) and Meyerhof et al. (1981). The ratios of the predicted over the measured ultimate moment, $R_m$ for the six tests are as follows:

- $R_m = (0.50 \sim 1.48)$ with $\overline{R_m} = 0.84$ for Brinch Hansen (1961) method, where $\overline{R_m}$ denotes the average value of $R_m$ for the six tests;
- $R_m = (0.76 \sim 2.20)$ with $\overline{R_m} = 1.22$ for Broms (1964b) method;
- $R_m = (0.56 \sim 1.60)$ with $\overline{R_m} = 0.89$ for Petrasovits and Awad (1972) method;
- $R_m = (0.32 \sim 0.81)$ with $\overline{R_m} = 0.51$ for Meyerhof et al. (1981) method; and
- The coefficients of variation of the ratio $R_m$ for the four methods were approximately (35 ~ 40) %.

2.2.3.3 Generic limiting force profile

Guo (2006) proposed a generic form of limiting force profile (LFP), in which the ultimate resistance per unit length varied monotonically with depth, being expressed as

$$p_u = A_r(z + \alpha_0)^n$$

(2.8)

where $A_r$ is the gradient of the LFP, $\alpha_0$ is an equivalent depth to consider the resistance at the ground surface, and $n (<3)$ is a power governing the shape of the LFP, as shown in Figure 2.7.
The $A_s$ is related to the soil properties in terms of density, internal friction angle for cohesionless soils, undrained shear strength for cohesive soils, and Uniaxial Compressive Strength (UCS) of intact rock and can be expressed through a non-dimensional parameter $N_g$.

$$A_s = (\bar{\gamma}_u, \gamma' d$, or $q_u^{1/2}) N_g d^{1-n} \quad \text{(any } n) \quad (2.9)$$

where $\gamma'_s$ is the average effective density of the overburden soil (unit weight above water table, and buoyant weight below), $\gamma'_d$ is the effective overburden stress at a depth of pile diameter, $\bar{\gamma}_u$ is the average undrained shear strength of cohesive soils over the slip depth under maximum load. The slip depth is the depth that plastic yield (thus slip) along the pile-soil interface occurs and soil resistance has reached the ultimate value, $p_u$, while below which the pile-soil interface is still in elastic state. Initially the slip depth may be taken as $(8\sim10)d$. $q_u$ is the maximum of the average UCS over $0.5l_c$ or that over the slip depth at the maximum imposed load (Guo, 2006, 2009d).

The value of $n$ may be taken as 0.7 for clay, 1.7 for uniform sand and 0.7~2.3 for rock, respectively, based on extensive back-analysis of the measured pile response, with the majority being field tests, including 32 piles in clay, 20 piles in sand, and 16 rock-socketed shafts. The power $n$ may be adjusted to take account of soil non-homogeneity for more general layered soil profiles. Guidelines for selecting the $N_g$, $n$ and $\alpha_0$ are tabulated in Table 2.5.

The generic limiting force profile (LFP) becomes that suggested for sand by Broms
by choosing an appropriate set of $N_g$, $\alpha_0$ and $n$. For example, substituting $N_g = 3K_\rho , \alpha_0 = 0 , n = 1$ into the equations (2.8) and (2.9), it becomes the Broms’ (1964b) LFP for sand, while giving $N_g = \gamma \cdot d / \bar{s}_u + 0.5 , \alpha_0 = 2d / N_g , \alpha_0 = 0 , n = 1$, it reduces to Matlock’s (1970) LFP for soft clay.

### 2.2.4 Ultimate lateral capacity

For any given limiting force profile, the ultimate lateral capacity can be calculated by equilibrium analysis of the horizontal force and moments, treating the soil as rigid plastic material and considering the different failure mechanisms shown in Figures 2.1 to 2.3. The analysis procedures have been described by Brinch Hansen (1961), Poulos and Davis (1980) and Poulos (1985). Various formula and design charts for determining the lateral capacity of a pile have been provided in terms of loading eccentricity, $e$, pile geometry and plastic moment, $M_p$ for both short and long piles under either restrained or unrestrained pile head fixity conditions (Broms, 1964a, 1964b; Poulos and Davis, 1980; Fleming et al., 2009). Guo (2006, 2008) presented two expressions for estimating the ultimate lateral capacity of a short and a long free-head pile. They are described below.

Firstly, the ultimate lateral capacity of a short free-head pile in a cohesionless soil with a linear limiting force profile ($n = 1$ and $\alpha_0 = 0$) described by equation (2.8) can be evaluated by

\[
\frac{T_u}{A_d l^2} = \left( \frac{z_r}{l} \right)^2 - 1
\]

where $T_u$ is the ultimate lateral load, $z_r$ is the depth of rotation point. $z_r$ can be
calculated by solving the following equation

\[
\left( \frac{z_e}{l} \right)^3 + \frac{3}{2} \frac{e}{l} \left( \frac{z_e}{l} \right)^2 - \frac{3}{4} \frac{e}{l} - \frac{1}{2} = 0
\]  

(2.11)

The form of equation (2.10) is shown to emphasise the independence of the normalized lateral capacity, \( T_u / A_l d l^2 \) from the pile diameter, but only its reliance on the load eccentricity ratio, \( e/l \) as shown in Figure 2.8.

Secondly, the ultimate lateral capacity for a long free-head pile associated with the generic limiting force profile can be calculated by

\[
\frac{M_p}{A_r} = \frac{1}{n + 2} \left[ \alpha_{n+1}^n + (n + 1) \frac{T_u}{A_r} \right]^{\frac{n+2}{n+1}} - \left( \frac{\alpha_{0}^{n+2}}{n+2} + \frac{\alpha_n T_u}{A_r} \right) + \frac{T_u e}{A_r}
\]  

(2.12)

This expression may be used to produce charts for estimating the lateral load capacity in terms of plastic moment, \( M_p \) of the pile and the parameters \( A_r, \alpha_0 \) and \( n \) for the LFP, as shown in Figure 2.9. These equations can be derived from simple limit equilibrium analysis. A different approach for deriving equation (2.12) can also be obtained using the upper bound method in limit analysis.

### 2.2.5 Analysis methods

Theoretical approaches for calculating the deflection of laterally loaded piles can be classified into two groups using: (1) the subgrade reaction approach; and (2) the continuum approach.

#### 2.2.5.1 Subgrade reaction approach

(a) Linear subgrade reaction approach
In the subgrade reaction approach, a laterally loaded pile is idealised as an elastic beam restrained by a series of infinitely closely spaced discrete linear springs distributed along the length of the pile as shown in Figure 2.10. The pile behaviour is governed by

\[ E_p I_p \frac{d^4 y}{dz^4} + ky = 0 \]  

(2.13)

in which \( k \) is the modulus of subgrade reaction, defined as

\[ k = \frac{p}{y} \]  

(2.14)

where \( p \) is soil reaction at a point on the pile per unit of length along the piles, \( y \) is the pile deflection at that point.

For a flexible pile subjected to a horizontal load, \( T_l \) and moment \( M_0 \) at ground level, exact analytical solutions to equation (2.13) have been given by Hetenyi (1946) for constant \( k \) with depth. Expressions for calculating the ground level deformation have also been given for \( k \) increasing proportionally with depth by Barber (1953) and Reese and Matlock (1956). Non-dimensional solutions have been obtained by Matlock and Reese (1960) for a general distribution of \( k \) with depth in a power form and in a polynomial form.

For a free-head, floating-base rigid pile in soils with a constant or a linear variation of the modulus of the subgrade reaction, analytical solutions have been presented by Scott (1981). These solutions are summarised in Table 2.6. It can be seen that the bending moment, shear force and soil reaction of a rigid pile are independent of the actual value of \( k \). In particular, the solutions give the relationship between the maximum bending moment, \( M_{\text{max}} \) and lateral load applied at the ground surface, \( T_l \), which can be expressed as
\[ M_{\text{max}} = (0.148 \sim 0.26) \, T \, l \]  
(2.15)

where \( z_{\text{max}} = (0.33 \sim 0.42) \, l \) is the depth, at which maximum bending moment, \( M_{\text{max}} \) occurs.

(b) Modulus of subgrade reaction

The modulus of subgrade reaction plays a significant role in the analysis of pile deformation under lateral loads. The key problem in the implementation of the linear subgrade reaction approach lies in assessing the modulus of subgrade reaction. Moreover, the shape of the \( p \sim y \) curves discussed in the next section is also controlled by the initial modulus of subgrade reaction at the start of loading. Thus the methods for determining the modulus of subgrade reaction are worth briefly reviewing.

Table 2.7 summarizes the available methods for determination of the modulus of subgrade reaction, \( k \). They can be classified into three main groups:

(1) experimental and empirical methods;

(2) direct use or modification of the \( k \) suggested for beams on elastic foundations; and

(3) simplified theoretical models.

In the experimental and empirical methods, the most direct way is to use the full-scale lateral loading tests on an instrumented pile so that the soil resistance and pile deflection along the pile can be measured. The \( k \) is also empirically related to other soil properties, generally Young’s modulus and undrained shear strength for cohesive soils and relative density for cohesionless soils, assessed on the basis of in-situ test data; such as pressuremeter, flat dilatometer and plate load tests. A useful summary of these methods for determining the \( k \) has been given by Terzaghi (1955), Jamiolkowski.

With the methods of the second group, average $k$ values were determined by direct use or modification of those suggested for beams on elastic foundations. Biot (1937) studied bending of an infinite beam under a concentrated load and resting on an elastic three-dimensional foundation. He proposed an expression for evaluation of the modulus of subgrade reaction, $k$, by only making the maximum bending moment in the beam evaluated from continuum elastic solutions identical to that obtained from subgrade reaction approach. Vesic (1961) extended Biot’s work to the case of loading by a couple and also presented an expression to calculate the value of $k$. The value for $k$ calculated from Vesic’s equation enables the distribution of deflection, slope, bending moment, shear force and pressure along the beam to be fairly close between the rigorous continuum elastic solution and the subgrade reaction solution. Bowles (1996) suggested that the value of $k$ in Vesic’s expression should be doubled for laterally loaded piles. Carter (1984) and Ashford and Juirnarongrit (2003) examined the available data from lateral load testing of piles of different diameters and modified Vesic’s equation to consider the effect of pile diameter for better prediction of the pile deflection. These proposed expressions are provided in Table 2.7. The modulus of subgrade reaction $k$ in these expressions is related to the Young’s modulus, $E_s$, Poisson’s ratio, $\nu_s$ of the soil, diameter or width $d$, of the pile and the bending rigidity, $E_p I_p$ of pile.

The third group consists of approximate analytical methods, in which the variational calculus is used to obtain the governing differential equations of the soil and pile
system, notably the modified two-parameter model by Sun (1994) and load transfer approach by Guo and Lee (2001). In Sun’s model, the modulus of subgrade reaction is related to the Young’s modulus, $E_s$, Poisson’s ratio, $\nu$, of the soil, while in Guo and Lee’s approach, it is only related to the shear modulus, $G_s$. Theoretical equations have been derived for estimating the value of $k$. They are also presented in Table 2.7. To use these equations, a non-dimensional parameter or load transfer factor, $\gamma$ has to be determined, which reflects the pile-soil relative stiffness, the form of loading (that is the eccentricity of the applied lateral load), and pile-head and pile-base boundary conditions.

The modulus of subgrade reaction determined from the above methods has been widely used in research and practice. However, they can all be criticised for certain limitations. For example, the experimental methods for determining $k$ are by and large empirical, since the modulus of subgrade reaction is not a property of soil. The pile geometry (e.g., diameter), pile head and base conditions (e.g. free-headed or fix-headed), and pile-soil relative stiffness can not be taken into account. On the other hand, Guo (2001) examined the adequacy of the available expressions for calculating $k$ suggested initially for beams on elastic foundations only. He revealed that the solutions presented by Biot and Vesic are not suitable for lateral pile analysis, particularly for rigid piles. The reasons are attributed to the significant difference between the soil deformation pattern around a laterally loaded pile and that of a beam on elastic foundation. Finally, the simple theoretical model presented by Guo and Lee (2001) provided a rational estimation of modulus of subgrade reaction. Nevertheless, it is only valid for elastic homogeneous isotropic soils. The use of Bessel functions may make it unappealing for practical application. Recently, Guo (2009d) proposed
two simple equations to estimate the modulus of subgrade reaction, thus avoiding the use of Bessel functions. He believes that they are sufficiently accurate for practical application.

Alternatively, the modulus of subgrade reaction may be back analysed from measured pile response using statistical analysis as done by Honjo et al. (2005) or using the available closed-form solutions for flexible piles (Guo 2006, 2009a; Guo and Zhu 2005a) and for rigid piles (Guo, 2008). An extensive back analysis of 51 rigid piles in cohesionless soils will be carried out in this thesis to determine modulus of subgrade reaction. Guidelines for estimation of the shear modulus are established. This will be discussed in chapter seven.

(c) \( p \sim y \) method

Extension of subgrade reaction approach to account for the non-linear soil property leads to the \( p \sim y \) method, devised by McClelland and Focht (1958). The relationship between the local unit soil resistance per unit pile length, \( p \) and pile deflection, \( y \) is generally referred as \( p \sim y \) curves in Figure 2.11. These curves were originally deduced from experimental results of carefully performed full-scale piles instrumented with strain gauges for the measurement of bending moment along their depth. Such semi-empirical curves are generally complex and may incorporate factors such as variation of the soil stiffness with depth, strain softening, dynamic and cyclic loading effects. During the past few decades, methods for constructing these curves for various types of soil and loading conditions (static and cyclic) have been developed. They include:

(1) soft clay with presence of free water by Matlock (1970);
(2) sand by Reese et al. (1974), Murchison and O’Neill (1984), Yan and Byrne (1992);
(3) stiff clay with free and no free water by Reese et al. (1975);
(4) stiff clay with no free water by Reese and Welch (1975);
(5) integrated clay method for all cohesive soils by Gazioglu and O’Neill (1984);
(6) layered soil by Georgiadis (1983);
(7) calcareous sand by Wesselink et al. (1988), Novello (1999), Dyson and Randolph (2001), and Erbrich (2004); and

Solutions to equation (2.13) with these $p \sim y$ curves for analysis of laterally loaded piles can be obtained using numerical approaches such as finite difference method (Reese, 1977; Gleser, 1984) and finite element method (McVay et al., 1995). Duncan et al. (1994) and Brettmann and Duncan (1996) have developed a characteristic load method by performing nonlinear $p \sim y$ analysis for a wide range of free-head and fixed-head piles and drilled shafts in clay and sand and derived simple equations using dimensional analysis to determine the load ~ deflection relationship at the groundline, the maximum bending moment along the shaft and its depth of occurrence. The method can be conveniently used in spreadsheet while still modelling the nonlinear behaviour of laterally loaded piles. However, it applies only to long piles in uniform soils. The development and practical application of the $p \sim y$ method have been well summarised by Reese and Van Impe (2001).

To synthesis the $p \sim y$ curves, a number of parameters are required to be properly determined and difficulties exist in choosing appropriate curves for a given combination of pile size and soil type. For instance, a number of $p \sim y$ curves have
been developed for both silica and calcareous sand (Reese et al., 1974; Murchison and O’Neill, 1984; API, 1993; Yan and Byrne, 1992; Wesselink et al., 1988; Novello, 1999; Dyson and Randolph, 2001; Guo and Zhu, 2005b). These proposed $p \sim y$ curves are widely used in practice, especially for offshore pile foundation analysis, and are summarised in Table 2.8. The proposed $p \sim y$ curves at a depth of $2d$ from these methods are depicted in Figure 2.12. Comparison of these $p \sim y$ curves reveals that significant difference exists in the shape and magnitudes of the soil reaction and pile deflection responses. For example, in the Reese et al. (1974) $p \sim y$ curves, the ultimate soil resistance $p_u$ is attained at pile deflection of $3d/80$ at any depth, while in the $p \sim y$ curves suggested by Yan and Byrne (1992), Wesselink et al. (1988), and Dyson and Randolph (2001), the soil reaction, $p$, is an ever-increasing fractional power function of $(y/d)^m$, with $m = 0.5\sim0.65$, and has no defined ultimate values. Indeed, Poulos and Hull (1989) and Poulos et al. (2001) argued that representation of the non-linear behaviour using complex curves may not be necessary for predicting lateral pile response accurately. In contrast, simple representation using elastic-plastic or hyperbolic curves (Wu et al., 1998, Georgiadis et al., 1992; Castelli and Maugeri, 2009) may be adequate to capture the non-linear pile load deflection response. This contention has been supported by the work of Guo and his co-workers (Guo, 2006, 2008, 2009a; Guo and Zhu, 2005a, 2005b). They have shown that solutions using elastic-plastic $p \sim y$ curves can give reasonably good predictions of pile performance under both static and cyclic lateral loading. These solutions are described as follows.

(1) **Solutions for flexible piles Guo (2006, 2009a)**

Using a load transfer model combined with the generic limiting force profile as discussed in section 2.2.3.3, Guo (2006) developed elastic-plastic solutions for
laterally loaded free-head flexible piles. The solutions will not be described in detail here, since a good description, validation and comments on use of the solutions are given by Guo (2006). The assumptions and features of the solutions are summarised in Table 2.9. The main concepts are explained herein.

A free-head pile is schematically shown in Figure 2.13, with a lateral load $T_l$ applied at an eccentricity, $e$ above ground line. The pile is free to deflect with no constraint imposed at the pile head and along the effective pile length except the soil resistance. The system is idealised and represented as a spring-slider-membrane system attached to the pile shaft as shown in Figure 2.14(a). The mechanical behaviour of a pair of spring-sliders at any depth is depicted by an elastic-plastic $p-y$ curve (see Figure 2.14(b)) in terms of the modulus of subgrade reaction, $k$ and the limiting force per unit length, $p_u$. The coupled effect among the discrete springs is captured by a hypothetical membrane with a tension, $N_m$. With increasing lateral load, the relative slip between the pile and surrounding soil or plastic zone initiates from the ground surface and progresses steadily down the pile to a depth, $x_p$. Below this depth, the soil is in the elastic state (zone), while above the depth it is in the plastic state (zone) and the interaction among the springs is ignored by taking $N_m = 0$.

Relying on the model, Guo (2006) presented closed-form solutions in succinct, explicit non-dimensional form. The solutions are characterised by the reciprocal of characteristics length of the pile, the slip depth and the three parameters, $A_r$, $\alpha_0$ and $n$, describing the limiting force profile. The key to implementing the solution lies in assessment of the slip depth, $x_p$, under a given lateral load. The sliding depth subsequently enables the pile deflection, bending moment and shear force in the
plastic zone to be derived, while the pile response in the elastic zone can be obtained by elastic solutions (Guo and Lee, 2001). With the solutions, extensive back-analysis of measured response of piles, with the majority being field pile tests, has been carried out and guidelines for selecting the parameters for the limiting force profile are provided for clay, sand and rock. The ranges of these parameters have been summarised in Table 2.5. The most important finding of this study is that response of laterally loaded piles is governed by the limiting force profile and the maximum slip depth and is insensitive to the shape of the limiting force profile if the total resistance within the maximum slip depth is maintained.

Reddy and Valsangkar (1970), Madhav et al. (1971), Dawson (1980), Rajani and Morgenstern (1993), Motta (1995), Alem and Gherbi (2000), Alem and Benamar (2003), Hsiung (2003), and Hsiung et al. (2006) have developed similar analytical solutions by treating the soil as elastic-plastic material, but their solutions have less versatility and variability since either a constant or linear increasing limiting force with depth was adopted. Guo’s (2006) solutions can be reduced to these special cases by taking \( n = 0 \) \((A_r \neq 0)\) or \( n = 1 \) in equation (2.8) for the generic limiting force profile. Randolph et al. (1988) have also developed an analytical approach for elastic-plastic analysis of lateral pile performance and assessing the likely formation of a gap around the piles, which allows for limiting force being mobilised over the upper part of the piles, while the response of the lower part of the piles is evaluated from elastic solutions.

Guo (2009a) extended the analysis by developing similar elastic-plastic solutions for a laterally loaded fixed-head pile. Along with the \( p \sim \) multiplier method suggested by
Brown et al. (1988), they have been used to analyse the non-linear response of pile groups.

(2) Solutions for rigid piles

Given a laterally loaded short rigid free-head pile, the pile rotates about a point as a rigid body and the displacement of the pile varies linearly with depth. The pile rotation and displacement can be determined from the equilibrium of horizontal force on the pile and bending moment about the pile-tip or pile-head, without solving the differential equation (2.13). Analysis for the rigid piles using the subgrade reaction approach has been described by Czerniak (1957), Scott (1981), Guo (2008) and Zhang (2009) and others. Scott (1981) presented the elastic-plastic solutions for a rigid pile under pure loading ($e = 0$) in light of a $k$ constant with depth and constant limiting force profile. The solutions may be used for the analysis of rigid piles in over-consolidated clay. Guo (2009c) recently extended these solutions to take the eccentricity ($e \neq 0$) into account and studied the response of rigid piles under combined lateral-moment loading. Zhang (2009) also developed a method for the nonlinear analysis of rigid piles in cohesionless soils, assuming a linear increasing soil resistance, $p_u$ and modulus of subgrade reaction, $k$ with depth. However, this method has limitations noted by the author. For instance, Zhang (2009) did not consider the effect of the eccentricity and pile diameter on the modulus of subgrade reaction, $k$. Guo (2008) defined the soil resistance per unit length along a pile mobilised at different load levels as the on-pile force profile and distinguished it from the limiting force profile. The difference between the on-pile force profile and limiting force profile is that the former depends on the lateral load levels and pile displacement, while the latter does not. He then presented closed-form solutions for both a constant
$k$ and Gibson $k$ (linearly increasing with depth) by stipulating a linear limiting force profile. For easy presentation, these solutions will be described in chapter 7. They are used for back calculation of measured responses of 51 rigid piles in cohesionless soils.

The $p \sim y$ method is the most practically useful procedure for the analysis and design of pile foundations under lateral loading. It provides a relatively simple means of analysis and enables factors such as nonlinearity, variation of soil stiffness with depth, and layering of the soil profile to be taken into account. Despite its wide application, the method is often criticised for its theoretical drawbacks and limitations. The most serious criticism directed against the $p \sim y$ method is that the soil is not treated as a continuum but as a series of discrete, uncoupled springs. Secondly, replacement of the soil continuum by discrete springs precludes the extension to the analysis of pile groups since the interaction between neighbouring piles can not be considered. However, because the recommendations for the construction of $p \sim y$ curve were generally based on the results of full-scale tests, Reese and Van Impe (2001) argued that the continuity of the soil is explicitly satisfied.

### 2.2.5.2 Continuum approach

In the continuum approach, soil is treated as a continuous medium with assumptions made to its stress-strain behaviour or constitutive relationships. There are three main means and solutions techniques in this approach for estimating the behaviour of laterally loaded piles: (a) boundary element method; (b) energy and variational principles; and (c) finite element method and finite difference method.

#### (a) Boundary element method
Poulos (1971) has presented solutions for the analysis of laterally loaded piles using the boundary element method. The pile is simulated as a thin rectangular vertical strip in an elastic homogeneous isotropic media. Pile-soil interaction is achieved from uniform distribution of normal force acting on each discretised pile element and any shear stress developed at the pile sides are neglected. Mindlin’s (1936) solutions for horizontal displacement induced by a horizontal load within a semi-infinite mass is used to evaluate the soil displacement, while pile deflection is obtained from the differential equation for bending of a thin beam, written in finite difference form. Poulos’ (1971) solutions have been used widely by many researchers in verifications of their solutions (Baguelin et al., 1977; Randolph, 1981; Verruijt and Kooijman, 1989; Sun, 1994; Guo and Lee, 2001). To model nonlinearity and soil yielding, Poulos (1973) extended the solutions by incorporating soil yielding pressure and variation of the elastic modulus by assuming elastic-plastic soil behaviour.

Banerjee and Davies (1978) have carried out similar boundary element analysis for a laterally loaded pile in a soil layer whose modulus increases linearly with depth. Davies and Budhu (1986) and Budhu and Davies (1987, 1988) improved Banerjee and Davies’s work by developing nonlinear analysis of laterally loaded piles in over-consolidated clays, cohesionless soils and soft clays. The soils are taken as elastic-plastic materials. The lateral pile soil pressure can not exceed an assigned limit pressure on the front, sides and back of the piles. The results from the boundary element analysis were curve-fitted to give simple equations for calculation of the pile-head displacement. Similar equations have also been developed and presented in algebraic expressions to facilitate practical computation by Poulos and Hull (1989).

(b) Energy and variational principles
Sun (1994) modified the Vlasov model, which was initially used for beams on elastic foundations, for the analysis of laterally loaded circular piles embedded in a homogeneous, isotropic linear elastic medium associated with an assumed displacement field as illustrated in Figure 2.15. The minimum potential energy and variational principles are adopted to derive the governing differential equation for pile deflection. Closed-form solutions have been obtained for different pile-head and base conditions. The solutions depend on a principal parameter, $\gamma$ determining the rapidity of horizontal soil displacement decaying with increasing distance from the pile. This parameter predominantly relies on the pile soil relative stiffness, loading characteristics and pile-head and base conditions and has to be evaluated iteratively. Zhang et al. (2000) extend this model for the analysis of laterally loaded shafts in rock underlying a layer of soil.

Guo and Lee (2001) examined the analysis using Sun’s approach and found that it provides unreliable results when the Poisson’s ratio of the soil is larger than 0.3. They introduced a new stress field as shown in Figure 2.16 and took the effect of Poisson’s ratio on the pile response into account by incorporating it into the equivalent modulus, $G^*$ as suggested by Randolph (1981). They then developed a new coupled load transfer model in a similar way to Sun (1994) and expressed the solutions in compact forms. In particular, the loading transfer factor correlating the displacement of pile and soil was evaluated and expressed in a simple statistical formula in terms of pile relative stiffness and ratio of pile length to radius, thus avoiding the need for iterative techniques. This model has been used to capture the pile-soil interaction in the elastic zone in the elastic-plastic solutions by Guo (2006, 2009a).
Basu and Salgado (2007) used Sun’s model to study the behaviour of a laterally loaded circular pile in multi-layered soils. Subsequently, they modified the model for piles with rectangular cross section and concluded that rectangular piles can be analysed using the solutions developed from the model for circular piles as long as they have the same bending rigidity, thus justifying the use of equivalent solid piles to lateral loading (Randolph, 1981; Guo and Lee, 2001; Basu and Salgado, 2008).

A virtue of the models discussed above is that they clearly show the interaction between the pile and surrounding soil, derivation of the differential equation for the pile deflection and determination of the soil displacement. However, these solutions are applicable only to linear elastic soils. The recourse to the iterative scheme to calculate the load transfer factors, for instance, a set of 6 factors (Basu et al., 2009) and the complicated mathematics involved may preclude their popularity with practitioners. Alternatively, Shen and Teh (2002, 2004) have presented a different way to analyse laterally loaded piles using the variational approach, with the displacement and reaction pressures along the pile shaft being represented by a finite series. Einav (2005) argued that all of these works are subjected to the constraints of the assumed non-dissipative nature of the soil. In the framework of modern thermomechanics and variational principles, he proposed an approach for the study of laterally loaded piles in dissipative soils. The method is theoretical, and still requires validation against experimental data.

\textbf{(c) Numerical methods}

Numerical methods comprising finite element method and finite difference method have been widely applied to model geotechnical problems as reviewed by Carter \textit{et al.}
(2000) and Potts (2003) and with no exception for piles under lateral loading.

Analysis was firstly carried out for piles in elastic medium using different forms of methods such as finite element method coupled with Fourier techniques (Randolph, 1981) and finite element method with substructuring techniques (Verruijt and Kooijman, 1989). Randolph (1981) presented approximate algebraic expressions for the response of flexible piles to lateral loading, based on parametric study of the results from finite element analysis. Carter and Kulhawy (1992) have presented similar equations for rigid piles. These solutions are convenient in practical use for estimating the response of piles.

With the rapid development in both computer hardware and geotechnical software, nonlinear analyses have been conducted with the soil behaviour being described by advanced constitutive models, including the von Mises model, Drucker-Prager model, Mohr-Coulomb model, and boundary surface plasticity models. In particular, they can capture the soil nonlinearity and the possible slippage and the separation between the pile and soil using interface elements, which are of critical concern to laterally loaded piles. These issues have been addressed by Brown and Shie (1990, 1991), Trochanis (1991a, 1991b), Wakai et al. (1999), Zhang et al. (1999), Zhang et al. (2000), Yang and Jeremic (2002, 2003, 2005), Comodromos and Pitilakis (2005), Dodds (2005), Fan and Long (2005, 2007), Karthigeyan et al. (2006, 2007), Tuladhar et al. (2008) and Comodromos et al. (2009) and others to improve the understanding of effects such as soil layering, pile-soil-pile interaction, group effect, cyclic loading, vertical load on the lateral response of piles in sand, and cracking of pile shaft, on the behaviour of laterally loaded piles or pile groups. Although sophisticated and rigorous
in modelling, analyses using these methods are still too costly and cumbersome for most routine applications and require extensive training because of the potential pitfalls associated with numerical analysis (Potts, 2003). Application of these methods also relies on their ability to model the soil, pile, and pile-soil interface behaviour appropriately. In many cases, the parameters required for the constitutive models of the soil and of the interface elements can not be easily determined and have to be estimated or approximated. These reasons may have prevented use of these numerical methods for routine analysis and design of laterally loaded piles in practical projects.

2.3 PASSIVE PILES

2.3.1 Failure mode and ultimate lateral capacity

Viggiani (1981) has suggested a limit equilibrium method for evaluating the ultimate lateral resistance of piles used to stabilise landslides in a two-layer purely cohesive soil. Six modes of failure, either within the soil or in pile, have been identified, depending on the geometry of the problem, on the yield moment of the pile section and on the strength of stable and sliding soils. Dimensionless solutions for the ultimate lateral resistance exerted by the pile on the sliding plane and the maximum bending moment on the pile for each failure mode have been developed. The range of occurrence of each mode and a set of formulae derived from the solutions are summarised in Table 2.10. They allow maximum bending moment to be related to lateral sliding force by assuming:

1. a constant sliding depth;
2. a uniform soil movement profile (without axial load on pile-head);
3. the shear strength of the soil remains constant with depth, but not necessarily equal, in the sliding and stable layers; and
(4) pile-soil interaction along the pile attains the ultimate state and ultimate resistance or limiting force between the pile and soil is achieved.

The solutions were subsequently amended by Chmoulian (2004) and are popularly used in the design of stabilising piles for slopes (Allison et al., 1991; O’Kelly et al., 2008).

Three main modes of failure within the soil may occur for a rigid pile, as shown in Figure 2.17, with full mobilisation of the soil resistance either in the sliding layer, or in the stable layer, or in both layers simultaneously. They are:

(1) ‘translational’ failure mode \( A \) – the whole pile translates together with the sliding soil;

(2) ‘rotational’ failure mode \( B \) – the pile rotates as a rigid body;

(3) ‘flow’ failure mode \( C \) – the pile is fixed in the stable layer and the sliding soil flows around the pile.

Within the ‘rotational’ failure mode, two local largest bending moments are developed with one above and another below the sliding plane. Figure 2.18 illustrates that additional three sub-failure modes of \( B_1 \), \( B_Y \), and \( B_2 \) may occur, with failure of pile itself by developing plastic hinges.

Poulos (1995) carried out numerical analysis for piles with soil movements arising from slope instability using a simplified boundary element analysis. The soil in the sliding layer is assumed to move as a rigid body. The underlying stable layer remains stationary. Some characteristics of pile behaviour have been revealed with practical implications. Poulos (1999) further provided design charts for estimation of maximum
shear force developed in a stabilising pile in a two-layer cohesive soil. The results of the analysis are presented in Figure 2.19. The following possible failure modes were identified.

1. ‘Flow’ failure mode – the sliding layer is shallow and the unstable soil becomes plastic and flows around the pile;
2. ‘Short-pile’ failure mode – the sliding layer is relatively deep and the length of the pile in the stable layer is relatively shallow;
3. ‘Intermediate’ failure mode – the soil strength in both sliding and stable layers is fully mobilised along the pile length; and
4. ‘Long-pile’ failure mode – the pile itself yields and could be associated with the previous three modes of soil failure.

It should be noted that the analyses presented by Viggiani (1981) and Poulos (1995) are performed with an assumption that the pile is unrestrained, i.e. the pile head is not fixed, connected to or restrained against a structure or piles. Lee et al. (1991) and Mostafa and Naggar (2006) have investigated the effect of seabed instability on pile foundations supporting offshore platforms with the pile head fixed against rotation and also described the various similar modes of pile behaviour. In using the above formulae and design charts, the key parameter is the ultimate soil resistance between the pile and soil, which will be discussed next.

### 2.3.2 Ultimate soil resistance for passive loading

Limited research work has been done into the estimation of the ultimate resistance $p_u$ for passive piles undergoing lateral soil movement. Some proposed methods are
summarised below.

For cohesive soils, the ultimate resistance $p_u$ may still be calculated using the expression given by equation (2.5). The different values suggested for piles in sliding cohesive soils are summarised in Table 2.11. On the basis of the results listed, some of the suggested values of $N_p$ are about half of those values at greater depth for active piles presented in Table 2.2. Poulos (1995) argued that there appears to be no reason why such a difference should exist other than proximity to the ground surface.

For cohesionless soils, Poulos (1995) used the approach suggested by Broms (1964b) to estimate the $p_u$ in which

$$p_u = \alpha K_p \gamma_s z d$$  \hspace{1cm} (2.16)

where $K_p$ is the Rankine passive earth pressure coefficient, $\alpha$ is a coefficient ranging between 3 and 5.

Poulos et al. (1995) took $\alpha = 4.6$ in back analysis of their model test results by matching the calculated bending moment with those measured in a single pile test.

Leung et al. (2000) have conducted a series of centrifuge tests on piles in sand subjected to excavation-induced soil movement with both stable and failed retaining walls. In particular, substantial soil movements were observed at the onset of wall collapse. Their numerical back analysis results show that the limiting force is significantly less than that calculated by equation (2.16) (Leung et al., 2002). Assuming the unstable sand behaving in a flow state like a ‘liquid’, they took the effective overburden pressure as the limiting soil pressure along the pile portion in the
soil failure zone. Theoretical prediction using this limiting pressure reveals significantly improved agreement between the calculated and measured pile responses. However, there are some disputes in relation to using the smaller limiting soil pressure at larger soil movements (Chen and Poulos, 2002).

Smethurst and Powrie (2007) have presented measured and calculated net pressure profiles for a pile used to stabilise a railway embankment. Results of their analysis indicate that the pile-soil pressure is much less than the limiting values following Broms (1964b) when soil would start to flow around the pile. For the typical pressure profiles shown in Figure 2.20(a), the measured net pressure is on average about half of the limiting pile-soil pressure. They concluded that the pile may fail firstly by formation of a plastic hinge before the ultimate lateral pressures are likely reached over much length of the pile in an infrastructure slope. However, this may not be a general result for the pile-soil interaction, which depends on the relative stiffness of the pile and soil, and also on the magnitude of the soil movements.

Guo (2009b) reported that the $p_u$ for passive loading was similar to that for laterally loaded piles. The $p_u$ was deduced from approximately 50 tests on instrumented model piles in uniform or progressively moving sand.

For the general case of a $c - \phi$ soil, Ito and Matsui (1975) have developed a theory and presented theoretical equations for estimating lateral forces acting on piles in a row through an unstable layer above the sliding plane. It is assumed that Mohr-Coulomb’s plastic condition occurs in the surrounding ground just around the piles and the lateral force imposing on the piles is independent of the state of equilibrium of the slope.
Figure 2.21 depicts the status of plastic deformation of the piles under lateral soil deformation. The lateral force imposing on a pile per unit length can be estimated by the following equation.

\[ p(z) = cA \left\{ \frac{1}{N_\phi \tan \phi} \left\{ \exp \left( \frac{D_1 - D_2}{D_2} N_\phi \tan \phi \tan \left( \frac{\pi}{8} + \frac{\phi}{4} \right) \right) \right\} - 2N_\phi^{1/2} \tan \phi - 1 \right\} + \frac{2 \tan \phi + 2N_\phi^{1/2} + N_\phi^{-1/2}}{N_\phi^{1/2} \tan \phi + N_\phi - 1} \right\] 

\[ - c \left\{ D_1 \frac{2 \tan \phi + 2N_\phi^{1/2} + N_\phi^{-1/2}}{N_\phi^{1/2} \tan \phi + N_\phi - 1} - 2D_2 N_\phi^{-1/2} \right\} \]

\[ + \frac{\gamma_s z}{N_\phi} \left\{ A \exp \left( \frac{D_1 - D_2}{D_2} N_\phi \tan \phi \tan \left( \frac{\pi}{8} + \frac{\phi}{4} \right) \right) - D_2 \right\} \]  

(2.17)

where \( N_\phi = \tan^2 (\pi/4 + \phi/2) \), \( A = D_1 (D_1 / D_2)^{\left( N_\phi^{1/2} \tan \phi + N_\phi^{-1/2} \right)} \), \( c \) is the cohesion of the soil, \( D_1 \) is centre to centre distance between piles, \( D_2 \) is the opening between the piles, \( \phi \) is internal frictional angle soil, \( \gamma_s \) is unit weight of the soil, \( z \) is depth from ground surface.

The solutions have formed the basis of some suggested design methods for the stability analysis of slopes with piles (Ito and Matsui, 1981 and Hassiotis et al., 1997). Nevertheless, the model has been developed for rigid piles with infinite length with the assumption that the soil is rigid and perfectly plastic. Thus this model may not represent the behaviour of actual piles in the field. Equation (2.17) implies that the lateral force per unit length increases linearly with the depth and is not influenced by the pile position in the slope. As pointed out by Poulos (1995), this equation is also only applicable for a limited range of pile spacing.
In addition, De Beer and Carpentier (1977) have also derived an alternative equation by modifying Ito and Matsui’s method. By taking $c = 0$ or $\phi = 0$ into equation (2.17), the lateral force can be estimated for cohesionless or cohesive soils, respectively.

2.3.3 Analysis methods

A number of methods have been developed for the analysis of piles subjected to lateral passive loading from lateral soil movement. Stewart et al. (1994) have presented a comprehensive review of various existing analytical and design methods to estimate the effect of lateral soil movement induced by the construction of approach embankment on piles supporting bridge abutments. They classify these methods into four categories.

(1) Empirical methods – the pile response is estimated in terms of maximum bending moment and pile cap deflection on the basis of charts developed from experimental data;

(2) Pressure based methods – a pressure distribution acting against the piles is estimated in a relatively simple manner, and is generally only used to calculate the maximum bending moment in the piles;

(3) Displacement based methods – the distribution of lateral soil displacement is input and the resulting pile deflection and bending moment distribution are calculated; and

(4) Finite element analyses – the piles are represented in the mesh and the overall soil-pile system response is calculated.

In the specific case of piles in unstable slopes, the pressure-based methods (Ito and
Matsui, 1975; De Beer and Carpentier, 1977; Viggiani, 1981) can be used to estimate the ultimate lateral resistance of stabilising piles. However, they apply to the ultimate state only and give no indication of pile-soil interaction and development of pile resistance with soil movement. In reality, the soil reaction distribution along the pile depends on the relative soil-pile displacements. Smethurst and Powrie (2007) and White et al. (2008) have measured negative upslope soil reaction (see Figure 2.20(a)) on free head piles near the surface as a result of pile displacement exceeding the soil displacement as shown in Figure 2.20(b). This so called ‘gripping’ restraint (Dobry et al., 2003; Brandenberg et al., 2005) on the pile can not be considered in the pressure-based methods. The effect of this negative pressure is to reduce the shear force at the sliding surface. Consequently, the pressure-based methods may over-predict the resistance that the pile can provide to arrest the sliding of the potentially unstable portion of the slope.

The displacement-based methods are more versatile in modelling the pile-soil interaction, since the soil displacement profile around the pile and development of the pile resistance with soil movement can be considered. These methods allow the bending moment and deflection profiles to be calculated.

Numerical analysis using the finite element and finite difference method is the most rigorous approach in capturing the behaviour of piles subjected to lateral soil movement. Complicated influencing factors such as soil stratigraphy, soil movement profiles, non-linear soil behaviour and pile-soil interaction can be well represented and simulated. Three-dimensional numerical analyses have been conducted to evaluate the response of piles due to slope movement and stability of a slope.
reinforced with piles (Cai and Ugai, 2000; Chen and Martin, 2002; Jeong et al., 2003; Martin and Chen, 2005; Won et al., 2005; Wei and Cheng, 2009). They will be discussed in the next section.

In this section, analyses using the displacement-based methods are discussed. The analysis procedure generally involves two major steps:

1. estimation of the free soil displacement profile in the absence of piles; and
2. analysis of the soil-pile interaction by superimposing these movements on the piles.

Similar to the analysis of piles under active loading, there are two models in schematization of the soil surrounding the piles: (1) the subgrade reaction approach; and (2) the continuum approach.

2.3.3.1 Subgrade reaction approach

Modification of the subgrade reaction approach to analyse piles subjected to lateral soil movement has been reported by many researchers (Marche, 1973; Byrne et al., 1984; Maugeri and Motta, 1991; Reese et al., 1992; Frank and Pouget, 2008; White et al., 2008). Similar to modelling piles under active lateral loading, the soil is replaced by a system of nonlinear springs. In this analysis, the lateral soil reaction is assumed to be a function of the relative displacement between the pile and soil, given by

\[ p = k(y - y_s) \]  

(2.18)

where \( y_s \) = free-field soil movement, \( p \) is the soil reaction, \( y \) is the pile deflection and \( k \) is the modulus of subgrade reaction. The response of the pile subjected to only free-field soil movement without externally applied lateral load can then be obtained by
solving the following differential equation

\[ E_p I_p \frac{d^4 y}{dz^4} - p(y - y_s) = 0 \]  

(2.19)

The use of this method requires imposing a known free-field soil displacement profile to the boundary ends of the springs as shown in Figure 2.22. Both displacement and moment response along the pile can then be calculated corresponding to input of free-field soil movement. When the soil movement is large enough to cause the ultimate pressure to be fully mobilised, the ultimate resistance discussed in section 2.3.2 is used. When predicting the response of piles using the subgrade reaction method recourse to a numerical approach is generally required. This analysis has been implemented via computer programs LPILE (Reese and Wang, 2000) and PILATE (Frank and Pouget, 2008).

Chow (1996) has also described a numerical model for the analysis of piles used for slope stabilisation. The piles are modelled using finite elements. The soil response at the individual piles is modelled using the subgrade reaction approach and pile-soil-pile interaction is considered using the theory of elasticity.

Some difference in the load transfer curves is noted between the active and passive piles. Bransby (1996) stated that the passive load transfer \( p \sim \Delta y \) curves (\( \Delta y = y_s - y, y_s \) is an equivalent uniform soil displacement, \( y \) is lateral pile displacement) characterise the local soil-shear deformation around the pile, whereas \( p \sim y \) curves used for the active loading also include the effects of both local and global soil behaviour and depend on pile-group geometry and loading. Bransby and Springman (1999) have further discussed the selection of \( p \sim \Delta y \) curves’ functions for passive lateral loading of pile groups. Little work has been done to investigate the load transfer curves used
to predict piles in unstable slopes.

Within the subgrade reaction approach, analytical methods have also been developed for predicting the response of passive piles. Table 2.12 summarises the condition and salient features of the various methods. They are only applicable to unrestrained free-head piles.

Fukuoka (1977) has presented elastic solutions for the response of piles with infinite length in landslides considering a uniform movement of the sliding layer. The loading on the pile induced by the landslides was replaced by a concentrated lateral force at the depth of the sliding plane. The pile segments in both the sliding layer and stable layers were taken as infinite beams embedded in elastic foundation. Their response was estimated separately with the continuity conditions of the pile enforced at the sliding surface.

Cai and Ugai (2003) extended Fukuoka’s solutions to take account of the influence of laterally linear movement of the sliding layer in Figure 2.23. They revealed that increasing the movement gradient of the sliding layer, $\tan \theta_0$, induces higher maximum bending moment in the stable layer and smaller maximum bending moment in the sliding layer. The use of these solutions and their accuracy relies on selecting the concentrated force, $H$ and the movement gradient of the sliding layer, $\tan \theta_0$, for which guidance has not been established. Moreover, the magnitude of the soil movement can not be considered, since pile displacement near the surface of the sliding layer is assumed to be identical to the movement of the sliding layer. In other words, it corresponds to the ultimate state of soil flowing past the pile head at the surface of the
sliding layer.

Guo (2003) has presented elastic-plastic solutions for passive piles based on the ‘equivalent load’ concept. The effect of the lateral soil movement is encapsulated into an equivalent load by stipulating rigid rotation of the pile in the sliding layer as shown in Figures 2.24. The assumption of rigid rotation may be suitable in the case of normal sliding as in Figure 2.24(c). Guo (2009b) revisited this approach and refined the solutions to simulate two predominate normal and deep sliding mechanisms, see Figure 2.24(d). The equivalent load is correlated to soil movement, pile deflection and rotation at the sliding surface, and parameters describing the limiting force profile in the sliding layer. Four methods; elastic-elastic coupled (E-E (coupled)), elastic-elastic (E-E), elastic-elastic and plastic (E-EP) and elastic-plastic and elastic-plastic (EP-EP) have been developed. They were verified using relevant boundary element analysis. These methods have been used to evaluate the measured pile response of eight field tests. The study reveals that the E-EP and EP-EP methods are sufficiently accurate to predict nonlinear response of passive piles in normal and deep sliding, respectively.

2.3.3.2 Continuum approach

Poulos (1973) has presented a boundary element method for analysis of single piles embedded in soil undergoing lateral movement. The method is an extension of the approach developed for lateral loaded piles based on elastic continuum theory (Poulos, 1971). The analysis requires an input of the distribution of horizontal soil movement, soil modulus with depth, and the ultimate soil pressure on the pile. Parametric studies were conducted to examine the effects of various factors on pile behaviour, including effect of relative pile flexibility, boundary conditions, soil movement distribution,
Young’s modulus and limiting pressure distribution. The method accounts for the continuity of soil, but a good prediction from the method depends on an accurate estimation of the magnitude of soil movement and distribution of lateral soil modulus and limiting lateral pile-soil pressure, which are difficult to be accurately determined. The approach has been modified to analyse and design reinforcing piles, to increase slope stability (Hull et al., 1991; Poulos, 1995, 1999; Lee et al., 1995; Madhav et al., 1997) and study the effect of seafloor instability on offshore pile foundations (Lee et al., 1991).

Chen and Poulos (1997) presented a number of elastic solutions in chart form generated using a boundary element program. These solutions assume that the soil remains elastic and they are only suitable for unrestrained free-head piles in uniform soil or ‘Gibson’ soil with either a uniform or linear soil movement profile. A reasonable estimation of pile behaviour may be obtained when the ratio of soil movement to pile diameter is less than 10%. The design charts tend to overestimate the maximum bending moment and pile head deflections with the increasing of this ratio. The numerical analysis using the ERCAP program by Poulos (1999) indicates that the ultimate conditions are reached when the soil movement exceeds about 60% of the pile diameter, regardless of the soil failure modes.

2.3.4 Piled-slope stability analysis

The use of piles to stabilise active landslides or to increase the stability of potentially unstable slopes has been applied successfully. Such applications have been extensively reported (Fukuoka, 1977; Ito and Matsui, 1975; Ito et al., 1979; Popescu, 1991; Bromhead, 1997; Zeng and Liang, 2002; Smethurst and Powrie, 2007; Frank
and Pouget, 2008; O’Kelly et al., 2008; Kang et al., 2009). The methods for the
design and analysis of slope stabilising piles can be generally classified into two
groups: (1) uncoupled analysis; and (2) coupled analysis.

2.3.4.1 Uncoupled analysis

In uncoupled analysis, it is assumed that piles provide an additional resistance at the
intersection of the critical sliding surface with the piles to increase the safety factor of
the slope. The slope stability and pile response are considered separately. The slope
stability is generally evaluated using the limit equilibrium method (Ito et al., 1979;
Lee et al., 1995; Hassiotis et al., 1997; Jeong et al., 2003) or limit analysis (Yu et al.,
1998; Ausilio et al., 2001; Nian et al., 2008). The pile response can be calculated
using either the subgrade reaction approach (Ito et al., 1979; Hassiotis et al., 1997;
Jeong et al., 2003; Nian et al., 2008) or the continuum approach (Lee et al., 1995;
Poulos, 1995). The main features of these methods are summarised in Table 2.13. The
results obtained from these studies are rather different and in some cases even
contradict each other, especially with respect to the most effective location of the pile
within the slope.

The design procedures for piles have been described by Viggiani (1981) and Poulos
(1995) and usually consist of three main steps:

1. evaluating the total shear force needed to increase the safety factor for the
   slope to the desired value;

2. evaluating the maximum shear force that each pile can provide to resist sliding
   of the potentially unstable portion of the slope; and

3. selecting the type and number of piles as well as the most suitable location for
these piles within the slope.

2.3.4.2 Coupled analysis

In the coupled approach, the pile-soil interaction and piled-slope stability are evaluated simultaneously using either three-dimensional finite element or finite difference analysis. Cai and Ugai (2000), Won et al. (2005), Wei and Cheng (2009) and Ellis et al. (2010) performed coupled analyses to predict the stability of a slope reinforced with piles. The shear strength reduction method (Zienkiewicz et al., 1975; Duncan, 1996) has been used to evaluate the safety factor of the slope. The effects of pile spacing, pile head conditions and bending stiffness and pile positions on the safety factor, the optimum pile location have been examined.

2.3.5 Laboratory and field tests

2.3.5.1 Laboratory tests

In this thesis, the behaviour of piles subjected to lateral soil movement is investigated through model tests and therefore it is worthwhile to review the previous experimental research that has been conducted in this area. Laboratory studies directly addressing the response of either single piles or pile groups due to lateral soil movement have been investigated extensively either by using centrifuge modelling techniques or small-scale laboratory model tests.

Performance of piles has been investigated through centrifuge modelling for:

- piles supporting bridge abutments or pile adjacent to an embankment (Springman, 1989; Stewart et al., 1994; Bransby and Springman, 1997; Ellis and Springman, 2001);
• single piles and pile groups in sand or clay due to excavation induced soil movements (Leung et al., 2000, 2003, 2006; Ong et al., 2006, 2009); and
• single piles and pile groups in liquefied and laterally spreading ground (Abdoun et al., 2003; Dobry et al., 2003; Dobry, 2007; Finn, 2005; Bhattacharya et al., 2004; Brandenberg et al., 2007).

Performance of piles has been investigated through laboratory model tests for piles subjected to lateral soil movements (Fukuoka, 1977; Chen, 1994; Poulos et al., 1995; Chen et al., 1997; Pan et al., 2000, 2002a; Tsuchiya et al., 2001; White et al., 2008). Only the recent work related to the current study is reviewed.

To examine the performance of piles subjected to lateral soil movements, Chen (1994) conducted laboratory model tests on single piles and pile groups embedded in calcareous sands. The experimental apparatus is illustrated in Figure 2.25. The soil movement was induced by rotating upper steel plates of a model container such that a triangular movement profile (with maximum at the top and zero at a certain depth) was achieved. It must be pointed out that this profile only refers to soil movement adjacent to the boundary of the container, while the soil movement at the location of the pile was not measured and remained unknown. The effect of pile head fixity condition, ratio of moving soil depth to pile embedded depth, and pile diameter on the maximum bending moment in single piles has been investigated. The effects of a number of parameters on the lateral response of a pile within a group, including pile spacing, pile configuration, number of piles and pile head fixity condition have also been examined. The group effect was quantified using group factors defined in terms of maximum bending moment. The results show that group effect might either
increase or decrease the maximum bending moment, depending on the above-
mentioned influencing factors. The work has been published by Poulos et al. (1995)
and Chen et al. (1997).

Pan et al. (2000) have conducted laboratory tests on single piles in soft clay to
determine the ultimate soil pressure acting on the pile shaft. The pile was
instrumented with pressure transducers mounted on the pile surface. Both the pile
head and tip were fully fixed against movement and rotation. The test results indicate
that the ultimate soil pressure for a single pile was about 10s_u, which was fully
mobilised at a relative soil-pile movement of 0.4 times of the pile diameter.

Leung et al. (2000) have carried out centrifuge model tests to investigate the
behaviour of piles and pile groups in sand due to excavation induced soil movement.
It is found that the maximum bending moment and pile deflection decrease
exponentially with increasing distance between the wall and pile. At an excavation
depth, the pile bending moment increases with the increasing soil movement until a
threshold is reached. In the case of a retaining wall collapse, a clear soil failure zone
was identified behind the wall. The piles within the failure zone suffer larger bending
moment and deflection than those outside the failure zone. Thus, it is reasonable to
take the boundary of the soil failure zone for demarcating the severity of detrimental
pile response (Leung et al., 2002).

Guo and Ghee (2004, 2005) developed an experimental apparatus, which allows
lateral soil movements and vertical load to be applied simultaneously on a pile. This
apparatus was used to investigate the behaviour of piles subjected to progressively
moving sand in this thesis. The details of the apparatus were described in chapter three. Guo and Ghee (2005) reported the results of four tests on a 32-mm diameter free-head pile undergoing a uniform soil movement profile. An increase in the axial load from 0 to 294 N on the pile-head leads to: (1) a 50% increase in maximum bending moment; (2) a 25% decrease in the depth of maximum bending moment; (3) a 30% lower soil movement at which the soil reactions reach maximum. They further indicated that when an axial load is applied, (1) the pile behaves as if rigid at the ratio of sliding layer depth to stable layer depth of 0.5 (keeping the pile length of 700 mm); (2) the pile head may deflect more than the soil movement; and (3) the pile deflects mainly by rotation, rather than by translation as was the case with no axial load.

Guo and Qin (2006) carried out four model tests using the same apparatus but on a 50-mm diameter free-head pile in sand subjected to a triangular profile of soil movement. These test results show that the soil sliding depth is the control parameter on the pile behaviour. An axial load of 294 N on the pile head can create additional bending moment and pile displacement, which also makes the mode of pile-soil interaction changing with soil sliding depth.

Guo and Qin (2005) compared the behaviour of a 50-mm diameter free-head pile subjected to triangular and uniform soil movement profiles. They found that the pile responses such as the distribution bending moment, and pile displacement are significantly affected by the soil movement profiles. For example, the maximum bending moment increases approximately 50% when the uniform soil movement profile is replaced by a triangular soil movement profile. In the tests with a triangular soil movement profile the bending moment increases with soil movement from 30
mm onward, and reaches maximum values at soil movement of 80 mm. In contrast, the moment reaches maximum at a much lower soil movement of 20 mm.

