A reflection on what adds up:

Teaching the mathematically disengaged

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This paper focuses on my experiences with a particularly difficult, disengaged middle school mathematics class who were disillusioned with the learning process. In response I introduced this class to Collective Argumentation, a structured approach for students to work collaboratively and participate in mathematical discussions. By giving the students a voice and an opportunity to talk their way to understanding an enhanced engagement was observable. The outcomes have implications for those teachers disillusioned by traditional teaching methods as they attempt to re-engage their students.

The problem or local context

Many students opt out of mathematics at their first opportunity without reaching their true potential. Nardi and Steward (2003) performed a one-year study interviewing and observing seventy disengaged 13/14 year olds who were not disruptive in mathematics classes. The students in the study resented rote-learning the rules and procedures. They said mathematics was difficult and tedious and didn’t allow them to work with their peers. The students requested relevance and variety in their activities and had a preference for collaboration and group work. Although Australian states’ Departments of Education strongly support co-operative learning, few schools systematically embed it in their pedagogical practices (Gillies, 2003). As a former classroom teacher and now teacher educator I have always been concerned about students underachieving. In this paper I reflect on my experience as a classroom teacher, on ways of helping disengaged students and discuss how Collective Argumentation helped provide a framework for a
difficult class to collaborate and engage more actively in mathematical discussions and thinking.

Middle school students are moving from concrete to abstract thinking (McInerney & McInerney, 2010) and can be intensely curious and capable of high achievement when engaged and challenged and can be taught to be self reflective (The Centre for Collaborative Education 2000 cited in Hilton & Hilton, 2005). Yet the students, such as those who are the focus of this paper, are often disengaged from schooling, with their behaviour characterised as problematic, and with low levels of achievement. Yet my study and those of others have found students can be helped to become more engaged and successful through the approaches taken by schools and teachers. Fredricks, Blumenfeld and Paris (2004) found in their review of school engagement that caring and supportive teachers can improve engagement by developing caring relationships with their students and therefore wield significant influence. As I found, teachers also need to believe substantively that all students want to think more productively and provide a classroom environment where all students can be actively engaged. If the teacher only ever transmissively presents information to students as the dominant top-down form of classroom discourse, higher order thinking is unlikely to occur (Hilton & Hilton, 2005). I had to encourage change in my classroom.

**Collective Argumentation: The conceptual context**

Lack of success and disengagement are closely linked to the absence of understanding. If students are operating from faulty premises, ongoing difficulties are likely to occur.
Schoenfeld (1992) argued that “doing mathematics [is] an act of making sense” (p 18). If students are to do mathematics and make sense of it, they need to be given opportunities to think about, talk and argue about their ideas in class. Teachers can support this type of exploratory learning by giving students a wide range of problems and situations from closed to open-ended. Students can construct or create new knowledge when they reflect on their physical and mental actions (McInerney & McInerney, 2010). When this knowledge is different from what students expect or where significant gaps occur, teachers need to encourage students to modify their existing schema and develop new concepts (Simon, 1995) — rather than to withdraw or rebel. Teachers can aid this process of assimilation or accommodation (see for example McInerney & McInerney, 2010) by providing a learning environment where “… children publicly express their thinking and more generally, engage in mathematical practice characterised by conjecture, argument and justification” (Cobb, Wood & Yackel, 1993, p 91). The teacher’s role is to encourage students to build on each other's explanations and to continue the conversation until the class has constructed and accepted a satisfactory group solution.

To create such a classroom culture, where students collectively solve problems and arrive at new understandings, the teacher needs to discuss with the students how to talk about mathematics and what their roles and responsibilities are in a dialogic conversation. The teacher’s role is not to evaluate the students’ responses per se, but to encourage them to share not only their solution but also the thinking that led to it (Cobb, Wood & Yackel, 1993; Solomon 2007). By judicious questioning and offering
suggestions to consider, students can learn to find their own errors and so build greater understandings. By providing students with a supportive environment students are able to move between the concrete and the abstract and are encouraged to discuss and defend their ideas.

