AN INVESTIGATION INTO THE USE OF STRUCTURED GAMES TO TEACH EARLY FRACTION CONCEPTS TO STUDENTS WHO ARE DEAF OR HARD OF HEARING

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I certify that the substance of this dissertation has not already been submitted for any degree and is not currently submitted for any other degree.

I certify that any help received I preparing this dissertation, and all sources, have been acknowledged in this dissertation.

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ABSTRACT

This study discusses the nature of mathematics and analyses current trends in mathematics education which focus on the learning process and its inter-relationship with the social and other contexts in which it occurs. In particular, the notion is presented that the development of mathematical understanding cannot be separated from the situation which nurtures it. The study relates these notions to the specific educational characteristics of students who are deaf or hard of hearing in relation to the development of understanding in mathematics education.

The particular educational focus of this study is the development of the concept of fractions with a group of four eleven and twelve year-old students who attend a special education unit for students who are deaf or hard of hearing. These students comprised half of the Year 6 and 7 classes in the unit at the time. The other students attended a regular hearing class for their mathematics instruction, but it had been determined that the students in the research project required an alternative program in mathematics.

The approach implemented in this study was a departure from traditional techniques for teaching fractions and relied extensively on the use of games with almost no formal instruction. It emphasised the development of appropriate language to facilitate an understanding of the notion of fractions through the investigation of concrete materials, pictorial representations and interactions between students and teacher. The approach had previously been used in a research project with hearing students and been found to develop an understanding of fractions. Each of the students is described and evidence of his/her participation is provided. The program is detailed through a sequence of games and activities and student outcomes. The specific role of games receives special attention in terms of mathematical and social outcomes.

Implications are drawn for approaches to developing understanding of fractions and mathematics generally for students who are deaf or hard of hearing which focus on the relationship between developing understanding and the language to describe that understanding. If there is to be an across the board improvement in "quantitative literacy" (Daniele, 1993) for students who are deaf or hard of hearing, it is suggested that sweeping changes are required at school level. This research provides a starting point for discussion among professionals as to their beliefs about mathematics, their expectations of their students and how best to approach the serious deficit of true understanding of mathematics among students who are deaf or hard of hearing.

CONTENTS

CHAPTER 1

Background to the study

Educating students who are deaf or hard of hearing

The challenge of teaching language (in our case, English) to deaf students is a difficult one, for despite efforts over many years, no solution has yet been found to the difficulties of developing standard English for students who are deaf or hard of hearing. Explanations for failure abound in the literature and they often centre on how teachers have failed to overcome these difficulties. Some suggested solutions lie in the limitations imposed by deafness and in the choice of a suitable mode of communication (Bouvet, 1990; Connor, 1986; Schlesinger, 1986) while others focus on the location where communication and curriculum meet (Davis, 1986; Green, 1990; Northcott, 1990; Ross, 1990). There is also a prevailing opinion that teacher expectations and communication style are at the heart of difficulties in developing appropriate language levels in students who are deaf or hard of hearing (Connor, 1986; Wood, Wood, Griffiths, & Howarth, 1986). In reality, the solution probably lies somewhere in all of these and, as a profession, educators of the deaf are still searching for it.

"One of the major goals of educating children with impaired hearing is to facilitate their ability to communicate effectively with others" (Luetke-Stahlman & Luckner, 1991, p. 4) and, to this end, the fundamental tool is language. "Language is the medium that allows people to transmit thoughts to other human beings, to identify their innermost feelings, to aid in solving personal problems, and to explore the world beyond their sight and current time frame" (Luetke-Stahlman & Luckner, 1991, p. 4). Language is empowering. It is fundamental to communication and understanding in every aspect of human endeavour. On the other hand, reduced facility with language confines humans to a very restricted world.

Because of the complex nature of deafness, a simple definition is not possible, so within this document, the term "deaf or hard of hearing" is used to encompass all aspects of hearing impairment in a very general sense: the term "deaf" generally meaning more significant hearing impairment and hard of hearing denoting a less significant hearing loss. In addressing what is meant by "deaf" and "hard of hearing", there are three aspects to be considered. Firstly, there is the clinical measurement of hearing. Modern technology and the skills of audiologists allow us to provide accurate measurement of hearing. This information then influences the amount and type of amplification that is provided for children. In turn, teachers are supplied with aided and unaided audiograms which indicate the amount of hearing available to individual students with and without hearing aids. As a result of information gained from the unaided audiogram, audiologists also describe students in terms of being mildly, moderately, severely or profoundly deaf. However, these classifications are not a watertight indication of how students will function auditorily and educationally. Therefore it is useful to qualify these descriptions in functional terms. For example, profound deafness does not preclude the use of audition in an educational setting and a moderate hearing impairment does not automatically indicate minimal educational difficulty.

Finally, the clinical description of hearing impairment need bear no relation to the cultural definitions of deafness. Some students who are deaf or hard of hearing are already part of a distinct cultural group, the Deaf community.

Members of this community have a cultural identity and they signify this by writing "Deaf" with a "capital d". It may be that they come from Deaf families who sign or they may identify with other Deaf people. They are proud to be Deaf and communicate through the language of the Deaf community, Auslan (Australian Sign Language). Many other students will become part of this community at some time in their lives. Students who are already nurtured by the Deaf community generally have a first language, Auslan, and this may contribute to a smoother transition to acquiring English. Similarly, these students have many Deaf role models to assist in promoting a positive self-image. Therefore, it can be seen that simple descriptions of students in terms of their clinically measured levels of hearing are not sufficient to describe attitudes towards deafness and individual educational needs.

Educationally, the impact of reduced hearing is greatest in the area of communication and language development. Just as the clinical classifications of hearing are not a watertight indicator of how students will function educationally, they are not reliable indicators of how language will develop (Boothroyd, 1982; Power, 1990). Many student characteristics interact when learning and language development occur. These may include the presence of additional disabilities, motivation to learn, age of onset of hearing impairment, exposure to language away from the school setting and, of course, home environment (Power, 1990).

In terms of formal education for students who are deaf or hard of hearing, the link between facility with language and literacy is well accepted (Power, 1990). The connection between language proficiency and thinking processes is also recognised (Wood et al., 1986). However, the importance of language in developing mathematical understanding and skills is not so widely embraced.

Therefore, another frontier of language development presents itself and a new challenge emerges, that of cultivating communication and understanding within the parameters of mathematics. If we revisit Luetke-Stahlman and Luckner's definition of language, we come across some of the essential skills at the core of mathematics, not only for students who are deaf or hard of hearing but for all students. Communicating thoughts, recognising feelings, solving problems, and looking beyond the immediate are all part of what are currently acknowledged as goals for school mathematics (*A National Statement on Mathematics for Australian Schools*, 1990). The very attributes that contribute to mastery of mathematics are those that lie at the heart of challenges within deaf education generally.

Mathematical achievement for students who are deaf or hard of hearing

What does this mean in terms of mathematics for students who are deaf or hard of hearing? Is proficiency in mathematics something many can never hope to achieve? What do students who are deaf or hard of hearing need mathematics for? In his article *Quantitative Literacy,* Daniele (1993) states what should be obvious to all — deaf people require the same skills in mathematics as hearing people if they are to be participating, contributing members of society. Therefore, he urges teachers and administrators to "assume responsibility for initiating and monitoring progress toward mathematics literacy" (Daniele, 1993, p. 81). In doing so, he advises that it is critical to resist the temptation to provide students "life skills" in mathematics if indeed those skills reflect little more than drill with whole numbers, computation and a little work with fractions, decimals and percentages. Instead, "course goals should move students toward becoming better problem solvers by communicating mathematically" (Daniele, 1993, p. 79).

Students who are deaf or hard of hearing cannot be expected to participate fully in our society without the same mathematical skills as their hearing counterparts. The key to developing these skills lies in being able to apply mathematical skills and discuss what they are doing. For this to occur, language enters the equation. Daniele, on considering what is at the heart of true understanding of mathematics, states "Mathematics can be characterised as the language we use to quantify concepts and express various relationships" and that "mathematics, language, thinking, communication, and success in the workplace cannot be separated" (p. 81).

Before delving further into this discussion, it is timely to examine what constitutes school mathematics. Daniele has hinted that it is much more than a study of formulas. It is an all-encompassing way of thinking, operating and communicating. *A National Statement on Mathematics for Australian Schools* (1990) interprets mathematics as "the science of patterns" which "involves observing, representing and investigating patterns and relationships in social and physical phenomena and between mathematical objects themselves" (p. 4). This document also illustrates the importance of mathematics in our society. Mathematics is embedded in our culture, it is part of the way we think and operate. We use it every day consciously and unconsciously. Understanding mathematics enables us to make judgements about significant issues in our society according to information that is disseminated, and to make informed decisions regarding these issues. We need mastery of mathematics to gain employment in many areas and also to function within employment. When we examine the uses of mathematics, they are all relevant to people who are deaf or hard of hearing. Indeed, they are more than relevant, they are essential.

Mathematical understanding and mastery generates the power to be informed, to be responsible, to be independent. Lack of these skills results in uncertainty and vulnerability.

Changes to mathematics education

Mathematics education has undergone enormous changes in recent times. The emphasis in teaching mathematics has moved from the more mechanical aspects of mathematics to active involvement in learning and developing understanding of mathematical concepts through "thinking, reflecting, organising and applying what is learned to other situations" (Griffiths & Clyne, 1994, p. 1). No longer is it sufficient for mathematics to be regarded as the passive transmission of knowledge from teacher to student or as students reproducing teacher knowledge: "This form of mathematical communication, with its emphasis on students learning a fixed set of mathematical concepts, skills and procedures, promotes the view that mathematics knowledge is a 'body of immutable and necessary truths' " (Ernest, 1991, p. 62) "that one must somehow store in one's memory for later access" (Frid, 1993, p. 27). This type of approach to mathematics teaching results in poor understanding and an inability to apply what is learned to novel situations, as is illustrated by students' own statements as recorded in Frid (1993, p. 27): "I have just memorised and I use it. I don't particularly understand it. ... And right now it's just a matter of being able to produce it on a test." Another student explained: "I can do it but I don't know why. Just do it because he says so". These students have been able to verbalise that they realise they don't understand mathematics, and implicit in their statements is a perception that understanding is possible and it would empower them to apply their mathematical knowledge and deliver success in the mathematics arena. However, it shows even more a failure in mathematics understanding when a student says to a teacher: "You talk, talk, too much. Just show me how to write it." This was announced recently by a deaf student during a mathematics session aimed at teaching and learning the division algorithm and what it represented. She was experiencing difficulty understanding, and therefore communicating, what the algorithm represented and demanded that the teacher revert to the method of teaching mathematics she had experienced in the past and in which she had found some measure of "success" (so called). Her statement indicated that she was uncomfortable with the expectation that she understand and communicate about mathematics and she preferred to simply follow a formula for computation because, from past experience, that was all that was necessary for success in mathematics. In all of these cases, the students' thoughts and perceptions of mathematics have been shaped by an approach to mathematics teaching which does not meet students' needs and is not appropriate in 1997.

The current approach to mathematics education has one of its foundations in the work of Vygotsky in Russia in the early part of this century. Among the theories he proposed was the belief that learning should be an active process and that an essential ingredient in this process is language. He likened the process of learning to the workings of labour changing materials into practical objects and he compared language to tools: tools necessary to mould the process of learning. "Vygotsky extended the mediational role of tools to psychological tools such as sign systems (language, writing, number systems). Vygotsky saw languages as playing a special role in the development of thought" (Confrey, 1995, p. 39). "Thus in Vygotskian theory, languages, sign systems, possess a central role in the development of higher cognitive thought" (p. 40). Vygotsky saw learning as moving from what is known to the next small step in new knowledge. To Vygotsky, learning was an active process undertaken by the learner and facilitated by the teacher. The teacher's role was not to impart knowledge but to create appropriate experiences and to guide the students to understanding through communication. Vygotsky believed that knowledge is something to understand and use, not merely to reproduce.

Central to Vygotsky's approach is the social aspect of learning. Recognising that learning occurs in a social context, Vygotsky highlighted the importance of interactions in the learning process. "Vygotsky places the role of social interactions at the forefront of his theories. For him, activity is inherently social, and it is through the engagement in activity, in the company of parents, peers, teachers and others, that intellectual development transpires" (Confrey, 1991, p. 28). The implication of this is that, once again, communication, and hence, language, emerges as crucial in the learning process. It is also implicit that the learning context must be a supportive and positive one for students to participate confidently and openly in this agenda.

The implications of the depth and breadth of Vygotsky's theories have become apparent in contemporary approaches to mathematics education. Currently, the process of teaching mathematics is regarded as "instruction that focuses on providing carefully structured opportunities for discovering rules and inventing algorithms, activities essential to developing flexibility in using numbers" (Markovits & Sowder, 1994, p. 5) by challenging students with interesting activities in an environment that encourages and fosters enquiry: one that is bathed in mathematical communication. The outcomes aspired to from this process are true mathematical understanding, an ability to apply and use this understanding, confidence in doing things mathematical, an ability to work and communicate cooperatively.

Just as the implications of Vygotsky's theories are clear for mathematics education generally, so too they have implications for the education of students who are deaf or hard of hearing in all areas, including mathematics. Because of the substantial difficulties that many students who are deaf or hard of hearing experience in literacy and because the same students are generally able to do simple computation, the notion occurs that these students achieve acceptable levels of mastery in mathematics. However, the reality is that they have long struggled with mathematics in the broader sense because it involves much more than memorisation and formulas. "It is incorrect to assume that deaf students as a group are doing acceptably well in mathematics" (Daniele, 1993). Wood et al. (1986) also acknowledge the "relatively slow progress in learning mathematics" among deaf children. Barham & Bishop (1991) open their investigation with the statement that "Most teachers of deaf children, at both primary and secondary level, would agree that mathematics is one subject with which their pupils find special difficulty" (p. 179). Stone (1988) adds her voice to the chorus of researchers recognising the problem as "the tendency of deaf students to know better how to perform a mathematical operation than when to perform it" (p. 67). Therefore, it is apparent that, as a general rule, students who are deaf or hard of hearing do not achieve in the area of mathematics at an acceptable level: certainly not at a level that would empower them to use mathematics in the manner outlined in *A National Statement on Mathematics for Australian Schools* (1990). For many of them, their skills are limited to simple computation and they lack the understanding to empower them to use the skills they have in everyday situations and may also be, therefore, severely limited in career options.

Mathematics education for students who are deaf or hard of hearing

Although deafness is a low incidence disability, there is, nonetheless, much research available on many different aspects of learning by deaf students—philosophical approaches, language development, audition, literacy, appropriate school settings and the like. However, there is a surprising lack of research regarding mathematics education for students who are deaf or hearing impaired. This tends to indicate that, within deaf education, mathematics education is not high on the agenda. It is possible that other pressing issues such as philosophical differences in educational approaches or developing language and literacy dominate teachers' attention despite the acknowledged importance of mathematics in daily living and career options. Indeed, Fridriksson & Stewart (1988, p. 51) believe that "mathematics is a neglected area in the total instructional component of a deaf child's education. ... Little attention is devoted to the conceptualisation of mathematics principles and teachers rely heavily on drill and practice techniques." This leads to the impression that teachers of the deaf may not expose students to the full spectrum of mathematical experiences, tending instead to concentrate on the easier areas of memorisation and avoiding the difficult areas of understanding and application. This argument is supported by Wood, who, after investigating mathematical achievements of deaf or hard of hearing students in a number of settings, believed that reduced overall mathematical achievement among these students occurred because "many teachers teach the same somewhat limited curriculum to deaf children" (1986, p. 164). Kluwin & Moores (1989) went so far as to suggest that teachers of the deaf are not as familiar with current trends in mathematics education as their counterparts in hearing classes and that individual locations need to look to the organisation of their mathematics programs as well as in-service training with a focus on the content of mathematics curriculum.

If there is a dearth of information available about mathematics generally for students who are deaf or hard of hearing, there is even less material concerning the teaching of fractions. The topic of fractions is the focus of this thesis and is generally an important element of all mathematics programs. Fractions play a significant part in our use of mathematics. "Rational numbers [such as fractions, decimals and percentages] have a variety of applicational meanings. Rational numbers are involved in representing and controlling part-whole situations and relationships; they are fundamental to measuring continuous quantities; they are involved where quantities, particularly continuous quantities, are divided; and they are involved in any quantitative comparisons of two quantities" (Pitkethly & Hunting, 1996, p. 5). Therefore, the understanding and application of fractions is a basic mathematical skill required by all participating members of our society for a variety of purposes. However, the notion of fractions is socially constructed, i.e., fractions are not as "obvious" as whole numbers, and exist within the context of our culture. Therefore, the associated language is complex and specific to the mathematics domain. Consequently, understanding fractions is a significant challenge for students who are deaf or hard of hearing. "A fractions curriculum is very challenging to teach. Some of the challenge is because fractions look different from any numbers the [deaf] children have seen up to this point" (Silva, 1986, p. 126). For whatever reason, it would appear that fractions are not high on the agenda in mathematics programs for students who are deaf or hard of hearing but are, nevertheless, an important and useful skill for participating in our society.

It is indeed fortunate for teachers of the deaf that current approaches to the teaching and learning of mathematics support the need for meaningful activity combined with appropriate language. Although there may be a surprising shortage of details about mathematics education for students who are deaf or hard of hearing, the situation is not as desperate as might at first appear. Many papers and programs based on current approaches to the teaching and learning of mathematics furnish appropriate information for mathematics programs designed for students who are hearing. Because of the emphasis on combining experience and appropriate language, they are also relevant for students who are deaf or hard of hearing. They offer teachers of these students the opportunity to shift the focus of mathematics programs from memorisation to understanding and, through the emphasis on communication and language, to enable teachers to transfer the skills employed in other curriculum areas to the mathematics domain. Nevertheless, it is always a challenge to modify approaches to teaching and expectations of students.

The beginnings of a resolution of the problem of developing appropriate mathematical mastery for students who are deaf or hard of hearing thus starts to emerge. It promises to materialise from the association between meaning, understanding and language. Hence, this project will investigate the implementation of an approach to teaching mathematics to students who are deaf or hard of hearing which is based on "constructivist" theories of learning (Cobb, Yackel & Wood, 1992; Herscovics & Berganon, 1984) which support building links between meaning, understanding and language. The mathematical content used will be beginning notions of fractions. The study will examine learning outcomes for students in terms of understanding of the mathematical content as well as the appropriateness of the approach to teaching and learning mathematics (particularly fractions) for students who are deaf or hard of hearing.

CHAPTER 2

Literature review

Mathematics in our lives

Mathematics is recognised as an important component of the educational process with outcomes that are not only valued by our society but are viewed as essential for informed and accomplished participation in that society. Furthermore, proficiency in mathematics is regarded as an avenue for enriching one's life and attaining fulfillment in personal and career aspirations. Yet this justification for mathematics tells us little about what it is and how it enables independence and fulfillment. There have been many formal definitions, such as "a science of space and number" or "a science of patterns" (*A National Statement on Mathematics for Australian Schools*, 1990, p. 4), "a symbolic language" (Luetke-Stahlman & Luckner, 1991, p. 318), "an abstract and highly symbolic subject where precision and formalism are critical" (Durkin, 1991, p. 3). However, these definitions fail to include what is possibly the most significant aspect of mathematics, and that is, "understanding". For whatever else mathematics may be, it is useful, and to use mathematics, it must be understood.

A more meaningful description of mathematics comes from Daniele (1993, p. 80), "Mathematics can be characterised as the language we use to quantify concepts and express various relationships". From this perspective, we are invited to consider mathematics as a means of communicating, describing and understanding things that happen around us, thus empowering us to predict what might happen in the future. Indeed, mathematics is more often described in terms of outcomes and applications. "It can enhance our understanding of our world and the quality of our participation in society" (*A National Statement on Mathematics for Australian Schools,* 1990, p. 5), in daily living, in civic life, at work and as part of our culture. Booker, Briggs, Davey and Nisbet (1992, p. viii) claim that using mathematics is one of the paramount outcomes of educational programs, so "problem solving should be central to the curriculum as the focus shifts from learning standard procedures to applying a variety of processes to real-life needs". Daniele (1993, p. 76) also recognises the power of mathematics in our society

> [by] preparing students for the workplace ... [it] also carries enormous political significance as it relates to critical thinking, empowerment, fostering informed voting, preserving culture, and maintaining cultural and economic control; and it further can be viewed simply from the perspective of adding dignity and richness to life itself.

Mirroring the value placed on mathematics within our culture, schools allocate a significant amount of time to mathematics instruction. Mathematics inservice programs for teachers are encouraged on a regular basis as teachers are expected to provide an environment that stimulates students to achieve a level of understanding and proficiency that will enable them to achieve these outcomes.

Changing conceptions of mathematics education

As beliefs about mathematics have shifted from viewing mathematics as a body of immutable truths that can be passed didactively from teacher to student to believing that mathematics is "a way of knowing" (Brissendon, 1980, p. 72) there have been fundamental changes in approaches to teaching and learning mathematics. Some of these changes can traced back to Vygotsky (1986) who considered that reality was continually constructed in social situations through communication and was not simply the learning of a preexisting body of knowledge. It was Vygotsky who recognised that learning occurred in a social context and the social context had an important influence on learning outcomes. He understood that while learning and context are separate, they are also inextricably intertwined, as learning is not only a process of understanding information, it is also a process of enculturation into our society. As we acquire information, we also discover what our culture regards as significant. Within this social context, communication is essential and the facilitator for this is language. Language, he theorised, facilitated a shift in the learning process from simply remembering to logical thinking. In particular, Vygotsky established a link between language and thought. He believed that language was a "psychological tool" and, as such, had a particular function in the development of logical thought. In this way, he postulated that learning was not simply imitation but depended on the social context combined with effective communication. This was formalised in his theory through which he described the roles of the teacher and the learner in the process of developing knowledge through the conception of the "zone of proximinal development" as a territory between those tasks which a student can undertake successfully independently and those which require assistance from an adult (Confrey, 1995, p. 40). It is one of the most fruitful modern conceptualizations of the relationship between teacher and learner.

> The "zone of proximinal development" [is] where students perform beyond the limits of their individual skill, supported by a more experienced person. In social interaction in the zone of proximinal development, children are able to participate in more advanced problem solving than they are capable of independently, and in doing

so they practise the skills that they internalise to advance what they can do independently (Tudge & Rogoff, 1984).

Constructivism

In contrast with an earlier view of developing mastery of mathematics which treated knowledge as "an integral, self-sufficient substance theoretically independent of the situations in which it is learned and used" (Brown, Collins, & Duguid, 1989, p. 32), it is now accepted that, in keeping with Vygotsky's theories, students actively construct their mathematical ways of knowing as they strive to be effective by restoring coherence to the worlds of their personal experience (Cobb, 1994). Therefore, learning is dependent on the social activity that surrounds it. Mathematical learning should be viewed as both a process of active individual construction and a process of enculturation into mathematical practices of wider society (Cobb, 1994). This approach to learning is referred to as "constructivism" and operates on very different suppositions about learning than do traditional approaches to instruction. One of the outcomes of traditional approaches to teaching mathematics is that students develop a conception of mathematics as a body of mysterious rules which are, basically, not easily understood and applied. Constructivist approaches, on the other hand, aim to develop understanding of mathematical concepts which can be applied and used in a variety of situations.

Within a constructivist framework, the mathematics classroom is full of activity and interactions because the view of mathematics used is that concepts or realities are best developed through the mental "conflicts" experienced while solving problems in activity. In this way, activity is initially valued above correct answers and errors are seen to be an important part of learning. Through activity, learning from errors and from others, students build their understanding of mathematics which is more meaningful than simply reproducing the teacher's ideas. Hence understanding leads to correct answers and students who are less likely to make mistakes. Through activities, mathematics can be used to describe and predict as well as solve problems. Mathematics involves logical thought which can be developed and used to solve problems and improve understanding. By participating in a variety of activities, students become aware that mathematics is part of our lives inside and outside the mathematics classroom (Griffiths & Clyne, 1994).

Activity facilitates learning in two ways. Students develop meaning from their own activity of enquiry and discovery and they also learn from interacting with other students or the teacher. Mathematical understanding is a process that builds upon itself through activity and reflection. Central to learning in this way is communication which, according to Vygotsky, can occur with others or with oneself (reflection), and the "tool" for such communication is language. Communication is significant in every aspect of learning in a constructivist classroom as meaning is negotiated and shared. Thus activity and communication are the basic structures of a complex web of characteristics of constructivist learning and growth of understanding.

An important aspect of communication is discussion, which is significant in mathematics for a number of reasons. Through discussion, the participants discover the background knowledge of a partner in learning and, from there, work towards mutual understanding (but not necessarily agreement) as they describe, justify, challenge and reflect and, in doing all these things, clarify their own thoughts (Ainley, 1988; Burnett, 1993). Discussion also furnishes the teacher with a valuable tool for evaluation by illuminating students' thinking and creating opportunities for clarifying understanding (Ernest, 1986). Additionally, the teacher can furnish much positive feedback to students and promote a positive attitude to mathematics through the teacher's involvement in discussion.

As they work with others, students discuss what they are doing and they listen to different ideas. In this way, they are able to expand their understanding and learn from others as well as clarify their own thinking through formulating and communicating their ideas in a meaningful way (Ernest, 1986). Shared activity creates meaning through a social context and involves watching and listening and cooperation. The social context of learning is not peripheral to the process but an integral part of it (Booker, 1996). As students discuss and negotiate meaning, they make sense of their activity and construct their own mathematical understanding according to their interpretation of their experiences. Participants use each others' contributions to the activity to develop and share meaning. Negotiation does not necessitate arguments. It implies discussion and sharing of meaning through communication. Therefore, meaning is not bestowed by the teacher, it is constructed by participants and continues to evolve with new endeavours and experiences.

Activity is part of the context for learning because it provides the reference point for what will grow into the abstractions of mathematics. There is no substitute for activity. "Representations that arise out of activity cannot easily (or perhaps at all) be replaced by descriptions" (Brown, Collins & Duguid, 1989, p. 36). Through activity, learning is embedded in meaning and in the culture of everyday ventures. Students are presented with opportunities to use their knowledge to further expand their understanding of mathematics. They realise that mathematics consists not of isolated, meaningless abstractions but is a practical and useful part of the way we live.

In the beginning, it is the materials themselves that capture the attention of the child. By continually engaging with these materials, thoughts about mathematics begin to take shape. Gradually, the materials become a vehicle for thinking rather than an end in themselves. In the same way as children gain satisfaction from re-reading a favourite book, children are motivated to continually revisit mathematical ideas (Pengelly, 1990).

Activity also provides students with opportunities to construct mathematical knowledge in a way not available through print. Through activity, students can see the mathematical notion modelled, mentally manipulate the notion, discuss the notion, negotiate the significance of the notion and, in so doing, build their own understanding.

Activity initiates students into collaboration and cooperation in the social setting which is an important part of the way we learn and work in our society. It shifts the focus from individual performance to shared outcomes and the process of collaboratively solving problems. Brown et al. (1989) propose a number of positive outcomes from collaborative activity. Groups not only provide a platform for accumulating knowledge but they give rise to insights which would not occur individually, and as a result, students discover how to cooperate with and learn from others. Group activities also generate opportunities for members to understand, rehearse and reflect on different roles involved in collaboration. Students discover when to take initiative, when to observe others, when to work individually or with others, or who will undertake different jobs (for example, record what is happening or be spokesperson; Slavin, 1983; 1987). Finally, students learn how to confront misconceptions and restructure their thinking in a group situation, a skill they will need as they continue through school and in many career situations. It is timely to reiterate that language and communication play a crucial role when students collaborate.

For group activity to be successful, students must find the endeavour meaningful and motivating, and although it may be challenging, they must feel that the task can be accomplished. Therefore, support should be available to guide students' learning. Blumenfeld (1992) suggests that in a cooperative activity situation students are motivated by the opportunity to be involved in independent learning and show a greater level of cooperation and individual mastery than in a traditional learning situation. She says, "Students construct understanding through actively participating in a community of learners; peers are seen as sources of information rather than a threat to one's self-esteem" (p. 277). As Webb (1991) discovered through her research, in some cases it can be mutually beneficial for students to support each other. Webb found that where explanations were given in response to a request for assistance, both the student who gave the explanation and the student who received the explanation profited from the activity. The student providing the explanation benefited from the opportunity to clarify his thinking and the student receiving the explanation benefited from an explanation in language pitched at his level and within his zone of proximinal development. Davidson (1990) lists other social outcomes of group activity for students. He notes that students' self-esteem improves in cooperative learning situations and that social acceptance of mainstreamed students with disabilities is improved in such cooperative learning circumstances. Finally, it is interesting and useful to note the findings of Cooper, Marquis, King and Moore

(1982) that the ability to cooperate improves over time, especially during middle childhood. Therefore, many of the expectations related to the social aspect of mathematics may be inappropriate with younger students because they are not ready to participate in cooperative activities.

Within a constructivist framework, teachers play a pivotal role in learning through providing activities that are appropriate and motivating, and by modelling meaningful discussion and coaching students experiencing difficulty (Leder, 1993). It is the teacher's responsibility to ensure that activities are pitched at a level that activates the students' prior knowledge but affords them the opportunity to build on it through experiencing similar ideas in a number of different ways. Students need to be exposed to and give justifications for their reasoning in problem solving. Teachers need to be aware of students' ways of thinking and to be able to assist with understanding errors and answering questions in a constructive way. Students need to be guided through the maze of mathematical language by using the familiar to initiate them into the unfamiliar. It is also the teacher's role to establish an atmosphere of acceptance of risk-taking and one that values all contributions by being "enthusiastic, knowledgeable, nurturing, respectful, and trustworthy" (Blumenfeld, p. 276).

The social dimensions of learning are supported by the institution of certain expectations in the classroom. For students, learning in this way constitutes risk-taking. By exposing their thinking and conceptualisation to others, students risk criticism and negativity from their peers. Therefore, it is important that there is an atmosphere of acceptance within the learning situation that promotes diversity, values difference and accepts all contributions, however wide of the mark (Cobb, McClain & Whitenack, 1995). In the situation where difference is valued, it is reflected in diversity of ideas and differences in learning styles. Students are encouraged to contribute to discussion and to listen, to participate in activity and to watch, to share their ideas and to accept others' contributions, to generate solutions to problems, to reflect and to reach consensus. When these expectations are accepted by students, they become comfortable with contributing to discussion and exposing their thinking. Another justification for instituting such classroom expectations is to establish norms of behaviour, thus minimising the breakdown of discourse and classroom organisation which can occur when differences arise in activity-centred learning (Voigt, 1994). "Our concern lies not with social skills for their own sake but only with the extent to which they contribute to a group's ability to learn from discussion. Each group is simultaneously negotiating social relationships and attempting the given task: if the former preempts their time and attention little learning will go on" (Barnes & Todd, 1977, p. 46).

At first glance, this approach may appear a "laissez-faire" method of learning and teaching, but within the framework of activity-based teaching, there are certain roles for the teacher and students. The teacher plays a crucial role in creating activities and an environment for children to develop concepts. True to Vygotsky's theory of the zone of proximinal development, it is the teacher's role to establish what the child knows and to create an environment in which she can build on this knowledge through activity and interaction. If the activities involve a conceptual leap that is beyond the student's current capability, she will not learn from the experience. By interacting with their students and listening to them, it is the teacher's function to encourage the children to explore the activities in a way that assists them to construct mathematical understanding. It is legitimate for the teacher to intervene when the children are in need of direction, affirmation or information through modelling and questioning. For teachers to do this, they must be familiar with the mathematical content they are promoting in their classroom. "The teacher or instructional designer approach[es] the double agenda as such, engaging students constructively in thinking both about *X* [the mathematical content] and about the learning process reflectively" (Perkins, 1991, p. 20).

The students' most significant role is to accept responsibility for their own learning. This is pivotal because the constructivist approach to teaching and learning mathematics is based on confronting the students with conceptual conflict and allowing them to find a way or ways of resolving that conflict and, in so doing, expanding on their mathematical understanding (Perkins, 1991). The classroom requires students to manage their behaviour and roles in the ways expected in a constructivist classroom. Autonomy is part of the student role in a learning situation where they must make choices regarding how they will participate in an activity and how they will interact with others. This capacity is not necessary in a more traditional learning situation where information is simply passed from teacher to pupil.