Guo et al. (2006) further examined the effect of soil movement profiles on vertically loaded piles in sand. Again four tests are presented for an instrumented 50-mm diameter free-head pile subjected to an arc, and a uniform profile of soil movement. The comparison of the test results indicates that the soil movement profiles dominate the distribution of bending moment along the pile, and pile displacement. 40 ~ 50% higher maximum bending moment could be induced by changing the uniform soil movement profile to an arc one, whereas the pile displacement may reduce by 2 ~ 3 times. Same percentage of increase in the moment is noted due to the addition of an axial load of 294 N, where increase in displacement due to an arc movement profile is observed, but decrease due to a uniform movement profile is resulted. They commented that the test results should be useful to practical design.

Following the work by Guo and Ghee (2004, 2005) and Guo et al (2006), Ghee (2010) conducted more model tests to investigate the response of a single pile in sand subjected to an axial load and a uniform profile of soil movement induced by a rectangular loading block. He investigated the effect of pile diameter, density of sand, axial load and two combinations of sliding and stable layer depths on the pile responses. He presented the results of individual tests but did not undertake analysis of the test data. He also conducted model tests on pile groups subjected to a uniform soil movement profile by varying the pile diameter, axial load per pile applied on the top of the pile cap, pile arrangement, and pile spacing, and two combinations of sliding and stable layer depths. These tests were all conducted in a capped-head pile
head fixity condition. He concluded that: (1) the maximum bending moment increases with the increase of the ratio of sliding layer depth to stable layer depth and the pile diameter; (2) the maximum soil pressures acting on the pile in the sliding layer and the stable layer of the soil did not exceed the limiting pressure proposed by Barton (1982); (3) a linear relationship exists between the maximum bending moment and maximum shear force.

As indicated above, the results of individual tests have been presented and published; only preliminary analysis of the test results has been carried out. Most importantly, the use of soil movement in these publications is arguable and can be misleading. The measurement for quantifying the magnitude of lateral soil movement was virtually the boundary movement of the shear box, rather than the movement of the soil surrounding the pile. In the literature on pile foundations subjected to lateral soil movements, for example, Poulos (1973, 1995), Byrne et al. (1984) and Cai and Ugai (2003), the lateral soil movements generally represent the movement of the soil immediately adjacent to the piles. In this thesis, frame movement, \( w_f \) (explained in chapter three) is used to replace the soil movement as presented in the previous publications (Guo and Ghee, 2004, 2005, Guo and Qin, 2005, 2006, Guo et al., 2006). This correction and clarification have made significant improvement on the interpretation and analysis of the test results and resulted in the development of Guo and Qin (2010) simple solutions for calculation of the maximum bending moment and maximum shear force in the model tests on piles undergoing lateral soil movement. Since the solutions were developed on the basis of test results, they will be presented after the analysis of the test results in chapter 4.
White *et al.* (2008) have conducted large-scale load tests on slender steel-reinforced, concrete piles subjected to free-field lateral soil movement. The piles were installed through a shear box into stable soil. The soil movement was induced by pushing the direct shear box, thus producing a uniform translational movement profile. The relative soil-pile displacements and pile response have been measured through instrumentation. The results show that pile head deflection exceeded soil movement even at the initial loading, hence demonstrating ‘flexible’ pile behaviour. Back-analysis of the lateral pile response using LPILE (Reese and Wang, 2000) indicates that the subgrade reaction approach within the displacement-based analysis methods is adequate and provides satisfactory prediction with experimental tests.

Rather limited experimental and numerical analysis have been performed to investigate the performance of piles under combined axial and lateral loading. Some recent studies are reviewed below.

Anagnostopoulos and Georgiadis (1993) have conducted model tests on aluminium closed-ended piles (free head) in clay. Both axial and lateral loads were applied at ground elevation following different loading consequence with varying magnitudes. The experimental results show that the lateral load results in an increase in the axial pile displacement and that the axial load has a negligible effect on the lateral pile response. Recently, Karthigeyan *et al.* (2006, 2007) studied the response of piles under the combined action of vertical and lateral loads in sand and clayey soil using three-dimensional finite element analysis. The results show that the vertical loads on the pile significantly increase the lateral capacity of the pile in sand and marginally decrease the capacity in clayey soils. However, none of the analysis has investigated
the combined effect of vertical loading and passive lateral loading due to lateral soil movement.

Chen et al. (2002) have examined the changes in vertical bearing capacity of piles due to horizontal ground movement. The experimental study concluded that the lateral ground deformation may cause increases in the axial bearing capacity due to the increase of shaft friction with the progress of ground movement.

Recently the influence of axial load on lateral pile response in liquefiable soils during earthquakes has received considerable attention including dynamic centrifuge testing using physical models and numerical simulations and analyses. Bhattacharya et al. (2004) have undertaken dynamic centrifuge tests to study the effect of axial load on a pile during soil liquefaction. The pile was fixed in an equivalent shear beam box in the centrifuge model. The experiments were conducted in level ground. The study reveals that axial load plays a dominant role in the collapse of piles due to seismic liquefaction. They concluded that the pile may fail by buckling instability without invoking lateral spreading of the soils, which induces bending in the pile. Knappett and Madabhushi (2009a, 2009b) have extended the work by investigating this mechanism in pile groups and examining the effect of pile length and group spacing on the collapse behaviour. They found that the effect of the axial loads is to amplify the lateral pile deflections.

O’Rourke et al. (1994) have described an analytical approach to evaluate the response of vertically loaded piles to liquefaction-induced lateral spreading. For a two-layer soil profile comprising a liquefiable soil overlying a non-liquefiable soil, two failure
mechanisms in the pile structure have been identified. The first one is buckling of the pile under the action of axial load owing to the insufficient lateral support due to the reduced stiffness of the liquefied soil. The other is bending failure by formation of plastic hinges as a result of excessive pile deflection induced by horizontal soil movement. Bhattacharya and Madabhushi (2008) have presented a comprehensive review of methods for pile design in seismically liquefiable soil and compared the two plausible mechanisms of pile failure in liquefiable soils. In addition, the pile may undergo excessive rigid body rotation and the liquefied soil may flow around the pile. They are similar to the ‘rotational’ and ‘flow’ modes described by Viggiani (1981).

2.3.5.2 Field tests

Instrumented field tests have also been reported on piles used for the treatment of landslides. The piles were usually instrumented with strain gauges to measure the bending moment and inclinometers installed within the piles to measure pile displacements. These tests are very useful in:

- evaluating the field performance of piles;
- verifying and modifying the design methods;
- identifying the mechanism of slope and pile interaction based on relative pile and soil movements; and
- assessing the effectiveness and improvement of factor of safety as a result of the remedial measures with the use of piles.

Some of these case histories have been reviewed and summarised by Fukuoka (1977), Chen and Poulos (1997), and Cai and Ugai (2003). Some well-documented and recent cases are described below.
Esu and D’Elia (1974) reported a field test in which a reinforced concrete pile was installed into a clay shale earthflow. The pile was instrumented with pressure cells and an inclinometer. The bending moment, shear force, pile displacement and rotation distribution have been derived from measured data. This test has been extensively analysed by many scholars using various methods (Mauger and Motta, 1991; Chow, 1996; Chen and Poulos, 1997; Cai and Ugai, 2003; Guo, 2009b). Different assumptions have been made regarding the soil properties and soil movement profile to match the measured pile response.

Kalteziotis et al. (1993) presented an experimental study of landslide stabilisation by large diameter piles. Two rows of large diameter (1.0 m in diameter and 12 m in length) were used to prevent the sliding mass. Among them three were replaced by steel pipe piles of the same geometry and inertia. Two of these piles, one at each row, were instrumented with vibrating wire strain gauges to measure bending moment and assess the force developed on the pile with time. Two inclinometer tubes were installed inside the instrumented piles to measure displacement profiles of piles. Another two inclinometers were installed between piles in each row to measure the soil movement profile. The results show that the soil movement profile adjacent to the pile may be taken as a triangular distribution and soil movement between piles may be reduced by the arching effect.

Smethurst and Powrie (2007) reported the monitoring and analysis of the bending behaviour of discrete piles used to stabilise a railway embankment. The piles were instrumented with strain gauges to measure the bending moments induced in the pile by slope movements. Inclinometer tubes were installed both inside the strain-gauged
piles and in the slope midway between each pair of instrumented piles to measure the relative movement between the piles and soil midway between the piles. Readings were taken over a period of 4 years after the pile installation. The mechanism of loading on the pile imposed by the soil movement has been clarified based on the relative pile and soil displacements obtained from field measurements. The measured pile displacement and bending behaviour suggest that much of piles’ movement was caused by the seasonal cycles of wetting and drying of the embankment and the progressive reduction in pile flexural rigidity due to cracking of the concrete section.

Frank and Pouget (2008) have analysed the lateral loading transfer mechanism of an experimental pile undergoing long duration thrust owing to a moving slope over 16 years. Experimental $p \sim \Delta y$ ($\Delta y = y - y_s$, is the relative displacement, $y$ is the pile displacement, $y_s$ is the free displacement of the soil) curves have been constructed. A Menard pressuremeter method and a self-boring pressuremeter method are proposed to predict the $p \sim \Delta y$ curves. Comparison of results from numerical analysis using three sets of reaction curves derived from the experimental measurement and the two suggested methods show that the $p \sim \Delta y$ curves determined from the Menard pressuremeter and the self-boring pressuremeter methods are too stiff. Thus the proposed methods lead to an overestimate of the pile displacements and bending moment. Further work needs to be done to determine and improve the $p \sim \Delta y$ curves.

2.4 ASSESSMENT OF SHEAR MODULUS OF SANDS

In the theoretical analysis of laterally loaded piles, the deformation parameters of the soils, such as Young’s modulus, $E_s$, shear modulus, $G_s$, or modulus of subgrade
reaction, $k$, and the ultimate soil resistance or limiting force, $p_u$ are required. Determination of the limiting force profile for active piles has been reviewed in sections 2.2.3.1 and 2.2.3.2 for cohesive and cohesionless soils, respectively. Estimation of the ultimate soil resistance for piles under passive loading from lateral soil movement has been discussed in section 2.3.2. Therefore, attention will be focused on the assessment of the deformation parameters. Additionally, the model pile tests in this thesis were conducted in local Queensland sand. Unlike the Ottawa sand, Toyoura sand and Dog’s bay sand, etc, which are widely used in the international geotechnical research community, the deformation properties of this sand have not been thoroughly investigated. The previous research results on assessment of the sand stiffness parameters may be beneficial for this research and are necessary to be reviewed. Among them, the modulus of subgrade reaction, $k$ can be evaluated by the expressions summarized in Table 2.7, while the Young’s modulus, $E_s$, can be calculated from the shear modulus, $G_s$ and Poisson’s ratio of the soil, $v_s$ by $E_s=2G_s(1+v_s)$. Therefore, only the relevant research on the shear modulus of sand is reviewed below.

The stress-strain-strength behaviour of soils is complex and depends on many factors including: stress level, stress path, anisotropy, loading rate, and drainage, etc (Atkinson and Sallfors, 1991). The small-strain shear modulus, $G_{\text{max}}$ is a fundamental parameter. In practice, $G_{\text{max}}$ is defined as the measured shear modulus at very low strains of the order of $10^{-6}$ (Seed et al., 1986). As stated by Poulos et al. (2001), the $G_{\text{max}}$ provides a universal reference or benchmark value of stiffness when applied to foundation system, because

- the rate of loading, static or dynamic, has no practical effect on $G_{\text{max}}$;
the $G_{max}$ is relatively insensitive to the static stress prehistory, expressed through
the overconsolidation ratio; and
- the $G_{max}$ is not influenced by the drainage condition and applies to deformations
under both undrained and drained conditions.

Many empirical relationships have been proposed to estimate the small-strain shear
modulus $G_{max}$. Several expressions for estimation of $G_{max}$ are summarised in Table
2.14. Inspection of the various relationships shows that for cohesionless soils, the
$G_{max}$ is primarily influenced by confining pressure and the void ratio. In particular,
Pestana and Salvati (2006) investigated the small-strain behaviour of granular soils
and demonstrated that if anisotropy is not considered, a general formulation for $G_{max}$
can be expressed as

$$
\frac{G_{max}}{p_{am}} = G_b \cdot f_1[e] \cdot f_2\left[\frac{\sigma'_m}{p_{am}}\right]
$$

(2.20)

where $\sigma'_m$ is the mean effective stress; $p_{am}$ is the atmospheric pressure; $e$ is void ratio;
and $G_b$ is material constant. Their evaluation shows that the formulation $f_1[e] = e^{-1.3}$,
provided the best match for the experimental data on various sand;

$$
f_2\left[\frac{\sigma'_m}{p_{am}}\right] = \left(\frac{\sigma'_m}{p_{am}}\right)^n
$$

with the power law exponent $n=0.5$ used for sands, and $n=0.5$-0.75 for gravels and materials with a significant amount of gravels. The value of $n$ is
consistent with the suggestion by Wroth and Houlsby (1985), who stated that a value
of $n=0.5$ may capture most of the important features of increased shear stiffness with
pressure. The value of $G_b$ is correlated with the angularity of the material for fairly
uniform sands and ranges from 400 to 800. Sands with more angular grains tend to
have higher values of $G_b$, while well-graded (the coefficient of uniformity $C_u >7$ )
sand and gravels tend to have lower values of $G_b$. Recently, Wichtmann and
Triantafyllidis (2009) studied the influence of particle-size distribution curve of quartz sand on the $G_{\text{max}}$. They revealed that the $G_{\text{max}}$ is not influenced by variations in the mean particle size, $d_{50}$, but it significantly decreases with increasing of coefficient of uniformity $C_u$ of the particle-size distribution curve. Four different correlations (method 1 to 4) have been developed for $G_{\text{max}}$ based on Hardin’s equations or the modulus coefficient $(K_2)_{\text{max}}$ (explained later on) in Seed et al. (1986) equation, as presented in Table 7.14. They concluded that their empirical formulas in methods 1 and 3 which are formulated in terms of void ratio are generally more precise than the equations in methods 2 and 4 which establish correlations of $G_{\text{max}}$ with relative density, $D_r$. Nevertheless, they suggest that for rough estimations, the empirical equations formulated in terms of relative density may be sufficient for practical purpose.

Laboratory testing has shown the stress-strain-strength behaviour of sands is highly nonlinear. The variation of secant shear modulus with increasing shear strain is customarily shown on a plot of $G_s/G_{\text{max}}$ against the shear strain on a logarithmic axis. Figure 2.26 plots the $G_s/G_{\text{max}}$ measured on laboratory reconstituted samples of clean sand under drained conditions. This normalization makes it possible to compare the relationships obtained by various investigators, and it also facilitates the use of the relationship in practice. The figure indicates that the shear modulus increases with confining pressure, but decays nonlinearly with increasing shear strains. The plot also indicates that the rate of reduction in shear modulus with strain becomes smaller when the confining pressure is greater and most sands exhibit similar $G_s/G_{\text{max}}$ reduction curves irrespective of the different testing conditions. Figure 2.27 presents the variation of soil stiffness for service deformation levels corresponding to different geostructural systems suggested by Mair (1993). It can be seen from this figure that
for application with low amplitude loading events, such as foundation vibrations and earthquakes, very high shear modulus values may be expected. On the other hand, at the other extreme involving lateral spreading and landslides, much lower shear modulus values should be used to predict the soil responses. In addition, Fahey (1999) stated that the soil strain may vary with location relative to the foundations. If some type of equivalent linear elastic model is to be used for deformation prediction, the appropriate value of secant stiffness to be used will vary depending on the location relative to the foundations. Several models have been proposed to describe the degradation of shear modulus either in terms of shear strain or shear stress level. Three of such models are presented below.

Hardin and Drnevich (1972) showed the nonlinear stress-strain behaviour of sand can be represented by a hyperbolic model, which may be expressed as

\[
\frac{G_s}{G_{\text{max}}} = \frac{1}{1 + \gamma / \gamma_r},
\]

(2.21)

where \(G_s\) is the secant shear modulus, \(\gamma\) is the shear strain, and \(\gamma_r\) is the reference shear strain defined as

\[
\gamma_r = \frac{\tau_{\text{max}}}{G_{\text{max}}},
\]

(2.22)

where \(\tau_{\text{max}}\) is shear strength.

Fahey (1992) showed that by substituting equation (2.22) and noting the definition of shear strain \(\gamma = \tau/G_s\) into equation (2.21), the hyperbolic model can be expressed in terms of shear stress rather than shear strain as

\[
\frac{G_s}{G_{\text{max}}} = 1 - \frac{\tau_{\text{max}}}{G_{\text{max}}},
\]

(2.23)
This produces a simple linear relationship, independent of confining stress level. Fahey and Carter (1993) further modified the relationship and gave the following formulation

\[
\frac{G_s}{G_{max}} = 1 - f \left( \frac{\sigma_{max}}{G_{max}} \right)^g
\]  

(2.24)

where \( f \) and \( g \) are fitting parameters. The value of \( f \) and \( g \) may be taken as 1 and 0.3 for sand.

Although the two approaches mentioned above have been put into practice, Poulos et al. (2001) pointed out that it may be preferable to consider the degradation relationship for shear modulus in terms of strain level, rather than stress level, since the strain level for different foundation types may be different, despite the fact that the stress level may be similar.

For practical purpose, Seed and Idriss (1970) and Seed et al. (1986) proposed that the secant shear modulus can be estimated by

\[
G_s = 218.8 K_2 (\sigma'_m)^{0.5}
\]  

(2.25)

where \( G_s \) and \( \sigma'_m \) are in kPa; and \( K_2 \) is a soil shear modulus coefficient, representing the influence of void ratio and strain amplitude. For any sand, this coefficient has a maximum value \( (K_2)_{max} \) at very low strains of the order of \( 10^{-6} \), i.e. \( G_s = G_{max} \) at \( K_2 = (K_2)_{max} \). \( (K_2)_{max} \) depends on the relative density, \( D_r \) and can be estimated from (Seed and Idriss, 1970; Yan and Byrne, 1992)

\[
(K_2)_{max} = 3.5 (D_r)^{2/3}
\]  

(2.26)

Seed et al. (1986) stated that the values of \( (K_2)_{max} \) range from 30 for loose sands to
about 75 for dense sand and they are 1.35 to 2.5 times greater for gravels than for sands. This model will be employed to evaluate the shear modulus of sand from back-analysis of laterally loaded pile tests in chapter 7.

2.5 CONCLUDING REMARKS

This chapter has given a brief review of the wide range of literature concerned with the behaviour of piles under both active and passive lateral loads. Some significant aspects have been covered and conclusions are drawn.

2.5.1 Active piles

- The generic limiting force profile (see equation (2.8)) proposed by Guo (2006) is versatile in describing the variation of the ultimate soil resistance with depth, with the three parameters $A_r, n (<3)$, and $\alpha_0$ controlling its gradient, shape and soil resistance at the ground surface. It reduces to the existing limiting force profiles (Matlock, 1970; Broms, 1964b; Barton, 1982) by selecting an appropriate set of parameters, which also enables associated closed-form solutions to be developed. This limiting force profile is independent of the failure mode of the soil surrounding the pile and the three parameters have to be obtained from back-analysis of measured pile responses.

- The $p \sim y$ method is the most practically used procedure for the design and analysis of piles under lateral loading. However, the $p \sim y$ curves may not necessarily be complex enough for accurate estimation of pile responses. Solutions using simplistic elastic-plastic curves can offer reasonably good prediction of pile performance under both active and cyclic loading.
The analytical approach adopted by Guo (2006) combines the uncoupled model for the upper part of the piles, where the limiting force or ultimate soil resistance has been mobilised, together with a coupled model for elastic analysis of the lower part of the piles. The elastic-plastic solutions subsequently developed can be used in evaluating the performance of the lateral piles, which can be implemented using spreadsheet program or by manual calculation.

The approach presented by Basu and Salgado (2007) and Basu et al. (2009) using variational principle can be used to study the behaviour of laterally loaded piles in multi-layered soils. However, their solutions are suitable only to elastic soils and can not allow for the nonlinearity of the soil and the possible slippage and yield at the pile-soil interface under relative larger lateral loading, which is an important feature of laterally loaded piles. The reliance on the iterative scheme to calculate a number of load transfer factors, for instance, 6 factors (Basu et al., 2009), may hamper their practical application.

2.5.2 Passive piles

- Limited work has been done with respect to the ultimate soil resistance under passive loading, which is one of the key parameters required in the numerical analysis of passive piles. Some controversial suggestions or even conflicting conclusions about whether difference in ultimate soil resistance between active piles and passive piles exists, which requires more research to be carried out.

- With assumed pressure distributed along a pile, pile responses including maximum shear force and bending moment can be obtained using the pressured-based method; but this method does not allow for displacement of
the piles and pile group effects to be taken into account. Additionally, the pressure itself generally depends on the relative displacement between the pile and the soil.

- The displacement-based method can account for the soil-pile interaction, effect of different soil displacement patterns. However, the free field soil movements are relatively difficult to determine in practice. The analytical solutions presented by Fukuoka (1977), Cai and Ugai (2003) and Guo (2009b) provide an alternative approach for the analysis of passive piles. However, they are only applicable to free-head piles and particularly estimation of the concentrated load at the sliding surface needs more exploration.

- The most rigorous approach to assess the behaviour of piles subjected to lateral soil movement is the use of three-dimensional numerical analysis with a sophisticated soil constitutive model to capture the behaviour of the soil. The soil displacement around the pile and group effect can be reasonably quantified. However, the reliability of the results depends on the accuracy and calibration of the soil constitutive models employed.

- The recent field tests reported by Smethurst and Powrie (2007) and Frank and Pouget (2008) provide valuable information on the performance of piles used for increasing slope stability or in a moving slope for a long duration with respect to the mechanism of soil-pile interaction based on the relative pile and soil movement, the development of limiting pile-soil pressure with soil movement and verification of the analysis and design methods.

- Although extensive laboratory studies have been conducted to investigate the response of passive loaded piles, relative little effort is made to assess the response of the piles under the action of the combined vertical loading and
lateral soil movements. In reality, piles may work under these loadings; such as, piles supporting bridge abutments by the approach embankment, pile foundations for high-rise buildings near excavation and piles supporting offshore structures subjected to submarine landslides.

- The apparatus developed by Guo and Ghee (2004) can be used to investigate the behaviour of piles under vertical load and lateral soil movements. Tests have been done on single piles and pile groups of different configuration undergoing a uniform soil movement profile using a rectangular loading block. Limited number of tests have also been conducted on single piles using a triangular loading block. Preliminary analysis of the previously conducted test results show that the soil movement profile can affect the responses of single pile. More tests are required to investigate the effect of the following parameters on the pile responses:
  (1) the distance between pile location and the source where free soil movement are induced;
  (2) the effect of axial load level on pile head;
  (3) the effect of loading block angle;
  (4) different combination of sliding and stable layer depths;
  (5) the effect of soil movement profiles on vertically loaded single piles; and
  (6) the response of pile groups subjected to progressively moving sand.

- Simple solutions are not available to facilitate preliminary assessment and design of piles in pre-failure state when subjected to soil movement. Therefore, more experimental and analytical work is necessary to improve the understanding the pile performance when piles are subjected to lateral soil movement.
Table 2.1 Expressions for calculating critical pile length

<table>
<thead>
<tr>
<th>References</th>
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<tbody>
<tr>
<td></td>
<td>Constant $k$ or $E_s$ ($k$, $E_s$ = constant)</td>
<td>Linearly increasing, $E_s$ ($E_s$ = mz)</td>
</tr>
<tr>
<td>Hetenyi (1946)</td>
<td>$4(E_p I_p / k)^{0.25}$</td>
<td></td>
</tr>
<tr>
<td>Vesic (1961)</td>
<td>$3.18(E_p I_p / k)^{0.25}$</td>
<td></td>
</tr>
<tr>
<td>Randolph (1981)</td>
<td>$d(E_p/G^*)^{2/7}$</td>
<td>$d(2E_p/m^*d)^{2/9}$</td>
</tr>
<tr>
<td>Gazioglu and O’Neill (1984)</td>
<td>$3(E_p I_p / E_d d^{0.5})^{0.286}$</td>
<td></td>
</tr>
<tr>
<td>Davies and Budhu (1986)</td>
<td>$1.5d(E_p/E_s)^{3/11}$</td>
<td></td>
</tr>
<tr>
<td>Budhu and Davies (1987, 1988)</td>
<td></td>
<td>$1.3d(E_p/md)^{2/9}$</td>
</tr>
<tr>
<td>Poulos and Hull (1989)</td>
<td>$4.44(E_p I_p / E_s)^{0.25}$</td>
<td>$3.30(E_p I_p / m)^{0.20}$</td>
</tr>
<tr>
<td>Guo and Lee (2001)</td>
<td>$1.05d(E_p/G)^{0.25}$</td>
<td></td>
</tr>
</tbody>
</table>

Note: $m^*$ = rate of increase of soil Young’s modulus with depth  
$m^*$ = rate of increase of $G^*$ with depth
Table 2.2 Variation of $N_p$ with depth for cohesive soils

<table>
<thead>
<tr>
<th>References</th>
<th>Value of $N_p$ at</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ground surface (z = 0)</td>
</tr>
<tr>
<td>Meyerhof (1951)</td>
<td>–</td>
</tr>
<tr>
<td>McClelland and Focht (1958)</td>
<td>–</td>
</tr>
<tr>
<td>Reese (1958)</td>
<td>2</td>
</tr>
<tr>
<td>Brinch Hansen (1961)</td>
<td>2.57</td>
</tr>
<tr>
<td>Broms (1964a)</td>
<td>2</td>
</tr>
<tr>
<td>Matlock (1970)</td>
<td>3</td>
</tr>
<tr>
<td>Reese et al. (1975)</td>
<td>2</td>
</tr>
<tr>
<td>Stevens and Audibert (1979)</td>
<td>4.8</td>
</tr>
<tr>
<td>Poulos and Davis (1980)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Gazioglu and O’Neill (1984)</td>
<td>3</td>
</tr>
<tr>
<td>Muff and Hamilton (1993)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2 (smooth)</td>
</tr>
<tr>
<td></td>
<td>3.5 (rough)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Fleming et al. (2009)</td>
<td>2</td>
</tr>
</tbody>
</table>

Note:

$^a$ Poulos and Davis (1980) provided curves. These values were reported by Randolph and Houlsby (1984).

$^b$ The soil-pile cohesion are ignored.

$^c$ Only the effect of soil-pile cohesion in front of the pile is considered.
### Table 2.3 Expressions for calculating ultimate resistance in sand

<table>
<thead>
<tr>
<th>References</th>
<th>Expressions for $p_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hansen (1961)</td>
<td>$p_u = K_q z \gamma' zd$</td>
</tr>
<tr>
<td>Broms (1964b)</td>
<td>$p_u = 3 K_{p} \gamma' z d$</td>
</tr>
</tbody>
</table>
| Reese et al. (1974)             | $p_u = (A_o or A_c) \min(p_{u1}, p_{u2})$  
\[ p_{u1} = \gamma' z \left( \frac{K_q z \tan \phi' \sin \beta}{\tan(\beta - \phi')} + \frac{\tan \beta}{\tan(\beta - \phi')} (d + z \tan \beta \tan \alpha) \right) \]  
\[ p_{u2} = K_d d \gamma' z (\tan^4 \beta - 1) + K_d d \gamma' z \tan \phi' \tan^4 \beta) \] |
| Bogard and Matlock (1980)       | $p_u = \min(p_{u1}, p_{u2})$  
\[ p_{u1} = (C_1 z + C_2 d) \gamma' z ; \quad p_{u2} = C_3 d \gamma' z \] |
| Barton (1982)                   | $p_u = K_b \gamma' zd$                                                              |
| Meyerhof (1995)                 | $p_u = K_b \gamma' zd \leq q_d$  
$K_b$ depends on depth and angle of internal friction at shallow depth.  
$K_b = K_0 N_q$ at greater depth. |
| Zhang et al. (2005)             | $p_u = (\eta \tau_{max} + \xi \tau_{max}) d$  
\[ \tau_{max} = K_{\gamma'} zd \tan \delta \] |
| Fleming et al. (2009)           | $p_u = N_\beta dz$                                                                  |

Note:  
$K_q = \text{Hansen earth pressure coefficient that depends on } z/d \text{ and the angle of internal friction } \gamma'$;  
$\alpha = \phi'/2, \quad \beta = 4\delta + \phi'/2, \quad A_o, A_c = \text{empirical adjustment factor for static and cyclic loading};  
\eta, \xi = \text{dimensionless shape factors};  
K = (0.5 \sim 2.0) K_b, \text{ lateral earth pressure coefficient (depends on pile type and construction methods)};  
\delta = \text{interface friction angle between the pile and the soil};  
K_{\gamma'} = \text{passive earth pressure coefficient};  
K_b = \text{active earth pressure coefficient};  
K_0 = \text{at rest earth pressure coefficient};  
K_{\gamma'} = \text{net lateral soil pressure coefficient for friction};  
N_\beta = \text{bearing capacity factor for a shallow strip footing};  
q_u = \text{ultimate bearing capacity of vertical strip footing under horizontal load};  
N_\beta = \text{gradient of the average ultimate resistance across the width of a pile}.$
Table 2.4 Capacity of lateral piles based on limit states (after Guo, 2008)

<table>
<thead>
<tr>
<th>Methods</th>
<th>( \frac{T_i}{A_i d l^2} )</th>
<th>( A_r )</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Using equilibrium against rotating point</td>
<td>( K_q \gamma_s \gamma_s' )</td>
<td></td>
<td>Brinch Hansen (1961)</td>
</tr>
<tr>
<td>2 ([6(1 + e/l)]^{-1})</td>
<td>( 3K_p \gamma_s \gamma_s' )</td>
<td></td>
<td>Boms (1964b)</td>
</tr>
<tr>
<td>3 ([2.13(1 + 1.5e/l)]^{-1})</td>
<td>(28 ~ 228) kPa</td>
<td></td>
<td>McCorkle (1969)</td>
</tr>
<tr>
<td>4 (0.5[2(z_r/l)^2 - 1])</td>
<td>( (3.7K_p - K_a)\gamma_s )</td>
<td></td>
<td>Petrasovits and Awad (1972)</td>
</tr>
<tr>
<td>5 ((1 + 1.4e/l)^{-1})</td>
<td>( F_b S_{b_a}(K_p - K_a)\gamma_s )</td>
<td></td>
<td>Meyerhof et al. (1981)</td>
</tr>
<tr>
<td>6 (\frac{1 - d/l}{[1 - 0.333\ln(l/d)e/d]} )</td>
<td>4.167 ( \gamma_s )</td>
<td></td>
<td>Dickin and Wei (1991)</td>
</tr>
<tr>
<td>7 (0.51\frac{z_r}{l} \left(1.59\frac{z_r}{l} - \frac{l}{d}\right))</td>
<td>( 0.8\gamma_s 10^{1.3\tan\phi_s + 0.3} )</td>
<td></td>
<td>Prasad and Chari (1999)</td>
</tr>
<tr>
<td>8 (\frac{0.1181}{1 + 1.146e/l})</td>
<td>( K_p^2 \gamma_s \gamma_s' )</td>
<td>Derived using ( z_0/l = 0.618 ) in eq. (C-1, Table 7.1).</td>
<td></td>
</tr>
</tbody>
</table>

Note: \( K_q \) = Hansen earth pressure coefficient that depends on \( z/d \) and the angle of internal friction \( \phi_s \)

\( F_b \) = lateral resistance factor (0.12 for uniform soil)

\( S_{b_a} \) = a shape factor that relies on the embedment depth \( l \) and the angle of internal friction \( \phi_s \)
Table 2.5 Parameters for generic limiting force profiles

<table>
<thead>
<tr>
<th>Media</th>
<th>Expressions</th>
<th>Parameters</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>$p_u = N_u \overline{F_u} d \left( \frac{z}{d} + \frac{\alpha_0}{d} \right)^n$</td>
<td>$0.6 \sim 4.79$</td>
<td>Guo (2006) Guo and Zhu (2005b)</td>
</tr>
<tr>
<td>Sand</td>
<td>$p_u = N_u \gamma_d d ^z \left( \frac{z}{d} + \frac{\alpha_0}{d} \right)^n$</td>
<td>$(0.4 \sim 2.5) K_p ^2$</td>
<td>Guo (2006)</td>
</tr>
<tr>
<td>Rock</td>
<td>$p_u = N_u q_u ^{1/n} d \left( \frac{z}{d} + \frac{\alpha_0}{d} \right)^n$</td>
<td>$580 \lambda d$</td>
<td>Guo (2009d)</td>
</tr>
</tbody>
</table>

Note: $\overline{F_u}$ = average undrained shear strength of the cohesive soil (initially taking 8d); $q_u$ = maximum of the average Uniaxial Compressive Strength (UCS) over 1.5 time of the critical pile length or that over the slip depth at the maximum imposed load; $\lambda = \sqrt{[k/(4E_s I_s)]}$, reciprocal of characteristic length of the pile or shaft.
Table 2.6 Solutions for rigid piles in elastic foundations (Scott, 1981)

<table>
<thead>
<tr>
<th>Load</th>
<th>$k = \text{constant}$</th>
<th>$k = k_0z$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u(z) = \frac{2T_z}{kl} (-3\frac{z}{l} + 2)$</td>
<td>$u(z) = \frac{18T_z}{k_0l^3} (-\frac{4}{3} + 1)$</td>
</tr>
<tr>
<td>$T_z$</td>
<td>$p(z) = ku(z) = \frac{2T_z}{l} (-3\frac{z}{l} + 2)$</td>
<td>$p(z) = k_0zu(z) = \frac{18T_z}{l^2} (-\frac{4}{3} + 1)$</td>
</tr>
<tr>
<td>$z$</td>
<td>$\frac{T(z)}{T_z} = 1 + 3(\frac{z}{l})^2 - 4\frac{z}{l}$</td>
<td>$\frac{T(z)}{T_z} = 1 + 8(\frac{z}{l})^3 - 9(\frac{z}{l})^2$</td>
</tr>
<tr>
<td></td>
<td>$\frac{M(z)}{T_zl} = (\frac{z}{l})^3 - 2(\frac{z}{l})^2 + \frac{z}{l}$</td>
<td>$\frac{M(z)}{T_zl} = 2(\frac{z}{l})^4 - 3(\frac{z}{l})^3 + \frac{z}{l}$</td>
</tr>
<tr>
<td></td>
<td>$M_{\max} = 0.148$ at $\frac{z_{\max}}{l} = \frac{1}{3}$</td>
<td>$M_{\max} = 0.26$ at $\frac{z_{\max}}{l} = \frac{27}{64} = 0.42$</td>
</tr>
</tbody>
</table>

Note: The $u(z)$, $p(z)$, $T(z)$, and $M(z)$ are the pile displacement, soil reaction, shear force and bending moment along the pile at a depth $z$, respectively.
Table 2.7 Expression for estimation of modulus of subgrade reaction

<table>
<thead>
<tr>
<th>Mtd</th>
<th>Description</th>
<th>Expressions</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Experimental and empirical methods</td>
<td>The $k$ is assessed from pile tests or correlated with other soil properties obtained from in-situ tests, such as pressuremeter, flat dilatometer and plate load tests.</td>
<td>Terzaghi (1955) Bowles (1996) Poulos and Davis (1980)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k = \frac{1.23E_s}{C(1-v_s^2)} \left[ \frac{E_s d^4}{C(1-v_s^2)E_f E_p} \right]^{0.11}$</td>
<td>Biot (1937)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k = \frac{0.65E_s}{1-v_s^2} \left[ \frac{E_s d^3}{E_f E_p} \right]^{1/2}$</td>
<td>Vesic (1961)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k = \frac{1.0E_s d_e}{(1-v_s^2)d_{ef}} \left[ \frac{E_s d^2}{E_f E_p} \right]^{1/2}$ (d_{ef}=1.0m)</td>
<td>Carter (1984) Ashord and Juinrarongrit (2003)</td>
</tr>
<tr>
<td>2</td>
<td>Direct use or modification of the $k$ suggested for beams on elastic foundations</td>
<td></td>
<td>Glick (1948)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k = \frac{22.24E_s(1-v_s)}{(1+v_s)(3-4v_s)[2ln(2l/d)-0.443]}$</td>
<td>Baguelin et al. (1977)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k = \frac{16\pi(1-v_s)G_s}{(3-4v_s)ln[(2l/d)]-[2/(3-4v_s)]}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k = \frac{(3-4v_s)}{(1+v_s)(1-2v_s)} \left[ \frac{E_s}{E_p} \right]^{1/2} \left[ \frac{l}{r_s} \right]^{2/3} \left( \frac{K_i(\gamma)}{K_s(\gamma)} \right)^2 \left[ \frac{1}{K_s(\gamma)} - 1 \right]$</td>
<td>Sun (1994)</td>
</tr>
<tr>
<td>3</td>
<td>Simplified theoretical models</td>
<td></td>
<td>Guo and Lee (2001)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\gamma = k \left( \frac{E_s}{G_s} \right)^{1/3} \left( \frac{l}{r_s} \right)^{1/2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{k}{G^*} = (8.4 + \frac{\bar{e}}{0.119 + 0.2427\bar{e}})(\frac{l}{d})^{-0.43(1-\frac{\bar{e}}{1-\bar{e}})}$ (Rigid piles)</td>
<td>Guo (2009d)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{k}{G^<em>} = (6.86 + \frac{\bar{e}}{0.1458 + 0.2843\bar{e}})(G^</em>)^{-0.087(1-\frac{\bar{e}}{1-\bar{e}})}$ (Flexible piles)</td>
<td></td>
</tr>
</tbody>
</table>

Note: $C =$ coefficient varying from 1 for uniform pressure distribution to 1.13 for uniform deflection. $R =$ radius of the outer rigid boundary of the elastic soil zone, depending on pile rigidity, loading type and pile head fixity condition (Baguelin et al., 1977). $K_i(\gamma) =$ modified Bessel function of second kind of $i$ th order ($i = 0, 1$). $\gamma =$ non-dimensional parameter (Sun, 1994) or load transfer factor (Guo and Lee, 2001) $k_i,k_2,k_3 =$ coefficients for estimating $\gamma$ (Guo and Lee, 2001).
Table 2.8 Summary of $p \sim y$ curves for sands

<table>
<thead>
<tr>
<th>References</th>
<th>$p \sim y$ functions</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reese et al. (1974)</td>
<td>Relatively complex curves consist of four segments with the first and second straight lines connected by a parabola and the third straight line starting at 3d/80.</td>
<td>(1) Developed from results of full scale field pile tests in clean fine sand; (2) The ultimate $p_u$ is attained at $y_u$ of 3d/80 for any given set of $p \sim y$ curves.</td>
</tr>
<tr>
<td>Murchison and O’Neill (1984) and API (1993)</td>
<td>$p = \eta A p_y \tanh \left( \frac{kz}{\eta A p_y} y \right)$, $A = 0.9$, for cyclic loading, $A = 3-0.8z/d \geq 0.9$, for static loading $\eta$ is a factor describing the pile shape</td>
<td>(1) Using a single analytical hyperbolic tangent function to describe the $p \sim y$ curves; (2) Having defined $p_u$, but the $y_u$ changing with depth; (3) The $p_u$ is calculated from Reese et al. (1974) or Bogard and Matlock (1980) methods.</td>
</tr>
<tr>
<td>Yan and Byrne (1992)</td>
<td>$p = E_{\text{max}} y$ $(y \leq \alpha^2 d)$, $p = \alpha d E_{\text{max}} (y/d)^{0.5}$ $(y &gt; \alpha^2 d)$, $\alpha = 5(D)^{0.8}$, $E_{\text{max}}$ is the maximum soil Young’s Modulus.</td>
<td>(1) Developed from results of small scale model tests in fine silica sand; (2) The shape of the $p \sim y$ curves is controlled by $E_{\text{max}}$, which varies with depth, but the $p \sim y$ curves have no defined $p_u$.</td>
</tr>
<tr>
<td>Wesselink et al. (1988)</td>
<td>$p = Rd \left( \frac{z}{z_0} \right)^{\alpha} \left( \frac{y}{d} \right)^{\beta}$, $z_0=1m$, $R=650kPa$, $n = 0.7$, $m = 0.65$, $\gamma = 2$, $n = 0.33$, $m = 0.5$, $R = 2.7$, $n = 0.72$, $m = 0.6$</td>
<td>(1) Empirical $p \sim y$ formulation based on centrifuge model pile tests in calcareous sand; (2) The ever increasing power curves never reach an ultimate resistance $p_u$.</td>
</tr>
<tr>
<td>Novello (1999)</td>
<td>$p = R/d \left( \frac{z}{z_0} \right)^{\alpha} \left( \frac{y}{d} \right)^{\beta} &lt; q_c d$, $R = 2$, $n = 0.33$, $m = 0.5$, $q_c = 20$, $d = 0.4$, $R = 2.7$, $n = 0.72$, $m = 0.6$, $R = 2$, $n = 0.33$, $m = 0.5$</td>
<td>(1) Empirical formulation using cone tip resistance, $q_c$, to normalize the results of small scale and prototype and centrifuge tests on pile; (2) The ultimate resistance $p_u$ is related to the cone tip resistance, $q_c$.</td>
</tr>
<tr>
<td>Dyson and Randolph (2001)</td>
<td>$p = R y ^{\alpha} \left( \frac{q}{q_{\gamma}} d \right)^{\beta} \left( \frac{y}{d} \right)^{\gamma}$, $R = 2.7$, $n = 0.72$, $m = 0.6$</td>
<td>(1) Developed from results of centrifuge model pile tests in calcareous sand; (2) The soil resistance, $p$ is linked to the soil strength through cone resistance $q_c$ and has no ultimate values.</td>
</tr>
<tr>
<td>Guo (2006)</td>
<td>$p = \min \left( k_y p_u, \gamma \right)$, $k = \text{constant}$, $p_u = A_y (z + \alpha_0)^a$</td>
<td>(1) Idealized elastic-perfectly plastic curves; (2) The ultimate resistance, $p_u$ varies in depth along the general limiting force profile.</td>
</tr>
<tr>
<td>Guo and Zhu (2005b)</td>
<td>$p = \frac{y}{k_y p_u}$, $k = \text{constant}$, $p_u = A_y (z + \alpha_0)^a$</td>
<td>(1) Using a hyperbolic function to describe the $p \sim y$ curves; (2) The ultimate resistance, $p_u$ can be evaluated using Reese et al. (1974) method.</td>
</tr>
</tbody>
</table>

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### Table 2.9 Assumptions and features of elatis-plastic solutions for flexible piles (after Guo, 2009a)

<table>
<thead>
<tr>
<th>Item</th>
<th>Assumptions</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \sim y$ curve and $x_p$</td>
<td>The springs are characterized by an idealized elastic-plastic $p \sim y$ curve ($y$ being rewritten as $w$ in Figure 2.14(b). The uncoupled and coupled load transfer models are utilized to portray the plastic and elastic zones (Figure 2.13).</td>
<td>Plastic and elastic zones encounter at a depth $x_p$ called slip depth</td>
</tr>
<tr>
<td>$G_s$ for $k$ and $N_m$</td>
<td>Average $G_s$ over a depth of $l_c$ is used to estimate the $k$ and $N_m$ for elastic state.</td>
<td>$k$ and $N_m$ deduced from a soil deflection mode</td>
</tr>
<tr>
<td>Liming force profile (LFP)</td>
<td>Average soil properties over a maximum slip depth are employed to estimate the LFP for plastic state.</td>
<td>The generic LFP is described by eq. (2.8) and (2.9).</td>
</tr>
<tr>
<td>Plastic zone</td>
<td>Interaction among the springs is negligible (i.e. $N_m=0$). Pile deflection $w(x)$ exceeds $w_p$, for which resistance per unit length $p$ is fully mobilized.</td>
<td>$N_m=0$ ( w(x)&gt;w_p ) ( p=p_u )</td>
</tr>
<tr>
<td>Pile deflection at slip depth $w_p$</td>
<td>At the slip depth, $x_p$, the pile deflection $w(x)$ equals. Below the $x_p$, the deflection and the soil resistance $p$ ($&lt;p_u$) is linearly proportional to the $k$.</td>
<td>( w_p=p_u/k ) ( p=kw(z) )</td>
</tr>
<tr>
<td>Slip</td>
<td>Pile-soil relative slip (e.g. value of $w(x)-w_p$) can only be initiated from mudline, and it can only be move downwards.</td>
<td>Allowing closed-form solutions to be generated</td>
</tr>
</tbody>
</table>
### Table 2.10 Equations for calculating ultimate lateral load for piles in a two-layer cohesive soil (after Viggiani, 1981 and Chmoulian, 2004)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Ultimate lateral load and Maximum bending moment</th>
<th>Occurrence range</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (Translation)</td>
<td>$T\lambda = \frac{\lambda}{k_c c dl_l}$</td>
<td>$\lambda &lt; \lambda'$ and $\lambda &lt; \bar{\lambda}$</td>
</tr>
<tr>
<td></td>
<td>$T\lambda = \frac{(1+\lambda)^2 + \lambda^2 + \lambda - 1}{1+\lambda}$</td>
<td>$\lambda' \leq \lambda \leq \lambda''$ and $M_1 \leq M_j$ and $M_2 \leq M_j$</td>
</tr>
<tr>
<td>B (Rotation)</td>
<td>$M_1 = \frac{1}{4} \left(1 - \frac{T\lambda}{k_c c dl_l} \right)^2$</td>
<td>$\lambda &lt; \lambda'$ and $\lambda &lt; \bar{\lambda}$</td>
</tr>
<tr>
<td></td>
<td>$M_2 = \frac{1}{4} \left(\lambda - \frac{T\lambda}{k_c c dl_l} \right)^2$</td>
<td>$M_j \leq M_j$ and $M_1 \leq M_j'$</td>
</tr>
<tr>
<td>C (Flow)</td>
<td>$Tc = \frac{1}{k_c c dl_l}$</td>
<td>$\lambda'' &lt; \lambda$ and $\lambda &lt; \bar{\lambda}$</td>
</tr>
<tr>
<td>B1</td>
<td>$T_{by} = 2 \left(1 + \frac{M_y}{1+\chi k_c c dl_l^2} \right)$</td>
<td>$\lambda'' &lt; \lambda$ and $\lambda &lt; \bar{\lambda}$</td>
</tr>
<tr>
<td>BY</td>
<td>$M_1 = \frac{1}{4} \left(1 - \frac{T_{by}}{k_c c dl_l} \right)^2$</td>
<td>$\lambda'' &lt; \lambda$ and $\lambda &lt; \bar{\lambda}$</td>
</tr>
<tr>
<td>B2</td>
<td>$T_{by} = \frac{1}{1+2\chi} \left(1 + (1+2\chi)(1+\frac{4M_y}{k_c c dl_l}) - 1 \right)$</td>
<td>$\lambda'' &lt; \lambda$ and $\lambda &lt; \bar{\lambda}$</td>
</tr>
<tr>
<td></td>
<td>$M_1 = \frac{1}{4} \left(1 - \frac{T_{by}}{k_c c dl_l} \right)^2$</td>
<td>$\lambda'' &lt; \lambda$ and $\lambda &lt; \bar{\lambda}$</td>
</tr>
</tbody>
</table>

**Non-dimensional parameters:**

- $\chi = \frac{k_1 c_1}{k_2 c_2}$, $\bar{\chi} = \frac{2M_y}{k_c c dl_l^2} - 1$
- $\lambda = \frac{l_2}{l_1}$, $\lambda' = \frac{\sqrt{\frac{2}{\chi} + 1} - 1}{1+2\chi}$, $\lambda'' = \chi + 2\chi^2 + 2\chi$, $\bar{\lambda} = \chi\left(\frac{1+\chi}{1+\chi}\right)$

**Note:** $l_1$ and $l_2$ are the pile length in the sliding and stable layers; $c_1$ and $c_2$ are the shear strength of the sliding and stable layers; $k_1$ and $k_2$ are the bearing capacity factors of the sliding and stable layers; $M_y$ is the yield moment of the pile section; $T_{by}, T_{by}, T_{by}, T_{by}, T_{by}$, and $M_j, M_j', M_j$ are defined in Figures 2.17 and 2.18.
Table 2.11 Values of $N_p$ for piles in sliding cohesive soils

<table>
<thead>
<tr>
<th>References</th>
<th>Value of $N_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bea (1971)</td>
<td>7 ~ 12</td>
</tr>
<tr>
<td>Smoltczyk (1973)</td>
<td>4</td>
</tr>
<tr>
<td>Ito and and Matsui (1975)</td>
<td>3.33</td>
</tr>
<tr>
<td>De Beer (1977)</td>
<td>2.8</td>
</tr>
<tr>
<td>Schapery and Dunlop (1978)</td>
<td>10 ~ 20</td>
</tr>
<tr>
<td>Viggiani (1981)</td>
<td>4</td>
</tr>
<tr>
<td>Towhata and Al-Hussain (1988)</td>
<td>12 ~ 20</td>
</tr>
<tr>
<td>Maugeri and Motta (1991)</td>
<td>3 ~ 3.5</td>
</tr>
<tr>
<td>Chen (1994)</td>
<td>11.4</td>
</tr>
<tr>
<td>Chow (1996)</td>
<td>3 ~ 4</td>
</tr>
<tr>
<td>Bransby and Springman (1999)</td>
<td>11.75</td>
</tr>
<tr>
<td>Pan et al. (2000)</td>
<td>10.6</td>
</tr>
<tr>
<td>Pan et al. (2002a)</td>
<td>10 (stiff pile)</td>
</tr>
<tr>
<td></td>
<td>10.8 (flexible pile)</td>
</tr>
<tr>
<td>Leung et al. (2006)</td>
<td>6</td>
</tr>
</tbody>
</table>
### Table 2.12 Conditions and salient features for various analytical solutions for passive piles (after Guo, 2009b)

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Values of $H_i$, $\theta_0$</td>
<td>Assumed</td>
<td>Calculated from $w_s$, $l_s$, and parameters for generic limiting force profile</td>
<td></td>
</tr>
<tr>
<td>Soil parameters</td>
<td>$G_{i1}$ and $G_{i2}$</td>
<td>$G_{i1}, G_{i2}$ and $w_s$</td>
<td></td>
</tr>
<tr>
<td>$w_s$ profile</td>
<td>Uniform $\theta_0=0$</td>
<td>Linear $\theta_0=0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$w_{i1}=0$</td>
<td>$w_s = (l_i - x_p)(\theta_{i2} + \theta_{i1}) + w_{i2}$</td>
</tr>
<tr>
<td>slope $\theta_{i1}, \theta_{i2}$</td>
<td>$\theta_{i1}=\theta_{i2}$</td>
<td>$\theta_{i1}+\theta_0=-\theta_{i2}$</td>
<td></td>
</tr>
<tr>
<td>Moment $M_0$</td>
<td>$M_{01}=M_{02}$</td>
<td>$M_{01}=0$ at $e_0i \neq 0$</td>
<td>$M_{01}=0$ at $e_0i = 0$</td>
</tr>
<tr>
<td>Common features</td>
<td>All methods adopted</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$H_2=-H_1$ and $I_1 &gt; I_2 + x_p$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comments</td>
<td>• $I_1 &gt; 1.2 I_{c1} + x_{p1}$</td>
<td>• $I_1 &lt; 1.2 I_{c1} + x_{p1}$</td>
<td>• $I_1 &gt; 1.2 I_{c1} + x_{p1}$</td>
</tr>
<tr>
<td></td>
<td>• Not related to value of $w_s$</td>
<td>• $w_s$ dependent</td>
<td>• $w_s$ dependent</td>
</tr>
<tr>
<td></td>
<td>• A given sliding depth</td>
<td>• Elastic for sliding layer, elastic-plastic analysis for stable layer</td>
<td>• Elastic-plastic analysis for both layer</td>
</tr>
<tr>
<td></td>
<td>• No dragging impact</td>
<td>• Dragging impact considered</td>
<td>• No Dragging impact</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Offering upper bound solutions</td>
</tr>
</tbody>
</table>

Note: $M_{0i} = H_i e_0i$, bending moment at the sliding level; $e_0i$ = distance from sliding level to incorporate dragging effect; $H_2 = H_1$, lateral load applied a sliding depth; $l_i$ = pile length in sliding layer ($i=1$) and stable layer ($i=2$); $l_{c1}$ = critical embedded pile length in sliding layer ($i=1$) and stable layer ($i=2$); $\theta_0 = \theta_0 = \theta_{i2}$ = differential angle of the pile between upper and lower layer at the sliding level; $\theta_{i1}$ = rotational angle of the pile at the sliding level; $w_{i1} = w_{i2} = \theta_{i1}$ = lateral pile deflection at the sliding level; $x_{p1}$ = sliding thickness from the elastic-plastic boundary to sliding level; $x_s$ = thickness of unloaded zone, at which soil movement is less than pile deflection; $w_s$ = magnitude of uniform soil movement; All these symbols are illustrated in Figures 2.23 and 2.24.
Table 2.13 Comparison of various methods for uncoupled piled-slope stability analysis

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope stability analysis</td>
<td>Limit equilibrium method</td>
<td>Friction circle method</td>
<td>Upper bound limit analysis</td>
<td>Bishop simplified method</td>
</tr>
<tr>
<td>Lateral force acting on pile section above the sliding surface</td>
<td>Using Ito and Matsui (1975) equation based on theory of plasticity</td>
<td>Assumed either rectangular or triangular distribution with depth</td>
<td>Generated from boundary element analysis of pile-soil interaction by applying external soil movement</td>
<td></td>
</tr>
<tr>
<td>Pile response analysis</td>
<td>Subgrade reaction approach</td>
<td>N/A</td>
<td></td>
<td>Continuum approach</td>
</tr>
<tr>
<td>Most effective pile position within the slope</td>
<td>Upper middle part of the slope</td>
<td>Near the toe of the slope</td>
<td>Homogeneous slope</td>
<td>Modified boundary element method</td>
</tr>
<tr>
<td>Common features for piled-slope analysis</td>
<td>The resistance of pile is incorporated at the intersection of the sliding surface with pile in piled-slope analysis.</td>
<td></td>
<td>Two-layer slope</td>
<td>Soil movement and pile-soil interaction with soil movement may occur.</td>
</tr>
<tr>
<td>Comments</td>
<td>Soil movement and pile-soil interaction with soil movement is not taken into account. Assumed that the pile is sufficiently strong and is embedded in a strong underlying stratum to sustain the loads arising from soil movement. Corresponding to the ultimate state of soil movement flowing past the pile section (i.e. soil movement exceeding the pile deflection) above the sliding surface. Implying only “flow mode” of soil failure occurs.</td>
<td>Soil movement and pile-soil interaction with soil movement is considered. Various soil failure modes and pile failure may occur.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.14 Formulations for small strain shear modulus $G_{max}$