Collective Argumentation (Brown, 2007; Brown & Renshaw, 2006) is a framework that allows students and their teacher to work collaboratively in the active solution of problems. The framework has four stages. The students initially individually represent their solution to (or ideas about) the question, problem, issue, statement, or task. This activity may be represented as a diagram, graph, algorithm or an idea for a method of solution. This step allows each student time to think about the problem and to have something to share in the subsequent discussion. The students then co-operatively compare their response with others in their group (2-5 students). As they take turns to explain and justify their response, the group is able to reach consensus and agree collaboratively on a common understanding and solution. The final response must be agreed to by all group members. All students have a responsibility to seek clarification from other group members if they do not understand and the other group members are obliged to help them. This final solution and the thinking and steps that led to their solution are presented to the class for discussion and validation. All other students are encouraged to engage in mathematical argument with the presenting group (Brown, 2007; Brown & Renshaw, 2006). This framework provides the support and structure for the students to engage in substantive mathematical discussions and to enhance ways of talking about mathematics.
The teacher’s overall role is to engage with the students and encourage them to participate in mounting mathematical arguments. By using appropriate evidence to question the students about the legitimacy of their conjectures and the usefulness of their solution strategies, the teacher is able to question the students about their processes (Lampert, 1990). To do this the teacher needs to listen to and observe before challenging students by asking questions or by seeking explanations and justifications. The teacher needs to ensure the mathematical knowledge is drawn out. This may mean rephrasing, paraphrasing and re-representing the students’ contributions. The teacher may also need to make larger connections between the student presentations and their previous work (Brown, 2007) showing the students that mathematical understanding adds up or builds. The teacher must also support students as they challenge the statements of others and in their use of the symbols of conventional mathematics.

To ensure the success of Collective Argumentation, the teacher will also need to help students develop the necessary behaviour patterns based on a set of classroom norms or charter of values. A suggested class charter of values could require students to (Brown, 2007):

• the courage required to state their ideas and opinions to others,
• the humility necessary to accept that their ideas may not always be adequate,
• the honesty essential to giving accurate feedback,
• the restraint integral to maintaining social cohesion,
• the persistence required to pursue ideas and views in the face of opposition, and
the generosity necessary to affirm the achievements of others.

A Case Study:
This study was part of a larger action research project where a group of teachers from different schools met with university researchers to reflect on their classroom experiences using Collective Argumentation. The teachers and students kept reflective journals and the teachers collected samples of student work, which were shared on professional development (PD) days. In the account that follows, I use excerpts from the discussion of my classroom practice during a Collective Argumentation PD day which were taped and transcribed (identified by number codes referring to the relevant paragraphs), excerpts from my reflective journal, and student work samples are used to illustrate the issues raised.

“I’ve never had a class like this!” I wrote in my journal. When I missed a class, supervising teachers assigned to teaching this class insisted on doing supervision only in pairs. My aim as a middle school mathematics teacher was to engage the students in this class in meaningful mathematical discussions so that they could talk their way to understanding. To do this successfully I needed to motivate the students so that they would want to participate more fully and make more sense of the mathematics. For this to occur, the students needed to see mathematics as relevant and useful to their daily lives and also to see that their ideas were valued by their classmates and me. In contrast, these students viewed mathematics as tedious and as Nardi and Steward (2003)
described an “irrelevant body of isolated non-transferable skills” (Nardi and Steward, 2003, p361).

I had been using Collective Argumentation in my teaching for three years. This was a Year 8 class who were disaffected with school; many were below the benchmark on State numeracy testing, had poor concentration spans and exhibited poor behaviour. For many of these students football was the one thing that kept them at school. “Well, yeh, obviously their football is more important because they call themselves the Bay Reps for life” (16 and see Figure 1).

Although the students had had four lessons on adding fractions, two with another teacher and two with me, I was sure that they still didn't understand the concept. So, as a challenge, I wrote up on the board, \[ \frac{2}{5} + \frac{3}{4} = \frac{5}{9} \] Why is it wrong?” When the initial reaction of some dominant students was “But it’s right Miss,” I changed the challenge to \[ \frac{2}{5} + \frac{3}{4} = \frac{5}{9} \] Why is it wrong or why is it right? Use diagrams and other methods to explain.” The aim of this task was to open up the problem and see what the students actually understood about adding fractions. It had become obvious that there were many long-held misconceptions about fractions. I asked the students to use Collective Argumentation and allowed them to form their own friendship groups. Each group attempted to add the fractions with some sort of algorithm and most had drawn diagrams of the fractions but did not necessarily consider the implications of their diagrams. For example, one group just added the numerators and added the denominators as in Figure
1. They drew a diagram of each fraction and as they were explaining another student commented “but \(\frac{5}{9}\) is smaller than \(\frac{3}{4}\) and you’re adding \(\frac{3}{4}\) and \(\frac{2}{5}\)”. The group decided that perhaps they were wrong. “And they decided that perhaps, yes, they were wrong because their pictures didn’t match, but they still didn't really want to stop adding the numerators and adding the denominators because they at least had a strategy they could use” (24).