The role of language in learning mathematics

It has been argued that language and communication are fundamental to developing understanding of mathematics because of the need for discussion about concepts and activities and the central role of learning from others. The relationship between the two is clear. Language arises from the need to communicate and is one of the most important facilitators of communication. Because language is standardised by a set of rules, understanding between speaker and listener can be achieved when those rules are mutually understood. However, there is a number of facets of language that are particular to mathematics, and although they are governed by the same set of syntactic rules as “ordinary talk”, they contribute to the complexities of understanding mathematics. Therefore, just as language is an important part of the solution to understanding mathematics, it can also provide difficulties. Pimm (1991) introduced the notion of the language of mathematics as a “register” or a “function” of language, for although “mathematics talk” is not a different language, it sometimes uses language in a different way from our commonplace use.

Hillegeist and Epstein (1989) discuss three different uses of language within the context of mathematics. There is the “language of mathematics” which includes technical terms, symbols, words and other representations used in a specifically mathematical sense. This use of language is very precise and lacks the redundancy of everyday language; for example, algebra completely lacks redundancy and yet is full of meaning; the fraction symbol, *4/6* means four parts out of six equal parts and this small symbol is rich in meaning. If any part of the message is missing, it is difficult to reconstruct the meaning. For students who benefit from the redundancy of conversational communication, it is difficult to adapt to such a concise use of language. Hillegeist and Epstein also discuss the “language of instruction” which is the register of English used in instruction and in textbooks. It may draw from a student’s everyday language but is, nevertheless, different. It is more concise than everyday language and resorts to the use of more complex structures to accommodate meaning. Finally, they acknowledge the importance of the level of language competence that students bring to mathematics (that is, the “language of learning”), and that there is great variation among students' proficiency in language. Therefore, students with a higher competence in language are better placed to cope with the linguistic demands of mathematics. The significance of differentiating the three uses of language in mathematics clarifies areas of possible difficulty in understanding mathematics. For example, a student who has English as a second language may be an excellent student of mathematics in their first language but struggle with grasping the concepts when it is taught in English. Similarly, a student who is deaf or hard of hearing and who has delayed language development may not achieve an appropriate level of mastery in mathematics because of difficulty with English. Additionally, "success in learning mathematics depends upon how successful each student is in integrating the language of mathematics, the language of instruction, and his or her own language of learning" (Hillegeist & Epstein, 1989, p. 705). As mathematical concepts become more abstract and the language used to describe them becomes more complex, the importance of language proficiency increases and any mismatch between the language of the learner and the languages of mathematics and instruction will disrupt the learning process.

The language of mathematics can be a very confusing domain (Spanos, Rhodes, Dale & Crandall, 1988). It contains words that have a specific mathematical application and are not used in any other domain (such terms as "divisor" and "quotient"). There is also an endless array of symbols which are part of the language of mathematics. These begin with numbers and develop into the complexities of algebra. Manipulation of these symbols is part of dealing with the abstraction of mathematics, but the symbols have no meaning without an understanding of the underlying mathematical principles. Students with reduced language proficiency will experience difficulty operating at a symbolic level, and because such use of language is specific to mathematics, it can be learned only in the context of mathematics. There are also words that have a different meaning when used in a mathematical context from an everyday context (such as "table" and "rational") and can be a source of confusion until the mathematical meaning of such words is made clear. Another difficult aspect of the language of mathematics is that there can be many different ways of signalling an operation. Words such as "combine", "sum", "increased by" all imply addition without actually saying it. Spanos et al. (1988) examine the syntactic, semantic and pragmatic features of English that are different in the mathematics register and they indicate that, at every level, the complexities of language used in a mathematical sense provide enormous challenges for the uninitiated. "Correctly manipulating the special vocabulary and phrases and the word order found in mathematics discourse is intricately tied to the ability to infer the correct mathematical meaning from the language" (Spanos et al., p. 230).

Language has another function that, although not specific to mathematics, serves an important function in this abstract system. "The symbol system provided by language (together with other systems, such as mathematics) is generative and creative" (Wood et al., 1986, p. 145). Wood et al. propose that language facilitates communication and, in doing so, allows us to move from the present concrete context to another which is removed from the here and now. In this way, language assists in generating a new use of mathematics and allows for the creation of more complex notions. This would appear to have a direct application to the use of symbols in mathematics as they are an abstract representation of a context often far removed from the visual representation.

Therefore, reduced language proficiency may result in difficulty with the symbolic aspects of mathematics. Language also expedites reflection and planning so that ideas can be generated and evaluated mentally without the need for a concrete context. "Language provides a vehicle for 'taking thought' " (Wood et al., 1986, p. 145). Generating ideas, planning solutions, reflecting and evaluating plans are all integral parts of problem solving, in and out of the mathematical arena. Language is indeed a powerful tool for the mathematician. However, using a natural language to express mathematical concepts requires mastery of complex language skills and to expect students to "pick up" the language as they read the texts and listen to explanations in class is to court the kind of failure that too many students meet in their mathematics courses (Spanos et al., p. 238).

Spanos et al. encourage teachers to look for ways to develop the different uses of language in a mathematics context, and so particular attention needs to be given to these linguistic aspects of learning within mathematics programs, especially for students who experience difficulties with language learning in general.

The power of language within the learning process lies in how it is used. Language can be used in an attempt to transmit information rather than to develop understanding, as von Glaserfeld (1991) indicates. Language frequently creates the illusion that ideas, concepts, and even whole chunks of knowledge are transmitted from a speaker to a listener. This illusion can be extraordinarily powerful because it springs from the belief that the meaning of words and phrases is fixed somewhere outside the users of the language.

When language is used in this way, learning and shared understanding are the illusions that trick the speaker and the listener into believing mathematical understanding has been achieved. However, in such cases, understanding of mathematics is the loser and all that is often achieved is the attempted transmission of that information which results in reduced mathematical achievement. Von Glaserfeld warns of the need to teach mathematics in a way that develops an understanding of mathematics and not just an ability to reproduce the right words without the associated concept development.

Assessment of mathematical outcomes

Because constructivism makes different assumptions about learning, evaluation of constructivist learning situations will take various forms; however, there is still the need to identify and describe learning outcomes. "Constructivism proposes that learning environments should support multiple perspectives or interpretations of reality, knowledge construction, context-rich, experience-based activities. Perhaps the thorniest issue yet to be resolved regarding the implications of constructivism for learning is how to evaluate the learning that emerges from those environments" (Jonassen, 1991, p. 28). Jonassen suggests that evaluation should be "more goal-free" (p. 29), avoiding setting too many goals before the learning process begins. He also suggests that the activities be evaluated in terms of describing their relevance to the student's individual needs as well as ensuring an appropriate level of complexity to meet these needs and to advance conceptual development. Assessment of student outcomes should be referenced to the context in which they occur and combine with descriptive evidence of learning. True to the constructivist philosophy, activities, ideally, engage students in understanding rather than reproducing information. In this way understanding is enhanced and students are provided with the opportunity to develop their understanding of concepts rather than only reproducing teacher talk. Consequently, evaluation of outcomes should assess thinking skills and provide a description of the context in which this evidence was observed (Love, 1988). Similarly, because diversity is encouraged and valued, this should be taken into consideration when evaluation procedures are set in place. As communication and negotiation of meaning are an integral part of learning, then evaluation of communication must include examples of each. Standard testing procedures do not offer opportunities to evaluate the many facets of constructivist outcomes and assessment techniques for this approach should reflect the goals of the learning situation.

The use of games in learning mathematics

Language exists because of a context, a need to communicate, and it develops through exposure and practice. In this case, the context under consideration is mathematics, so there is a need to create an interest in and a context for communicating about mathematics and for students to be exposed to and to practise using the language of mathematics. One way to generate interest is through one of children's favourite activities, games. This match between the behaviours freely committed to game-playing and the expectations for involvement in mathematical learning has led to the inclusion of many games and related structured activities in mathematics programs. Although this has been chiefly for motivation or to reward children for progress, use can also be made of the interest generated by games to assist children to generalise to the more abstract recorded forms and higher level mathematical ideas (Booker, 1996b).

Play is an important avenue through which children learn to make sense of the world. Through play, they learn the social conventions of society. Games are an engaging and functional part of all children's worlds and can have an application within the classroom as well. Play is children's way of making sense of the world around them through situations which are engaging, familiar and non-threatening. Furthermore, games create a social setting that is rich with real world experiences (Burnett, 1993). According to Kanes (1991), there are certain common characteristics of games.

> A game is an activity involving one or more players in which ... actions are constrained by rules and result in the transformation of the game from one state of play to another. It is conventional to refer to such admissible actions as *moves* in the game, and any given state of play as an interim or provisional outcome of the game. A game is *played* by moving through a sequence of admissible moves (p. 229).

The introduction of games to the classroom is likely to generate excitement and involvement of most children. It is not uncommon for students to say that they prefer to play games than participate in written activities. In addition to motivating students to be actively involved in mathematical activity, games also add variety to the mathematics program. Because of the interactions that occur between participants, games provide a prompt for discussion among students and between students and the teacher. In terms of constructivist theories, games afford a stimulating and rich social context for mathematical learning (Booker, 1996b).

Oldfield (1991) identifies a number of objectives for and outcomes of using games. Firstly, games can be used to drill skills that require repeated practice over a long period of time. Students sometimes fail to realise that they have practised the same skills many times previously because they are preoccupied with participating in an interesting game. Similarly, a variety of games may be used to practise the same skill without raising objections from the students. Practising basic mathematics facts is an example of this type of game. Games can also be specifically designed to assist students in developing and extending mathematical concepts. Through such games, real experiences are brought to the classroom to enrich students' background knowledge and to reinforce their understanding of particular mathematical concepts through these experiences. An example of such a game is the *Colour-me Fractions* Game implemented in this research study and described in Chapter 4. Observing students playing games also affords the teacher insights into their thinking and an opportunity to intervene if needed through questioning, discussion and modelling. Another outcome of using games that develop concepts is that students can be presented with cognitive conflicts which they attempt to resolve. Therefore, as they are building mathematical understanding, they are also developing problem-solving strategies. Games which incorporate the use of strategies fall into this category because to achieve success students must plan and predict. Participation can be at an individual level, but at team level, all the dynamics of discussion and collaboration can come into play. An example of such a game is the *Blocking Game* which was also incorporated in the major research project and will be discussed in Chapter 4. Games are useful to stimulate mathematical discussion and, in an environment modelled on constructivist principles, almost any game can be used for this purpose. Onslow concludes that "a game can provide the embodiment for a meaningful discussion to assist students in accommodating new concepts" (1990, p. 591).

At another level, the motivation and enjoyment fostered by mathematical games can reap other benefits. Competitive games that involve an element of chance or luck in winning give students who are struggling with the concepts an equal chance of success as those children who are already comfortable with the concepts. In this way, they are afforded an opportunity to feel good about themselves and their ability to achieve success, because it appears to them that their success at the game was a direct result of their mastery of mathematics (Booker, 1996b).

Because they create an authentic context for children, mathematics and associated games have value for them (Ainley, 1988). Ainley reminds us that mathematical games provide a context for enjoying and using mathematics in much the same way that books and comics provide a context for enjoying and using reading. Because of the importance of the game context for them, children want to acquire and practise skills of cooperation that are invaluable in and out of the classroom. Because a game involves taking turns, children must observe this rule in playing a game. In some games which include an opportunity for teamwork, there is also the opportunity to justify, conjecture and predict before reaching a consensus about a move. Predicting is a very empowering skill as it gives a sense of control over the environment (Ainley 1988). Similarly, depending on the way a game is set up, students can assist others to move through a game with no concrete benefit to themselves other than the satisfaction of assisting others.

The characteristics of a beneficial mathematical game could well begin with Onslow's (1990, p. 590) discussion of research into mathematical games:

> Solely playing the game had little effect in assisting any of the students in overcoming their conceptual difficulties ... for a game to produce learning it should be constructed so that the skills, insights, or facts to be learned are clearly needed by the players in order to succeed.

This implies that the students need to perceive a valued context for using mathematics games. Additionally, the facility to promote discussion, success, collaboration and a positive attitude to mathematics are also possible outcomes of playing games. As we can see, games offer the opportunity for a wide range of mathematical activity and sit very comfortably within a constructivist approach.

Teaching and learning fraction concepts

Despite the widespread use of calculators for performing operations with fractions, an understanding of rational numbers (decimals, fractions, percents, ratio and proportion) is still regarded as important because of their application in

> representing and controlling part-whole situations and relationships; they are fundamental to measuring continuous quantities; they are involved where quantities, particularly continuous quantities, are divided; and they are involved in any comparisons of two quantities (Pitkethly & Hunting, 1996, p. 5).

Further, as Titus (1995) points out, this kind of understanding has particular application to technical courses, as well as being an important understanding for success in primary and secondary level mathematics. Yet, on the other hand, it has been observed that "the domain of rational numbers is considered to be one of the most complex mathematical domains that students encounter in their presecondary school years" (Mack, 1993, p. 86). Most children come to the learning situation of fractions with some background knowledge of how numbers and mathematics are used in our society.

There does not appear to be consensus among researchers as to the most appropriate approach for teaching fractions, but there are, however, some points of agreement across researchers (Pitkethly & Hunting, 1996). There is no disagreement as to the value of teaching students about fractions and researchers agree that instruction in the area of fractions should be based on the principles of constructivism and should aim for understanding of the notion of parts and wholes. There also seems to be agreement that fractions are not easy for students to understand and one of the difficulties can be found in their lack of knowledge of whole numbers. The part-whole subconstruct that is often used to introduce the concept of parts of a whole can leave students with the misconception that the parts of the whole are measured as whole numbers instead of measuring a unit that has been divided up. However, the knowledge of number that students bring to fractions is also of assistance when counting parts.

> "One" is the unit in the whole number scheme. It is able to be counted, replicated and grouped. "One" is also the unit on which the fractional scheme is based. The fractional unit can be counted, replicated and grouped. However, it is also a measuring unit which can be "fractured" or divided up (Pitkethly & Hunting, 1996, p. 27).

Understanding the complexities of the notion of fractions depends on integrating counting and partitioning. However, it is also necessary to understand that partitioning results in a quantity that is symbolised by a new number. Without the appropriate understanding of parts of a whole, fraction symbols become confusing and just another hurdle to mastering the concept of fractions; for example, students may read the fraction symbol as two separate numbers instead of one.

The symbols used to represent fractions also present a number of complexities for students to comprehend. It is not uncommon for students to believe that fractions with a larger denominator are larger than those with a smaller denominator when, in fact, the opposite is true. If they understood the notion of partitioning and what the symbol represented, this simplistic idea based on their experience with the whole number system would not occur. Similarly, if students do not understand the notion of equal parts of a whole, then fraction symbols have no meaning and are perceived and acted upon as two distinct numbers. Although the symbol is a visual representation, it is not a concrete representation and does not make the notion of fractions clear. It is simply a way of representing fractions in a written form and is meaningless unless the notion it represents is understood. For this reason, Mack (in Pitkethly & Hunting, 1996) and Pitkethly and Hunting suggest delaying the introduction of symbols until students have a strong conceptual foundation of understanding fractions.

Other ideas regarding the teaching of fractions are touched upon in Pitkethly and Hunting's overview. The first is the role of language in the development of a concept of fractions. Although this is mentioned, a strong connection is not made between understanding and using the language of fractions and understanding the basic concepts of fractions. Students need many experiences of fractions matched to the appropriate, meaningful language of fractions to see the connection between their experiences of fractions and the language to describe them.

Language allows them to describe their ideas and to adapt their ideas as their concept of fractions expands. It facilitates the transition from a concrete representation to simplified language to a formalised idea that can be represented and manipulated by language. Within a constructivist framework, language lies at the heart of mastering mathematics and, therefore, commands a high profile in the literature concerning the teaching and learning of fractions. Kerslake (1991, p.86) recognises the importance of language in this context. "It can be argued that many of the difficulties which pupils experience are due to the restricted and imprecise use of language describing fractions".

There are other interesting ideas presented in Pitkethly & Hunting's overview. Ball (in Pitkethly & Hunting, 1996) discovered in her work with young students that the "rectangular model" (which depicts the whole as a rectangle) was the most effective model in representing the partitioning process and was much easier to use than the traditional "circular model" (which depicts the whole as a circle). Saenz-Ludlow (1992) structured her work to allow the students not only to partition but also to see the parts recast as a whole. Other researchers, Kieran, Mason and Pirie (1992), Mack (1993) and Steffe and Olive (1993) also recognise the importance of this experience in developing a full understanding of the part-whole subconstruct. Streefland (1991) feels that it is important that students have many concrete experiences of early fraction concepts, and if these kinds of experiences are abandoned too early, it results in student errors. Many of the researchers recognise that to develop a broad understanding of fractions, there needs to be experiences of continuous and discrete contexts; that is, discrete contexts such as the rectangular models of fractions as well as continuous contexts such as "half a cup of milk".

Through their work focussing on various aspects of the concept of fractions, researchers have highlighted areas of difficulty and possible approaches to avoid these. However, there is not one general approach for teaching fractions that results from their work. Some place a strong emphasis on partitioning parts of the whole while others feel that this limits students' understanding of fractions in the long term. Some recognise the importance of language in the development of the notion of fractions while others give it no mention. Some recognise the importance of establishing a strong conceptual basis for fractions before introducing the use of symbols while others do not consider it a significant issue.

Teaching fractions

Though there are confusing messages from researchers, there is, at the same time, support for the philosophy of constructivism with a solid base in the psychology of learning that is available to guide program planners. One outcome of such an approach is described in *Teaching Primary Mathematics* by Booker et al. (1992). They see fractions as an important part of any mathematics program not only because of their application to real life situations but also because an understanding of fractions fosters valuable intellectual development.

From the start, the conundrum of how to consider partitioning is confronted. Ball (1993) and Booker et al. state that partitioning is very important in the early conceptualisation of fractions but that children need to do the partitioning themselves and to see the parts of a whole. This, then, removes from instruction the traditional circle already cut into pieces of varying sizes and replaces it with materials the students can divide themselves. The model chosen for students to manipulate in this approach is that advocated by Ball (1993).

Through many experiences with this model and other real life materials, students can be exposed to meaningful language to describe the notion of fractions and use it themselves in a meaningful context. Students are given opportunities to shade parts of a whole, and, in so doing, can see, simultaneously, both the parts and the whole and the relationship between them. They are also taken through experiences where they discover the importance of the parts being equal before they can be considered fractions and can be described using the language of fractions. Once this understanding is stabilised, various other models can be introduced and the students can generalise their understanding of fractions across various contexts. While building up an understanding of parts of a whole, the students at the same time gain experience of the fact that these parts can be less than a whole or the same as a whole. Thus, 3 sevenths is less than a whole and 7 sevenths is a whole. Fractions that are more than one whole are dealt with in much the same way, with the students shading regional models that are more than one whole, and, thus seeing the relationship between the parts and the whole at the same time, as well as using both forms of appropriate language; that is, 7 fourths and 1 and 3 fourths. Many experiences are necessary for students to appreciate the intricacies of this notion and to understand how to rename fractions.

The language used to describe fractions can be used to lead into the introduction of the symbol: 4 sixths is 4 out of 6 equal parts and that lends itself to the recording of 4/6. Equivalent fractions are introduced and reinforced through experiences with folding and shading regional models. In this way, students can see the relationship between fractions of a whole and fractions of a group as well as generating the language for renaming fractions. Just as comparison is an important part of fathoming whole numbers, it is also a significant part of understanding fractions. Understanding of early fraction concepts and equivalent fractions as well as comparison of whole numbers is also a prerequisite to being able to identify what is "greater than", "less than" or "equal to".

This research project aims to investigate these fundamental ideas with students who are deaf or hard of hearing and the implementation of this program is described in detail in Chapter 4. This program of teaching and learning the early notions of fractions has been successfully implemented in another research project with hearing students (Booker, 1996). The content and sequence for this approach to teaching and learning fractions is found in *Teaching Primary Mathematics* by Booker et al. (1992) and the games and activities were devised at the Mathematics Assistance Clinic at Griffith University.

The education of students who are deaf or hard of hearing

It is now time to turn our attention to the central focus of this research, students who are deaf or hard of hearing. The education of students who are deaf or hard of hearing has a history dating back centuries and, in that time, many philosophies and methodologies have been proposed, implemented and disappeared. Education of the deaf is never static for those involved in the search for the answers to questions that have been pondered for centuries.

Despite differences in opinions about other issues, most professionals working in the education of students who are deaf or hard of hearing would agree that language is their central concern because it is involved in almost every aspect of human development.

> Language is the medium that allows people to transmit thoughts to other human beings, to identify their innermost feelings, to aid in solving personal problems, and to explore the world beyond their sight and current time frame. In addition, language has been considered by many to be the singularly most important tool for obtaining knowledge and skills in our society. Modern evidence has shown that language must be defined and understood as a dynamic interaction between the cognitive, linguistic, and communicative domains (Luetke-Stahlman & Luckner, 1991, p. 4).

The impact of delays in language development on these children is also widely acknowledged.

> The professional literature in education of the hearing impaired is replete with a common concern — the devastating effects that severe, early-onset hearing impairment have on language acquisition (Kretschmer & Kretschmer, 1986, p. 131).

Although it is accepted that students who are deaf or hard of hearing develop cognitively in a similar way to their hearing peers, the impact of linguistic delay can be seen on higher-level cognitive tasks. Greenberg and Kusche (1989, p. 101) concluded that:

> For the average deaf child, the understanding of conceptual information does not appear to override visual perception until adolescence, while this same transition occurs between the ages of 5 and 7 for the average hearing child. As a result, the elementary school-age deaf child thinks predominately in a preoperational manner in some areas, but processes information in a concrete way in

> others .... Given the research currently available, it appears that language deprivation influences the way information is processed, which in turn results in a type of experiential deprivation [as well as affecting] the development of further concept formation .... We believe that language has a strong effect on concrete and formal operational modes of thinking, while it has relatively less influence on sensorimotor and preoperational thought.

This formulation will have an impact on thinking about concept formation in mathematics as mathematical concepts are abstract and although they can be depicted in a concrete form until approximately the middle primary school years, students are by then expected to have sufficient understanding of early mathematical concepts on which to base more complex notions. If students who are deaf or hard of hearing experience difficulties in understanding mathematical concepts beyond the middle primary school years, then their progress in mathematics will be restricted.

Quigley and Paul (1984) submit that although language and cognitive development are primarily separate functions, the two become so closely intertwined that it is difficult to separate the two for practical purposes. Dolman (1983) deduced from his research that there was a relationship between syntactic competence in English and deaf students' ability to think operationally. Livingstone (1986) contends that, although students who are deaf or hard of hearing have few difficulties thinking independently, problems arise because any associated learning has limited linguistic representation. Because of this, sharing discoveries is problematic, learning is not enhanced and important building blocks for future learning are lost. Thus, language delay experienced by students who are deaf or hard of hearing has an effect on cognitive functioning which manifests itself more in terms of a delay in cognitive development rather than a difference in cognitive development. The implications for this research are significant for mastery of mathematics in terms of currently accepted approaches to teaching and learning mathematics. Firstly, because of problems with language, students who are deaf or hard of hearing can be expected to experience difficulty in exchanging information on a strictly linguistic level; that is, in communicating their thoughts and understanding meaning as expressed by others. Secondly, these students can also be expected to experience difficulty in moving from concrete to more abstract ways of thinking. Although this may appear to be a good reason to avoid approaches which emphasise discussion, students will only develop the language to facilitate improved mathematical understanding through using it and seeing it used.

It has been claimed that memory functioning in deaf or hard of hearing students is different from and/or delayed in development compared with normally developing hearing children. In the literature, there are three aspects to examining memory. There is short-term memory in which information is stored temporarily prior to its transfer to long-term memory. It is also where information is manipulated and used after retrieval from long-term memory. Long-term memory is where existing knowledge is permanently stored. Finally, the relationship between short and long-term memory is also important for learning. The function of memory in students who are deaf has been the focus of research for most of this century and there is a confusing array of information available that appears to indicate similarities and differences between deaf and hearing students.

The manner in which information is coded in short-term memory appears to affect efficient storage and retrieval of information. Paul and Jackson (1993) describe various types of codes for storing information in short-term memory. Information can be coded phonologically and this system relies on the articulatory and auditory aspects of language. Another type of coding used in short-term memory is visual coding, which relies on the configuration of printed words. Signing, finger spelling and speechreading are coded in a different way again, using a visual-spatial basis.

Investigations into the various types of coding for short-term memory indicate that students who are deaf or hard of hearing may code information differently from hearing students. The students' experience of information presented in different modes appears to effect preferred modes of coding. O'Connor and Hermelin (1972) found that, quantitatively, there was little difference between memory performance of deaf and hearing subjects. However, they noted marked differences between the groups in the way the information was coded. They found that the hearing children tended to code information auditorily while the students who were deaf were likely to depend on spatial coding. Further investigations by these researchers (1973) reinforced these findings and they provide support for the comparability of the two ways of coding information as far as performance on tasks is concerned. However, it has been suggested (Marschark, 1993) that the deaf students who participated in this research were most likely educated within an oral system and therefore a question remains about the applicability of this research to students who communicate manually. Marschark also raises the question of suitability of using digits and number concepts as test information to compare students who are hearing and deaf (as did

Hermelin and O'Connor) as the deaf students may not have age-appropriate familiarity or experience with them.

Wallace and Corballis (1973) looked at differences in types of coding for short-term memory among manual, oral and hearing students. They found that when shorter stimuli were presented to their subjects, oral and manual deaf students relied on visual coding while hearing students relied on phonological coding. However, when the length of the stimuli was increased, oral students evidenced phonological as well as visual coding. They found that visual coding was most important to deaf students, while phonological coding was the preferred mode for hearing students. They also found that the deaf students had poorer recall than hearing students and that slower presentation of stimuli assisted manual students.

Harris and Arnold (1984) examined memory for experience-based stimuli. They compared short-term memory for letters and lip shapes to memory for hand shapes. Their pattern of results indicated that oral ability improves memory for orally applicable information and that manual ability improves memory for manually related stimuli. These results would tend to support the suggestion (Furth, 1966) that experience and familiarity with the information affect how efficiently it is coded into short-term memory. Similarly, Liben and Drury (1977) found through their research that the more meaningful the stimuli, the more likely they were to be remembered; meaningfulness lessening the differences among deaf and hearing subjects.

In general terms, it would thus appear that deaf and hearing students have qualitatively different ways of organising their experiences. Hearing students are more inclined to code information phonologically while deaf students are more likely to code information visual-spatially. However, it has also been found that there are processing differences among deaf students, some of who use various coding strategies. It is also likely that there is an inverse relationship between degree of hearing loss and short-term memory performance and phonological skills are likely to assist in memory performance. It also seems likely that experience and background knowledge as well as language proficiency have a positive effect on memory functioning.

The effectiveness of coding information into short-term memory is directly related to the functioning of long-term memory. It is in this section of memory that information is classified, cross-referenced, and stored in various ways. "The degree to which information in the STM (short-term memory) is efficiently coded, organised, and enriched increases its chances of being properly stored within the LTM (long-term memory) and it being later recalled" (Mann & Sabatino, 1985, p. 175). Comparisons between deaf and hearing students reveal differences in LTM coding. Marschark (1993) states that language fluency effects the functioning of long-term memory. The development of skills in vocabulary and grammar and relating meanings of words and sentences improves comprehension of information and assists in memory functioning.

Furth (1961) examined long-term memory in terms of paired-associate learning. He found no differences in functioning between hearing and deaf children who were 7 to 10 years old but significant differences between hearing and deaf students 11 to 12 years old. From this, Furth concluded that visual memory of younger deaf children is similar to that of hearing children. Blanton, Nunally and Odom (1967) researched long-term memory functioning by contrasting semantically related, rhyming and graphemically similar words for hearing and deaf students. They found that deaf students surpassed their hearing peers in all categories. MacDougall and Rabinovitch (1971) investigated visual versus phonological long-term memory codes among deaf students from oral and manual communication groups as well as hearing students and found no differences among the three groups. Similarly, Allen (1970) found that children with greater hearing losses remembered more than those with less significant losses and she concluded that their predominant use of visual coding allowed them to bypass the inconsistencies of acoustic information. She surmised that deaf students use a primary visual code and a secondary phonological code for long-term memory.

Other researchers have found quantitative as well as qualitative differences in comparisons of long-term memory of deaf and hearing students. Conlin and Paivio (1975) and Bonvillian (1983) found that although deaf students, to accommodate their reduced auditory experience, use different coding strategies for long-term memory from hearing students, there is still a difference in recall favouring hearing students. Both researchers found that high signability of information assisted deaf students and that high imagery words assisted memory more than low imagery words. Frumkin and Anisfeld (1977) looked at types of coding for deaf students in long term memory and found that both semantic and visual coding is used but that semantic coding led to better recall. They hypothesised that the formational properties of signs do not appear to provide for deaf children the same coding efficiency as speech does for hearing students.

Marschark (1989) states that despite the differences between the strategies that deaf and hearing students use to code information into long-term memory they may be equally effective. He proposes that different organisation of information based on different experiences does not constitute memory inadequacy. However, research focussing on recall of relational versus distinctive information (Marschark, DeBeni, Polazzo & Cornoldi, 1992) indicated that, in general terms, deaf students recall less than hearing students and they also recall less relational material as opposed to item specific information.

The indications from all this appear to be that there are qualitative and quantitative differences between hearing and deaf students in long-term memory functioning and that the transfer of information between long- and short-term memory is facilitated by proficiency in a language. There also appear to be indications that phonological coding has a role to play in an efficient long-term memory, so with lessened access to phonological data, deaf or hard of hearing children may be delayed in achieving levels necessary for fully effective functioning.

Within mathematics learning, memory has an important contribution to make. Firstly, there is the need to consider Marschark's observation that students who are deaf or hard of hearing may not have the same facility with number as hearing students because of reduced experience and different experience of mathematical concepts. Therefore, even at this most basic level of mathematics, deaf students may be at a disadvantage.

Most obviously, memory has a role to play in the memorisation of mathematics facts, for although students can be shown strategies for calculating these facts, in the long term they must be memorised and become automatic. It would appear to be the case that students who are deaf or hard of hearing may experience difficulty in this area of mathematics because of memory differences from hearing students. Indeed, many teachers comment on the fact that their students experience difficulty in this area in contradiction to the generally held former belief that this is a strength of these students.

Memory also plays an important part in developing understanding of mathematical concepts. Initially, students are required to draw on their experiences in and out of the classroom to build the complex understanding of number, of operations and other mathematical concepts. In addition they need to make relationships between these understandings at very early stages of mathematical learning; e.g., the relationship between tens and ones or the relationship between the concepts of "before" and "after" and the number system. Without early development of understanding, subsequent concept development will be hindered. And so, as one concept builds on the understanding of another, the relationship of mathematical concepts to each other is built around the organisation of memory and the retrieval of information.

The current focus of mathematics education is problem solving and the basic tool of problem solving is memory and the manner in which it facilitates the relating of information. Successful problem solving involves understanding the relationship between a number of pieces of information, drawing on experience, making relationships between the information and mathematical concepts, predicting and estimating and, finally, carrying out appropriate mathematical calculations. Each step in the process utilises memory function in different ways and it can be concluded that difficulty with memory must affect problem solving ability.

It has been shown that, in terms of short- and long-term memory, students who are deaf or hard of hearing function differently from hearing students when required to code and recall information and it has also been shown that they do not always spontaneously make relationships between pieces of information. It could well be that differences in the functioning of memory have significant effects on mathematical achievements of these students.

The implications of this research are significant for mastery of mathematics in terms of traditional approaches to teaching and learning mathematics. Firstly, because of problems with language, students who are deaf or hard of hearing can be expected to experience difficulty in exchanging information on a strictly linguistic level; that is, in communicating their thoughts to others and understanding meaning as expressed by them. Secondly, these, students can also be expected to experience difficulty in moving from concrete to more abstract ways of thinking. Although this may appear to be a good reason to avoid approaches which emphasise discussion, students will only develop the language to facilitate improved mathematical understanding through using it and seeing it used. Gamesing approaches such as used in the present research can help overcome these problems by providing many opportunities for discussion of mathematical concepts in clearly understandable contexts.

The research evidence surveyed above suggests that memory function is delayed for students who are deaf. There is also evidence to indicate that memory is aided when information is presented in a meaningful way. Deaf students are traditionally disadvantaged by mathematics instruction that relies on memorisation and does little to promote understanding. Therefore, this lends support to the implementation of an approach to mathematics instruction that focuses on the development of meaning and understanding such as the approach used in this research which, in the long term, will be more effective in promoting mathematical competency for deaf students.

The area of mathematics most reliant on memory function is recalling mathematics facts. Although students are shown strategies to assist them in calculating these facts until they are automatic, the final consideration is that they need to remember them and be able to do so with speed. Any difficulty in recalling mathematics facts effects students' proficiency in completing operations and in using those operations in a problem solving situation. Therefore, it an important area of mathematics.