<table>
<thead>
<tr>
<th>References</th>
<th>Expressions</th>
<th>Unit for $G_{max}$ and $\sigma^e_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardin and Richart (1963)</td>
<td>$G_{max} = 700 \left( \frac{2.17 - e}{1+e} \right)^2 (\sigma^e_m)^{1.85} \text{ at } \gamma = 10^{-4}$</td>
<td>kg/cm²</td>
</tr>
<tr>
<td></td>
<td>(for dry round grained Ottawa sand)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$G_{max} = 325 \left( \frac{2.97 - e}{1+e} \right)^2 (\sigma^e_m)^{1.85} \text{ at } \gamma = 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(for angular dry crushed quartz sands)</td>
<td></td>
</tr>
<tr>
<td>Seed and Idriss (1970) Seed et al. (1986)</td>
<td>$G_{max} = 1000(K_2)_{max}(\sigma^e_m)^{1.85}$</td>
<td>psf</td>
</tr>
<tr>
<td></td>
<td>$(K_2)_{max}$ ranging from 30 for loose sands to 75 for dense sand</td>
<td></td>
</tr>
<tr>
<td>Hardin and Drnevich (1972)</td>
<td>$G_{max} = \frac{321(2.97 - e)^2}{1+e}$ $\text{OCR}^2 \left( \frac{\sigma^e_m}{\sigma^e_{1.8^5}} \right)$</td>
<td>kPa</td>
</tr>
<tr>
<td></td>
<td>$R = 0 \sim 0.50$, depending on the plasticity Index.</td>
<td></td>
</tr>
<tr>
<td>Iwasaki et al. (1978)</td>
<td>$G_{max} = 850 \left( \frac{2.17 - e}{1+e} \right)^2 (\sigma^e_m)^{1.84} \text{ at } \gamma = 10^{-5}$</td>
<td>kg/cm²</td>
</tr>
<tr>
<td></td>
<td>$G_{max} = 900 \left( \frac{2.17 - e}{1+e} \right)^2 (\sigma^e_m)^{1.85} \text{ at } \gamma = 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>Atkinson (2000)</td>
<td>$\frac{G_{max}}{p_s} = A \left( \frac{p - p'}{p_s} \right)^a \left( \frac{p_{max}}{p'} \right)^{m}$ $p_s$ = reference pressure, taken as 1 kPa or as atmospheric pressure, $p'$ = current effective stress; $p'_{max}$ = the maximum past effective stress. For coarse grained soils, $A = 4000$ and $n = 0.58$, the value of $m$ depends on the history of overconsolidation.</td>
<td>kPa</td>
</tr>
<tr>
<td>Pestana and Salvati (2006)</td>
<td>$G_{max} = G_s e^{-0.5} \left( \frac{\sigma^e_m}{\sigma^e_{1.8^5}} \right)$ $n = 0.5$ for sand; $n = 0.5 \sim 0.75$ for gravels and materials with a significant amount of gravels; $G_s = 400 \sim 800$, depending on the angularity of the material.</td>
<td>kPa</td>
</tr>
<tr>
<td>Wichmann and Triantafyllidis (2009) Method 1</td>
<td>$G_{max} = A \left( \frac{a - e}{1+e} \right)^2 p_{max}^n (\sigma^e_m)^n$ $n = 0.40 C_v^{0.18}$</td>
<td>kPa</td>
</tr>
<tr>
<td></td>
<td>$a = 1.94 \exp(-0.066 C_v)$ $A = 1563 + 3.13 C_v^{2.06}$</td>
<td></td>
</tr>
<tr>
<td>Method 2</td>
<td>$G_{max} = 177000 \left( \frac{1 + D_r/100}{173 - D_r/100} \right)^{0.08} p_{max}^n (\sigma^e_m)^n$</td>
<td></td>
</tr>
<tr>
<td>Method 3</td>
<td>$G_{max} = 218.8 A_2 (a_2 - e)^2 (\sigma^e_m)^{0.5}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a_2 = 1.94 \exp(-0.066 C_v)$ $A_2 = 69.9 + 0.21 C_v^{2.84}$</td>
<td></td>
</tr>
<tr>
<td>Method 4</td>
<td>$G_{max} = 1509720 \left( \frac{1 + D_r/100}{16.1 - D_r/100} \right)^{0.08}$</td>
<td></td>
</tr>
</tbody>
</table>

Note: $e$ = void ratio; $\sigma^e_m$ = mean principal stress; $\gamma$ = shear strain; $p_{atm}$ = atmospheric pressure; $C_v$ = coefficient of uniformity; $D_r$ = relative density; OCR = overconsolidation ratio.
Figure 2.1 Failure modes for free-headed piles (after Broms, 1964a)

(a) Long pile  
(b) Short pile

Figure 2.2 Failure modes for perfectly restrained piles (after Broms, 1964a)

(a) No hinge  
(b) 1 hinge  
(c) 2 hinges

Figure 2.2 Failure modes for perfectly restrained piles (after Broms, 1964a)
(a) Soil movement near ground level  
(after Broms, 1964a)  

(b) Flow mechanism of soils at a great depth  
(after Fleming et al., 2009)  

Figure 2.4 Soil movements around a laterally loaded pile
Figure 2.5 Limiting force profiles for cohesive soils (after (a) Broms, 1964a; (b) Fleming et al., 2009)

Figure 2.6 Comparison of limiting force profiles for cohesionless soils (after Fleming et al., 2009)
Figure 2.7 Schematic generic limiting force profiles

Figure 2.8 Normalised ultimate lateral capacity for a rigid pile  (after Guo, 2008)
Figure 2.9 Normalised load versus maximum bending moment \((e = 0)\) (after Guo, 2006)
Figure 2.10 Subgrade reaction model of soil around pile (after Fleming et al., 2009)

Figure 2.11 A set of $p \sim y$ curves (after Prakash and Sharma, 1990)
Figure 2.12 Comparison of $p - y$ curves for sands at depth $z = 2d$ (after (a) Dyson and Randolph, 2001; (b) Guo and Zhu, 2005b)
Figure 2.13 Schematic of pile soil system for a free head pile (after Guo, 2006)

(a) Coupled load transfer model   (b) Normalised p – y curve

Figure 2.14 Coupled load transfer analysis (after Guo, 2006)
Figure 2.15 A pile under lateral loading. (a) pile-soil system; and (b) soil displacement components (after Sun, 1994)

Figure 2.16 Stress and displacement field adopted in the load transfer analysis: (a) cylindrical coordinate system with displacements and stresses; (b) vertical loading; (c) torsional loading; (d) lateral loading. (after Guo and Lee, 2001)
<table>
<thead>
<tr>
<th>Displacement</th>
<th>Soil reaction</th>
<th>Shear force</th>
<th>Bending moment</th>
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<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Figure 2.17 Failure modes of stabilising rigid piles  
(a) Translation; (b) Rotation; (c) Flow. (after Viggiani, 1981)
Figure 2.18 Failure modes of rigid piles with plastic hinges (after Viggiani, 1981)
Figure 2.19 Pile behaviour characteristics for various modes
(after Poulos, 1995)
Figure 2.20 Measured and calculated pile response under soil movement (after Smethurst and Powrie, 2007)
Figure 2.21 State of plastic deformation in the ground around piles (after Ito and Matsui, 1975)

Figure 2.22 The $p - y$ analysis model for pile subjected to lateral soil movement (after Juirnarongrit and Ashford, 2006)
Figure 2.23 Scheme of analysed model for a flexible pile in landslides (after Cai and Ugai, 2003)

Note: \( L_i \geq L_{ci} \)
\( w_s \approx w_{k1} + w_{k2} \)

Note: \( L_1 < L_{ci} \) & \( L_1 < L_{c1} \)
\( w_s \approx w_{k2} + |\theta_{g2}|(L_1 - x_s) \)

(a) The problem  (b) The imaginary pile  (c) Deep sliding  (d) Normal sliding

Figure 2.24 An equivalent load model for a passive pile (after Guo, 2009b)
Figure 2.25 Experimental apparatus (after Chen, 1994)
Figure 2.26 Variation of shear modulus with shear strain for sands (after Kokusho, 1987)

Figure 2.27 Variation of the soil stiffness for service deformation levels corresponding to different geostructural systems (after Mair, 1993, De Vos and Whenham, 2006)
3 TEST APPARATUS, PROCEDURES AND PROGRAM

3.1 INTRODUCTION

Physical modelling using small scale tests can bring valuable insights into understanding the three-dimensional complexity of the mechanisms of pile-soil interactions. It can be used to identify mechanisms, clarify and quantify key parameters, assess the applicability of and reveal the limitations of analytical models (Randolph and House, 2001; Abdoun et al., 2003). The results of model tests may be used to calibrate conceptual models and analytical methods and demonstrate the appropriateness of these models to the design or evaluation of the response of piles in practical situations (Dobry et al., 2003). Physical modelling has played an important role in solving geotechnical problems and will continue in the future.

To investigate the response of vertically loaded piles and pile groups subjected to lateral soil movement, Guo and Ghee (2004) developed a new experimental apparatus. Later Qin and Guo (2007) modified the apparatus to investigate the pile behaviour under static and cyclic lateral loading. A series of model tests on piles subjected to both passive loading form lateral soil movement and active loading at pile head have been carried out using the apparatus. This chapter mainly presents the test apparatus, data acquisition, testing procedures, and program for the model tests on piles subjected to lateral soil movement. For ease of presentation, the modification of the
apparatus for applying lateral loading was presented in chapter eight.

In this chapter, firstly, the experimental apparatus and loading system for generating soil movement are introduced. Secondly, the model ground preparation, properties of sand used, instrumentation of the model piles and the data acquisition system are presented. Thirdly, experimental procedures for conducting the passive pile tests are elaborated. Fourthly, test program and test details for single piles and pile groups undergoing lateral soil movement are described. Finally, processes and preliminary analysis of the collected data via the data acquisition system are discussed. The experimental results are presented in the subsequent chapters.

3.2 TESTING APPARATUS

The experimental apparatus primarily consists of a shear box, a loading system, and a measurement system. The model ground was prepared in the shear box, in which a single pile or pile group was installed. The loading system allows lateral soil movement and vertical load to be applied simultaneously to the single pile or pile group embedded in the model ground. The measurement system consists of a data acquisition system, pile instrumentation, dial gauges and reference board for measuring the displacement of the boundary of the shear box. The experimental data obtained from the measurement system permits the real-time response of the piles and soil-pile-shear box interaction to be captured and the effect of lateral soil movement
on piles to be investigated. The experimental apparatus, sand properties, preparation of model ground, and model piles instrumentation are presented. Detailed information about the apparatus was given by Guo and Ghee (2004) and Ghee (2010).

3.2.1 Shear box

Figure 3.1 shows an overview of the experimental apparatus and the loading system. The shear box measures 1 m both in length and width. The upper section of the shear box consists of a series of stacked 25-mm-deep square laminar steel frames. Using the stacked laminar frames has the advantage of allowing separate sliding of each frame, and larger frame movements to be achieved. The frames, which are allowed to slide, contain the ‘sliding layer of soil’ of thickness $L_m$. The lower section of the shear box comprises a 400-mm-high fixed timber box and the desired number of laminar steel frames that are fixed, so that a ‘stable layer of soil’ of thickness $L_s$ ($\geq 400$ mm) can be guaranteed. Changing the number of moveable frames in the upper section, the thickness of the stable and moving layers are varied accordingly. Note that the $L_m$ and $L_s$ are defined at loading location, and they do vary across the shear box. The actual sliding depth $L_m$ around the test pile is unknown.

3.2.2 Lateral loading system

The lateral loading system encompasses a horizontal hydraulic jack system and a loading block that exerts lateral force on the upper movable laminar frames at one side of the shear box shown in Figure 3.1(b). The hydraulic jack system primarily
consists of a cylinder, hand pump, pressure gauge and hose. The specifications for the main components of the jack system were presented in Table 3.1. The hydraulic jack cylinder was mounted on a horizontal beam supported by two vertical columns fastened on the laboratory strong floor. The horizontal beam can be moved up and down along the two vertical columns so that various sliding layer depth, \( L_m \), can be achieved using the same loading block. The pressure gauge provided readings of hydraulic working pressure to mobilise the sliding layer. The pressure multiplied by the cylinder effective area gave the lateral force on the loading block. The pumping speed of the hydraulic jack controls the movement rate of the loading block, thus the soil movement in the shear box.

3.2.3 Sand properties and preparation

Medium oven-dried quartz sand from the Logan River, Carbrook, Queensland was used in this study. In order to prepare uniform model ground, the sand raining technique was adopted to achieve a consistent density of the sand. This technique has been widely used in laboratory model tests in the study of capacity and load-displacement characteristics of pile foundations under axial and lateral loads (Turner and Kulhawy, 1994; Kim \textit{et al.}, 2004; Mezazigh and Levacher, 1998; Vendure \textit{et al.}, 2003; Rosquoet \textit{et al.}, 2009) and pile loading tests with ground displacements (Chen, 1994; Tsuchiya \textit{et al.}, 2001; Chen \textit{et al.}, 2002). It is well recognised that this method of sand placement can create the most uniform and reproducible deposits (Bieganousky and Marcuson, 1976). A sand rainer was used for the present study. The
The sand rainer was fabricated from plywood, having internal dimensions of 1 m by 1 m, and 0.15 m in height. The base of the sand rainer was fabricated from a fixed 18-mm thick timber plate underlying by a moveable 6-mm-thick plastic plate. Both plates were perforated with 5 mm diameter holes on a 35 mm by 50 mm grid pattern. The moveable plastic plate can slide along the slots on both sides of the sand rainer. When the plastic plate was pulled out and the holes in the two base plates coincided, the sand was discharged from the sand rainer.

Figure 3.2(a) shows raining of sand into the shear box. The sand rainer was suspended over the shear box by four slings at the four corners, which were connected to an overhead crane (already exist in the laboratory). When the sand rainer was filled with sand, it was moved above the shear box by the crane. The sand rainer could be lifted or lowered so that the designed sand falling height was achieved. The falling height was measured from the bottom of the sand rainer to the surface of the sand in the shear box. When the raining started, the position of the sand rainer was adjusted to ensure that the sand was evenly rained into the entire range of the shear box. The sand rainer was raised to a new position as the sand deposition layer in the shear box increased. This raining process was repeated until the shear box was full of sand.

The sand density was controlled by the intensity and velocity of deposition of the sand particles, which in turn depend on the diameter of the holes of the sand rainer and the sand falling height. Prior to the tests in this thesis, Guo and Ghee (2004) and Ghee
(2010) had conducted a number of sand raining tests to determine the relationship between the sand density and falling height. Their results were re-plotted in Figure 3.3. The density of the sand increased from 15.89 kN/m$^3$ to 16.27 kN/m$^3$ as the sand falling height increased from 400 mm to 600 mm. Ghee (2010) also reported a number of calibration tests to investigate the uniformity and reproducibility of the model ground using the raining technique. His investigation showed that the variation in density in various tests using the same falling height was less than 0.1 kN/m$^3$. He also checked the uniformity of the sand by raining the sand into eight 101.6 mm diameter moulds, placed at eight locations, but at the same height within the shear box. These tests were performed using different sand falling height and intensity of deposition. His evaluation demonstrated that the variation in sand density was less than 0.1 kN/m$^3$ within the shear box for each sand falling height.

Following their work, the sand falling height was selected as 600 mm in the current tests, giving a relative density, $D_r$, of 89%, unit weight, $\gamma_s$ of 16.27 kN/m$^3$. The same falling height and sand density were used in the tests reported by Guo and Ghee (2004, 2005), Guo and Qin (2005), Guo et al. (2006), and Ghee (2010). This selection of the sand falling height ensures that the results from the current tests can be compared with those from the previous tests. At the selected sand density, key characteristics of the sand are summarised in Table 3.2. The particle size distribution of the sand determined using dry sieving method was plotted in Figure 3.4. The peak angle of internal friction $\phi_s$ was evaluated as 38º from three parallel series of direct shear
tests with the normal stress ranging from 26.7 kPa to 67.6 kPa (Ghee, 2010).

### 3.2.4 Model piles

Aluminium pipe piles of three different diameters were used throughout the experimental study. Their geometrical properties and bending rigidity were presented in Table 3.3. The majority of the model pile tests were conducted using the 32-mm diameter pile. The different characteristics of the three piles allowed the effects of relative pile stiffness to be investigated.

### 3.2.5 Instrumentation and data acquisition

The piles were instrumented with strain gauges to measure the bending moment along the pile induced by soil movement. Figure 3.5(a) shows a schematic of pile instrumentation as well as the strain gauges locations on the pile. On the two vertical lines of the front and rear sides with respect to the soil movement direction, ten pairs of electrical resistance strain gauges were installed on the outer surface of a tested pile with a spacing of 100 mm shown in Figure 3.5(b). The ten pairs of strain gauges were numbered from $SG_1$ to $SG_{10}$, starting from the pile tip. Correspondingly, the bending moment calculated from these strain gauges measurement were numbered from $M_1$ to $M_{10}$, which were used in the calculation of other pile responses as explained in section 3.5. In order to protect from damage during the installation of pile and testing under soil movement, the strain gauges were sealed with a protective coating by 1 mm epoxy resin shown in Figure 3.5(b). Figure 3.5(c) shows a cross section at a
measurement level. The gauges wires were wrapped and fixed on the outer surface of
the pile using electronic tapes. Then the whole pile surface was wrapped using
electronic tapes shown in Figure 3.5(d). The specifications of the strain gauges were
presented in Table 3.4.

A data acquisition system (DAQ) from National Instruments™ was used to collect,
process, transmit and store data from strain gauge measurement for each pile test. The
DAQ consists of three SCXI-1520 universal strain gauge input modules and the
associated accessory SCXI-1314 front-mounting terminal blocks (Figure 3.5(d)). Each
strain gauge module has eight simultaneously sampled analog input channels that
could be configured into a quarter-bridge, half-bridge, or full-bridge configuration of
a Wheatstone bridge. The quarter-bridge configuration (Figure 3.6) was used to
connect each strain gauge attached to the model pile to a different channel. Figure 3.7
shows how a strain gauge was connected to a channel in a SCXI-1314 front-mounting
terminal block. Initially, two linear variable differential transformers (LVDT)s were
used to measure the horizontal displacements at two separate points on the pile head.
Later on, to make more channels for the strain gauges, the LVDTs were replaced by
mechanical dial gauges in the tests conducted in this thesis.

A LabVIEW program was written to control the data acquisition system, which
enables the function of null offset, shunt calibration and recording data be done
automatically. Figure 3.8 presents a graphic interface captured by the program during
a test, allowing the operator to view graphical displays of changing conditions during testing. The data obtained from the strain gauges measurements were recorded and stored in a computer, which can be retrieved in the form of spreadsheets, such as Excel. Details about the data acquisition system and the LabVIEW program were specified in Ghee (2010).

### 3.2.6 Calibration of test piles

In order to ensure an appropriate relation between the strain gauge output and bending moment, all the test piles have been calibrated in bending by testing the pile as a simply supported beam with both ends of the pile clamped to the ball supports. Figure 3.9 shows the test setup for the calibration of a model pile. Load was applied at the middle of the pile. A load series consisting of six load levels of 10 kg, from 0 to 50 kg were conducted. The piles were carefully orientated so that the loads were applied in the plane of the strain gauges. The measured strain at each strain gauge location was compared with the actual calculated bending moment. This comparison provided a direct calibration between the change in voltage and the bending stress at the strain gauge locations.

### 3.3 EXPERIMENTAL PROCEDURES

#### 3.3.1 Model ground preparation

As described in section 3.2.3, the model ground was prepared using the sand raining
technique. Upon completion of the model ground preparation, the ground surface was levelled by rolling a 50 mm diameter pile. A 40 by 40 mm grid was made using dry coloured sand as shown previously in Figure 3.2(b). The coloured sand was made by mixing the sand with food colour additive. The grid were prepared by using a transparent plastic plate (4 mm thick) cut with longitudinal openings of 4 mm width and spaced at 40 mm (centre to centre). Two lines $AB$ and $CD$ in the form of a cross in Figure 3.2(b) were marked through the centre of the model ground surface for the ease of locating the position for installation of the pile. The shear box was carefully moved by a pallet truck and secured under the frames for installation of the pile.

3.3.2 Pile installation

Figure 3.10 shows the installation of a two-pile group. A vertical jack system was used for the installation of the pile and pile group. The specifications for the main components of this jack system were also presented in Table 3.1. The ENERPAC PATG-1102N turbo air hydraulic pump connected with an air compressor (already exists in the laboratory) was used to install the pile into the model ground to a desired embedded length. Prior to the installation of the pile, the pile tip was pushed into the model ground manually. A spirit level was used to check the verticality of the pile when positioned prior to and during the installation. Any adjustment to the verticality of the pile was made manually. Great care was taken to make sure that the line joining the centre of the pair of strain gauges in Figure 3.5(c) coincides the soil movement direction so that the maximum bending stress induced by the soil movement on the
pile was measured. A cylinder (80 mm in diameter and 20 mm in height) was connected to the end of the hydraulic jack cylinder and secured on the pile head. The pressure measured from the jack gauge was recorded at every 100 mm increment until the desired embedded length was reached. The hydraulic jack has a maximum stroke of 300 mm. Four extension rods were used to extend the travel distance of the jack stroke. The jack pressure was released every time an extension rod was connected. The jack-in pressure for a single pile and a two-pile group was recorded during installation of the piles. However, the force required to install a four-pile group exceeds the capacity of the jack. Therefore the piles in these groups were installed individually.

3.3.3 Loading application

The lateral hydraulic jack and the loading block were first set up. Axial load was applied on the pile-head using the desired number of weights, of 10 kg each (see Figure 3.1). These weights were secured by a sling at 500 mm above the ground surface. This method of load application simulated a free-head fixity condition since the use of weights provided no lateral restraints on the pile head.

3.3.4 Measurement system set up

Figure 3.11 shows the setting up of the measurement system. Firstly, the mechanical dial gauges were connected to the pile (Figure 3.11(a)). Secondly, the front mounting terminal blocks containing the strain/bridge modules were connected to the data
acquisition system (Figure 3.11(b)). Thirdly, a ruler was attached on the top frame and a reference board with graph paper was adjusted to the initial position (Figure 3.11(c)). The strain gauges and dial gauges readings were reinitialised before conducting the tests. The pile or pile group was ready to be tested under the designed conditions.

3.3.5 Generation of lateral soil movement and data recording

As described in section 3.2.2, pumping the lateral hydraulic jack exerted force on the loading block, which pushed the contacted frames in the sliding layer to move into the sand and to generate soil movement. The strain gauge, dial gauge readings and hydraulic working pressure obtained from the hydraulic jack gauge were taken at every 10 mm movement of the top laminar frame. This movement was measured from the reference board (Figure 3.11 (c)) and called frame movement, $w_f$ in this thesis. Tests were terminated at frame movement, $w_f$ of 120 ~150 mm.

3.3.6 Completion of test

Upon completion of test, firstly, the loading system and measurement devices were removed. Then the pile was withdrawn. Finally sand was emptied through an outlet at the base of the box and kept in a storage box.
3.4 TEST PROGRAM

3.4.1 Previous tests

As mentioned in section 1.2, an experimental investigation on piles embedded in soils subjected to lateral soil movement has been outlined in the ARC project.

At the first stage, the experimental investigation was carried out on single piles and pile groups subjected to uniform lateral soil movement induced by a rectangular loading block. Each test needed two persons and it took about seven hours to do one test: four hours to prepare a model ground due to the relatively large size of the shear box, and three hours to set up the loading and measurement systems and to do the test. Enghow Ghee (Ghee, 2010) together with the author conducted the majority of these tests, when the author worked as a visiting research scholar. These tests included twenty tests on single piles and twenty-one tests on pile groups conducted in sand undergoing a uniform soil movement profile with the following parameters investigated

- two densities of sand;
- three pile diameters;
- two combinations of sliding layer depth and stable layer depth;
- two axial load levels on a single pile or a pile in a group;
- three different pile group configuration;
- two pile spacings between piles in a group; and
two types of pile head fixity conditions.

The results of all of these tests have been reported in the thesis of Ghee (2010). Among them, the results of eight tests have been published.

- Guo and Qin (2005) presented results of RS50-0 and RS50-294.

The results of the eight tests were further analysed in this thesis (presented in chapter five) using the solution developed by Guo and Qin (2010).

Later, in order to investigate the effect of soil movement profile on the pile responses, the author did eight tests (TS32-0, TS32-294, TD32-0, TD32-294, TS50-0, TS50-294, TD50-0, TD50-294) using a triangular loading block with the assistance of Enghow Ghee. The results of these tests were reported by Guo and Qin (2006) and Guo and Qin (2010). Ghee (2010) also presented the bending moment profiles and pile displacement profiles at $w_f = 60$ mm to study the influence of pile diameter and axial load. The results were further analysed in this thesis.

The tests presented in this thesis, similar in appearance to the previous tests in Ghee (2010), are all new and were designed to fulfil the aim of proposed ARC project highlighted in section 1.2. They are briefly explained below (with the test program being presented in next section):
• The previous studies involved instrumented model piles installed in sand subjected to a uniform soil movement profile using a rectangular loading block. The current tests primarily used a triangular loading block, which results in a progressive soil movement profile followed by a trapezoidal profile.

• The current tests are the first to reveal large lateral soil movement (excluding earthquake-induced liquefaction and lateral spreading) in the laboratory. Soil movement near the loading side at the model ground surface may reach up to 150 mm.

• The effects of the following parameters on pile responses were explored. (1) pile characteristics (pile diameter, pile bending rigidity, pile group configuration, and pile head fixity); (2) varying depths of sliding layer and stable layer, profiles of soil movements (uniform, triangular, trapezoidal); (3) vertical load level; (4) distance from the pile locations to the boundary of soil movement; (5) the triangular loading block angle; and (6) the magnitude of soil movement.

The investigations were novel and critical to: (1) pile failure mode; (2) determine modulus of subgrade reaction; and (3) verify analytical solutions as highlighted in the ARC project aims. For instance, all the test results were analysed using the solution developed by Guo and Qin (2010) (discussed later on). The solution is
practically useful since the relationship between the maximum bending moment and maximum shear force is for the first time established for piles subjected to moving soil. The maximum shear force and maximum bending moment are required in the design of reinforcing piles to increase slope stability.

3.4.2 Current tests

3.4.2.1 Single piles subjected to soil movement

In total, 25 tests on single piles were conducted subjected to soil movement and two tests without piles. Figure 3.12 illustrates a model test set up for a single pile. Figure 3.13 shows a schematic model pile under testing. The details of these tests were summarised in Table 3.5. Each test was designated by a combination of letters and numbers to signify ‘shape of loading block’, ‘predetermined moving soil depth’, ‘pile diameter’, ‘axial load level’ and other factors examined. For example, tests RS25-0, TS32-0, and TD50-294 are explained: (1) ‘T’ and ‘R’ signify the triangular and rectangular loading block; (2) the ‘S’ refers to a predetermined sliding layer depth, \( L_m \) of 200 mm and stable layer depth, \( L_s \) of 500; while ‘D’ refers to a deep sliding depth, \( L_m \) of 350 mm and \( L_s \) of 350 mm; (3) ‘25’, ‘32’ or ‘50’ indicates 25 mm, 32 mm or 50 mm in outer diameter of the pile; and (4) ‘0’ or ‘294’ represents 0 or 294 N axial load applied at the pile head, respectively. The sample model ground was prepared in the way described previously and was maintained 800 mm in depth for each test. The model pile was installed to an embedded length, \( L \), of 700 mm below the ground surface to avoid interference with the bottom of the shear box. If unspecified, the pile
was always installed in the centre of the shear box, i.e. \( S_b = 500 \) mm, as shown in Figure 3.12(b).

The first 24 tests in Table 3.5 were categorised into seven groups, including the ‘general tests’ group and six parametric study groups of pile location, axial load, loading block angle, varying sliding depth, diameter and distance, loading block shape. Tests 1 to 21 were conducted using the triangular loading block 1 with an angle of 15°, except tests TS32-0 (22.5°) and TS32-0 (30°) which were conducted using the triangular loading block 2 and 3 as shown in Figure 3.14. Tests 22 to 24 were carried out using a rectangular block.

(1) Group 1: The 8 **General tests** were conducted to bring out the effect of soil movement on the pile behaviour: under 2 axial loading levels (0 or 294 N); 2 combinations of predetermined sliding and stable layer depth \( (L_m = 200 \) and \( L_s = 500 \) mm or \( L_m = 350 \) and \( L_s = 350 \) mm); and 2 pile diameters (32 or 50 mm). In particular, test TS32-0 was designated as the ‘standard test’ for convenience in comparison with other tests.

(2) Group 2: The **Pile location** tests including TS32-0-340, TS32-0 and TS32-0-660 were conducted to explore the effect of distance from the free soil movement source at the loading side to the pile location by installing the pile at three distances \( S_b \) of 340 mm, 500 mm and 660 mm.
(3) Group 3: The **Axial load** tests comprising TS32-0, TS32-294, TS32-588, and TS32-735 were carried out to examine the effect of varying the magnitude of axial load applying at pile head.

(4) Group 4: The **Loading block angle** tests comprising TS32-0, TS32-0 (22.5°) and TS32-0 (30°) were conducted to examine the effect of the loading block angle.

(5) Group 5: The **Varying sliding depth** tests consisting of T32-0 (L_m=125), TS32-0, T32-0 (L_m=250), T32-0 (L_m=300), T32-0 (L_m=350) were conducted to investigate the effect of variation of sliding layer depth, L_m. Note that Test T32-0 (L_m=350) was conducted under identical conditions to test TD32-0. This test was also conducted to confirm the repeatability of the model tests.

(6) Group 6: The **Diameter and distance** tests including TS25-300, TS25-0-500 and TS25-700 were conducted to explore the diameter effect as well as the distance S_b on the piles.

(7) Group 7: The three **Loading block shape** tests were conducted using a rectangular loading block to investigate the effect of soil movement profiles on the pile response, together with the **General tests**.
(8) Group 8: The two tests No. 25 and No. 26 were conducted without piles using the triangular loading block 1 and a rectangular loading block to assess the soil movement and lateral forces needed to mobilize the sliding soil layer without piles.

(9) The supplementary test 27, RS50-294(S), was undertaken to confirm the repeatability of the test RS50-294 reported previously by Guo and Qin (2005).

The results of the tests in Groups 1 to 5 were presented and discussed in chapter four to explore the responses of rigid piles in progressively moving sand. The results of the tests in Groups 6 to 7 and the supplementary test RS50-294(S) were presented in chapter five to investigate the effect of soil movement profiles on the pile responses.

3.4.2.2 Pile group subjected to soil movement

In practice, piles are usually used in groups. Hence, model tests were also carried out on pile groups to investigate the performance of piles within a group of different arrangement, pile head condition, and centre-to-centre spacing. Three basic types of pile arrangements were considered, as defined by Chen et al. (1997). They are: (1) piles in a row perpendicular to the direction of soil movement; (2) piles in a line parallel to the direction of soil movement; and (3) piles in a square arrangement. Figure 3.15 illustrates the experimental set up for a four-pile in square group. Both free-head and capped-head group conditions were taken into account. A pile group is defined as having a free-head fixity condition, if the pile heads within the group have
no restraints. Otherwise, the pile group is defined as having a capped-head fixity condition, if the pile heads within the group are connected by a pile cap. No axial load was applied on the pile head or pile cap.

Twenty-one pile group tests were carried out. Figure 3.16 shows a schematic capped-head pile group under testing. The triangular loading block 1 (Figure 3.14(a)) was used for all the pile group tests. The pile group configurations and pile head conditions for these tests were summarised in Table 3.6. They were described below.

Tests G1 to G10 were conducted with two piles in a row. Figure 3.17(a) and Figure 3.17(b) show the experimental setting up for a free-head and capped-head group, respectively. Both piles were instrumented. Among them, tests G1 to G6 were carried out with a centre-to-centre spacing of \(3d\), i.e. \(S_b/d = 3\), to investigate the effect of distance, \(S_b\) between the loading side where free soil movement was induced and the pile group location on the pile responses. Tests G7 to G10 together with tests G3 and G4 were conducted to examine the effect of pile spacing on the responses of the piles in a row while maintaining constant distance \(S_b = 500\) mm to the loading side.

Tests G11 to G16 were conducted with two piles in a line. Figure 3.18(a) and Figure 3.18(b) show the experimental setting up for a free-head and capped-head group, respectively. The pile closer to the loading side is referred as the ‘front’ pile, while the other is called ‘back’ pile for convenience, as illustrated by the insert in Table 3.6. The
front piles were always installed at a distance $S_b = 340$ mm from the shear box boundary. This series of tests were designed to investigate the effect of pile spacing and pile head fixity conditions on the responses of piles arranged in a line.

Tests G17 to G21 were conducted with four piles (2×2) in a square. Figure 3.19(a) and Figure 3.19(b) show the experimental setting up for a free-head and capped-head group, respectively. Similar to the two piles in a line, the piles closer to the loading side are referred as the ‘front’ piles, while the other are called ‘back’ piles. The front row of piles were also installed at a distance $S_b = 340$ mm from the shear box boundary. Due to the symmetrical arrangement of the pile configuration and limited channels of the data acquisition system, only two piles were instrumented with strain gauges, as shown in Figure 3.19 and Table 3.6.

The piles used in the groups were 32 mm in diameter and 1200 mm in length. The instrumentation of the piles was the same as that of the single piles described in the preceding section. Two types of pile caps with different spacing had been designed and manufactured and the details of the pile caps have been described by Ghee (2010). The pile caps having a centre-to-centre spacing of three pile diameters are illustrated in Figure 3.20. The pile caps were made of solid aluminium blocks 50 mm thick, into which two to four holes of 25 mm deep with diameter of 32.5 mm were drilled. The diameters of the holes were made slightly larger (+0.5 mm) than the pile diameters.
3.5 DATA ANALYSIS AND PROCESSING

As described in section 3.2.5, the piles were instrumented with strain gauges. The pile displacements above the ground surface were measured by dial gauges. The strains measured from the 10 pairs of strain gauges, dial gauge readings, and lateral force measurements from the lateral hydraulic jack were analysed to study the pile responses and the shear box-soil-pile interaction occurring in each model test. The procedure for the data analysis and processing was explained in detail in the following sections.

3.5.1 Strain and bending moment relationship

The purpose of attaching strain gauges on the model piles is to measure the strain developed in the pile during testing. The strain measurements are necessary and useful for calculating other pile responses. The strain measurements recorded at the ten locations in Figure 3.5 were used to determine the bending moments at the same elevations. The flexural stress, $\sigma_z$, is first calculated from the measured strain, $\varepsilon_z$, using Hooke’s Law (equation 3.1). Then discrete bending moment, $M$ is calculated from the flexural stress, $\sigma_z$ by applying the elastic flexure formula (equation (3.2)) for a beam under pure bending in mechanics of materials. Substituting for $\sigma_z$ from (3.1) into (3.2), the relationship between bending moment and strain is obtained and is presented in equation (3.3). This equation is only valid while the pile material remains within the elastic stress range.
\[\sigma_z = E_p \varepsilon_z \]  
\[\sigma_z = \frac{M d}{I_p} \]  
\[M(z) = \frac{2E_p I_p}{d} \varepsilon_z\]

where \(\sigma_z\) is the flexural stress, \(\varepsilon_z\) is the measured strain, \(M(z)\) is the bending moment of the pile at a depth \(z\), \(E_p\) is the Young’s Modulus of the pile, and \(I_p\) is the moment of inertial of the cross section of the pile, \(d\) is the outer diameter of the pile.

### 3.5.2 Beam theory

Beam theory provides the relationships which describe the five pile responses, including displacement, rotation, bending moment, shear force and soil reaction. These relationships can be best explained and illustrated with a set of successive curves shown in Figure 3.21. All the pile responses are functions of depth. Equations (3.4) to (3.7) describe the rest four pile responses derived from the bending moment within a pile.

**Displacement**

\[y(z) = \int \left( \int \frac{M(z)}{E_p I_p} \, dz \right) \, dz\]  
\[S(z) = \int \frac{M(z)}{E_p I_p} \, dz\]

**Rotation**

**Shear force**

\[T(z) = \frac{dM(z)}{dz}\]

**Soil reaction**

\[p(z) = \frac{d^2 M(z)}{dz^2}\]

where the depth \(z\) is measured downward from the soil surface, \(y(z)\) is the lateral displacement of the pile, \(S(z)\) is the rotation of the elastic curve defined by the axis of
the pile (in radian), \( T(z) \) is the shear force in the pile, \( p(z) \) is the soil reaction per unit length.

### 3.5.3 Sign convention

Before presenting a method for determining the pile rotation, displacement, shear force, and soil reaction, it is first necessary to establish a sign convention so as to define “positive” and “negative” pile responses in the interpretation of test results. Figure 3.21 illustrates the positive directions of the pile responses. They are explained below.

- **Frame (soil) movement**
  The hydraulic jack and loading block system generates positive frame movement thus soil movement in the loading direction across the shear box.
- **Pile displacement**
  The pile displacement away from the loading side is positive.
- **Pile rotation**
  The pile rotation in the clockwise direction is deemed positive.
- **Bending moment**
  The internal moment which causes tension in the front side (closer to the loading direction) of the pile is taken as positive.
- **Shear force**
  The internal shear force which causes a counter-clockwise rotation of the pile segment on which it acts is positive.
• Soil reaction

The soil reaction which acts in the direction away from the loading side, i.e. causing positive pile displacement, is deemed positive.

3.5.4 Boundary condition

Two integration constants are required in performing double integration of bending moment profile in equation (3.4) to determine the pile displacement profile. These integration constants are physically equivalent to the displacement, \( y_0 \) and rotation, \( S_0 \) of the pile at the ground surface \( (z = 0) \), respectively. There are several ways in which the two unknown constants can be determined. One approach is to measure these values directly during experimental testing. However, this approach was deemed as not feasible since the significant sand movement at the soil surface at large frame movements may disturb the dial gauges. An alternative approach is to use known solutions to solve analytically for the two unknown constants. However, no such analytical solutions are available for piles subjected to lateral soil movement.

In the current analysis, it is assumed that the pile displaced rigidly above the ground surface. This assumption is appropriate because no lateral restraints were applied on this portion of the pile. As shown in Figure 3.22, the displacements of the pile, \( y_A \) and \( y_B \) at the two points \( A \) and \( B \), where the two dial gauges contacted the pile, can be calculated from the two dial gauge readings at the corresponding elevations, \( H_A \) and \( H_B \) above the ground surface. With the assumption of rigid pile displacement, the
displacement, $y_0$ and rotation, $S_0$ at the soil surface were determined by equations 3.8 and 3.9.

Rotation

$$S_0 = \frac{y_B - y_A}{H_B - H_A}$$ (3.8)

Displacement

$$y_0 = y_A - S_0H_A$$ (3.9)

Additionally, the test results by Guo and Ghee (2004, 2005) indicate the pile was generally rigid. The bending moment and shear force at pile tip were zero, i.e., $M(L)=0$, and $T(L)=0$.

3.5.5 Analysis of discrete bending moment results

The discrete measured strains need to be fitted by a continuous analytical function, to gain bending moment distribution along the pile length. Polynomial and spline functions are always preferred, such as fifth or sixth order polynomial functions (Stewart, 1992; Chen, 1994) and fourth or fifth order spline functions (Smethurst and Powrie, 2007; Frank and Pouget, 2008), probably because they are easy to be integrated and differentiated. A general polynomial can be expressed as

$$M(z) = c_1 + c_2z + c_3z^2 + c_4z^3 + \ldots + c_mz^{m-1}$$ (3.10)

The bending moment functions must satisfy several physical constraints, depending on the type of loading, i.e., active loading from pile head or passive loading from lateral soil movement. These constraints include the moment, shear force and soil reaction occurring at the ground surface, which is explained below.

Substitution of equation (3.10) into equations (3.6) and (3.7) gives
For a pile subjected to only lateral soil movement in sand, the moment, shear force and soil reaction at the ground surface are zero, i.e. at $z=0$, $M(0)=0$, $T(0)=0$, and $p(0)=0$. Consequently, taking $z=0$ in equations (3.10), (3.11) and (3.12) gives

\[ M(0) = c_1 = 0 \]  
\[ T(0) = c_2 = 0 \]  
\[ p(0)=2c_3 = 0 \text{ (for cohesionless soil)} \]

Substituting the $c_1=c_2=c_3=0$ into equation (3.10) leads to

\[ M(z) = c_4 z^3 + \ldots + c_m z^{m-1} \]

Using data analysis software, the method of least squares can be used to solve for the unknown coefficients within the polynomial functions of equation (3.14).

The technique of polynomial curve fitting may not be suitable for the analysis of the discrete bending moment results in the current model tests. Taking the calculated bending moment of test TS32-0 as an example (results presented in Figure 4.4(a)), the bending moment profile at $w_t < 50$ mm is difficult to be fitted well using equation (3.14), while higher order polynomial ranging from 4 to 6 is necessary to accurately fit the bending moment profiles at $w_t \geq 50$ mm. This approach resulted in inconsistent results at different magnitude of frame movements. Numerical integration and differential were thus used to derive the pile rotation, displacement, shear force, and soil reaction, as discussed next.
3.5.6 Numerical integration for calculating pile rotation and displacement

As discussed in section 3.5.2, the pile rotation and displacement are calculated from the first and second order of integration of the bending moment profile on the basis of beam theory. Numerical integration using the trapezoidal rule was used to compute the pile rotation profile with the pile rotation at the ground surface as the input boundary condition. Referring to Figure 3.23, the pile rotation at a specific point was calculated by the following generalized equations

\[
S_i = S_0 - \left( M_0 + M_i \right) \frac{\Delta z}{2E_p I_p}
\]

where \( \Delta z \) is the subinterval for dividing the pile length, equalling to 100 mm, or the spacing of the strain gauges; \( M_i \) is the discrete bending moment. The bending moment at pile tip, \( M(L) \) is assumed to be zero. It should be noted that the distance, \( \Delta z \) between the pile tip and the first pair of strain gauges for measuring \( M_1 \) equals to 50 mm. The same distance was also noted between the ground surface and the seventh pair of strain gauges for measuring \( M_7 \). This variation in the subinterval, \( \Delta z \) must be taken into account in the calculation when using equations 3.15(a) ~ 3.15(d). The variation in \( \Delta z \) had been considered in the spreadsheet program for processing the test data, which was presented in section 3.5.8. With the calculated pile rotation at...
each point, the pile rotation profile was obtained.

The derived pile rotation profile was integrated using the trapezoidal rule to calculate
the pile displacement with the pile displacement at the ground surface calculated from
equation (3.8). Referring to Figure 3.23, the pile displacement at a specific point was
calculated by equations 3.16(a) ~ 3.16(d). Again, the variation in the subinterval, \( \Delta z \)
immediately below the soil surface and above the pile tip must be considered in the
calculation.

Ground surface: \( y_0 \) (measured at ground surface) \( \text{(3.16a)} \)

\[
y_i \text{ (i=7)} = y_0 + \left( S_0 + S_T \right) \frac{\Delta z}{2} \quad \text{(3.16b)}
\]

\[
y_i \text{ (i=1,2,3,4,5,6)} = y_0 + \left( S_0 + 2 \sum_{k=1}^{7} S_k + S_T \right) \frac{\Delta z}{2} \quad \text{(3.16c)}
\]

Pile tip: \( y(L) = y_0 + \left( S_0 + 2 \sum_{k=1}^{7} S_k + S(L) \right) \frac{\Delta z}{2} \quad \text{(3.16d)}
\]

3.5.7 Numerical differentiation for deriving shear force and soil reaction

The shear force, \( T_i \) and soil reaction, \( p_i \) at the points corresponding to the strain gauge
locations along the pile are determined by the finite difference form of the derivatives
of equations (3.6) and (3.7) from the bending moment distribution, which are
equations (3.17) and (3.18), respectively.

\[
T_i = \frac{M_{i+1} - M_{i-1}}{2\Delta z} \quad \text{(3.17)}
\]

\[
p_i = \frac{M_{i+1} - 2M_i + M_{i-1}}{\Delta z^2} \quad \text{(3.18)}
\]
Equation (3.17) expresses the central difference for the first derivative and was used to calculate the shear force. The central difference yields more accurate approximation than the forward difference and backward difference since its error is proportional to the square of the interval $\Delta z$.

Use of equation (3.18) for calculating the soil reaction is arguable. It is well known that double differential of discrete bending moment data points tends to amplify measurement errors and thus leads to an inaccurate soil reaction. Presently, there is no generally accepted standard method for deducing soil reaction. Nevertheless, Levachev et al. (2002) noted that the method using (along with the method of least squares) the values of $M(z)$ at successive five points on the bending moment profile offers more reliable and exact results than the usual method of numerical differential given by equation (3.18). In this method, the soil reaction is calculated by equation (3.19)

$$p_i = \frac{1}{7(\Delta z)^2}(2M_{i+2} - M_{i+1} - 2M_i - M_{i-1} + 2M_{i-2})$$

In essence, this method for computing the soil reaction is equivalent to fitting a cubic polynomial by least squares to successive sets of five equally spaced measured bending moment data points and then differentiating this polynomial at the central point.

Equation (3.19) was used to calculate soil reaction. Its accuracy and errors are discussed below.
Yang and Liang (2006) evaluated the accuracy and applicability of four methods for deducing the soil reaction from measured strain data, including the global fifth-order polynomial curve fitting, piecewise cubic polynomial curve fitting, weighted residuals, and smoothed weighted residual method. Their assessment reveals that the piecewise cubic polynomial curve fitting provides the smallest error in deriving the soil reaction, while the global fifth-order polynomial curve fitting gives the largest error.

Matlock (1958) performed sensitivity study on the effect of error in moment measurement in deducing soil reaction by the two methods presented in equations (3.18) and (3.19). His analysis results were presented in Table 3.7. His investigation showed that if the value of the bending moment at the central point, $M_i$, changes by 1%, the calculated soil reaction by equation (3.19) has smaller errors for a specific subinterval length, $\Delta z$. For instance, at $\Delta z = 6$ in (=152.4 mm), the 1% error in moment, $M_i$, at the central point (increasing from 773.52 in.-kips to 781.26 in.-kips) makes the computed soil reaction increase by 187% using equation (3.18) (the absolute value of soil reaction, $p_i$, increasing from 0.230 kips/inch to 0.660 kips/inch); or by 27% using equation (3.19) (the absolute value of soil reaction, $p_i$, increasing from 0.229 kips/inch to 0.291 kips/inch). The results also indicate that errors in soil reaction are relatively much greater than errors in bending moment.

Calculation of soil reaction by equation (3.19) requires successive five measured
bending moment points of equal spacing along the pile. The soil reaction, $p_i$ (i=3 to 8), corresponding to the same elevation where strain gauges were installed, can be calculated directly from the calculated bending moment, $M_i$ (i=1 to 10) shown in Figure 3.23(a). However, the determination of the soil reaction at the two points near the pile tip, $p_1$ and $p_2$ needs special treatment. Two fictitious bending moment data, $M_{11}$ and $M_{12}$ below the pile tip were introduced in order to construct the finite difference calculation as in the method suggested by Matlock and Reese (1960) and Gleser (1984). They can be determined from the boundary condition of shear force, $T(L)=0$ at the pile tip. From equation (3.17), taking $\Delta z = 50\text{mm}$, we can obtain $T(L) = \frac{M_1 - M_{-1}}{2\Delta z} = 0$, thus $M_{-1} = M_1$. Similarly, taking $\Delta z = 150\text{mm}$, we have $M_{-2} = M_2$. With the two fictitious points, the soil reaction at the two points, $p_1$ and $p_2$ can be conveniently calculated by equation (3.19).

3.5.8 Spreadsheet program for data processing

A spreadsheet program via Microsoft Excel VBA was written to process and analyse the measured data on the basis of the methods mentioned above. Pile response profiles of bending moment, shear force, soil reaction, rotation, and displacement can be deduced and plotted automatically for a single pile or a pile within a group at each frame movement. This program has greatly facilitated the analysis of test results.
3.6 REPEATABILITY

To check the repeatability of the tests using the triangular loading block, test T32-0 (L_m=350) was conducted under the same condition as test TD32-0. The test results at \( w_f = 120 \) mm were presented in Table 3.8. The maximum bending moment, \( M_{\text{max}} \), and maximum shear force, \( T_{\text{max}} \), from T32-0 (L_m=350) are 1.2% and 5% less than those from TD32-0. Additionally, a supplementary test RS50-294(S) was conducted to confirm the results of test RS50-294 reported by Guo and Qin (2005). The results of the two tests at \( w_f = 20 \) mm were also presented in Table 3.8. The \( M_{\text{max}} \) from RS50-294 is 6% less than that from RS50-294(S). The \( T_{\text{max}} \) in the sliding layer is about 180.5 N in the two tests, while the \( T_{\text{max}} \) in the stable layer in RS50-294 is about 20% less than that from RS50-294(S). The results of these tests show the repeatability of the tests.

3.7 SOIL MOVEMENT IN THE SHEAR BOX

In order to measure the sand movement on the soil surface, grid with each zone of 40 mm by 40 mm was made using coloured sand as shown in Figure 3.2. The distortion of the coloured grid gives an indication of the soil movement at the ground surface during the test.

To investigate the soil movement and the locations of failure surface in the shear box, coloured sand columns were inserted into the sand using a steel tube of 20 mm in
diameter at different locations in the shear box prior to testing. Each column extended over the full height of the sliding layer and into the underlying stable layer. The sand was carefully excavated around the columns after the completion of the tests to identify depths where the sand columns were sheared and try to map the slip surface. Difficulty was encountered during the excavation and measurement because of the sliding of the sand. Consequently, this method was abandoned in the following tests. Obviously, without pile, the current model tests are similar to a model wall test, if the boundary of the frame at the loading side is taken as a wall, with the wall being pushed into the sand. Therefore, the pattern of soil movement behind a wall associated with the passive earth pressure problem may provide insight into the soil movement in the shear box.

3.7.1 Soil movement associated with passive earth pressure

Research on the passive earth pressure problem has been carried out experimentally, analytically, and numerically by Roscoe (1970), Fang et al. (1994), Gudehus and Nubel (2004), Potts and Fourie (1986), Benmebarek et al. (2008) among others.

In the tenth Rankine lecture, Roscoe (1970) presented extensive experimental study and theoretical methods for solving passive pressure retaining wall problems carried out at Cambridge University in the 1960s. The strain field patterns behind a wall of different movement modes have been determined experimentally by radiographs. One of such radiographs is illustrated in Figure 3.24, showing the rupture surfaces induced
by a wall translated horizontally into dense sand with a horizontal free surface. Similar failure surfaces have also been obtained by Gudehus and Nubel (2004) in their experiments. According to these investigation, the sand behind the wall moves with the wall $AB$ (see Figure 3.24) in the beginning. A first rupture surface is formed horizontally from the wall base as the movement of the wall increases and the rupture zone grows progressively until finally the rupture zone extends to point $C$ on the free surface of the sand. Thereafter, while sliding continues on this primary rupture surface $BC$, the radial rupture surface develops.

Fang et al. (1994) investigated passive earth pressures with various wall movements. Their test results for a wall under translational movement show that at the ultimate state, the distance of the rupture surface to the wall is about $2.26$ times the wall height.

Lemnitzer et al. (2009) reported full-scale tests on bridge abutment wall pushed into abutment backfill sand. Small diameter gypsum columns were installed in the backfill prior to testing, which enable detailed mapping of ground deformation patterns within the backfill. Figure 3.25 shows the post trench with the coloured gypsum columns and the inferred failure surface marked by the dash line. Their test results show that the distance between the principal failure surface and wall is approximately $2.5 \sim 3.0$ times the wall height. They also concluded that the shape of the failure surface was approximately log-spiral near the wall, and it extended below the base elevation of the wall.
Based on the above research, it is reasonable to assume that the soil fails by the formation of a log-spiral failure surface in the shear box in the series of tests using a rectangular loading block. Furthermore, at the ultimate state, if the distance between the failure surface and the wall is taken as 2.5 times the wall height as an approximation, it means that the pile installed at the centre of the shear box will be just outside the failure zone in the tests with $L_{m}=200\text{m}$, as schematically shown in Figure 3.26. Thus, it is necessary to install the pile closer to the loading side so that relatively larger soil movement can be imposed on the pile. This has been achieved by conducting the groups of ‘pile location’, ‘diameter and distance’, and ‘loading block shape’ tests as discussed in section 3.4.1.

### 3.7.2 Limit equilibrium approach for estimating soil movement

As mentioned previously, regrettably, the soil movement in the shear box was not measured during testing in the current model tests due to technique limitations. This has significantly hampered the interpretation and analysis of the test results since the behaviour of the pile and the pile-soil interaction rely on the relative movement between the pile and soil.

Lesniewska and Mroz (2000, 2001, 2003) proposed a limit equilibrium approach to study the evolutions of shear band in sand retained by flexible wall in the course of progressive sand deformation. In their investigation, the active soil wedges or shear
bands development during excavation is simulated as a wall-spring-rigid blocks system shown in Figure 3.27. The behaviour of the soil adjacent to the wall is modelled by a set of rigid blocks, cut off from the soil body by lines parallel to the classical Coulomb wedge ‘slip line’ and sliding subsequently on each other (see Figure 3.27 (a)). The sand was assumed as a rigid-plastic softening material and the soil wedge motion is affected by the soil-wall elasticity and by soil softening expressed in terms of the friction angle decreasing with the slip along the active shear band. To account for the elastic compliance of the retaining wall, the soil-wall interaction is modelled by a set of elastic springs of uniform stiffness at the interface (see Figure 3.27 (c)). They have applied the model in the analysis of shear band pattern observed in dredged model tests on cantilever walls and shown that the model can provide realistic simulation of consecutive shear band formation. They concluded that the dominate parameter in this problem is the increasing height of excavation on one side of the wall, thus inducing the transition of the cantilever wall support and the associated variation of wall compliance.

Following the limit equilibrium approach proposed by Lesniewska and Mroz (2000), effort has been made to extend the model in Figure 3.27 to simulate the evolutions of soil movements and the formation of passive soil wedges between the frame boundary and the pile in the shear box induced by the progressive frame movements. Although this study have given some insight into the formation in the soil wedges qualitatively, accurate measurement of soil movement is required for precise prediction.
Furthermore, the assumption of rigid plastic soil behaviour makes the soil movement difficult to be connected with the responses of piles.