![Figure 1: Student response where the numerators and denominators have been added.](image)

Another group realised that they needed a common denominator so they decided that, as 5 was the biggest number, it would be fifths. But to make \(\frac{3}{4}\) conform to the same denominator they added one to each of the numerators and the denominators so that \(\frac{3}{4}\) became \(\frac{4}{5}\). They then added \(\frac{2}{5} + \frac{4}{5} = 1\frac{1}{5}\) as shown in Figure 2. I talked with this group
about what they had done, asking them to draw diagrams. They hadn't understood that fifths are smaller than quarters so I drew the fifths and quarters above each other and tried to align them so that they could see that one fifth is actually smaller than one quarter. This misunderstanding confirmed that these students had a problem with fractions and they hadn't really understood the concept of a fraction as part of the whole (55).

\[
\frac{2}{5} + \frac{3}{4} = \frac{5}{9}
\]

(This is wrong because the denominator is not right. You got to make \( \frac{3}{4} \) to be \( \frac{4}{5} \) by adding +1 to the number it will add up to \( \frac{2}{5} + \frac{4}{5} \) which will equal \( 1\frac{1}{5} \).)

Figure 2 Student group response (with explanation retyped for ease of interpretation)

where a common denominator has been achieved by adding 1 to both the numerator and denominator.
Having students present their work for discussion (validation) has shown the importance of students talking about mathematics. When students use multiple ways of representing their ideas, by using symbols, diagrams and by talking about them it is possible to get a more comprehensive insight into the students’ mathematical understanding (Van der Walle, 2010) and address their misconceptions.

Each group of students presented their work to the class and each was incorrect. However, the students and I had some very productive discussions because they were actually thinking about the fractions (36 to 38). The students were engaged as they compared, explained, justified and agreed upon their group response. This was observable by their discussions and the consensus that had been reached before validation.

Once the students realised that everyone had a problem with fractions - even the students they considered mathematically capable - they showed increased motivation to learn. I introduced fraction blocks to help them understand that fractions consisted of equal parts of a whole, and for adding fractions with common denominators, for example, one third plus two thirds equal three thirds (28). Many students came voluntarily to lunchtime tutoring where we played with the fraction blocks; however, they didn't want other students to know. This is understandable because they wanted to be labeled as unsuccessful at mathematics (33). One group even hid outside the door until the previous group had left (33). We spent considerable time, over a number of lunchtimes,
building visual representations of what fractions were and how to add them. We also
explored equivalent fractions by overlaying the blocks. Although I tried working with
small groups in the classroom, we had limited success because they were surrounded by
unsupportive peers. Only students who came for extra help were successful in righting
their understandings (30).

This class took more time than I had anticipated developing the skills to successfully
participate in Collective Argumentation. Their first attempt was a disaster. Only three of
the ten groups managed to produce a group solution for the timeline they were asked to
draw and even these groups refused to share their timelines with the class. Initially we
concentrated on developing the skill to work with a partner and produce solutions and
explanations to problems on which they both agreed (consensus). To keep the students
on task and to stimulate discussion I asked the students questions in the form, “Please
explain why you have done it this way. Is that the only way you could answer the
question? Is there another way of doing it?” After much discussion about how to open
ideas up for class discussion and how to argue against others’ ideas without any personal
reference, we began with some groups presenting their findings for class validation.
Actually sitting quietly and listening respectfully to their peers proved very challenging
for many of the students.

Following the fraction lesson most students were much more willing to engage in
mathematical discussions when using Collective Argumentation.
Conclusion

By using Collective Argumentation I provided these middle-school students with a framework that helped them to participate in mathematical discussions more productively. They were beginning to learn that they could express and share their personal understandings and even re-consider and co-author them without losing face. Using this strategy helped many of the previously disengaged students to identify their own misconceptions about adding fractions. Many students also became motivated to seek assistance outside of regular lesson time. This framework has implications for those teachers disillusioned by traditional teaching methods as they attempt to re-engage their students with mathematics. Collective Argumentation provides a structured approach for students to work collaboratively. As students work through the steps of representing, comparing, explaining, justifying, agreeing and validating, they become involved in mathematical discussions which help them develop greater understandings. Class members may find it empowering to learn to use their student voice. Being able to present their work and to have the opportunity to justify their choices may help students to buy into an environment where everyone is valued not just for sporting prowess or for misbehaviour but for understanding what adds up. More research in this area is necessary to track development of student skills over a longer period.

References