Language proficiency can be appraised and measured in each of the uses we make of it, but it is only a tool, an agent to expedite something more dynamic and rudimentary. Language thrusts people into the communication process whereby they convey their thoughts and ideas and they understand others' perceptions and feelings and communication and language also facilitates discussion with oneself. This dynamic process begins at birth and continues to enrich us throughout our lives. Effective communication occurs when there is an exchange of meaning. It requires the people involved either to take the initiative or follow the initiative and to take turns at doing this. Wood et al. (1986) describe it as a "partnership". The pragmatics of communication are more difficult to identify and measure. They include transmitting meaning, understanding meaning, supplementing meaning, turn-taking, appropriate agreement and disagreement strategies, asking and answering questions, appropriate means of changing the direction of the conversation, to name just a few. There is no doubt that language is a tool that is used to implement these pragmatic skills and, furthermore, the more sophisticated the pragmatic skill the greater the need for well developed language proficiency. Hence, there is the need to look beyond mere language proficiency in a syntactic sense and to examine the effectiveness of the uses to which it is being put. The dynamics of conversation are relevant to current approaches to teaching and learning mathematics which emphasise discussion and learning from others. For this to occur, the assumption is made that the dynamics of the interactions will facilitate learning and enrichment and not stifle it. Therefore, it is worthwhile to investigate these dynamics to ascertain the quality of interactions and, hence, the learning experiences in the mathematics class.

To this end, Wood, in association with various other researchers (Wood, Griffiths, Howarth & Howarth, 1982; Wood, 1984; Wood, Griffiths & Howarth, 1986) has explored the nature of conversation and the pragmatics that occur within that framework. The focus of their work was conversations between teachers of students who were deaf or hard of hearing and those students and the effects of varying levels of teacher "control" in conversation on communication outcomes. They were looking to relate student responses to teacher initiatives, and in so doing, gain insight into what was happening in the communication process between the two groups. They found that certain pragmatic initiatives could be identified and quantified by relating them to specific syntactic structures and pragmatic moves (Wood, 1985).

During their research into the dynamics of teacher/pupil conversations, Wood and his associates identified a number of important aspects of communication with students who are deaf or hard of hearing that are specific to this population. Firstly, they acknowledge that "the effect of profound, prelingual deafness on the linguistic abilities of children is often devastating" (Wood et al., 1982, p. 295). They point out that students who are deaf or hard of hearing often evidence an ever-increasing discrepancy between what they know and what they can communicate about and that "this growing gap between knowledge and communication often dislocates the processes of social interaction, teaching and learning" (Wood et al., 1986, p. 6). They also remind the reader that, mostly, communication with deaf people is face to face and that much casual dialogue, such as from room to room or as the speaker is walking away, is not part of their experience. They surmise that a result of this is could be the regular communication breakdowns that occur between teachers and deaf students when discussion centres on experiences that are outside the present context of communication.

Wood and his colleagues stress the importance of one subtle but significant difference in communicating with people who are deaf or hard of hearing when a "reference", such as an object, is part of the discussion. In describing such a circumstance, Wood (1985, p. 13) describes "the concept of a 'referential triangle'. It involves two minds, each at some level, sharing a common focus of attention." To make a connection between the reference and the associated information, the person who is deaf or hard of hearing must consciously move his attention from the reference to the speaker many times and remember the connection between referent and language. Therefore, these people must "do much more 'discovering' [of] the relationships between two very different visual experiences that are displaced in time" (Wood et al., 1986, p. 22). The child must "divide his attention between two spatio-temporally distinct experiences and then integrate or co-ordinate them" (Wood, 1985, p. 14). One of the outcomes of this is that the reference and the associated information are separated in time and the learning cycle is disrupted.

Therefore, it can be seen that there are subtle but significant differences between communicating with hearing students and those who are deaf or hard of hearing. Firstly, there is the possibility of language delay in deaf students and this is often combined with a growing discrepancy between what is understood and what can be communicated. It has also been found that the process of communication is different for students who are deaf or hard of hearing because of their inability to listen and look at the same time, and, so, communication is separated from its referent. Additionally, it has been found that when no referent exists, communication often breaks down, as deaf students have difficulty understanding information about something not in the immediate context. Therefore, it is clear that the communication process is not a smooth one for these students.

As a result of these communication differences, Wood and his associates found that teachers attempted to compensate by "controlling" conversations with their students and undertaking frequent "repair" of conversations when there is a breakdown in understanding. In an attempt to take control when not able to achieve shared understanding, teachers restrict interactions through the types of questions they ask and the time they are prepared to wait for a response from the child, in the comments they make and by dominating conversations. The outcome of this process is reduced participation by deaf children. Wood (1985) found that there was a direct correlation between teacher control and student participation in conversations; that is, the more control exerted by the teacher, the less often students take the initiative within a conversation. Similarly, when teacher control is low, children are more likely to contribute their own comments, answer questions with elaborations and ask questions. Significantly, they also found that when teacher control is high, students are less likely to listen to their peers and more likely to respond to their peers in a dismissive manner. Therefore, the way in which teachers communicate with their students influences the flow of communication, the tone of the conversation and the amount of student participation. Nevertheless, they also found that situations in which there was no structure or ground rules for conversation were equally unproductive. "Thus, the whole 'tenor' of a group conversation is directly influenced by very specific, and very simple, features of teaching style" (Wood et al., 1986, p. 57). Wood's research results in England were supported by similar findings about teachers' conversational styles in Australia, *inter alia*, in Signed English, the method of communication used by teacher and students in the present study (Power, Wood & Wood, 1990).

After investigating the nature of conversation, Wood and his colleagues were able to identify the linguistic features which indicated various levels of control and a means of linking them to student responses. They concluded that most control is exerted when the teacher requires the student to repeat what the teacher has said. It such situations, students must simply reproduce what has been said by the teacher and are not afforded the opportunity to formulate their own responses and ideas. The next level of control is characterised by the teacher asking two-choice questions. These questions require either a yes or a no answer or a one-word response. They have found that students rarely elaborate in their responses to two-choice questions and, because of this, their answers are the subject of much misunderstanding. When teachers use Wh questions (who, what, where, when, why, how), there is less but still a significant level of control. Wood points out that questions, by their very nature, are controlling, because they require the listener to listen, consider and respond to the questioner. He feels that a constant cycle of questions and answers is unlikely to allow students the opportunity to develop appropriate skills in the communication partnership. However, Wh questions do allow some freedom to give elaborated responses and, as such, are less controlling. Lower on the control scale are personal contributions from the teacher, for example, "I like icecream too". These occur when the teacher gives a personal reaction or an opinion, and so indicates involvement and interest in the conversation without controlling it and opens the way for children to take some initiative by responding with a question, elaborating on what has been said or even taking control of the conversation. The lowest level of control is when the teacher uses "phatics" (for example, "I see" or "Oh dear") to indicate involvement in the conversation — a definite step back from control. It is a signal that the initiative within the conversation is available and a change of topic may be appropriate.

On the other hand, student responsive moves can be classified according to their suitability and include appropriate responses, an appropriate response accompanied by an elaboration, misunderstanding, no response and a response that was not understood. When both aspects of analysis are related, a picture emerges of the pragmatics of the conversation and the level of teacher control can be calculated. Such an analysis gives an indication of how topics are generated, how turn taking is respected, how changes to the direction of the conversation are achieved and if understanding and communication are outcomes. Wood accepts that the pragmatics of an explicit teaching situation will be different from that of a conversation and that there is a place for more teacher control at such times. However, he is concerned that teachers of students who are deaf or hard of hearing may become so focused on teaching and learning, that they "try to 'teach' speech, vocabulary, syntax, and phonology at the same time as they attempt to converse with children" (Wood, 1985, p. 45) and that the outcome of this is that interactions are seen by teachers only as vehicles for instruction and, consequently, students do not learn what communication is and how to communicate effectively.

When teachers relinquish control of conversations there are two outcomes. Firstly, students begin to take more initiative and make more contributions to the conversation and, secondly, teachers are able to show students that their competence in participating in a conversation is valued and productive. Therefore, teachers need to be aware of the purpose of interactions and respect the associated ground rules. In the final analysis, it should result in a change of emphasis within the classroom with teachers relinquishing some control and initiative and students taking up the conversational space thus provided.

The implications of these findings for this research are significant. The approach to developing understanding used in this research is through games and, although it is formally a teaching situation and not strictly a conversation, high levels of teacher control still would interfere with the learning process by limiting the students' opportunities to interact and learn for themselves and from each other. It is still possible within such a "gamesing" approach for a teacher to take control of interactions and reduce the learning situation to that of high teacher control and telling information. Therefore, the questions and comments made by teachers will either promote discussion and interaction or stifle it. The simple process of using a game does not ensure that understanding is developed unless suitable interactions occur.

Students who are deaf or hard of hearing and mathematical learning

On a more specific level, students who are deaf or hard of hearing demonstrate generalised reduced achievement in the area of mathematics. It has already been established that mathematics is of primary importance in functioning in our society at whatever level of participation, a favoured position shared with literacy. Fundamental changes have occurred in what is regarded as mathematical achievement. (*A National Statement on Mathematics for Australian Schools*, 1990; Booker et al., 1992; Daniele, 1993). Skills in computation no longer solely constitute mastery of mathematics. Mastery of mathematics has been expanded to include understanding of mathematical concepts evidenced by application of concepts in problem solving situations and an ability to discuss and justify mathematical thinking.

Bearing in mind the empowerment of those in our society who become proficient in mathematics, it can be concluded that those in our community who do not achieve a suitable level of competence in mathematics are denied a similar level of informed participation, enrichment in their lives and opportunities to enhance our society. The overwhelming voice of research into the mathematical achievement of students who are deaf or hard of hearing indicates that the majority of these students fall into the category of underachievers and this has been the case for some time. According to Daniele (1993) educators of students who are deaf or hard of hearing must "face the reality that deaf students have traditionally struggled with quantitative literacy. Educators must combat the myth that deaf students have far less difficulty with mathematics than other disciplines" (p. 78). Barham and Bishop (1991, p. 179) also agree that "most teachers of deaf children, at both primary and secondary level, would agree that mathematics is one subject with which their pupils find special difficulty." Support for this notion comes from other researchers working in the area (Hartman, 1994; Hoyt Kidd & Lamb, 1993; Hoyt Kidd, Madsen & Lamb, 1993; Titus, 1995; Wood et al., 1986) leaving us in little doubt as to the low status of mathematical achievement among students who are deaf or hard of hearing.

In keeping with research findings on their development in other areas of cognition, it appears that students who are deaf or hard of hearing develop mathematical reasoning in much the same way as their hearing peers but at a slower rate. Wood et al. (1986) found that although they developed at a delayed rate, the mathematical reasoning processes of deaf students were qualitatively similar to those of hearing children. So once again the issue is delay, not difference. A number of reasons can be proposed as to why this is the case. Firstly, if these students experience delays in cognitive development, similar delays will be experienced in the development of mathematical reasoning. Also, if delays in language development impact on the development of higher cognitive skills, it is to be expected that delays in language development will affect mastery of mathematics. Similarly, if effective communication is accepted as an important mathematical skill and students who are deaf or hard of hearing experience difficulties in this area, it would be expected that mathematical achievement would be affected. However, another hypothesis has been proposed as a significant contributor to the underachievement of deaf students in mathematics. It has been suggested that teachers in this specialised area of education do not always have current knowledge of the process of mathematics teaching and learning and, therefore, do not have appropriate expectations of the children with whom they work (Daniele, 1993; Stone, 1988; Fridriksson &

Stewart, 1988; Kluwin & Moores, 1989). This consideration has been raised by a number of researchers and merits some reflection.

Daniele (1993, p. 76) warns that the face of mathematics has undergone changes in recent years and it can no longer be interpreted in the same narrow terms as in the past and the danger exists that by "limiting the manner in which mathematics literacy is viewed, educators limit what students are expected to know and do, and, in turn limit career options and possibilities available to them". Kluwin and Moores (1989) explored mathematical achievement of students who were deaf or hard of hearing across various academic placements and they concluded that, after accounting for individual differences among students, those in regular classes had the highest level of mathematical achievement. At the same time, they were able to state that it was the quality of the instruction that was the determinant of mathematical achievement and not the placement as such. Therefore, they suggested that teachers of the deaf operating within self-contained classes needed more knowledge about current developments in mathematics education and should be required to participate in similar professional development activities as their colleagues in hearing classes. Strong words were used by Fridriksson and Stewart (1988) to introduce their thesis on teaching mathematics to children who are deaf. They declared that

> mathematics instruction is a neglected area in the total instructional component of a deaf child's education .... mathematics is not seen as important [and] appears a mere bystander in the total education program for the deaf child. As a result, progress in mathematical understanding is not stressed and success can be assured by testing the child on principles already thoroughly mastered (p. 51).

They continued to say that mathematics programs for many deaf children consist of textbooks and worksheets with the incorporation of some computer programs. They suggest that teachers in this area of education should become acquainted with all facets of mathematics and open up a new dimension of the world for their students by incorporating all aspects of mathematics into their program and having higher expectations of their students. Equally as damning are the conclusions drawn by Wood et al. (1986). They suggest that, in the area of mathematics, teachers are failing to capitalise on the abilities possessed by students who are deaf or hard of hearing. They found that teachers in this area were able to predict what their students were able to do quite accurately but placed a disappointed interpretation on this fact. "Perhaps teachers can predict what students cannot do because they know the children have never been taught those things" (p. 164). They suggest that teachers of these students teach only limited parts of the mathematics curriculum and do not allow their students the opportunity to understand the full range of concepts usually covered in the regular curriculum.

Teaching and learning fraction concepts for students who are deaf or hard of hearing

If it is true that the development of mathematical understanding does not assume a high place in the education of students who are deaf or hard of hearing, then it would follow that an understanding of fractions is also not a high priority of teachers of these students. Titus (1995) notes that although very little is written about teaching students who are deaf or hard of hearing about rational numbers "there is clearly a need for [deaf] students to be meaningfully educated in rational number topics" (p. 255). In fact, educators of these students at high school and tertiary level have specifically identified rational numbers as requiring increased attention from teachers. Titus found that students who were deaf and students who were hard of hearing displayed a similar mastery of fractions to that of younger hearing students. Students who were deaf or hard of hearing did not improve in their understanding of fractions as they progressed through school. An alarming finding was that older students demonstrated "whole number dominance" (Titus, 1995, p. 258); that is, when estimating fraction size, they were influenced by the size of the numbers in the fraction rather than reading the fraction numbers as a whole. This would indicate a lack of understanding of the meaning of the fraction symbol and, possibly, the notion of fractions. Therefore, students who were deaf or hard of hearing did not show an age-related increase in fractional ordering skills while older hearing students were not greatly swayed by the size of the numbers in the fraction when evaluating size. Titus considers a contributing factor to this situation to be the language used to describe fractions. Such language is a source of difficulty for hearing students and, as such, is of even greater significance for deaf students.

Titus also indicates concern regarding the current state of mathematical knowledge of teachers of the deaf and points to the possibility that it is a contributing factor in their students' delayed understanding of fractions. She suggests that the use of different modes of presentation (concrete, pictorial and symbolic) are important and that materials which can be manipulated should play significant role in instruction in this area. In addition, this should be accompanied by appropriate language in association with the symbolic representation so that students can move between different modes of representation and, hence, build on their understanding. She highlights students' need to be able to communicate mathematically, to explain their thinking and understanding and so go beyond the concrete to reflect on their thinking. She concludes that much of what has been found to be effective with hearing students in understanding fractions should also be useful for students who are deaf or hard of hearing.

Silva (1986) published a "fractions curriculum" for students who are deaf. She acknowledged the challenge of teaching fractions to these students and, it would appear, decided that a completely new approach was called for. From the beginning, students make wholes using the "rectangular model". However, they are required to call them "gzorkes" and discuss fractions as part of a "gzorke". The next step in the curriculum is to introduce addition of like fractions as a way of defining a gzorke. Therefore, 1 gzorke = ½ of a gzorke + ½ of a gzorke. Then the students, somehow, guess that 16/16 = 32/32, thus indicating an understanding of equivalent fractions. They are then ready to use fraction bars to find answers to such problems as: Find 1/8 of a gzorke + 3/8 of a gzorke. Renaming fractions was described in terms of "fattening" and "reducing" and multiplication of fractions was easily mastered by the students — and all of this in three to four weeks. Silva found that the students were able to successfully complete a test a few weeks later and she was assured of the effectiveness of her program.

Silva's "fractions curriculum" is a departure from all that is accepted as good practice for teaching mathematics and teaching students who are deaf. Within this "curriculum", there does not appear to be a sequential development of starting from what the students already know to establish what they need to know. It appears that the students move very quickly from learning about gzorkes, to calculating equivalent fractions, to calculating with equivalent fractions. The basic notion of fractions is completely by-passed by the substitution of the term gzorke for whole. It is difficult to understand why a new and irrelevant term such as this would be used with students who are acknowledged as experiencing difficulty with language in the first place. Surely, the use of such a term would only complicate the learning process and distance the students from the meaning of fractions. These students require a tightly sequenced program of activities matched with meaningful language and many opportunities to use and be exposed to that language. It would appear that Silva's "curriculum" does not come close to these criteria.

It has been shown that mathematics in the current educational climate emphasizes the importance of developing understanding through experiencing the concepts and developing and using the appropriate language. Literature has also highlighted the need for approaches to mathematics education for students who are deaf or hard of hearing to look to current approaches to mathematics teaching and learning to improve educational outcomes for these students. The particular mathematical focus of the present research is the development of initial fraction concepts with a group of children who are deaf or hard of hearing. The approach used reflects constructivist principles and is centred around the use of games and activities to develop concepts and provide practice in these concepts and the associated language. Mathematical interactions are promoted, as are the processes of learning from each other and reflection upon what has been done. In this way the research investigates the viability of using such an approach to teaching and learning mathematics with students who are deaf or hard of hearing.

CHAPTER 3

The pilot study

Research into mathematical learning has shown that "games can be used to teach a variety of content in a variety of instructional settings" (Bright, Harvey & Wheeler, 1985, p. 133). Taking into account research into the use of games within mathematics programs, it is a useful proposition to study the use of games within a mathematics program for students who are deaf or hard of hearing. Games have the potential to create a motivating context for students in which they spontaneously discuss mathematical content and use language to describe the concepts associated with the game. Because of the critical need to develop and use appropriate language in the process of developing mathematical understanding and because of the concrete and motivating nature of games, they appear to offer the facility to engage students in activities which promote the use of appropriate mathematical language within a meaningful and stimulating context.

However, despite the promise of a meaningful context for promoting language development and understanding through the use of mathematical games, there is a query as to the effectiveness of games because of the difficulties with "divided attention" as discussed by Wood (1986); that is, the constant need for students to shift attention from the games to the signer/speaker and back to the games and, hence, a possible interference with the learning process.

The first step in the present research was therefore to conduct a pilot study to test the viability of a "games" perspective. In this way, it was hoped to gain experience in conducting research with students who were deaf or hard of hearing as well as in the use of a program of mathematical activities with such students.

It was decided to investigate a topic that was familiar to the students as the time available for the pilot study was short and the tasks would need to be relatively easy for the students to master. In this way, the focus could be on the implementation of the games rather than specific mathematical concepts. In view of this, the subject of multiplication was chosen. Each of the students had been introduced to multiplication a number of years previously, and as such, it was expected they would master the content of the program easily. It is significant to note at this stage that while all of the students had previous experience with multiplication, in this study different language was used to promote mastery of multiplication. Therefore, an element of new content was introduced. The students were selected on the basis of pretesting using the numeration section of the *Booker Profiles in Mathematics* (Booker, 1995) to ensure that numeration skills were sufficiently well established for multiplication to be attempted. The students were also pretested using the addition section of the *Booker Profiles in Mathematics* to ensure that their mastery of addition was sufficient to accommodate an understanding of multiplication. They were also selected on the basis of their reliance on the signed component of the linguistic signal. The study comprised the pretesting, eight one-hour weekly sessions with the group of four students and post-testing on mastery of multiplication. Each of the eight sessions was videotaped and later analysed. During each session the researcher also attempted to note instances of significant interest which could be followed up during analysis of the videotape. The sessions consisted of instruction followed by a related game. At the end of each game, the students were asked to write their thoughts about the game.

Throughout this document, pseudonyms are used to protect the anonymity of the students participating in both the pilot and the full research study. The four students who participated in this study attended a primary Special Education Unit for students who are deaf or hard of hearing. Within Education Queensland, it has been accepted that their educational needs have been ascertained as requiring an alternative program because of the significance of their hearing impairment prior to enrollment in such a unit. Each student was profoundly deaf according to hearing tests conducted by paediatric audiologists from Australian Hearing Services. They were enrolled in a program which followed a philosophy of Total Communication and, in this case, it was implemented through the simultaneous use of Australasian Signed English and spoken English. Two of the students were eleven years old, one student was twelve years old and the other turned thirteen during the study. The group consisted of two girls and two boys. Two of the students, Bill and Rose, were to progress to high school in the following year, while the other two students, Felicity and Gerry, were to remain at the primary unit for another year. Bill was described as experiencing Attention Deficit Disorder and most certainly found it difficult to remain focussed on a task for any length of time. He also found the many materials used in the study a source of distraction as he constantly played with the pencils, blocks, games or whatever was close at hand. Bill was not a gracious loser and tried every trick in the book to win. On one occasion, he became involved in a disagreement with Felicity over a game and stormed out of the room. The other students were more cooperative and conciliatory and rarely provoked disagreement. They were able to discuss their differences and accept assistance from each other gracefully.

Though the concept of multiplication was familiar to these students, it is, nonetheless, a difficult one for students to master because it is so closely aligned with that of addition, and yet must be differentiated from it. It is developed from the notion of repeated addition but goes beyond that level of understanding. Therefore, students need to be competent with the addition concept and associated skills before they can be expected to expand on their understanding to encompass the multiplication concept. Pretesting showed that these students had the necessary prerequisites of an understanding of the addition concept and associated skills.

An understanding of multiplication can be achieved through the use of arrays to represent problem situations requiring the application of multiplication. Arrays provide an excellent visual representation of multiplication and evoke simple and appropriate language. Thus, three groups of birds with two birds in each group can be visually represented as:

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* *
* *

and the appropriate language of *3 twos* can be represented. This language is in contrast to the traditional language of *2 multiplied by 3* or *2 times 3* which has no intrinsic relationship to the multiplication concept. In addition, arrays can be used to clearly illustrate the relationship between *3 twos* and *2 threes*. Materials clearly illustrate multiplication situations and develop the concept of multiplication as an identity which is independent of other operations. Arrays can be used to complement this process by representing the materials in pictorial form and providing a consistent representation for eliciting consistent and meaningful language. A connection can be made between the pictorial representation of arrays and the symbolic recording of multiplication. Therefore, *3 twos* can be represented in arrays as:

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* *
* *
* *
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and lead to the symbolic representation as:

2 counters in each row
3 rows
6 counters altogether

and later to

$$\begin{array}{r} 2 \\ \underline{\times 3} \\ 6 \end{array}$$

and is read as 3 twos are 6.

The students were introduced to the notion of multiplication through arrays. They were given Unifix blocks and A3 size sheets that contained a large grid of squares. Then they were familiarised with the language associated with arrays by gradually making an array with the blocks on the sheet starting with 1 five and adding fives to it until there were 6 fives. For example,

```
*****     1 five

*****     2 fives
*****

*****     3 fives
*****
*****
```

This process was repeated with different arrays until it was felt that the students had sufficient understanding and could use the appropriate language. They were then introduced to the *Array Game* which is played with the same materials using two dice to control "plays". One dice was ten-sided, containing the numbers zero to nine, while the other dice was six-sided, with terms such as *sevens, twos* or *sixes* written on each side. When both dice were rolled, the language for an array was presented, such as *3 sixes*. The students then made that array with the blocks on their grid and continued in the same way until their grid was filled. It did not take long for the conundrum to arise as to what to do with an array such as *6 threes* when no such space existed on their sheet. It was then that the students were guided to the discovery that arrays could be split so that *6 threes* was the same as *2 threes* and *4 threes.* Once the students were comfortable using the blocks, the format of the game changed to colouring the arrays on grid paper and recording the name on the coloured array. This game was played over a number of weeks, giving the students many experiences of arrays and also of splitting arrays. While the students were playing the game and using the dice, the appropriate language was used in a spontaneous and a meaningful way. It took the students some time to establish the difference between arrays using similar language, such as *4 fives* and *5 fours*. At first, there was uncertainty as to which way to represent particular arrays, but this was slowly resolved as the students gained more experience. Bill was the last to master this notion, possibly due to his distracted behaviour and lack of attention to discussions among other members of the group. The students appeared to enjoy the game, especially for the first few times they played it.

The *Array Game* could be played in a number of different ways. It could be played in teams, with each team having its own set of dice. It could be played individually with each individual having his/her own set of dice, or it could be played with pairs of players sharing the set of dice. From the dynamics that developed in this project, it appeared that the best interactions occurred when the students played in teams or when the dice was shared. When the students played in teams, they discussed the arrays with each other and when the dice was shared they were interested in what others rolled. On one occasion, Rose and Felicity were sharing a joke, Bill looked over

Gerry *She cheat. Look. 5 five. Five.*

Although it was not obvious from the video exactly what had occurred, it is clear that Gerry and Bill felt they had discovered a discrepancy between what was rolled and what was observed and they were prepared to justify it. Bill experienced difficulty with the *Array Game* and often appeared more preoccupied with playing with the pencils and distracting the group than with the task in hand.

The next area of multiplication discussed was multiplication basic facts. The facts for multiplication provide a challenge for students. The answers involved larger numbers than previously encountered and the thinking strategies for remembering them are more difficult to understand and remember than for addition facts. Games provide an avenue for practising the facts in many different ways in an enjoyable context. Increasing the speed of recalling facts is also possible through the use of appropriate games and the practice provided through the playing of games.

The first facts to be discussed were the twos facts. They were approached as a revision of the addition doubles facts. It was at this point that there was no escaping recording the multiplication algorithm. The students were familiar with the doubles facts and had no difficulty in making the association with the twos facts. Bill, on one occasion, recorded *2 sevens are fourteen* using the addition sign instead of the multiplication sign. It appeared that he was indeed making the association between the addition doubles and the twos multiplication facts and quickly corrected the sign. Rose made the correct response when asked to write *double 4* another way but read it as *2 multiplied by 4 equals 8*. When prompted for another way of saying it, she was able to say *2 fours 8*.

The game used to practise the twos facts was called *Stinker*. Firstly the meaning of *stink* and *stinker* was discussed and all of the students decided they did not want to end up as the stinker. At first, Rose did not understand the explanation of *stink* given by the teacher, but Felicity explained it to her and the game proceeded. It was played with a set of cards. Five cards were taken from the pack and used to make a house which was built on top of the other cards which were spread on the table. Each student took a card from the pile of cards under the house when they correctly answered a fact and tried not to knock the house down. The player who knocked the house down was *Stinker*. The game proceeded and the students were totally engrossed. Even Bill sat very still so as not to knock over the house of cards. During this game, the students were eager to answer their fact so they could have their turn and it proceeded charged with excitement. Some of the comments about this game were:

Felicity	*It good fun because it help me and make me happy.*
Gerry	*My breakfast came to my throat. I feel it. I have to swallow not vomit and then card fall down.*
Bill	*Fun.*
Rose	*Good. Fun.*

It should be noted that students' comments in all transcripts are quoted verbatim and have not been corrected for grammar.

The next session was devoted to the fives facts. The strategy for remembering these was to relate the facts to reading the clock in five minute intervals. Once again, the students appeared to be familiar with telling the time and had no difficulty in making the association with the fives facts. However, problems were experienced in recording the facts and all students needed prompting to use the appropriate language. Both Rose and Felicity reverted to the horizontal format of recording the facts. The turn-arounds of the fives facts were discussed using arrays and this did not present any difficulties. The game used to practise the facts was a track game following a basketball theme. It was called *Slam Dunk* and contained basketball terminology and the pictures of their basketball heroes. The boys were avid basketball fans and the girls were familiar with the sport. The boys noticed the game before the session began and were eager to play it from the start. A ten-sided dice was rolled to determine how far a player could move but, before this was done, a fives fact had to be answered correctly. The students participated well in this game but it did not generate the excitement of *Stinker*. However, they requested to play it again next session.

The nines facts were the focus of the next session. Initially, they were described in terms of arrays to ensure the students could match the language to the facts. Following this, the nines facts that were already familiar were recorded on the blackboard and the students were asked to search for a pattern in order to calculate the unknown facts. They were not able to see the pattern even with some obvious hints and so they were shown the strategy for calculating the nines facts. "Since 9 is near ten, the tens digit will be 1 less than the number nine is multiplied by; the ones digit and the tens digit always sum to nine" (Booker et al., 1992, p. 143). After practising and applying the thinking strategy, the students were challenged to a game of *Flasher*. For this game, each fact was written on a card with the answer on the reverse side. The students were shown the facts one by one and raised their hands when they wished to answer the fact. The first person with his hand raised was chosen to answer the fact and, if he was correct, he kept the card. When all the cards were distributed, the player with the most cards was declared the winner. This game was chosen first to inject excitement into the tedious process of practising facts and also to encourage speed in recall of the facts. However, on this occasion, the excitement was too stimulating for Bill who became frustrated at not being chosen often enough to answer the facts and at Felicity's success. After *Flasher*, *Slam Dunk* was played as requested from the previous week. However, the nines facts were practised this time. As the students were familiar with the game, the teacher relinquished control of the game and became an observer. One result of this was that the students watched each other's moves very carefully. Another result was that Bill quickly moved to assume control of the game by taking hold of the cards containing the facts. Felicity commented that he was a pest which agitated Bill. He accused her of cheating, knocked the dice out of her hand and onto the floor then stormed out of the room. Without teacher intervention, the game degenerated into arguments and attempts at cheating. It was also noticed that Felicity became impatient when any of the students were slow to reply and supplied the answer for them. The girls were asked for their preference of game themes for the following session and they requested something about horses.

The final session concentrated on square number facts. Arrays were used to illustrate the meaning of square numbers. The students were asked to colour 5 fives on an array sheet.

Teacher *What shape is that?*

Bill *Mean 5 fives.*

Teacher *Yes. Look. What shape did you make?*

Bill playing with his pencils.

Teacher *Will you watch me please? What shape did you make?*

Bill *Like square.*

Teacher *Yes, it's a square.*

The students had no problem understanding the notion of square numbers but benefited from the opportunity to practise the facts. It was noted that all of the students thought that *1 one* was two and yet knew some of the more difficult square number facts. In accordance with the girls' wishes, a track game featuring the theme of horses was played and was called *Hoofs and Manes.* The square number facts were practised during this game. Previously learned terminology continued to appear. Bill commented that *8 eight equal 64.* The game was played in a positive way and the square number facts were practised almost without realising it.

A final game was played for which they were put in teams. The girls chose to play together as did the boys. Each team was given a sheet with a grid of numbers that were the answers to all multiplication facts and three ten sided dice. All dice were rolled but the team members had to decide which two numbers they would multiply and then cross the product off their sheet. The first team to cross off four numbers in a row was the winner. This game enabled the students to access all multiplication facts and stimulated much discussion among team members as they justified their choice of facts and products to each other. Bill appeared to be enjoying this game as he explained his opinions to Gerry. Rose and Felicity worked well together and listened to each other's explanations. Although Gerry's initial reaction was that this game would be *boring*, he played it happily with Bill.

The approach used in these sessions was based on the recommendations in *Teaching Primary Mathematics* (Booker et al., 1992). However, the time span for the study allowed only eight one hour sessions one week apart. It turned out not to be possible to complete the program of teaching multiplication in this time or to work on any area thoroughly enough to achieve any real changes in the students' understanding. Nonetheless, their comments and interactions indicated that the approach motivated them to be involved in the activities and fostered discussion and exchange of ideas.

Learning through mathematical games was meaningful for the students and gave them the opportunity to be exposed to and use appropriate language to describe the area of mathematics in which they were currently engaged. Through games, they could describe their understanding, justify their understanding and learn from others engaged in the same process. Through games, they could also come to see mathematics as a way of thinking and as an arena for applying their skills and understanding. Early in this study, when the concept of arrays was being established, two of the students were involved in a discussion with the teacher about the meaning of 4 ones. Felicity felt that 4 ones was fourteen while Rose disagreed with her.

Felicity *No. 4 ones.*

Rose *Means multiply.*

In this case, we can see Rose's reasoning and her attempts to justify it to Felicity. She knew that the array indicated multiplication and considered that the answer must be larger, so she guessed that a number containing both digits might be close. Felicity did not agree and tried to show her the error of her ways. Later in the day, Rose correctly identified 2 sevens and 4 sixes indicating a change in her understanding of arrays. At this point, Bill was using the correct language while Felicity sometimes omitted the plural *s* in such terms as 2 five. Another discussion occurred when Rose asked Felicity which way to arrange her array for 4 fives. Felicity thought for a while, then suggested it could go either way and told her, "You decide". Within the span of the eight sessions of this project, the students would use the appropriate language of arrays when prompted.