The formation of shear bands associated with strain localisation and bifurcation is a fundamental characteristic of granular material behaviour. Accurate estimation of the evolutions of shear bands in sand may require an adequate constitutive model, such as the hypoplastic model discussed by Gudehus and Nubel (2004), in numerical analysis. By bearing in mind that the focus of this study was on the responses of piles and the current testing conditions, endeavour was not pursued in this direction.
Table 3.1 Specifications for the hydraulic jacks

<table>
<thead>
<tr>
<th>Terms</th>
<th>Horizontal jack system</th>
<th>Vertical jack system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>ENERPAC RC-108</td>
<td>ENERPAC RC-59 (Single-acting)</td>
</tr>
<tr>
<td>Effective area</td>
<td>14.5 cm²</td>
<td>6.5 cm²</td>
</tr>
<tr>
<td>Capacity</td>
<td>101 kN</td>
<td>45 kN</td>
</tr>
<tr>
<td>Maximum stroke</td>
<td>150 mm</td>
<td>300 mm</td>
</tr>
<tr>
<td>Maximum operating pressure</td>
<td>700 bar</td>
<td>700 bar</td>
</tr>
<tr>
<td>Pump</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>ENERPAC P-392 Hand pump</td>
<td>ENERPAC Turbo II AIRハイドロパック PATG-1102N</td>
</tr>
</tbody>
</table>

Note: Force = hydraulic working pressure × cylinder effective area

Table 3.2 Properties of the quartz sand

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective particle size, $D_{10}$ (mm)</td>
<td>0.12</td>
</tr>
<tr>
<td>Coefficient of uniformity, $C_u$</td>
<td>2.92</td>
</tr>
<tr>
<td>Coefficient of curvature, $C_c$</td>
<td>1.15</td>
</tr>
<tr>
<td>Unit weight, $\gamma'$ (kN/m^3)$^1$</td>
<td>16.27</td>
</tr>
<tr>
<td>Relative density index, $D_r$</td>
<td>0.89</td>
</tr>
<tr>
<td>Internal friction angle, (degree)</td>
<td>38</td>
</tr>
</tbody>
</table>

^1 The falling height of sand was 600 mm.
### Table 3.3 Properties of the instrumented model piles

<table>
<thead>
<tr>
<th>Pile</th>
<th>Length (mm)</th>
<th>Outer diameter (mm)</th>
<th>Wall thickness (mm)</th>
<th>Bending rigidity $E_A p$ (kNmm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1200</td>
<td>25</td>
<td>1.6</td>
<td>$0.57 \times 10^6$</td>
</tr>
<tr>
<td>2</td>
<td>1200</td>
<td>32</td>
<td>1.5</td>
<td>$1.17 \times 10^6$</td>
</tr>
<tr>
<td>3</td>
<td>1200</td>
<td>50</td>
<td>2.0</td>
<td>$6.10 \times 10^6$</td>
</tr>
</tbody>
</table>

### Table 3.4 Specifications of strain gauges

<table>
<thead>
<tr>
<th>Type</th>
<th>KFG-5-120-C1-23L3M2R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature compensation for</td>
<td>Aluminium</td>
</tr>
<tr>
<td>Gauge length</td>
<td>5 mm</td>
</tr>
<tr>
<td>Gauge resistance (24°C,50% RH)</td>
<td>$120.4 \pm 0.4 \Omega$</td>
</tr>
<tr>
<td>Gauge factor (24°C,50% RH)</td>
<td>$2.14 \pm 1.0 %$</td>
</tr>
<tr>
<td>Adoptable thermal expansion</td>
<td>23.4 PPM/°C</td>
</tr>
<tr>
<td>Transverse sensitivity (24°C,50% RH)</td>
<td>0.4 %</td>
</tr>
<tr>
<td>Applicable gauge cement</td>
<td>CC-33A, EP-34B</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>Kyowa Electronic Instruments, CO. LTD, Japan</td>
</tr>
</tbody>
</table>
Table 3.5 Details of the single pile tests

<table>
<thead>
<tr>
<th>Test number</th>
<th>Test description</th>
<th>Outer diameter (mm)</th>
<th>Axial load (N)</th>
<th>Sliding layer depth (mm)</th>
<th>Stable layer depth (mm)</th>
<th>Test group</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TS32-0</td>
<td>32</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>TS32-294</td>
<td>32</td>
<td>294</td>
<td>200</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>TD32-0</td>
<td>32</td>
<td>0</td>
<td>350</td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>TD32-294</td>
<td>32</td>
<td>294</td>
<td>350</td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>TS50-0</td>
<td>50</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>TS50-294</td>
<td>50</td>
<td>294</td>
<td>200</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>TD50-0</td>
<td>50</td>
<td>0</td>
<td>350</td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>TD50-294</td>
<td>50</td>
<td>294</td>
<td>350</td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>TS32-0-340*</td>
<td>32</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td>Group 2: Pile location</td>
</tr>
<tr>
<td>10</td>
<td>TS32-0-660*</td>
<td>32</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>TS32-588</td>
<td>32</td>
<td>588</td>
<td>200</td>
<td>500</td>
<td>Group 3: Axial load</td>
</tr>
<tr>
<td>12</td>
<td>TS32-735</td>
<td>32</td>
<td>735</td>
<td>200</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>TS32-(22.5°)‡</td>
<td>32</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td>Group 4: Loading block angle</td>
</tr>
<tr>
<td>14</td>
<td>TS32-(30°)‡</td>
<td>32</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>T32-0 (L_m=125)</td>
<td>32</td>
<td>0</td>
<td>125</td>
<td>575</td>
<td>Group 5: Varying sliding depth</td>
</tr>
<tr>
<td>16</td>
<td>T32-0 (L_m=250)</td>
<td>32</td>
<td>0</td>
<td>250</td>
<td>450</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>T32-0 (L_m=300)</td>
<td>32</td>
<td>0</td>
<td>300</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>T32-0 (L_m=350)</td>
<td>32</td>
<td>0</td>
<td>350</td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>TS25-0-300</td>
<td>25</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td>Group 6: Diameter and distance</td>
</tr>
<tr>
<td>20</td>
<td>TS25-0-500</td>
<td>25</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>TS25-0-700</td>
<td>25</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>RS25-0-300</td>
<td>25</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td>Group 7: Loading block shape</td>
</tr>
<tr>
<td>23</td>
<td>RS25-0-500</td>
<td>25</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>RS25-0-700</td>
<td>25</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Test No. 25</td>
<td>No piles with triangular block</td>
<td>200</td>
<td>500</td>
<td>Group 8: Without piles</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Test No. 26</td>
<td>No piles with rectangular block</td>
<td>200</td>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>RS50-294(S)</td>
<td>50</td>
<td>294</td>
<td>200</td>
<td>500</td>
<td>Supplementary test</td>
</tr>
</tbody>
</table>

* Pile location, S_b=340 mm and 660 mm
‡ Loading block angle of 22.5°, 30.0°
### Table 3.6 Pile group configuration

<table>
<thead>
<tr>
<th>Pile group configuration</th>
<th>Test no.</th>
<th>Distance (mm)</th>
<th>Spacing (mm)</th>
<th>Pile head condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Two piles in a row</td>
<td>G1</td>
<td></td>
<td></td>
<td>Free–head</td>
</tr>
<tr>
<td></td>
<td>G2</td>
<td></td>
<td></td>
<td>Capped–head</td>
</tr>
<tr>
<td></td>
<td>G3</td>
<td></td>
<td></td>
<td>Free–head</td>
</tr>
<tr>
<td></td>
<td>G4</td>
<td></td>
<td></td>
<td>Capped–head</td>
</tr>
<tr>
<td></td>
<td>G5</td>
<td></td>
<td>S_b=500</td>
<td>Free–head</td>
</tr>
<tr>
<td></td>
<td>G6</td>
<td></td>
<td>S_b=660</td>
<td>Capped–head</td>
</tr>
<tr>
<td></td>
<td>G7</td>
<td></td>
<td>S_b=500</td>
<td>Free–head</td>
</tr>
<tr>
<td></td>
<td>G8</td>
<td></td>
<td>S_b=500</td>
<td>Capped–head</td>
</tr>
<tr>
<td></td>
<td>G9</td>
<td></td>
<td>S_b=500</td>
<td>Free–head</td>
</tr>
<tr>
<td></td>
<td>G10</td>
<td></td>
<td>S_b=500</td>
<td>Capped–head</td>
</tr>
<tr>
<td>(b) Two piles in a line</td>
<td>G11</td>
<td></td>
<td></td>
<td>Free–head</td>
</tr>
<tr>
<td></td>
<td>G12</td>
<td></td>
<td></td>
<td>Capped–head</td>
</tr>
<tr>
<td></td>
<td>G13</td>
<td></td>
<td>S_b=500</td>
<td>Free–head</td>
</tr>
<tr>
<td></td>
<td>G14</td>
<td></td>
<td>S_b=500</td>
<td>Capped–head</td>
</tr>
<tr>
<td></td>
<td>G15</td>
<td></td>
<td>S_b=500</td>
<td>Free–head</td>
</tr>
<tr>
<td></td>
<td>G16</td>
<td></td>
<td>S_b=500</td>
<td>Capped–head</td>
</tr>
<tr>
<td>(c) Four piles in a square</td>
<td>G17</td>
<td></td>
<td></td>
<td>Free–head</td>
</tr>
<tr>
<td></td>
<td>G18</td>
<td></td>
<td></td>
<td>Capped–head</td>
</tr>
<tr>
<td></td>
<td>G19</td>
<td></td>
<td>S_b=500</td>
<td>Free–head</td>
</tr>
<tr>
<td></td>
<td>G20</td>
<td></td>
<td>S_b=500</td>
<td>Capped–head</td>
</tr>
<tr>
<td></td>
<td>G21</td>
<td></td>
<td>S_b=500</td>
<td>Free–head</td>
</tr>
</tbody>
</table>

Note: The symbols ● indicate that the piles at the locations were instrumented. The pile diameter, \( d \) was 32mm. The predetermined final sliding layer depth, \( L_m \) and the stable layer depth, \( L_s \) were 200mm and 500 mm, respectively.
Table 3.7 Effect of error in test data on soil reaction (Matlock, 1958)

<table>
<thead>
<tr>
<th>Method</th>
<th>Strain gauges spacing in inches</th>
<th>Soil reaction, in kips per inch</th>
<th>Percentage change in computed soil reaction due to error in moment of central point $M_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Calculated from “exact” moments</td>
<td>With 1% error in moment of central point $M_i$ at 78-in depth</td>
</tr>
<tr>
<td></td>
<td>Δ$z$</td>
<td>$p_1$</td>
<td>$p_2$</td>
</tr>
<tr>
<td>Divided differences Eq. (3.16)</td>
<td>6</td>
<td>-0.230</td>
<td>-0.660</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>-0.229</td>
<td>-0.337</td>
</tr>
<tr>
<td>Least squares, 5-point cubic Eq.(3.17)</td>
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<td>-0.229</td>
<td>-0.291</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>-0.226</td>
<td>-0.242</td>
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</table>

Note: $Error = \frac{p_2 - p_1}{p_1} \times 100$

Table 3.8 Results of repeated tests

<table>
<thead>
<tr>
<th>Test description</th>
<th>Frame movement $w_f$ (mm)</th>
<th>Maximum bending moment $M_{max}$ (kNmm)</th>
<th>Depth of $M_{max}$ (mm)</th>
<th>Maximum Shear force $T_{max}$ (N)</th>
<th>Pile deflection $y_0$ (mm)</th>
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<tr>
<td>TD32-0</td>
<td>120</td>
<td>119.5</td>
<td>450</td>
<td>495.9</td>
<td>414.8</td>
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<td>T32-0 (L_m=350)</td>
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<td>118.1</td>
<td>475</td>
<td>471.7</td>
<td>406.7</td>
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<td>RS50-294</td>
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<td>450</td>
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<td>180.5</td>
</tr>
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<td>RS50-294(S)</td>
<td>20</td>
<td>47.8</td>
<td>450</td>
<td>179.7</td>
<td>180.4</td>
</tr>
</tbody>
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(b) Colored grid (40×40 mm)
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Diameter, d = 50 mm
Diameter, d = 32 mm
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Sealed gauges 100 mm
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(b) Cross section at a measurement level

Soil movement direction
strain gauges

SCXI-1314 Front-mounting terminal block

Figure 3.5 Instrumented model pile
Figure 3.6 Wheatstone bridge electrical circuit (Quarter Bridge) (Ghee 2010)

\[ e_0 = \frac{E}{4} K_s \cdot \varepsilon_0 \]

No temperature compensation; \( x1 \) output.

Figure 3.7 Wiring diagram for strain gauges

Quarter Bridge

Strain Gauge

SCXI-1314
Front-mounting terminal block
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4 RESPONSE OF RIGID PILES IN PROGRESSIVE MOVING SAND

4.1 INTRODUCTION

This chapter presented the results of the first to fifth group (see Table 3.5) of single pile tests subjected to soil movement. The test results are presented in the form of the profiles of bending moment, shear force and deflection along the pile; and the development of maximum bending moment, maximum shear force and pile displacement at the ground surface against frame movement. They are analysed in order to:

- show the behaviour of piles in progressively moving sand;
- examine the effect of the distance between the pile location and free soil movement source, axial load level, varying sliding depth, and loading block angle on the response of the piles;
- establish the relationship between maximum bending moment and maximum shear force;
- examine the development of the moment and shear force with progressively moving soil; and
- develop simple solutions for calculating response of passive piles and propose simple model for explanation of equivalent load for passive piles.
4.2 RESULTS OF GENERAL TESTS

The driving force during the installation of piles, the lateral force on the frames and the mobilisation of frames are presented and discussed in this section first. The results of the standard test TS32-0 are then presented. Finally, the responses of the piles at shallow and deep sliding depth are assessed.

The critical pile responses at typical frame movement are summarised in Table 4.1, including maximum bending moment, $M_{\text{max}}$, depth of the maximum bending moment maximum occurring, $z_{\text{max}}$, maximum shear force, $T_{\text{max}}$, and pile displacement at the ground surface, $y_0$.

4.2.1 Driving force

The jack-in forces for six typical tests were recorded during the installation of piles. They are plotted in Figure 4.1. This figure shows that the driving forces increase approximately linearly with the pile penetrations. At the final penetration of 700 mm, the average total forces of the same diameter piles reach 5.4 kN ($d = 50$ mm) and 3.8 kN ($d = 32$ mm), respectively, with a variation of $\pm 20\%$. This reflects the possible variations in model ground properties, as the jack-in procedure was consistent. (Note the axial load of 294 N on the pile head was 7% ~ 9% of the final jacking resistance.) The final average shaft friction for the installation is estimated as 54 kPa ($d = 32$ mm) and 49.1 kPa ($d = 50$ mm), respectively, ignoring the end-resistances and plug-in effect on the open-ended piles.
4.2.2 Lateral force on frames

Total lateral forces applied via the lateral hydraulic jack and loading block on the frames were recorded upon each 10 mm movement of the top frame (see Figure 3.11(c)). This measured top frame movement is designated as, $w_f$ in this thesis. The lateral forces against the frame movement, $w_f$ for the six tests are plotted in Figure 4.2. The lateral force measured in test No. 25 without pile (see Table 3.5) is also included for comparison. Examination of the figure reveals:

- the force in general linearly increases with the frame movement until it attains constant values;

- shear modulus of the sand $G_s$ is deduced as 15~20 kPa using the initial linear portion of each lateral force-frame movement curve. The maximum shear stress $\tau$ is estimated as 4.5~5.0 kPa ($=4.5~5.0$ kN on the loading block/shear area of 1.0 m$^2$). The maximum shear strain is evaluated as 0.25~0.3 (=$w_f/L_m$, with $w_f = 50~60$ mm and $L_m = 200$ mm), assuming the shear force is transferred across the sliding plane at the depth $L_m$ of 200 mm;

- the average overburden stress $\sigma_v$ for the sliding layer of 200 mm is about 1.63 kPa ($\approx16.27*0.1$). At this low stress level, sand dilatancy is evident and a number of ‘heaves’ (Figure 4.3(j)) were observed on the model ground surface associated with the progress of frames; and

- the lateral force attained a maximum of around either $w_f = 50~60$ mm (TS series) or 90~120 mm (TD series), and dropped slightly after these frame movements. The slight fluctuation in the measured force during large frame movement reflects
stress building up and being redistributed around the pile, and possibly adjustment of the shear box-pile-soil system, featured by the gradual formation of ‘heaves’. The pile response, however, attained the maxima at a higher $w_f$ of either 70–90 mm (TS series) or 120 mm (TD series), indicating a retardation of ~ 30 mm in $w_f$ in transferring the applied force to the piles. This delay in the development of maximum pile response is related to the mobilisation process of the frames induced by the triangular loading block, which is interpreted in the next section.

- The measured maximum force of 4.5 ~ 5 kN on the loading block in the TS series of tests is about 18% ~ 31% greater than the measured 3.8 kN in the test No. 25 without pile, depending on the pile diameter, which shows the reinforcement effect of the pile to lateral soil movement. The maximum lateral force in tests TD32-0 and TD32-294 is about 1.78 ~ 2.0 times greater than that measured in test without pile, indicating that larger sliding layer depth induces greater sliding force and requires larger reinforcement force.

4.2.3 Mobilisation of frames

The sequential frame movements are illustrated in Figure 4.3 by some photographs taken at typical $w_f$ during test TS50-294. The number of fully mobilised frames and the depth of soil movement induced at typical frame movement, $w_f$ is provided in Table 4.2 for both the TS and TD series of tests. The mobilisation of the frames in the TS series is explained as below for clarification.
(1) $w_f < 60 \text{ mm}$: Progressive mobilisation of frames downwards before pre-determined sliding depth ($L_m \leq 200 \text{ mm}$)

The advancement of the loading block not only induces lateral movement of the frames but also gradually mobilises the deeper frames. As indicated by Figures 4.3(b) and 4.3(c), increasing the frame movement, $w_f$ from 20 mm to 30 mm, forces the No. 4 frames to move. Consequently, the depth of moving soil increases to 100 mm ($= 25 \times 4$) since a total of 4 frames are mobilised. To quantify the impact of the depth of moving soil, a sliding depth ratio $R_L$ is introduced and defined as the ratio of thickness of moving soil depth over the pile embedment length. Thus, the sliding depth ratio $R_L$ increases with the frame movement $w_f$ in this stage and a $w_f < 60 \text{ mm}$ would correspond to a triangular profile to a sliding depth $L_m$ of 200 mm. It should be noted that this depth refers to the sliding of soil at the boundary near the loading block side. The movement of the soil immediately surrounding the pile remains unknown and may not up to this depth.

(2) $w_f = 60 \text{ mm}$: Frame movement at the onset of determined sliding depth ($L_m = 200 \text{ mm}$)

At $w_f = 60 \text{ mm}$ the predetermined sliding depth, $L_m$ of 200 mm was reached since the No. 8 frame was just mobilised by the loading block. If the boundaries of the mobilised frames at loading side are taken as a retaining wall, this state corresponds to that of the wall rotating around its toe about 16.7° ($= \arctan(60/120) = 16.7^\circ$). At this moment, the sliding depth ratio $R_L$ equals to 0.285 (i.e. $R_L = L_m/L = 200/700 = 0.285$).

(3) $w_f > 60 \text{ mm}$: Soil sliding depth remaining at the pre-determined sliding depth
Continuing advancement of the loading block and frames causes the 8 mobilised frames to move simultaneously and therefore a trapezoid frame movement profile was enforced (Figures. 4.3(e) ~ (h)). Increasing frame movement, \( w_f \), does not induce any increment of \( R_L \) since the depth of moving soil \( L_m \) remains 200 mm and hence \( R_L = 0.29 \).

### 4.2.4 Typical test results of TS32-0

Typical pile responses of the standard test TS32-0 are presented in Figure 4.4. As explained in chapter 3, this test was undertaken without axial load under a sliding layer depth \( L_m = 200 \) mm and a stable layer \( L_s = 500 \) mm thus a final sliding depth ratio of \( R_L = 0.29 \). These figures plot the bending moment, shear force, soil reaction, pile rotation and deflection profiles at each 10 mm of frame movement until \( w_f = 120 \) mm and the measured bending moment, \( M \), the deduced shear force, \( T \), pile deflection, \( y_0 \) and rotation angle at the ground surface associated with the progress of frame movement, \( w_f \).

Figure 4.4(a) shows that the bending moment profiles are analogous to a parabolic shape at \( w_f \geq 50 \) mm. The measured maximum bending moment, \( M_{\text{max}} \), occurs at a depth of 350 mm down the pile below the ground surface. From the shear force profiles in Figure 4.4(b) we see that the two largest local shear forces were obtained at depths of 250 mm and 550 mm. The free-headed pile deflected mainly by rotation.
around pile tip in Figure 4.4(c). The inconsistent behaviour of the pile displacement at $w_f = 60$ mm is attributed to the mobilization of the No. 8 frame (explained in section 4.2.3), which results in a substantial increment in the pile responses. As expected, the soil movement results in positive soil pressure on the pile above the sliding depth in Figure 4.4(c), showing the passive loading in the moving layer, while the soil in the middle part of the pile (from depth 200 mm to 550 mm) provides active resistance above the reverse point at a depth of 550 mm.

Figures 4.4(f) to 4.4(h) show that the frame (thus soil) movement at $w_f \leq 40$ mm causes negligible to little response of the pile. Then the pile responses increase rapidly with increasing frame movement, $w_f$ and reach the peak values at $w_f = 70$ mm. For example, the $M_{max}$ sharply rises from 2.7 kNmm ($w_f = 40$ mm) to 50 kNmm ($w_f = 70$ mm). Afterwards, they decrease slightly and remain more or less constant after $w_f > 70$ mm. At $w_f = 120$ mm, the pile deflection $y_0$ of 9.8 mm is significantly less than the frame movement. The actual movement of soil at the pile location maybe less than 120 mm, but it was observed that the sand at the ground surface had flowed around the pile.

4.2.5 Response at shallow sliding depth

Figure 4.5 provides the typical profiles of bending moment, shear force and pile deflection on the 32-mm-diameter pile with an axial load of 294N deduced from test TS32-294. Figure 4.4 and Figure 4.5 reflect the impact of enforcing the triangular
movement profile \((w_f \leq 60\, \text{mm})\) and that of trapezoid profile afterwards. Critical responses of maximum bending moment \(M_{\max}\), the depth \(z_{\max}\), and the pile deflection at ground surface \(y_0\) are obtained, such as

- At the maximum response state and without axial load (TS32-0): \(M_{\max} = 49.7\,\text{kNmm}\), \(z_{\max} = 370\,\text{mm}\), and \(y_0 = 10\,\text{mm}\). The pile mainly rotated about the pile tip.

- Imposing the axial load, the \(M_{\max}\) increase to 78.6\,\text{kNmm} (60% increase compared to without axial load) (see Figure 4.5). Negative bending moment was observed around the ground surface at the initial stage with \(w_f = 37\,\text{mm}\) \((R_L < 0.17)\). The pile rotated about a depth of 550~700 mm (pile-tip level), and induced a deflection \(y_0\) of ~13 mm (30% increase).

The largest response profiles occurred around \(w_f = 70~90\,\text{mm}\) for the two tested piles. They are plotted in Figures 4.6(a) and 4.6(b) together with those reported previously for the \(d = 50\,\text{mm}\) piles (Guo and Qin, 2006). The evolution of the maximum bending moment \(M_{\max}\) and the maximum shear force \(T_{\max}\) are plotted in Figures 4.7(a) and 4.7(b) against the frame movement \(w_f\) for the \(d = 50\,\text{mm}\) and 32 mm piles, respectively. The figures demonstrate that:

- a small thickness of moving soil with \((R_L < 0.17, \text{thus } L_m < 120\,\text{mm})\) did not render sand to move around the piles located at \(S_b = 500\,\text{mm}\). The initial frame movement \(w_f\) (denoted by \(w_i\) hereafter) of 37 mm causes trivial responses on each of the four tested piles. An effective frame movement, \(w_e\), should be \(w_f - w_i\) (mm);

- as the \(w_f\) increase from 37 to 80 mm \((R_L = 0.17 \sim 0.29)\), the \(M_{\max}\) for all tests increases proportionally, irrespective of axial loads, perhaps the \(w_f\) is prevailed by...
a triangular soil movement profile. At higher \( w_f (>80\text{--}90\text{ mm}) \), \( M_{\text{max}} \) maintains at a constant value and conforms to a trapezoid soil movement; and

- given \( w_f =37\text{--}80\text{ mm} \), the \( M_{\text{max}} \) in tests with axial load (e.g. TS32-294, and TS50-294) exhibits a ‘delayed’ stiff increase and attains a high ultimate value, compared to the pile without the load. The effect is even more remarkable in the TD series tests with deep sliding depth, shown subsequently. A 60% increase in the \( M_{\text{max}} \) owing to axial load of 294N for the 32 mm diameter pile is noted in comparison with the 30% for the 50 mm diameter piles.

The \( w_l \) captures the impact of the evolution of wedges induced by the loading block. For instance, at \( w_f =30\text{ mm} \), \( L_m =105\text{ mm} \), the lateral extent of the wedge is calculated as 215 mm (=105tan\((45^\circ+38^\circ)/2\)). This extent is vindicated by a number of ‘heaves’ mentioned earlier (see Figure 4.3 (j)). This figure also shows that the sand has flowed past the pile at the soil surface, resulting in constant pressure acting on the pile, which also implies that the sand along the potential failure surfaces in the shear box may have attained critical state. The correlation between maximum shear force \( T_{\text{max}} \) and the \( M_{\text{max}} \) is discussed later.

### 4.2.6 Response at deep sliding depth

Figures 4.8 and 4.9 provide the response profiles obtained at the deepest pre-determined sliding depth of 350 mm using the triangular loading block (TD32-0 and TD32-294). Without axial load, the 32 mm diameter pile principally rotated about the middle pile embedment, and the deflection \( y_0 \) reached 46 mm at \( w_f =110\text{ mm} \).
Imposing the axial load of 294N (TD32-294), the pile translated and rotated around the pile tip and the $y_0$ reached 62.5 mm. The moment and shear force profiles for the 32 mm piles and for the 50 mm piles at maximum state ($w_f = 120$ mm) are depicted in Figures 4.10(a) and 4.10(b), respectively. The evolution of the maximum bending moments and shear forces with the advance of the frame movement, $w_f$ is illustrate in Figure 4.11(a) and 4.11(b). These figures show the following features:

- reaction from the 50 mm piles is negligible within a $w_i$ of 30 mm, which is slightly less than 37 mm for the 32 mm diameters piles;

- the axial load causes nearly constant bending moment down to a depth of 200 mm in the upper sliding layer (see Figure 4.10(a)) below which, the distribution of bending moment with depth resembles that from TS tests Figure 4.6(a). It also renders the $M_{max}$ increase to $\sim 143$ kNmm that occurs at a depth $z_{max}$ of 0.465 m. The development of constant bending moment in the upper sliding layer may be related to the high free-stand length (500mm) of the pile above the soil surface. Under the soil movement, the pile displaces as shown in Figure 4.9 (c). Then the dead weights applied on the top of the pile generate a moment, $M_0$ at the soil surface. This is the so called $P \cdot \Delta$ effects, i.e. axial load inducing extra bending moment. Because the low lateral resistance provided by the soil near the surface, this moment has to be transferred to a deep depth in the soil. Comparison of the bending moment profiles in Figure 4.10 (a) indicates that the moment, $M_0$ may dominate the pile bending stress in approximate 200 mm depth below the soil surface. The results of test TD32-294 in Figure 4.9 indicates that at $w_f = 120$mm,
the additional bending moment at the soil surface due to $P \cdot \Delta$ effects is about 21.7 kNmm (294 N times the pile displacement at the soil surface of 73.8 mm), which is very close to the measured bending moment $M_0 = 19.4$ kNmm. This $P \cdot \Delta$ effects is approximately 16% of the maximum bending moment, $M_{\text{max}}$ of 124.6 kNmm measured in the stable layer for the pile undergoing soil movement. This illustrates that the $P \cdot \Delta$ effects induced by the 294 N is of secondary importance to the soil movement on the pile response in this test.

- The thrust $T_{\text{max}}$ and $M_{\text{max}}$ in the pile will in general attain higher values than those obtained in the figures, as the frame movement $w_f$ of 120 mm just mobilises a sliding depth $L_m$ 350 mm (see Table 4.2).

4.3 RESULTS OF PARAMETRIC STUDY TESTS

In this section the results of the second to fifth groups of the parametric study tests (see Table 3.5) are presented in a similar way to the general tests. The critical pile responses are also summarised in Table 4.1.

4.3.1 Effect of pile location in the shear box

The effect of the distance from soil movement source at the loading side to the pile location was investigated in ‘pile location’ test series. The 32-mm-diameter pile was installed at a distance $S_b$ of 340 mm (TS32-0-340), 500 mm (TS32-0) and 600 mm (TS32-0-660) from the loading side with a pre-specified sliding depth of 200 mm (see
Figure 3.8(b)). The pile was installed at the centre of the shear box at $S_b = 500$ mm. The pile responses for the three tests are shown in Figure 4.12. The pile behaviour in tests TS32-0-340 and TS32-0-660 are similar as those observed in TS32-0. Figure 4.12(a) shows that negligible pile responses were induced at $w_f < 37$ mm ($R_L < 0.17$) and the gradient of the linear segment of the $M_{\text{max}} \sim w_f$ curves at $w_f = 40$–80 mm decreased with the increasing distance $S_b$.

The peak values of $M_{\text{max}}$ are plotted in Figure 4.13(a) against the distance $S_b$. A reduction of $\sim 32$ kNm in $M_{\text{max}}$ is obtained as the pile was relocated from $S_b = 340$ mm to 500 mm, while a reduction of $\sim 10$ kNm from $S_b = 500$ mm to 600 mm. The reason for the reduction of the $M_{\text{max}}$ is due to the fact that the soil pressure transferring from the loading side decreases with the distance, $S_b$ owing to the shear and friction in the sand. Figure 4.13(b) plots the maximum soil reaction profiles for the three tests. The maximum soil pressure, $p$ in the sliding layer decreases about 67% from approximately 1.2 N/mm to 0.4 N/mm, as $S_b$ increases from 340 mm to 660 mm, i.e., pile installed further away from the loading side. The initial frame movements, $w_i$ are plotted in Figure 4.13(c) against the distance $S_b$. It can be seen that changing the pile location in the shear box does not have significant impact on the initial frame movements $w_i$.

Apparently, the movement of frames induced passive failure zones when they were pushed into the sand. This has been justified by the failure wedges observed in front
of the frames in the shear box. Five wedges characterised by ‘sand heaves’ were observed on the ground surface with the furthest one measured about 500 mm from the loading block side as shown in Figure 4.3(j). The location of this wedge on the ground surface was originally 650 mm (=500+150, mm) away from the loading side. The soil movement field at the ground surface indicates that sand may have flowed around the pile within the failure zone. Beyond the failure zone, however, the ground surface remained intact and no sand movement was observed (see Figure 4.3(j)). In test TS32-0-340, the pile was 340 mm away from the loading block side and within the failure zone, whereas in test TS32-0-660 the pile was 660 mm away from the loading side and lay outside the failure zone even at a very large frame movement of 150 mm. The attenuation of the magnitude of soil movement from the loading side to the pile location may contribute to the reduction in the maximum bending moment, due to the non-uniform mobilisation of soil movement across the shear box.

4.3.2 Effect of axial load

The effect of the magnitude of axial load applied at the pile head on the pile response was examined in the third group of ‘axial load’ tests by varying the axial load between 0N and 735N. Two additional tests TS32-588 and TS32-735 were conducted with an axial load of 588N and 735N, respectively. Along with TS32-0 and TS32-294, they enable the responses of axial loaded piles subjected to lateral soil movement to be preliminarily assessed. The results of this group are plotted in Figure 4.14. At the ground surface the axial load causes a small bending moment due to the $P \cdot \Delta$
effects as explained previously, generally less than 20% of the maximum bending moment, $M_{\text{max}}$, but it does not change the bending moment and shear force profiles and the evolvement pattern of $M_{\text{max}}$ and $T_{\text{max}}$ with frame movement $w_f$. The pile deflected mainly by rotation about the pile tip in TS32-0 and TS32-294, but in TS32-735 the pile rotated at a depth of 500 mm (about 0.7$L$) below the ground surface and the pile tip ‘kicked out’ about 3.8 mm in the opposite direction. The axial load applied at pile head generally enlarges the critical pile responses. An increase in the axial load from 0 N to 735N on the pile head leads to: (1) an 80% increase in $M_{\text{max}}$; (2) an 80% and 37% increase in $T_{\text{max}}$ in the stable layer and sliding layer, respectively.

4.3.3 Effect of loading block angle

In order to examine the dependency of the pile behaviour on the loading block angle $\theta$, another two loading blocks were made with an angle of 22.5° and 30°, respectively (see Figure 3.10). Tests TS32-0 (22.5°) and TS32-0 (30°) were conducted using block 2 ($\theta = 22.5°$) and block 3 ($\theta = 30°$), respectively, under the same conditions as the ‘standard’ test TS32-0. The results of this group of ‘loading block angle’ tests are presented in Figure 4.15. Similar characteristics are displayed for the three tests. Nevertheless, Figure 4.15(a) shows that the initial frame movement $w_i$ caused little bending moment on the pile increases to $w_i = 50$ mm in TS32-0 (22.5°) and $w_i = 80$ mm in TS32-0 (30°). These values are 1.35 and 2.16 times the 37 mm in the standard test TS32-0.
The depth of soil movement with the frame movement using the two new loading blocks is also provided in Table 4.2. They are plotted in Figure 4.16(a). Changing the loading block 1 ($\theta = 15^\circ$) to block 2 ($\theta = 22.5^\circ$) or block 3 ($\theta = 30^\circ$) leads to the measured frame movement $w_f$, upon which the pre-determined final depth $L_m = 200$ mm was just attained, increasing from 60 mm (TS32-0 (15$^\circ$)) to 90 mm (TS32-0 (22.5$^\circ$)) or 110 mm (TS32-0 (30$^\circ$)), respectively. In the extreme case of $\theta = 0^\circ$, a triangular loading block degrades to a rectangular one that generates a uniform translational frame movement pattern as discussed in next chapter. The peak values of $M_{max}$ and the initial frame movement, $w_i$ are plotted in Figure 4.16(b) against the loading block angle, $\theta$. Value of $M_{max}$ for $\theta = 0^\circ$ was obtained from test RS32-0 reported by Guo and Ghee (2005). The figure shows that the peak values of $M_{max}$ and $w_i$ strongly depend on the loading block angle, $\theta$, and can be approximately evaluated by the following expressions

$$M_{max} = 1.4*\theta + 25$$

(4.1)

$$w_i = 2.8*\theta$$

(4.2)

where $M_{max}$ is peak value of maximum bending moment (kNmm), and $\theta$ is the loading block angle (degree).

As reviewed in Section 2.3.3.1, Cai and Ugai (2003) studied the response of flexible piles under linearly varying soil movement profiles (inverse triangular distribution with zero movement at the base of sliding layer) by using an analytical method. They demonstrated that increasing the inclination, $\theta_0$ between the pile axis and the soil
movement profile shown in Figure 2.23 can lead to higher maximum bending moment in the stable layer. However, it must be emphasised that the angle $\theta_0$ in their study refers to the inclination of the soil movement profile at the pile location, which is different from the angle of the loading block in the current tests.

Chen (1994) conducted similar laboratory tests to study the effect of lateral soil movements on pile foundations. The details of his investigation have been reviewed in section 2.3.5.1. Figure 4.17 plots the peak values of $M_{\text{max}}$ against loading block angle $\theta$ obtained from the current tests and those deduced from the model tests conducted by Chen (1994). The current tests were conducted in progressive moving sand by using a triangular block having a fixed angle $\theta$. Chen’s tests were originally conducted to investigate the effect of $L_m$ on bending moment with fixed $L_s$. They are reassessed and interpreted in terms of boundary rotational angle, $\theta$ as his tests may be seen as conducted by rotation of a wall about its toe. The rotation angle of the wall was calculated from dividing the soil surface movement $w_f$ by the sliding layer depth $L_m$. Chen’s test results show that the maximum bending moment $M_{\text{max}}$ increases with the increasing rotation angle $\theta$. At a specific rotation angle $\theta$, increasing the sliding layer depth, $L_m$ results in larger maximum bending moment $M_{\text{max}}$ when the stable layer depth, $L_s$ remains constant. For instance, at $\theta = 5^\circ$ and $L_s = 325$ mm, $M_{\text{max}}$ increases from 3.63 kNmm to 28.8 kNmm as $L_m$ increases from 200 mm to 350 mm. These test results are further used to confirm the relationship between maximum bending moment, $M_{\text{max}}$ and maximum shear force, $T_{\text{max}}$ in chapter 5.
4.3.4 Effect of varying sliding depth

In order to evaluate the effect of variation of sliding layer depth $L_m$ on the pile response, four tests (tests 15 to 18 in Table 3.5) were conducted on the $d = 32$ mm piles without axial load to a pre-determined final sliding depth of $L_m = 125$ ($R_L = 0.179$), 250 (0.357), 300 (0.429) and 350 (0.50) mm. The results of this group of ‘varying sliding depth’ tests are plotted in Figure 4.18. The magnitudes of $M_{max}$ are 5.2, 62.6, 115.3, and 118.1 kNmm at the frame movement upon which the pre-determined $L_m$ was attained. They finally reached 5.7, 123.5, 175.0, and 140.0 (not yet to limit) kNmm, respectively.

To examine the effect of progressively moving sand on the pile, the sliding depth ratio, $R_L$, was calculated from the observed frame movement, $w_f$, and provided in Table 4.2. The evolution of $M_{max}$ for the eight general tests with the normalised sliding depth $R_L$ is given in Figure 4.19(a). It shows three distinct stages:

- a small value of $M_{max}$ at $0 \leq w_f \leq 37$ mm ($R_L < 0.17$);
- the linear increase in $M_{max}$ owing to the triangular movement profile with $37$ mm $\leq w_f \leq 60$ mm ($L_m = 200$ mm, $R_L = 0.29$) or with $37$ mm $\leq w_f \leq 120$ mm ($L_m = 350$ mm, $R_L = 0.50$); and
- the moment rises at either $R_L = 0.29$ or 0.50 caused by uniform frame movement beyond the triangular movement.

The $M_{max}$ of the group of ‘varying sliding depth’ tests are plotted in Figure 4.19(b) against the $R_L$, together with the results of test TS32-0 ($R_L = 0.29$). The peak values of
$M_{\text{max}}$ obtained at the five sliding depth ratios are connected, as shown by the peak $M_{\text{max}}$ enveloping curve in Figure 4.19(b). This curve serves as an envelope covering the measured data from the tests conducted on the 32-mm piles in this study. Note that the $M_{\text{max}}$ and $T_{\text{max}}$ from T32-0 ($L_m = 350$ mm) at $w_f = 120$ mm are 1.2% and ~5% less than those obtained from TD32-0, showing the repeatability and accuracy of the current tests.

Poulos et al. (1995) presented the results of similar model tests on a single pile (25 mm in diameter) embedded in calcareous sand subjected to lateral soil movement. In contrast with the current tests with constant pile embedment depth $L$, the model tests in their experimental investigation were conducted under varying sliding layer depths $L_m$ (from 200 mm~350 mm) but a fixed stable layer depth $L_s$ (=325 mm), thus varying pile embedment depth, $L$ by imposing a triangular profile of lateral soil movement on the model pile. The lateral soil movement was generated by rotating a steel plate about a fixed sliding depth in a container. They also conducted another series of seven tests under a fixed sliding layer depth $L_m = 350$ mm and varying stable layer depth $L_s$ (from 250 mm~325 mm) to examine the combined effects of $L_m$ and $L_s$ on the pile response at sliding depth ratio $R_L > 0.5$. Their test results are also plotted in Figure 4.19(b) for comparison. It is seen from their test results that the $M_{\text{max}}$ increases with sliding depth ratio $R_L$ ($0.38 \leq R_L \leq 0.52$) and reaches its peak value at $R_L = 0.52$, thereafter decreases gradually when $R_L$ is greater than 0.52. The variation of $M_{\text{max}}$ versus sliding depth ratio $R_L$ in the current tested range of $R_L = 0$~0.5 is consistent with
the findings obtained from Poulos et al. (1995). However, their test results did not disclose the features of pile behaviour shown in the third stage of the current investigation, i.e. $M_{max}$ increasing at the final $R_L$ due to the translation of the frames. This may be due to the different soil movement patterns induced in the two experimental investigations.

4.4 RELATIONSHIP BETWEEN $M_{max}$ AND $T_{max}$

The calculated maximum shear force in the stable layer, $T_{max}$ are plotted against the measured maximum bending moment, $M_{max}$ in Figure 4.20(a) for the eight general tests and Figure 4.21(a) for the 10 parametric tests. They are linearly correlated and can be approximated by

\[
M_{max} = 0.357T_{max}L \quad \text{or} \quad M_{max} = T_{max}L/2.8 \tag{4.3}
\]

where $L$ (= 0.7 m ) is the pile embedment length. This relationship is useful because the maximum shear force and maximum bending moment are required in the design of reinforcing piles to increase slope stability.

Figures 4.20(b) and 4.21(b) plot the maximum shear force in the sliding layer, $T_{max2}$ against the maximum bending moment, $M_{max}$ for these tests. Again, approximately linear relationships were obtained between the $M_{max}$ and $T_{max2}$, giving

\[
M_{max} = 0.385T_{max}L \quad \text{or} \quad M_{max} = T_{max}L/2.6 \tag{4.4}
\]

Depending on the sign convention, the coefficient for best fitting between the
maximum bending moment, $M_{\text{max}}$, and maximum shear force differs by less than 8%, regardless of the sign of the maximum shear force.

### 4.5 Calculation of $M_{\text{max}}$ and $T_{\text{max}}$

#### 4.5.1 Guo and Qin (2010) solutions

Guo and Qin (2010) proposed simple solutions to estimate the evolution of maximum bending moment, $M_{\text{max}}$, and maximum shear force, $T_{\text{max}}$, with the frame movement, $w_f$, observed in the model tests. The solutions are directly used for the current analysis. The development of Guo and Qin (2010) solutions are explained below.

1. The solutions were developed on the basis of the test results presented in section 4.2 and 4.3;

2. The analytical solutions for rigid piles in elastic foundations presented by Scott (1981) were used to study the passive piles due to lateral soil movement. The pile displacement at ground surface $y_0$ for a laterally loaded free-head rigid pile can be calculated using the elastic solutions with constant modulus of subgrade reaction from

$$y_0 = \frac{4T_i}{(kL)}$$

$$M_{\text{max}} = 0.148T_iL$$

in which $T_i$ is the applied lateral load at the ground surface, $k$ is the modulus of subgrade reaction, $L$ is the pile embedment length, $M_{\text{max}}$ is the maximum bending moment. (Scott (1981) solutions have been summarized in Table 2.6.)
(3) The maximum shear force $T_{\text{max}}$ was used as the equivalent load, $T_i$ in equation (4.6) for passive piles owing to lateral soil movement. The use of equivalent load is further explained in section 4.6 in this thesis.

(4) The equations (4.3) and (4.4) were used to correlate the maximum shear force with the maximum bending moment.

(5) The maximum shear force and maximum bending moment were calculated from

\[ T_{\text{max}} = (w_f - w_i)kL/4 \]  
\[ M_{\text{max}} = (w_f - w_i)kL^2/(10 \sim 27) \]

where $w_f$ is the frame movement, $w_i$ is the initial frame movement that depends on loading manner and pile-soil relative movement. The modulus of subgrade reaction $k$ is deduced from the overall sliding process characterised by sand-pile-shear box interaction. The denominator values of 10 ($\approx 4/0.385$) and 27 ($= 4/0.148$) correspond to the current model test data in sliding layer and the elastic case of constant $k$ in the solution by Scott (1981). A value of 11.2 ($= 4/0.357$) should be used in the calculation of $M_{\text{max}}$ if the maximum shear force in the stable layer is correlated to $M_{\text{max}}$.

Guo and Qin (2010) solutions were developed with three hypotheses.

(1) The initial frame movement, $w_i$ using the triangular loading block causes very small reaction in the piles.

(2) The effective frame movement, $w_f - w_i$ causes linear increasing pile displacement during the passive loading process, which is based on the data in Figures 4.7(c),
4.11(c), 4.12(c), 4.14 (c), 4.15 (c) and 4.18 (c). The pile deflection can be estimated from the effective frame movement of \((w_f - w_i)\) using curve fitting by

\[
y_0 = (0.19\sim1.0)(w_f - w_i)
\] (4.9)

Any pile soil relative rigid movement was simply incorporated into the initial frame movement, \(w_i\) and modulus of subgrade reaction, \(k\).

(3) The calculated \(T_{\text{max}}\) and \(M_{\text{max}}\) must be capped by the values deduced using ultimate state via elastic-plastic solutions, otherwise unrealistic higher values can be obtained.

4.5.2 Example calculations using Guo and Qin (2010) solutions

Guo and Qin (2010) provide example calculation of the \(M_{\text{max}}\) and \(T_{\text{max}}\) for the current model tests and the model tests (Poulos et al., 1995). They are explained below.

(1) TS and TD general tests: The measured curves of \(M_{\text{max}} ~ w_f\) and \(T_{\text{max}} ~ w_f\) (see Figures 4.7 and 4.11) were simulated regarding the pre-specified final sliding depths of 200 mm (TS tests) and 350 mm (TD tests), respectively. Elastic theory offers (Guo, 2008): \(k/G_s = 2.841 (d = 50 \text{ mm})\) and \(2.516 (d = 32 \text{ mm})\). The \(G_s\) was deduced as 15~21 kPa from the overall response of sand-pile-shear box interaction, thus the \(k\) was obtained as 45~60kPa. Given \(w_f = 30 \text{ mm} (\text{TD50-294})\) and \(w_f = 37 \text{ mm} (\text{TS50-294})\) the maximum bending moment and shear force are thus calculated using

\[
M_{\text{max}} = (w_f - w_i)kL^2/11.2\text{ and } T_{\text{max}} = (w_f - w_i)kL/4.
\]

They are plotted in Figures 4.7 and 4.11 using solid lines and agree well with measured data. As the pile diameter is changed to 32...
mm, the $k$ reduces to 25–35 kPa, because of its proportional reduction to the diameter (resulting in 28.8–38.4 kPa), and further to a ratio of 2.516/2.841. The values of $k$ offer good estimations for the $d = 32$ mm tests as well, as shown in the figures.

(2) Translational loading with variable sliding depths (constant $L$): The measured $M_{max}$ of piles in TD32-0 and T32-0 ($L_m = 350$) tested to a final sliding depth of 350 mm is presented in Table 4.3. It was modelled using $M_{max} = (w_f - 0.037)kL^2/11.2$, and $k = 35$ kPa. The estimated $M_{max}$ for a series of $w_f$ (or $R_L$) are also provided in Table 4.3 and plotted in Figure 4.19(a). The $R_L$ was based on actual observation during the tests, which may be slightly different from those calculated by $R_L = 3.3w_f/L$. The same calculation is also plotted in Figure 4.19(b). Especially the moment increase at $R_L = 0.5$ (for $w_f > 120$ mm) was estimated using an additional movement of 30 mm (=150-120 mm) beyond the $w_f$ of 120 mm to show the capped ultimate state. Table 4.3 shows that the calculated values agree with the two sets of measured $M_{max}$, in view of using the same $w_f$ of 37 mm for either test.

(3) Rotational loading about a fixed sliding depth: The $M_{max}$ was obtained in model pile tests by loading with rotation about a fixed sliding depth (thus a typical $R_L$) (Poulos et al., 1995). The results for a series of $R_L$ were depicted in Figure 4.19(b) and tabulated in Table 4.4. The measured $M_{max}$ is simulated via the following steps:

- The ratio of $k/G_s$ was obtained as 2.39–2.79 using the expression given by Guo and Lee (2001), which itself is a function of a factor $\gamma (=1.05d/L)$. 
• Shear modulus was calculated approximately by \( G_s = 10z \) (\( G_s \) in kPa, \( z = L_s + L_m \) in m), from the Young’s Modulus \( E_s = 0.025z \) (in MPa, where \( z \) is the depth below the soil surface, in mm) and the Poisson’s ratio \( \nu = 0.3 \) (Poulos et al., 1995). Then the \( k \) was calculated from the shear modulus \( G_s \).

• With \( w_i = 0 \) (as observed), the \( T_{\text{max}} \) was estimated using equation (4.7) for \( w_f = 37 \) mm \((R_L < 0.5)\) or 60 mm \((R_L > 0.5)\), respectively.

• The \( M_{\text{max}} \) was calculated by \( M_{\text{max}} = w_f k L^2 / 10 \) as per equation (4.8).

The tested piles are of lengths 375 ~ 675 mm, and the \( G_s \) was 3.75 ~ 6.75 kPa. The values of \( M_{\text{max}} \) calculated for the 10 model piles are also provided in Table 4.4. They are plotted against the ratio \( R_L \) in Figure 4.19(b). The simple calculation approach provides satisfactory estimation for the 10 tests.

Guo and Qin (2010) concluded that the 3 ~ 5 times difference in the \( M_{\text{max}} \) between the current model tests and those in Poulos et al. (1995) is likely owing to the impact of pile dimensions, the subgrade reaction modulus, \( k \), the effective movement, \( w_f - w_i \), and the loading manner.

4.5.3 Further verification

To further verify the validity of Guo and Qin (2010) simple solutions, calculations were also undertaken for the parametric study tests presented in section 4.3. In performing the estimation, the following assumptions are made.

• The initial frame movement \( w_i \), causing little reaction on the pile, is estimated
based on the measured curves of $M_{\text{max}} \sim w_i$ (Figures 4.12 (a), 4.14 (a), 4.15 (a) and 4.18 (a)). For instance, $w_i = 37$ mm for test T32-0 ($L_m = 300$), while it may be taken as 89 mm for test TS32-0 (30°).

- The $M_{\text{max}}$ is correlated to $T_{\text{max}}$ given by equation (4.3).
- $k = (2.4~3) \, G_s$, $G_s$ is deduced from the overall shear process of the pile-soil-shear box system.

The calculated $M_{\text{max}}$ were plotted in Figures 4.12 (a), 4.14 (a), 4.15 (a) and 4.18 (a) using solid lines. The values of $k$ for each test are summarised in Table 4.1. The following conclusions are made from the estimation:

- The modulus, $k$ reduces by 54% as the distance $S_b$ increases from 340 mm to 660 mm.
- The loading block angle may only affect the initial frame movement $w_i$, but not the $k$.
- The variation of sliding depth ratio $R_L$ from 0.179 to 0.50 does not significantly affect the initial frame movement $w_i$, and the deduced $k$ falls in a range of 38 ~ 45 kPa, varying less than ±10% than the 42kPa obtained in the standard test TS32-0.
- The increasing of $k$ in test TS32-588 and TS32-735 may be attributed to the additional bending moment generated by the axial load.
- The calculated $M_{\text{max}}$ and $T_{\text{max}}$ must be kept within the bounds of the peak values, for instance, as shown in Figure 4.19(b).
4.6 A SIMPLE MODEL EXPLAINING THE USE OF EQUIVALENT LOAD

In view of the significance of the use of equivalent load to the analysis of the test results, interpretation and development of the simple solutions presented below, a simple conceptual model is proposed to explain the use of equivalent load for passive piles in the current model tests.

The soil reaction profiles in Figures 4.4 (e) and 4.5 (e) appear to be complicated. No attempt was made herein to study the soil reaction profiles. Instead, it is worth noting that the bending moment profiles in Figures 4.4 (a) and 4.5 (a), and the shear force profiles in Figures 4.4 (b) and 4.5 (b) are very similar to the bending moment and shear force profiles of a rigid pile subjected to lateral soil movement as shown in Figure 4.22. This figure illustrates a schematic of a rigid pile undergoing lateral soil movement with the top layer of soil of thickness, \( L_m \), sliding on a firm underlying soil of thickness, \( L_s \), along a slip surface.

By means of limit equilibrium analysis, the maximum shear force, \( T_{max1} \) and \( T_{max2} \), and the maximum bending moment, \( M_{max} \), in Figure 4.22 can be found:

\[
T_{max2} = p_2 L_m \quad (4.10a)
\]

\[
T_{max1} = p_3 (L - z_1) = p_1 (z_1 - L_m) - p_2 L_m \quad (4.10b)
\]

\[
M_{max} = \frac{T_{max2} z_{max}}{2} \quad (4.10c)
\]

where \( p_1, p_2 \) and \( p_3 \) are the soil reaction on the three portions of the pile; \( z_1 \) is the
depth at which the maximum shear force, $T_{max1}$ occurs; $z_{max}$ is the depth of maximum bending moment. The maximum shear force $T_{max2} (= p_2 L_m)$ may be taken as the total passive loading in the sliding layer, the force ($= p_1(z_1 - L_m)$) is the active resistance above the reversal point in the stable layer, $T_{max1}$ is the ‘reversed’ loading due to “kicking out” of the pile tip. Comparison of the bending moment and shear forces profiles in Figures 4.4 and 4.22 indicates that the maximum bending moment, $M_{max}$ and maximum shear force, $T_{max1}$ and $T_{max2}$ in the current model pile tests are likely to be relatively insensitive to the precise distribution of soil reaction. This behaviour of the pile has also been noted in their evaluation of the bending moment of piles subjected to lateral spreading by Dobry et al. (2003) and assessment of full-height piled bridge abutments constructed on soft clay by Ellis and Springman (2001). Thus, the maximum bending moment, $M_{max}$ and maximum shear force, $T_{max1}$ and $T_{max2}$ in the current model pile tests could be estimated using the assumed soil reaction profile shown in the simple model.

Relying on the simple model in Figure 4.22, the relationship between the maximum bending moment and maximum shear force can be established in two approaches.

**4.6.1 Approach based on test results**

As a first approximation, the statistical analysis of $z_{max}$ of the 18 tests in Table 4.1 gives $z_{max} = (325 \sim 475)$ mm, with an average value of 410 mm. Thus the average ratio $z_{max}/L = 410/700 = 0.586$. Substituting $z_{max} = 0.586L$ into equation (4.10c) leads to
This constant 0.293 is approximately 24% less than the 0.385 in equation (4.4) obtained from the experimental results discussed in section 4.4.

4.6.2 Approach based on limit equilibrium analysis

Alternatively, the passive loading on the pile in the sliding layer, \( p_2 L_m \) may be replaced by a concentrated lateral load, \( T_{\text{max}2} \), applied at an eccentricity, \( e \) above the sliding surface as shown in Figure 4.23. The static equilibrium expressions of the pile derived from Figure 4.23 are presented below.

Horizontal force equilibrium:

\[
T_{\text{max}2} = p_1 z_0 - p_3 (L_s - z_0) \quad (4.11)
\]

Moment equilibrium about pile tip:

\[
T_{\text{max}2} (e + L_s) + p_3 (L_s - z_0)^2 / 2 - p_1 z_0 (L_s - z_0 / 2) = 0 \quad (4.12)
\]

Substituting \( T_{\text{max}2} \) in equation (4.11) into equation (4.12) allow \( z_0/L_s \) to be determined as

\[
\frac{z_0}{L_s} = -\frac{e}{L_s} + \sqrt{\left(\frac{e}{L_s}\right)^2 + \frac{2 p_3 e}{p_1 + p_3 L_s} + \frac{p_3}{p_1 + p_3}} \quad (4.13)
\]

where \( z_0 = z_1 - L_m \), is introduced for convenience in presentation, and \( e = L_m / 2 \). The maximum bending moment, \( M_{\text{max}} \) occurs at a depth \( z_{m0} (=z_{\text{max}} - L_m, \text{see Figure 4.23}) \) below the sliding surface at which point the shear force is zero, i.e. \( T(z_{m0}) = T_{\text{max}2} - p_1 z_{m0} = 0 \), or \( z_{m0} = T_{\text{max}2}/p_1 \). Along with equation (4.11), we can obtain

\[
\frac{z_{m0}}{p_i} = \frac{T_{\text{max}2}}{p_i} = \frac{(p_1 + p_3) z_0 - p_3 L_s}{p_1} \quad (4.14)
\]
Thus, the maximum bending moment, $M_{\text{max}}$ for the uniform distribution of soil pressure in Figure 4.23 can be determined by

$$M_{\text{max}} = T_{\text{max}2}(e + z_{m0} / 2)$$  \hspace{1cm} (4.15)$$

Substitution of $z_{m0}$ in equation (4.14) into equation (4.15) leads to

$$M_{\text{max}} = T_{\text{max}2} \left( e + \frac{p_1 + p_3}{2p_1} z_0 - \frac{p_3 L_s}{p_1} \right)$$  \hspace{1cm} (4.16a)$$

It is appropriate to normalise the maximum bending moment, $M_{\text{max}}$ by the product of $T_{\text{max}2}L$ as done in equation (4.3), since $T_{\text{max}2}$ represents the passive loading on the pile, therefore

$$\frac{M_{\text{max}}}{T_{\text{max}2}L} = \frac{L_s}{L} \left( e + \frac{1}{2} \left( \frac{1}{p_3} \frac{z_0}{L_s} - \frac{p_3}{p_1} \right) \right)$$  \hspace{1cm} (4.16b)$$

If the relative magnitude of $p_3$ and $p_1$ is known, the terms on the right-hand side of equation (4.16) is determinate for a combination of $L_m$ and $L_s$. Nonetheless, we can take $p_1 = p_3$ as an approximation on the basis of test results (see Figure 4.4 e) and study the relationship between the maximum bending moment, $M_{\text{max}}$ and maximum shear force, $T_{\text{max}2}$. In accordance with the current model tests, two cases are outlined as follows.

(a) Shallow sliding depth with $L_m/L_s = 200/500$ and pile length $L = 700\text{mm}$

$$\frac{e}{L_s} = \frac{1}{2} \frac{L_m}{L_s} = \frac{1}{2} \frac{200}{500} = \frac{1}{5}$$

$$\frac{z_0}{L_s} = -\frac{1}{5} + \sqrt{\left(\frac{1}{5}\right)^2 + \frac{1}{5} + \frac{1}{2}} = 0.66$$

$$\frac{M_{\text{max}}}{T_{\text{max}2}L} = \frac{500}{700} \times \left( \frac{1}{5} + \frac{1}{2} (1 + 1) \times 0.66 - 1 \right) = 0.257$$
(b) Deep sliding depth with $L_{m}/L_{s}=350/350$ and pile length $L=700\text{mm}$

Following the same calculation procedure as shallow sliding, the $e/L_{s}$, $z_{0}/L_{s}$, and $M_{\text{max}}/T_{\text{max}2L}$ are computed as 0.5, 0.618 and 0.309, respectively.

The value of, $M_{\text{max}}/T_{\text{max}2L}$ of 0.257 and 0.309 is 34% and 20% less than the 0.385 obtained from the experimental results. In this simple model, the ratio $M_{\text{max}}/T_{\text{max}2L}$ increases from 0.257 to 0.309 as the $e/L_{s}$ ($=0.5L_{m}/L_{s}$) increases from 0.2 to 0.5 or the sliding depth ratio, $R_{L}$ ($=L_{m}/L$) increases from 0.286 to 0.5, while keeping the pile embedded length $L=700\text{mm}$. This variation of $M_{\text{max}}/T_{\text{max}2L}$ with $R_{L}$ obtained from the simple model is different from the nearly constant $M_{\text{max}}/T_{\text{max}2L}=0.385$ as shown in Figure 4.20(b) and Figure 4.21(b). The exact reason for this discrepancy is not clear and probably because of the assumption of $p_{1}=p_{3}$; and the non-uniform soil movement in the shear box, resulting in the soil movement around the pile not as assumed in the current simple model.

With this simple model, it can be concluded that the linear relationship between the maximum bending moment, $M_{\text{max}}$ and maximum shear force, $T_{\text{max}}$ is established and the use of maximum shear force as equivalent load for the passive piles in the current model tests is explained. In view of the complexity of the shear box-soil-pile interaction, numerical methods are required to do more precise analysis.
4.7. DISCUSSIONS

As reviewed in section 2.3.3, a number of methods have been developed for the analysis of piles subjected to passive loading from lateral soil movement. This section discusses the limitations and practical values of the simple solution developed by Guo and Qin (2010), in comparison with solutions developed by other researchers. Since the current solution was developed from the results of model tests on relatively rigid piles and falls in the category of elastic solutions, it is appropriate only to compare it with those developed under similar conditions. Such solutions are very limited. To the author’s knowledge, the elastic solutions for unrestrained free-head pile presented by Chen and Poulos (1997) appear to be the most suitable solutions to be compared with.

4.7.1 Comparison with the solutions by Chen and Poulos (1997)

Chen and Poulos (1997) present a number of elastic solutions in chart form to estimate the response of piles subjected to lateral soil movement. These design charts were generated using a boundary element program PALLS. Two series of such charts are presented, one for a pile in a soil subjected to a uniform movement with depth, and the other for a soil in which the horizontal movement decreases linearly with depth, from a maximum at the surface to zero at a depth. The solutions assume that the soil remains elastic, and they therefore generally give an upper bound estimate of the pile moment and deflection. The use of these solutions relies on accurate estimation of the soil movement immediately surrounding the pile.
Guo and Qin (2010) solutions fall in the category of subgrade reaction approach. The calculation of the maximum bending moment is related to the boundary movement, which may represent the movement of the ground at a distance to the location of the pile. Accurate estimation of the shear modulus is required in using the current solution.

The two solutions may be further compared in estimation of the maximum bending moment for a single pile due to lateral soil movement.

Chen and Poulos (1997) used their design charts to predict the development of bending moment with soil movement for a model test on a single pile reported by Chen (1994) and Poulos et al. (1995). The calculated maximum bending moment, together with those from full analysis by PALLS and the measured values are plotted against the measured soil surface movement in Figure 4.24(a). It can be seen that their elastic chart solution gives a substantial overestimation of the measured bending moments. The maximum bending moments are 4.2 and 6.4 times the measured values at soil surface movement of 5 mm and 60 mm, respectively. The exact reasons for this discrepancy were not clear and presumably attributed to nonlinear effect by Chen and Poulos (1997).

Following the example calculation in Section 4.5.2, the $M_{\text{max}}$ is calculated by $M_{\text{max}} = \frac{w_f k L^2}{10}$. The $k$ is determined as 16.2 kPa (see Table 4.4). Thus, $M_{\text{max}} = 0.738w_f$
(kNmm, \( w_f \) in mm). Given \( w_f = 10, 20, 30, 40, 50, 60 \) mm, the \( M_{\text{max}} \) is calculated as 7.38, 14.76, 22.24, 29.52, 36.9, 44.28 kNmm, respectively. The calculated \( M_{\text{max}} \) is plotted against the soil surface movement in Figure 4.24(b). The measured data are also included for comparison. A fairly good agreement with the measured \( M_{\text{max}} \) can be seen, with an underestimation of 10% on average using Guo and Qin (2010) solution. This estimation can be improved by using a slightly high modulus of subgrade reaction, \( k \), for instance, \( k = 18.5 \) kPa.

**4.7.2 Limitations of the solution**

There are three main limitations for the equivalent elastic solution of Guo and Qin (2010).

Firstly, as suggested by its name, the solution is elastic and it is only applicable for the calculation of the initial linear development of \( M_{\text{max}} \) and \( T_{\text{max}} \) with frame movement, say, at \( w_f \leq 80 \) mm, as shown in Figure 4.7. As stated in the hypothesis 3 in Section 4.5.1, the calculated \( M_{\text{max}} \) and \( T_{\text{max}} \) must be capped by the ultimate values; otherwise, they will keep increasing with the frame movements, which is unrealistic in comparison with the test results.

Secondly, the frame or boundary movement beyond which the solution loses validity is difficult to be properly determined. The calculation using Guo and Qin (2010) solutions relies primarily on the results from the model tests. Clearly, an accurate
simulation of the complex shear box-soil-pile interaction is necessary. The complexity of the interaction also illustrates the necessity for caution in extrapolating the simple solutions beyond the ranges of the experimental validation.

Lastly, the accuracy of the calculation of the $M_{\text{max}}$ and $T_{\text{max}}$ relies significantly on the proper evaluation of the modulus of subgrade reaction, $k$, which in turn can be calculated from shear modulus, $G_s$ of the soil. Guidelines for determining an appropriate shear modulus to be used in the solutions for piles under soil movement are not available. It is noted that the shear modulus used in the current solution for calculating the maximum bending moment, $M_{\text{max}}$ is very low (about 15 ~ 20 kPa). What is surprising is that the shear modulus used in their theoretical analysis reported by Poulos et al. (1995) and Chen and Poulos (1997) is even lower for the similar experimental investigation. Their reported shear modulus is about only 3.75 ~ 6.75 kPa at pile tip as presented in Table 4.4. These values were obtained from the Young’s modulus, $E_s=0.025z$ (in MPa, where $z$ is the depth below the soil surface, in mm) and Poisson’s ratio $\nu_s = 0.3$, as reported by Chen and Poulos (1997). As reviewed in section 2.4, the shear modulus can be assessed on interpretation of lateral loaded pile test data. This will be achieved by back-analysis of laterally loaded rigid piles in sand presented in chapter 7.

4.7.3 Practical values of the solution

The most significant contribution in the solution is that the linear relationship between
the maximum bending moment and maximum shear force is established from the results of model tests on rigid piles in progressively moving sand. As pointed out by Poulos (1995), the maximum shear force is required in the design of reinforcing piles to increase slope stability. On the other hand, the maximum bending moment is required in the structural design of the pile. Thus, once the required shear force is evaluated, the maximum bending moment can be readily calculated. Then the pile property, such as the material and cross-section, can be determined for design.

In Guo and Qin (2010) solution, the pile responses such as maximum bending moment and maximum shear force are calculated from the frame movement or movement of the boundary for inducing soil movement. In reality, the soil movement surrounding the piles may not be easily measured and available. On the other hand, the boundary movement can be readily measured, such as the deflection and rotation of a retaining wall in the case of pile subjected to excavation-induced soil movement (Leung et al., 2000). The solution has been extended to evaluate the behaviour of piles due to excavation-induced soil movement from a series of centrifuge model tests conducted by Leung et al. (2000, 2006) and Ong et al. (2006). Preliminary analysis of their test results show that Guo and Qin (2010) solution can give satisfactory estimation of the development of maximum bending moment, $M_{\text{max}}$ and maximum shear force, $T_{\text{max}}$ with the excavation depth (not shown here).
4.8. CONCLUDING REMARKS

This chapter presents the results of model tests on single piles in progressive moving sand. The test results are provided for the progressive development of applied force, bending moment, shear force and deflection along the piles. The tests enable simple solutions to be proposed for predicting the pile response.

The model tests show the following features.

- The $M_{\text{max}}$ is largely linearly related to the maximum shear force $T_{\text{max}}$ and can be approximately expressed as $M_{\text{max}} = T_{\text{max}}L/2.8$.
- Maximum bending moment increases by 60% for the 32 mm diameter piles or 30% for the 50 mm piles, and its depth by ~50%, upon applying a static load of 7~9% the maximum driving force.
- 3~5 times different bending moment can occur given similar size of model piles but different loading manner, compared with the model pile test results of Poulos et al. (1995).
- Increasing the distance between the pile location and loading side leads to a reduction in $M_{\text{max}}$ and subgrade modulus $k$.
- The axial load results in additional maximum bending moment in the model pile.
- The sliding depth ratio, $R_L$, dominates the critical pile responses.
- The loading block angle affects the initial frame movement $w_i$, which causes little pile reaction. The maximum bending moment $M_{\text{max}}$ and $w_i$ increases with the increasing of loading block angle.
With respect to the solutions, the following conclusions were drawn.

- Equation (4.8) may be used to estimate the maximum bending moment $M_{\text{max}}$, for which the sliding thrust $T_{\text{max}}$ is calculated using equation (4.7). The estimate should adopt an effective frame movement of $w_f - w_i$, in which the $w_i$ depends on the pile diameter, pile position from the free soil movement source $S_b$, loading manner and loading block shape.

- The subgrade modulus $k$ may be estimated using the theoretical ratio of $k/G_s$ and shear modulus $G_s$ (e.g. 15~20 kPa in the current tests). The $G_s$ is pertinent to the overall shear process of the pile-soil-shear box system or local pile-soil interaction. The $k$ varies with diameter and should be considered accordingly.

- The modulus of subgrade reaction deduced from the current model tests was 30~65 kPa using Guo and Qin (2010) solutions.

- Using the equivalent elastic pile-soil interaction, the $T_{\text{max}}$ from equation (4.7) must be capped by ultimate plastic state.

- A simple conceptual model was proposed to explain the use of maximum shear force as equivalent loading for passive piles. Two approaches were presented to correlate the $M_{\text{max}}$ with $T_{\text{max}}$. The first approach based on the current test results gives $M_{\text{max}} = 0.293T_{\text{max}}L$. The second approach based on the limit equilibrium analysis provides $M_{\text{max}} = (0.257 \sim 0.309)T_{\text{max}}L$.

- The limitations and practical values of Guo and Qin (2010) were discussed. By predicting the maximum bending of a model pile test, limitations of existing method were also highlighted.
<table>
<thead>
<tr>
<th>Test description</th>
<th>Frame movement (w_r) (mm)</th>
<th>Maximum bending moment (M_{\text{max}}) (kNmm)</th>
<th>Depth of (M_{\text{max}}) (z_{\text{max}}) (mm)</th>
<th>Maximum Shear force (T_{\text{max}}) (N)</th>
<th>Pile deflection (y_0) (mm)</th>
<th>(k) (kPa)</th>
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<td>Comp. side</td>
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<td>5.2</td>
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Table 4.1 Summary of single pile test results
Table 4.2 Frame movement versus sliding depth ratio

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<tr>
<th>Triangular (Final $L_m=200\text{ mm (15°)}$)</th>
<th>Frame movement, $w_f$ (mm)</th>
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<td>8</td>
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<td>8</td>
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<td>Depth of soil movement, $L_m$ (mm)</td>
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<td>150</td>
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<td>200</td>
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</tr>
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<td></td>
<td>Sliding depth ratio, $R_L$</td>
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<td>0.10</td>
<td>0.14</td>
<td>0.21</td>
<td>0.29</td>
<td>0.29</td>
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<table>
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<tr>
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<table>
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<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
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<th>80</th>
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<td>0.14</td>
<td>0.18</td>
<td>0.21</td>
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</table>

<table>
<thead>
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<th>Frame movement, $w_f$ (mm)</th>
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<th>40</th>
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<th>70</th>
<th>90</th>
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Table 4.3 Calculation for translating pile test TD 32-0 and T32-0 ($L_m=350$)

<table>
<thead>
<tr>
<th>Input data</th>
<th>Calculated†</th>
<th>Measured $M_{max}$ (kNmm)</th>
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<tr>
<td>$w_f$ (mm)</td>
<td>$G_s$ (kPa)</td>
<td>$R_L$</td>
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<tr>
<td>30</td>
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Note
† $w_f=37$ mm, $\gamma=0.048$, $k/G_s=2.516$, $L=0.7$ m, $d=32$ mm
‡ Trapezoidal movement profile
Table 4.4 Calculation for rotating tests (Poulos et al., 1995)

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<tr>
<th>Embedded length $L$ (mm)</th>
<th>$w_f$ (mm)</th>
<th>$G_s$ (kPa)</th>
<th>$R_L$</th>
<th>$\gamma$</th>
<th>$k/G_s$</th>
<th>$k$ (kPa)</th>
<th>$T_{max}$ (kN)</th>
<th>$M_{max}$ (kNmm)</th>
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Figure 4.1 Jack in resistance during pile installation

Figure 4.2 Lateral force on frames against frame movements
Figure 4.3 Progressive moving sand induced by a triangular loading block
Figure 4.4  Response of pile during TS32-0 (Standard test)
Figure 4.5 Response of pile during TS32-294
Figure 4.6 Maximum response profiles of piles (final sliding depth $L_m = 200$ mm)
Equi. elastic calculation:
\[ M_{max} = (w_f - 0.037)kL^2 / 11.2 \]
\( k = 34 \text{kPa}, \ d = 32 \text{mm} \)

Equi. elastic calculation:
\[ M_{max} = (w_f - 0.037)kL^2 / 11.2 \]
\( k = 60 \text{kPa}, \ d = 50 \text{mm} \)

\( G_s = 21.1 \text{kPa} \)

Maximum bending moment, \( M_{max} \) (kNmm)
Frame movement, \( w_f \) (mm)
(a) Measured \( M_{max} \)

TS32-0
TS32-294
TS50-0
TS50-294

Equi. elastic calculation:
\[ T_{max} = (w_f - w_i)kL / 4 \]
\( k = 34 \text{kPa}, \ d = 32 \text{mm} \)

\( L = 0.7 \text{m} \)

Maximum shear force, \( T_{max} \) (N)
Frame movement, \( w_f \) (mm)
(b) Measured \( T_{max} \)

TS32-0
TS32-294
TS50-0
TS50-294

Equi. elastic calculation:
\[ y_0 = 0.19(w_f - w_i) \]
\( w_i = 40 \text{mm} \)

\( y_0 = 0.35(w_f - w_i) \)
\( w_i = 40 \text{mm} \)

Deflection at ground surface, \( y_0 \) (mm)
Frame movement, \( w_f \) (mm)
(c) Measured \( y_0 \)

TS32-0
TS32-294
TS50-0
TS50-294

Figure 4.7 Evolution of maximum response of piles (final sliding depth \( L_m = 200 \text{ mm} \))
Figure 4.8 Response of pile during TD32-0
Figure 4.9 Response of pile during TD32-294
Figure 4.10 Maximum response profiles of piles at $w_f = 120$ mm (final sliding depth $L_m = 350$ mm)
Figure 4.11 Evolution of maximum response of piles (final sliding depth $L_m = 350$ mm)

Equi. elastic calculation:

$M_{\text{max}} = (w_f - 0.03)kL/11.2$

- $k=25\text{kPa}, d=32\text{mm}$
- $k=45\text{kPa}, d=50\text{mm}$

Maximum bending moment, $M_{\text{max}}$ (kNmm)
Frame movement, $w_f$ (mm)

Maximum shear force, $T_{\text{max}}$ (N)
Frame movement, $w_f$ (mm)

Deflection at ground surface, $y_0$ (mm)
Frame movement, $w_f$ (mm)
Figure 4.12 Pile responses at varying distances of pile location
($S_b = 340\text{mm}, 500\text{mm}, \text{and } 660\text{ mm}$)
Figure 4.13 Variation of pile responses with distance $S_b$
Figure 4.14 Pile responses under varying axial load levels
(P = 0 N, 294 N, 588 N, and 735 N)
Figure 4.15 Pile responses at different loading block angles 
($\theta = 15^\circ$, 22.5$^\circ$, and 30$^\circ$)
Figure 4.16 Variation of pile responses with loading block angles
Figure 4.17 Variation of $M_{\text{max}}$ with wall rotation or loading block angle

Chen (1994) Changing $\theta$ and varying $L_{m}$, $L_{s}$=325mm
- ■ $L_{m}$=350mm
- ▲ $L_{m}$=300mm
- ● $L_{m}$=250mm
- ◆ $L_{m}$=200mm

Current tests
Fixed $\theta$, $L_{m}$=200mm, $L_{s}$=500mm
- ■ $M_{\text{max}}$ at $w_{a}$
- ■ Peak $M_{\text{max}}$ at $w_{p}$

<table>
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<tr>
<th>$\theta$ (deg)</th>
<th>15</th>
<th>22.5</th>
<th>30</th>
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<tr>
<td>$w_{a}$ (mm)</td>
<td>60</td>
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<td>110</td>
</tr>
<tr>
<td>$w_{p}$ (mm)</td>
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<td>100</td>
<td>120</td>
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</tbody>
</table>

Maximum bending moment, $M_{\text{max}}$ (kNmm)
Wall rotation or loading block angle, $\theta$ (degree)
Figure 4.18 Pile responses at varying sliding depths
(L_m = 125mm, 200mm, 250mm, 300mm, and 350 mm)
Figure 4.19 Variation of $M_{\text{max}}$ with sliding depth ratio $R_L$

Elastic calculation:

$M_{\text{max}} = (w_f - 0.037)kL^2/15.4$

$k = 2.52G_s$ (kPa)

Equi. elastic calculation:

$M_{\text{max}} = (w_f - 0.037)kL^2/11.2$

$k = 2.52G_s$ (kPa)

Maximum bending moment, $M_{\text{max}}$ (kNmm)

Sliding depth ratio, $R_L$

(a) Measured data
- TS32-0
- TS32-294
- TS50-0
- TS50-294
- TD32-0
- TD32-294
- TD50-0
- TD50-294

(b) Various sliding depths
- $L_m = 125$ mm
- $L_m = 200$ mm
- $L_m = 250$ mm
- $L_m = 300$ mm
- $L_m = 350$ mm

$M_{\text{max}} = wL^2/10$

$w = 37$ mm $(R < 0.5)$

$w = 60$ mm $(R >= 0.5)$

$k = (2.4 - 2.8)G_s$ (kPa)

Figure 4.20 Maximum shear forces versus maximum bending moments
(Results of eight general tests)
Figure 4.21 Maximum shear forces versus maximum bending moments
(Results of ten parametric study tests)
Figure 4.22 Pile response profiles under equivalent load for the model tests

Figure 4.23 Schematic explanation of equivalent load
Figure 4.24 Predicted and measured maximum bending moment for model test on single pile
(a) prediction by design charts and full analysis (Chen and Poulos, 1997)
(b) calculation using current equivalent elastic solution

\[
M_{\text{max}} = w_k L^2 / 10
\]
\[k = 16.2 \text{ kPa}, L = 675 \text{ mm}\]
5 EFFECT OF SOIL MOVEMENT PROFILES ON VERTICALLY LOADED SINGLE PILES

5.1 INTRODUCTION

The study in chapter four reveals that (1) only the effective frame movement causes significant responses on rigid piles in progressively moving sand, whereas the initial frame movement reflecting the impact of distance between pile location and the loading side, loading block angle, and loading manner and so on, has negligible impact on the piles; (2) a linear correlation exists between the maximum bending moment and the maximum shear force; and (3) Guo and Qin’s (2010) simple solutions provide satisfactory estimation of the evolution of the maximum bending moment and shear force with frame movement. These findings were achieved from the model test results conducted on 32 mm and 50 mm diameter piles in sand undergoing soil movement induced by a triangular loading block. The effect of other soil movement profiles on the relationship between the $M_{max}$ and $T_{max}$ requires further verification to facilitate practical design.

This chapter attempts to examine the responses of piles in uniform and progressively moving soil movement profiles. Firstly, the results of the sixth and seventh groups of the parametric study tests (see Table 3.5) are presented to further clarify the effective frame movement concept. This is restricted to assessment of the impact of the distance between the free soil movement source and pile location. Secondly, the previous published results of eight typical tests using a rectangular loading block are compared with those using the triangular block to examine the effect of soil movement profiles. Thirdly, the linear relationship between maximum bending
moment $M_{max}$ and the thrust $T_{max}$ are further explored, which renders Guo and Qin’s (2010) solutions to be extended to piles in uniformly moving sand.

5.2 VERTICALLY LOADED PILES IN UNIFORM MOVING SAND

As explained in chapter 1, model tests have been undertaken on single piles and pile groups in sand to investigate the pile response under lateral soil movement using a rectangular loading block. Some test results have already been published (Guo and Ghee, 2004; Guo and Ghee, 2005; Guo and Qin, 2005; and Guo et al., 2006). Among them, eight tests are summarised in Table 5.1. For completeness, the details of these tests are explained below.

These tests were conducted under the same conditions as the general tests presented in Table 3.5, but replacing the triangular loading block 1 ($\theta = 15^\circ$) with a rectangular one. The pile was always installed to an embedded depth of 700 mm at the centre of the shear box, i.e. $S_b = 500$ mm as shown in Figure 3.8. A schematic of the pile under testing is illustrated in Figure 3.9. The sand used in the model ground has a unit weight $\gamma'_s$ of 16.27kN/m$^3$ and angle of internal friction $\phi'_s = 38^\circ$. Following the specifications in Chapter three, each test was also denoted by two letters and two numbers, e.g. RS32-0 or RD50-294, where ‘R’ signifies tests using the rectangular loading block; ‘S’ represents that a shallow sliding layer depth $L_m = 200$ mm, and stable layer of $L_s= 500$ mm and ‘D’ indicates that a deep sliding layer $L_m = 400$ mm
and \( L_s = 300 \text{ mm} \); ‘32’ or ‘50’ indicates 32 or 50 mm in pile diameter, and ‘0’ or ‘294’ represents 0 N or 294 N applied at the pile head, respectively. These tests will be categorised as ‘RS’ and ‘RD’ series for convenience.

The measured critical pile responses are presented in Table 5.2, including: (1) the peak values of maximum bending moment \( M_{\text{max}} \); (2) the frame movement \( w_f \) at which the peak value of \( M_{\text{max}} \) is attained; (3) depth of maximum bending moment along the pile \( z_{\text{max}} \); (4) maximum shear force \( T_{\text{max}} \); (5) pile deflection at the ground surface \( y_0 \).

**5.3 PILE RESPONSES DUE TO EFFECTIVE FRAME MOVEMENT**

The sixth and seventh groups of tests in Table 3.5 were carried out to examine the effect of pile diameter and to further explore the impact of the distance between free soil movement source and pile location. The results of these tests may be used to clarify effective soil movement. Due to space limitation, only the development of the maximum bending moment, \( M_{\text{max}} \), shear force, \( T_{\text{max}} \) and pile deflection at the ground surface are plotted against the frame movement, \( w_f \); and the largest bending moment, shear force and deflection profiles are provided.

**5.3.1 Response of \( M_{\text{max}} \), \( T_{\text{max}} \) and \( y_0 \) versus \( w_f \)**

The development of the maximum bending moment, \( M_{\text{max}} \), shear force, \( T_{\text{max}} \) and pile deflection at ground line, \( y_0 \) for the sixth group of three tests (RS25-0-300,
RS25-0-500 and RS25-0-700) using the rectangular loading block are plotted against frame movement, $w_f$ in Figures 5.1(a), 5.1(b) and 5.1(c). These figures show that the frame (thus the soil) movement immediately induced pile response, regardless of the distance $S_b$. The $M_{\text{max}}$, $T_{\text{max}}$ and $y_0$ increase linearly with the frame movement until $w_f = 20 \sim 30$ mm, then remain constant except in test RS25-0-500, indicating a constant soil pressure transferred on the pile at larger $w_f$.

The same responses of the piles in the seventh groups of tests TS25-0-300, TS25-0-500 and TS25-0-700 using the triangular block are plotted in Figures 5.2(a), 5.2(b) and 5.2(c). In contrast with the tests using the rectangular block, these figures demonstrate that an initial portion of frame movement of $20 \sim 40$ mm, depending on the distance $S_b$, causes negligible or little pile response using the triangular block. Then the $M_{\text{max}}$, $T_{\text{max}}$ and $y_0$ increase sharply with the frame movement and attained their peak values at $w_f = 65$, 80 and 85 mm in the three tests, respectively.

**5.3.2 Maximum pile response profiles**

The largest bending moment, shear force and pile deflection profiles with depth for the tests using the rectangular block are provided in Figures 5.1(d), 5.1(e) and 5.1(f). The profiles for the tests using the triangular block are presented in Figures 5.2(d), 5.2(e) and 5.2(f). These figures demonstrate that:

- The bending moment profiles of these tests are similar amongst themselves and analogous to parabolic. The maximum bending moment occurs at a depth of $350 \sim 450$
mm below the ground surface with an average of 400 mm, which is about 0.57 times of the pile’s embedded length.

- The shear force profiles are also of similar shape. Two largest local shear forces of approximately equal magnitudes but of converse signs are deduced from the bending moment profiles.
- Piles deflect mainly by rotation around pile tip or a depth near the pile tip.

5.3.3 Effect of distance $S_b$ on the $M_{max}$

The peak values of maximum bending moment, $M_{max}$, shear force, $T_{max}$ and pile deflection at ground surface, $y_0$ are summarised in Table 5.2. The $M_{max}$ values are plotted against the distance $S_b$ in Figure 5.3. By using the rectangular loading block, the $M_{max}$ reduces about $\sim 5 \text{kNmm}$ as the pile was installed in the shear box from $S_b = 300 \text{mm}$ to $S_b = 500 \text{mm}$, and further reduces approximately $\sim 17 \text{kNmm}$ from $S_b = 500 \text{mm}$ to $S_b = 700 \text{mm}$. With the use of the triangular block, the $M_{max}$ decreases about $\sim 24 \text{kNmm}$ from $78 \text{kNmm}$ at $S_b = 300 \text{mm}$ to $53.9 \text{kNmm}$ at $S_b = 700 \text{mm}$. These findings are consistent with those from the second group of ‘pile location’ tests conducted on the 32-mm diameter model pile as discussed in section 4.3.1. The results of the three tests are also plotted in Figure 5.3 for comparison. It may be concluded that the $M_{max}$ decreases with increasing distance $S_b$ between the pile location and loading side where free soil movement is generated. The reasons are due to the non-uniform soil movement across the shear box and attenuation of the magnitude of soil movement from the loading side to the pile locations as discussed
previously. The current test results corroborate the consistency of the model tests.

5.3.4 Effective frame movement

In order to quantify the effect of soil movement on the pile responses induced by the rectangular and triangular loading blocks using Guo and Qin (2010) solutions, an effective frame movement, \( w_e \) is introduced and expressed as

\[
    w_e = w_f - w_i
\]  

(5.1)

where \( w_i \) is an initial frame movement causing negligible pile response. For the current tests, the value of \( w_i \) can be taken as 20 mm for TS25-0-300 and as 40 mm for TS25-0-500 and TS25-0-700 (see Figure 5.2(a)), whereas it is 0 mm for the three tests using the rectangular block as shown in Figure 5.1(a) and independent of the distance, \( S_b \). Therefore, the initial frame movement \( w_i \) may also be taken as 0 mm for the tests listed in Table 5.1, i.e. \( w_e = w_f \) using the rectangular block.

The measured \( M_{max} \) and \( T_{max} \) in Figures 5.1(a) and 5.2(a) and 5.1(b) and 5.2(b) are plotted together for each test in Figures 5.4(a) and 5.4(b) against the effective frame movement, \( w_e \). It can be observed that the \( M_{max} \sim w_e \) and \( T_{max} \sim w_e \) curves are similar and independent of the shape of the block and distance \( S_b \), but the gradient of the initial segment of these curves reduces with the increasing distance \( S_b \).
5.4 EFFECT OF SOIL MOVEMENT PROFILES ON PILE RESPONSES

In this section, the results of the tests presented in Table 5.1 are compared with the eight general tests reported in chapter 4 to explore the effect of soil movement profiles on the performance of piles.

5.4.1 Generation of soil movement profiles

The sand movement profiles generated in the shear box adjacent to the loading block side can be summarised as follows:

- Employing the rectangular block, a constant displaced depth of $L_m = 200$ mm (RS series) or $L_m = 400$ mm (RD series) was enforced all the time for the frame movement of $w_f$, as shown in Figure 5.5. The induced soil movement is said to have a uniform profile.

- Using the triangular block 1, the mobilisation of the frames has been explained in section 4.2.3. The loading block induces a progressively moving triangular soil movement profile followed by a trapezoid profile.

It must be stressed that the soil movement profiles mentioned here refers to those occurring near the loading block location. They varied across the shear box, but the soil movement profile around the pile was unknown in all the tests.

5.4.2 Lateral force on frames

The lateral forces on frames were measured only in tests RS50-0 and RS50-294 for
tests using the rectangular block. They are plotted in Figure 5.6. The measured forces in the six tests (TS32-0, TS32-294, TS50-0, TS50-294, TD32-0, and TD32-294) and in the tests No. 25 and 26 without piles (see Table 3.5) are also plotted for comparison. The figure demonstrates the following features:

● The lateral force imposed by the hydraulic jack on the frames attained the maximum values for all the tests irrespective of loading blocks, and post peak softening was observed in tests RS50-0, RS50-294 and TS50-294.

● Using the rectangular block, the lateral forces increased with the frame movement and attained maximum values at $w_f = 20 \sim 30$ mm. They dropped afterwards by about 20% in RS50-0 and RS50-294.

● At the shallow sliding depth of $L_m = 200$ mm, the maximum lateral forces on the frames in the triangular series of tests (TS50-0, TS50-294, TS32-0, and TS32-294) are approximately 1.30 ~ 1.41 times greater than those in the rectangular series of tests (RS50-0 and RS50-294). The possible reasons may be that at $w_f \geq 60$ mm, the trapezoidal movement of the frames mobilizes more sands and results in a larger failure surface in the shear box.

● The change of the measured maximum lateral force in the tests with and without piles using the rectangular loading block is immaterial. This is possibly due to that the pile was installed in the centre of the shear box and nearly beyond the failure zone induced by the rectangular loading block in the shear box, as discussed in section 4.5.2.
In addition, the shear modulus of the sand, $G_s$, may be grossly estimated from the lateral force frame movement curves. More specifically, for the tests RS50-0 and RS50-294 using the rectangular loading block, the shear stress is $(3.5 \sim 3.9) \text{kPa} = (3.5 \sim 3.9) \text{kN/}1.0\text{m}^2$, and shear strain is $0.1 = 20 \text{mm}/200 \text{mm}$, assuming a linear shear displacement from sand surface to sliding depth of 200 mm. Therefore, the shear modulus $G_s$ is about $(35 \sim 39) \text{kPa}$.

5.4.3 Response profiles of bending moment, shear force and deflection

5.4.3.1 Shallow sliding depth

The measured largest bending moment, shear force and deflection distributions with depth for both the TS ($d = 32 \text{ mm}$) and RS ($d = 32 \text{ mm}$) series of tests are provided in Figures 5.7(a), 5.7(b) and 5.7(c). The same profiles for the $d = 50 \text{ mm}$ piles are presented in Figures 5.8(a), 5.8(b), and 5.8(c). Both Figures 5.7(a) and 5.8(a) show that the bending moment profiles for each test are parabolic in shape and independent of the loading blocks, but the magnitude of the bending moment changes, depending on the shape of the loading block and the axial load level. The maximum bending moment, $M_{\text{max}}$, occurs at $350 \sim 450 \text{ mm}$ below the ground line, which is approximately $(0.5 \sim 0.65)L$, ($L = 700 \text{ mm}$, the embedded length of the pile). Figures 5.7(c) and 5.8(c) indicate that the pile deflected mainly by rotation around the pile tip. These figures also show that under the same testing conditions, the tests with the triangular loading block generally have larger displacements at the ground surface. For example, the $y_0 = 7.2 \text{ mm}$ in TS50-0 in comparison of the $y_0 = 2.4 \text{ mm}$ in RS50-0.
This is due to the fact that at the ultimate state more pressure may have been transferred from the moving sand to the piles using the triangular loading block than using the rectangular loading block, resulting in greater pile responses, including the maximum bending moment, maximum shear force and pile deflection, as demonstrated in Figures 5.7 and 5.8. This fact is also supported by the greater lateral force on the triangular loading block measured from the lateral hydraulic jack.

5.4.3.2 Deep sliding depth

Figures 5.9 and 5.10 provide the three profiles for $d = 32$ mm and $d = 50$ mm model piles under the deep sliding depth of $L_m = 350$ mm using the triangular loading block (TD series) and of $L_m = 400$ mm using the rectangular loading block (RD series), respectively. Compared with the pile responses under the shallow sliding depth, the shape of loading block thus the mobilisation pattern of the frames has significant impact on the pile response under the deep sliding depth. As discussed in chapter 4, the bending moment profiles in the tests (TD50-0, TD50-294, TD32-0, and TD32-294) using the triangular loading block are similar in shape to a parabola, indicating a single curvature characterising the shape of the elastic deformation of the pile. In contrast, Figures 5.9(a) and 5.10(a) show that two largest local bending moment developed in the sliding layer and stable layer in tests RD32-0 and RD50-0 using the rectangular loading block, showing that the elastic deformation of the pile has double curvatures and producing an ‘intermediate mode’ of failure (Poulos, 1995). Poulos (1995) stated that if the piles are used to increase slope stability, they should be
designed with the ‘intermediate mode’ being in the working state since the piles provide the largest shear force and bending moment in this mode of failure. The maximum negative bending moment in test RD50-0 was measured as -35.9 kNmm at \(w_f = 120\) mm, occurring at a depth of 250 mm. This value is approximately 40% of the positive maximum bending moment of 87.5 kNmm that occurred at 550 mm down the pile. Similarly, a small magnitude of negative maximum bending moment of -8.4 kNmm was measured in test RD32-0. In contrast, no negative bending moment was measured in the sliding layer in the TD series of tests.

Figures 5.9(c) and 5.10(c) illustrate that the pile deflection modes are relatively complicated, depending on the loading block shapes and axial load level. The piles mainly deflected by rotation around a certain depth for the tests without axial load (e.g. in TD32-0, RD32-0, and RD50-0), while they rotated and translated simultaneously under axial load of 294N (e.g. in TD32-294 and RD50-294).

Using the triangular loading block, the pile deflection at the ground line was generally less than the corresponding frame movement. For instance, in test TD32-0 the pile displacement at the ground surface was about 58.7 mm at \(w_f = 120\) mm, whereas the pile base kicked out about 35.2 mm. However, the pile deflection at the ground line may exceed the frame movement for the tests using the rectangular loading block. In particular, in test RD32-0, the pile base deflected about -160.9 mm at \(w_f = 120\) mm, which was even larger than the pile deflection \(y_0\) of 155.7 mm at the ground level. The
pile deflection profile in Figure 5.9(c) indicates that the pile deflects mainly by rigid rotation. The similar magnitude of the sliding layer depth $L_m = 400$ mm and stable layer depth $L_s = 300$ mm may result in a ‘rotational’ failure mode B shown in Figure 2.17(b), as identified by Viggiani (1981), or the ‘intermediate’ failure mode demonstrated in Figure 2.19(b), as assessed by Poulos (1995). In this case of failure mode, the pile at the ground level and at pile-tip may have similar magnitude of displacement but opposite direction. This behaviour of the pile is also supported by the relatively shallow rotation point at a depth of 300 mm in RD32-0 shown in Figure 5.9(c).

5.4.4 Evolution of $M_{max}$ and $T_{max}$ with effective frame movement $w_e$

The evolution of the $M_{max}$ and $T_{max}$ in moving sand using the rectangular and triangular loading blocks may be explored in terms of the effective frame movement, $w_e$. The measured $M_{max}$ and $T_{max}$ of TS32-0 and TS32-294 (originally plotted in Figures 4.7 (a) and 4.7(b)) are now re-plotted in Figures 5.11(a) and 5.11(b) against the effective frame movement, $w_e$. The effective frame movement, $w_e$ was calculated from equation (5.1) by subtracting the initial frame movement $w_i$ from $w_f$. The measured data from the corresponding two tests RS32-0 and RS32-294 using the rectangular block are also included for comparison. Similar $M_{max} \sim w_e$ and $T_{max} \sim w_e$ curves are plotted in Figures 5.12(a) and 5.12(b) for $d = 50$ mm piles at shallow sliding depth, in Figures 5.13(a) and 5.13(b) for $d = 32$ mm piles at deep sliding depth, and in Figures 5.14(a) and 5.14(b) for $d = 50$ mm piles at deep sliding depth.
Figures 5.11 and 5.12 show that at the shallow sliding depth:

- The $M_{\text{max}} \sim w_e$ and $T_{\text{max}} \sim w_e$ curves of the TS series of tests are analogous to those of the RS series of tests. Each curve consists of two straight-line segments.

- The $M_{\text{max}}$ and $T_{\text{max}}$ increase linearly with $w_e$ until 20 mm using the rectangular loading block or 40 mm using the triangular loading block and reach the ultimate values afterwards.

- Changing the loading block from rectangular to triangular, the ultimate $M_{\text{max}}$ increases about 114% ($d = 32$ mm) and 50% ($d = 50$ mm) without axial load, and 115% ($d = 32$ mm) and 152% ($d = 50$ mm) with an axial load of 294N.

Figures 5.13 and 5.14 show that at the deep sliding depth:

- Unlike the trend under shallow sliding depth, no general trend of $M_{\text{max}} \sim w_e$ and $T_{\text{max}} \sim w_e$ curves was observed.

- The $M_{\text{max}}$ and $T_{\text{max}}$ keep increasing with the effective of frame movement $w_e$, and have not reached the ultimate state even at $w_e = 120$ mm for the majority of tests. This is possibly because the soil pressure induced by the soil movement on the pile has not reached the ultimate value in the tested range of frame movement.

5.4.5 Effect of axial load

Compared with the influence of sliding layer depth and loading block shape on the pile behaviour, an axial load of 294 N (7% ~ 9% of the driving force) on the pile head has limited impact on the maximum bending moment profiles. Some salient features of the tests results are observed.
Firstly, small positive bending moments were obtained at the ground surface for the tests with axial load as shown in Figures 5.7(a) to 5.10(a). For instance, the moment at the ground surface, $M_0$, is measured as 31.8 kNmm in test RD50-294, which is close to the moment of 35.28 kNmm due to the $P \cdot \Delta_e$ effect as explained in section 4.2.6. The measured $M_0$ at the ground surface in the tests with an axial load of 294 N is generally less than 30% of the positive maximum bending moment $M_{\text{max}}$ along the embedded pile.

Secondly, the strong resemblance between the bending moment profiles of RD32-0 and RD32-294 in Figure 5.9(a) indicates the magnifying effect of the axial load on the bending moment along the pile, which may enforce the measured negative bending moments in the sliding layer in tests RD32-0 rise to positive values in test RD32-294. Similar observations are made between the bending moment profiles in tests RD50-0 and RD50-294 as show Figure 5.10(a).

Thirdly, the 294 N axial load causes larger peak values of the maximum bending moment $M_{\text{max}}$ and $T_{\text{max}}$ in the piles (except tests RS50-0 and RS50-294) as can be seen from Figures 5.11(a) to 5.14(a). Applying a static load of 294N on the pile head, with respect to the RS tests, $M_{\text{max}}$ increases by 60% for the $d = 32$ mm piles or decreases 25% for the $d = 50$ mm piles. As for the RD tests, $M_{\text{max}}$ increases about 44% and 40% for the $d = 32$ mm and $d = 50$ mm piles, respectively. It should be noted that the $M_{\text{max}}$ of 44.9 kNmm in RS50-294 is less than the $M_{\text{max}}$ of 61.7 kNmm in RS50-0, which is unusual compared with the trend showing by the other test results. In order to check
the results of these tests, a supplementary test RS50-294(S) was conducted and the results are also presented in Figure 5.8 and Figure 5.12. The results of RS50-294 and RS50-294(S) are similar. The measured maximum bending moment, $M_{\text{max}}$ of 47.8 kNmm is still 22% less than the value of 60.4 kNmm measured in RS50-0. The possible reasons maybe due to different pile deflection profiles as shown in Figure 5.8(c).

The axial load of 294 N may greatly affect the pile deflection mode, especially combined with a deep sliding layer. Without the axial load, the pile in test RD32-0 (Figure 5.9(c)) and RD50-0 (Figure 5.10(c)) deflected mainly by rotation. A direct comparison of the pile deflection profile in test RD50-294 with that of RD50-0 in Figure 5.10(c) shows the stabilising effect of the axial load, which enabled the soil to carry the pile forward and caused translational deflection in test RD50-294, while the pile deflected mainly by rotation in test RD50-0.

5.5 CALCULATION USING SIMPLE SOLUTIONS

5.5.1 Extension of simple solutions

The simple solutions proposed by Guo and Qin (2010) can be used to calculate the development of $M_{\text{max}}$ and $T_{\text{max}}$ with effective frame movement, $w_e$. The solutions can be generalised and expressed as:

$$M_{\text{max}} = T_{\text{max}} L / m$$ \hspace{1cm} (5.2)
\[ T_{\text{max}} = w_{f}kL/4 \quad (5.3) \]
\[ M_{\text{max}} = w_{f}kL^2/(4m) \quad (5.4) \]

where \( L (=0.7\text{m}) \), is the embedded length of the pile; \( k=(2.4-3)G_s \), is the modulus of subgrade reaction and \( G_s \) is the shear modulus of the sand (Guo, 2008) and \( m \) is a non-dimensional constant for the best curve fitting between \( T_{\text{max}} \) and \( M_{\text{max}} \).

The measured \( M_{\text{max}} \) (Figures 5.1(a) and 5.2(a)) for the \( d = 25\text{ mm} \) pile tests are plotted together in Figure 5.15(a) against the corresponding \( T_{\text{max}} \) (Figures 5.1(b) and 5.2(b)). Figure 5.15(a) demonstrates a remarkably good linear relationship between the \( T_{\text{max}} \) and \( M_{\text{max}} \) under any \( w_f \) and independent of the shape of loading block. The value of \( m \) is estimated as 2.8 as shown by the solid line.

Similarly, the measured \( M_{\text{max}} \) and \( T_{\text{max}} \) for the \( d = 32 \) and \( 50\text{ mm} \) pile tests are plotted together in Figure 5.15(b). The linear relationship is evident again and also offers \( m = 2.8 \) for best match with a variation of \( \pm5\% \).

**5.5.2 Example calculation**

The measured \( M_{\text{max}} \) and \( T_{\text{max}} \) of the piles in RS25-0-300 and TS25-0-700 tested to a frame movement, \( w_f \) of 20 mm and 90 mm are presented in Table 5.3. The \( T_{\text{max}} \) and \( M_{\text{max}} \) are estimated for the two tests in the following calculation sequence.

- The ratio of \( k/G_s \) was calculated as 2.375 using the expression given by Guo and Lee (2001) (see Table 2.7), with the load transfer factor being calculated as 0.0382 using \( L = 0.7\text{ m} \) and \( d = 0.025\text{ m} \).
• The shear modulus $G_s$ was estimated as 22.7 kPa and 11.4 kPa. The corresponding $k$ was calculated as 54 kPa and 27 kPa, respectively.

• With $w_i=0$ for the rectangular loading block and $w_i = 40$ mm for the triangular loading block (as observed), the $T_{\text{max}}$ was estimated in terms of $w_e$, using equation (5.2).

• The $M_{\text{max}}$ was calculated as $M_{\text{max}} = w_e k L^2/11.2$, with $m = 2.8$ as per equation (5.3).

The calculated values of $M_{\text{max}}$ and $T_{\text{max}}$ for both tests are provided in Table 5.3 and plotted against $w_e$ in Figure 5.4. They compare well with the measured data. In the same way, calculations were performed for the $d = 32$ mm and 50 mm pile tests. The calculated $M_{\text{max}}$ and $T_{\text{max}}$ are also plotted in Figures 5.11 to 5.14. The used modulus of subgrade reaction $k$ for each test is summarised in Table 5.2.

5.5.3 Comments on the current calculations

• Guo and Qin (2010)’s solutions were developed to cater to the initial linear segment of $M_{\text{max}} \sim w_e$, and $T_{\text{max}} \sim w_e$ curves. The estimated $M_{\text{max}}$ and $T_{\text{max}}$ must be capped by those deduced for ultimate state as discussed in section 4.4.2.

• Under the shallow sliding depth (RS and TS series), Figures 5.11 and 5.12 show that the deduced $k$ is independent of the shape of the employed loading block (rectangular or triangular) and the two axial loading levels (0 and 294 N), but depends on the pile diameter. The values of $k$ are plotted against pile diameter in Figure 5.16, including those deduced from the $d = 25$ mm tests. The $k$ increases
about 33% from 45 kPa to 60 kPa as the pile diameter increases from 32 mm to 50 mm.

- Under the deep sliding depth (RD and TD series), Figure 5.13 and 5.14 show some scatter in the measured data, especially for the $d = 50$ mm pile tests. The simple solutions may still be used to calculate the $M_{\text{max}}$ and $T_{\text{max}}$, which offer the $k$ in a range of 22.5 ~ 35 kPa and 15 ~ 45 kPa for the $d = 32$ and 50 mm pile tests, respectively.

- The simple calculations reveal that the distance $S_b$ between the pile location and free soil movement source at loading side has a great impact on the $k$. For the six tests conducted on the 25 mm pile, the $k$ varies in a range of 27 ~ 54 kPa as presented in Table 5.2. They are plotted in Figure 5.17 against the $S_b$. The value of $k$ is doubled as the $S_b$ is reduced from 700 mm to 300 mm (the pile was installed closer to the loading side). This is consistent with the findings from the ‘pile location’ tests presented in section 4.3.1.

### 5.6 Calibration Against In Situ and Centrifuge Tests

It is well-recognized that geotechnical models may be prone to significant scale effect primarily because of the difference in stress level between the model tests and field tests and the nature of soils, especially granular soils, for which the internal friction angle decreases with increased normal stress (Vesic, 1973; Turner and Kulhawy,
1994). Therefore, the results obtained from 1g small scale model tests can not be scaled directly to prototype values. Additionally, the current experimental investigation involves two types of soil movement profiles, i.e. a uniform and progressively moving profile. The model tests may replicate the situation that the piles are installed close to the embedded retaining walls in a foundation pit, offering the benefit of reducing the retaining wall displacements during excavation, as described by Yap and Pound (2003) and Katzenbach et al. (2005, 2006). In reality, the soil movement movement profiles always remain unknown. Therefore, the relationship between the $M_{\text{max}}$ and $T_{\text{max}}$ needs to be calibrated using the results of in-situ, small scale and centrifuge tests under different soil movement profiles.

5.6.1 In situ tests

Guo and Qin (2010) have validated the simple correlation between the $M_{\text{max}}$ and $T_{\text{max}}$ using measured response of eight in situ test piles, one centrifuge test pile and fourteen model test piles subjected to soil movement of various profiles, as reviewed in chapter 4 and 5. Their investigation showed that

(1) The maximum bending moment $M_{\text{max}}$ induced by the soil movements was measured in all the tests;

(2) The shear force, $T_{\text{max}}$ was only measured for three piles. The $T_{\text{max}}$ for the remaining six piles were deduced using elastic and elastic–plastic solutions presented by Cai and Ugai (2003) and Guo (2009b).
(3) For flexible piles, the length $L$ for each pile was taken as the smallest values of the $L_i$ and $L_{ci}$ (i = 1, 2), where $L_i$ and $L_{ci}$ are the thickness of sliding layer and equivalent length for rigid pile in sliding layer; $L_2$ and $L_{c2}$ are the thickness of stable layer and equivalent length for rigid pile in stable layer (see Figure 2.24).

(4) The ratio $M_{max}/(T_{max}L)$ for each case was calculated. The obtained ratios fall into the range of 0.148 ~ 0.4, as established in section 4.4.1 in Chapter 4. The results are re-plotted in Figure 5.18.

(5) The ratio $M_{max}/(T_{max}L)$ for the in situ pile (Frank and Pouget, 2008) in the sliding layer stays around 0.25, with respect to the “pre-pull back” (behaving as free head) and “after pull back” (fixed head) situation for 16 years’ test duration.

Based on the above investigation and assessment, they concluded that the ratio $M_{max}/(T_{max}L)$ is independent of loading level for either the model tests or the field tests.

Further analysis were conducted and presented here using the results from 10 small scale model tests (Chen, 1994) and 3 centrifuge tests (Leung et al., 2000).

5.6.2 Small scale model tests

Chen (1994) reported ten laboratory tests on a single free-head pile of 25-mm-diameter subjected to lateral soil movement. The details of these tests are present in Table 5.4. The bending moment profiles of these tests were fitted using 5th
order polynomial functions and the shear force profiles were derived from first
derivation of these functions. The measured maximum bending moment $M_{\text{max}}$ and
deduced maximum shear force $T_{\text{max}}$ are tabulated in Table 5.4. It should be noted that
the embedded pile length varies in each test. This allows the ratio $M_{\text{max}}/(T_{\text{max}}L)$ to be
evaluated. The ratios of $M_{\text{max}}/(T_{\text{max}}L)$ are plotted against the sliding depth ratio $R_L$ in
Figure 5.19. They range from 0.23 to 0.34 with an average value of 0.28. Accordingly,
this average value renders $m = 3.55$, which is about 27% higher than 2.8 obtained
from the results of the current model tests. This difference may reflect the influence of
the scale effect and variation of soil movement profiles, which requires independent
evaluation using numerical analysis.

5.6.3 Centrifuge tests

Leung et al. (2000) presented the results from centrifuge model tests on a single pile
in sand due to unstrutted deep excavation induced soil movement. The pile responses
in the case of both stable and collapsed retaining wall were investigated. The
free-field soil movements, pile bending moment and deflection profiles at different
depths of excavation were presented for three tests. Again, the shear force distribution
was back-calculated by fitting a 5th order polynomial function to a measured bending
moment profile. The test details, the measured maximum bending moment $M_{\text{max}}$, and
the deduced maximum shear force, $T_{\text{max}}$ at various excavation depths are tabulated in
Table 5.5. Figure 5.20 plots the maximum bending moment $M_{\text{max}}$ against the
maximum shear force $T_{\text{max}}$ in the sliding layers for the three tests. Again, the linear
relationship is apparent between the $M_{\text{max}}$ and $T_{\text{max}}$, allowing $m = T_{\text{max}}/M_{\text{max}} = 4.18$, with the $L$ (=12.5 m) taken as the pile embedded depth. Because the values of the $T_{\text{max}}$ in the sliding layer and stable layer were of similar magnitude, only the $T_{\text{max}}$ in the sliding layer were used for calculating the ratio $M_{\text{max}}/T_{\text{max}}L$, at each excavation for the three tests, without loss of generality.

The sliding depth ratio $R_L$ was defined as the ratio of the excavation depth adjacent to the wall, $L_m$ over the pile embedment length $L$. They were also presented in Table 5.5. The ratios $M_{\text{max}}/(T_{\text{max}}L)$ at different excavation depths for the three tests are plotted against the sliding depth ratio $R_L$ in Figure 5.21. The ratio $M_{\text{max}}/(T_{\text{max}}L)$ stays approximately at an average value of 0.24 and is independent of the magnitude of the soil movement and excavation depth. It also falls into the range of 0.148 ~ 0.40, as demonstrated in equation (4.6). This finding is identical to that obtained from the results of rigid piles in progressively moving sand presented in Chapter 4. Nevertheless, the value of the constant $m = 4.18$ obtained from these centrifuge tests increases about 50% compared with the $m = 2.8$ derived from the current model tests. This may be primarily due to the difference in the two investigations in terms of stress level (1 g to 50 g), the behaviour of the piles (rigid to flexible), source of soil movements and soil movement profiles (passive loading in level model ground to excavation).
5.7 CONCLUDING REMARKS

This chapter examines the pile response due to effective soil movement and the effect of soil movement profiles. The results of two groups of six tests on a 25-mm diameter pile using a rectangular and triangular block are first presented to further clarify the concept of effective frame movement. The results of previously conducted eight tests using a rectangular block (2 pile diameters, 2 axial load levels and 2 combination of sliding and stable layer depth) are compared with the eight general tests (Table 3.5) with a triangular block to quantify the effect of soil movement profiles. Guo and Qin’s (2010) simple solutions are used to calculate the measured $M_{\text{max}}$ and $T_{\text{max}}$ with $w_e$ and comments are made concerning the calculations using the solutions.