Teacher *So, what do you need* (to finish the game)*?*

Gerry *2 two.*

Teacher *And ...*

Gerry *1 2 3 4 5 6. 1 six.*

However, Gerry slipped back into the use of "times" and "multiplied by" when responding spontaneously. When relating the doubles facts for addition to the twos facts for multiplication, a discussion occurred which illustrated this point.

Teacher *What's another way to write double 4?*

Rose wrote $\begin{array}{r} 4 \\ \underline{\times 2} \\ 8 \end{array}$

Teacher *And how do we read that?*

Rose *Two multiply four equal 8.*

Felicity shakes her head.

Teacher *Do you know the other way we were saying?*

Felicity *2 fours eight.*

Teacher *Good.*

In this case, Rose had reverted to the familiar reading of the algorithm while Felicity remembered the introduced language. Similarly, when recording multiplication, the students reverted to the horizontal format of recording when left to their own devices. At various times, Felicity recorded 2 sixes as $2 \times 6 = 12$ and 2 fives as $2 \times 5 = 10$.

Even at the end of the project, the students experienced difficulty with reading multiplication correctly. There is no doubt that three of the students very quickly grasped the idea that 3 sevens and 7 threes gave the same numerical result but this appeared to occur before they fully grasped the significance of the individual terms.

Throughout the study, the students were given many opportunities to discuss, justify and use their skills. The main avenue for this was the *Array Game*. It was the means for developing the concept of multiplication, illustrating and eliciting the appropriate language and stimulating discussion among the students. It was central to the entire process. The other games were simply a cover for practising the facts in an enjoyable context. They, too, elicited discussion but they did not aim to develop understanding, just to provide practice.

As a general rule, the students' comments about the games were positive and reflected original thoughts as well as some of the teacher's ideas. Surprisingly, all of Bill's comments relayed a feeling of enjoyment and achievement. Not surprisingly, they also conveyed a clear message that he liked to win. He wrote about the *Array Game*, *I love this game because I always win and it help me to understand.* He also wrote, *I like the game "it is fun". I played and won, I love this game.* About *Stinker*, he wrote *I like this game because it make me fun and happy.* With regard to *Slam Dunk*, he said *I like Slam Dunk* *game because I like Shaq O' Neil.* Gerry was honest about his thoughts. He found *Hooves and Manes* boring but of the *Array Game,* he said *I like the game. I think it help me. I think it is good.* On another occasion he commented *I like the game. Last time I lost.* Therefore, his enjoyment was not dependent on winning. He also felt that the *Array Game* had something to offer him in terms of *I like the game because it make people know more and learn well.* He also enjoyed *Slam Dunk* because *It have basketball player on it.*

The incident involving Felicity and Bill received a mention in both the girls' comments. Rose reported that day that *It* [the game] *is bored because we fright* [fight] and Felicity recounted the following week *I like it because no more fight.* Rose's comments were straightforward. She liked games because they were *fun* or *easy* and she disliked games because they were *bore.* One of the boring games was the *Array Game,* which was surprising considering her observed involvement in the activities. Felicity's opinions were honest. They reflected a desire for variety and new games. Of the *Array Game*, she wrote after playing it for a few weeks, *I like this but it is bored for me and I want to tell you why. I have been doing it in Junction Park school. I hate to reapert* [repeat] *again and again.* Initially, she had felt that the *Array Game help me to think. It is great.* When *Slam Dunk* was repeated, she commented *I don't like it because it is bored and I had it two weeks ago.* At that time she had reported that the game *make me happy today.*

Post-testing followed the eight week program and was the multiplication section of the *Booker Profiles in Mathematics*. Through this test, information was gained regarding the students' understanding of the concept of multiplication, proficiency in the basic facts, competence with the algorithm, and ability to use multiplication in a problem solving context. Bill was unavailable for testing on two occasions so no post-testing was possible for him. Felicity's test results indicated a student with a good grasp of all aspects of multiplication. In the test situation, she consistently used the appropriate language and recording, which had not been the case in the study. Gerry also showed a good grasp of all areas except for multiplication by more than one digit. He had no idea what to do with the additional digits and made no attempt to use them.

Rose experienced difficulty with the conceptual subtest. This section of the test was designed to investigate the student's grasp of the concept of multiplication through the interpretation of pictures in terms of the concept of multiplication. Her multiplication facts were incorrect on one item and, although she attempted the items in the correct way, she agonised over every move. Therefore the researcher was led to believe that her understanding of the multiplication concept was not solid. It is also uncertain how much of this difficulty related to language issues. Rose's facts were slow but reasonably accurate. She appeared to have developed a number of strategies to compensate for not remembering the facts. In terms of the algorithms, it appeared that Rose followed a formula and, when dealing with larger numbers, became confused with the assortment of numbers that required some action. She attempted, in some cases correctly, multiplication by two and three digits but confused herself with the recording. She attempted the problems correctly but did not seem confident of what she should do. On one occasion, she asked if the algorithm was "times".

In terms of the three students who participated in the post-test, two of the students revealed a good understanding of multiplication. Despite the use of a traditional approach to their conceptual development, they appeared to have grasped the notion and were able to apply it in problem solving situations. They were also able to accommodate the use of arrays and new language in the multiplication context. The third student tried hard but was not convincing. She struggled through the conceptual and problem solving sections, made very hard work of the facts and sometimes confused herself with the recording of the algorithm.

It was difficult to tell if the eight week program had any impact on the students' mastery of multiplication, particularly since eight one-hour sessions, sometimes more than a week apart, were not ideal for learning. However, their comments indicated they enjoyed the experience and felt they gained something from it. There were other outcomes which were positive. The researcher gained valuable experience in conducting research of this nature and many lessons were learned. The first and, probably, most important lesson, related to the use of a video camera and lighting considerations. The videos of a number of the sessions of the pilot study ranged from a little dark to shadowy outlines only. This created enormous difficulties when observing students who sign and whose speech is not always intelligible and severely limited scrutiny of those sessions. As well, because of the small size of the room, it was difficult to include all students in the picture and, hence, obtain a complete record of interactions. Similarly, it was impossible to monitor interactions when one student whose speech was not always easy to follow was hidden from view. These difficulties were noted and more suitable settings devised for the major study.

While discussing the reading of signed communication, another complication arose. The researcher was a visitor to the school and was unfamiliar with the students. Throughout the study, even with the assistance of a videotaped replay, Bill's signing was difficult to follow. His signs were fast, very large and interspersed with other movements. The situation was complicated by the researcher having worked away from the classroom for quite some time. This had resulted in reduced capacity to read signed communication in general and a greater difficulty in reading the signed communication of a signer such as Bill. By the end of the eight sessions, the researcher was improving in understanding his signed communication but only after consulting with his teacher and receiving some helpful tips for following his signs.

The result of this experience was the decision to seek a transfer to the classroom for the purpose of the major research project. This would achieve a number of outcomes. Firstly, signing competence would be improved by constant use and reading of signs. As well, the researcher would be more familiar with the students in the research project and it would be part of their school program and not something a stranger conducted once a week.

The issue of familiarity with the mathematical content is a vexed one. Despite using ingrained language from a traditional approach to instruction, at least two of the students demonstrated through post-testing a good grasp of all aspects of multiplication while another student gave many indications of inconsistencies. Any impact of this program on these students' mastery of multiplication was doubtful, but it is felt that more conclusive results may have been possible if the content had been completely new. The time span of eight hours spread over approximately twelve weeks was unsuitable for modifying ingrained ideas and uses of language. It was very difficult to introduce a new way of thinking about multiplication, and hence new language, in such a short space of time. Therefore, it was resolved that, for the larger project, a mathematical topic not familiar to the students would be selected. An interesting aside to the study was another instructional situation with which the researcher was concurrently involved. This was with a group of four students who were deaf or hard of hearing and who also had additional disabilities. They were being introduced to multiplication for the first time. No pre- or post-testing was done and learning was more prolonged, but it was the researcher's experience that they accepted the language of arrays and accommodated the concept of multiplication more easily than the research group. This is in keeping with a finding of Markovits & Sowder (1994) who reported that "where robust knowledge and procedures already exist, ... there is less likelihood of change because the prior knowledge is already accessible" (p. 24). They go on to report other researchers who have made similar findings.

In terms of the way in which games were implemented some lessons were learned. It appeared that team games or games that required the students to work together provoked more interactions than when the students were able to work alone. Therefore, it was noted that, for the full study, all games and activities should require interactions among students and not activities to be completed alone. It was also noticed that when the researcher relinquished control of the game the students became much more involved. Unfortunately, this was also when tensions mounted and disagreements got out of control. However, it may be worth exploring this possibility further. The theme of the game was important to the enjoyment of particular students. The boys liked *Slam Dunk* because they were involved in the then current basketball card craze but the girls were not very interested. Similarly, the girls liked *Hooves and Manes* because they had chosen the theme, but the boys were not so enthusiastic. It is interesting to note that both games were track games and generated using the same principles but with a different track design and theme. The issue of using competitive games with children as volatile as Bill requires further investigation. The fact that games involve the use of a number of different materials suggests that there are more items to distract attention and competition is inclined to distract from the learning process because it often leads such students on a tangent into pointless arguments. However, the games appeared to have the opposite effect on the other students who found them fun and a source of interest.

There were instances of cooperation and discussion during the study. During one session, three of the students noticed that they had the same pattern on their array sheet. Discussion then followed about what was needed to complete the array and different ways of splitting arrays to complete the game. On another occasion, Bill was recording on the whiteboard and used the addition symbol instead of the multiplication symbol. Felicity explained mathematically why he was incorrect and the appropriate changes were made. On another occasion, Rose was experiencing difficulty with formulating an answer during a game, so Felicity encouraged her by saying "You can do it." Rose did not initiate many interactions but watched what was happening in the group and appeared to benefit from it. When asked a question she would respond, but did not appear to want to be the focus of attention. The girls cooperated well with each other and so did the boys. In each partnership, there was one dominant and one less competitive member of the partnership. During one session, when open conflict eventuated, Rose expressed her disaffection with the situation and this led to a discussion where it was agreed that conflict spoiled everyone's enjoyment of games. Following this discussion, cooperation between group members improved. It may be significant that the group cooperated well during *Stinker,* the game that they all voted their favourite.

Sometimes the students found it difficult to generate their own opinions and feelings about the games. It is felt that this was more of an indication of lack of language to facilitate this kind of communication rather than lack of ideas. They also found justifying their ideas difficult at times. As a result, the researcher had to contribute suggestions which invariably manifested themselves in the students' written comments. Therefore, it is difficult to know if comments such as *help me to think, help me to understand, learn well* and the like are the students' original ideas or a reflection of the input of the researcher. It appears that the students need more experience in expressing and justifying their opinions in a more general sense before they can be required to give insights in a situation such as this.

The final issue to become evident from this study was the implications of problems with "divided attention" for the use of games and activities with students who are deaf or hard of hearing. Because of the visual nature of their mode of communication, these students could not attend to discussion and participate in an activity at the same time. They had to choose one or the other. Therefore, discussion and explanation were separated from activity and it was more difficult to make the connection between activity and meaning. Thus a contradiction became apparent. These students required visual input of information and, in mathematics; that is, *hands on* experiences through activities and games. They also required visual input of communication which involved speechreading and/or signing. So, although the practical aspects of this approach to teaching mathematics were eminently suitable, the effectiveness is reduced because it is more difficult for these students to extract meaning because the activity and the explanation are separated in time. There does not appear to be any possible solution to this situation. However, in an effort to offset this predicament, these students require more experience, more time and more discussion to meet their educational needs.

Implications for the thesis study

The experience of this pilot study influenced the design of the major research project. It was felt that the mathematical content of the full study should be completely new for the students involved to avoid the complication of relearning previously ingrained concepts and language and enabling a focus on the development of understanding. It was also clear that the time span of the full study would need to be significantly longer and more intensive than that of the pilot study, thus creating more appropriate conditions for effective learning. For this to be achieved, the full research study would need to be part of the students' daily mathematics program and not an extra activity conducted by a stranger with sessions spaced a week apart or more. For the major research project, greater care was required with the technical aspects of videotaping sessions so as to avoid the shortcomings of the pilot study. Having touched upon the potential that games offered for promoting discussion and interactions among students who are deaf or hard of hearing, it was important that in the full research project, more opportunities be created for sharing ideas and decision making through discussion. An ideal way of doing this is through team participation in the games and activities as opposed to individual endeavour. By their very nature, the use of games and concrete materials moves students who are deaf or hard of hearing into the domain of "divided attention" and, thus, runs the risk of reducing the impact of the activities. Careful attention would also be needed to ensure that the students were given time to move their attention from one information source to another to allow for the maximum impact of the activity. These important points and implications not only give specific pointers for the full study, they already provide suggestions and insights that will be helpful to teachers of students who are deaf or hard of hearing.

CHAPTER 4

The research study

The students

The students who participated in the project are described in terms of their individual characteristics and differences. Four students participated in the project and they comprised half the then current Year 6/7 class in the unit. They were not selected individually for the project but comprised a group of students who needed to become familiar with fractions. The other Year 6/7 students were integrated into a regular Year 7 class for their mathematics program while the students participating in the project remained in the Special Education Unit. This indicates that these particular students did not grasp mathematical concepts easily. There were two boys and two girls - Suzanne, Bert, Rachel and Alex.

The four students covered a variety of combinations of hearing impairment and preferred modes of communication. Two of the students were profoundly deaf and relied heavily on signed communication as would be expected of most students attending a special education unit, while the other two students had sufficient hearing to operate orally. These two students could sign but found it easier to communicate without signed communication. As well, two of the students had attended special education units for their entire school life, while the other two had spent a significant length of time in hearing classes. The latter two students showed strong indications of poor self-confidence when it came to mastering new mathematics: one was well practised in avoidance strategies while the other all too readily denigrated his own efforts at mathematics. It is also interesting to note that two students came from deaf families and two from hearing families. This is a higher proportion of students from deaf families than would normally be found in a group of students in such a unit. It is also significant to note that one of the students who came from a deaf family was more comfortable with oral communication than Total Communication and was often the initiator of spontaneous oral conversations with the teacher, a fact which resulted in the exclusion from that conversation of students who required signed communication. Therefore, there was not a neat distinction between students from deaf families needing signed communication and students from hearing families preferring oral communication. It is also worthy of note that all students were comfortable with the use of Signed English and that this did not present any difficulties for the students from deaf families who also used Auslan. (Auslan is Australian Sign Language which is the recognised language of the Deaf community in Australia.)

As a result of mathematical assessment with the *Booker Profiles in Mathematics* (Booker, 1995), all students indicated a general lack of understanding of the number system, operations on numbers and problem solving applications. Nonetheless, even within this ability level, one student, Bert, demonstrated a superior level of understanding compared to his peers. However, there were still significant gaps in his understanding of number and his problem solving abilities. Overall, although the students were nominally in Year 6/7 they were actually operating at approximately a Year 3/4 level in mathematics.

As the program was built around using various games to develop a basic understanding of fractions, there was a heavy emphasis on thinking, discussing, and justifying as well as enjoyment. Apart from the mastery of the fraction work, changes to some negative classroom behaviours were noted. One student ceased avoidance behaviours for the term of the project, another student lessened disruptive behaviours and another began to verbalise confidence in himself and pleasure at his success in mathematics. These were by-products of the mathematics program which enriched the learning process and the students' self concept as well.

The program of teaching and learning fractions

The mathematics program itself was tightly sequenced to move the students through a number of steps, expanding their notion of fractions as they went. Previously, this sequence of activities and learning steps had been the subject of a research project with hearing students which had produced high levels of understanding and skill (Booker, 1996a). It offered these students the opportunity to experience fractions in a very concrete way, surrounded by appropriate language and the opportunity to use this language. At first, the students became acquainted with the idea of parts needing to be equal. At the same time they were becoming familiar with the conventions of the language of fractions. Games were provided which required the students to make a match between the appropriate language and pictorial representation of fractions. They were also required to compare fractions, match equivalent fractions and work with fractions of a group. The entire program consisted of games and activities with minimal time allocated to traditional styles of instruction such as much teacher talk with less student participation.

A profile of participating students

Bert was thirteen years old and had experienced a profound bilateral hearing impairment from birth. He had attended a number of units for students who are deaf throughout Queensland. His educational needs had been ascertained as requiring an alternative program because of his hearing impairment. Bert relied on spoken and Signed English for communication. His speech was intelligible and considering the level of his hearing impairment, he used his hearing very well. Nevertheless, his understanding of speech was heavily supported by signed communication. He had been described in reports as experiencing "Attention Deficit Disorder" and, without doubt, his behaviour was very unsettled, particularly during the early stages of the project. However, even when he was at his most unsettled, Bert was always focussed on the activities and keen to be involved. At about Day 12 of the project, his behaviour became noticeably more calm and conciliatory. Discussions with his mother confirmed that he often took a while to settle into new routines. Bert's mathematics program had previously been centred on the usual classroom routines of manipulating materials, focussing on the blackboard and individual bookwork. Therefore, a mathematics program that was comprised entirely of games removed familiar structure and control from the mathematics classroom as well as changing the routine, thus leading to unsettled behaviour early in the program.

Results of the *Numeration Profile* indicated that he had not developed a full understanding of expanded notation and he, like the others, found difficult the item testing ordinal numbers in one digit numbers which required comprehension of very sophisticated language. The procedure of processing language and then following a two-step instruction proved too difficult for him. Although it seemed as if he had been exposed to rounding, he found it difficult. He was able to move without difficulty between the language, the symbols and the materials related to four digit numbers. His counting was good except for one problem counting backwards in ones across hundreds.

In terms of computation, testing showed that his concept of addition was shaky as he could not generate problems to match the algorithms on the test. Nevertheless, he had no difficulty with the items related to the subtraction concept. His basic facts in all areas of computation were accurate but he often resorted to using his fingers when facts were not automatic. He was generally accurate doing addition algorithms and appeared to understand regrouping. Addition problems couched in simple language demonstrated his accuracy, but he stumbled on the third item which contained more information. He had more difficulty with the subtraction algorithm and was successful only with the first item which tested understanding of the concept. No attempt was made at the third problem as he found the language too difficult and the irrelevant information confusing.

The results of Bert's *Mathematics Profile* reveal a student with some areas of understanding and some areas of weakness. He found it very difficult to apply his knowledge of mathematics to a problem solving situation. Bert's approach to this lengthy activity was one of enthusiasm and pleasure as he liked mathematics and had experienced success in the area. He liked to understand mathematics and liked to discuss his answers. When he found an item difficult, he merely indicated he did not know and happily moved on, confident of eventual success.

Bert enjoyed the program of activities and games as presented in the project. There were times when he was not at his most cheerful and, on one occasion, he opted out of participating in a game because he had lost the previous one. However, despite any struggles with his disposition, Bert was always involved in and following the activities. On another occasion, he had completed the game and he was waiting for the others to finish. He was still watching the game closely and contributing his ideas. When playing *Fish,* he would ask for what he had missed to be repeated as he was keen to follow everything that happened. When asked what he thought of playing games he responded: *Makes you think. Good.* When pushed a little further on the subject, he continued: *They make you think about things to make sure it is right. How many are there? What is 9 ninths? What is one whole? Make me clever.* So, not only did he enjoy the program, he also felt he had learned from it.

Bert brought to the project his interest in mathematics and his expectation of success. From the first day of the project, Bert understood and articulated every aspect of the fraction work we were attempting. From Day 1 of the project, Bert began to use the *th* ending of fraction names and was generally consistent in its use. On Day 2, after a brief encounter with fractions, he was able to describe all aspects of fourths.

Teacher	*How many pieces can we cut this* (chocolate) *into?*
Rachel	*Four.*
Teacher	*Fourths?*
Bert	*One, two, three* (indicating the number of cuts). *One, two, three, four* (indicating the number of pieces). *Chop, chop, chop. Cut there, there, there.*
Teacher	*I thought we could cut it into smaller pieces.*
Suzanne	*No, big one this time.*

As Alex cuts the chocolate:

Teacher	*Bert, tell me about fourths.*
Bert	*They must be the same size.*
Teacher	*What?*
Bert	*They must be the same size.*

Teacher	*Yes, and how many of them are there?*
Bert	*There could be six.*
Teacher	*What?*
Bert	*Four.*
Teacher	*Yes, there are four. And what do 4 fourths make?*
Bert	*Um* (distracted by Alex cutting the chocolate).
Teacher	*4 fourths make what?*
Bert	*1 whole.*
Teacher	*So, how many parts make that whole?*
Bert	*Four.*
Teacher	*And what do we call them?*
Bert	*Fourths.*

In this exchange, which took place on the second day of the project, Bert was comfortable with the fact that fractions were composed of equal parts and the number of parts was related to the names of the parts. He also was comfortable with the concept of 1 whole. On the same day, he was able to generate other fraction names when they were needed after experiencing other fractions.

Bert always could explain his thinking. There was always a reasonable explanation for his answers even when they were incorrect. On one occasion, Bert drew a card in a game that required him to nominate if the parts were equal or unequal. He studied the card for some time and, still uncertain of the correct status of the parts, he took out his ruler and measured the width of the parts.

Bert	*Equal.*
Teacher	*Can I see it? Equal?*
Bert	*They are the same size. That one centimetre, one centimetre, one centimetre, one centimetre.*

Teacher *Look at that part. Now look at that part.*

Bert *Unequal.*

Teacher *Yes, they are different sizes.*

Later in the project, after some experiences of equivalent fractions, he was able to judge equivalent fractions for himself. Bert was able to do this confidently while the other students were still coming to grips with this notion.

Teacher *You rolled 2 tenths. What is the same as 2 tenths? Is there something the same as that?*

Bert (After studying the sheet.) *Fifth. 1 fifth.*

Bert was confident in his own understanding and, unlike some of the others, would not be dissuaded when he knew he was correct. Here, he was justifying equal parts under pressure.

Bert *Equal. Same size. Halves.*

Teacher *Are you sure about that?*

Bert *Part are equal.*

The teacher then asked the other children if anyone thought the parts were the same and if anyone thought they were different. Bert remained sure of himself.

Bert *They are equal. Same size.*

Teacher *That's right. Good boy.*

Earlier on the same day, Bert had used the sides of parts being straight as a justification for them being equal and had learned from his mistake.

Bert *Unequal. They are not the same size.*

Teacher *Are you sure?*

Bert *Yes. Big, big, small.*

Teacher *But they are straight.*

Bert *Yes.*

Teacher *Are they equal?*

Bert *Yes. No. Big, big, small.*

Early in the project, he was able to justify why parts were unequal. Bert not only articulated his understanding spontaneously, he also was able to justify and explain why he made decisions or why things had happened.

Bert *Unequal because they are not the same size.*

Teacher *What's different about them?*

Bert *(That's) smaller. (That's) bigger than that.*

On another occasion,

Bert *We got 1 whole already.*

Teacher *What did you roll?*

Bert *9 ninths.*

On another occasion, he was able to explain his thinking behind a mistake.

Teacher *How many halves make one whole?*

Bert *Four, two.*

Teacher *Why did you say four? What were you thinking about?*

Bert *I think about how to make 2 wholes.*

He was confident in his own understanding and would not be intimidated into changing his mind. As well, he would think through situations aloud and, when required to explain how he had marked his sheet, his response was: *4 fourths, 1 whole.* (He then checked his sheet.*) 8 fourths, sorry. 1 whole, 2 whole.*

Later, he was able to calculate that 2 fourths make 1 half, 2 fourths make 4 eighths, and how many ninths make 1, 2 and 3 thirds. During his introduction to fractions of a group, it was as if he was completely familiar with the concept, and yet he had not been exposed to this work before.

Teacher *Tell me about the cows.*

Bert *There are four cows and there two cows colour. 2 fourths.* (He then matched the picture to the written language and the symbol.)

At no time during the project did Bert show any significant confusion or indications of difficulty.

As well as his proficiency with the fraction work, Bert was notable for his progression from unsettled to more settled behaviour. As his behaviour became more calm, his cooperation improved and his interactions with the other students became more positive. On Day 5 of the project, Bert decided to opt out of a game because he had just lost a game and *Other people are smart at this game*. By Day 18, he was sporting enough to say *Good on you Alex* instead of a show of displeasure when Alex made a pair that he had been attempting to make. Although the early part of the project was punctuated by attention to Bert's behaviour, his interest never wavered from the activities. Even when he was most unsettled, he wanted to be involved in the activities and was communicating and contributing to the discussions about fractions.

Bert was able to benefit from the guidance of the teacher and follow a line of questioning to improve his own understanding.

Teacher *Tell me about these.*

Bert *They are not equal.*

Teacher *Why are they unequal?*

Bert was unable to answer.

Teacher (Holding another card.) *Why are these equal?*

Bert *Because they are the same size.*

Teacher (Returning to the original card.) *Why are these unequal?*

Bert *Because they are not the same size.*

A similar situation occurred during a game that required 1 whole to be completed. Bert had already participated in games where parts (for example) 5 fifths, needed to be rolled to complete a whole, but on this occasion it had slipped his mind.

Bert	*Miss Louth, you don't have that* (1 whole) *on it* (the dice).
Teacher	*Yes, but you can make it with other things.*
Bert	*9 ninths?*
Teacher	*Um.*
Bert	*Oh yeah.*

Bert learned new language quickly in this context. On one occasion, the teacher had mentioned that parts were unequal because the area was different. Later that day, Bert attempted to use the language in this new context.

Bert	(The parts are) *unequal.*
Teacher	*Are you sure?*
Bert	*Yes, they are not the same size, the area.*

Bert liked to win the games. In one way, this kept him focussed as he wanted to see for himself that everything that happened was fair. On the other hand, he was very competitive and was not pleased when he lost a game. On one occasion he was discovered cheating at the blocking game, so desperate was he to win at all costs. When the group had been distracted by another student experiencing difficulty, he had moved one of his blocks to put himself in a winning position. When asked what he liked about a particular game, his response was : *I like it when they lose*. On another occasion, when he needed ninths to win a game of *Colour-me-fractions* in which a dice was rolled containing the names of fractions to be matched to pictorial representations on a sheet, Bert set the dice to ninths pretending he had rolled it. He was noticed by

Suzanne and chastised for cheating. When the focus moved to the next student, he smiled to himself and signed *I try.* It was obviously worth the risk. The intrinsic rewards of participation were still a future pleasure for this young man. He also made very clear which students he preferred as team partners. His first choice was always Alex and he cooperated with and supported him. When watching Alex's efforts at colouring the fraction bars, he once commented, *Perfect! Fantastic!* He also allowed Alex to make some decisions.

Alex had rolled 9 tenths.

Bert *9 tenth. Where is that?*

Alex and Bert search together and Alex finds it.

Alex *Oo, oo.*

Bert *Yes.*

They both push to colour it.

Bert *I will.*

Alex lifts the pen from Bert's hand.

Bert *Why?*

Alex indicates another colour pen.

Bert *Alright. You.*

His ability to cooperate with and respect other students was again illustrated by this exchange with Alex.

Bert *5 eighth, where is that? One, two, three, four, five, six* (counts the parts of some wholes). *No. No. Where the eighth?*

Alex then showed Bert the eighths.

Bert *Good. One, two, three, four, five.*

He also played happily with Suzanne and helped her to find fractions without any discord. On one occasion he and Suzanne were teamed together for *Colour-me-*

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fractions. Bert rolled ninths and Suzanne indicated the parts she intended to colour. Bert shook his head so Suzanne checked the sheet again and found the correct parts. Bert commented to her *I know where they are. I count them before. That one is ninths. That one is ninths. Two of them.* However, he was not happy to play with Rachel and he would sabotage their games with go-slow tactics and poor cooperation. It is not clear why this occurred, possibly something to do with disagreements outside the classroom.

Bert was a contrast of two attributes which are not usually regarded as complementary — pleasing achievement and disruptive behaviour. At times, he was moody and difficult to work with. At other times, he was positive and cooperative. He was the only student who required frequent attention to his unsettled behaviour and he was the student who, at the end of the project, showed greatest evidence of solid mastery of every aspect of the work and who was most able to communicate this understanding.

When the project had been completed, it was clear that Bert had mastered all the mathematical content that had been presented over the twenty days. However, in retrospect, the focus had been more on dealing with his behaviour and less on how well he had done. Certainly, he was by far the most active and noisy member of the group and the most likely to disrupt. But, on viewing the videos more closely, he was on task almost one hundred percent of the time and it was his outspoken manner that provided so much feedback about his mastery of the work.

Suzanne spent the first five years of her school life in regular classes. She was originally ascertained as requiring modification of the regular school program because of her hearing impairment. She was eleven years old and had a severe loss in her right ear and a moderate to profound deteriorating loss in her left ear. In 1995, Suzanne was reascertained as requiring an alternative program and enrolled in the special education unit for students who are deaf. The change in her ascertainment indicated that she required an alternative educational program because of her hearing impairment. She was expected to spend one more year at the Special Education Unit before entering high school.

She was a child belonging to two cultures — hearing and Deaf. She had enough hearing to be oral and aural in good auditory conditions and she also fitted comfortably with the Deaf community. Suzanne's parents were both Deaf and her brother, who was Deaf, also attended the special education unit. Socially, Suzanne mixed with the Deaf community and hearing friends. Because she was proficient with both modes of communication, Suzanne communicated effectively with all the students in the group.

Although she displayed an ability to learn mathematics and an interest in mathematics, there were large gaps in her knowledge of mathematics. Her *Numeration Profile* indicated that place value and sequencing were areas of difficulty for Suzanne. She had little grasp of expanded notation and also had difficulty with some of the language associated with understanding number such as *least, after, greatest.* When questioned, she appeared to ignore the information she found confusing and then answered questions according to what she had understood. In counting activities, Suzanne sometimes lost her place and then became confused. She was unfamiliar with rounding. In the area of computation, Suzanne exhibited similar difficulties with addition and subtraction. Her understanding of the concepts was patchy and needed consolidation. Because of this, she had difficulty in developing problems of her own relating to these concepts. Suzanne solved the addition problems, but this was in the context of her general difficulty with problem solving. She took some clues from the fact that all addition items were tested together and likewise for subtraction. Therefore, if she had been tested on addition facts, she deduced that the subsequent problems required her to carry out an addition algorithm. This indicated that her understanding of the concepts of addition and subtraction were insufficient to be applied to a problem situation. As well, she once verbalised that *Altogether means add it up* which indicates applying a procedure instead of true understanding. On another occasion, after adding instead of subtracting, she justified it by saying *Because it says 'How many more students?'*. Suzanne was following a procedure instead of applying understanding and knowing what the problem was asking of her. Her basic facts were a problem in that she had developed a complex system of using her fingers and was most reluctant to rely on her memory for facts. Therefore, she was very slow and sometimes inaccurate. Most of the mistakes she made in algorithms were related to basic facts, so Suzanne's understanding of mathematics was patchy. In almost every area, she had developed some skills but not enough understanding to be successful in a regular classroom.

Suzanne was erratic in her mastery of the fraction work. There were some aspects of her participation in the project that indicated that she was experiencing the learning process exactly as planned. However, basic inconsistencies in her use of the language of fractions remained to the end of the project. In some ways, this was not surprising when her academic history was considered. Suzanne spent the first five years of her school life in hearing classes. In that time, she had learned many things related to the curriculum, but she also appeared to have learned how to avoid challenging herself and meeting mathematics head on. When the work became demanding and Suzanne began to feel vulnerable, her reaction could be to request a visit to the toilet or to threaten to be sick. She initiated conversations about irrelevant topics, took a long time to settle to a task or distracted other students. There was also the question of many unexplained headaches in class. Suzanne was not confident in herself nor in the system's ability to support her through her difficulties. She was in limbo. She was a student who would usually be regarded as too oral for a unit using Simultaneous Communication but had difficulty in the noisy and dynamic environment of a regular classroom.

There was a number of pleasing aspects to Suzanne's involvement in the project. One was the level of understanding of fractions she achieved. This is discussed further later in this chapter. Another was the complete absence of avoidance behaviour for the entire period of the project. There were a couple of days when she was genuinely unwell with asthma or a virus, but, even so, she was still involved in the activities. I feel that this was very much related to the fact that Suzanne was confident that she could handle the work and that it was interesting and fun for her. Another pleasing aspect of Suzanne's involvement in the project was that she communicated effectively about mathematics by commenting and asking and answering questions. In some ways, this was facilitated by her oral skills and her chatty nature. Had she been less oral, it would have been difficult for her to question and comment so freely because the teacher would have to be attending directly to Suzanne to understand her message. However, because she was oral, the teacher could hear and understand her message even when focused on other students and Suzanne understood this.