Comparison of the model test results reveals the following features:

- The $M_{\text{max}}$ reduces about 50% or 31% for the $d = 25$ mm pile tests using the rectangular or triangular block as the distance $S_b$ increases from 300 mm to 700 mm.
- The impact of the distance $S_b$ between the pile location and free soil movement source on the pile response can be well captured by the effective frame movement, $w_e$, which relies on the shape of the loading block and distance $S_h$.
- The $M_{\text{max}} \sim w_e$ or $T_{\text{max}} \sim w_e$ curves in the tests using the triangular loading block resembles those using the rectangular block.
- The soil movement depth dominates the bending moment, shear force and deflection profiles.
• The axial load of 294N (7% ~ 9% of the driving force) generally causes an increase in $M_{\text{max}}$ and may alter the pile deflection mode.

Some practical implications may be drawn with respect to the simple solutions.

• The $M_{\text{max}}$ is linearly related to the $T_{\text{max}}$. The relationship can be described by equation (5.2) with $m = 2.8$ and is independent of the shape of loading block and the distance $S_b$.

• The linear relationship between the $M_{\text{max}}$ and $T_{\text{max}}$ is calibrated using the results from in-situ tests, similar small scale model tests and centrifuge tests. The studies show that the value of $m$ may increase about 28% ~ 50%. These discrepancies may be due to the scale effect, the variation of soil movement profiles and different pile responses to soil movements.

• Equations (5.2) to (5.4) give reasonably good estimation of the variation of $M_{\text{max}}$ and $T_{\text{max}}$ with $w_c$ at the shallow sliding depth. The $k$ decreases about 50% as the distance $S_b$ increase from 300 mm to 700 mm. The estimated $M_{\text{max}}$ and $T_{\text{max}}$ must be capped at larger frame movements; otherwise unrealistic higher values will be obtained.

• For the RS and TS series of tests, the modulus of subgrade reaction $k$ is independent of the shape of the employed loading block and the axial loading levels, but depends on the pile diameter.
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<th>Sliding layer depth (mm)</th>
<th>Stable layer depth (mm)</th>
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<td>294</td>
<td>400</td>
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<td>200</td>
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Table 5.2 Summary of 15 test results

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<th>( M_{\text{max}} ) (kNmm)</th>
<th>( z_{\text{max}} ) (mm)</th>
<th>( T_{\text{max}} ) (N)</th>
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### Table 5.3 Calculation for test RS25-0-300 and TS25-0-700

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Note: $w_e = w_f - w_i$, $\gamma = 0.0382$, $k/G_s = 2.375$, $L = 0.7$ m, $d = 25$ mm
Table 5.4 Calculation for $M_{\text{max}}/(T_{\text{max}}L)$ for model tests (data from Chen, 1994)

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<th>$L_s$ (mm)</th>
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### Table 5.5 Calculation for $M_{\text{max}}/(T_{\text{max}}L)$ for centrifuge tests (data from Leung et al., 2000)

<table>
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<tr>
<th>Test</th>
<th>Excavation depth $L_m$ (m)</th>
<th>Embedded length $L$ (m)</th>
<th>$R_L$</th>
<th>$T_{\text{max}}$ (kN)</th>
<th>$M_{\text{max}}$ (kNm)</th>
<th>$M_{\text{max}}/T_{\text{max}}L$</th>
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Figure 5.1 Pile responses at varying distance of pile locations
(S_b=300mm, 500mm, and 700mm, Rectangular profile)
Figure 5.2 Pile responses at varying distance of pile locations
($S_b=300\text{mm}, 500\text{mm}, \text{and} 700\text{mm}, \text{Triangular profile}$)
Figure 5.3 Variation of maximum bending moment, $M_{\text{max}}$ with distance, $S_b$
Figure 5.4 Evolution of $M_{\text{max}}$ and $T_{\text{max}}$ with effective frame movement, $w_e$
Figure 5.5 Photographs showing mobilization of frames using a rectangular block

(a) $w_f=30$ mm  (b) $w_f=40$ mm

Figure 5.6 Lateral forces on frames against frame movements
Figure 5.7 Response profiles of piles (d=32mm, shallow sliding depth)
Figure 5.8 Response profiles of piles (d=50mm, shallow sliding depth)
Figure 5.9 Response profiles of piles (d=32mm, deep sliding depth)
Figure 5.10 Response profiles of piles (d=50mm, deep sliding depth)
Equi. elastic calculation

\[ M_{\text{max}} = w_e k L^2 / 11.2 \]
\[ k = 45 \text{kPa} \]
\[ d = 32 \text{mm} \]

(a) Measured data
- - - RS32-0
- - - RS32-294
- - - TS32-0 (\(w_i = 37\)mm)
- - - TS32-294 (\(w_i = 37\)mm)

Maximum bending moment, \(M_{\text{max}}\) (kNmm)
Effective frame movement, \(w_e\) (mm)

(b) Measured data
- - - RS32-0
- - - RS32-294
- - - TS32-0 (\(w_i = 37\)mm)
- - - TS32-294 (\(w_i = 37\)mm)

Maximum shear force, \(T_{\text{max}}\) (N)
Effective frame movement, \(w_e\) (mm)

Figure 5.11 Evolution of \(M_{\text{max}}\) and \(T_{\text{max}}\) with \(w_e\)
(d = 32mm, shallow sliding depth)
Figure 5.12 Evolution of $M_{\text{max}}$ and $T_{\text{max}}$ with $w_e$
(d=50mm, shallow sliding depth)
Figure 5.13 Evolution of $M_{\text{max}}$ and $T_{\text{max}}$ with $w_e$
(d=32mm, deep sliding depth)

Equi. elastic calculation
\[ M_{\text{max}} = w_e k L^2 / 11.2, \quad d=32\text{mm} \]
\[ k=35, 28, 22.5 \text{ kPa} \]

Equi. elastic calculation
\[ T_{\text{max}} = w_e k L / 4, \quad d=32\text{mm} \]
\[ k=35, 28, 22.5 \text{ kPa} \]
Figure 5.14 Evolution of of $M_{\text{max}}$ and $T_{\text{max}}$ with $w_e$
(d=50mm, deep sliding depth)
Figure 5.15  Maximum shear forces versus maximum bending moments
Figure 5.16 Variation of $k$ with pile diameter $d$

Figure 5.17 Variation of $k$ with distance $S_b$
Figure 5.18 Calculated versus measured ratios of $M_{\text{max}}/(T_{\text{max}}L)$ (Guo and Qin, 2010)

Figure 5.19 Variation of $M_{\text{max}}/(T_{\text{max}}L)$ with $R_L$ (data from Chen, 1994)
Figure 5.20  Measured $M_{\text{max}}$ versus $T_{\text{max}}$ (data from Leung et al., 2000)

Figure 5.21 Variation of $M_{\text{max}}/(T_{\text{max}} L)$ with $R_L$ (data from Leung et al., 2000)
6 RESPONSES OF RIGID PILE GROUPS IN PROGRESSIVE MOVING SAND

6.1 INTRODUCTION

The responses of single piles in progressively moving sand and the effect of soil movement profiles on pile behaviour have been investigated in the previous chapters. This investigation provides a better understanding of the performance of single piles subjected to lateral soil movement. In practice, piles are often used in groups. Therefore, model tests were also conducted on pile groups subjected to lateral soil movement and these test results are presented in this chapter.

The experimental details and procedures have been described in section 3.3 in chapter three. A series of model tests on three typical pile group configurations were performed to study the behaviour of pile groups undergoing progressively moving soil using the triangular loading block 1 ($\theta = 15^\circ$). Based on the analyses of the single pile tests presented in the previous chapters, the pile group tests were conducted under a shallow sliding depth of $L_m = 200$ mm with a stable layer depth of $L_s = 500$ mm. In total, 21 group tests were conducted in this part of the study. The layout of pile groups in each test is schematically shown in Table 3.6. These model tests are categorised into three groups, i.e. two piles in a row, two piles in a line and four piles in a square. The experimental results are presented in the next sections.
6.2 EXPERIMENTAL RESULTS

For the pile group tests conducted in the present study, it was found that in all the tests, the bending moment, shear force, soil reaction, rotation and deflection profiles along each pile within a group at a frame movement were quite similar to the ‘standard’ single pile test TS32-0. Similar to the presentation of the parametric study tests on single piles, only the development of maximum bending moment, $M_{\text{max}}$, maximum shear force, $T_{\text{max}}$, and pile deflection at the ground surface, $y_0$ with frame movement, $w_f$ are presented, along with the largest bending moment, shear force and deflection profiles.

6.2.1 Lateral force on frames

The lateral forces exerted via the lateral hydraulic jack and loading block on the frames were recorded at each 10 mm of frame movement, $w_f$ during the tests. They are plotted in Figure 6.1 against the frame movement, $w_f$ for the 21 pile group tests. The lateral force measured in the single pile test TS32-0 and test 25 conducted without piles are also included for comparison. The figures show:

- The force linearly increases with the frame movement until the maximum values were attained at $w_f = 60$–$70$ mm for all of these tests.
- The maximum lateral force on the loading block varies in a range of 4–5 kN, depending on the pile arrangement configuration (two piles in a row, two piles in a line or four piles in square), pile head condition (free or capped), pile spacing, and the distance of pile group installation position to the loading side, $S_b$. 
The measured maximum lateral force in the pile group tests are approximately 5% ~ 30% greater than the maximum force of 3.8 kN in the test without piles, showing the reinforcement effect of the pile groups to soil movement.

6.2.2 Sand movement surrounding the pile groups

Figures 6.2(a), 6.2(b), 6.2(c), and 6.2(d) provide photographs taken during typical tests at \( w_f = 120 \) mm for a single pile, two piles in a row \( (S_v/d = 5) \), two piles in a line \( (S_h/d = 5) \) and four piles in square \( (S_v/d = S_h/d = 5) \), respectively. They gave some indication of the failure pattern of the model ground surrounding the pile groups for different pile arrangements. In general, significant sand heave up was observed in front of the pile groups near the loading side. Figure 6.2(b) shows the sand heaving to be about 80 mm at \( w_f = 120 \) mm for the two piles in a row group, while the model ground remains intact behind the piles, showing the reinforcement effect of the pile groups. The sand tended to flow around the piles and the periphery of the pile groups at large magnitudes of soil movement, as seen in Figures 6.2(b) to 6.2(d). By comparing the sand movement pattern around the piles in a group with that of a single pile in Figure 6.2(a), it is found that the soil displacement field around an individual pile in the group overlaps in front of the piles and clearly demonstrates the group effect of piles in resisting the lateral soil movement. However, Figure 6.2(c) show less heaves in front of the piles in comparison with the single pile test in Figure 6.2(a). The exact reasons for this characteristic of the soil movement in the shear box are not clear and may possibly due to the pile arrangement. Since the distance between the
front pile and the shear box boundary, \( S_b \) was kept constant as 340 mm in the two piles in a line groups, which is less than the \( S_b = 500 \) mm in the single pile test in Figure 6.2 (a), this variation and pile arrangement may possibly change the soil movement field in the shear box. The quantitative analysis of the difference requires advanced measurement techniques, such as particle image velocimetry (PIV) (Whittle et al., 2003).

6.2.3 Common features of the tested pile groups

6.2.3.1 Development of \( M_{\text{max}} \), \( T_{\text{max}} \) and \( y_0 \) with frame movement \( w_f \)

The development of measured maximum bending moment, \( M_{\text{max}} \), maximum positive shear force measured in the stable layer, \( T_{\text{max}} \) and pile deflection at groundline, \( y_0 \) induced in a pile within a row of two piles (with different distance \( S_b = 340, 500, \) and 700 mm) are plotted respectively in Figures 6.3(a), 6.3(b) and 6.3(c) against the frame movement, \( w_f \). The same development is plotted in Figures 6.4(a), 6.4(b) and 6.4(c) for two piles in a row (\( S_v/d = 3, 5, \) and 7), in Figures 6.5(a), 6.5(b) and 6.5(c) for two piles in a line (free-head, \( S_h/d = 3, 5, \) and 7), in Figures 6.6(a), 6.6(b) and 6.6(c) for two piles in a line (capped-head, \( S_h/d = 3, 5, \) and 7), in Figures 6.7(a), 6.7(b) and 6.7(c) for four piles in square (free-head, \( S_v/d = S_h/d = 3, 5, \) and 7), and in Figures 6.8(a), 6.8(b) and 6.8(c) for four piles in square (capped-head, \( S_v/d = S_h/d = 3 \) and 5). The same responses of the single pile TS32-0 are also plotted in Figures 6.3(a~c) to 6.8 (a~c) for comparison. The peak values of \( M_{\text{max}}, T_{\text{max}} \) and \( y_0 \) induced in the individual piles in a pile group were picked up from the curves in Figures 6.3(a~c) to 6.8 (a~c). If these
curves do not give clear indication of peak values, the values of $M_{\text{max}}$, $T_{\text{max}}$ and $y_0$ at $w_f = 70$ mm are selected. This is due to the fact that the pile responses generally have reached the maxima at $w_f = 70$ mm noted in chapter 4 using the same triangular loading block on single piles. They are summarised in Table 6.1. The following observations are made from these figures.

- The development of $M_{\text{max}}$, $T_{\text{max}}$ and $y_0$ with frame movement $w_f$ for an individual pile in a pile group is similar to that measured in the single pile test TS32-0, irrespective of the pile configuration, pile head fixity condition and pile spacing.

- The initial portion of frame movement, $w_i$, of 30–40 mm induces marginal responses on each pile in the tested pile groups.

- In the majority of piles, the $M_{\text{max}}$, $T_{\text{max}}$ and $y_0$ increase sharply at $w_f \geq 30$–40 mm and reach their peak values at $w_f = (60$–70) mm, followed by a slight fluctuation and then stabilise at the limit values.

- The pile configuration, pile head condition and pile spacing have little impact on the development of $M_{\text{max}}$, $T_{\text{max}}$ and $y_0$ with frame movement $w_f$. However, the pile position in a group, i.e. the front or back pile, and the distance to the loading side $S_b$, may affect the pile behaviour as the $M_{\text{max}}$ did not reach the peak values in some piles as shown in Figures 6.3(a) and 6.5(a).

- The pile deflection at the groundline, $y_0$ was much less than the frame movement, $w_f$. 
6.2.3.2 Bending moment, shear force and pile deflection profiles

Figures 6.3(d), 6.3(e) and 6.3(f) present the largest bending moment, shear force and pile deflection profiles for a pile in the two piles in a row group at $S_b = 340$, 500, and 700 mm, respectively. The same profiles have been plotted in Figures 6.4(d), 6.4(e) and 6.4(f) for two piles in a row ($S_v/d = 3$, 5, and 7), in Figures 6.5(d), 6.5(e) and 6.5(f) for two piles in a line (free-head, $S_h/d = 3$, 5, and 7), in Figures 6.6(d), 6.6(e) and 6.6(f) for two piles in a line (capped-head, $S_h/d = 3$, 5, and 7), in Figures 6.7(d), 6.7(e) and 6.7(f) for four piles in square (free-head, $S_v/d = S_h/d = 3$ and 5), and in Figures 6.8(d), 6.8(e) and 6.8(f) for four piles in square (capped-head, $S_v/d = S_h/d = 3$ and 5). The same responses of the single pile TS32-0 are also plotted in Figures 6.3(d~f) to 6.8 (d~f) for comparison. The bending moment, shear force and pile deflection profiles for an individual pile in a pile group are similar in shape to the corresponding profiles measured in the single pile test TS32-0. Nonetheless, the position of a pile in a group, pile configuration, and pile head fixity conditions have some impact on the bending moment and pile deflection profiles. They are discussed below.

- The bending moment profiles are of parabolic shape. The maximum bending moment occurs around 350–450 mm below the sliding plane in the stable layer, which is about 0.5 and 0.65 times the pile embedment length. For the pile groups with capped head (G12, G14, G16, G18 and G20), negative bending moment was measured near the groundline in the sliding layer in the front piles, while positive bending moment was detected near the groundline in the back piles.

- The largest negative shear force developed at a depth of 250 mm, which was 25%
deeper than the predetermined sliding depth, while the maximum positive shear force occurred at a depth of 500 mm in the stable layer.

- The piles deflected mainly by rotation about a depth near the pile tip.

6.2.3.3 $M_{\text{max}}$ versus $T_{\text{max}}$

The measured $M_{\text{max}}$ and $T_{\text{max}}$ for the piles in each group are plotted together in Figure 6.9 under any frame movement, $w_f$. The linear relationship between the maximum bending moment, $M_{\text{max}}$, and maximum shear force, $T_{\text{max}}$, deduced from the stable layer is evident in Figure 6.9(a) and maybe expressed as

$$M_{\text{max}} = \frac{T_{\text{max}}L}{m}$$  \hspace{1cm} (6.1)

with $m = 2.8$ for best fitting of the data.

The maximum bending, $M_{\text{max}}$ was also plotted against the maximum shear force obtained for the sliding layer in Figure 6.9(b). The same expression may be used to fit the data with $m = (2.0\text{~}3.5)$ and an average value of 2.8.

6.2.4 Two piles in a row

In order to investigate the distance between the loading side and the pile group location in the shear box, six tests were conducted on two piles in a row with free-head (G1, G3, and G5) and capped-head (G2, G4, and G6) with a spacing of $S_s/d = 3$ at three different locations in the shear box, i.e. $S_s = 340$ mm, 500 mm and 600 mm (Table 3.6). Figure 6.3 shows the measured pile responses of these tests. The
measured responses of the two instrumented piles are essentially identical due to the symmetrical pile configuration. Herein only the responses of one pile are presented. Figure 6.10(a) plots the maximum bending moment, $M_{\text{max}}$ against the distance $S_b$. The test results for a single pile located at the same positions, as reported in chapter 4, is also included for comparison. The maximum bending moment, $M_{\text{max}}$ decreases with an increasing distance, $S_b$ for both head conditions. For instance, $M_{\text{max}}$ reduces about 37% and 63% for the free-head and capped-head cases, respectively, as $S_b$ increases from 340 mm to 660 mm. At the same location in the shear box, the values of $M_{\text{max}}$ the piles in the capped-head pile groups are slightly less than those obtained from the piles in the free-head group. Moreover, they are less than those of the single pile, showing the pile group effect on the pile performance.

The effects of pile spacing on the responses of two piles in a row were also investigated by conducting tests on both free-head (G3, G7 and G9) and capped-head (G4, G8 and G10) groups for three different spacing $S_v/d = 3, 5$ and 7 at $S_b = 500 \text{ mm}$. The responses of these pile groups are presented in Figure 6.4. The variation of maximum bending moment, $M_{\text{max}}$, is plotted against the normalised pile spacing, $S_v/d$ in Figure 6.10(b). The $M_{\text{max}}$ increases about 52% and 67% for the free-head and capped-head groups as the spacing, $S_v$ rises from $3d$ to $7d$. The development pattern of $M_{\text{max}}$, $T_{\text{max}}$, and $y_0$ with frame movement and the response profiles for the piles in the capped-head groups in Figures 6.3 and 6.4 are similar to those of the free-head cases, revealing that the pile cap and pile-soil-pile interaction have insignificant impact on
the performance of two piles in a row. It should be noted that these tests were conducted on free-standing and floating-base pile groups. If the pile cap is sitting on the ground surface, the pile responses may be different, which requires further investigation.

6.2.5 Two piles in a line

Six tests (G11~G16) were conducted on pile groups arranged in a line parallel to the direction of soil movement (see Table 3.6). The pile responses for the free-head groups G11, G13, and G15 are presented in Figure 6.5. The same responses deduced for the capped-head groups G12, G14, and G16 are provided in Figure 6.6. Both Figures 6.5(a) and 6.6(a) show that the induced maximum bending moments, $M_{\text{max}}$, for the front piles are higher than those in the back piles at the same magnitude of frame movement, indicating the ‘shadowing’ effect of the front piles on the back piles when subjected to soil movement. For instance, for the free-head pile group with a spacing of the third test, G13, the $M_{\text{max}}$ in the back pile is about 30\% less than that in the front pile at $w_f = 70$ mm. Figure 6.5(d) shows that the bending moment profiles of the front and back piles at $w_f = 70$ mm for different spacings were found to be similar in shape and the measured bending moments in the back piles are less than those of the corresponding front piles in the same group. It reveals that the presence of the front pile reduces the detrimental effect induced by soil movement on the back piles. Meanwhile, Figure 6.6(d) shows that small negative bending moments developed in the upper portion of the front piles in the sliding layer due to the restraint imposed by
pile cap, whereas positive bending moments were measured near the groundline for the back piles. The different behaviour of the front and back piles indicate interaction between the two piles through the pile cap.

6.2.6 Four piles in a square

Five tests G17~G21 were carried out on four piles in a square (2 × 2) group. Only one pile in each row was instrumented to measure the pile responses (see Table 3.6). The 2 × 2 pile group may be taken as a combination of the two piles in a row and two piles in a line. Figures 6.7 and 6.8 show the pile responses. Again, the maximum bending moment in the front piles are found to be greater than those in the corresponding back piles for both head fixity conditions at a specific frame movement, w_f. Figure 6.8(d) shows that negative bending moments developed in the upper portion of the front piles with capped-head pile. Figure 6.10(d) plots the maximum positive bending moment $M_{max}$ for the free-head pile groups G17, G19 and G21 against the normalised spacing, $S_b/d$ ($S_v/d$). The $M_{max}$ remains constant at approximately 40kNmm for the front piles with the three pile spacing ($S_b/d = S_v/d = 3, 5, 7$), which is approximately 15% less than that obtained in the two piles in a row group G1 with the same distance $S_b = 340$ mm to the loading side, indicating that the pile interaction in multi-rows alleviates the impact of soil movement on the piles. However, it decreases with increasing pile spacing, $S_b/d$ for the back piles, which is similar to the findings from the two piles in a line group shown in Figure 6.10(c).
6.3 PREDICTIONS OF $M_{\text{max}}$ AND $T_{\text{max}}$

The methods developed for the single piles described in chapters 4 and 5 may be extended to predict the development of maximum bending moment, $M_{\text{max}}$ and shear force, $T_{\text{max}}$ associated with frame movement, $w_f$ for a pile in a group. The expressions are presented below in terms of effective frame movement, $w_e$.

\[
T_{\text{max}} = w_e k_g L / 4 \quad (6.2)
\]
\[
M_{\text{max}} = w_e k_g L^2 / (4m) \quad (6.3)
\]

where $w_e = w_f - w_i$ is the effective frame movement, $w_i$ is an initial frame movement that causes little pile response by moving sand, depending on the $S_b$, loading manner, and pile-soil relative movement, $m = 2.8$ and $k_g$ is the subgrade reaction modulus for an individual pile in a group.

The $M_{\text{max}}$ and $T_{\text{max}}$ obtained from the piles in Figures 6.3(a–b) to Figures 6.8(a–b) are re-plotted against the effective frame movement, $w_e$ in Figure 6.11(a–b) to Figure 6.16(a–b). The $M_{\text{max}}$ and $T_{\text{max}}$ were calculated under a set of $w_e$ for all the pile group tests. They are also plotted together with the measured data in these figures. The simple predictions generally capture the measured initial linear segment of these curves up to the elastic limit. However, the estimated $M_{\text{max}}$ and $T_{\text{max}}$ must be capped by full plastic responses. The values of $k_g$ and $w_i$ deduced for each pile are tabulated in Table 6.1. The variations of the $k_g$ are plotted against the distance, $S_b$ in Figure 6.17(a) for the two piles in a row, against the normalised spacing, $S_r/d$ in Figure 6.17(b) for the two piles in a row, against the normalised spacing, $S_h/d$ in Figure ...
6.17(c) for the two piles in a line, and against the normalised spacing, $S_h/d$ in Figure 6.17(d) for the four piles in square. The impact of these factors on the modulus of subgrade reaction will be discussed in terms of group factors in the next section.

### 6.4 GROUP EFFECT

In this section, pile group factors are first introduced to quantify the group effect. Then the pile group behaviour for the three pile arrangement configuration are analysed, focusing on comparison of the maximum bending moment, $M_{\text{max}}$ with that of the ‘standard’ single pile test TS32-0.

#### 6.4.1 Group factors

In order to assess the group effect of piles subjected to lateral soil movement, the critical pile responses of a pile within a group, such as the maximum bending moment, pile head deflection and limiting soil pressure (Chen and Poulos, 1997; Pan et al., 2002a) are compared with those of a single pile. Chen et al. (1997) have demonstrated that the group effect assessed in terms of measured maximum bending moment may be more reliable and consistent. In the present study, a group factor, $F_m$ is introduced to quantify the group effect. This compares the peak value of the maximum positive bending moment of a pile in a group with that of the ‘standard’ single pile at the same predetermined final sliding depth, $L_m$ and is described by the following expression:

$$F_m = \frac{M_{g(\text{max})}}{M_{s(\text{max})}}$$  \hspace{1cm} (6.4)
where $F_m = \text{group factor}$; $M_{g(\text{max})} = \text{peak value of maximum bending moment for a pile in a group}$; and $M_{s(\text{max})} = \text{peak value of maximum bending moment of the ‘standard’ single pile TS32-0}$.

In order to assess the pile group effect on the subgrade modulus $k$, another group factor $F_k$ is introduced by comparing the subgrade modulus for a pile in a group with that of the single ‘standard’ test TS32-0. In this respect, the factor $F_k$ may be expressed as:

$$F_k = \frac{k_g}{k_s}$$  \hfill (6.5)

where $F_k = \text{group factor}$; $k_g = \text{subgrade modulus for a pile in a group}$; and $k_s = \text{subgrade modulus deduced from the ‘standard’ single pile TS32-0}$.

\textbf{6.4.2 Impact of distance $S_B$ and pile spacing on the group factors}

In order to calculate the group factors, the measured maximum bending moment and the deduced modulus of subgrade reaction for the pile group tests (see Table 6.1) were normalised by the measured $M_{s(\text{max})} = 49.7 \text{kNmm}$ and $k_s = 42 \text{kPa}$ derived for the ‘standard’ single pile TS32-0, respectively. A value of $F_m$ or $F_k$ other than one for a pile indicates the existence of the group effect. Table 6.2 summarises the group factors obtained from the 21 pile group tests. The variation of the $F_m$ and $F_k$ with $S_B$, $S_v/d$ and $S_h/d$ for the current pile group tests are plotted in Figures 6.18 and 6.19. The following characteristics have been noted.

- Figures 6.18(a) and 6.19(a) show that the group factors, $F_m$ and $F_k$ decrease with
the increasing distance, $S_b$, indicating the decaying effect of the soil movement on the pile response as the piles were installed further away from the loading side. The group factor $F_k$ decreases about 50% from 1.0 to 0.48 as $S_b$ increases from 340 mm to 660 mm for both pile head fixity conditions.

- The group factor, $F_m$, increases with increasing pile spacing, $S_v$ for the two piles in a row with both head fixity conditions as seen in Figure 6.18(b), showing gradual declining of the pile-soil-pile interaction as the pile spacing increases. The $F_m$ at $S_v/d = 3$ are about 35% and 40% less than those at $S_v/d = 7$ for the free-head and capped-head cases, respectively. Moreover, the factors $F_m$ for the free-head cases are less than those of the capped-head cases for the three pile spacing with the largest difference of 0.31 occurring at $S_v/d = 5$. This finding is consistent with the analysis results obtained from the two piles in a row (parallel to wall) groups subjected to excavation induced soil movement (Leung et al., 2003). In comparison, it appears that the spacing $S_v$ has less impact on the group factor, $F_k$ than on the $F_m$. The value of $F_k$ may be taken as 0.75 on average, regardless of the spacing and pile head fixity condition.

- For the two piles in a line groups, the group factors, $F_m$, are plotted against the normalised spacing, $S_h/d$ in Figure 6.18(c). For the free-head pile groups, the $F_m$ of the front piles may be more or less than unity, depending on the spacing, $S_h/d$, with the largest value of 1.1 being obtained at $S_h/d = 5$. Nevertheless, the $F_m$ of the back piles decreases from 0.76 to 0.55 as the $S_h/d$ increases from 3 to 7. This tendency is compatible with the results obtained from the two piles in a row
groups installed at varying distance $S_b$ presented in Figure 6.18(a). For the capped-head pile groups, it is found that the group factor $F_m$ for the front piles are approximately 1.0 for the three pile spacing with a variation of 10%. However, the $F_m$ increases about 15% from 0.85 to 1.0 with the spacing $S_h$ rising from 3$d$ to 7$d$. The group factors, $F_k$ of the front piles are approximately (20~25)% greater than those deduced for the back piles at the same pile spacing $S_h$. For the free-head pile groups, the $F_k$ increases about 71% and 64% for the front piles and back piles as pile spacing $S_h$ increases from 3$d$ to 7$d$, respectively. However, the $F_k$ for the capped-head cases is less sensitive to the spacing than those for the free-head cases.

- For the four piles in a square (free-head), the $F_m$ remains constant at 0.8 for the front piles as seen in Figure 6.18(d). It decreases about 33% from 0.6 to 0.41 for the back piles as the $S_h/d$ increases from 3 to 7. The deduced $F_k$ decreases with the increasing spacing, $S_h/d$ ($S_v/d$) for both the front and back piles. Figure 6.19(d) shows that the $F_k$ decrease about 30% and 50% for the front piles and the back piles as the $S_h/d$ increases from 3 to 7, respectively. At the same pile spacing, values of $F_k$ for the front piles are (0.42~0.95) times greater than those deduced for the back piles, depending on the pile spacing.

6.4.3 Comparison of group factors

Group effects on the lateral response of vertical piles to lateral soil movements have also been studied experimentally by Chen (1994), Chen et al. (1997) and Pan et al.
(2002a) and numerically by Chen and Poulos (1997) and Jeong et al. (2003). The group effect was quantified by group factors. In their studies, the group factors were defined in terms of either maximum bending moment, $M_{\text{max}}$ at the same amount of free-field soil movement (Chen, 1994; Jeong et al., 2003) or limiting soil pressure, $p_u$ (Chen and Poulos, 1997; Pan et al., 2002a). These group factors are summarised in Table 6.3.

For the two piles in a row, the group factors are plotted together against the normalised pile spacing in Figure 6.20. The group factors obtained from the current tests are consistent with the previous experimental and numerical analysis results, although different group factors are used. Figure 6.20 shows that the group factors decrease as pile spacing decreases for two piles in a row, except the results presented by Chen and Poulos (1997) from the plane-strain finite element analysis for an infinitely long row of piles in clay.

Figure 6.21(a) presents the group factors of the front piles for the two piles in a line group. The results from the model tests conducted by Chen (1994) show that the value of the group factor $F_m$ for the free-head front piles is greater than unity for the three different normalised spacings of $S_h/d = 2.5, 5, \text{ and } 7.5$, with the largest value of about 1.6 occurring at $S_h/d = 5$. The results of the current tests also show that the largest group factor $F_m$ may be obtained at the spacing of $5d$, but with a lower value of 1.08. For the capped-head pile groups, the value of $F_m$ is less sensitive to the pile spacing in
the current tests. However, it increases about 35% from 0.93 to 1.25 as the normalised spacing $S_{h}/d$ increases from 3 to 5, then decreases about 50% to 0.64 at the normalised spacing of 7.

Figure 6.21(b) presents the group factors of the back piles for the two piles in a line group. The results from both the current model tests and those conducted by Chen (1994) show that the value of group factor $F_m$ for the free-head back piles remains stable or increases less than 10% as the normalised spacing, $S_{h}/d$ increases from 2.5~3 to 5, then decreases about 27~37% with $S_{h}/d$ increasing further to 7~7.5. For the capped-head cases, the $F_m$ determined from the current tests increases about 18% from 0.85 to 1.02 as the normalised spacing, $S_{h}/d$ increases from 3 to 7. The $F_m$ obtained by Chen (1994) may be more or less than unity, depending on the pile spacing, with the largest values of 1.36 at $S_{h}/d = 5$. It appears that the pile caps have a significant influence on the group factors for two piles in a line. It must be borne in mind that the front piles were always installed at a distance $S_b = 340$ mm from the loading side in the current tests, whereas the single pile was installed at $S_b = 500$ mm, i.e. in the center of the shear box. The distance of the pile position from the loading side, $S_b$ has a great impact on the tests results, which has been clarified in preceding sections. However, it is not clear about the position of the piles from the loading side in the model tests presented by Chen (1994).
6.5 CONCLUDING REMARKS

Model tests were carried out on pile groups in sand undergoing lateral soil movement. Twenty one (21) tests were presented concerning free-head and capped-head piles groups consisting of two piles in a row, two piles in a line and four piles in a square. They were conducted without axial load, at a sliding depth of 0.29L and the soil movement was imposed by a triangular loading block. The simple solutions were extended for predicting the responses of pile groups due to lateral soil movement. The group effect is quantified in terms of group factors, which are compared with the previous experimental and numerical analysis results. The following conclusions can be drawn.

- The development of $M_{\text{max}}$, $T_{\text{max}}$ and $y_0$ of a pile in a group with frame movement $w_f$, is similar to those of a single pile. The pile configuration, pile head condition and pile spacing have little impact on the evolvement pattern of $M_{\text{max}}$, $T_{\text{max}}$ and $y_0$ with frame movement, $w_f$. An initial portion of frame movement, $w_i$, of 30~40 mm induces negligible responses on the piles. The effective frame movement, $w_e$, of 30~50 mm is required to mobilise the maximum pile responses.

- The bending moment profiles of an individual pile in a group are more or less of parabolic shape. The maximum bending moment, $M_{\text{max}}$ occurs at approximately $(0.5~0.65)L$.

- The $M_{\text{max}}$ is linearly related to the maximum positive shear force $T_{\text{max}}$ and can be approximately described as $M_{\text{max}} = T_{\text{max}}L/2.8$. This relationship is identical to that derived from the results of single pile tests.
Extension of the simple solutions renders satisfactory prediction of the $M_{\text{max}}$ and $T_{\text{max}}$ with effective frame movements, $w$, for an individual pile in a group, provided that the modulus of subgrade reaction $k_g$ is appropriately determined.

For the two piles in a row group, the maximum bending moment, $M_{\text{max}}$ decreases with an increasing distance, $S_b$ from the free field soil movement source to the pile location. It reduces about 37% and 63% for the free-head and capped-head cases as $S_b$ increases from 340 mm to 660 mm. The $M_{\text{max}}$ increases about 52% and 67% for the free-head and capped-head groups with the pile spacing increasing from $3d$ to $7d$. The bending moment and deflection profiles of the capped-head groups are similar to those of the free-head cases, indicating that the pile cap has a limited impact on the behaviour of the piles in a row perpendicular to the direction of the soil movement.

For the two piles in a line group and four piles in a square group, the $M_{\text{max}}$ of the front piles are generally greater than those in the back piles at the same magnitude of frame movement. It indicates that the piles closer to the free-field soil movement source will reduce the detrimental effects arising from lateral soil movement on the back piles. The rigid pile cap has a significant influence on the pile responses. It leads to negative bending moments developed near the groundline in the upper portion of the front piles. Due to the interaction among the frames, piles, pile cap and soil, the variation of $M_{\text{max}}$ with the pile spacing is rather complicated for the front piles. However, it generally decreases with increasing pile spacing for the back piles.
• For the two piles in a row group, group factors $F_m$ and $F_k$ decrease with increasing distance $S_b$. The factors $F_m$ for the free-head cases are less than those of the capped-head cases, with the largest difference of 0.31 occurring at $S_v/d = 5$. The spacing $S_v$ has less impact on the group factor, $F_k$ than on the $F_m$. The $F_k$ may be taken as 0.75 on average and independent of the pile spacing and head fixity conditions. The group factors obtained from the current tests and the previous experimental and numerical analysis results show that the group factors decrease as pile spacing decreases.

• For the two piles in a line group of free-head fixity condition, the $F_m$ of the front pile may be more or less than unity, depending on the spacing, while the $F_m$ of the back piles decreases from 0.76 to 0.55 as the spacing $S_b/d$ increase from 3 to 7. The group factors, $F_k$ of the front piles are approximately (20 ~ 25)% greater than those deduced for the back piles at the same pile spacing $S_b$. For the free-head pile groups, the $F_k$ increases about 71% and 64% for the front piles and back piles as pile spacing $S_b/d$ increases from 3 to 7. The $F_k$ for the capped-head cases is less sensitive to the spacing than those for the free-head cases.

• For the four piles in a square group (free-head), the $F_m$ remains constant at 0.8 for the front piles and decreases about 33% from 0.60 to 0.41 for the back piles as the $S_b/d$ increases from 3 to 7. The $F_k$ decreases with the increasing spacing for both the front and back piles. At the same pile spacing, the $F_k$ for the front pile are (0.42 ~ 0.95) times greater than those deduced for the back piles.
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## Table 6.2 Summary of group factors

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Table 6.3 Summary of group factors from previous study

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<th>Soil type</th>
<th>Pile group</th>
<th>Pile head condition</th>
<th>Spacing</th>
<th>F_m or F_p †</th>
<th>Reference</th>
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<td></td>
<td>Two piles in a row</td>
<td>Free-head</td>
<td>S_v/d=2.5</td>
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<td>Chen (1994)</td>
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<td></td>
<td>S_v/d=7.5</td>
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<td>Front piles</td>
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<td>S_b/d=5</td>
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<td>S_v/d=7</td>
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<td>Piles in one infinitely long row</td>
<td>Free-head</td>
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<td>Chen and Poulos (1997)</td>
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<tr>
<td>Clay †</td>
<td>Two piles in a row</td>
<td>Front piles</td>
<td>Head-tip-fixed</td>
<td>S_v/d=3</td>
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</tr>
<tr>
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<td></td>
</tr>
<tr>
<td></td>
<td>Back piles</td>
<td>Head-tip-fixed</td>
<td>S_v/d=3</td>
<td>0.77</td>
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</tr>
<tr>
<td></td>
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<td>S_v/d=3</td>
<td>0.41</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>S_v/d=5</td>
<td>0.76</td>
<td></td>
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</tbody>
</table>

† Group factor F_p = p_m/p_u, where p_m = p_u for a pile in the group; and p_u = p_a for a single isolated pile.

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Figure 6.1 Force on loading block against frame movements
(a) A single pile

(b) Two piles in a row

(c) Two piles in a line

(d) Four piles in a square

Figure 6.2 Soil movement surrounding piles ($w_f = 120$ mm)
Figure 6.3 Responses of two piles in a row (Tests G1, G2, G3, G4, G5 and G6)
Figure 6.4 Responses of two piles in a row (Tests G3, G4, G7, G8, G9 and G10)
Figure 6.5 Responses of two piles in a line (Tests G11, G13 and G15)
Figure 6.6 Responses of two piles in a line (Tests G12, G14 and G16)
Figure 6.7 Responses of four piles in a square (Tests G17, G19 and G21)
Figure 6.8 Responses of four piles in a square (Tests G18 and G20)
Figure 6.9 Maximum shear forces versus maximum bending moments
(a) $T_{\text{max}}$ from stable layer; and (b) $T_{\text{max}}$ from sliding layer
Figure 6.10 Variation of $M_{\text{max}}$ with $S_b$, $S_h/d$ or $S_v/d$
Figure 6.11 Evolution of $M_{\text{max}}$ and $T_{\text{max}}$ with $w_e$ for two piles in a row (Tests G1, G2, G3, G4, G5 and G6)
Figure 6.12 Evolution of $M_{\text{max}}$ and $T_{\text{max}}$ with $w_e$ for two piles in a row (Tests G3, G4, G7, G8, G9 and G10)
Figure 6.13 Evolution of $M_{\text{max}}$ and $T_{\text{max}}$ with $w_e$ for two piles in a line (Tests G11, G13 and G15)
Figure 6.14 Evolution of $M_{\text{max}}$ and $T_{\text{max}}$ with $w_e$ for two piles in a line (Tests G12, G14 and G16)
Figure 6.15 Evolution of $M_{\text{max}}$ and $T_{\text{max}}$ with $w_e$ for four piles in a square (Tests G17, G19 and G21)
Figure 6.16 Evolution of $M_{\text{max}}$ and $T_{\text{max}}$ with $w_e$ for four piles in a square (Tests G18 and G20)
Figure 6.17 Variation of k with $S_b$, $S_{b/d}$ or $S_{v/d}$
Figure 6.18 Variation of $F_m$ with $S_b$, $S_h/d$ or $S_v/d$
Figure 6.19 Variation of $F_k$ with $S_b$, $S_b/d$ or $S_f/d$
Figure 6.20 Comparison of group factors for two piles in a row
Figure 6.21 Comparison of group factors for two piles in a line
7 DESIGN PARAMETERS FOR LATERALLY LOADED RIGID PILES IN SAND

7.1 INTRODUCTION

In chapters 4 to 6, the responses of relatively rigid piles and pile groups subjected to lateral soil movements have been investigated using the model tests. The test results have been analyzed using Guo and Qin (2010) simple solutions. The solutions rely on appropriate estimation of the modulus of subgrade reaction, which is correlated with the shear modulus of the soil. As discussed in section 4.7.2, it is noted that Chen and Poulos (1997) used very low shear modulus of the sand (about 3.75 ~ 6.75 kPa) in their prediction of the responses of a model pile subjected to soil movement. The Young’s modulus (thus shear modulus) of the sand in their calculation was evaluated on the basis of back-calculation from laterally loaded pile tests. The purpose of this chapter was to assess the shear modulus of sand through back-calculation of laterally loaded rigid pile tests; and to investigate the difference in the modulus of subgrade reaction for piles subjected to lateral loading and soil movement.

Extensive theoretical and in situ full-scale studies, as well as laboratory model tests have been carried out on laterally loaded rigid piles in cohesionless soils. Several methods have been developed for predicting lateral capacity of rigid piles based on an assumed profile of soil resistance per unit length along a pile (Brinch Hansen, 1961; Broms, 1964b; Petrasovits and Awad, 1972; Meyerhof et al., 1981; Fleming et al., 2009; Prasad and Chari, 1999). These methods allow different lateral capacities to be determined, as
discussed in section 2.2.3.2, but not the deflection of piles at working loads. Elastic solutions have been presented by Broms (1964b), Poulos (1971), and Carter and Kulhaway (1992) to predict the elastic deflection of rigid piles under lateral loading, but not the nonlinear response of the piles. Guo (2008) established elastic-plastic solutions for the analysis of laterally loaded rigid piles. A constant modulus of subgrade reaction, $k$ or a linearly increasing modulus with depth, Gibson $k$, is assumed in developing the solutions, while a linear limiting force profile (LFP) is stipulated. Presented in explicit expressions in terms of the slip depth mobilized from the ground line or pile tip, the solutions enable nonlinear response of piles to be predicted and displacement-based capacity to be estimated. These solutions were used in the back-analysis of lateral loaded rigid piles in sand in this chapter.

The employed elastic-plastic solutions were firstly introduced and explained in relation to the calculation of nonlinear responses of the piles. The back-calculation were then carried out against measured data of 51 free-headed rigid piles in cohesionless soils, including 16 full-scale field tests, 12 centrifuge tests and 23 laboratory model tests. An example study on a full-scale field test was elaborated to demonstrate the calculation of the pile response at pile tip yield state. The criterion and reliability of the back-calculation were discussed. Normalized nonlinear load–displacement or moment–rotation responses were provided for all tests. The impact of load eccentricity on the nonlinear pile responses and pile capacity was highlighted. Estimation of shear modulus of sands from the back-analysis was discussed in detail. Guidelines for selecting design parameters for laterally loaded rigid piles in sand may be developed from this study.
7.2 ELASTIC-PLASTIC SOLUTIONS FOR LATERALLY LOADED RIGID PILES

7.2.1 Problem definition

A free-headed pile with a lateral load, $T_l$, applied at an eccentricity, $e$, above the groundline is schematically shown in Figure 7.1(a). The pile is defined as rigid if the pile-soil relative stiffness, $E_P/G_s$, exceeds a critical ratio, $(E_P/G_s)_c$, where $(E_P/G_s)_c = 0.052(l/r_0)^4$ (Guo and Lee, 2001) and $E_P$ is Young’s modulus of an equivalent solid pile, $G_s$ is the shear modulus of the soil, $l$ is the pile embedded length, and $r_0$ is an outer radius of a cylindrical pile.

7.2.2 Modulus of subgrade reaction

The pile-soil interaction is characterized by a series of springs distributed along the shaft. Each spring has an idealised elastic-plastic $p$–$y(u)$ curve at any depth shown in Figure 7.1(b). The soil resistance, $p$, is proportional to the local displacement, $u$ at the depth and to the modulus of subgrade reaction, $kd$, by

$$p = kdu \quad \text{(Elastic state)}$$

(7.1)

The magnitude of $k$, is related to the shear modulus of the soil by

$$kd = \frac{3\pi \overline{G}_s}{2} \left\{ 2\gamma \frac{K_1(\gamma)}{K_0(\gamma)} - \gamma^2 \left[ \frac{K_1(\gamma)}{K_0(\gamma)} \right]^2 - 1 \right\}$$

(7.2)

where $d$ is outer diameter of the pile, $\overline{G}_s$ is an average shear modulus of the soil over the pile embedded length, $K_i(\gamma)$ is the modified Bessel function of second kind of $i\text{th}$ order.
(i = 0, 1); \( \gamma \) is a non-dimensional factor given by \( \gamma = \frac{k_i}{l} \), \( k_i = 2.14 \) and 3.8, respectively for pure lateral load \( (e = 0) \), and pure moment loading \( (e = \infty) \). The value of \( k_i \) can be approximately estimated by \( k_i = 2.14 + e/l/(0.2 + 0.6e/l) \) (Guo, 2009c), increasing from 2.14 to 3.8 as \( e \) increase from 0 to \( \infty \). The \( k \) may be written as \( k_0 \omega^m \) \( [k_0, \text{FL}^{-m \cdot 3}] \), with \( m = 0 \) and 1 being referred to as constant \( k \) \( (k = k_0) \) and Gibson \( k \) \( (k = k_0z) \) hereafter. It should be pointed out that in order to keep the equations in Tables 7.1 and 7.2 consistent with those presented by Guo (2008), the diameter of the pile, \( d \) is incorporated into equation (7.1), thus the modulus of subgrade reaction is the product of \( k \) and \( d \) in this chapter. For the constant \( k \) and Gibson \( k \), the \( k \) and \( k_0 \) have a unit of MN/m\(^3\) and MN/m\(^4\), respectively.

### 7.2.3 Limiting force profile

Once the local pile displacement, \( u \) exceeds a threshold value of \( u^* \) as seen in Figure 7.1(b), \( p \) reaches its limiting value, \( p_u \) and the pile–soil relative slip is initiated. It is assumed that the \( p_u \) increases linearly with depth \( z \) for a rigid pile in a cohesionless soil and may be described by

\[
\frac{z}{p_u} = A_r z d \quad \text{(Plastic state)}
\]

(7.3)

where \( A_r z \) is net limiting pressure on the pile surface, and \( A_r \) may be expressed as

\[
A_r = N_t \gamma_s K_p^2
\]

(7.4)

where \( \gamma_s \) is an effective unit weight of the soil (dry weight above water table, and buoyant weight below); \( K_p = \tan^2(45^\circ + \phi_s / 2) \), is the coefficient of passive earth
pressure; \( \phi' \) is an effective frictional angle of soil; \( N_g \) is a non-dimensional parameter.

The actual \( N_g \) may be back calculated from the measured pile responses as shown later.

### 7.2.4 Explicit expressions for the solutions

Guo (2008) developed elastic-plastic solutions for either a Gibson \( k \), or a constant \( k \), by stipulating a linear limiting force profile (LFP) described by equation (7.3). These solutions are presented in explicit expressions characterised by slip depths. Their non-dimensional forms are presented in Table 7.1 for pre-tip yield and tip yield states, and in Table 7.2 for post-tip yield and yield at rotation point states (YRP) (explained later on), in terms of pile-head load \( T_i/(A_r \cdot dl^2) \); groundline displacement \( u_0k_0/A_r \) (Gibson \( k \)) or \( u_0k/(lA_r) \) (constant \( k \)); rotation angle \( \omega k_0/A_r \) (Gibson \( k \)) or \( \omega k/A_r \) (constant \( k \)); depth of maximum bending moment, \( \bar{z} \); and maximum bending moment \( M_{max}/(A_r \cdot dl^3) \). The equations to determine the distribution of the shear force and bending moment for the corresponding yield states are provided in Table 7.3 and Table 7.4. These equations were not shown in the paper by Guo (2008) and were also derived independently by the author. The four typical pile soil interaction states depending on the pile displacement are explained below.

1. The pile has a displacement \( u = \omega \varepsilon + u_0 \), where \( u_0 \) is the pile displacement at groundline, \( \omega \) is the rotational angle. It rotates about a depth \( z_r = -u_0/\omega \), where the local pile deflection \( u = 0 \).
(2) The soil resistance \( p \) attains \( p_u \) once the deflection \( u \) exceeds \( u^* \) \([= A_r/k_0 \) (Gibson \( k \) or \( = A_rz_0/k \) (constant \( k \))]. In particular, once the pile tip displacement \( u \) \((z = l)\) touches \(-u^*\) (Gibson \( k \)) or \(-u^* l/z_0\) (constant \( k \)), or the \( p \) \((z = l)\) at the pile tip touches \( A_rld \), the pile is said at tip yield state. The soil resistance per unit length along the pile, i.e., the on-pile force profile at tip yield state is illustrated in Figure 7.2(a).

(3) The on-pile force per unit length, \( p \), follows the positive \( p_u \) profile given by equation (7.3) to the slip depth \( z_0 \) from groundline. Below the \( z_0 \), it is described by equation (7.1) at the pre-tip yield state since the pile–soil interaction is still in elastic state. After the pile tip yields, or post-tip yield, pile-soil relative slip may also initiate from the pile tip and expand upwards to another slip depth, \( z_1 \) as illustrated in Figure 7.2(b) and finally approach the practically unachievable yield at rotation point (YRP) in Figure 7.2(c).

7.2.5 Spreadsheet programs

Guo (2008) implemented the solutions into a spreadsheet program called GASLSPICS. The author also coded a spreadsheet program independently based on the expressions presented in Tables 7.1 to 7.4, with which the calculations were undertaken in this thesis. User-defined functions for calculating some critical terms were written using macros in Microsoft Excel VBA. The following parameters are required to use the program:

- pile dimensions of \( d \) and \( l \), and soil parameters \( \phi_s \) and \( \gamma_s \);
- eccentricity \( e \), of the applied lateral load \( T_l \); and
- the parameters \( A_r, \) \( k_0 \) and \( k \).
The program can be used in two different ways. First, for a given set of input soil and pile properties, it can be used to predict the nonlinear response and estimate the ultimate lateral capacity of the pile. Secondly, it can be used to deduce the parameters $A_r$, $k$, and $k_0$ that describe the modulus of subgrade reaction, and gradient of the linear limiting force profile by matching the predicted with measured pile response.

7.3 BACK CALCULATION OF MEASURED PILE RESPONSES

7.3.1 Characteristics of pile tests

In total, the measured data from 60 laterally loaded piles constructed in horizontal ground, near and on slopes in cohesionless soils were collected. Only the 51 pile tests in horizontal ground were studied, comprising 16 full-scale field tests, 12 centrifuge tests and 23 model tests. The pile geometry of diameter, $d$, embedded length, $l$, and loading eccentricities, $e$ for each test are summarized in Table 7.5. The properties of sand including the relative density, $D_r$, the internal friction angle, $\phi$, and unit weight, $\gamma$, are given in Table 7.6. Statistical analysis of the pile and soil properties is provided in Table 7.7. The measured pile responses are plotted by scatter points in Figures 7.3 to 7.22.

7.3.2 Back calculation criteria

Theoretically two measured response ($T_t \sim u_0$ and $T_t \sim M_{max}$ (or $\omega$) curves) can be fitted by using the program to uniquely deduce the parameters $A_r$, $k$ and $k_0$. For the majority of pile tests, however, only one measured pile load-displacement relationship; either $T_t \sim u_0$ ($u_t$) or $M_0 \sim \omega$ is available. Back calculation may still be carried out as the initial elastic
portion and subsequent nonlinear portion of the measured \( T_t \sim u_t(u_t) \) or \( M_0 \sim \omega \) curves may serve the purpose, but it may result in some difference in the determination of the \( A_r \), and \( k (k_0) \).

Back calculations were carried out by best matching with the measured responses of the 51 test piles. The deduced parameters \( A_r \), and \( k (k_0) \) were tabulated in Table 7.6. The calculated pile responses with a Gibson \( k \) and constant \( k \) were plotted in Figures 7.3 to 7.22 using dotted and solid lines, respectively. Moreover, the calculated pile responses at tip yield state were highlighted using hollow dot points \( \circ \), and solid dots \( \bullet \) for Gibson \( k \) and constant \( k \) in these figures. Two typical case studies were elaborated next.

7.3.3 Case study 1 – Full-scale field tests of steel pole foundations in loose sand (F1)

Haldar et al. (2000) conducted eight full-scale field tests on fully instrumented steel transmission pole foundations. Each pole consisted of top and bottom sections with diameters of 0.779 m and 0.740 m (an average diameter, \( d \), of 0.76 m). The two parts were joined together by bolted connections. The typical cross section of the pole was a 12-sided polygon. The embedded length, \( l \), of the pole varied from 2.36 m to 3.2 m. The lateral loads were applied at an eccentricity, \( e \), of approximately 23.0 m to investigate the responses of pole foundations under a large moment. Each pole was instrumented to measure the applied load at the top of pole, and deflections near the ground line. The rotation of the pole was derived from the deflection of the pole at two different distances. Ten strain gauges were installed at different sections of the pole to measure distribution of the bending moment at selected depths. Lateral loads were applied in an incremental
manner until either the safe structural capacity of the pole was reached or a greater
deformation at ground line was observed.

The poles were tested in four different types of backfills, namely; sand, in situ gravelly
sand, crushed stone and flowable material. The loose to medium dense sand backfill
(F1~F5) had relative densities, $D_r$ of 22%~56%, effective unit weight, $\gamma'$ of 16.4~17.6
kN/m$^3$, and effective internal frictional angle, $\phi'$ of 32.6°~39.2°, respectively. The dense
crushed stone (F6) and in situ gravelly sand (F7) have a relatively density of 85% with
larger effective frictional angles of 49.8° and 42.7°, respectively.