Throughout the project, she was generally interested and involved in the activities. Her concentration did not wander and there was not one indication of avoidance behaviour in twenty sessions. There were times when she checked other students' responses and asked them to justify their actions. On one occasion, she noticed inconsistencies with Alex's pairs of cards. *Alex, that's wrong. Four eighth, two eighth.* On another occasion, she reminded Bert that he had already had his turn.

Suzanne *You got six. You got six.*

Bert *Six?*

Suzanne *Yes, before. You got it over there.*

Another time, she checked Rachel colouring a fraction and advised, *No, not that one*. These were the actions of a student interested in the activities and following every aspect of the games.

Suzanne was learning to explain her thinking and was building confidence in her own ability to do so. She was not accustomed to this kind of questioning and was uncomfortable with it initially. There were times when Suzanne changed her opinion simply because she had been asked to explain her response. This would indicate that she was not solid in her understanding and lacked confidence in her decisions. However, on another occasion, when asked to explain why parts were unequal, she replied while indicating the parts, *Bigger, bigger, smaller.* Suzanne appeared to enjoy the success she achieved and was easily motivated to discover more about fractions. During the first experience she had with the *Colour me fractions* game using two dice, the following exchange took place.

Suzanne *When we roll, do we colour both of them* (both dice) *or one side* (one dice).

Teacher	*It depends on what you roll.*
Suzanne	*Oh, I see. If I roll twelve, do I have to colour, colour, colour, colour?* (pointing across the rows)
Teacher	*Today you will use two dice.*
Suzanne	*Oh, I know what you mean.*
Teacher	*What does that say?* (Indicating the two dice.)
Suzanne	*Seven and fifth.*
Teacher	*What's that?*
Suzanne	*Fifth.*
Teacher	*No, what number is it?*
Suzanne	*5 sevenths.*
Teacher	*Can you show me 5 sevenths?*
Suzanne	(pointing to fifths and sevenths) *Five and seven.*
Teacher	*No, where are your sevenths?*
Suzanne	*Sevenths? There.*
Teacher	*Now, you have to colour five of them.*
Suzanne	*Where 5 sevenths?*
Teacher	*You have to colour five of your sevenths.*
Suzanne	*Like one two three four five stop?*
Teacher	*Yes and colour them. Oh, what have we got here?*
Suzanne	*9 sevenths.*
Teacher	*Now show me your sevenths. You have to colour nine of them.*
Suzanne	*You can't.*
Teacher	*Can't you?*
Suzanne	*Nope.*
Teacher	*See if you can show me 9 sevenths.*
Suzanne	*There* (pointing to the ninths). *Nine?*

Teacher *No, sevenths. Show me the sevenths. How many is there?*

Suzanne *One two three four five six seven.*

Teacher *But there's more.*

Suzanne *Eight nine?*

Teacher *Um. OK?*

Suzanne *Yep.*

Teacher *So it's more than one whole.*

Later Suzanne rolled 2 sixths and said to herself: *Sixths and colour two of them.* In this exchange, it can be seen that Suzanne progressed in her understanding of fractions. She had been comfortable with naming parts but had been absent for the first session when the other students had been introduced to using two dice and identifying the number of parts. Suzanne's first reaction was to indicate that she was comfortable with the notion but, when probed by the teacher, it was seen that she needed assistance to master the language and the notion. As expected, she was uncertain of the meaning of the language indicating the number of parts, but following a line of questioning she was able to decipher the code for herself and manifest the first steps of understanding. In the process of doing this, she also came to grips with fractions that were more than one whole. Through listening, questioning and manipulating the materials, Suzanne's first experience of fractions that were more than one whole was a positive and an enriching one. She was also clarifying the language of fractions; that is, that the parts were named as well as the number of parts. Through the questions of the teacher, she moved from what was known to the unknown.

Suzanne listened to other students contributions and learned from them.

Alex *Eight equal parts.*

Suzanne *No.*

Alex *Yes. One, two, three, four, five....*

Suzanne *How come the other one said eleven? That's the same one.*

Teacher *Those parts are bigger.*

Suzanne *And the other one smaller. I see.*

This discussion showed that she had been following the game and as well as her own moves. We also see that she is confident enough to continue to challenge Alex for clarification and to consider his replies critically.

She appeared to enjoy the program and, when asked for her opinion about the games, she replied that, [It] *makes you clever. It makes you think about halves, thirds, whatever.* Generally, her perception of games were that they were good and she commented once that games were interesting. Like the others, she preferred playing team games and her reason for this was that they were easier to win.

By the end of the project, Suzanne showed that she knew when parts were equal and unequal. She understood the concept of a whole being made up of parts. As well, she understood the concept of equivalent fractions and attempted to find them. She had been exposed to fractions of a group but required more experiences to consolidate understanding. However, some difficulties persisted. Confusion between fractions with similar names persisted; for example, *4 sevenths* and *7 fourths*. She was also inconsistent in her use of the *th* ending, consistently omitting the *th* ending from fraction names in spoken and signed communication. Sometimes she would say the correct fraction name and sign it incorrectly, for example, say *1 sixth* and sign *one six*. This last practice had implications for those students who relied on signed communication. Even though she had used the correct language in her spoken communication, the students who relied on signed communication received incorrect information from her. After a two week break from the fraction work, Suzanne reverted to substituting *2 three* for 2 thirds, *6 eight* for 6 eighths and *4 nine* for 4 ninths in oral and signed communication. It seemed that the language of fractions was still not completely within her grasp.

Suzanne was a pleasure to have in the group. She was mature enough to transcend personality differences and worked without conflict with every member of the group as well as having a calming effect on Bert. She was an open book, freely expressing her thoughts for example, [This game] *it's wasting our time because we're not winning anything.* She involved herself in the activities and enjoyed them. Although she did not verbalise it in the same way as Alex, her actions indicated that she experienced success and enjoyed the experience. This was indicated by the complete absence of avoidance behaviour for the entire time and was a very positive aspect of the project. It showed that, when supported appropriately, Suzanne was interested in mathematics and could stay on task in an age-appropriate manner.

Rachel was a mature girl who liked to be challenged and to learn new things. Her motivation came from within herself and she was not dependent on teacher control to maintain a focus on the activities. When this project took place, Rachel had only recently arrived from another school and was still a little shy with the teacher and the boys. She socialised regularly with Suzanne's family and, therefore, preferred to work with her friend.

Rachel was eleven years old and had been profoundly deaf from birth. Her parents were deaf and she had one older hearing sister and one younger deaf sister. Because of this and because she mixed mostly within the Deaf community, she was very comfortable with her deafness and very confident in herself. As reported by her mother, Rachel accepted much responsibility for running the home — more so than her siblings. Accordingly, at school she was very independent, and showed initiative and impatience with others who did not, in her opinion, pull their weight. She used Signed English at school but her communication with her family was through a different mode, probably Auslan. She was ascertained as requiring an alternative program because of her hearing impairment. Her family had recently moved and she had, consequently, changed schools. She arrived with the reputation of being a high achiever, mature, responsible and headstrong.

The results of Rachel's *Numeration Profile* indicated some gaps in her understanding of number. She demonstrated significant difficulties in the area of place value beyond two digit numbers. Because she seemed to have no idea about what the questions were asking of her, it is suspected that she was not experienced in this way of thinking about number. In fact, after the project was completed, she mastered this information reasonably well and quickly. She did not understand rounding, and, it would appear that she had not experienced this way of using numbers. Her *Computation Profile* showed that she was proficient with addition and subtraction facts. She completed addition algorithms well but had not mastered regrouping in subtraction. The concept of addition appeared well established but not subtraction. Her problem solving skills in addition and subtraction were poor. Generally, Rachel's performance painted a picture of a student who could perform most of the more automatic aspects of mathematics but who lacked the understanding necessary to apply and use it.

After completing the project, initial impressions were that outcomes for Rachel were not positive. There was no memory of anything outstanding in her performance. However, after viewing the videos, it was clear that she showed good understanding and came to grips with all aspects of the fraction work quite effortlessly. Because other students were either very demanding (indicating confusion), or very oral and talkative, it seems Rachel's progress was initially overlooked. The difference between Rachel and Bert (who also mastered the material well) was not so much in their understanding but in the ways in which they indicated this understanding. Whereas Bert is very outgoing and was constantly verbalising his thoughts, Rachel was much more self-contained and most of her communication with the teacher was answering questions. Early in the project, she was asked to indicate if parts were equal or unequal and to justify her answer. She was able to do this without hesitation. *Unequal. Because some are big and other one are little* or *Unequal. That same as that.* [That is] *different.* However, in practical terms, she indicated understanding through her participation in all the games and activities. In fact, it may be that the approach of using only games and activities enabled her to indicate understanding in a much more effective way because it was focussed very much on hands-on performance as well as manipulating the appropriate language.

Rachel was involved in the activities and the accompanying interactions throughout the project. She had adapted to her deafness and was visually focused in a way the others were not, watching when it was her turn, watching other students' moves and communication and watching the teacher's interactions with other students. She would often answer questions directed at another student. On other occasions, she moved her chair to be closer to games so she could be more involved. After the project had been completed and the opportunity arose to get to know Rachel more closely, it became apparent that she had very high expectations of herself and enjoyed new challenges.

When paired for team games, Rachel kept her partner on task or indicated they should work more quickly. This occurred on different occasions with Suzanne and Bert. She did not hesitate to remind Bert to stay focussed on an activity even though it wasn't always well received. When paired with Alex, there was very little communication. This appeared to be more related to both being uncomfortable with each other's preferred mode of communication. Although Rachel was an excellent speechreader, she was much more comfortable with signed communication and received much more information through that mode. Although Alex was competent in reading signed communication, he was not comfortable in signing complete language. As a result, communication between Rachel and Alex was mostly limited to one word utterances, pointing and gestures. Rachel preferred to be paired with Suzanne. They communicated effectively and worked well together. It was difficult to understand much of what passed between them as it wasn't Signed English. I suspect it was Auslan and the language they probably use when they socialise outside of school. I also suspect that a couple of conversations that occurred when the teacher was involved in lengthy interactions with other students did not relate to mathematics, but possibly boyfriends and the like.

Although Rachel preferred to work independently, she also benefitted from interactions with the teacher as well as from observing interactions between others. Early in her introduction to equivalent fractions, the following exchange took place.

Rachel	(Having rolled the dice.) *3 fifth.*
Teacher	*3 fifths. 3 fifths, what is that the same as?*
Rachel	(Thinks and points to the sheet.)
Teacher	*Fifths.* (Then outlines 3 fifths.) *3 fifths is the same as what?*
Rachel	(Thinks.) *1 whole.*
Teacher	*No, not one whole.*
Rachel	(Thinks.) *Eight.* (Then checks the sheet again.) *Six.*
Teacher	*Six?*
Rachel	*6 ten* (meaning 6 tenths).

Significant in this exchange was the fact that Rachel was steered to work out an equivalent fraction. As well as that, it was positive to observe her concentration and her application. At no point was there any indication that she wanted to be told the answer.

The greatest difficulty observed was that which Rachel experienced in mastering the language of fractions. Probably it was more of a problem for the observer than for her as she gave every indication of understanding fraction names but simply omitted the *th* ending. Over the time of the project, Rachel regularly omitted the *th* ending to indicate a fraction. She experienced more difficulty than the others using the structure *Ten equal parts are tenths,* although she managed almost every variation on this structure, for example, *4 parts equals fourths* or *6 parts sixths equals.* On Day 15 of the project, she rolled 9 sevenths but signed *nine and seven.* For a student of Rachel's ability and considering the attention the *th* ending had received, it would reasonably be expected she master it by that time. At various other times, she would sign the fraction name without the *th* ending, e.g., *eight ten* for 8 tenths and *five seven* for 5 sevenths. She also signed such things as *two five* for 2 fifths. All of these indicated that she had not completely accepted the need for the conventional language of fractions in its regular and irregular forms.

At the same time, she was indicating understanding through her participation in the games and activities. She was always accurate in her interpretation of fractions in the activities such as the *Colour me fractions* games. So she appeared to know what she was doing and understood the fraction names without difficulty. She was accurate working from receptive language to expressing fractions concretely, but she was inaccurate moving from the concrete form to accurate expressive language. So it appeared that she understood fractions but had difficulty with the intricacies of the language. There could be a number of reasons for inconsistencies with the *th* ending:

- She did not fully understand the concept of fractions;
- It was inconvenient to have to fingerspell the *th* ending or the irregular form;
- The introduction of the signed form of the symbol was unhelpful at a time when the correct language was not completely mastered;
- It was a developmental stage in coming to completely control the information and, had the project continued for longer, she would have been a consistent user of the conventional language form;
- The representation of number sign and *th* ending is used exclusively in Auslan to represent dates.

The fact that Rachel was able to adapt to fractions of a group so effortlessly seems to indicate that her understanding was good despite any inconsistencies in the expressive language of fractions.

With the assistance of concrete materials, Rachel was able to discover equivalent fractions. She made the transition from manipulating parts of a whole to early manipulation of equivalent fractions without any significant difficulty. She would take time to try to identify equivalent fractions. No doubt, she was also motivated by the desire to finish the games first. Like the others, Rachel liked to win. She was also mature enough to handle defeat gracefully. On her first exposure to fractions of a group, Rachel was able to say without prompting that 4 sixths of the group of frogs was coloured.

Although she was self-contained and not given to spontaneous shows of expression, Rachel responded when asked for her opinion about games that they were *Good because they make me interest.* This comment is supported in a concrete way by her consistent focus throughout the project and her insistence that team partners do the same. On Day 18 of the project, Rachel was challenged by a difficult question,

Teacher	*What do you know about fractions?*
Rachel	*Easy to do. 5 sixth.*
Teacher	*What does that mean?*
Rachel	*Five colour.*
Teacher	*Out of...?*
Rachel	*Six.*

Rachel did well to attempt to reply to such an abstract question and then to handle ellipsis. From this interaction, we can see that she is confident in her mastery of fractions and her understanding is sufficient to facilitate answering difficult questions.

In the project, Rachel progressed from having a passing exposure to fractions such as *a half* and *a quarter* to indicating a broader understanding of the concept of fractions, understanding the names and parts of fractions, understanding and identifying, with the aid of concrete materials, equivalent fractions and fractions that were more than one whole. She also adapted effortlessly to fractions of a group.

Rachel was a good student and mastered most areas quickly and it is felt that she benefitted greatly from learning in this way. In contrast with other students in the project, Rachel was confident in herself and mastered the fraction work relatively easily. It is felt that her understanding was improved through the range of experiences provided by such an approach. The experience was enjoyable and lacked any pervading sense of inadequacy so often experienced by students who are deaf. In much the same way, this approach also provided many challenges for Rachel, enough to maintain her involvement and the satisfaction of learning new things. Although she did not verbalise it, it can confidently be reported that she also enjoyed the experience.

Alex was different from the other students in the group in a number of ways. Most obviously, his hearing impairment was far less significant than those of the others. He had a moderate, bilateral hearing loss and his aided audiogram indicated that in ideal listening conditions he could detect all speech sounds. Alex spent the first four years of his school life in regular classes. At that time, he attended mathematics and language classes conducted by a Learning Support Teacher. At the commencement of the project, he was twelve years old and had been enrolled for three years in a special education unit for students who are deaf. His educational needs were ascertained in the area of hearing impairment as requiring modification to the regular school program because of his hearing impairment, and ascertained also as requiring additional educational support because of learning difficulties unrelated to his hearing impairment. There was some doubt as to the validity of his level of ascertainment for hearing impairment and the impact of his hearing impairment as opposed to his learning difficulty. In an educational sense, Alex was not deaf but hard of hearing. This would normally mean that his education could proceed in the regular classroom with support from a teacher of the deaf. However, it had always been difficult to find an appropriate placement for him because of his additional language processing difficulties. With his hearing aids on, Alex could hear most things and when using an FM aid (which occurred most days) he missed very little. His speech was perfectly clear with no indication of a hearing impairment. All high frequency sounds were present in his speech. Therefore, Alex's preferred mode of communication was speech and listening. Because Alex had attended the unit for three years, he had become proficient in reading signs but he was very uncomfortable when required to use more than a few signs at a time.

The other noticeable difference from the other students was the difficulty Alex had in mastering mathematics. Everything was a struggle for him and he was aware of it. He had experienced it for a long time. He was very conscious of his lack of success and was overjoyed whenever he achieved success. At some times Alex did surprisingly well, while at other times he had difficulty with work he, seemingly, had mastered previously. This was consistent with Alex's academic experience generally. When it came to memorising, he was successful. His spelling and mathematics facts were better than most of the students.

However, when understanding became an issue, Alex always struggled and this was particularly apparent in mathematics.

Previous assessments by a Speech and Language Pathologist indicated delays in the areas of receptive and expressive language. Alex also had difficulty in the test situation following instructions with three or more elements. Because of the pattern of his language errors in the test situation, there was a very strong suspicion that Alex's difficulties in understanding and using language were more related to language processing difficulties than hearing impairment. He experienced greater difficulty in this area than his deaf peers. After seeking the advice of a Speech and Language Pathologist who specialised in language processing difficulties, a few simple strategies were suggested in an attempt to assist Alex to learn by taking into account his specific needs in language processing. They were:

- Keeping language simple, avoiding long sentences and embedded clauses and phrases;
- Using visual stimuli as much as possible;
- Reducing activities that include a sequence of steps to single step activities;
- Using an FM system with him to reduce the interference of background noise (as students with language processing problems are easily distracted by background noise).

Although these strategies are very close to those suggested for students who are deaf or hard of hearing, by the end of primary school many such students are expected to follow a sequence of instructions and comprehend language of increasing complexity; whereas Alex was not able to do this and became confused and agitated when faced with information presented in such a way.

The results of the *Numeration Profile* (Booker, 1995) indicated that Alex had significant difficulty with place value, particularly in three and four digit numbers. Although his ability with one and two numbers was quite acceptable, Alex experienced great difficulty with the item in one digit numbers that required him to manipulate complex language; *show the dinosaur that is the fourth after the one that is underlined.* Given his difficulties with language processing, it is possible that the linguistic demands of this question caused him more difficulty than knowledge of ordinal numbers. This was reinforced by observations of his use of ordinal numbers in other situations, for example, citing the order of students to finish a game, which indicated no significant difficulties in this area. There were also indications that he had difficulty with sequencing numbers so that simple terms such as *the number after* and *least* confused Alex and it appeared that he had little experience of rounding numbers. Nonetheless, his knowledge of basic place value appeared to be reasonably secure, giving him the foundations for numeration understanding.

In the area of computation, Alex was strongest in addition. According to the profile, he understood the concept of addition, his basic facts were of an acceptable standard and he could implement the algorithm with only minor problems with regrouping. His mistakes in addition algorithms appeared to be more related to carelessness and wanting to complete the work as quickly as possible. In the area of problem solving Alex experienced significant difficulty. Again, it appeared that Alex found the concentration of information in words very confusing. He was not sure what the information meant, nor was he sure about what the question was asking of him. However, Alex did have some success with completing the addition problems because he operated on the premise of *if in* *doubt, add* and *how many always means add.* On the other hand, an addition problem which contained more words and more information confused him and his strategies were of no use when there was an overwhelming amount of language.

In the area of subtraction, Alex had greater difficulty. His understanding of the subtraction concept was not strong. It was better when presented with visual information (a picture) but he could not generate his own subtraction problems. Alex's subtraction facts were good although he resorted to using his fingers a couple of times. He tended to rush through subtraction algorithms and there were many mistakes with regrouping. In the area of problem solving, he had difficulty gaining meaning from the information supplied. There appeared to be too much information and too many words for Alex to begin to make any sense of it. The results of Alex's *Mathematics Profile* indicate a student with some skills but large gaps in understanding and application of mathematics. Alex appeared to operate on the assumption that mathematics was a set of formulas and details to be remembered.

In terms of the project, one of the most obvious outcomes was Alex's reactions to success and failure. His life in the mathematics class was a roller coaster ride between exhilaration when he achieved success and subdued sadness or negative comments when he was aware that he was experiencing difficulty. Alex communicated his pleasure at success on a number of occasions. There was no mistaking the pleasure he derived from the success he had achieved. After winning a game when teamed with Bert, his reaction was: *We won, won, won! We won! We won! We won!* He then stood up and did "high fives" with Bert, that is, they slapped two hands together in the air as a sign of winning and success similar to their American basketball heroes. On another occasion, when teamed with Suzanne, they won a game that had become prolonged. Alex proclaimed his pleasure by: *Hurray, Suzanne! We got it.*

On another occasion, Alex was very pleased with himself when he had done very well at identifying some equivalent fractions in a game. The teacher had praised him by saying: *Good thinking. Good boy.* Alex then turned to his partner, Bert, and said: *I told you we can do it.* His success became more apparent to him as the minutes passed and his pleasure increased. Alex turned to Bert and said: *Did you see what I just did? I said "We can do it. Two wholes and then those".* Still later, after the teacher had commented on how well he had done identifying the equivalent fractions, Alex gleefully replied: *I should go to high school now.* The reference to high school was significant because it was an ever-present unknown in the back of Alex's mind. He was not certain which high school he would attend and, although he wanted to progress on to high school, he was concerned about how he would cope.

However, Alex's mood changed when he was experiencing difficulty. He often became very quiet and hung his head. He was very vulnerable to any suggestion of failure whether real or imagined. It was a practice during the playing of team games for partners to check each others' responses. However, on one occasion when the teacher had suggested to Suzanne to check Alex's answer, he became annoyed and said: *I know it's tenths.* It seemed that Alex felt so vulnerable that he was sensitive to imagined criticism. On another occasion, he made more comments indicating how vulnerable he felt. The teacher had just commented to Rachel that she was clever and Alex reacted by saying: *Yes, she knows more than us.* Then as if to prove he was clever as well, he threw in a completely unrelated piece of information and asked: *What's one thousand times* *one thousand?* Also, Alex could be very self-deprecating. After realising he had made a mistake, he once commented to himself: *I'm stupid.* Although every effort was made to bolster his confidence, his self-esteem fluctuated with his success in mathematics. There was no quick solution to Alex's long-term lack of confidence.

The requirement to use complete Signed English for the benefit of Rachel and Bert was a cause of concern for Alex. He found it difficult to sign and speak at the same time. He verbalised this discomfort on one occasion when he was teamed with Rachel and was reminded by the teacher that he needed to sign what he was saying so Rachel could understand. His reaction was: *Next year, I'm not going to sign when I go to high school. I'm not going to sign. Just sit there like a log.*

The other outstanding aspect of the project work for Alex was the inconsistency of his responses. At times he gave every indication of having mastered the fraction work presented to him, while at other times he gave indications of confusion and lack of understanding. Considering his language processing difficulties and the complications often created by language overload, the following exchange with the teacher was surprisingly fruitful.

Teacher	*Have a guess, Alex, how many 8ths make 1 fourth?*
Alex	*Four.*
Teacher	*Show me 1 fourth. Now, how many eighths is the same as that? Have a look at your eighths. Put them up against it.*
Alex	*Two.*
Teacher	*So, how many eighths is the same as 1 fourth?*
Alex	*2 eighths.*

Teacher *2 eighths. Now show me one half. Good boy. Now, before, what did you say? How many fourths is the same as 1 half?*

Alex *2 fourths.*

Teacher *Yes. Now, how many eighths will be the same as 1 half?*

Alex *What was it?*

Teacher *How many eighths, these ones, how many of them is the same as 1 half?*

Alex *Four. 4 eighths.*

Teacher *Good boy. 4 eighths is the same as 1 half.*

In this exchange, Alex was able to decode some very complex language about relationships and emerge indicating understanding. It would have been a good outcome with any of the students, much more one with significant language processing difficulties. He was also able to justify his responses at times. When asked why parts were unequal, his reply was clear: *They're not the same size. Those two are the same and they're not. They're unequal.*

On another occasion, a short but very significant exchange occurred between Alex and the teacher. Alex had just identified correctly parts as sixteenths. What followed was a linguistically complicated question from the teacher (conditional negative) which often would be guaranteed to confuse most of the students. He not only understood but also answered correctly and then continued to improve his understanding by questioning the teacher.

Teacher *Alex, if they were not equal, would they be sixteenths?*

Alex *No. What would you call them?*

Teacher *Just parts.*

Unfortunately, these very encouraging indications of understanding were punctuated by incidents when Alex floundered. When asked if parts were equal or unequal there was obvious confusion where previously there had appeared to be understanding.

Alex	*Equal. Unequal. Equal.*
Teacher	*What do you think?*
Alex	*Unequal.*
Teacher	*Why?*
Alex	*Equal, I mean.*
Teacher	*What do you mean? Are they the same or not the same?*
Alex	*Not the same.*
Teacher	*Are you having a good look at them? Are they the same or not the same?*
Alex	(Thinks for a long time) *They're the same.*
Teacher	*Yes. So, are they equal or unequal?*
Alex	*Equal.*

This interaction is also an example of how, when he was experiencing difficulty, Alex benefited from the information being broken down into very small steps, thus enabling him to process the information successfully.

Throughout the project, Alex regularly experienced difficulty in correctly identifying fractions that contained the same numbers but had different meanings, for example *8 fifths* and *5 eighths*. There was no indication in his *Numeration Profile* of problems with ordinal numbers, so no link could be established with that particular aspect of the language of mathematics. When teamed with Bert, he verbalised: *But look. 5 fourths, 4 fifths, we can do that, the same.* Once he also requested *4 fourths* when he needed *8 eighths*.

Under normal circumstances, this kind of confusion would be interpreted simply as lack of understanding of the meaning of fraction names and the strategy for remediation would be more experiences of making, identifying and naming fractions. However, in this case, it was never clear how much of the problem was related to not understanding fractions and how much was confusion created by the similarity of the fraction names. In some respects, the language appeared to add to his confusion rather than endow meaning. Certainly, the other students appeared to have sufficient experience with fractions to understand the difference. Even when the teacher attempted to guide him to the correct answer through a sequence of questioning, Alex sometimes continued to experience difficulty.

Alex	*Is that 3 fourths? 7 fourths?*
Teacher	*How much is coloured, Alex?*
Alex	*Four.*
Teacher	*And how many altogether?*
Alex	*Seven.*
Teacher	*So what do you reckon it is?*
Alex	*7 fourths.*
Teacher	*No.*
Alex	*4 sevenths.*

In this situation, it is still uncertain if Alex took a gamble that the only alternative must have been right or if he actually understood what he was describing. Alex experienced many confusions such as this.

Suzanne	*Who did seven?*
Alex	*Seven five. Yes, seven five.*
Teacher	*Did you have fifths? Did you roll 7 fifths?*
Alex	*Yes.*
Teacher	*Well, they're sevenths not fifths.*
Alex	*We had fif* (meaning "five") *sevenths.*

Teacher *Oh, 5 sevenths.*

Alex *Well, I was right.*

In view of the differences in understanding between Alex's mastery of fractions and that of his deaf peers it may be that Alex's difficulties in understanding and manipulating fractions and the problems of inconsistency in performance were more closely related to his language processing difficulties than to his hearing impairment.

There were also times during the project when, although Alex did not initiate correct responses, he was able to work things out for himself with a little guidance from the teacher.

Alex *Can we cross out 1 whole?*

Teacher *No, but you can make it another way.*

Alex *How?*

Teacher *You have a think.*

Next turn Alex rolled 9 ninths (the students were throwing the dice during the game).

Alex *Oh, 1 whole.*

This was a positive indication of understanding from Alex. He was not given the solution to his problem, only a clue, and he was able to find it for himself. Similarly, when Alex had rolled 6 thirds and wasn't sure what to do:

Teacher *Where are the thirds?* (Alex points.) *How many are there?*

Alex *Three.*

Teacher *Are there any more thirds?* (Alex points.) *How many are there?*

Alex *Three.*

Teacher *Three and three. 3 thirds and 3 thirds make what?*

Alex *1 whole, 6 thirds. I should colour that* (1 whole) *and that* (1 whole).

Later, Alex explained to Bert,

Alex *6 thirds can make that. Same. Three and three.*

Alex exhibited good skills in cooperating with his peers. He liked to contribute and be involved but he did not attempt to impose his ideas on his partner. Over the time of the project, he worked well with all of the students and was not involved in any significant differences of opinion. He listened to what they had to say and was open to learn from the experience. On one occasion, Bert rolled 8 ninths and Alex was colouring an incorrect fraction. Bert commented that he was wrong and then indicated the correct response for Alex.

On the basis of Alex's own communication, there seems little doubt that he enjoyed learning in this way and in his own estimation achieved success. One day, early in the project, Alex asked if he should take out his maths book (for written work).

Teacher: *Not right now.*

Alex *I don't want to do work today. I want this* [game] *today.*

Teacher *Isn't this work?*

Alex *No, I want to play this instead of work, writing.*

Teacher *This is work.*

Alex *That's work?*

He also recognised that he benefited from the activities and games. Typical of all the students, he liked to win but he was also able to look beyond that point.

Teacher *What do you think of playing games?*

Alex *Good* (without hesitation).

Teacher *Why is it good?*

Alex *Just won.*

Suzanne *Makes you clever.*

Alex *And think.*

One of the most important comments he made with regard to his feelings about the work he was doing came when he quietly volunteered that *It makes you happy.*

Alex's involvement in the project provided a number of different perspectives. Firstly, he was the student with the least significant hearing impairment and the greatest difficulty in coming to grips with understanding fractions. Secondly, he was the student with the lowest self-esteem and the only one to verbalise it. Thirdly, it is felt that he was the student who benefitted most from the nonthreatening way in which the activities and the information were presented because he enjoyed it and felt successful. Fourthly, it is still unclear what is the most effective way in which to present information to promote understanding for Alex, while it is more apparent that this style of learning experiences was effective for the other deaf students.

It is difficult to say what Alex had mastered at the end of the project. At various times he gave indications of understanding all aspects of the fraction work including recognising equal and unequal parts, naming parts, identifying parts that were more and less than a whole, understanding what makes a whole, equivalent fractions and fractions of a group. However, at other times, he experienced difficulty with work of which there had been previous evidence of mastery.

Methods of communication in the education of students who are deaf or hard of hearing

It is opportune to devote attention to the issue of preferred mode of communication for deaf or hard of hearing students. The combination of students presented in this project is unusual. Within special education units for students who are deaf or hard of hearing in Queensland, it is rare to find students with as much hearing as Suzanne and Alex. It is even more atypical to find two such students in one group. Students such as Suzanne and Alex are normally educated in regular classes. One of the differences between students who are deaf or hard of hearing in regular classes and those placed in special education units is in their communication needs. Alex and Suzanne were oral and aural in good auditory conditions and enjoyed a passing chat or comment in class. They did not need signed communication to learn and understand. Bert, although profoundly deaf, used his hearing well. He heard some of what was said and his speech was intelligible but he required and requested signed communication to ensure optimum understanding. Rachel, because of her background and her needs, was a user of signed communication. Alex and Suzanne sometimes passed comments and drew the teacher into asides about the activities in a manner that was natural to them. However, when this occurred, Rachel and Bert often missed the information. Rachel, in particular, had to be aware that communication was occurring, it needed to be signed communication and she needed to be attending to it before she could be involved. Meeting the communication needs of the various members of the group was a challenge.

Within the group, there were some students who required signed communication to be part of what was happening and to learn from what was discussed. There were other students who were more comfortable using oral communication and not signing. For them, spontaneous communication was spoken. An additional complication was that two of the students were Auslan users outside school and one of these students was more comfortable using oral communication than Signed English.

The experience of this study was that it sometimes happened that oral students drew the teacher into spoken asides not involving the whole group. In spite of the initial intention to include all students in discussions, sometimes the discussion developed using only spoken language. It was then that the teacher had to make a decision whether to interrupt the spontaneity of the discussion and sign what had already been discussed or risk losing the students who had missed the early part of the conversation by attempting to include them in developments in the hope they would understand.

Therefore, the issue at stake was not the method of signed communication, that is, Auslan or Signed English. It was the difficulty of concurrently meeting the communication needs of students who communicated spontaneously in two very different ways.

In order to meet the needs of all students as far as possible, a solution was found through the use of Total Communication; that is, the concurrent use of spoken and Signed English, the oral students could access signed and spoken communication and the students who required signed communication could also access what was being discussed. Therefore, it is not considered a major issue that the children who used Auslan away from school were being disadvantaged as they were very familiar with this method of communication.