In the case of test F1, the pole was tested in loose sand backfill. The diameter of the pole,
$d$, and embedded depth, $l$ is 0.7545 m and 3.2 m, respectively. The lateral load was
applied at an eccentricity, $e$, of 22.25 m. The measured $M_0$ ~ rotation $\omega$ curve is plotted
in Figure 7.3(a). The measured pole displacement and bending moment distribution with
depth at ground line moment, $M_0$ of 245 kNm, 365 kNm, 485 kNm, and 685 kNm are
plotted in Figures 7.3(b) to 7.3(c). The measured soil pressure on the pile using pressure
cells at $M_0 = 685$ kNm is plotted in Figure 7.3(d).

The back estimated pile responses for best fitting the measured curves are also plotted in
Figures 7.3(a) to 7.3(d). This was achieved by taking $A_r = 400$ kN/m$^3$, $k_0 = 26.1$ MN/m$^4$,
and $k = 41.0$ MN/m$^3$. With these deduced parameters, the computation of the pole
responses at tip yield state with Gibson $k$ and constant $k$ are presented in Table 7.8 and
Table 7.9, respectively. The following features are observed from these figures.
(1) The pile responses at the tip yield state are highlighted in Figure 7.3(a) using hollow dot points ○, and solid dots ● for Gibson $k$ and constant $k$, respectively. Taking the same value of $A_r$, back calculation using the solutions with a constant $k$ gives better match of the measured $M_0 \sim \omega$ relationships.

(2) Figures 7.3(b) and 7.3(c) show that pile deflections were well predicted, while the bending moment distributions were slightly overestimated especially at high load levels for both $k$.

(3) The measured moment at ground line $M_0$ of 685 kNm is close to the estimated $M_0 = 682.4$ kNm (Gibson $k$) and $M_0 = 751.65$ kNm (constant $k$) (see Table 7.8) at the tip yield state. Therefore, the on-pile force profiles at tip yield state are also plotted in Figure 7.3(d) for comparison. These profiles were constructed by drawing lines in sequence between adjacent points $(0,0), (A_r, z_0^r, z_0), (0,z_r)$ and $(-A_r, l,l)$ using $A_r$ and the corresponding calculated $z_0^r, z_r$ at tip yield state presented in Table 7.8 for both $k$. The soil pressure distribution proposed by Prasad and Chari (1999) is also included in Figure 7.3(d). The measured data fall within the zones enclosed by the individual soil pressure profile. This finding further confirms that the pole was at pre-tip yield or close tip yield state.

(4) The ultimate ground line moment of the pole was estimated as 875.7 kNm. This value is 2.4% greater than the reported ultimate moment of 855 kNm at 5° rotation of the pole.
Case study 2 - Centrifuge test of steel pipe pile in medium dense sand (C2)

Three centrifuge model tests were conducted on laterally loaded piles in uniform fine-grained dry sand at an acceleration of 50g (Georgiadis et al., 1992). Each pile was instrumented with five pairs of strain gauges to measure the bending moments along the pile length. Among them, a prototype steel pipe pile (C2, pile P2) was 9.05 m in length, 1.224 m in outer diameter and 17.25 mm in wall thickness with flexural stiffness $E_p I_p = 2495.0$ MN.m$^2$. The sand was compacted to medium dense to dense state with a relative density, $D_r$ of 60%. The average unit weight $\gamma'$ and effective internal frictional angle, $\phi'$ was 16.3 kN/m$^3$ and 36°, respectively. Load was applied at an eccentricity of 1.25 m above the ground line. The measured load $T_t$ – pile head displacement $u_t$ curve is plotted in Figure 7.4(a). The distribution of bending moment, shear force and pile displacement with depth at $T_t = 1304$ kN are plotted in Figures 7.4(b) to 7.4(d).

With the recommended $k_0 d = 11$ MN/m$^3$ (Georgiadis et al., 1992) and $E_p I_p = 2495.0$ MN.m$^2$, the dimensionless length $\eta l$ of the pile is calculated as 3, in which $\eta = (k_0 d / E_p I_p)^{1/5} = 0.34$/m. The pile is neither short ($\eta l \leq 2$ ) nor long ($\eta l \geq 2$ ) (Broms, 1964b), but of medium length. Although the pile is not strictly rigid, back calculations have been carried out to explore the flexibility effect on the pile response. With the deduced parameters $A_r = 280$ kN/m$^3$, $k_0 = 3$ MN/m$^4$, $k = 10$ MN/m$^3$, it can be concluded that the predicted $T_t \sim u_t$, and distributions of bending moment and shear force at $T_t = 1304$ kN compare well with the measured curves. The predictions generally bracket the measured pile response using the proposed Gibson $k$ and constant $k$, with a marginally
better match using the Gibson \( k \). However, it is noted that the solutions over predict the pile displacement around pile tip in Figure 7.4(b). This discrepancy may be attributed to the relatively flexible features of the pile.

### 7.4 DISCUSSIONS

#### 7.4.1 Reliability of the back calculation

The 51 pile tests may be divided into three groups based on the number of measured pile response curves.

1. Eight tests with two or more response curves
   
   They are piles in tests F1 (Figure 7.3), F12~13 (Figures 7.7 and 7.8), C1~2, (Figures 7.4 and 7.10) and M21~23 (Figures 7.20 to 7.22).

2. Thirteen tests with the load displacement curve, \( T_i \sim u_0 \) \( (u_i) \) or \( M_0 \sim \omega \) ranging from elastic to a clear ultimate state
   
   They are piles in tests F14~16 (Figure 7.9) and C3 ~ 12 (Figures 7.11 and 7.12).

3. The remaining 30 piles tests
   
   Only the load ~ displacement curves were measured in these tests and they have no clear indication of failure.

The back-estimated \( A_e, k \) and \( k_0 \) for the 21 tests in the first and second groups are warranted and reliable as discussed in section 7.3.2. These have been corroborated by good agreement between the back estimated curves with those measured in the
corresponding figures. The back estimated results in the third group may vary to some extent if additional measured responses are available.

Figures 7.3 to 7.22 show that the calculations with a constant $k$ generally offer a better estimation against measured nonlinear responses of the piles than those based on Gibson $k$, in light of the linear limiting force profile with the same gradient $A_r$. However, the back calculation also reveals that the elastic-plastic solutions for both $k$ cannot accurately predict the pile responses in tests M3, M8, M10 and M18. In these tests, the piles were embedded in loose sand with low relative density and the measured load-displacement curves show stress hardening characteristics. The following discussions will focus on the results from back calculation using the solutions with constant $k$.

7.4.2 Effect of $e/l$ on nonlinear pile response

In order to investigate the effect of $e/l$ on the pile responses, the measured load ~ displacement, $T_t \sim u_o(u_t)$ and $M_o \sim \omega$ data for each test were normalised by $A_r dl^2$, $A_r/l/k$, $A_r dl^2$ and $A_r/k$, respectively, using the deduced $A_r$ and constant $k$ tabulated in Table 7.6. The normalized lateral load versus groundline displacement or pile head displacement data were plotted using linear plot and log-log plot in Figure 7.23(a) and 7.23(b), respectively. Similarly, the normalized moment versus groundline rotation data were plotted in Figure 7.24. The non-dimensional $T_t/(A_r dl^2) \sim u_o k/(A_r l)$ and $M_o/(A_r dl^3) \sim -\omega k/A_r$ curves at various $e/l$ ratios obtained from the solutions with constant $k$ are also plotted using bold solid lines in these figures. At a specific $e/l$, the normalised measured $T_t \sim u_o(u_t)$ or $M_o \sim \omega$ curves merge together or fall within a very narrow band around the
solid line, regardless of soil properties. It can be seen that the ratio $e/l$ has a very significant impact on the normalised load $T_i/(A, dl^2)$, which reduces with the increase of $e/l$. For instance, at $u_0k/(A,l)=2$, the reduction of $T_i/(A, dl^2)$ is about 50% from 0.09 to 0.048 as $e/l$ increases from 0 to 0.714.

**7.4.3 Effect of $e/l$ on pile capacity $T_0$ and $M_0$**

The measured ultimate lateral capacities of 29 tests were reported in terms of either lateral load, $T_0$ or ground line moment, $M_0$, as tabulated in Table 7.5. These ultimate capacities were estimated based on the following two criteria:

1. the load under which the lateral load – pile head displacement curve becomes linear or substantially linear (Meyerhof et al., 1981; Chari and Meyerhof, 1983; Meyerhof and Sastry, 1983; Prasad and Chari, 1996, 1999; and Lee et al., 2010); or
2. the lateral load/moment inducing a certain rotation angle, say $3.5^\circ$–$5.5^\circ$ (Laman et al., 1999; Dickin and Laman, 2003) or $5^\circ$ (Haldar et al., 2000).

Figure 7.25(a) and Figure 7.25(b) show the normalised measured pile capacity $T_0/(A, dl^2)$ and moment $M_0/(A, dl^3)$ against normalised eccentricity $e/l$, respectively, with those at tip yield and yield at rotation point (YRP) presented by Guo (2008). Figure 7.25(a) suggests that the measured ultimate lateral load, $T_u$, is generally less than the predicted capacities at tip yield state. By contrast, Figure 7.25(b) shows that the measured ultimate ground line moment, $M_u$ falls in the range of the capacity between tip yield state and yield at rotation point, except in M14 and M16. In these two tests, the measured
values of $M_u$ were obtained at a much lower pile rotation angle, $\omega$ of around 1.5°. Overall, the pile capacities at the yield at rotation point provide upper bound estimation.

### 7.4.4 Estimation of shear modulus $G_s$

The modulus of subgrade reaction, $k_d$ is related to the average shear modulus $G_s$ of the soil over the embedded length of the pile through equation (7.2). Conversely, the shear modulus of the sands can be deduced from the back-calculated modulus of subgrade reaction. On the other hand, the small-strain shear modulus, $G_{max}$ may serve as a benchmark or reference value for the estimation of shear modulus, as reviewed in section 2.4, and many empirical equations have been suggested to estimate the $G_{max}$ (see Table 2.14). As listed in Table 7.6, in the majority of the 51 tests, only the relative density, $D_r$ of the soils is provided. Therefore, the method proposed by Seed and Idriss (1970) is used first to calculate the $G_{max}$.

Seed and Idriss (1970) and Seed et al. (1986) suggested that the shear modulus of cohesionless soils can be estimated using the following equation:

$$G_s = 218.8K_2(\sigma_m')^{0.5}$$ (7.5)

where $G_s$ is in kPa; $\sigma_m'$ is the effective mean principal stress in kPa, which is related to the vertical effective stress $\sigma_v'$ by $\sigma_m' = [(1 + 2K_0)/3]\sigma_v'$, where $K_0 = 1 - \sin \phi'$, is the coefficient of earth pressure at rest. The vertical effective stress, $\sigma_v'$ around a pile varies with depth; therefore, an appropriate choice is to use the average vertical effective stress along the embedded length of the pile. $K_2$ is a shear modulus coefficient describing the influence of void ratio and strain amplitude developed in the soils. At very low shear
strains (less than or equal to $10^{-4}\%$), $G_s = G_{\text{max}}$ and $K_2$ approaches a maximum value $(K_2)_{\text{max}}$ that depends on the relative density, $D_r$, and can be estimated from (Seed and Idriss, 1970; Yan and Byrne, 1992)

$$(K_2)_{\text{max}} = 3.5(D_r)^{2/3} \quad (7.6)$$

Seed et al. (1986) stated that the values of $(K_2)_{\text{max}}$ range from about 30 for loose sands to about 75 for dense sand and they were 1.35 to 2.5 times greater for gravels than for sands. To estimate the $G_{\text{max}}$, given the relative density $D_r$, the value of $(K_2)_{\text{max}}$ can be estimated from equation (7.6). Substituting the calculated $(K_2)_{\text{max}}$ into (7.5), the small-strain shear modulus $G_{\text{max}}$ is calculated.

The second and fourth methods presented by Wichtmann and Triantafyllidis (2009) (see Table 2.14) have also been used to calculate the $G_{\text{max}}$ and to provide an order-of-magnitude check against the calculated values of $G_{\text{max}}$ from Seed and Idriss (1970) method.

It should be noted that the sands in tests F6 ~ F11 include dense crushed stone, dense gravelly sand and dense gravelly layers. The values of $(K_2)_{\text{max}}$ calculated from (7.6) in these tests are doubled (the approximate average value of 1.35 ~ 2.5) in the calculation on the basis of the suggestions of Seed et al. (1986). At the same time, a shear modulus $G_s$ can be deduced using equation (7.2) from the back-calculated $kd$. Calculation of the $G_{\text{max}}$ and $G_s$ has been performed for each pile test and the values of $G_{\text{max}}$ and $G_s$ are tabulated in Table 7.6. The detailed calculation of $G_s$ and $G_{\text{max}}$ for the 51 tests is provided in Tables 7.10 and 7.11.
Table 7.10 shows that

- For the full scale field tests and centrifuge tests C1 and C2, $\frac{k_d}{G_s} = 3.27 \sim 6.91$, with an average of 5.0. For the model tests, $\frac{k_d}{G_s} = 2.37 \sim 5.12$, with an average of 3.7. Greater values of $\frac{k_d}{G_s}$ with an average value of 13.32 for the centrifuge tests C4 $\sim$ C12 were obtained. The main reason for the high values of $\frac{k_d}{G_s}$ in these tests is due to the rectangular cross section of the piers. The equation (7.2) was originally presented by Guo and Lee (2001) and used in the current back calculation. The equation was derived from a two-parameter load transfer model for a circular solid pile embedded in linear, homogeneous and isotropic soils. Strictly speaking, it is not applicable for piles with other shapes of cross sections rather than circular (Basu and Salgado, 2008), especially the rectangular pier of large width from 1 m to 6 m in C4 $\sim$ C12. Therefore, the back-calculated shear modulus from these tests will not be included in the statistical analysis conducted later on. The same reason may also partly explain the relatively high values of $\frac{k_d}{G_s}$ in tests F1 $\sim$ F7 with the 12-sided polygonal pole. The use of equation (7.2) is a compromise between simplicity and consistence.

- The effect of loading eccentricity and pile diameter on the shear modulus has been considered in the back calculation. When the pile diameter and embedded length are kept constant, the increase of loading eccentricity generally results in an increase of $\frac{k_d}{G_s}$. For instance, the $\frac{k_d}{G_s}$ increases from 3.93 to 4.47 as the eccentricity increases from 0.15 m in test F16 to 2 m in test F15. Since the pile diameter is incorporated into the modulus of subgrade reaction, the ratio $\frac{k_d}{G_s}$ appears to increase with increasing pile diameter. For example, in the series of tests M5 $\sim$ M7
reported by Adams and Radhakrishna (1973), when the pile diameter increases from 0.0508 m to 0.1016 m, the \( kd/G_s \) increases from 3.83 to 5.09.

Table 7.11 shows that

- The values of \( G_{max} \) calculated using the second method and fourth methods presented by Wichtmann and Triantafyllidis (2009) are within approximately ±25% and ±20% of those calculated by the equations (7.5) and (7.6) suggested by Seed and Idriss (1970) and Seed et al. (1986). Thus the \( G_{max} \) calculated by equations (7.5) and (7.6) is reliable and can be used as a benchmark value for the estimation of shear modulus.

- The ratio of \( G_s/G_{max} \) for the three tests F14 ~ F16 reported by Lee et al. (2010) is much greater than those obtained in the rest of the full scale field tests. The back-calculated \( G_s \) for test F16 is even 22% greater than the calculated \( G_{max} \), which appears to be unrealistic. The reason for this discrepancy is possibly due to that the piles were tested in clayey sand. Unfortunately, detailed properties of the clayey sand were not reported. The clayey sand may have much higher plasticity than the cohesionless soils. Vucetic and Dobry (1987) show that the plasticity index is the main factor controlling \( G_s/G_{max} \) for a wide variety of soils ranging from clays to sands and greater plasticity index is associated with less reduction of \( G_s/G_{max} \). This study also indicates that the equations suggested by Seed and Idriss (1970) and Seed et al. (1986) may not be suitable for estimating the \( G_{max} \) for the clayey sand.

- The model tests M5 ~ M7 were conducted in extremely dense sand with the relative density \( D_r =100\% \). The ratio of \( G_s/G_{max} \) for the three tests is approximately 0.12. This value is about 8.7 times greater than the average value of 0.014 for the \( G_s/G_{max} \).
obtained from the rest of the model tests. The exact reasons for the large difference are not clear.

- Due to the reasons mentioned above, the analysis results of the 15 tests, F14 ~ F16, C4 ~ C12 and M5 ~ M7, were excluded in the statistical analysis presented next. In addition, results from the three tests, F12, F13 and C3, were excluded as well since the relative density, $D_r$ for the sands were not available. The ratio of $G_s/G_{\text{max}}$ is plotted against the relative density $D_r$ for the rest of 33 tests in Figure 7.26. The plot shows that the back-calculated $G_s$ was approximately (3~20)% of $G_{\text{max}}$ with an average of 11.3% for the 11 full-scale field tests (F1 ~ F11) and 2 centrifuge tests (C1 ~ C2), while it was (0.8~2.6)% of $G_{\text{max}}$ with an average of 1.4% for the 20 model tests. The difference in the $G_s/G_{\text{max}}$ between the full-scale field tests and model tests may be attributed to the scale effect, stress and strain levels in the soils and other issues. They are discussed below.

Firstly, due to the scale effect, the stress and strain level of the sands in the field tests, centrifuge tests, and model tests may be different. As presented in Table 7.11, the average vertical effective stress, $\sigma'_m$ for the 11 full-scale field tests (F1~F11) is calculated as 18.3 kPa, while it is approximately only 2.4 kPa for the 23 model tests. In order to explain the effect of stress level on the $G_s/G_{\text{max}}$, Figure 7.27 shows typical test results from the study made by Kokusho (1980) on the Japanese Toyoura sand ($D_{50} = 0.19$ mm, $C_u =1.3$). The plot indicates that the rate of reduction in shear modulus with strain becomes greater as the confining pressure decreases. With the same set of data for Toyoura sand, Pestana and Salvati (2006) further demonstrated
that the effect of confining pressure is much greater than the effect of density on the small-strain nonlinearity. As also illustrated in Figure 7.27, the increase in the confining pressure leads to an increase in the shear modulus at a given strain level. On the other hand, Poulos et al. (2001) stated that even though the stress level may be similar for different foundations, the strain level in the soil may be different. The back calculated $G_s$ may correspond to different stress and strain levels. The strain dependency of shear modulus and difference in confining pressure or vertical effective stress may contribute partly to the varying degree of reduction of $G_s$ in comparison of $G_{max}$ in the field and model tests.

Secondly, the piles were tested in various soil types in the 51 tests, including fine clean dry loose sand, silty sand, silty and silty sand with gravel layers, coarse uniform angular sand, etc. The natural soils in the field tests may be nonlinear, inhomogeneous and anisotropic. As discussed previously, equation (7.2) is suitable for linear, homogeneous and isotropic soil. It provides rough estimation.

Thirdly, the field tests consist of bored piles, cast-in-place piers and drilled piers etc. The lateral loading on the piles and piers may be applied at different rate. The experimental results by Dyson and Randolph (2001), Kim et al. (2004) and Kong and Zhang (2006) show that the method of installation and loading rate can affect the behavior of the piles subjected to lateral loading. These factors may have some effect on the back calculated results, which warrant further research.
Fourthly, the possible improvement of the analysis lies in the estimation of the maximum shear modulus, $G_{\text{max}}$. As reviewed in section 2.4, the $G_{\text{max}}$ for sand is primarily controlled by the relative density and confining pressure. Several expressions relating $G_{\text{max}}$ to other soil parameters, such as void ratio, $e$, coefficient of uniformity, $C_u$, and relative density, $D_r$, have been devised for various granular soils ranging from crushed quartz sands to gravels. The recent study by Wichmann and Triantafyllidis (2009) shows that the correlations of $G_{\text{max}}$ in terms of void ratio are more precise than the correlations of $G_{\text{max}}$ with relative density. The rough empirical equations formulated in terms of relative density may be sufficient for practical purpose. The range in the ratio $G_s/G_{\text{max}}$ may be narrowed if more pile tests are available along with the properties of the soils, such as void ratio and coefficient of uniformity, are known. Further work need to be done in the future.

To illustrate the effect of shear modulus on the load–pile head displacement relationship, parametric studies were undertaken for test M20. For this test, the $G_{\text{max}}$ was determined as 23.341 MPa from Seed and Idriss (197) method (see Table 7.11). The current back calculation to the measured data (Figure 7.19) provided $A_r= 890$ kN/m$^3$, $k=16.960$ MN/m$^3$, allowing $N_y= 1.362$ and $G_s= 0.413$ MPa or $G_s/G_{\text{max}}= 1.77\%$ to be deduced. Guo (2008) also analyzed this test and gave $A_r= 739$ kN/m$^3$, $k=16.960$ MN/m$^3$. With $A_r$ kept constant as 890 kN/m$^3$, the effect of changing the shear modulus for $G_s/G_{\text{max}}= 1.0\%$, 1.77\%, 2.0\%, and 3.0\% has been explored. The calculated results in terms of pile head displacement and load capacity at pile tip yield,
and the ultimate capacity at yield at rotation point from these analyses are presented in Table 7.12.

Figure 7.28 plots the measured data and the calculated load ~ pile head displacement curves from analysis 1 ~ 4. As expected, greater shear modulus gives stiffer pile response. Because $A_r$ was kept constant, equation (C-2) combined with equation (C-8) in Table 7.1 indicates that the pile head displacement at pile tip yield state is inversely proportional to the $k$, thus to the shear modulus $G_s$ according to equation (7.2). For instance, the calculated pile head displacement, $u_t$ of 162.5 mm from analysis 1 with $G_s/G_{\text{max}} = 1.0\%$ was three times that of 54.17 mm from analysis 3 with $G_s/G_{\text{max}} = 3.0\%$. Nevertheless, the variation of shear modulus has no effect on the predicted capacities at the yield at rotation point and pile tip yield states. They depends only on the ratio of loading eccentricity to pile embedment length, $e/l$ in (see equations (C-1) and (C-8) in Table 7.1 and equations in Table 7.2).

Analyses have also been undertaken using the parameters provided by Guo (2008). The load ~ pile head displacement curve is plotted as ‘Guo (2008)’ in Figure 7.28. With the $k$ taken as 16.96 MN/m$^3$ in both studies, the current calculated $A_r$ increases by 20.4% from 739.0 kN/m$^3$ to 890 kN/m$^3$ in test M20, resulting in 21% increase in the predicted ultimate lateral capacity. This difference is due to that the current back-calculation was based on best match with the measured pile responses.
7.4.5 Difference in modulus of subgrade reaction between lateral loading and soil movement

The modulus of subgrade reaction can be calculated from the shear modulus by equation (7.2). Therefore, the difference in the modulus of subgrade reaction between lateral loading and soil movement may be discussed in terms of shear modulus.

The following seven tests are excluded: (1) M5 ~ M7 ($D_r=100\%$); (2) M8 and M9 (with a small pile embedment length of 0.2 m); (3) M17 (conducted in crushed stone); and (4) M18 ($D_r=25\%$). The shear modulus deduced from the rest of the 16 model tests ranges from 181 kPa to 600 kPa, with an average of 316.7 kPa. This value is approximately 8.6% less than the average value of the shear modulus of 367 kPa deduced from tests M21 to M23 reported by Qin and Guo (2007). By considering the similar order of the pile size and relative density of the sands, the value of the shear modulus for the sand deduced from the laterally loaded pile tests is much greater than 15 ~ 20 kPa used in Guo and Qin (2010) solutions for piles subjected to lateral soil movement and 3.75 ~ 6.75 kPa provided by Chen and Poulos (1997) for estimation of the single model pile tests, as discussed in section 4.7.2. The possible reasons for the difference are discussed below.

As reviewed in section 2.4, the shear modulus values for sands are primarily influenced by three main factors: (1) the confining pressure; (2) the void ratio (or relative density); and (3) the strain amplitude (Seed et al., 1986). The tests on piles subjected to lateral soil movement in chapters 4 to 6 and the static loaded pile tests M21 to M23 were conducted using the same apparatus and sand. The model ground was prepared under identical
falling height, thus the same relatively density. The variation of confining pressure and relative density of sand and their influence on the shear modulus may be assumed to be negligible. The remaining main factor influencing the shear modulus is the shear strain level. Prakash and Kumar (1996) show that the average shear strain around laterally loaded piles may fall in the range of 0.002 ~ 0.02 under working load condition. Guo and Qin (2010) show that the maximum shear strain at the loading side was approximately 0.25 ~ 0.3 at \( w_f = 50 \sim 60 \) mm in the passive model pile tests with a sliding depth of 200 mm using the triangular loading block 1 (Figure 3.14(a)). Based on the test data for Ottawa sand reported by Hardin and Kalinski (2005), Carter (2006) carried out analyses and extrapolated the results beyond the range of shear strains measured in the experimental tests, but he warned that it was potentially dangerous to do so. Figure 7.29 presents the extrapolated results given by Carter (2006), showing typical variation of Young’s Modulus (hence also shear modulus) with confining pressure and shear strain for sand. It can be seen that the Young’s Modulus at shear strain of 0.1 is much less that when the shear strain is in the range of 0.001 ~ 0.01. The large discrepancy in the shear strain level may result in the difference in the shear modulus used in the solutions proposed by Guo and Qin (2010) and in the analysis of laterally loaded piles. This explanation is likely to be reasonable because the pile responses of maximum bending moment and maximum shear force in Guo and Qin (2010) solutions were correlated to the frame movement of the shear box at which free soil movement was induced. The shear modulus should be deduced from the overall sliding process characterized by the sand pile–shear box interaction rather than the local pile-soil interaction (Guo and Qin, 2010). Further research is warranted.
7.4.6 Estimation of $N_g$

In order to provide guidelines for estimating $p_u$ for rigid piles in cohesionless soils, the value of the dimensionless parameter $N_g$ was deduced from the back-calculated $A_r$ for each test using equation (7.4) and tabulated in Table 7.6. By comparing equations (7.3) and (7.4) with the expressions for ultimate resistance in sand presented in Table 2.3, the limiting force profile (LFP) described by equation (7.3) may be termed as Broms LFP if $N_g = 3/K_p$ or as Barton LFP if $N_g = 1$. Table 7.6 shows that:

- Excluding the five tests, (1) F2 (pile tested in very loose sand); (2) F6 (in dense crushed stone); and (3) F14 ~ F16 (in clayey sand due to the reasons noted in section 7.4.5), the value of $N_g$ was evaluated as 1.0 ~ 3.0, with an average of 1.41, from the rest 11 full-scale field tests and the three centrifuge tests C1, C2 and C3. The gradient of the limiting force profile for rigid piles in sand is thus approximately 41% higher than that in Barton LFP on average.

- The $N_g$ decreases with increase in pile diameter or width, $d$. In particular, $N_g$ reduces from 0.63 to 0.44 as the width of the rectangular pier increasing from 1m (test C4) to 6m (test C8) for the centrifuge tests reported by Dickin and Laman (2003). In view of the large pier width, it is reasonable to take the rectangular pier as a rigid wall rather than a pile, thus reducing the three-dimensional effect.

- The value of $N_g$ varies from 0.70 to 4.77 with an average value of approximately 2.0 for the 23 model tests. The difference in the value of $N_g$ between the field tests and model tests is probably related to the scale effect and stress levels in the soils.

- The range for $N_g$ deduced from the current analysis on rigid piles is consistent with that obtained by Guo and Zhu (2010) from the study on 20 flexible piles in...
sand. They provided $N_g = 0.4 \sim 2.8$, with an average of 1.29, but with $n = 1.7$ (see equation 2.8) to reflect the influence of the pile flexibility.

Excluding the three tests F14, F15, and F16 in clayey sand, the back-calculated $N_g$ from the rest of 48 tests was plotted against the normalized pile diameter, $d/d_{ref}$ in Figure 7.30. The variation of $N_g$ with pile diameter may be estimated by

$$N_g = (0.4-1.8)(d/d_{ref})^{-0.25}$$

(7.7)

where $d$ is the pile diameter in meter and $d_{ref} = 1.0$ m.

To investigate the effect of $N_g$ thus the gradient of limiting force profile (LFP) on the pile response, comparative analyses have been undertaken for test M20 using Broms LFP and Barton LFP by taking $N_g = 0.502$ and 1.0, respectively. The $k$ and $G_s$ were taken as 16.960 MN/m$^3$ and 0.413 MPa. The calculated results were also presented in Table 7.12. The calculated load ~ pile head displacement curves were plotted in Figure 7.31 as ‘Broms LFP’ and ‘Barton LFP’. It can be seen that the initial stiffness remains unchanged, but the predicted ultimate capacity increases with the increasing $N_g$. Using Broms LFP and Barton LFP, the ultimate lateral capacity was underestimated by 171% and 36.5% in comparison with the current calculation for this pile.

### 7.5 CONCLUDING REMARKS

Nonlinear analyses using a spreadsheet program based on elastic-plastic solutions presented by Guo (2008) have been carried out for measured response of 51 laterally
loaded rigid piles in cohesionless soil, including 16 full-scale in situ tests, 12 centrifuge tests, and 23 laboratory model tests. These analyses allow the parameters $A_r$, $k$ and $k_0$ describing the limiting force profile and modulus of subgrade reaction to be deduced. The study shows:

- The elastic–plastic solutions (Guo, 2008) with a constant $k$ by stipulating a linear limiting force profile with the same gradient, $A_r$, generally give better estimation against measured nonlinear response for rigid piles in cohesionless soils than those with a Gibson $k$ (a linear increase $k$ with depth).

- The normalised load $T_i/(A_i d l^2)$ reduces with the increase of $e/l$. At $u_0 k/(A_i l) = 2$, the $T_i/(A_i d l^2)$ reduces about 50% from 0.09 to 0.048 as $e/l$ increases from 0 to 0.714.

- The measured ultimate lateral load capacity, $T_u$ using the previously suggested criteria (Meyerhof et al., 1981) is less than the capacity estimated at tip yield state using the solutions presented by Guo (2008).

- $k_d/G_s = 3.27 \sim 6.91$, with an average of 5.0, for the 16 full scale field tests and 2 centrifuge tests; and $k_d/G_s = 2.37 \sim 5.12$, with an average of 3.7, for the 23 laboratory model tests.

- $G_s = (3 \sim 20)\% G_{max}$ for the 11 full-scale and 2 centrifuge tests; and $G_s = (0.8 \sim 2.6)\% G_{max}$ for the 20 small scale laboratory model tests, where $G_{max}$ was estimated from the relative density of the sand by equations 7.5 and 7.6, as proposed by Seed and Idriss (1970).

- Analyses using Guo (2008) solutions with constant $k$ show that (1) variation of shear modulus only affects the initial stiffness of the load–pile head displacement relationship and has no effect on the predicted capacities at the yield at rotation point
and pile tip yield states, which only depends on the ratio $e/l$; and (2) the pile head displacement at pile tip yield state is inversely proportional to the $k$, thus to the shear modulus $G_s$, when the gradient of the limiting force profile, $A_r$ is kept constant.

- $G_s = 181 \sim 600$ kPa, with an average of 316.7 kPa for 16 laterally loaded model piles tests, which is greater than the $G_s = 15 \sim 20$ kPa used in Guo and Qin (2010) solution for passive piles. The difference may be attributed to the large discrepancy in the shear strain level.

- The $N_g$ is related to pile diameter, $d$ and can be estimated by $N_g = (0.4\sim1.8)(d/d_{ref})^{-0.25}$. The ultimate pile capacity increases with the increasing $N_g$. The parametric study on test M20 show that the ultimate lateral capacity may be underestimated by 171% and 36.5% using Broms LFP and Barton LFP.

- The analysis results in this study on the responses of 51 rigid piles are consistent with those obtained conducted by Guo and Zhu (2010) on 20 flexible piles in sand. They are useful in developing guidelines in estimating the shear modulus and gradient of the limiting force or soil resistance profile for laterally loaded piles in cohesionless soils.
Table 7.1 Solutions for Pre-tip and tip yield state (Guo, 2008)

\[ u = \omega \varepsilon + u_0 \] and \( z_r / l = -u_0 / \omega l \)

\[ p = k d u , \quad p_u = A, dz \], kd is the modulus of subgrade reaction, k is written as \( k_0 \varepsilon^m \).

\[ k = k_0 \text{ for constant } k (m=0) \].

\[ \frac{u_0 k_0}{A} = \frac{3 + 2(2 + z_0^3) \varepsilon + z_0^4}{(2 + z_0)(2 \varepsilon + z_0) + 3(1 - z_0)^2} \] \hspace{1cm} (G-2)

\[ \frac{k_0 l}{A} = \frac{-2(2 + 3 \varepsilon)}{[2 + z_0)(2 \varepsilon + z_0) + 3(1 - z_0)^2} \] \hspace{1cm} (G-3)

\[ \bar{z}_m = \sqrt{2T_i / (A, dl^2)} \quad (z_m \leq z_0) \] \hspace{1cm} (G-4)

\[ \bar{T}_i = \frac{1 + 2z_0 + 3z_0^2 + \varepsilon^2 + z_0^4}{6(2 + z_0)(2 \varepsilon + z_0) + 3} \] \hspace{1cm} (C-1)

\[ \frac{u_0 k}{A, l} = \frac{(2 + 3 \varepsilon)z_0}{(2 + 3 \varepsilon + z_0)(1 - z_0)^2} \] \hspace{1cm} (C-2)

\[ \frac{k}{A} = \frac{z_0^3 + 3(z_0 - 2 \varepsilon - 3)}{2(2 + 3 \varepsilon + z_0)(1 - z_0)^2} \] \hspace{1cm} (C-3)

\[ \bar{z}_m = \sqrt{3(1 + 2 \varepsilon) - (z_0 + 3 \varepsilon)z_0} \] \hspace{1cm} (C-4)

\[ \bar{z}_m = \frac{1}{3} \left( \frac{1}{3} + \frac{1}{3} z_0^3 \right) - \frac{1}{3} \left( \frac{1}{3} + \frac{1}{3} z_0^3 \right) \] \hspace{1cm} (C-5)

\[ M_m = \frac{(2z_m^3 / 3 + e)T_i}{z_m \leq z_0} \] \hspace{1cm} (G-6)

\[ M_m = \frac{(2z_m^3 / 3 + e)T_i}{z_m \leq z_0} \] \hspace{1cm} (C-6)

\[ M_m = \frac{(2z_m^3 / 3 + e)T_i}{z_m \leq z_0} \] \hspace{1cm} (C-7)

\[ \bar{z}_m = \frac{(2 \varepsilon + 3z_m^2)z_0 - 2z_m^2(1 - z_0) + (2 \varepsilon + 3z_m^2)(1 - z_0)}{2(1 - z_0)(2 + z_0)} \] \hspace{1cm} (G-7)

\[ \bar{T}_i = \frac{(2 \varepsilon + 3z_m^2)z_0 - 2z_m^2(1 - z_0) + (2 \varepsilon + 3z_m^2)(1 - z_0)}{2(1 - z_0)(2 + z_0)} \] \hspace{1cm} (C-8)

At tip-yield state

\[ ((2 \varepsilon + 3z_m^2)z_0) + (2 \varepsilon + 3z_m^2)(1 - z_0) = 0 \] \hspace{1cm} (Solving numerically)

Note: \( T_i, u, u_0, \omega, z, z_0, z_r, e \) and \( l \) are defined in Figure 7.1. \( z_m \) is the depth of maximum bending moment \( M_m, z_0^r \) is the slip depth \( z_0 \) at tip yield state. \( z_0 = z_0 / l, \bar{z}_m = z_m / l, \bar{\varepsilon} = e / l, z_0^r = z_0^r / l \).
Table 7.2 Solutions for Post-tip yield and YRP states (Guo, 2008)

<table>
<thead>
<tr>
<th>Gibson k (m = 1)</th>
<th>Constant k (m = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_r/l = \sqrt[3]{A_0} + \sqrt[3]{A_1 - D_1}/6$</td>
<td>$z_r/l = 0.5(1-C^2)(\sqrt[3]{A_0} + \sqrt[3]{A_1 + D_0})$</td>
</tr>
<tr>
<td>$A_j = D_0/8 - D_1^2/216 + (-1)^j[(27D_0^2 - 2D_0D_1)/1728]^{1/2}$</td>
<td>$A_j = (D_0^3 + D_1) + (-1)^j[D_1(2D_0^2 + D_1)]^{1/2}$</td>
</tr>
<tr>
<td>$D_1 = (3 + C^2)e/(1+C^2)$</td>
<td>$D_1 = (2 + 3e)/(1-C^2)$</td>
</tr>
<tr>
<td>$D_0 = (2 + 3e)/(1+C^2)$</td>
<td>$D_0 = -e$</td>
</tr>
<tr>
<td>($j = 0, 1$)</td>
<td>($j = 0, 1$)</td>
</tr>
</tbody>
</table>

| $z_0 = z_r(1 - C)$ | $z_0 = z_r/(1 + C)$ |
| $z_1 = z_r(1 + C)$ | $z_1 = z_r/(1 - C)$ |

\[
\frac{T_i}{A_0l^2} = \left(1 + \frac{C^2}{3}\right)\left(\frac{z_r}{l}\right)^2 - \frac{1}{2}
\]

\[
\frac{T_i}{A_0l^2} = \frac{1}{2} \left[\left(\frac{2}{1-C^2}\right)\left(\frac{z_r}{l}\right)^2 - 1\right]
\]

| $u_0 = A_r/(kC)$, or $C = A_r/(u_0k_0)$ | $u_0 = A_rz_r/(kC)$, or $C = A_rz_r/(ku_0)$ |

$\omega = -u_0/z_r$

Yield at rotation point (YRP) (both k)

| $z_0 = z_1 = z_r$ |

$u_0 = \infty$, $\omega = \pi/2$

\[
\frac{z_m}{l} = \left[2\left(\frac{z_r}{l}\right)^2 - 1\right]^{0.5}
\]

\[
\frac{T_i}{A_0l^2} = 0.5\left(\frac{z_m}{l}\right)^2
\]

\[
\frac{M_m}{A_0l^3} = \frac{T_i}{A_0l^2} l + \frac{1}{3}\left(\frac{z_m}{l}\right)^3
\]

Note: $C \leq C_y$, $C_y$ is the value of $C$ at the tip yield state at which $z_0 = z_0^t$. 
Table 7.3 Solutions for $T(z)$ and $M(z)$ (Pre-tip and tip yield states)

<table>
<thead>
<tr>
<th>Gibson $k$</th>
<th>Constant $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u^* = A_r/k_0$</td>
<td>$u^* = A_r z_0/k$</td>
</tr>
</tbody>
</table>

(1) $z \leq z_0$

\[
\frac{T_i(z)}{A_r dl^2} = \frac{T_i}{A_r dl^2} - \frac{1}{2} \left( \frac{z}{l} \right)^2 \quad \frac{T_i(z)}{A_r dl^2} = \frac{T_i}{A_r dl^2} - \frac{1}{2} \left( \frac{z}{l} \right)^2
\]

\[
\frac{M_i(z)}{A_r dl^3} = \frac{T_i}{A_r dl^2} \left( \frac{e}{l} + \frac{z}{l} \right) - \frac{1}{6} \left( \frac{z}{l} \right)^3 \quad \frac{M_i(z)}{A_r dl^3} = \frac{T_i}{A_r dl^2} \left( \frac{e}{l} + \frac{z}{l} \right) - \frac{1}{6} \left( \frac{z}{l} \right)^3
\]

(2) $z > z_0$

\[
\frac{T_i(z)}{A_r dl^2} = \frac{\omega l}{3u} \left[ 1 - \left( \frac{z}{l} \right)^3 \right] + \frac{u_0}{2u} \left[ 1 - \left( \frac{z}{l} \right)^2 \right] \quad \frac{T_i(z)}{A_r dl^2} = \frac{\omega_0 z}{2u} \left[ 1 - \left( \frac{z}{l} \right)^2 \right] + \frac{u_0 z}{u l} \left[ 1 - \frac{z}{l} \right]
\]

\[
\frac{M_i(z)}{A_r dl^3} = \frac{T_i}{A_r dl^2} \left( \frac{e}{l} + \frac{z}{l} \right) - \frac{1}{2} \left( \frac{z}{l} \right)^2 \left( \frac{z_0}{l} \right) + \frac{1}{3} \left( \frac{z_0}{l} \right)^3
\]

\[
\frac{M_i(z)}{A_r dl^3} = \frac{T_i}{A_r dl^2} \left( \frac{e}{l} + \frac{z}{l} \right) - \frac{1}{2} \left( \frac{z}{l} \right)^2 \left( \frac{z_0}{l} \right) + \frac{1}{3} \left( \frac{z_0}{l} \right)^3
\]

\[
+ \frac{\omega l}{3u} \left[ 1 \left( \frac{z_0}{l} \right)^3 - \frac{1}{4} \left( \frac{z_0}{l} \right)^4 \right] - \frac{1}{12} \left( \frac{z}{l} \right)^4 \left( \frac{z_0}{l} \right)
\]

\[
+ \frac{\omega_0 z}{2u} \left[ 1 \left( \frac{z_0}{l} \right)^2 - \frac{1}{3} \left( \frac{z_0}{l} \right)^3 \right] - \frac{1}{6} \left( \frac{z}{l} \right)^3 \left( \frac{z_0}{l} \right)
\]

\[
+ \frac{u_0}{u} \left[ 1 \left( \frac{z_0}{l} \right)^3 - \frac{1}{6} \left( \frac{z_0}{l} \right)^3 \right] - \frac{1}{2} \frac{u_0 z_0}{u l} \left( \frac{z_0}{l} \right)^2 \left( \frac{z}{l} \right)
\]
Table 7.4 Solutions for \( T(z) \) and \( M(z) \) (Post-tip yield state)

<table>
<thead>
<tr>
<th>Gibson ( k ) (( m = 1 ))</th>
<th>Constant ( k ) (( m = 0 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u^* = A_r/k_0 )</td>
<td>( u^* = A_r z_0/k )</td>
</tr>
</tbody>
</table>

1. \( z \leq z_0 \)

\[
\begin{align*}
\frac{T_i(z)}{A,dl^2} &= \frac{T_i}{A,dl^2} - \frac{1}{2} \left( \frac{z}{l} \right)^2 \\
\frac{M_i(z)}{A,dl^3} &= \frac{T_i}{A,dl^3} \left( \frac{e}{l} + \frac{z}{l} \right) - \frac{1}{6} \left( \frac{z}{l} \right)^3
\end{align*}
\]

2. \( z_0 < z \leq z_1 \)

\[
\begin{align*}
\frac{T_i(z)}{A,dl^2} &= \frac{T_i}{A,dl^2} - \frac{1}{2} \left( \frac{z_0}{l} \right)^2 + \frac{ez_0}{3u} \left[ \left( \frac{z_0}{l} \right)^2 - \left( \frac{z}{l} \right)^2 \right] - \frac{u_0}{2u} \left( \frac{z_0}{l} \right)^3 - \frac{u_0}{2u} \left[ \frac{z_0}{l} - \frac{z}{l} \right] \\
\frac{M_i(z)}{A,dl^3} &= \frac{T_i}{A,dl^3} \left( \frac{e}{l} + \frac{z}{l} \right) - \frac{1}{2} \left( \frac{z_0}{l} \right)^2 + \frac{1}{3} \left( \frac{z_0}{l} \right)^3 + \frac{1}{6} \left( \frac{z}{l} \right)^3
\end{align*}
\]

3. \( z_1 < z \leq l \)

\[
\begin{align*}
\frac{T_i(z)}{A,dl^2} &= \frac{T_i}{A,dl^2} + \frac{1}{2} \left[ \left( \frac{z}{l} \right)^2 - \left( \frac{z_0}{l} \right)^2 - \left( \frac{z_1}{l} \right)^2 \right] + \frac{ez_0}{3u} \left[ \left( \frac{z_0}{l} \right)^2 - \left( \frac{z}{l} \right)^2 \right] + \frac{u_0}{2u} \left[ \frac{z_0}{l} - \frac{z_1}{l} \right] \\
\frac{M_i(z)}{A,dl^3} &= \frac{T_i}{A,dl^3} \left( \frac{e}{l} + \frac{z}{l} \right) + \frac{1}{3} \left( \frac{z_0}{l} \right)^3 + \frac{1}{6} \left( \frac{z}{l} \right)^3
\end{align*}
\]

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Table 7.5 Characteristics of pile tests

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Reference</th>
<th>Pile reference</th>
<th>Pile type</th>
<th>e  (m)</th>
<th>l   (m)</th>
<th>d   (m)</th>
<th>Measured curves</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Full-scale field tests</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F1</td>
<td>Haldar <em>et al.</em> (2000)</td>
<td>4</td>
<td>Steel 12-sided polygonal pole</td>
<td>22.25</td>
<td>3.20</td>
<td>0.755</td>
<td>855 kNm, ( M_{0} \sim \omega ), ( M(z), u(z) )</td>
</tr>
<tr>
<td>F2</td>
<td></td>
<td>1</td>
<td></td>
<td>22.93</td>
<td>2.52</td>
<td>0.760</td>
<td>137 kNm</td>
</tr>
<tr>
<td>F3</td>
<td></td>
<td>2</td>
<td></td>
<td>22.94</td>
<td>2.52</td>
<td>0.760</td>
<td>253 kNm</td>
</tr>
<tr>
<td>F4</td>
<td></td>
<td>2A</td>
<td></td>
<td>22.86</td>
<td>2.59</td>
<td>0.761</td>
<td>615 kNm</td>
</tr>
<tr>
<td>F5</td>
<td></td>
<td>3</td>
<td></td>
<td>22.86</td>
<td>2.59</td>
<td>0.759</td>
<td>726 kNm</td>
</tr>
<tr>
<td>F6</td>
<td></td>
<td>5</td>
<td></td>
<td>22.88</td>
<td>2.57</td>
<td>0.759</td>
<td>654 kNm</td>
</tr>
<tr>
<td>F7</td>
<td></td>
<td>7</td>
<td></td>
<td>22.86</td>
<td>2.59</td>
<td>0.759</td>
<td>674 kNm</td>
</tr>
<tr>
<td>F8</td>
<td>Bhushan <em>et al.</em> (1981)</td>
<td>4</td>
<td>Cast-in-place piers</td>
<td>0</td>
<td>5.50</td>
<td>0.610</td>
<td></td>
</tr>
<tr>
<td>F9</td>
<td></td>
<td>5</td>
<td></td>
<td>0</td>
<td>5.50</td>
<td>0.915</td>
<td></td>
</tr>
<tr>
<td>F10</td>
<td></td>
<td>6</td>
<td></td>
<td>0</td>
<td>5.50</td>
<td>0.915</td>
<td></td>
</tr>
<tr>
<td>F11</td>
<td></td>
<td>7</td>
<td></td>
<td>0</td>
<td>5.50</td>
<td>1.220</td>
<td></td>
</tr>
<tr>
<td>F12</td>
<td>Ismael and Klym (1981)</td>
<td>1</td>
<td>Fine to medium sand</td>
<td>0</td>
<td>6.40</td>
<td>0.9144</td>
<td>530kN, ( T \sim u_0 ), ( M(z), u(z) )</td>
</tr>
<tr>
<td>F13</td>
<td>Pender and Matuschka (1988)</td>
<td>4</td>
<td>Field test</td>
<td>5.4</td>
<td>1.97</td>
<td>0.750</td>
<td>21.8kN, ( T \sim u_0 ), ( T \sim \omega )</td>
</tr>
<tr>
<td>F14</td>
<td>Lee <em>et al.</em> (2010)</td>
<td>T1</td>
<td>Bored piles</td>
<td>2.0</td>
<td>2.0</td>
<td>0.40</td>
<td>22kN, ( T \sim u_0 ), ( T \sim \omega )</td>
</tr>
<tr>
<td>F15</td>
<td></td>
<td>T2</td>
<td></td>
<td>2.0</td>
<td>2.40</td>
<td>0.40</td>
<td>50kN, ( T \sim u_0 ), ( T \sim \omega )</td>
</tr>
<tr>
<td>F16</td>
<td></td>
<td>T3</td>
<td></td>
<td>0.15</td>
<td>2.40</td>
<td>0.40</td>
<td>210kN, ( T \sim u_0 ), ( T \sim \omega )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Centrifuge tests</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>Georgiadis <em>et al.</em> (1992)</td>
<td>Pile P1</td>
<td>Stainless steel pipe</td>
<td>1.25</td>
<td>9.05</td>
<td>1.092</td>
<td>( M(z), T(z), u(z) )</td>
</tr>
<tr>
<td>C2</td>
<td></td>
<td>Pile P2</td>
<td></td>
<td>1.25</td>
<td>9.05</td>
<td>1.224</td>
<td>( T \sim u_0, M(z), T(z), u(z) )</td>
</tr>
<tr>
<td>C3</td>
<td>Laman <em>et al.</em> (1999)</td>
<td>1</td>
<td>Circular pier</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>400 kNm, ( M_{0} \sim \omega )</td>
</tr>
<tr>
<td>C4</td>
<td>Dickin and Laman (2003)</td>
<td>d/l=0.33</td>
<td>Rough steel rectangular pier</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>( M_{0} \sim \omega )</td>
</tr>
<tr>
<td>C5</td>
<td></td>
<td>d/l=0.6</td>
<td></td>
<td>6</td>
<td>3</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>C6</td>
<td></td>
<td>d/l=1</td>
<td></td>
<td>6</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>C7</td>
<td></td>
<td>d/l=1.33</td>
<td></td>
<td>6</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>C8</td>
<td></td>
<td>d/l=2</td>
<td></td>
<td>6</td>
<td>3</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>C9</td>
<td></td>
<td>d/l=0.33</td>
<td></td>
<td>6</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>C10</td>
<td></td>
<td>d/l=1</td>
<td></td>
<td>6</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>C11</td>
<td></td>
<td>d/l=1.33</td>
<td></td>
<td>6</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>C12</td>
<td></td>
<td>d/l=2</td>
<td></td>
<td>6</td>
<td>3</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Test No.</td>
<td>Reference</td>
<td>Pile reference</td>
<td>Pile type</td>
<td>e (m)</td>
<td>l (m)</td>
<td>d (m)</td>
<td>Measured Mu or Tu</td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
<td>----------------</td>
<td>-----------</td>
<td>-------</td>
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<th>$A_r$ (kN/m³)</th>
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Table 7.6 Soil properties and deduced parameters (continued)

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<td>91.0</td>
<td>36.5</td>
<td>0.39</td>
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*Average values of the two layers. †Reported by Prasad and Chari (1999).*
Table 7.7 Statistical characteristics of pile tests and deduced parameters

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<tr>
<th>Items</th>
<th>16 field tests</th>
<th>12 centrifuge tests</th>
<th>23 model tests</th>
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<td></td>
<td>Range</td>
<td>Average</td>
<td>Range</td>
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<td>e (m)</td>
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<td>10.6</td>
<td>1.25~6.0</td>
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<tr>
<td>l (m)</td>
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<td>d (m)</td>
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<td>l/d</td>
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<td>0.5~8.29</td>
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<td>D_s (%)</td>
<td>22~86</td>
<td>55.6</td>
<td>37~85</td>
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<td>ϕ_s (°)</td>
<td>30~49.8</td>
<td>37.4</td>
<td>36~49</td>
</tr>
<tr>
<td>γ_s (kN/m³)</td>
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<td>14.6~16.4</td>
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<td>A_t (kN/m³)</td>
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<td>N_g</td>
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<td>0.43~1.41</td>
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<tr>
<td>k_0 (MN/m⁴)</td>
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<td>k (MN/m³)</td>
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<td>Gs † (MPa)</td>
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<td>3.03~4.26</td>
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<td>G_max † (MPa)</td>
<td>21.88~175</td>
<td>93.82</td>
<td>30.32~78.6</td>
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</table>

Note: The 23 laboratory model tests were in circular solid or pipe cross section. 9 of the 12 centrifuge tests were in rectangular cross section. 7 of the 16 full scale field tests were 12-sided polygonal steel pole and 5 tests were cast in-place piers. † Tests F14 ~ F16, C4 ~ C12 and M5 ~ M7 are not included.
Firstly, substituting $\frac{e}{l} = 22.25/3.2 = 6.953$ into equation (G-8) to calculate $\bar{z}_0^y$.

$$ (\bar{z}_0^y)^3 + (2\bar{e} + 1)(\bar{z}_0^y)^2 + (2\bar{e} + 1)\bar{z}_0^y - (\bar{e} + 1) = 0 $$

Solving numerically, we can obtain $\bar{z}_0^y = 0.383$ (G-8)

$$ T_e = \frac{1}{A_d l^2} = \frac{1 + 2\bar{z}_0 + 3\bar{z}_0^2}{6 (2 + \bar{z}_0)(2\bar{e} + \bar{z}_0) + 3} = \frac{1 + 2 \times 0.383 + 3 \times 0.383^2}{6 (2 + 0.383)(2 \times 6.953 + 0.383) + 3} = 0.009924 $$

$$ \frac{u_0 k_0}{A_e} = \frac{3 + 2(2 + \bar{z}_0^y)\bar{e} + \bar{z}_0^4}{[2 + \bar{z}_0^y](2\bar{e} + \bar{z}_0) + 3] (1 - \bar{z}_0^y)^2} = \frac{3 + 2(2 + 0.383)^3 \times 6.953 + 0.383^4}{(2 + 0.383)(2 \times 6.953 + 0.383) + 3] (1 - 0.383)^2} = 2.2417 $$

$$ \frac{k_d}{A_r} = \frac{-2(2 + 3\bar{e})}{(2 + \bar{z}_0^y)(2\bar{e} + \bar{z}_0) + 3] (1 - \bar{z}_0^y)^2} = \frac{-2(2 + 3 \times 6.953)}{(2 + 0.383)(2 \times 6.953 + 0.383) + 3] (1 - 0.383)^2} = -3.2417 $$

$$ \bar{z}_m = \sqrt{\frac{2 T_e}{(A_d l^2)}} = \sqrt{2 \times 0.009924} = 0.14088 \quad (\bar{z}_m \leq \bar{z}_0) $$

As $\bar{z}_m < \bar{z}_0^y (= 0.383)$, $\bar{z}_m = 0.14088$

$$ \frac{M_m}{A_d l^2} = \frac{(2 \bar{z}_m^3 / 3 + \bar{e}) T_e}{A_d l^2} = (2 \times 0.14088^3 / 3 + 6.953) \times 0.009924 = 0.0699 \quad (\bar{z}_m \leq \bar{z}_0) $$

Using pile properties: $d = 0.7545$ m, $l = 3.2$ m, $e = 22.25$ m, and the deduced parameters $A_e = 400$ kN/m$^3$, $k_0 = 26.1$ MN/m$^4$, $k = 41.0$ MN/m$^3$, pile responses were calculated.