The mathematics program

The program for teaching fractions implemented in this project was developed at Griffith University where it had been used in a previous study in developing understanding of fractions with hearing students (Booker, 1996a). It centred on the use of games to provide students with appropriate experiences to build understanding and focused on developing language to facilitate communication and understanding about fractions. Because of these considerations, it appeared to offer some resolution to two identified causes of difficulty in the area of mathematics for students who are deaf or hard of hearing, those being the need for many concrete experiences to facilitate concept development and an added emphasis on the development of the language appropriate to mathematics.

The concept of fractions was first introduced to the students through facilitating experiences of dividing wholes into equal parts. In this case, the students eagerly divided different items of food into equal parts with the promise of an ensuing party. This gave them the added incentive of ensuring that the parts were equal because none of them wanted a smaller share. They divided into pieces chocolate, a cake, cheese, fruit, a pizza and they divided a packet of biscuits into equal groups of biscuits. They were particularly attentive to the division of chocolate. Through these activities, they were able to generate fraction names and to experience making parts from a whole. The activity also generated interest and many opportunities to be exposed to and use appropriate language for fractions. By communicating about the activities, the students were sharing meaning and testing their ideas on each other. The following activity focussed on dividing a jam roll into ten equal parts.

Teacher	*Now, we have something big here. We can cut it into more parts. How many parts will we cut that into?*
Suzanne	*Ten*
Teacher	*Can you do it?*
Suzanne	*No, not me.*

This reaction was not unusual for Suzanne who lacked confidence in her ability to master mathematics and often employed many avoidance techniques in an effort to evade failure. On the other hand, she found language activities much easier because of her more advanced level of language development and she enjoyed and requested language and literacy activities. On this occasion the teacher had used a rhetorical question and handed Suzanne the cake.

Suzanne accepts the cake.

Teacher	*Ten equal pieces. Work it out first.*
Suzanne	*Like that?* (Indicates where to cut.)
Alex	*That's six.*
Suzanne	*One two three four five six seven eight nine ten.* (indicating ten cuts.)
Teacher	*That's eleven. If you cut it ten times, you will have eleven parts.*
Alex	*That's ten.* (Checks the marks made on the cake by Suzanne.)
Teacher	*Is it?*
Alex	*Ten lines.*
Teacher	*Yes, but how many parts?*
Suzanne	*Ten.*
Bert	*Eleven.*
Teacher	*One two three four five six seven eight nine ten eleven. So you will need nine cuts.*

It would have been opportune to have had a similar discussion regarding a smaller number of parts before attempting *tenths*. In such a case, the students' instinctive knowledge of the relationship between the number of cuts and the number of parts could be used as a basis for a formal understanding of this relationship. Subsequently, this kind of discussion would be much easier.

Suzanne *Four. Four pieces.* (Suggesting an easier option.)

Was this another attempt to negotiate avoiding the difficult relationship between nine cuts and ten pieces?

Teacher	*No. Ten. Ten pieces. So, put the knife down and let's work it out. Can you work out how many cuts for ten parts.*
Suzanne	*You have to make all these nine lines.*
Teacher	*Yes. What's an easy way?*
Suzanne	*Cut that way.* (Indicating along rather than across the cake.)
Teacher	*No. If you cut it in half....*
Suzanne	*Like that?*
Teacher	*Well, that's not half. Is that half?*
Bert	*Yes. Chop. Chop.*
Teacher	*If you cut it in half, how many parts will you have?*
Suzanne	*Two.*
Teacher	*Yes, so, you will have to cut both halves some more times.*
Suzanne	*So you cut that in half* (then unintelligible speech).
Bert	*Yes.*
Teacher	*OK. Cut it in half.*
Suzanne	*There.*
Teacher	*Now, you need ten parts.*

Suzanne *So, put one cut there ....*

Teacher *So, how many parts do you need from each half?*

Rachel shakes her head.

Suzanne *Um. I cut it like that then there.*

Rachel *Five. Five.* (Indicating five parts from each half.)

Teacher *Suzanne, put the knife down for a minute.*

Suzanne *Five.*

Teacher *Yes, five parts from each half. So, you need to cut each half into how many pieces?*

Bert *Five.*

Teacher *Yes, five. Can you do that? It's easier to work out.*

Suzanne *So, one two three four.*

So the difficulty created by the need to make ten pieces disappeared when the number of pieces was reduced and Suzanne could picture what was needed.

Teacher *Yes, four cuts the same.*

(Suzanne works out where the cuts will go.)

Teacher *Are they about the same?*

Suzanne *Not really.*

Teacher *Make them close.*

Alex *She's a bit confused.*

Suzanne *There.* (indicating where the cuts go.)

Teacher *OK. Make them about the same.*

Suzanne *One. Oh boy. There.*

Teacher *You need three pieces from that big one. You need three more.*

Suzanne *One two.*

Teacher *Oo, that's a big one!*

Suzanne	*Yeah, I know. One two three.*
Alex	*It's very hard.* (Coming to Suzanne's defence.)
Suzanne	*Yes, it's hard because it's a soft one* (cake). *That's why.*

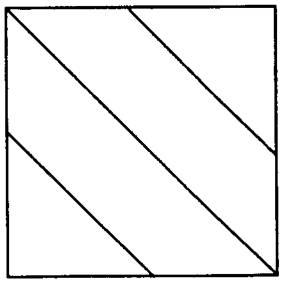
Suzanne was correct. Although the sponge cake was large enough to be cut into ten pieces, its texture was not conducive to making equal parts.

Teacher	*So that's five parts. You need to do the same to the other half.*
Suzanne	*There.* (Indicates where to cut.)
Bert	*Um.* (Nods in agreement.)
Suzanne	*OK, now. They're fifths. One two three. Oh boy.* (Completes the cutting.)
Teacher	*Thanks, Suzanne. We have to pretend that they are the same size and they are very close. That was good.*
Suzanne	*But this one smaller one.*
Teacher	*That's OK. You tried to do it. How many parts are there?*
Rachel	*Ten.*
Teacher	*Yes, there are ten parts. What are they called?*
Alex	*Tenths.*
Teacher	*Tenths. Don't forget the ths on the end. Tenths.*
Bert	*Tenths.*

After all the discussion surrounding making ten equal parts, the label, *tenths*, that was central to the activity, came automatically and effortlessly. In this activity, it can be seen that every student contributed something. The video showed intense interest from each student that did not waver through the entire activity. Much of the discussion focussed on how to make ten pieces. Within that framework, the students found that it was not as expected. Ten cuts did not make ten pieces. This issue had not arisen in previous situations when making fractions with a smaller number of parts. On those occasions, the students had been able to make the correct number of parts without consciously planning the number of cuts needed. They found that to make this task manageable, it was easier to divide the cake in half and then cut each half into five pieces. There was also a focus on ten parts being called *tenths* and the *th* ending was emphasised. At one stage, Suzanne verbalised *fifths* in relation to the number of parts required from each half which she viewed as a whole. The pragmatics of dividing a soft sponge cake into ten equal slices proved difficult (as explained by Suzanne). However, the students showed that, in this context, they understood what equal pieces were and they were aware that Suzanne had attempted to make the pieces equal. The students then went on to discuss the names of other fractions. Over two days of dissecting, discussing and consuming items of food, each student experienced making fractions from a whole and discussing fractions using appropriate language.

Following the experience of making different fractions of a whole and discussing appropriate names for those fractions, the emphasis shifted to honing the students' understanding of equal parts. This proved valuable because it seemed that in the past, the students had experienced *equal* in terms of facts and algorithms but they did not immediately generalise this across to fractions. Opportunities for experiencing and identifying equal and unequal parts were provided through games. The *Dinosaur Game* required the students to correctly identify if parts of different shapes were equal or unequal and then they had the opportunity to participate in a blocking game which generated intense interest.

At first, the students were not consistently able to identify equal and unequal parts. This is illustrated by one of Bert's responses to this shape.



He was not sure if the parts were equal or unequal so he measured the width of the parts with his ruler. Because the parts were all the same width, he decided that they were equal. His early definition of equal parts was consistent with the above response.

Teacher *How do you know if parts are equal?*

Bert *They are the same size and they are straight.*

Later, Bert's mastery had progressed to:

Bert *They are not equal.*

Teacher *Why?*

Bert *Because they are not the same tall.*

Teacher *The distance between them is the same.*

Bert *But big, small.* (Indicating different sized parts.)

Teacher *That's right. The area is different.*

Alex was inconsistent, showing both confusion and accurate mathematical thinking and communication. On one occasion, he was unable to decide independently if parts were equal or unequal. Even when the language was changed to familiar terms, it was not always helpful to him.

Alex *Equal. Equal. Equal. Unequal. Unequal.*

Teacher *You decide. You have to think are they the same size. Are they all the same?*

Alex *Yeah.*

Teacher *Is that the same as that?*

Alex *No. Unequal.*

Yet, a few minutes later, Alex was able to substantiate his response.

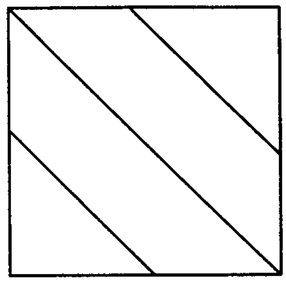
Alex *Unequal.*

Teacher *Why?*

Alex *Because they're all not the same size. They're the same, those two, and they're not.*

Teacher *That's right.*

The following day, Alex responded to the same configuration



as *Equal because the pairs are the same*. Alex's inconsistency was bewildering from the start. Suzanne experienced similar difficulties with the same configuration but hers were resolved.

Suzanne *But they are equal because, look, small big.*

Teacher *Some parts there are equal.*

Suzanne *Wider, smaller. Oh yeah.*

Through this game, the students keenly experienced and discussed parts of a whole almost without realising it because of the intense interest generated by the blocking game. In this game, they were able to place a block on a grid after successfully identifying equal or unequal parts. The aim of the game was to have four blocks in a row. Each block could be used either to build the line or block an opponent. A concern arose that, after playing the game a few times, the students might have been memorising the correct responses rather than indicating understanding. Therefore, new cards containing different shapes and fractions were produced and used with the same blocking format. This resulted in confirmation of the students' growing facility to identify equal parts. Variation to the activities was introduced by using a concentration game. In this game, students had to match the written terms *equal* and *unequal* to assorted shapes and parts.

When the students' responses were referenced to number correct for *equal* and *unequal*, it appeared that they were more often correct for parts that were equal. They were more unsure with parts that were unequal. This was consistent with findings regarding difficulties experienced with negatives in a mathematical context by students who are deaf or hard of hearing (Barham & Bishop, 1991).

Following the focus on parts being equal, this was combined with reinforcing fraction names. The game chosen for this purpose was *Grand Prix.* This was a track game. To progress around the track, the students had to correctly identify a fraction from a card and move the number of spaces rolled on a dice. The language structure to describe the fractions was similar to *Twelve equal parts are twelfths*. Rachel had the greatest difficulty in using this language, finding every variation on the structure. Her favourite variation was *Four parts equal fourths*. However, she gave every other indication of correctly identifying fractions. She always used the correct name and seemed to accept the need for parts to be equal. It appeared to be the language structure that confused her not the content. Another game used to offer variety was *Garfield Rules*. It, too, was a track game and it required the same identification and language as the previous game. Alex and Rachel sometimes required the assistance of structured questioning and comments from the teacher to guide them to use the desired language.

1. Are the parts equal?
2. How many of them are there?
3. We say *six equal parts are sixths*.

At this stage of the project, it appeared that using the fraction names was becoming automatic as the procedure became routine and the students were absorbed in the games. They did what they had to do and said what they had to say almost without thinking because they had understood and adopted the language and were more interested in the progress of the game.

The next stage in the development of the fraction concept involved relating the pictorial representation to the written language. The first step in this process was the initial *Colour-me-fractions* game. In this game, the students had to match the fraction name rolled on a large dice marked with the fraction names (for example, fifths) to drawings on a sheet of the same sized fraction bar with different numbers of equal parts. In this game, the fraction bars were arranged in order starting from the fractions with the fewest parts to the fractions with the largest number of parts. When the match was made, the fraction bar which was divided into the fraction rolled was coloured. This game could be played individually or in teams. Each student or team had a sheet which contained the same fractions. The first student or team to complete colouring every fraction bar on the sheet won the game. Before playing the game, students needed to be familiarised with the written form of the fraction name. The students enjoyed the game and the competition but more communication was generated when they played in teams. A slight variation to this game used the same fraction bars but had them out of order to oblige the students to examine the parts closely and to count the parts themselves.

In requiring students to justify their selection of fraction bars, sometimes it was helpful to use contrast to assist students to generate the explanation. In this case, Alex had selected the correct fraction bar but found it difficult to justify his response.

Teacher	*How do you know it's thirds, Alex?*
Alex	(Shrugs.)
Teacher	*Is that right?*
Alex	(Nods.)
Teacher	*Why isn't it sevenths or tenths? How do you know it's thirds?*
Alex	*Because it's got three.*

An aside to this incident came when the students requested to see themselves on the videotape. On seeing himself in difficulty, Alex commented: *I have trouble saying that. Every time I talk, I get mixed up with everything.* It seemed that, at least on this occasion, Alex understood what was happening and why he experienced so much difficulty. At other times, no assistance was required for Alex to accurately describe his understanding of equivalence of fractions.

Teacher	*How do you know it's sixths?*
Suzanne	*Oh, because it's got six parts in it.*
Teacher	*How do you know that's right?*
Bert	*There are eight parts.*

The subsequent step in expanding the students' understanding of fractions being equal parts of a whole was to change the pictorial representations of a whole but to use the same game format of *Colour-me-fractions*. The pictorial representations of the whole were varied from the fraction bars to various shapes with different equal parts. This enabled the students to understand that the language of fractions was constant through different shapes and sizes. As well, a new perspective was introduced, that of using two dice to generate the language of the number of equal parts, for example, 2 fifths. One dice carried the numbers one to ten, while the other dice carried fraction names corresponding with the fractions on the *Colour-me-fractions* sheet. In this way, the students were required to indicate the relationship between this elaborated language of fractions and a pictorial representation.

Most of the students made the transition relatively easily. However, initially Alex experienced difficulty with reading and extracting meaning from the language generated by two dice. His response was to read the dice back to front and to colour, for example, *fourths* instead of *sixths* for *4 sixths*. This response was consistent with the advice given previously by the Speech and Language Pathologist regarding students with language processing difficulties following multi-step procedures. Therefore, to assist him in separating the information into meaningful chunks, it was helpful to break down the information into single step operations.

1. How many sixths are there?

2. How many of them will you colour?

He also responded to 8 halves as *8 twoths*. This is interesting, in itself, because the written form that Alex saw was *8 halves*. So, his response indicated that he had made a cognitive connection between the term *halves* and two parts, but his expression of this idea reverted to unconventional language. Later in the day, Alex was able to justify his selection of parts.

Teacher *Where are the twelfths?*

Alex *Here.*

Teacher *How come you're so sure?*

Alex *Look. One two three four five six. One two three four five six.* (Indicates 2 sixes.)

Another response on the first day of using the elaborated language of fractions indicated an easy transition from naming parts to correctly identifying numbers of parts.

Bert *3 sevenths. There.* (Pointing to sevenths.) *Three of them.*

The next step in developing the concept of fractions was to use the same *Colour-me-fractions* procedure with different pictorial representation of fractions. This time, fraction bars were used but with two wholes divided into equal parts. This facilitated discussion about fractions of more than one whole, for example, 9 fifths. Therefore, when *8 sixths* was rolled, it could be identified and the students could make the relationship between the language and the pictorial representation themselves. Initial reactions were predictable. Alex rolled 9 thirds and coloured 3 ninths.

Teacher *Where are your thirds?*

Alex *Here.*

Teacher *They're not thirds. They're ninths. How many parts do thirds have?*

Alex *Three.*

Teacher *How many thirds have you got there?*

Alex *Three.*

Teacher *No, you've got more than three.*

Alex *Six.*

Teacher *Yes* (To the class.) *Alex has a problem. He rolled 9 thirds. Does he have 9 thirds? How many thirds has he got there?*

Alex *Six.*

Suzanne *You can't do that.*

The opportunity arose to introduce equivalent fractions but it was decided to delay it until the students were more comfortable with fractions of more than 1 whole and not to overload them with an excess of new information. At this point, they were becoming familiar with fractions of more than 1 whole and it was decided that requiring them to sort out equivalent fractions as well may have caused confusion.

Another requirement that arose from the game was to colour *1 whole*. It was not possible to roll *1 whole* using the dice. The students needed to discern that to fill one whole all the parts of the whole needed to be coloured, for example, 6 sixths.

Bert *We don't have it.* (Meaning 1 whole on the dice.)

Teacher *You can make it with other things.*

Bert *9 ninths. Oh yeah.*

Alex *Can we cross out 1 whole?*

Teacher *No. You have to roll it.*

Alex *There's no 1 whole* (on the dice).

Teacher *There's no 1 whole on there but you can make it another way. How? How do you think?*

Suzanne *I know how. 1 one.*

Teacher *There's no 1 one on there* (indicating the dice).

Suzanne *Um.*

Alex (After thinking for a while.) *9 ninths. Oh. 1 whole.*

Suzanne started to make the connection a little later.

Suzanne *8 eighths. How?*

Teacher *There's something else you can colour for 8 eighths.*

Suzanne points to the wholes.

Teacher *Yes.*

Suzanne *Both of them?*

Teacher *Just one. 8 eighths makes 1 whole.*

By playing this game and experiencing fractions in a concrete way, the students were able to work out for themselves how to account for the wholes in their game. It all occurred so naturally and without any fuss. They were able to make this cognitive leap because of the understanding they had previously developed through making and discussing equal parts.

At this point, it was pleasing to see Alex indicate enough confidence in himself and his ideas to question another student. This also created the opportunity to clarify his ideas through interactions with others. Bert had rolled 8 sixths.

Teacher *Is that more than 1 whole?*

Bert *Yes.*

Alex *No. Yes, it's more than one whole. You have to get eight and eight to make one whole* (means two wholes?).

Teacher *How many sixths make 1 whole?*

Alex *Six.*

Bert *6 sixths.*

Teacher *So you have more than 1 whole.*

Bert indicates to Rachel to colour 1 whole and two more sixths.

Although the students benefited from playing this game individually, there were many more interactions and the students reported increased enjoyment when it was played in teams. On one occasion, Alex and Bert were teamed together and Bert noticed that Alex had coloured *9 eighths* instead of *8 ninths*. Bert indicated the correct response and the game continued without incident. At another time, Alex rolled 5 eighths and Suzanne commented that she had rolled 5 eighths before. Bert asked where their eighths were and Alex showed him. Another opportunity for interactions arose when Bert reached for the dice from Suzanne. She asked him to wait because she was still considering what to do. Bert informed her that she had rolled zero so there was nothing she could do. It was easier for each student to follow what happened in the game when teams played. The students were interested in the other teams' responses and progress and this sparked discussion and requests for clarification.

At this stage of the project, the students were asked what they thought of playing mathematics games and the following discussion took place.

Alex	*Good.*
Teacher	*Why is it good?*
Suzanne	*Makes you clever and think.*
Alex	*And use your brain.*
Teacher	*How does it make you think?*
Alex	*Yes.*
Suzanne	*You have to think halves, thirds whatever. Make sure the right number. You have to make sure they're the right shape, size.*
Rachel	*Interesting.*

Bert	*Good. They make me think about things. Make sure it is right. Make sure how many are there. What is 9 ninths. 1 whole.*
Teacher	*Why is that important?*
Suzanne	*You have to put your thinking cap on before you start it.*

In this discussion, the students were bouncing ideas off each other and, in doing so, further developing their notions of fractions and what was significant about them. They were also experimenting with how best to express their thoughts, and through this process, clarifying their understanding. They also indicated that they found the games interesting and stimulating and, therefore, felt they were learning new things and were pleased by their progress.

After many experiences with fractions of more than one whole, it was time to introduce equivalent fractions. It was decided to introduce this notion by manipulating actual parts. Through this, it was hoped that by putting parts next to each other, the students would be able to generate the notion of equivalent fractions. *The Colour-me-fractions* sheet that had been used for fractions greater than one whole was enlarged so that the pieces were easy to manipulate for the students. They then cut the fraction bars into pieces and compared equivalent fractions. The students enjoyed this activity, discovering yet another aspect of fractions. The starting point was the familiar territory of parts making a whole then the students were invited to a challenge.

Teacher	*Bert, would you like to have a guess, how many fourths make one half?*
Bert	(There was a long silence while he looked at the pieces.) *Four.*
Teacher	*What do you think, Alex? How many ...* (Alex then answers.)
Alex	*4 fourths.*

Teacher	*Make 1 half not 1 whole.*
Alex	*Oh. Um.*
Teacher	(To Rachel.) *What do you think? How many* ... (Rachel then answers.)
Rachel	*Four.*
Alex	*Two?*
Teacher	*Check your pieces, Alex. Show me 1 half. Now how many fourths are the same as 1 half.*
Alex	*Four. Two.*
Teacher	*Show me. Show me 1 half. Now, show me the same in fourths. So how many fourths make 1 half?*
Bert	*Two.*

Similarly:

Teacher	*Rachel, have a guess. How many eighths will make 1 half?*
Rachel	*Four.*
Teacher	*How many eighths will make 1 fourth?*
Rachel	(Considers her answer first.) *Two.*

Nevertheless, initially, the different forms of language used to indicate equivalence caused some confusion. The language of *<u>How many</u> fourths <u>make</u> 1 whole?* was familiar to the students from previous discussions. However, the language of *<u>How many</u> fourths <u>is the same as</u> 1 whole?* caused some confusion with the students appearing to focus on *the same* rather than deriving meaning from the whole structure.

Teacher	*How many halves make one whole?*
Suzanne	*Two.*
Teacher	*How many halves are the same as one whole?*
Suzanne	*1 half.*

Alex	*Is that right what I just said?*
Teacher	*What did you say?*
Alex	*1 half.*
Suzanne	*2 halves.*
Teacher	*How many halves are the same as 1 whole? Have a look at them.*
Alex	*One.*
Teacher	*That's 1 half.*
Alex	*1 whole?*
Teacher	*What's the same as 1 whole? How many halves is the same as that?*
Alex	*Two.*
Teacher	*So, 2 halves make 1 whole. 2 halves is the same as 1 whole.*

Once the students were alert to look for equivalent fractions, the *Colour-me-fractions* game gathered new interest. It was positive that the students were content to play the *Colour-me-fractions* game many times in its various forms and maintain interest in it. Alex and Bert were teamed together and cooperated well. Alex rolled 7 halves and gave the dice to the other team. He was disappointed at the thought of being unable to make it. Finally, he and Bert were able to work out some equivalent fractions which meant progress in the game and a sense of satisfaction and achievement.

Alex	*Can't do it. One two three four.... Yes we can. 1 whole.*
Teacher	*How many halves is that, Alex?*
Alex	*One.*
Teacher	*No, there's more than one.*
Bert	*Two, four.*

Alex *Six, seven.*

Teacher *Good boy, good thinking.*

Alex *I told you we could do it.*

Teacher *So you've coloured in one two three four five six seven. Good.*

Alex (Whispers to himself.) *Yes.*

Bert used equivalent fractions successfully on an individual level.

Bert *8 fourth. That mean 1 whole.*

Teacher *8 fourths means more than 1 whole.*

Bert *2 whole.*

Teacher *Yes.*

The students were interested in each others' discoveries as they hoped to learn something that would give them an advantage when playing the game. The teacher rolled 7 fourths. Rachel checked the fourths fraction bars.

Rachel (The fourths are) *Full.*

Teacher *What about eighths? 7 fourths is the same as how many eighths?*

Rachel (Checks the fraction bars for some time.) *Fourteen.*

Teacher *14 eighths.* (To the whole group.) *She just found out that 7 fourths is the same as 14 eighths.*

Bert *The same as 14 eighths. Do you know that?*

Teacher *She can show you.*

Alex *Yeah. I can see it.*

Teacher *Can you?*

Alex *Yeah.*

Teacher *Are you sure she's right?*

Alex *Of course.*

Teacher *You'd better check it.*

Alex *One two three four five six seven eight nine ten eleven twelve thirteen fourteen.*

Alex was counting the number of eighths that equal 7 fourths as he looked at the fraction bars. This indicates that he was coming to grips with the idea of equivalent fractions. However, when required to indicate this information in reverse, the exact number of fourths eludes him.

Teacher *So, how many fourths is that the same as?*

Alex *Eight.*

Teacher *Seven.*

A number of developments can be observed in this small exchange. Initially, Rachel accepted that the fourths were full so no move could be made in the game. It only took the suggestion that an equivalent fraction be found in *eighths* for her determine that *14 eighths* is the same as *7 fourths*. Bert was surprised at her discovery and this drew Alex into the conversation. He, then, was asked to indicate his understanding of the relationship between them through the pictorial representation. This series of discoveries related to fractions of more than *1 whole* but pictorial representations used in the game facilitated this process.

Different ways of experiencing the same content were provided through two games. The first was a *Concentration* game where the students needed to match the pictorial representation to the written language and the symbols. The same cards were used for a *Fish* game where the students had to request matching cards from other members of the group. *Fish* promoted more interactions and mathematical discussion because the game required students to request and sometimes explain what they wanted. However, it also provided the opportunity to cheat (which was done in generous proportions).

Alex	*Do you have 4 fourths?*
Suzanne	*What?*
Alex	*4 fourths.* (Signs slowly and carefully.)
Suzanne	*I don't have 4 fourths.*
Alex	*Yes.*
Suzanne	*No.*
Alex	*Miss Louth, hasn't Suzanne got 4 fourths?* (Holds up his card that has 8 eighths. Also, Alex knew Suzanne had 8 eighths because he had already checked!)
Teacher	*That's not 4 fourths.*
Alex	*Oh.* (Looks embarrassed.)
Teacher	*What's that? What do you want? Do you want that or 4 fourths?*
Alex	*I thought it was 4 fourths.*
Teacher	*What is it?*
Alex	*8 eighths.*

In this situation, we can observe that one student has to decide what to request and make that request. The other student must understand the request, search through their cards and decide if they have the nominated card. Discussion and information processing is facilitated by the format of the game. On the other hand, *Concentration* could be played alone without the need to communicate meaning to another student. As well, because it involves requests and the presumption of honesty, *Fish* creates opportunities for students to deny having cards already in their hand as well as looking at other students' cards and discovering where needed cards are held. Although this move backfired on Alex on this occasion, it was impossible to convince some of the students to play this game honestly. The students preferred to play *Concentration* where they were only required to say what they had turned up. Any mathematical discussion usually stopped there.

As the project continued, the students were becoming comfortable and more skilful at justifying and describing.

Teacher *What does that mean?*

Children *4 tenths.*

Teacher *That means four pieces out of ten. Four pieces out of ten pieces.*

Bert *That means four pieces are colour(ed).*

Teacher *What does that say?*

Rachel *2 eight(hs).*

Teacher *What do you think that means?*

Rachel *Two colour in eight.*

Teacher *What does that mean?*

Bert *Seven pieces are coloured.*

Teacher *Out of how many?*

Bert *Out of seven.*

Alex was self correcting. He had asked for 2 tenths.

Alex *I asked for 2 tenths. I was supposed to ask for 3 tenths. Look, this one there.*

The next step in the project was comparing fractions. This was difficult for the students because they did not appear to have a solid grasp of the terms *greater than, less than, equal to* in their general mathematics performance. The *Numeration Profiles* of Rachel and Suzanne indicated problems in this area. A game which required the students to compare fractions was used. The students rolled a dice containing comparative statements, for example, *more than 1 third*, and they had to match it with a pictorial fraction on their individual sheet. It required a two step process — first interpreting what was on the dice and then interpreting that information in terms of a picture. An extra complication to this game was that there was no answer to some of the questions and this added to the students' difficulties. For example, there was no fraction matching *less than 1 fourth*. What had once appeared to be understood by most of the students, that is, the language of fractions, 1 fourth, was now a source of confusion in this context.

Bert rolled less than 1 fourth.

Teacher *What does that mean?* (No reply.) *What does that mean?* (No reply.) *Bigger than 1 fourth?*

Bert *Smaller.*

Teacher *Yes, smaller than 1 fourth.*

Bert points to 3 fourths.

Teacher *This? See the white part there, that's 1 fourth.*

Bert points to another 3 fourths.

Teacher *That's 3 fourths. Is that less?*

Bert (After thinking for a long time.) *No.*

Suzanne's responses reflected the general difficulty experienced by the group. Rachel had rolled less than 1 fourth.

Teacher *What does that mean?*

Rachel *Less than 1 fourth.*

Teacher (To Suzanne.) *Can you help her? What does less than 1 fourth mean?*

Rachel *Less three whole. Three off.*

Teacher *What does less than 1 third mean?*

Rachel *No more off.* (Shrugs.)

Teacher (To Suzanne.) *Listen to my question. What does less than 1 third mean?*

Suzanne *Mean ... not ... um ...can't 3 third.*

Teacher *What does less than mean?*

Suzanne *I don't know any more.*

Suzanne's comments about this game were: *Boring because I can't make it.* It was not good for her to feel inadequate when doing mathematical activities. On a more positive note, some progress appeared the following day. Suzanne had rolled *greater than 1 fourth* and Rachel signed it.

Teacher *What does that mean?*

Rachel *More than.*

Suzanne *Bigger.*

Teacher *Good.*

Rachel *More than 1 fourth.*

The final part of this project was introducing fractions of a group. This was done using a card game. The cards contained pictures of groups of objects and animals with some of them coloured to make the fraction. For example, one card had a picture of five cows and three of them were coloured. This represented 3 fifths. It was anticipated that the students would experience some difficulty making the transition from fractions of wholes to fractions of a group. However, this was not the case. The students effortlessly made the transition. This was possible because they had developed a good understanding of fractions from their experiences and built up an acceptable level of expertise using the appropriate language to describe and manipulate fractions.

The first discussion that occurred when fractions of a group was introduced was as follows.

Teacher *Look at that picture.*

Rachel *4 sixth*

Teacher *Good girl. Excellent. How many frogs are there?*

Class *Six.*

Teacher *How many frogs are coloured?*

Bert *Four.*

Teacher *So, 4 sixths are coloured. Four out of six. Tell me about the cows.*

Bert *There are four cows and there are two coloured. 2 fourths.*

Teacher *2 fourths of the cows are coloured. Find it for me* (meaning the matching cards of symbols and words).

The appropriate cards were found. The teacher then checked with Alex to ascertain that he was not confused by the variation to our previous work. He quickly and correctly identified the fraction of a group of calves as *1 third.* Therefore, the cards were used for a *Concentration* game where the students were required to match three cards for each fraction, that is, the picture, the symbol and the written language.

Although the students preferred to play *Concentration*, there was no check to ensure that what Alex was conceptualising and requesting was accurate. The discrepancies in signing fractions were still apparent. The students preferred to use the sign for the written symbol rather than the appropriate language and this signing was often inaccurate, for example, *1 five* for *1 fifth.* However, it must be said that the students understood the communication whether it was signed correctly or not. On this occasion, Alex was using the appropriate language and also signing it most consistently. However, his difficulties in matching the language to the meaning surfaced again.

Alex *Do you have 7 sixths?*

Teacher *Seven what?*

Alex *7 sixths.*

Teacher *That's not quite right. Have another look.*

Alex *Oh, 6 sevenths.*

Teacher *Yes, 6 sevenths.*

The students concentrated well on the game and Bert was able to manage a situation that, at one point, would have angered him.

Bert *My turn. Do you have 6 eighths?*

Suzanne *No.*

Bert *You do have 6 eighths.*

Suzanne (Checks her cards again.) *No.* (Then smiles and gives Bert the card.)

Alex managed to relax and to play with language which was positive after his general difficulties with language.

Alex *I've got 3 thirds (meaning three sets of matching cards).*

Teacher *You mean 3 threes.*

It was then discovered that Rachel has 5 threes.

Alex *Well, I've got 4 fours.*

The last day of the project came after two weeks school holidays and it was used to observe what the students had retained over that time. The students were very settled and focussed. They watched, listened and followed the game of *Fish* intently. It used the set of cards which illustrated fractions of a group. On two occasions, students spoke their requests without signing and Rachel immediately asked to be told what was said. This indicated that she wanted to be involved in the game and valued the information the other students were exchanging. The first game was played using only the picture and the symbol. The students were consistent with their strengths and weaknesses from before the holidays. They had no difficulty remembering the work and showed every sign of solid understanding. However, the inconsistencies with the signing of fractions continued despite a reminder from the teacher. On a lighter note, Alex commented that Bert should be the *Fish* queen because he had won the game with the most pairs. This was a reference to a previous day when Rachel had been nominated the *Concentration* queen because she had done so well. This fun with language was encouraging from a number of points of view. Firstly, it involved remembering a passing comment from the day before. Secondly, it was an appropriate comment from one twelve year old boy to another. As well, it involved playing with language which Alex rarely did. Also, it showed that he was relaxed and confident in himself and the work that was being done.

For the next game, cards with the written language were substituted for the cards with the symbols. Therefore this time the students had to match a picture with the written language. It was observed that although some inconsistencies in signing continued, accurate signing of the fractions increased. It was possible that the students used appropriate language when it was presented to them on the card and continued to use inaccurate signing when presented with the picture. However, the amount of accurate signing of the fractions increased and the correct language was reinforced. It was also true that on a number of occasions, Suzanne, Bert and Rachel said the correct fraction when their sign was inaccurate.

Throughout the project, the students were continually required to explain their thinking and to justify their answers. Initially, they were very uncomfortable with this procedure and interpreted it as an indication of an incorrect answer, and so promptly changed their responses.

Alex	*Equal. Unequal. Equal.*
Teacher	*What do you think?*
Alex	*Unequal.*
Teacher	*Why?*
Alex	*Equal, I mean.*

However, after some time, they realised that they would be asked to justify their answers when they were correct and incorrect and it became accepted as routine.

The use of games within the project

The use of educational games is certainly nothing new. Plato wrestled with the notion of making learning fun and amusing through games. In more recent times, the focus of literature describing mathematical games has shifted from proposing their use to analysis of the outcomes and types of games. Oldfield (1991) discusses various categories of mathematical games. He defines twelve different types ranging from puzzle games to games which practise skills, from games requiring the use of strategies to multicultural games, from mental games to computer games. In the context of this project, the following types of games were used. The game using the blocking strategy was a puzzle game. The various *Colour me fractions* games were to reinforce concepts. The track games, the bingo type game and the *Fish* and *Concentration* games were to practise skills. The most effective and enjoyable version of the *Colour me fractions* game was played in a collaborative form. An additional aim of all the games is the encouragement of mathematical discussion. Finally, the students' competitive streaks contributed to the success of the game format. A number of different types of games and activities was used. The notion of fractions was introduced through dividing up items of food in preparation for a party. The students were interested to ensure that all pieces were equal for fear of receiving the smaller portion themselves when the food was apportioned. Following this, they played games that required them to identify equal and unequal parts before they could move through a track game or participate in a strategy game of placing blocks on a grid. The game moves were not related to the idea of equal and unequal parts but provided the motivation for the students to participate in the activity of identifying the pieces as well as repeating and persisting with what could become a monotonous activity. The next step in the project involved naming parts of a whole and this was achieved through similar kinds of activities. The students were required to name the parts and then were able to able to participate in track games. The students were happy to participate in these activities and barely noticed that they were practising mathematics while their interest focused on the progress of the games.

In addition to discussing the outcomes of the project in terms of the mathematical understanding that was attempted and achieved and each student's participation in the project, another perspective emerges. It is significant that the mathematical outcomes of the project were achieved through the use of games. There was almost no "instruction" in the traditional sense. All understanding of the notion of fractions was constructed through the use of games and activities and associated discussion. Therefore, it can be seen that students who are deaf or hard of hearing can manage their own learning without the traditional varieties of teacher intervention. It has also been shown that the alternative role of mathematics teacher as planner and experienced guide has much to offer these students.

The next step in developing an understanding of fractions involved identifying parts from the written name. In this case, the game was directly related to the development of the mathematical concept as the students were required to roll a dice containing the names of fractions and then to find and colour the appropriate fractions on a sheet. Following this, the students rolled two dice and generated the fraction name and the number of parts, for example, *3 fifths*. They then identified and coloured the fraction they had rolled. It was during this game that the concept of equivalent fractions emerged as the students needed to fill *1 whole* but it was not represented on the dice. They also became aware, through playing this game, that another fraction was the same size as the one they had rolled and, hence, could be used instead, thus speeding their progress through the game. This understanding grew out of a need to participate in the game and was directly related to the content of the game. These games could be played individually or in teams. The students communicated that they enjoyed team games the most and it created the best opportunities for sharing mathematical knowledge as well as for discussion and justification of mathematical thinking. The students not only watched their own team's progress, they also watched the opposition as much as possible and did not hesitate to request justification when they suspected foul play or a mistake.

The concept of comparing fractions was developed through a board game where a card was turned up and a box on the board was matched to the information on the card. For example, the player was required to identify a fraction on the board that was *less than 1 half* and place a block on the pictorial representation of it. This was a form of *Bingo* as the player with three blocks in a row was the winner. This game was directly related to the mathematical concepts to be explored.

The symbolic representation of fractions was introduced and developed through card games that required the students to match the symbolic, pictorial and written representations of fractions. The cards could be used for *Fish* and *Concentration.* The format of *Fish* encouraged greater use of mathematical language and provided more opportunities for interactions, but both games provided a challenge for the students to make the connections between these formats and the signed or spoken language.

Through the variety of games and activities used in the project, the students maintained interest in and developed a basic understanding of the notion of fractions. The games were devised to meet specific instructional goals and, at the same time, they made the learning process enjoyable and boosted self-esteem. They also provided a vehicle for the teacher to gain insights into how the students were thinking and to contribute to the construction of their understanding through discussion related to the games and activities.

Of some interest were the interactions that occurred within the project. The transcripts show many interactions and interchanges of a mathematical nature. Students questioned, answered and commented on what was happening around them. Each student, in his own way and at his own pace, was involved in and contributed to discussions about mathematics. Each student was involved in interactions of a mathematical nature with other students. However, it is significant to note that a large number of mathematical interactions was between a student and the teacher. As illustrated in the student profiles, there were only a few interactions between students which were of a purely mathematical nature.

This was not an outcome expected from a program centred around games and activities. The program using games was designed to provide experiences for developing an understanding of fractions, to motivate students to be involved in the learning process, to make the learning process enjoyable and positive and, also, to facilitate mathematical discussion between students. In the context of this project, there was a number of interactions between students but most of these interactions related to procedural matters such as instructions written on track games, which colour pen to use, accusations of cheating or whose turn it was.

Ernest (1986) contends that the advantages of playing games in mathematics are that they generate enthusiasm, excitement, total involvement and enjoyment. He also highlights the need for students to discuss mathematics and proposes that games encourage discussion between groups of children and also between pupil and teacher. The experience of this project is that all of the above outcomes occurred except for mathematical discussion between students. On looking through the transcripts of interactions from each student, it is very clear that most of the mathematical discussions were between one student and the teacher, not between students. The students interacted on a mathematical basis especially when playing in teams, but it was usually a single sentence contribution rather than a discussion. For example, when playing the *Colour me fractions* game, Rachel rolled *6 thirds* and then pointed to the appropriate fraction bars for Bert to colour or, after Rachel rolled *6 fifths*, Bert commented that it was *1 whole and one*. Although this type of contribution is an indication of understanding, it does not constitute discussion. Students were more likely to engage in discussion in team situations and the *Colour me fractions* game provided the best opportunities for this as is seen in this exchange between Bert and Alex. Alex had rolled 9 tenths:

Alex *9 tenth. Where is that?*

Alex and Bert search together and Alex finds it.

Alex *Oo, oo.*

Bert *Yes.*

They both push to colour it.

Bert *I will.*

Alex lifts the pen from Bert's hand.

Bert *Why?*

Alex indicates another colour pen.

Bert *Alright. You.*

But even on this occasion, the discussion that took place focussed more on the choice of pen rather than the mathematics involved. It is also salient to note two students who are capable of using much more complex language, communicating in one and two word utterances. The reason for this is unclear.

On another occasion, Rachel and Suzanne were playing *Colour me fractions* as a team. Rachel noticed that Suzanne had made a mistake when attempting to colour *9 sevenths*.

Rachel *9 sevenths*

Suzanne *Where is nine?* (And starts to colour.)

Rachel *No, 9 sevenths.*

Suzanne *Yeah 9.* (Pointing to the ninths.)

Rachel *9 seven*(ths).

Suzanne *I don't know what you're talking about.*

Rachel	(To teacher.) *I said 9 seven*(ths). *She colour there, not there.*
Teacher	*Where are the sevenths?*
Suzanne	*There.*
Teacher	*So you have to colour nine of them.*
Suzanne	*Nine?*
Teacher	*How many sevenths in one whole?*
Suzanne	*Seven.*
Teacher	*And how many more will you colour?*
Suzanne	*Seven, eight, nine.* [indicating the correct response on the sheet].
Teacher	*That's right. Good girl.*

In this incident, there was a short discussion when Rachel attempted to clarify the situation with Suzanne who failed to follow what she was trying to explain. This discussion terminated with the students looking to the teacher to intervene because they had not been able to agree on the meaning of 9 sevenths.

The reasons for the scarcity of discussion are open to speculation. Is it that these students lacked experience in mathematical justification and reflection and, hence, the skills of discussing mathematics? It is likely that this may well have been a significant factor in the scarcity of mathematical discussion as these students were accustomed to manipulating materials, but in response to teacher directions and not as a stimulus to discussion. Is it that diminished mathematical interactions are associated with discussing abstract concepts within the framework of delayed language development? This is also a possibility as none of the students had age-appropriate language development. According to language testing carried out by the Speech and Language Pathologist, none of the children functioned above a seven or an eight year old level of language development. Was student discussion stifled by inappropriate levels of teacher control? This query was investigated in some depth.

The issue of teacher control is not a new one among teachers of the deaf. As noted before in the early 1980s, Wood and his associates in England were concerned at the level of teacher control exercised by teachers of the deaf during conversations with their students. Consequently, they developed a system for analysing and measuring the relationship between teacher moves and student responses. They were able to do this by relating student responses to teacher participation in conversation. The analysis syntactically coded teacher moves and assessed the associated student responses. In this way, the researchers built up a picture of the amount of teacher control and the ways in which that control was exercised. Using this measure, they were able to show a relationship between certain forms of questioning and reduced student participation. "As teacher control over the conversation increases, pupil initiative and length of turn decrease and pupil misunderstanding increases" (Power, Wood & Wood, 1990, p. 12). In parallel with Wood and Wood in England, Power compiled a similar study of control levels of teachers of the deaf in Australia. The findings of these studies (Power et al., 1990) show little difference between the levels of control of teachers of the deaf in both countries and also provide benchmarks for comparison.

Wood and Wood (1984) analysed conversations between students who were deaf and their teachers. They were particularly interested in the role of teachers in such conversations and the levels of control they exerted on these conversations by the contributions they made. They generated five levels of control ranging from very high to low levels of control. Teacher utterances were classified according to the following categories.

1. Those that required the student to repeat what the teacher had said.
2. Two choice questions.
3. Wh-type questions.
4. Personal contributions.
5. Phatics.

They found that enforced repetition was the highest level of control exerted by a teacher and, by using more phatics, teachers exerted the lowest levels of control. They found that when teachers used enforced repetition, students did not have to generate a response or think for themselves. When teachers used two choice questions, the students rarely elaborate their responses. Wh-questions produce similar responses from students, rarely eliciting elaboration or changes in the direction of the conversation. On the other hand, the use of a personal contribution in a conversation such as "Oh, did you?", leaves students feeling free to continue the conversation or to ask questions and show initiative. Finally, when a teacher uses phatics, such as "Great", a student is more likely to change the direction of the conversation. Thus, Wood and Wood developed a tool for analysing and calculating the amount of control teachers exert over conversations with their students. It must be emphasised that this is a tool for analysing conversations and the findings of Wood and Wood in England and Power in Australia are based on analysing conversations and not instructional situations such as this research. It could be expected that interactions of an instructional nature would result in higher levels of teacher control because of the nature of the role of the teacher.

Although the interactions in this project were in an instructional situation and not a conversation, when the consideration of teacher control affecting students' interactions arose, it was decided to use the Wood method of analysis to try to find some indication as to whether the teacher in this particular project was excessively controlling and hence interfering with the students' learning in a program based on constructivist principles. The decision was made on the basis that this tool was designed specifically to code interactions between teachers of the deaf and their students and it would be useful to see the outcomes of an instructional situation where more control was expected than in conversations. Thirty consecutive pairs of utterances were linked and analysed according to Wood's *Moves Matrix* (Wood, 1985). Within the context of this project, teacher control was found to be at 50%. On comparison with the findings from the Australian situation, the results of the matrix indicated the level of power moves to be within the medium range and slightly lower than Australian counterparts in conversations (56%). Student initiative, which was rated at 53%, was also within the medium range and slightly lower than the findings with Australian students in conversations (57%).

These results closely paralleled levels of control for conversation but were in an instructional situation where a higher level of teacher control would be expected. Hence it appeared that the teacher was not over-controlling interactions to the point where they stifled student initiatives. Wood and Wood (1984, p. 56) acknowledge the role of teacher control in didactic situations. "Thus, in situations where ... a teacher needs to ... lead children through a difficult line of reasoning ... then question after question will keep the focus of power in the hands of the person best qualified to wield it." Because, in the framework of this project, the level of teacher control in an instructional situation approximated an acceptable level of control in a conversational context, it is considered that teacher control was unlikely to be a pivotal factor in the low level of mathematical discussion among students and was not responsible for the low number of student interactions.

It seems likely that high levels of teacher control would reduce the effectiveness of the use of games. Within a constructivist framework, games provide a social context for learning, thus encouraging students to cooperate, discuss and reason mathematically. This would not be enhanced by high levels of teacher control. Another proposed outcome of games is for enjoyment of mathematics, which again would not be promoted by high levels of teacher control. Students need room to make mistakes and to enjoy themselves without teacher interference. Oldfield (1991) has found that "the value of many mathematical games can be considerably enhanced by suitable questioning" (p. 9). He continues to say that, if questioned in a certain way, students will find themselves not only answering questions but providing reasons for their answers. This would seem to indicate that enforced repetitions and two choice questions, and so high levels of teacher control, do not have a place in the use of games in mathematics education.

The differences of preferred modes of communication among students may have interfered with the development of spontaneous discussion across the group. Alex was a reluctant user of Signed English, Bert was often happy to speak without signing and could follow some communication without Signed English, Suzanne did not require signing and was more proficient in Auslan than Signed English, while any signing was Rachel's communicative lifeline. This cocktail of communication preferences created a complex situation in which to foster an exacting skill such as mathematical discussion. However, as noted previously, this complexity was dealt with by the use of Total Communication, i.e., signing and speaking , in order to meet the needs of the students in this group.

Another important aspect in this enquiry into the type of interactions that occurred in the project is that of "dual attention" as previously outlined (Wood et al., 1986). When signed or spoken communication occurred, the students had to choose where to focus their attention, on the signer/speaker or on the materials being discussed. This is a cognitively challenging task as they had to make the semantic connection between two things separated in time and space, that is, what was said and the materials being discussed. As well, when a student wanted to contribute to the discussion or make a comment, they were only understood by those who were watching if the message was signed or those who could hear if they spoke without signing. These were not ideal conditions for spontaneous sharing of information which was challenging in itself. To minimise the effects of this situation, certain factors must come into play. Firstly, there must be awareness of student needs in this regard by the teacher who can modify communication style by giving students time to shift attention from communicator to referent, thus accommodating this special need. Secondly, students themselves have a role to play by similarly modifying their communication style for their peers and, thirdly, by taking responsibility for watching appropriately when important matters are being discussed.

Further consideration in the focus on interactions within the project is the fact that each of the students had very different needs in terms of support from the teacher. Alex and Suzanne required careful questioning to assist them to develop communication preferences created a complex situation in which to foster an exacting skill such as mathematical discussion. However, as noted previously, this complexity was dealt with by the use of Total Communication, i.e., signing and speaking , in order to meet the needs of the students in this group.

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Further consideration in the focus on interactions within the project is the fact that each of the students had very different needs in terms of support from the teacher. Alex and Suzanne required careful questioning to assist them to develop understanding of the content of the project. Rachel benefitted from encouragement to contribute her ideas to the project, while Bert responded readily to challenges put to him. Each student had significant individual needs related to mathematics and learning in general. This is not surprising considering their placement at a Special Education Unit. Therefore much of the assistance offered to Alex and Suzanne was of no interest to Bert and Rachel. Equally significant is the fact that when the teacher was interacting with one student, other students were focussed on the game and three of them could not access a conversation at the same time whether it was oral or visual in nature.

These students had all experienced the use of games as part of their mathematics program previously but the games had been used more as a vehicle for drill and reinforcement than to develop the concepts of mathematics. In this project, the development of the notion of fractions occurred through the games themselves with little instruction arising outside that framework. This was a departure from their usual mode of instruction and learning and it may be that, with more prolonged use of instructional games and the expectation of mathematical discussion, these students would become more skilled in discussing mathematics and attending to each other.

Summary

This project focussed on introducing fractions to a group of four deaf or hard of hearing students in Years 6 and 7 at a Special Education Unit in Queensland who were selected to remain in the unit for their mathematics program while an equal number of their peers went to hearing classes for their mathematics program. These students had been judged as requiring an alternative program in mathematics. Because of their specific needs, it was important to use an approach which placed a heavy emphasis on experiences generating meaning and appropriate language. The approach selected was developed at Griffith University and had already been trialled with hearing students. Central to the approach was the use of games to provide the experiences and generate the language to support an understanding of the notion of fractions.

The group of students in the study was heterogeneous and an unlikely combination of hearing impairment and preferred modes of communication for this setting. This created some difficulties in sharing information equally among the students. One of the students relied totally on signed communication, one student followed and used some oral/aural communication but still relied on signed communication, while two of the students did not require any signed communication. Unfortunately, sometimes this resulted in the signing only student not being part of all communication as the other students engaged in spoken conversations and messages. This was significant for two reasons. Firstly, it isolated this student socially at times. Secondly, she missed some exchanges of information which were valuable in constructing the notion of fractions.

The students were introduced to the concept of equal parts. The value of focussing on equal parts became evident when students initially experienced some difficulty with the idea. Experiencing equal parts first centred on the experience of dividing items of food into equal parts and then moved to a regional model of fractions. With the added experiences of games and classifying fractions, equal parts were understood. Prior to recording fraction names, the students were immersed in language and experiences of fractions to establish a relationship between the two. Once the students saw the pattern of fraction names, they were able to generate the regular fraction names themselves. They had all met halves before so it required little introduction except to stress the importance of equal parts. The names of the fractions are consistent with ordinal numbers except for halves so it was not difficult to make the transition from signed or spoken language to written language such as *2 thirds*. It was not long before fraction names became automatic for all the students except for the student with the least significant hearing impairment. His responses were inconsistent to the end.

Relating written fraction names to regional models and also the spoken and signed language already in use was done incidentally through the different versions of the *Colour-me-fractions* game. The students had to match the language rolled on a dice to regional models on game sheets. At first, the students simply had to match the fraction name, for example, *fifths*, to a regional model.

Following this, they were required to match the language describing the number of parts, for example, *2 fourths*, to the regional model. During this activity, they demonstrated an understanding of what constitutes a *whole*. The dice used in these games did not contain *1 whole*: therefore, to fill *1 whole*, the students needed to roll *4 fourths, 5 fifths, 6 sixths* and so on.

The next step was to develop an idea of equivalent fractions. Although the word *equivalent* was not used at this point, the students were able to understand that some fractions were the same when they were compared. Just as the language *4 fourths makes 1 whole* was used to indicate parts making a whole, so the language *2 fourths makes 1 half* was used to describe the idea of equivalent fractions. This concept added new interest to the *Colour-me-fractions* game as the students realised there was another means of quickly filling their sheet and, thus, winning the game. They also watched each other's moves intently as they kept a check on each other and tried to learn from each other. This aspect of the project promoted more discussion, interest and questioning than any other.

The introduction of the fraction symbol, for example, 2/3, developed as a vexed question. The students had been using signed and spoken language to describe fractions in the same manner as hearing students. It is suggested that this continues for a greater period of time to secure the relationship between appropriate language and the concept of fractions. However, an additional option exists for students who sign to represent the symbol in a signed form. It is quicker and easier to sign the symbol than to sign the words in the appropriate language. When the written symbol was introduced into the program, the students were shown how to sign the symbol and reminded of the language to describe the symbol. When presented with these options, the students eagerly adopted the first, but, in some cases, it resulted in the use of inappropriate language, for example, *3 five* instead of *3 fifths*.

The students experienced some difficulties when comparing fractions. The terms *less than* and *greater than* created some difficulty. The students were improving in their mastery of this aspect of fractions but would have benefitted from more experiences. They were beginning to identify fractions that were *equal to, greater than* and *less than* and they were explaining their answers.

The final step in the project was considering fractions of a group. The students made the transition from fractions of a whole to fractions of a group without any effort. They required almost no assistance from the teacher and made the conceptual leap themselves. They also retained this understanding over a two week holiday and returned to the games as if they had continued playing them.

At the completion of the project, the students understood that fractions were made up of equal parts, the number of which could change. They understood the conventions of fraction names, although some of the students were not consistent in using them expressively. The students showed that they understood and manipulated fractions that were more and less than one whole. They were intrigued by the idea of equivalent fractions and attempted to identify them. They also grasped the concept of fractions of a group and manipulated them without difficulty. The project provided a motivating and effective approach for teaching a basic understanding of fractions. It is worthwhile to attempt to relate the outcomes of this project with outcomes of other programs teaching fractions to students who are deaf.

A summary of the fractions program and the associated games is in Table 1.

Sequence of learning outcomes	**Activities/games**
• Experiencing equal parts and naming equal parts.	Dividing food.
• Contrasting equal and unequal parts.	*Equal Parts Bingo* and *Concentration.*
• Consolidating equal parts and naming equal parts.	*Grand Prix* and *Garfield Rules.*
• Relating pictorial representation (fraction bars) to the written form e.g., *sixths.*	*Colour-me-fractions* (1&2) using one dice.
• Relating the pictorial representation (different shapes) to the written form.	*Colour-me-fractions* (3) using one dice.
• Developing the understanding and language for the number of equal parts, e.g., *4 sevenths*, relating the pictorial representation to the written form.	*Colour-me-fractions* (3) using two dice.
• Developing the understanding and language for parts making a whole *(6 sixths)* and parts making more than one whole *(9 fifths).*	*Colour-me-fractions* (4&5) using two dice, *Concentration, Fish.*
• Developing the understanding and language for equivalent fractions.	*Colour-me-fractions* (4&5) using two dice.
• Introducing the symbolic representation of fractions, e.g., *2/3.*	*Concentration, Fish.*
• Comparing fractions.	*Bingo.*
• Introducing fractions of a group.	*Concentration, Fish.*

Comparison with other programs

So, how does this program compare with other programs for teaching fractions to students who are deaf? Some research which outlined a program for teaching fractions to students who are deaf, *A Fractions Curriculum for Deaf Children* was conducted by Silva (1986). This program used fraction bars to introduce the notion of parts to the students and, strangely, avoided using the term *whole* by substituting the term *gzorke*. So the students learned about a *half of a gzorke* and *a fourth of a gzorke*. The program quickly moved into operations with fractions and did not appear to allocate much time to the basic understanding of fractions. As the author stated: "A fractions curriculum is very challenging to teach." If this is so, it is difficult to understand why the term *gzorke* was used. There appeared to be a great chance of confusing the students or overloading them with such an obscure and meaningless term. The author suggested that the entire program of understanding fractions and operations with fractions would easily be covered in three to four weeks with students who were nine to eleven years old and deaf. There is the suspicion that a true understanding of fractions is difficult to achieve in the absence of many in-depth experiences of the various aspects of the fraction concept and, without this understanding, successfully completing operations on fractions would be impossible. According to the author, a test, which is not described, was given to the students. "The test included problems of all types" and the students made very few errors which did not include the usual errors of adding top and bottom numbers when given fractions to add. The author was happy with the results of the program and recommended it to other teachers. The experience of the present project was that the students needed at least four weeks to come to grips with an understanding of the notion of fractions without any attention to computation with fractions. Therefore, it is difficult to accept how a program of teaching fractions which actually steers the students away from the important notion of wholes and parts of a whole has merit.

Other information about current programs for teaching fractions to students who are deaf was obtained from discussion with teachers and asking them what they considered a satisfactory level of understanding of fractions for students leaving primary school. From this, two things became apparent. Firstly, it was very difficult, almost impossible, to define an average student who was deaf and, hence, an average level of mastery of fractions for students who are deaf. Each student has such individual and special needs that every time an "average" student was identified, some particular characteristic prompted uncertainty as to whether the student in question was "average". The second result of the conversations indicated that the approach to fractions had been in a practical, "life skills" mode. As a basic expectation, if students had been able to identify and use in different situations (e.g., cooking or woodwork), one half, one quarter and one tenth, then teachers were content with the students' proficiency in fractions. This approach engenders concerns about the actual level of understanding of the notion of fractions the students developed. It is a very high expectation to require a student to use fractions in a practical situation without a true understanding of what fractions mean.

Therefore, it is felt that a program such as implemented in this project promotes an understanding of the nature of fractions. Once this has been achieved, using fractions in practical situations and computation with fractions is based on understanding and is much more meaningful for the students and success is more likely to be achieved.

The students participated well in the project and indicated a solid understanding of the aspects of fractions that were covered at the completion of the twenty days. Despite their impending teenage years and a customary perception that everything was "boring", the students participated enthusiastically in the games and activities. The videos revealed a group of students who were, generally, focussed, interested and actively involved in learning. Their comments imparted a positive attitude to the fraction work as well as to themselves and their expertise in mathematics. Motivation was never an issue as the students happily involved themselves in the games and activities. Alex, who had experienced more failure than most, was the keenest. In fact, one of the most notable achievements of the project was that Alex enjoyed mathematics, experienced success and disclosed that he felt good about himself.

In the light of this information, the outcomes of the project were encouraging. Within approximately twenty-five hours, students who were considered slow developers in a mathematical sense, understood and used fractions in a practical and a useful way. Each of them participated at his/her own level, according to his/her own skills and preferred mode of communication. Each of them, in general terms, enjoyed the project, felt good about his/her personal mathematical outcomes and his/her ability to understand the new concepts presented to them. In these terms, then, the approach of presenting appropriate and meaningful experiences with conventional language of mathematics has been successful, in some small way, in revealing to students difficult mathematical concepts and personal accomplishments.

CHAPTER 5

Implications of the study

Although students who are deaf or hard of hearing are characterised by many of the same attributes as their hearing peers there are also fundamental differences between them. Similarly, although students who are deaf or hard of hearing have been found to develop mathematical thinking in much the same way as hearing students, they also have specific needs with regard to learning and understanding mathematics. Because of their special needs, students who are deaf or hard of hearing depend on the implementation of an appropriate program using strategies that meet these special needs, and while the use of appropriate language is an important aspect of developing mathematical understanding for all students, it is crucial for students who are deaf or hard of hearing.

The program of teaching fractions implemented in this study was first used with hearing students and found to be successful (Booker, 1996). In this study, it was used to in the search for an approach to teaching mathematics to students who were deaf or hard of hearing which achieved understanding as well as the development of meaningful language to build and describe such understanding. Current constructivist approaches to mathematics education appeared to offer a sound basis for this endeavour.

General mathematical outcomes

In terms of the quest for an effective approach to develop mathematical understanding for students who are deaf or hard of hearing, there were a number of optimistic findings. The students evidenced understanding of the mathematical content through the their use of appropriate language in the activities of the program. The activities and games were successful at different levels. The learning process began at a concrete level, with initial activities of dividing wholes into equal parts and naming them. The activities and games then moved to another level by providing reinforcement through pictorial representation at a semi-abstract level through card games and the *Colour-me-fractions* games. Only then were symbols introduced so that the students could recognise and fully interpret the meaning of fraction symbols and associate a symbol with the correct pictorial representation. Games were used to maintain the students' interest in practising and reinforcing previously learned content. As reported in Chapter 4, there was a number of examples of the development of mathematical understanding as the students were able to learn from the games and activities, such as the notions of fractions as equal parts of a whole and the idea of a whole, the naming of fractions, fractions of more than a whole, equivalent fractions and fractions of a group.

The value of an approach such as this for students who are deaf or hard of hearing is evident because at every level meaningful language was promoted and the students were given opportunities to observe its use and encouraged to employ it in their endeavours to exchange meaning in the group. The examples of their use of the language of fractions, as detailed in Chapter 4, give cause for confidence in this approach to developing meaning and using appropriate language in association with each other. Certainly, the students required further experiences and reinforcement in the use of mathematical language, but it is clear that their understanding and use of the germane language was developing well at the end of the study.

In keeping with the expectation that interactions are desirable because they facilitate the use of language, enhance negotiation of meaning and provide clarification of concepts, it was found that there were many interactions where these dynamics occurred. The games and activities involved the students in the learning process and motivated them to maintain their interest. There were examples of students clarifying their own thinking and questioning each other's understanding through interactions such as the use of team games which created the need for the students to listen to others and describe their own understanding because of the need to make decisions and negotiate meaning. The students' competitive tendency also compelled them to watch the moves of the opposing team lest it gain an unfair advantage, again providing opportunities for meaningful use of language.

Although there were many examples of positive interactions during the study, a significant proportion of them were between a student and the teacher and interactions among students tended to be single sentence contributions rather than longer discussions. The reason for this is uncertain, but it is likely that it was, in part, due to their lack of experience with this style of learning, and it is anticipated that as the students became more experienced with this style of learning and gained confidence in their peers accepting their contributions without censure, they would be likely to hold more lengthy interactions about the games and activities. It also needs to be highlighted that certain games resulted in more discussion than others. The team games worked best because the students enjoyed each of the activities and also because their competitive streak urged them to participate in order to win. It was found that games that compelled the students to make requests of others facilitated more interactions than those which needed only the manipulation of materials.

Teacher style of interacting with students emerged as an important issue. Although this is not an innovative concern in this study, this issue takes on a whole new perspective when it is appreciated that the success of this approach to learning depends heavily on a two-way process of communication. It has been shown that teachers working with students who are deaf or hard of hearing can easily fall into the trap of taking control of interactions in an effort to compensate for their students' difficulties, thus limiting opportunities for them to experience normal interactions and associated learning (Chapter 2). Therefore, it is particularly important for teachers to be sensitive to styles of interacting with students that foster and not hinder interactions with students.

Students in this study showed evidence of responding positively to the affective aspects of learning made possible by an approach such as this. They indicated by their words and actions that they had a positive attitude to themselves and what they were learning. Three of the four students evidenced significant changes in behaviour during the study. The student who was well practised in avoidance techniques did not once attempt to evade any part of the mathematics program despite the fact that she was being challenged by the mathematical content. The student who appeared to have the lowest self-esteem and expectation of success verbalised his preference for the activities instead of written exercises as well as verbalising his delight at his instances of success. His self-deprecating comments and his fears were not extinguished but were outnumbered by regular expressions of enjoyment and pleasure at his own success. The student who experienced the greatest difficulty with cooperation and exhibiting calm focussed behaviour achieved a gradual but noticeable improvement in his cooperation and application to the activities over the time of the study. It is felt that these improvements were expedited by the fact that students could learn at their own pace and in their own way without pressure to keep up with others. Similarly, they also benefitted from individual interactions with the teacher which were made possible not only by the small number of students in the group but also by the other students' interest in the activities which sustained them while the teacher was focused on one student. Therefore, it is clear that the games and activities not only provided a suitable basis for mathematical learning that was motivating and interesting and met the students' educational needs, they also created the basis for a way of learning that was nonthreatening, and so boosted self-esteem as well as a positive attitude to mathematics.

The matter of "divided attention" is an integral problem in educating students who are deaf or hard of hearing and, as such, cannot be ignored. Within the boundaries of this study, students had to choose where to focus their attention when communication occurred, on the signer/speaker or the activity. Learning in this way is a cognitively challenging task as they had to make the semantic connection between two things separated in time and space; that is, what was signed/spoken and the materials being discussed (which were mostly out of the focus of the signed /spoken communication). As well, when a student wanted to contribute to the discussion or make a comment, they were only understood by those who were watching at the time. These are not ideal conditions for spontaneous sharing of information, an activity which is cognitively challenging enough in itself. Equally significant was the fact that when the teacher was interacting with one student and other students were focussed on the game, they could not access the teacher's information, whether it was oral or visual in nature. Within the larger group, there were two groups of students in terms of communication needs; two of the students required signed communication while the others preferred to use their hearing and speech-reading. Thus one group needed to be watching the signed communication because it was their only avenue of accessing communication. The other students needed to be supplementing auditory information with speech-reading. Therefore, it can be seen that, even with the best of intentions and skills on the part of any teacher, the fact that communication and activity are separated in time and that all communication is not able to be accessed readily reduces the effectiveness of the teaching/learning situation.

However, the issue of "divided attention" is not a reason to abandon this approach to developing mastery of mathematics. Students are faced with similar difficulties when involved in science experiments, in group reading situations, when watching interpreted videos, when skills are demonstrated in a physical education lesson, and so on. The question is one of awareness and how teachers can best manage the learning situation to accommodate their students' needs in this regard.