$\bar{z}_0^y = 1.2256$ m, $T_e = 30.67$ kN, $u_0 = 34.4$ mm, $\omega = -0.8895^\circ$, $z_m = 0.4508$ m, $M_m = 691.6$ kNm, $M_0 = T_e \times e = 682.4$ kNm, and $z_r = -u_0 / \omega = 2.212$ m.
Firstly, substituting \( \bar{e} = e/l = 22.25 / 3.2 = 6.953 \) into equation (C-8) to calculate \( z_0' \).

\[
z_0' = -(1.5\bar{e} + 0.5) + 0.5\sqrt{5 + 12\bar{e} + 9\bar{e}^2} = -(1.5 \times 6.953 + 0.5) + 0.5\sqrt{5 + 12 \times 6.953 + 9 \times 6.953^2} = 0.51
\]  
\( \text{(C-8)} \)

\[
\frac{T_i}{A_d l^2} = \frac{\bar{z}_0}{2(2 + 3\bar{e} + \bar{z}_0)} = \frac{0.51}{2(2 + 3 \times 6.953 + 0.51)} = 0.0109 \quad \text{(C-1)}
\]

\[
\frac{u_0 k_0}{A_d l} = \frac{(2 + 3\bar{e})\bar{z}_0}{(2 + 3\bar{e} + \bar{z}_0)(1 - \bar{z}_0^2)} = \frac{(2 + 3 \times 6.953) \times 0.51}{(2 + 3 \times 6.953 + 0.51)(1 - 0.51^2)} = 2.0894 \quad \text{(C-2)}
\]

\[
\omega \frac{k}{A_r} = \frac{\bar{z}_0^2 + 3(\bar{z}_0 - 2\bar{e} - 3)}{[2 + 3\bar{e} + \bar{z}_0](1 - \bar{z}_0^2)} = 0.51 \times \frac{0.51^2 + 3 \times (0.51 - 2) \times 6.953 - 3}{[2 + 3 \times 6.953 + 0.51](1 - 0.51^2)} = -3.0894 \quad \text{(C-3)}
\]

\[
\bar{z}_m = \sqrt{2T_i/(A_d l^2)} = \sqrt{2 \times 0.0109} = 0.1476 \quad (z_m \leq z_0)
\]

As \( \bar{z}_m < \bar{z}_0' (= 0.51) \), \( \bar{z}_m = 0.1476 \)  
\( \text{(C-4)} \)

\[
\frac{M_m}{A_d l^2} = \frac{(2\bar{z}_m / 3 + \bar{e}) T_i}{A_d l^2} = \frac{(2 \times 0.1476 / 3 + 6.953) \times 0.0109}{(2 \times 0.1476 / 3 + 6.953) \times 0.0109} = 0.0771 \quad (z_m \leq z_0)
\]  
\( \text{(C-6)} \)

Using pile properties: \( d = 0.7545 \text{ m}, l = 3.2 \text{ m}, e = 22.25 \text{ m}, \) and the deduced parameters \( A_r = 400 \text{ kN/m}^3, k_0 = 26.1 \text{ MN/m}^4, k = 41.0 \text{ MN/m}^3 \), pile responses were calculated.

\( z_0' = 1.635 \text{ m}, T_i = 33.68 \text{ kN}, u_0 = 65.23 \text{ mm}, \omega = -1.727^\circ, z_m = 0.473 \text{ m}, \) and \( M_m = 762.3 \text{ kNm} \),  
\( M_0 = T_i e = 751.65 \text{ kNm}, \) and \( z_r = -u_0 / \omega = 2.164 \text{ m}. \)
### Table 7.10 Computation of shear modulus $G_s$ from the back calculated constant $k$

<table>
<thead>
<tr>
<th>Test No</th>
<th>$e$ (m)</th>
<th>$l$ (m)</th>
<th>$d$ (m)</th>
<th>$r/l$</th>
<th>$r/l$</th>
<th>$k_1$</th>
<th>$\gamma$</th>
<th>$K_1(\gamma)$</th>
<th>$K_0(\gamma)$</th>
<th>$kd/G_s$</th>
<th>$k$ (MN/m$^3$)</th>
<th>$G_s$ (MPa)</th>
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<td>0.565</td>
<td>1.412</td>
<td>0.825</td>
<td>6.21</td>
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<td>0.760</td>
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<td>0.595</td>
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<td>2.379</td>
<td>0.086</td>
<td>0.072</td>
<td>15.42</td>
<td>12.5</td>
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### Table 7.10 Computation of shear modulus $G_s$ from the back calculated constant $k$ (continued)

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Note: $K_i(\gamma)=\text{modified Bessel function of second kind of }i\text{ th order } (i=0,1)$.

$\gamma=k_1r_0/l$, load transfer factor.

$r_0=\text{outer radius of a pile}$.

$k_i=2.14+e/l/(0.2+0.6e/l)$, coefficients for estimating $\gamma$.
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Table 7.11 Computation of shear modulus $G_{\text{max}}$ (continued)

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<td>2.627</td>
<td>20.952</td>
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<td>20.952</td>
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<td>20.704</td>
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<td>18.419</td>
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Note: MTD1: $G_{\text{max}} = A_D \left(1 + D_r/100\right) \left(\frac{1+p_{\text{atm}}}{\sigma'_{\text{w}}}\right)^n$ (MPa), $A_D=177$, $a_D=17.3$, $n=0.48$, $p_{\text{atm}}=100$ kPa, atmospheric pressure, and

MTD2: $(K_2)_{\text{max}} = A_{KD} \left(1 + D_r/100\right) \left(\frac{1}{(a_{KD} - D_r/100)}\right)^2$, $A_{KD}=6900$, $a_{KD}=16.1$. $G_{\text{max}} = 218.8(K_2)_{\text{max}} (\sigma'_{\text{w}})^{0.5}$, (kPa). Wichtmann and Triantafyllidis (2009)

MTD3: See equations (7.5) and (7.6). Seed and Idriss (1970), and Seed et al. (1986). † $G_{\text{max}}$ calculated from MTD 3.
<table>
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<th>k (MN/m³)</th>
<th>( N_g )</th>
<th>( A_r ) (kN/m³)</th>
<th>( u_t ) (mm)</th>
<th>( T_t ) (kN)</th>
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<td>2.78</td>
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Note: \( G_{max} = 23.341 \) MPa (see Table 7.11).
Figure 7.1 Schematic analysis for a rigid pile (after Guo, 2008)

T_l = lateral load; e = eccentricity; u_0 = pile displacement at ground line;
ω = angle of rotation (in radian); z = depth from ground line;
l = embedded length; z_0 = depth of slip; z_r = depth of rotation point;
p = soil resistance per unit length; p_u = ultimate soil resistance per unit length;
A_r = gradient of limiting force profile; d = outer diameter of the pile;
u = pile displacement; u^* = local threshold u above which pile soil relative slip is initiated;
k, k_0 = modulus of subgrade reaction, k = k_0 z^m, m = 0, and 1 for constant and Gibson k.
Figure 7.2 Schematic limiting force profile, and pile deformation. (a) Tip-yield state. (b) Post-tip yield state. (c) Impossible yield at rotation point (after Guo, 2008)
Figure 7.3 Predicted and measured response of pile F1 (Haldar et al., 2000)
Figure 7.4 Predicted and measured response of pile C2 (Georgiadis et al., 1992)
Figure 7.5 Predicted and measured response of pile F2-F7 (Haldar et al., 2000)
Figure 7.6 Predicted and measured response of pile F8-F11 (Bushan et al., 1981)
Figure 7.7 Predicted and measured response of pile F12 (Ismael and Klym, 1981)

Figure 7.8 Predicted and measured response of pile F13 (Pender and Matuschka, 1988)
Figure 7.9 Predicted and measured response of pile F14-16 (Lee et al., 2010)
Figure 7.10 Predicted and measured response of pile C1 (Georgiadis et al., 1992)
Figure 7.11 Predicted and measured response of pile C3 (Laman et al., 1999)

Figure 7.12 Predicted and measured response of pile C4-C12 (Dickin et al., 2003)
(a) Dense sand  (b) Loose sand
Figure 7.13 Predicted and measured response of piles M 1-M3 (Petrasovits and Awad, 1972)

Figure 7.14 Predicted and measured response of piles M 4-M7 (Adams and Radhakrishna, 1973)

Figure 7.15 Predicted and measured response of piles M 8-M9 (Meyerhof et al., 1981)

Figure 7.16 Predicted and measured response of pile M10 (Chari and Meyerhof, 1983)
Figure 7.17 Predicted and measured response of pile M11-M13 (Swane, 1983)

Figure 7.18 Predicted and measured response of pile M14-M17 (Prasad and Chari, 1996)

Figure 7.19 Predicted and measured response of pile M18-M20 (Prasad and Chari, 1999)
Figure 7.20 Predicted and measured response of pile M21 (Qin and Guo, 2007)
Figure 7.21 Predicted and measured response of pile M 22 (Qin and Guo, 2007)
Figure 7.22 Predicted and measured response of pile M 23 (Qin and Guo, 2007)
Figure 7.23 Normalised nonlinear load and displacement relationship
Figure 7.24 Normalised nonlinear moment and pile rotation relationship
Figure 7.25 Normalized pile capacity at critical yield states
Figure 7.26 $G_s/G_{\text{max}}$ vs. $D_r$ relationship

Figure 7.27 $G_s/G_{\text{max}}$ versus shear strain relationships for dense Toyoura sand with different confining pressures (after Kokusho, 1980)
Figure 7.28 Effect of shear modulus on the load - displacement relationships

Figure 7.29 Typical variation of Young’s modulus with pressure and shear strain for sand (after Carter, 2006)
Figure 7.30 $N_g \sim d/d_{ref}$ relationship

Figure 7.31 Effect of $N_g$ on the load - displacement relationships
8 STATIC AND CYCLIC BEHAVIOUR OF LATERALLY LOADED PILES IN SAND

8.1 INTRODUCTION

In chapter 7, the difference in the modulus of subgrade reaction for piles subjected to lateral loading and soil movements has been investigated. Another issue related to Guo and Qin (2010)’s solutions needs to be validated. As explained in section 4.5, the analytical solutions presented by Scott (1981) for laterally loaded piles was used by Guo and Qin (2010) to study passive piles. The maximum shear force, $T_{\text{max}}$ was used as the equivalent load $T_i$ and then the relationship between the maximum bending moment, $M_{\text{max}}$ and maximum shear force, $T_{\text{max}}$ was established. Broms (1964b), Scott (1981) and Guo (2008) have shown theoretically that the maximum bending moment linearly depends on the lateral load for rigid piles in sand. This has never been experimentally verified. Therefore, static and cyclic laterally loaded pile tests were conducted to provide measured relationship between the maximum bending moment and the applied lateral load.

In this chapter, firstly, the modification of the existing testing apparatus for applying lateral load on a single pile was described. Secondly, the results of static and cyclic laterally loaded model tests were presented. The test results were analysed to show the effects of cyclic magnitude and the numbers of cycles on the maximum bending moment and accumulated pile deformations. The relationship between the measured...
maximum bending and the applied lateral load was determined. The effect of cyclic loading on the pile responses via design parameters was examined.

8.2 TESTING APPARATUS AND PROCEDURES

Information regarding the apparatus has been reported previously in chapter 3. Thus only relevant parts and modifications of the loading system for applying lateral cyclic load were explained here. The model ground was prepared in the same way as for the passive loading tests described in chapter 3.

8.2.1 Storage box and lateral loading system

Figure 8.1 shows a photograph of the experimental set up. Figure 8.2 illustrates a schematic cross section of the storage box and loading system. The internal dimensions of the storage box are 1 m by 1 m, and 0.8 m in height. The upper part of the box is made of a series of 25-mm-thick square laminar steel frames underlying a 400-mm-high fixed timber box. A vertical jack is used to install piles into the storage box. A triangular steel frame with an aluminium pulley was manufactured and clamped on the vertical columns to support the lateral loading system. The pile was loaded laterally by means of weights added to a loading pan carried by flexible wire acting over the pulley and attached to the pile at an eccentricity above the ground surface. A hydraulic jack was placed underneath the loading pan. Pumping the hydraulic jack lift the weights up thus unloading, while releasing the jack loaded the
pile. Great care was taken when releasing the jack to ensure the pile was gradually loaded without impact. Thus, any inertia effect and rate effect are negligible.

8.2.2 Test procedures

The storage box was first moved into position beneath the sand rainer, which was suspended by an overhead crane in the laboratory. The storage box was secured, and sand then rained into the shear box from a height of 600 mm.

Secondly, the storage box was carefully moved into a position under the vertical loading frame using a pallet truck.

Thirdly, the pile was installed into the sand with the vertical jack to a desired embedded length.

Fourthly, the triangular steel frame (Figure 8.1) with the pulley was clamped on the vertical columns. The lateral loading devices were setup.

Fifthly, dial gauges were connected to the pile and strain gauges connected to a data acquisition system (which was controlled by a computer). By then, the pile was ready to be tested under static or cyclic loading.

Upon finishing each test, the loading system and measuring devices were first removed, then the pile was withdrawn. Finally sand was emptied through an outlet at the base of the box.
8.3 TESTING DETAILS

Seven (2 static and 5 cyclic loading) tests are presented. Figure 8.3 shows a schematic of a pile under testing. The pile was restrained by soil (free-headed) and was subjected to lateral loading only. The 32-mm-diameter instrumented model pile was used in this study. The pile properties are given in Table 3.2. The pile was always installed at the centre of the storage box. The horizontal distance from the inner surface of the shear box to the pile centre was about fifteen times (= 500/32= 15.6) the pile diameter. Therefore, the boundary effect was assumed to be insignificant. The test details are summarised in Table 8.1.

Tests TS1 and TS2 were conducted under a gradually increased static loading until failure. The two tests were performed to determine the ultimate lateral capacity of the pile. The static response of the pile also provided a reference back-bone curve for the cyclic tests results.

Tests TC1 to TC5 were conducted under one-way cyclic loading sequence in which the load increased from 0 to \( T_{\text{max}} \) and returned to \( T_{\text{min}} \) with no reversal of load direction (Long and Vanneste, 1994). Characteristics of the cyclic loading pattern can have a significant impact on the behaviour of the laterally loaded piles. Following McManus and Kulhawy (1994), and Al-Douri and Poulos (1995), two types of cyclic loading patterns; i.e. uniform amplitude cyclic and non-uniform amplitude (storm) cyclic loading, were designed in an attempt to simulate the main features of current or
wind load. Details of individual tests are described below.

Tests TC1 to TC3 were conducted under uniform amplitude cyclic loading conditions. Specifically, in test TC1, lateral load was increased from a minimum value of $T_{\text{imin}} = 0$ to a maximum of $T_{\text{imax}} = 215$ N and returned to $T_{\text{imin}} = 0$. This process was repeated 50 times (cycles). Afterwards, the pile was loaded monotonically until failure. The same loading procedure was used in TC2 but with $T_{\text{imax}} = 410$ N and $T_{\text{imin}} = 0$. In test TC3, the number of cycles was maintained as 50 with the $T_{\text{imax}} = 410$ N and $T_{\text{imin}} = 215$ N in the first 30 cycles, and $T_{\text{imax}} = 545$ N and $T_{\text{imin}} = 410$ N in the last 20 cycles.

Tests TC4 and TC5 were conducted under non-uniform amplitude (storm) loading conditions. ‘Storm loading’ may represent the extreme environmental loading pattern to simulate storm wave or current induced loading on offshore piles (Al Douri and Poulos, 1995). Test TC4 consists of three loading parcels in which the maximum load $T_{\text{imax}}$ increased gradually. In the first and second load parcels, the one-way cycles were repeated for 30 times under $T_{\text{imax}} = 115$ N and 215 N, respectively. In the third parcel, the maximum load $T_{\text{imax}}$ was increased to 410 N and the cycles were repeated for 50 times. Test TC5 was conducted under similar conditions to TC4 but with the pile embedment depth, $L$, changed to 400 mm.
8.4 TEST RESULTS

Results of three typical tests are presented below to illustrate the static, uniform amplitude cyclic loading and storm loading response of the piles, including the load ~ groundline deflection curves, the bending moment and pile deflection profiles. Table 8.2 summarises the critical responses of maximum bending moment, $M_{max}$ and pile displacement at the groundline, $y_0$ at the specified cycles of loadings.

8.4.1 Static loading tests

Test TS1 was conducted to determine the ultimate lateral loading capacity of the pile. Figure 8.4(a) shows the relationship between the lateral load and displacement at the ground surface. The nonlinearity of the load displacement curves was attributed to the nonlinear stress-strain properties of the sand, since the pile itself is unlikely to have such large deflection. A clearly defined ultimate lateral load has not been obtained even at displacements of 40 mm ($y_0/d = 1.25$). The ultimate lateral load capacity was determined as 740 N from the load–displacement curve. Figure 8.4(b) shows the distribution of bending moment along the pile. The bending moment increases monotonically with the increase of lateral load (from 50 N to 150 N, and so on). For example, the maximum bending moment was deduced as 64.5 Nm at lateral load of 450 N, which occurred at a depth of 150 mm below the ground. Figure 8.4(c) indicates that the pile deflected mainly by rotating about a depth of 340 mm down the pile.
8.4.2 Uniform amplitude cyclic loading tests

The responses of the pile when subjected to uniform amplitude cyclic loading are shown in Figure 8.5 for the case of test TC1 at the specified number of cycles. The strain gauges’ and dial gauge’s readings were recorded for each of the first five cycles, and every five cycles afterwards, which was the same for the rest of cyclic loading tests presented later. Figure 8.5(a) shows that the total pile displacement increased with increasing numbers of cycles. The total pile displacements at ground level were 3.1 mm and 4.9 mm for the first and 50th cycles, respectively. With the imposed cyclic load of 215 N, a slight increase in bending moment along the pile was measured at larger numbers of cycles. The maximum bending moment, \( M_{\text{max}} \), was again observed occurring and remaining at a depth of 150 mm. Sand flowing around the pile was observed when the pile was loaded. The separation between the pile and sand in Figure 8.6 indicated a ‘conical depression’ formed behind the pile under the cyclic loading. This formation is owing to displacement of the pile in the loading direction. It was noticed that with the number of cycles increasing, the ‘depression’ size enlarged and its depth increased. After 50 cycles, the depth of the depression was approximately 24 mm, and the intact sand was about 2.5 pile diameters (79 mm) distance to the model pile at the model ground surface.

8.4.3 Storm loading tests

As an example, the responses of piles when subjected to ‘storm’ loading are shown in Figure 8.7 for the case of test TC4 including three loading parcels with the \( T_{\text{max}} = 115 \),
215 and 410 N, respectively. Figure 8.7(a) shows the measured load ~ groundline deflection curve of the pile. The plot shows that the lateral deflection progressively accumulated during the cyclic loading. The first loading cycle induces a larger deflection than the subsequent one. The initial tangent stiffness of the first loading cycle in each parcel is lower than those of following cycles. The cyclic tangent stiffness of the pile response increases with the number of cycles. The plot also illustrates that the piles’ behaviour is nearly elastic when reloaded and the pile–soil systems tend to progressively stabilise to an elastic response with increasing numbers of cycles. This typical behaviour of the pile in dry sand under one-way cyclic loading was also observed by Swane (1983). However, this phenomenon can be expected only at low load levels, i.e. only when the magnitude of the cyclic loading is small in comparison with the failure load of the piles, otherwise, incremental collapse will result (Poulos, 1982; Swane, 1983; Levy et al., 2009).

Figure 8.7(b) provides the bending moment profiles of the pile for the three loading levels at the specified number of cycles. From this figure, the measured maximum bending moment $M_{max}$ of 23.7, 40.0 and 78.7 kNmm was determined for the first cycle of each loading stages. Similar to test TS1, the number of cycles has a minor impact on the $M_{max}$ in each stage. The depth at which the $M_{max}$ occurred was located around a depth of 150 mm along the pile below the ground surface and remained stable during the cyclic loading.
8.4.4 Effect of number of cycles and cyclic load level

The pile deflection at ground line $y_0$ and maximum bending moment, $M_{max}$ are plotted against the number of cycles for the cyclic loading tests in Figure 8.8 and Figure 8.9. As would be expected, both deflection $y_0$ and moment $M_{max}$ increase with an increasing number of cycles and cyclic load level. The increase in $M_{max}$ is not as great as for the deflection. For a specific cyclic loading condition, the accumulated pile deflection increases with the increasing number of cycles, but at a decreasing rate. The effect of the first 10 cycles is more significant than that of subsequent ones. For the uniform cyclic loading, from the first to the 50th cycle, the $y_0$ increases 60% and 85% in tests TC1 and TC2, respectively. For the ‘storm’ loading, under the maximum applied load of 215 N (second parcel) the $y_0$ increases 40% and 68% in tests TC4 and TC5 from the first to the 30th cycle. Figure 8.9 demonstrates that increasing the number of cycles has less effect on the $M_{max}$, which remains constant at a specific load level with the exception of in TC3. For this test, the $M_{max}$ increases by less than 20% between the first loading and the 30th cycle.

The cyclic load level has a more profound impact on the pile response than the number of cycles. For uniform amplitude cyclic loading tests, as the magnitude of cyclic load increases from 215 N to 410 N, the $M_{max}$ is approximately doubled, while the pile head displacement increases about 2.6 times for the first cycle, and is tripled for the 50th cycle. The ultimate lateral loading capacity of TC1 and TC2 after 50 cycles was approximately 10% higher than that deduced from static loading test TS1,
which shows slight densification of sand under cyclic loading condition. For the ‘storm’ loading test TC4, pile displacement $y_0$ increases marginally and approaches the ultimate values in the first and second load parcels with the applied maximum load $T_{t_{max}}$ of 215 N being 25% less than the predicted ultimate loading capacity $T_u$ of 884 N (see Table 8.2). Nevertheless, as the maximum load $T_{t_{max}}$ increases to 410 N, that is about 46% of the $T_u$ in the third parcel, the $y_0$ keeps increasing with the loading cycles and does not stabilise to a constant displacement. It appears that larger number of cycles may be required for the pile response to be stabilised to the ultimate state, or the pile displacement will continue to increase and eventually lead to failure of the pile-soil system.

The maximum bending moment, $M_{max}$ primarily depends on the maximum lateral load applied, $T_{t_{max}}$. For instance, the measured $M_{max}$ of 40.0 kNmm in the second parcel in test TC4 compares fairly well with the value obtained in tests TC1 with the $T_{t_{max}} = 215$ N.

### 8.5 MAXIMUM BENDING MOMENT VERSUS LATERAL LOAD

Figures 8.10(d) to 8.16(d) plotted the lateral load, $T_t$ against the measured maximum bending moment, $M_{max}$. Guo (2008) revealed that the $M_{max}$ of a laterally loaded rigid pile in sand depends on the load $T_t$, the eccentricity $e$ and the depth $z_m$ at which $M_{max}$ occurs. With the assumption of a linearly varying limiting force profile commonly
adopted for sand, the $M_{\text{max}}$ may be estimated by (Guo 2008):

$$M_{\text{max}} = \left( \frac{2}{3} z_m + e \right) T_t \quad \text{or} \quad M_{\text{max}} = \frac{2}{3} T_t z_m + M_0$$  \hspace{1cm} (8.1)

in which $M_0 = T_t e$, the bending moment at the ground surface.

With the $z_m$ determined from the bending moment distribution curves and an eccentricity $e$ of 115 mm, the calculated values of $M_{\text{max}}$ are also shown in Figure 8.10(d) to 8.16(d). Although the agreement between the calculated $T_t \sim M_{\text{max}}$ curves and measured data is good at lower load level, the calculated curves tend to diverge from the measured data and to overpredict the measured $M_{\text{max}}$ as the load increases. This divergence may be attributed to several factors, such as the calibration factors of the strain gauges for calculating the moment, the used, $z_m$ or the eccentricity, $e$. Only the used eccentricity $e$ is discussed because the ratio of load eccentricity to the embedded pile length has a significant impact on the behaviour of laterally loaded rigid piles as revealed in chapter seven.

Figures 8.10(b) to 8.16(b) reveal that the measured moment $M_0$ at ground surface is not consistent with the theoretical values of $M_0 = T_t e$. Because no strain gauges on the pile were located exactly at the ground surface, the measured $M_0$ was determined from the average readings of the strain gauges adjacent to the surface. Figure 8.17 plots the theoretical values against the measured values of $M_0$. The measured $M_0$ was about 63~100% with an average of 85% of the theoretical values. The specific source of this discrepancy is not fully understood by the author, but may be caused by the loading
eccentricity. The use of the annular pipe fastened on the pile (see Figure 8.1) for applying lateral load may cause the actual eccentricity less than the theoretical 115 mm for all the tests. If the eccentricity is modified by multiplying a factor $\alpha$ ($\alpha \leq 1$), the agreement between the measured and calculated $M_{\text{max}}$ will be reasonably well as shown by the dashed lines in Figures 8.10(d) to 8.16(d). The values of $\alpha$ vary in a range of 0.6 to 0.85, with the exceptional low value of 0.45 for test TS1. The measured linear correlation between the maximum bending moment, $M_{\text{max}}$ and the applied load, $T_t$ for the first time validates the simple equation (8.1) and the impact of loading eccentricity, $e$ on the maximum bending moment $M_{\text{max}}$. The current analysis also suggests that equation (8.1) can be used to check or verify the results from numerical analysis on rigid piles in sand, such as the mono piles for offshore wind farms. The procedures for carrying out the verification are given below.

The key in using equation (8.1) lies in the determination of the depth, $z_m$, at which the maximum bending moment occurs. Figures 8.10 (b) to 8.16 (b) show that the depth, $z_m$, remains nearly at a constant depth along the pile under different load levels in each test. Guo (2008) shows that the depth $z_m$ is insensitive to the states of yield between the pile and soil, especially at post-tip yield state (which is defined in section 7.2.4) under large load level, but significantly depends on the ratio of loading eccentricity, $e$ to the pile embedment depth, $l$. For example, at $e = 0$, as the yield extends from pile tip (see Figure 7.2 (a) in Chapter 7) to the rotation point (see Figure 7.2 (c)) the ratio $z_m/l$ increases by 7.2% from 0.4756 to 0.5098 for Gibson $k$; or by 4.9% from 0.4859
to 0.5098 for constant \(k\). In contrast, at tip yield state, as the ratio \(e/l\) increases from 0 to 1000, the ratio \(z_m/l\) reduces from approximately 0.48 to 0, i.e. to the ground surface, for both Gibson and constant \(k\), as shown in Figure 8.18. The depth \(z_m\) can be estimated from the curves presented in Figure 8.18, irrespective of the yield states between the pile and soil. From the depth \(z_m\), the maximum bending moment, \(M_{max}\) can be readily calculated by equation (8.1) with the known lateral load, \(T_t\), and eccentricity, \(e\). This is illustrated by an example calculation presented next.

Sorensen et al. (2009) carried out six laboratory tests on two instrumented closed-ended aluminium pipe piles with outer diameters, \(d\) of 60 mm and 80 mm. The wall thickness is 5 mm. Both piles have a slenderness \(l/d = 5\), corresponding to an embedded length, \(l\), of 0.3 m and 0.4 m, respectively. The horizontal load was applied at an eccentricity, \(e\) of 0.370 m above the soil surface. The piles were instrumented with strain gauges in five levels beneath the soil surface and tested in a pressure tank containing 0.69 m of fully saturated sand under an overburden pressure of 0, 50, and 100 kPa, respectively. They also analysed the test results using FLAC 3D and calibrated their model constructed in FLAC 3D. Only the measured and calibrated load-displacement relationship for a model pile test with the pile diameter of 80 mm and overburden pressure of 100 kPa were provided in their paper. They were reproduced in Figure 8.19 (a). Figure 8.19 (b) presents the calibrated and measured bending moment distribution at a lateral load, \(T_t\), of 2100 N for the same test. Following the previous discussion, the maximum bending moment, \(M_{max}\) is estimated
as follows.

- The $z_m/l$ is evaluated approximately as 0.30 from Figure 8.18, as $e/l = 0.37/0.4 = 0.925$. Thus $z_m = 0.3 \times 0.4 = 0.12$ m.

- The $M_{max}$ is calculated from equation (8.1) as

$$M_{max} = \left(2/3 \times z_m + e\right)Te = \left(2/3 \times 0.12 + 0.37\right) \times 2100 = 945.0 \text{ Nm.}$$

The calculated $M_{max}$ of 945.0 Nm overestimates the measured $M_{max}$ of 887.3 Nm by about 6.5%, while the $M_{max}$ of 796.4 Nm obtained from numerical analysis by Sorensen et al. (2009) using FLAC 3D underestimates the measured $M_{max}$ by about 10%. The calculated $M_{max}$ from equation (8.1) is also plotted in Figure 8.19 (b) using hollow dot point ○ for comparison. The applied lateral load, loading eccentricity, measured and calculated depth of maximum bending moment and the corresponding maximum bending moment were summarised in Table 8.3.

After calibration using the small-scale tests, Sorensen et al. (2009) extended their numerical model in FLAC 3D to full-scale wind turbine foundations. Three steel pipe piles with pile diameters of 3 m, 5 m, and 7 m were analysed. The piles were assumed to have an embedded length of 20 m and a load eccentricity of 15 m. Figures 8.20(a) and 8.20(b) present the lateral pile deflection and distribution of bending moment for the three piles at the applied lateral load of 6.4 MN, 8.8 MN, and 21.4 MN, respectively. From the ratio $e/l=15/20=0.75$, the $z_m/l$ was evaluated approximately as 0.32 from the curves in Figure 8.18. The depth of maximum bending moment $z_m$ was calculated as 6.4 m and the maximum bending moments were computed as 110.5
MNm, 153.2MNm, and 381.3 MNm from equation (8.1) for the three piles. They were presented in Table 8.3, together with the numerical analysis results by Sorensen et al. (2009). The computed maximum bending moment, $M_{max}$ from equation (8.1) was approximately (8~12) %, with an average value of 10%, greater than Sorensen et al. (2009)’s numerical analysis results. However, the computational effort using FLAC 3D will be tremendous and may requires days to run, based on the author’s experience in using FLAC 3D. Therefore, the simple equation (8.1) permits quick check and useful parametric studies to be made in the preliminary stage of design.

8.6 COMPARISON WITH THEORETICAL CALCULATIONS

8.6.1 Back analysis of pile responses

The measured pile response was back calculated using the spreadsheet program based on the elastic–plastic solutions developed by Guo (2008). The solutions and the calculation procedures have been explained in chapter seven. The work presented herein is an extension of the investigation of laterally loaded rigid piles in cohesionless soil described in the previous chapter.

Comparisons between the measured load-deflection at the ground surface curves, distributions of bending moment, deflection with depth for the tests are shown in Figure 8.10 to Figure 8.16. The measured load-deflection curves were generally bracketed by the predictions with the assumed constant and Gibson distribution of
modulus of subgrade reaction. However, the calculated curves tend to overestimate the bending moment and this tendency is most marked for tests TS1, TS2 and TC5. This discrepancy will be further explored later. The back calculated parameters $A_r$, $N_g$, $k$ and $k_0$ from the tests are summarised in Table 8.2.

### 8.6.2 Cyclic response

The load-deflection curves of the cyclic loading tests in Figures 8.10(a) to 8.16(a) show that the tangent stiffness of the pile response increases with increasing numbers of cycles. The stiffer pile response at larger numbers of cycles indicates increased resistance experience by the pile and an increasing of the modulus of subgrade reaction of the sand after a number of unloading-reloading cycles in the one-way cyclic loading tests.

To study this aspect of the behaviour of the pile, the reloading-deflection at a specific cycle was back-analysed through matching the calculated and measured load deflection curves. They are plotted in Figure 8.21. For clarity, only one of the calculated curves is included for each test in these figures. The match between the back-calculated and measured load-deflection curves is reasonably well. The deduced parameters are also presented in Table 8.2.

The values of the modulus of subgrade for constant $k$, and the parameter $k_0$ for unloading-reloading are $1.63\sim2.88$ and $1.60\sim2.0$ times that of the primary loading.
The increase of modulus of subgrade reaction is probably due to the densification of the sand around the pile, resulting in the increase in the stiffness of the pile. The increase in stiffness with the number of cycles contrasts with the cyclic load effects proposed by several researchers, where the modulus of subgrade reaction is degraded to account for cyclic loading by introducing a reduction coefficient, $R_n$, as

$$k = R_n n_h z$$

(8.2)

where $k$ is the modulus of subgrade reaction, $n_h$ is the soil reaction coefficient, $z$ is depth. For instance, Prakash (1961) proposes that $R_n = 0.70$ when the number of cycles is less than 50. Broms (1964b) suggests that for 40 cycles of load, $R_n$ equals to 0.75 and 0.50 for loose and dense sand, respectively.

The $N_g$ reduces by 10% compared with that for the primary loading. This means that $N_g$ for cyclic loading is about 0.9 times that for static loading. This conforms to the value of the reduction factor $A = 0.9$ on the ultimate soil resistance for cyclic loading, suggested by Murchison and O’Neill (1984) and API (1993) in the construction of $p$-$y$ curves in sand as presented in Table 2.8.

Guo and Zhu (2005b) investigate the static and cyclic response of six laterally loaded free-head flexible piles in calcareous sand by using the solutions presented by Guo (2006) with idealized elastic-plastic $p$-$y$ curves. Their assessment show that the value of $N_g$ for cyclic loading, in equation (2.9) for the generic limiting force profile described by equation (2.8) with $n=1.7$ and $\alpha_0 = 0$, is $(0.56 \sim 0.64)$ times that for
static loading. They attributed this degradation of the limiting force to the effect of ‘gap’ developed between the pile and soil under cyclic loading. This effect may also explain the reduction of \( N_g \) under cyclic loading for the current small-scale model tests, as evidenced by the depression or ‘gap’ behind the pile in Figure 8.6. Due to the limited test data, further investigation are needed to confirm this finding, and to extend for practical use in predicting the behaviour of rigid piles in sand.

8.7 CONCLUDING REMARKS

Static and cyclic behaviour of laterally loaded piles in dry sand was investigated through model tests. The responses of the piles were back calculated using a spreadsheet program based on the elastic-plastic solutions developed by Guo (2008) for rigid piles. The following conclusions are drawn from the experimental study.

- Equation \( M_{\text{max}} = \left( \frac{2}{3} z_m + e \right) T_e \) can be used to estimate the maximum bending moment \( M_{\text{max}} \) from the load, \( T_e \), eccentricity of loading, \( e \), and the depth \( z_m \), where \( M_{\text{max}} \) occurs. This equation can provide equally accurate \( M_{\text{max}} \), compared with FLAC 3D analysis presented by Sorensen et al. (2009).

- The use of annular pipe fastened on the pile for applying lateral loading causes the actual eccentricity less than the theoretical value. Nevertheless, the maximum bending moment \( M_{\text{max}} \) is still approximately linear related to the applied lateral load \( T_e \), independent of the number of cycles for all the tests. This effect can be considered by reducing the eccentricity, which gives \( M_{\text{max}} = \left( \frac{2}{3} z_m + \alpha e \right) T_e \), with
the factor $\alpha = 0.60 \sim 0.85$ for the current tests.

- The accumulated pile displacement increases with increasing numbers of cycles but at a decreasing rate during the cyclic loading.

- The cyclic load level has a greater impact on the pile behaviour than the number of cycles.

- Under cyclic loading, the modulus of subgrade for constant $k$, and the parameter $k_0$ for unloading-reloading are 1.63–2.88 and 1.60–2.0 times that of the primary loading.

- The $N_g$ for cyclic loading is approximately 0.9 times that of the primary loading. This reduction is consistent with the suggestions by Murchison and O’Neill (1984) and API (1993) for the construction of p-y curves in sand.
Table 8.1 Details for static and cyclic loading tests

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<th>Test</th>
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<th>( T_{min} ) (N)</th>
<th>( L ) (mm)</th>
<th>( e ) (mm)</th>
<th>Number of cycles</th>
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‡ Predicted capacity at ultimate yield state using the elastic plastic solutions (Guo, 2008);
† Numerator presents the values deduced from the primary loading, while denominator presents the values deduced from the nth cycles ($n = 50$ for TC1 and TC2, $n = 30$, 110, and 60 for TC3, TC4 and TC5).
Table 8.3 Calculation of $M_{\text{max}}$ (Data from Sorensen et al., 2009)

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<th>$l$ (m)</th>
<th>$e$ (m)</th>
<th>$z_m$ (m)</th>
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‡ Numerical analysis conducted by Sorensen et al., 2009;

† $z_m/l$ determined from the curves in Figure 8.18 with $e/l$. 

- 411 -
Figure 8.1 Experimental set up for lateral cyclic loading tests
$e = \text{eccentricity, } L = \text{embedded length of the pile.}$

**Figure 8.2 Schematic experimental setting up**
Figure 8.3 Schematic of a laterally loaded pile test
Figure 8.4 Response of pile during test TS1 (a) Load ~ displacement relationship; (b) bending moment distributions; and (c) pile deflection
Figure 8.5 Response of pile during test TC1 (a) Load – displacement relationship; (b) bending moment distributions; and (c) pile deflection
Figure 8.6 Soil displacement around pile (measured in tests TC1)
Figure 8.7 Response of pile during test TC4 (a) Load ~ displacement relationship; (b) bending moment distributions; and (c) pile deflection
Figure 8.8 Groundline displacements versus number of cycles
Figure 8.9 Maximum bending moment versus number of cycles
Figure 8.10 Predicted and measured response of test TS1
(a) Load ~ displacement relationship; (b) bending moment distributions;
(c) pile deflection; and (d) load versus maximum bending moment
Figure 8.11 Predicted and measured response of test TS2
(a) Load ~ displacement relationship; (b) bending moment distributions;
(c) pile deflection; and (d) load versus maximum bending moment
Figure 8.12 Predicted and measured response of test TC1
(a) Load ~ displacement relationship; (b) bending moment distributions;
(c) pile deflection; and (d) load versus maximum bending moment
Figure 8.13 Predicted and measured response of test TC2
(a) Load ~ displacement relationship; (b) bending moment distributions; (c) pile deflection; and (d) load versus maximum bending moment
Figure 8.14 Predicted and measured response of test TC3
(a) Load ~ displacement relationship; (b) bending moment distributions; (c) pile deflection; and (d) load versus maximum bending moment
Figure 8.15 Predicted and measured response of test TC4
(a) Load ~ displacement relationship; (b) bending moment distributions;
(c) pile deflection; and (d) load versus maximum bending moment

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Figure 8.16 Predicted and measured response of test TC5
(a) Load ~ displacement relationship; (b) bending moment distributions;
(c) pile deflection; and (d) load versus maximum bending moment
Figure 8.17 Calculated versus measured bending moment at groundline

Figure 8.18 Normalized depth of maximum bending moment at tip yield state (after Guo 2008)
Figure 8.19 (a) Load ~ displacement relationships; and (b) Predicted and measured $M_{\text{max}}$ (Data from Sorensen et al., 2009)
Figure 8.20 Comparison with FLAC 3D analysis (a) Pile deflection; and (b) Comparison of $M_{\text{max}}$ (Data from Sorensen et al., 2009)
Figure 8.21 Predicted and measured reloading responses of cyclic loading tests
Tests: (a) TC1; (b) TC2; (c) TC3; (d) TC4; and (e) TC5
9 CONCLUSIONS AND FURTHER RESEARCH

9.1 INTRODUCTION

This research provides an investigation of the responses of pile foundations due to lateral force and soil movements. The major achievements of this research are as follows:

(1) development of a simple solution for predicting the responses of piles subjected to lateral soil movement;
(2) establishment of the relationship between maximum bending moment and maximum shear force for passive piles;
(3) provision of design parameters for laterally loaded rigid piles in sand;
(4) identification of the difference of modulus of subgrade reaction between active and passive piles; and
(5) verification of the relationship between the maximum bending moment and lateral load for rigid piles, which is used in the solutions for active and passive piles.

These major findings have been developed through an extensive experimental investigation on piles in moving sand, including 43 tests on single pile (25 tests in this thesis, along with the previous 8 tests by Ghee (2010) and 10 tests by Chen (1994)), 21 tests on pile groups, 7 tests on single piles subjected to static and cyclic lateral loading, and a back analysis of 51 laterally loaded piles tests. On the basis of the test results in this thesis, Guo and Qin (2010) developed simple solution for passive piles. The relationship between the maximum bending moment and shear force found from this experimental investigation has been verified by the results of 8 in situ test piles and 3 centrifuge test
piles. The relationship between the maximum bending moment and lateral load for rigid piles is for the first time verified experimentally. This chapter summarises the principal finding from this research and discusses on future research.

9.2 FINDINGS OF RESEARCH

9.2.1 Active piles

The measured response of 51 laterally loaded rigid piles in cohesionless soil, including 16 full-scale in situ tests, 12 centrifuge tests, and 23 laboratory model tests have been analysed. The static and cyclic behaviour of 7 laterally loaded piles in sand has also been studied. The major conclusions from this study were:

• For laterally loaded rigid piles in sand, the maximum bending moment, $M_{\text{max}}$ can be approximately estimated by $M_{\text{max}} = \left(\frac{2}{3} z_m + e\right)T_t$ from the applied lateral load, $T_t$, eccentricity of the loading, $e$ and the depth $z_m$ at which maximum bending moment, $M_{\text{max}}$ occurs. The $z_m$ can be estimated from the curves in Figure 8.18. This equation can provide equally accurate $M_{\text{max}}$, compared with the FLAC 3D analysis presented by Sorensen et al. (2009).

• The use of annular pipe fastened on the pile for applying lateral loading causes the actual eccentricity less than the theoretical value in this study. Nevertheless, the maximum bending moment $M_{\text{max}}$ is still approximately linear related to the applied lateral load $T_t$, independent of the number of cycles for all the tests. This effect can be
considered by reducing the eccentricity, which gives  
\[ M_{\text{max}} = \left( \frac{2}{3} z_m + \alpha \omega \right) T, \]
with the factor \( \alpha = 0.60 \sim 0.85 \) for the current 7 tests.

- The load transfer approach using simplistic elastic-plastic \( p \sim y \) curves, such as the elastic-plastic solutions developed by Guo (2008) can offer good to excellent prediction of the nonlinear response of laterally loaded 51 piles in cohesionless soils.

- The elastic–plastic solutions (Guo, 2008) with a constant \( k \) by stipulating a linear limiting force profile with the same gradient, \( A_n \), generally give better estimation against measured nonlinear response for rigid piles in cohesionless soils than those with a Gibson \( k \) (a linear increase \( k \) with depth).

- \( k_d/G_s=3.27 \sim 6.91 \), with an average of 5.0, for the 16 full scale field tests and 2 centrifuge tests; and \( k_d/G_s=2.37 \sim 5.12 \), with an average of 3.7, for the 23 laboratory model tests.

- \( G_s= (3\sim20)\%G_{\text{max}} \) for the 11 full-scale and 2 centrifuge tests; and \( G_s=(0.8\sim2.6)\%G_{\text{max}} \) for the 20 small scale laboratory model tests, where \( G_{\text{max}} \) was estimated from the relative density of the sand by equations 7.5 and 7.6, as proposed by Seed and Idriss (1970).

- \( G_s= 181 \sim 600 \) kPa, with an average of 316.7 kPa for 16 laterally loaded model piles tests, which is much greater than the \( G_s=15 \sim 20 \) kPa deduced from the model tests on piles subjected to lateral soil movement. The difference is attributed to the large discrepancy in the shear strain level.

- The \( N_g \) is related to pile diameter, \( d \) and can be estimated by \( N_g = (0.4 \sim 1.8)(d/d_{\text{ref}})^{0.25} \). The ultimate pile capacity increases with the increasing \( N_g \). The parametric study
show that the ultimate lateral capacity may be underestimated by 171% and 36.5% using the soil resistance suggested by Broms (1964b) and Barton (1982).

- The Ng for cyclic loading is approximately 0.9 times that of the primary loading.
- The modulus of subgrade for constant $k$, and the parameter $k_0$ for unloading-reloading are 1.63–2.88 and 1.60–2.0 times that of the primary loading for piles under cyclic loading.
- The analysis results from the responses of 51 rigid piles are consistent with those obtained conducted by Guo and Zhu (2010) on 20 flexible piles in sand. They are useful in developing guidelines in estimating the shear modulus and gradient of the limiting force or soil resistance profile for laterally loaded piles in cohesionless soils.

### 9.2.2 Passive piles

A new apparatus developed by Guo and Ghee (2004) was used to investigate the behaviour of vertically loaded piles subjected to lateral soil movement. In total, 25 tests on single piles and 21 test on pile groups have been conducted. Guo and Qin (2010) developed new simple solutions for passive piles based on the test results. A new conceptual model was developed to explain the use of equivalent load for passive piles in this thesis. Modulus of subgrade reaction was deduced from the test results for passive piles in moving sand. This study showed that:

- The maximum bending moment, $M_{\text{max}}$, induced by the lateral load from soil movement is approximately linearly related to the maximum shear force, $T_{\text{max}}$, and can be evaluated by $M_{\text{max}} = T_{\text{max}}L/m$, with $m = 2.8$, regardless of the magnitude of soil movement. This relationship was found by Guo and Qin (2010) from the results of
eight test results. It is further justified by the extra six series of 16 tests on single piles by varying the distance between the ‘free field’ soil movement source at the loading side and pile location, the magnitude of axial load, the loading block angle, sliding depth, pile diameter and loading block shape. It is found that the $m (=2.8)$ is independent of these testing parameters. The linear relationship is calibrated using the results from 8 in situ tests, 10 similar small scale model tests and 3 centrifuge tests. The studies show that the value of $m$ may increase approximately 28% ~ 50% due to scale effect, variation of soil movement profiles and different pile behaviour to soil movements. For instance, $m=4.18$ for piles subjected to excavation induced soil movement. This relationship is useful in the design of piles to increase slope stability.

- A simple conceptual model was proposed to explain the use of maximum shear force as equivalent loading for passive piles. Two approaches were presented to correlate the $M_{max}$ with $T_{max}$. The first approach based on the current test results gives $M_{max} = 0.293T_{max}^2L$. The second approach based on the limit equilibrium analysis provides $M_{max} = (0.257 ~ 0.309)T_{max}^2L$. This simple model further proves the linear relationship between $M_{max}$ and $T_{max}$.

- Guo and Qin (2010) simple solution can be used to calculate the maximum bending moment, $M_{max}$ (equation 4.8) and maximum shear force, $T_{max}$ (equation 4.7) for passive piles. They have been extended to examine the effect of soil movement profiles induced by a rectangular loading block and a triangular loading block and group effect on the piles in terms of effective frame movement, $w_e = w_f - w_i$. It is found that the effective frame movement, $w_e$ causes perceptible pile response, whereas the initial frame movement, $w_i$ has trivial impact on the piles. The $w_i$ depends
on the pile diameter, pile position in the shear box, loading manner, angle of the
loading block and loading block shape (rectangle or triangle).

- Guo and Qin (2010) solution is suitable for the calculation of the initial linear
development of $M_{\text{max}}$ and $T_{\text{max}}$ with frame movement. Nevertheless, the linear
relationship between the $M_{\text{max}}$ and $T_{\text{max}}$ used in the solution is valid for the full range
of frame movement during the tests. In comparison with existing solutions for passive
piles, Guo and Qin (2010) solution can give a better prediction of the development of
$M_{\text{max}}$ with soil movement. The limitations of existing solutions were highlighted.

- Using Guo and Qin (2010) solutions, the modulus of subgrade reaction for passive
piles, $k$ was deduced as $15 \sim 65$ kPa from 33 single piles in moving sand. The
subgrade modulus $k$ can be estimated using the theoretical ratio of $k/G_s$ provided by
Guo (2008) and shear modulus $G_s$. The $G_s$ is pertinent to the overall shear process of
the pile-soil-shear box system.

- Guo and Qin (2010) solution was extended to analyse the response of pile groups in
moving sand. The modulus of subgrade reaction, $k_g$ was deduced as $18 \sim 60$ kPa from
the individual piles of 21 pile groups. These values are useful in assessing the pile
group effect.

- For a vertical pile under the combined loading of axial load and lateral soil movement,
increasing the axial load generally results in additional maximum bending moment
and excessive pile deflection in the model tests. For the 32 mm-diameter pile under a
pre-determined final sliding depth ratio, $R_L$ of 0.286, an increase in the axial load
from 0 N to 735 N (about 20% of the final jacking resistance) on the pile head leads
to: (1) an 80% increase in $M_{\text{max}}$; and (2) an 80% and 37% increase in $T_{\text{max}}$ in the stable layer and sliding layer, respectively.

- The sliding depth ratio, $R_L$, has a dominant impact on the pile response. When the magnitude of the $R_L$ is less than about 0.17, the induced pile bending moments and deflections were relative small. Once this threshold is exceeded, the moments and deflections increase substantially. The maximum bending moment, $M_{\text{max}}$, increases from 5.2 kNmm to 175.0 kNmm as the $R_L$ rises from 0.179 to 0.429 for a 32 mm model pile. The results of the present model tests also reveals that the $M_{\text{max}}$ may increase about 10% ~ 97% with an average of 48% at the final sliding depth due to the trapezoidal translational movement of the frames induced by the triangular loading block.

- The distance, $S_b$, between the pile location in the shear box and the loading side where ‘free field’ soil movements are generated may be reasonably used to quantify the severity of the detrimental effect of lateral soil movement on piles. The Maximum bending moment, $M_{\text{max}}$, Maximum shear force, $T_{\text{max}}$ and pile displacement at ground surface decreases with increasing distance, $S_b$.

- The angle of the triangular loading block, $\theta$ significantly affects the initial frame movement, $w_i$. The $M_{\text{max}}$ and $w_i$ increase linearly with the angle $\theta$ and can be evaluated by $M_{\text{max}} = 1.4\theta + 25$ and $w_i = 2.8\theta$ ($\theta$ in degrees) for the present model tests.

- The rectangular loading block induces a uniform soil movement profile adjacent to the loading side. By contrast, the triangular generates a progressive moving soil movement profile. Plotted against the effective frame movement, $w_e$, the $M_{\text{max}} \sim w_e$.
and $T_{\text{max}} \sim w_e$ curves are similar using the two types of blocks. The distribution of bending moment and pile displacement with depth are also similar in the RS and TS series of tests at a relatively shallow sliding depth ratio, $R_t$ of 0.286.

- The development of $M_{\text{max}}$, $T_{\text{max}}$ and $y_0$ of a pile in a group with frame movement, $w_f$ is similar to those of a single pile. The pile arrangement configuration, pile head condition and pile spacing have little impact on the evolvement pattern of $M_{\text{max}}$, $T_{\text{max}}$ and $y_0$ with frame movement, $w_f$.

- For the two piles in a row group, the maximum bending moment, $M_{\text{max}}$ decreases with an increasing distance, $S_b$ from the free field soil movement source to the pile location. The bending moment and deflections profiles of the capped-head groups are similar to those of the free-head cases, indicating that the pile cap has a limited impact on the behaviour of the piles in a row perpendicular to the direction of the soil movement.

- For the two piles in a line group and four piles in a square group, the $M_{\text{max}}$ of the front piles are general larger than those in the back piles at the same magnitude of frame movement. It indicates that the piles closer to the free-field soil movement source will reduce the detrimental effects arising from lateral soil movement on the back piles. The rigid pile cap has a significant influence on the pile responses. It leads to negative bending moment developed near ground line in the upper portion of the front piles. Due to the interaction among the frames, piles, pile cap and soil, the variation of $M_{\text{max}}$ with the pile spacing is rather complicated for the front piles. However, it generally decreases with increasing pile spacing for the back piles.

- For the two piles in a row group, group factors $F_m$ and $F_k$ decrease with increasing distance $S_b$. The factors $F_m$ for the free-head cases are less than those of the capped-
head cases for the three pile spacing, with the largest difference of 0.31 occurring at $S_v/d = 5$. The spacing $S_v$ has less impact on the group factor, $F_k$ than on the $F_m$. The $F_k$ may be taken as 0.75 on average and independent of the pile spacing and head fixity conditions. The group factors obtained from the current tests and the previous experimental and numerical analysis results show that the group factors decrease as pile spacing decreases.

- For the two piles in a line group of free-head fixity condition, the $F_m$ of the front pile may be more or less than unity, depending on the spacing, while the $F_m$ of the back piles decreases from 0.76 to 0.55 as the spacing $S_h/d$ increase from 3 to 7. The group factors, $F_k$ of the front piles are approximately (20 ~ 25)% greater than those deduced for the back piles at the same pile spacing $S_h$. For the free-head pile groups, the $F_k$ increases about 71% and 64% for the front piles and back piles as pile spacing $S_h/d$ increases from 3 to 7. The $F_k$ for the capped-head cases is less sensitive to the spacing than those for the free-head cases.

- For the four piles in a square (free-head), the $F_m$ remains constant at 0.8 for the front piles and decreases about 33% from 0.60 to 0.41 for the back piles as the $S_h/d$ increases from 3 to 7. The $F_k$ decreases with the increasing spacing for both the front and back piles. At the same pile spacing, the $F_k$ for the front pile are (0.42 ~ 0.95) times greater than those deduced for the back piles.
9.3 RECOMMENDATIONS FOR FURTHER RESEARCH

Despite the fact that a great deal of research has been done to investigate the performance of pile foundations due to lateral force and soil movements, many uncertainties still remain and therefore require further research. Some suggestions are given below:

- In the present study, rigid piles are used. It is necessary to assess the behaviour of relatively flexible piles subjected to lateral soil movement. Additionally, different soils may be used, such as clay or silts.

- The pile groups were tested under relative high free-standing with free-head and capped-head fixity conditions. Further research is required to investigate the influence of the degree of pile-head fixity conditions and reduction of the free-standing length above the ground line on the behaviour of pile groups.

- The simple solutions developed by Guo and Qin (2010) and the relationship between the maximum bending moment and lateral sliding force have been justified by the experimental results from two typical soil movement profiles. The simple solutions may be used for vertical piles subjected to excavation induced lateral soil movements; however, the value of $m$ is different.

- Numerical analysis should be carried out to simulate the frame movement, pile and soil interaction as observed during the tests to assess the factors influencing the pile behaviour, such as the frame movement pattern, pile head fixity, soil sliding depth.

- The present work may be extended to study the influence of axial load on lateral pile response in liquefiable soils during earthquake; and to study the impact of submarine slides on pipelines in the offshore environment.
REFERENCES


Barber, E.S. (1953). Discussion to Paper by S.M. Gleser. ASTM, STP 154, 96-99.


Brinch Hansen, J. (1961). The ultimate resistance of rigid piles against transversal forces. The Danish Geotechnical Institute, Copenhagen, Denmark, Bulletin No.12, 5-9.


Ghee, E. H. (2010). The response of axially loaded piles subjected to lateral soil


of one or two piles. *Journal of Geotechnical Engineering, ASCE*, 117(3), 448-466.


Wei, W. B., and Cheng, Y. M. (2009). Strength reduction analysis for slope reinforced