Linking with this issue is one that emerged clearly from this study, and that is the complications of catering for the needs of students who have different preferred modes of communication. Although both groups needed to be attending to benefit from communication, students using oral communication were at an advantage over those using signed communication. The students who were able to communicate without sign drew the teacher into asides which were beneficial to them, but because there was no signed communication accompanying this speech, could not be accessed by the other students. However, because they understood signed communication, they had the opportunity to access all signed conversations. The solution to this dilemma seems obvious — all communication should be signed. However, the issue is *preferred* modes of communication and it is not always so easy to implement such generalities when students operate differently in this regard. Two of the students, although not hearing, preferred not to sign and often requested assistance on a one-to-one basis and were most comfortable without signing. Therefore, it needs to be recognised that in situations such as this, there are difficulties in meeting the communication needs of both groups of children.

Despite such shortcomings (which are not peculiar to this approach to teaching), there appears to be weighty evidence which points to the suitability of an approach such as this for developing mathematical understanding with students who are deaf or hard of hearing. In addition, it emerges as a potential key to meeting the specific needs of such students who require an alternative mathematics program because of the way in which it relates meaning and language and allows students to work at their own pace.

The conclusions that can be drawn from these findings are that:

- it is possible for students who are deaf or hard of hearing to develop understanding of abstract mathematical concepts through games and activities;
- the development of the language associated with these concepts is a natural outcome of this approach;

- the games and activities are very motivating for the students who are actively involved in the learning process;
- although many interactions are sparked by this approach, there is the need to investigate how to encourage more and lengthy interactions between students;
- students respond positively to the affective aspects of this approach;
- the impact of "divided attention" on this approach to teaching and learning mathematics requires more investigation;
- it is difficult to meet the needs of students with different preferred modes of communication in the one group.

Specific outcomes related to the fractions program

As has been shown, there is substantial evidence that three of the students achieved competency in the areas of fractions completed during the study and one fluctuated almost daily in his mastery of the notions. Mastery of the following concepts was observed:

1. the notion of equal parts and the significance of fractions
2. the notion of a whole
3. the names of fractions
4. relating the language to a pictorial representation of fractions
5. understanding fractions of more than a whole
6. the notion of equivalent fractions
7. understanding fractions of a group

However, the area of comparing fractions was a cause of confusion for all of the students and required more attention, possibly at the level of comparison of whole numbers, before returning to comparison of fractions. As well as mastery of concepts, the students also evidenced an understanding of the language of fractions and a certain proficiency in this language. The th/ths ending on fraction names was not always spontaneously used by the students and it is felt that its use should be encouraged for the purposes of clearly conveying meaning to others and an expression of accurate understanding on the part of the student.

In most instances, the students were confident and relaxed about their knowledge of the various areas of fraction work. There was evidence of understanding, meaningful manipulation of materials and use of appropriate language which provided these students with a basis for expanding their conceptualisation of fractions. The particular games and activities used proved to be effective in developing understanding and they were motivating for the students. In fact, at the specific level of understanding the notion of fractions these games and activities appeared to be very effective and time-efficient in building an understanding of beginning fraction concepts relatively quickly.

This leads us to a point that requires more investigation and that is how soon to introduce the Signed English representation of the fraction symbol. It was through the games of *Concentration* and *Fish* that the students were confronted by the symbolic form of fractions. Until then, they had only needed to use the language to describe the fractions. At this stage of the project, they needed to learn what the symbols meant to be able to use the cards for the games. There are two means of representing fractions in Australasian Signed English. The first is by using the conventional language, for example, *2 fifths*, and this corresponds to spoken language. It is time consuming to sign fractions in this way. The second is by a visual representation of the symbol which corresponds to the written symbol. In such a case, *2 fifths* would be represented by spatially separating the two numbers in the symbol. Two would be signed first and above where the five would be signed, thus making a visual representation of the symbolic form of 2/5. This form of signing fractions is faster. However, if numbers are not separated spatially, the sign can appear to be *two five* which is very close to the language of multiplication, *2 fives.* It was at this stage of the project that the visual representation of the symbol was introduced to complement the written symbolic form. In retrospect, it may have been a mistake to introduce it at this time as the students were not consistently using the appropriate language to describe fractions. It is felt that it is the language of fractions that will facilitate a sufficient mastery of fractions, not the symbol. In hindsight, it would have been possible to relate the language to the written symbol without any exposure to the signed representation of the symbol at this point. The students quickly adopted the signed representation of the symbol because it was quicker and easier to sign but did little to enhance their understanding of fractions. This suggests that introduction of the signed representation of the symbol should be delayed until the students show evidence of the complete mastery of the conventional language of fractions. It may be that the signed representation of the symbol could be avoided completely and the students continue using only the conventional language.

The conclusions that can be drawn from these findings are that:

- understanding of the early notions of fractions can be achieved by most students who are deaf or hard of hearing;
- comparison of fractions and, possibly, whole numbers required additional attention;
- the *th/ths* ending of fraction names required special attention;

- the concrete and visual nature of the games and activities built understanding of the fraction concepts;
- this approach fosters students' confidence in their facility to master the fraction concepts; and
- the Signed English representation of fractions should be delayed until students have mastered the language of fractions.

Future directions

The quest for improvements in the levels of mathematical understanding for students who are deaf or hard of hearing is a long way from completion. It is, simply, not acceptable for significant numbers of students who are deaf or hard of hearing to complete school education with little or no real understanding of what mathematics is and its uses, because, if this happens, any disadvantage experienced in our society because of their disability will be magnified. As we have seen, competency in mathematics is as important in our society as competency in literacy. They are complementary skills which empower members to be informed, to participate at an acceptable level and to improve themselves. Therefore teachers must make a professional commitment to "quantitative literacy" (Daniele, 1993) to improve the current situation of underachievement in mathematics by these students. However, for this to occur, some things must change. It is acknowledged that, as with other areas of academic endeavour, students who are deaf or hard of hearing experience more difficulty than most in achieving mastery of mathematics. Therefore, it is suggested that teachers of these students will require better than average knowledge of mathematics education and familiarity with techniques in identification and remediation of difficulties in mathematics. A change in the attitudes of teachers of the deaf is

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Experience has shown that past efforts in professional development and emphases in mathematics programs have not been sufficiently effective in developing "quantitative literacy" for these students, so our attention needs to be directed to a different tack. Constructivist approaches to teaching and learning mathematics appear to offer a worthwhile alternative for further study and consideration. Therefore, additional research into the implementation of constructivist approaches to teaching mathematics to students who are deaf or hard of hearing needs to be undertaken to relate to the information generated by this research.

In keeping with this approach to teaching and learning and the emphasis on discussion and exchange of meaning between students, further information regarding strategies or techniques for promoting discussion among students who are deaf or hard of hearing would be beneficial. No doubt, an environment of acceptance and the teacher's use of low control contributions to discussions can play a part in fostering dialogue among students but there appear to be, nonetheless, other factors affecting the students' lack of propensity to involve themselves in discussion. Information regarding the nature of these factors may advance the implementation of such an approach and improve learning outcomes across the board.

There can be no doubt about the importance of language within the mathematical arena as a means of building and conveying understanding and meaning. Similarly, it is accepted that developing language is a source of significant difficulty for students who are deaf or hard of hearing. Accordingly, it would be most valuable to first, identify the kinds of linguistic structures and features of greatest consequence in the various levels of mathematics and second, to relate this information to the wealth of information about language development of students who are deaf or hard of hearing. This information could alert teachers to possible sources of difficulty for students and content areas for special attention.

Just as language is a source of difficulty for most students who are deaf or hard of hearing, so, then, are symbols. There is often confusion as to the value of symbols because they are often regarded as a source of visual information, the meaning of which is obvious and stands alone. Symbols are an abstraction and, as such, have the potential to be the source of a great dilemma for these students. Mathematics is built around symbols which provide a concise means of manipulating mathematical concepts. However, the symbols have no meaning in themselves, they are endowed with meaning by the ideas that lie behind them. Furthermore, fraction symbols are a departure from the whole number symbol system the students have experienced and have the potential to be even more confusing if the meaning behind them is not understood. Consequently, the impact of the symbol system in mathematics and strategies for introducing and using symbols with these students would provide engrossing reading for those involved in teaching mathematics.

The impact of difficulties with "divided attention" is felt in all subject areas. There is awareness of its existence and some provision for the different learning styles it necessitates. Nevertheless, details regarding the most effective techniques and strategies for reducing its impact are scarce. It appears logical that those who would have most to offer in this regard are Deaf people themselves and those who would have most to gain would be students who are deaf or hard of hearing and their teachers and parents. Further research into styles of teaching and, in particular, using materials which minimise the difficulties arising from "divided attention" would be most helpful.

Suggestions for future directions for improvements in mathematical outcomes for students who are deaf or hard of hearing are:

- professional development that focuses on current developments in mathematics education as well as techniques in diagnosis and remediation for students experiencing difficulties in mathematics;
- further research into the utilisation of constructivist approaches with students who are deaf or hard of hearing in other areas of mathematics;
- more information regarding factors affecting the seeming lack propensity of students who are deaf or hard of hearing to engage in discussions of length;
- further research regarding the effective use of mathematical symbols with students who are deaf or hard of hearing;
- information regarding the most effective strategies for reducing the impact of "divided attention" for students who are deaf or hard of hearing.

So the challenge continues in many different guises. Language is not only a subject within itself, it is an integral part of everything in the education of these students. Mathematics is not just numbers and operations on numbers, it is also a different application of language which, in turn, fosters understanding and improvement in mathematics. Somewhere in the alliance between language and understanding mathematics lies the key to improved mathematical outcomes for students who are deaf or hard of hearing and the empowerment that accompanies it.

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APPENDICES

Appendix A

No illustration available

Equal parts bingo

The aim of this game was to practise using fraction names.

Each student has a sheet which contains the names of different fractions in a grid formation. In turn, students turn over one from a pack of cards which contains pictorial representation of parts of a whole. The students identify the names of the parts using the language, e.g., *four equal parts are fourths*, and then mark the appropriate name on their sheet with a block. The winner is the first player to cover four names in a row.

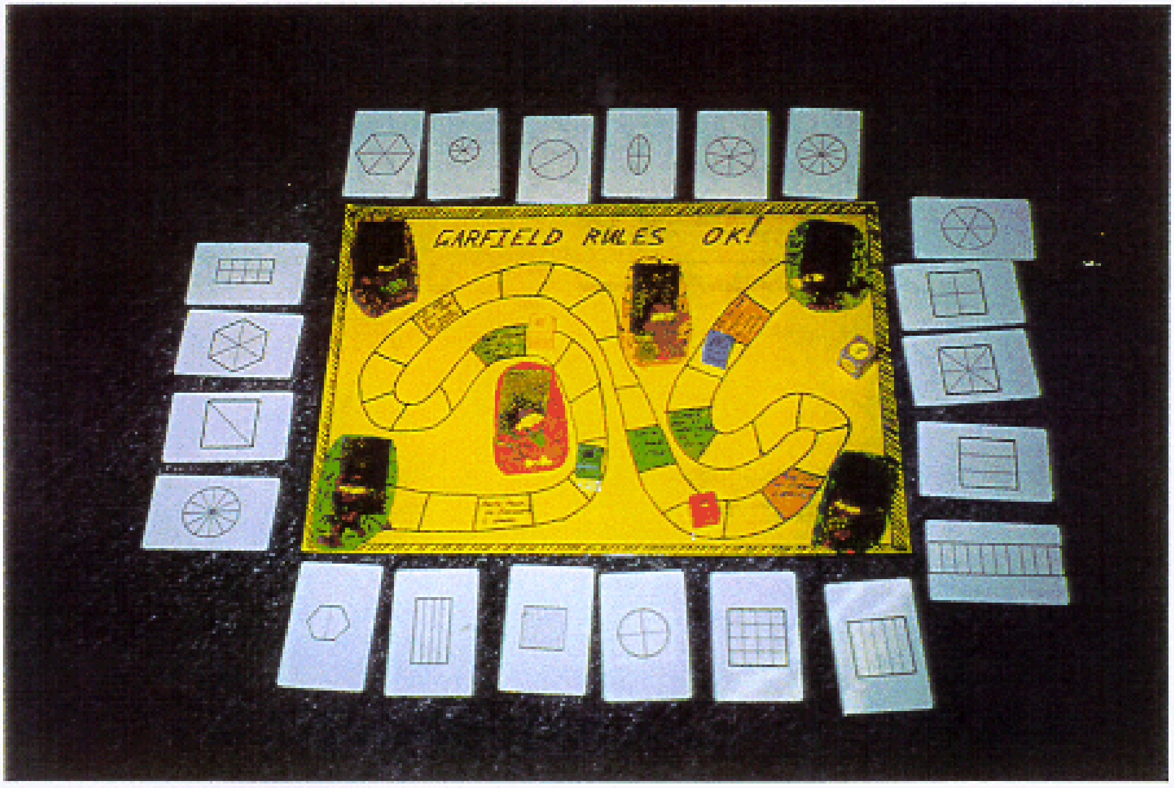
Appendix B

Concentration (equal and unequal parts)

The aim of this game was to identify parts which were either equal or unequal and match them to the appropriate word.

A set of cards is used which contain pictures of different shapes that are divided into parts that are *equal* and *unequal* as well as cards containing the words equal and unequal. The cards are set out on the table face down and students have to match the pictorial representation with the appropriate word. The winner is the student with the most pairs at the end of the game.

Appendix C



Garfield Rules

The aim of this game is to identify equal and unequal parts.

Students sit around a game board which contains a grid of squares. It is decorated by illustrations of the cartoon character, Garfield, hence its name. Each student is given the same number of cubes which are placed on the grid when a correct answer is given. Students take it in turns to turn up the top card of a pile of cards which contain pictures of different shapes divided into equal and unequal parts. They must then identify if the parts are equal or unequal. When their answer is correct, they can place a cube on any square and the winner is the first student to have four blocks in a row.

Appendix D

No illustration available

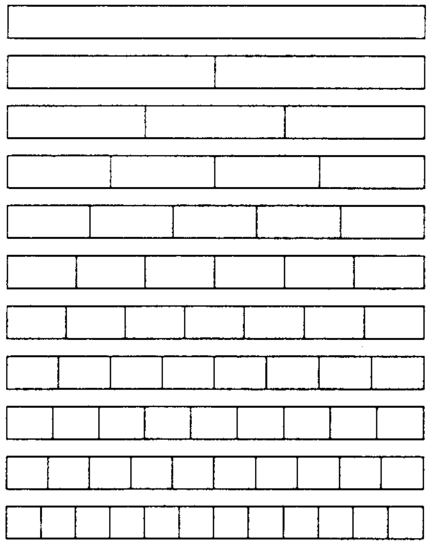
Grand Prix

The aim of this game is to name parts of a whole.

The students sit around a game board that is a simple track on the theme of car racing. Each student has a marker to denote their place in the game. The track is dotted with spaces that contain instructions such as *Oil leak. Pit stop. Miss a turn.* or *Refuelled. Go forward three spaces.* In order to progress through the game, the students must turn up the top card of a pile and correctly name the parts of the shape on the card, e.g., *three equal parts are thirds.* They move forward the number of spaces rolled on a dice. The winner is the first student to reach the finish line.

Appendix E

<u>Colour-me-fractions</u>



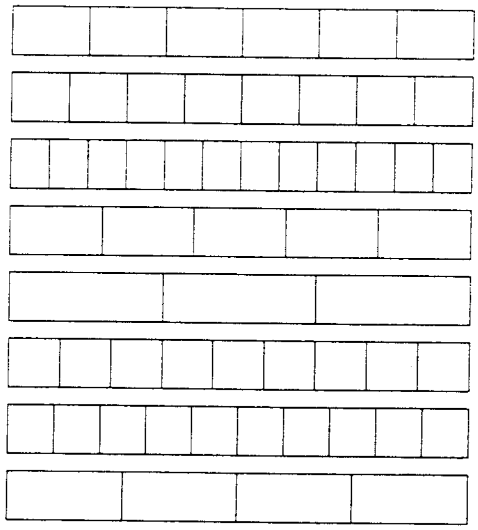
Colour-me-fractions (Number 1)

The aim of this game is to identify parts of a whole and match them to the correct language.

Each student is given a sheet which contains bars divided into parts. Students also need colouring pencils. The game is played by rolling a large ten-sided dice which has the fraction names on the sides, such as *sixths* or *eighths*. When the dice is rolled and a name is revealed, the students colour the appropriate parts on their sheet. This game can be played individually or in teams. The student or team with all parts coloured first is the winner. The sheet for this game features fraction bars with the parts in numerical order, e.g., halves, thirds, fourths etc.

Appendix F

Colour-me-fractions



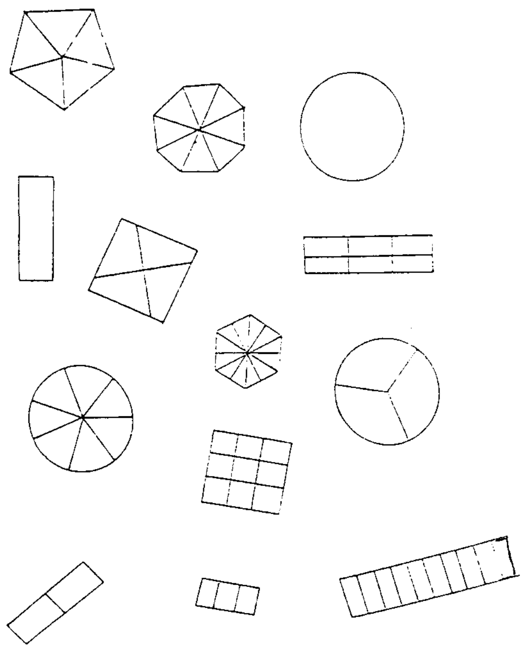
Colour-me-fractions (Number 2)

The aim of this game is to identify parts of a whole and match them to the correct language.

This game is played in the same way as the previous version. The sheet for this game features fraction bars out of numerical sequence, thus ensuring the students count the parts.

Appendix G

Colour-me-fractions



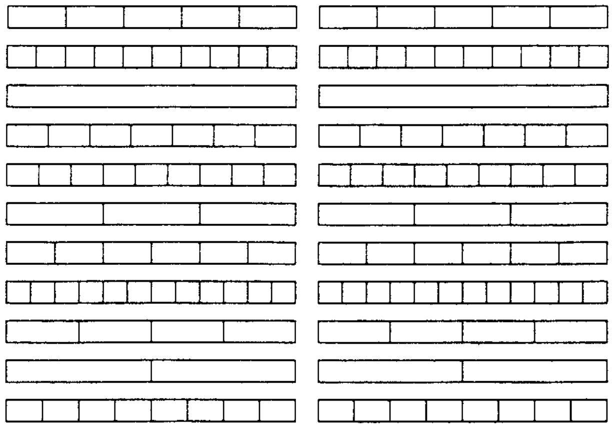
Colour-me-fractions (Number 3)

The aim of this game is to introduce the notion of a number of equal parts and to relate it to the appropriate language.

In this game, the students are presented with a sheet that contains different shapes divided into different numbers of equal parts and, significantly, wholes as well. For this game, the students roll two dice, the same large ten-sided dice they have used in other games and a smaller ten sided dice which contains the numbers 0-9. When they roll both dice, they will find a combination such as *3 fifths* which they then colour on their sheet. Because of the presence of wholes on this sheet, they need to roll *4 fourths, 5 fifths, 6 sixths* etc. to fill these shapes. The winner is the first person or team with a portion of each shape coloured.

Appendix H

Colour-me-fractions



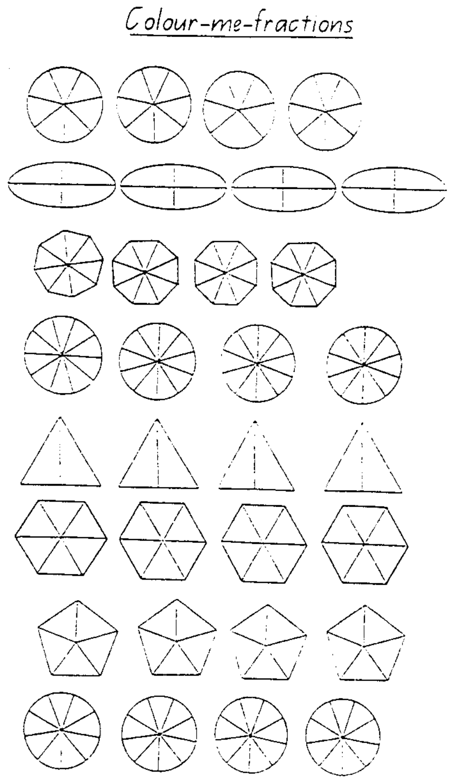
Colour-me-fractions (Number 4)

The aim of this game is to introduce the notion of fractions greater than one whole and to reinforce the notion of fractions of less than a whole in relation to fraction bars.

In this game, students are presented with a sheet that contains fraction bars ranging from 1 whole to tenths and each line contains two wholes divided into equal parts. They roll two dice as in the previous game and so it is possible to roll and identify *7 fifths* which is more than 1 whole. The students relate the written language to the pictorial representation. The first student or team with something coloured on each line is the winner.

While playing this game, the need arises to find a way to finish the game more quickly and equivalent fractions allow for this. By looking at their sheet, students can see that, e.g., *3 sixths* is the same as *1 half* as well as the previously discovered notion of wholes. So this format of the game can be played a number of times, still generating the same interest, as the students discover and experience these notions.

Appendix I

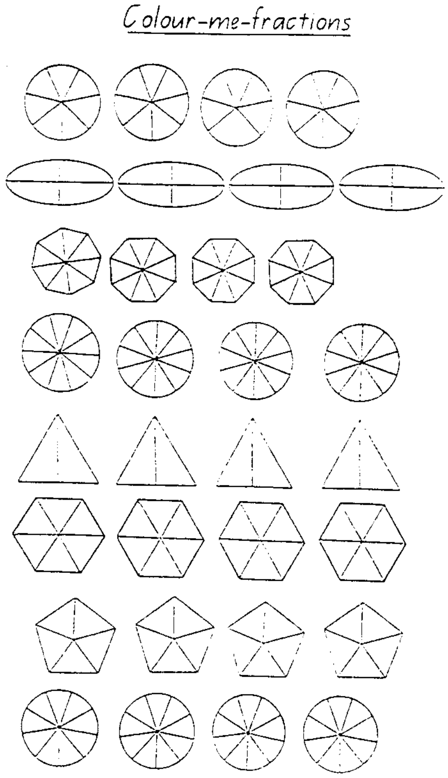


Colour-me-fractions (Number 5)

The aim of this game is to introduce the notion of fractions greater than one whole and to reinforce the notion of fractions of less than a whole in relation to different shapes.

This game follows the same procedure as the previous game.

Appendix I

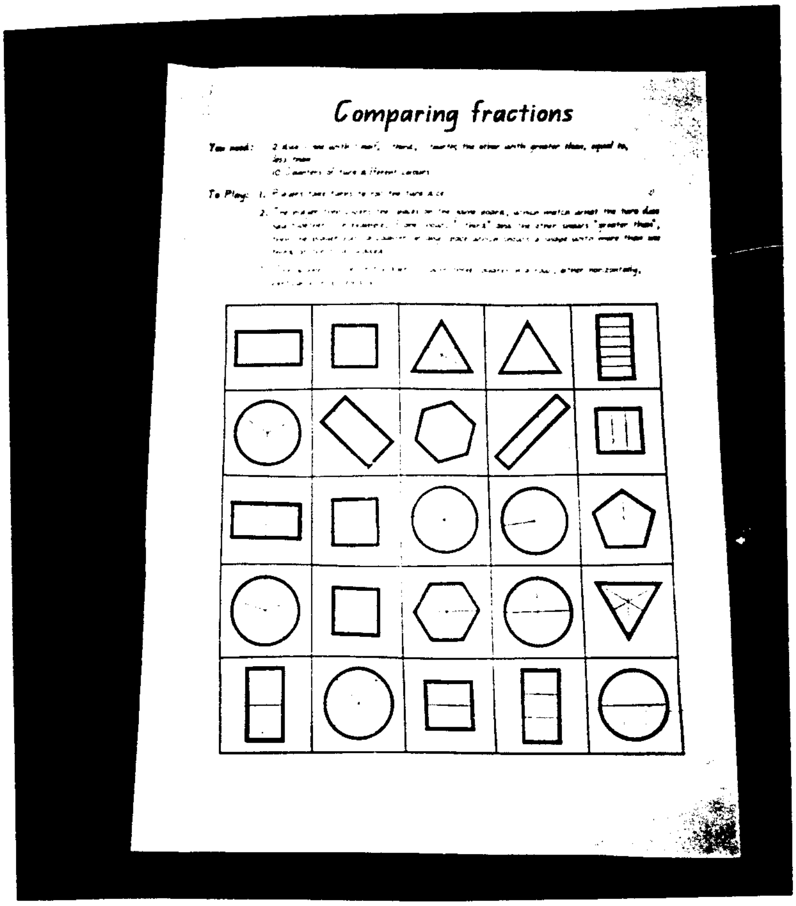


Colour-me-fractions (Number 5)

The aim of this game is to introduce the notion of fractions greater than one whole and to reinforce the notion of fractions of less than a whole in relation to different shapes.

This game follows the same procedure as the previous game.

Appendix J

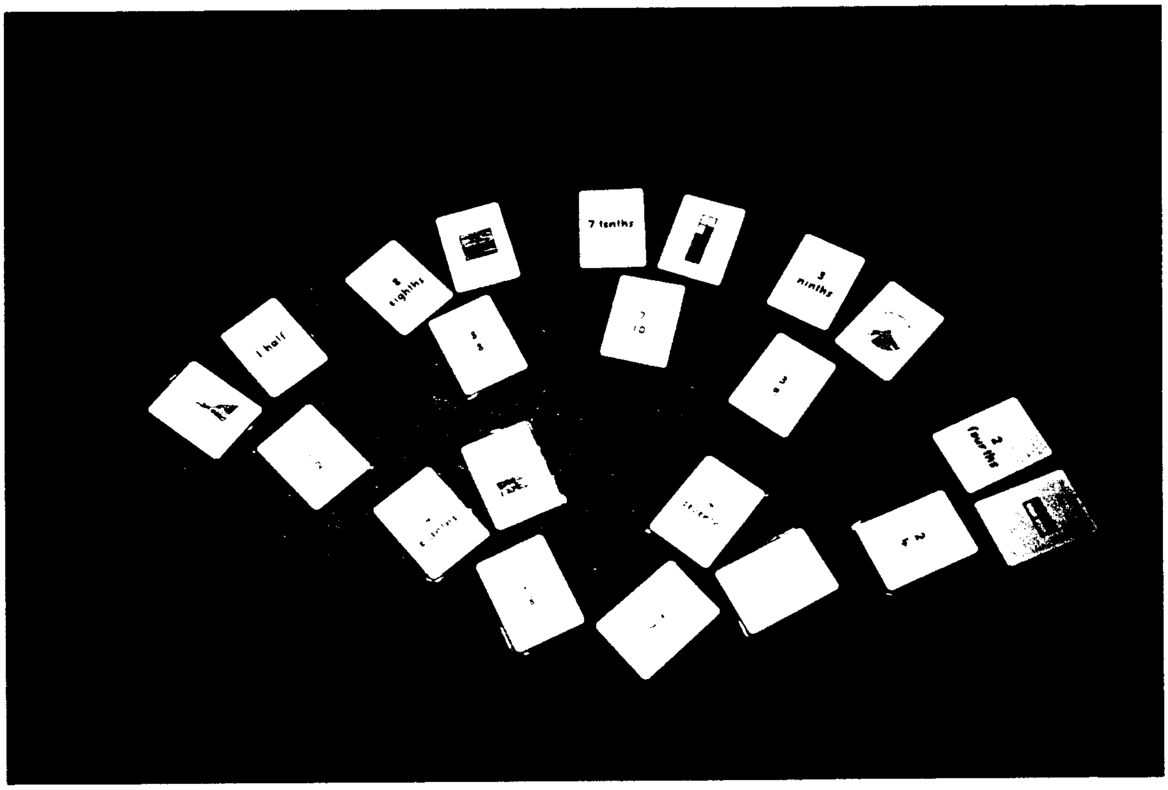


Comparing fractions

The aim of this game is to reinforce the notion of *greater than, less than* and *equal to* relating to fractions.

In this game, the students sit around a gameboard which contains the names of various fractions. They each have some blocks and they use two six-sided dice. One dice is marked with the terms *greater than*, *less than* and *equal to*, while the other is marked with the names *1 half*, *1 third* and *1 fourth*. The students roll both dice to obtain a combination such as *less than 1 half* and then must identify a fraction on the gameboard which matches this description. When this is done, they mark that fraction with a block and the first student or team with a row of four blocks is the winner.

Appendix K



Concentration/Fish (various representations of fractions)

The aim of this game is to make the connection between the pictorial representation, the written language and the symbolic representation of fractions.

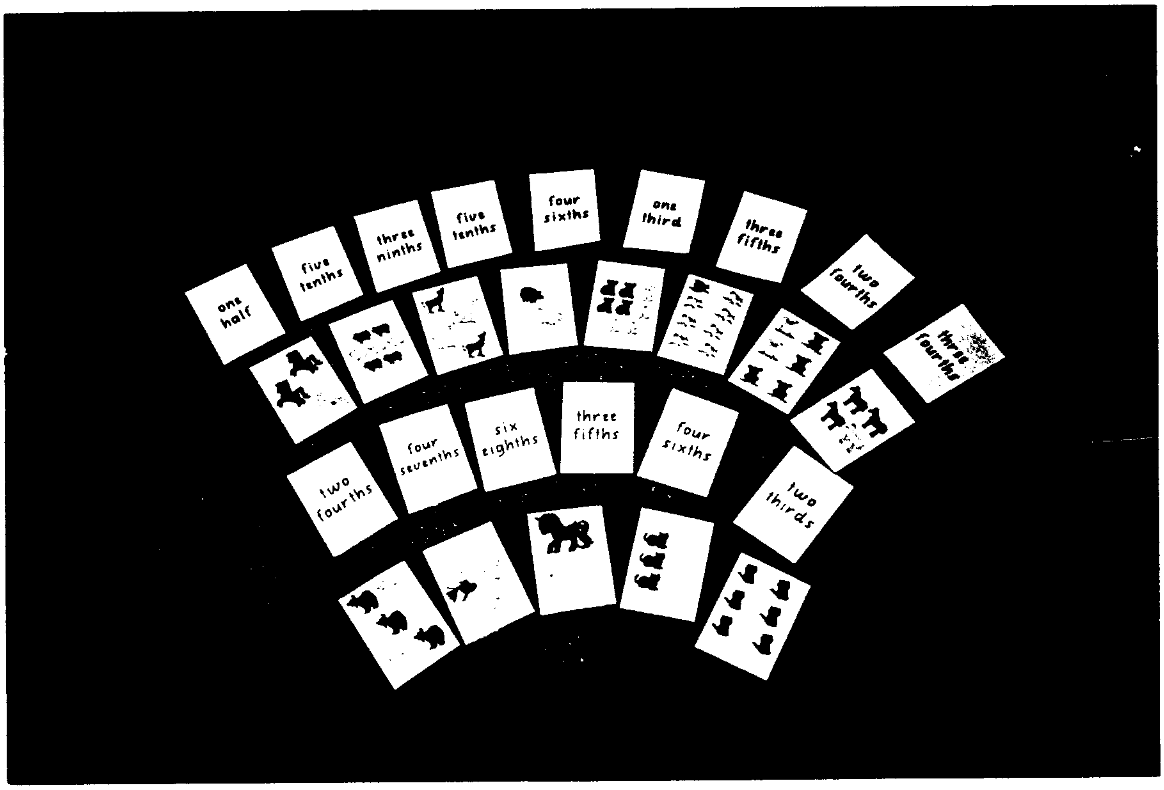
Concentration

A set of cards containing the pictorial representation, the written language and the symbolic representation of fractions is set out face down on the table and students need to make either matching pairs or matching groups of three cards. The student with the most pairs/groups of threes at the end of the game is the winner.

Fish

Using the same set of cards, the students are dealt six cards each and the other cards are placed on the table. Students must make pairs of matching cards by asking each other for the needed cards or picking up a card from the pile of cards. The student with the most pairs at the end of the game is the winner.

Appendix L



Concentration/ Fish (Fractions of a group)

This game is played with a set of cards that are marked with the pictorial representation, the written language and the symbol. The cards contain pictures of groups of objects and animals with some of them coloured to make the fraction. For example, one card has a picture of five cows and three of them are coloured. This represents 3 fifths. The cards can be used for the games of *Concentration* or *Fish* as described previously.