Novel Applications of the Wavelet Transform For Analysis of P Waves in Clinical ECG Recordings

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Abstract

In recent years there has been renewed interest in the importance and analysis of the clinical P wave in the development of cardiovascular disease. Current research has focused upon P wave morphology and its diagnostic importance in the assessment of atrial disorders and cardiac conditions in general. The analysis of P waves in clinical electrocardiogram recordings presents several problems arising from the poor signal to noise ratio of the P wave. The variability of the P wave, its susceptibility to influences from anterior processes and physiological alterations in elderly patients all affect the conspicuousness of P wave features. The current interest in the morphology of the P wave is due to advances in morphological analysis of other physiological signals and the inadequacies of classic cardiological measures of the clinical P wave.

Processing of the ECG has been a research topic within the signal processing field for several decades, accelerated by the more recent widespread acceptance of digital recording techniques in clinical cardiology practices. Improvements in digital processing and recording technologies coupled with reduction in their costs have seen a proliferation of signal processing concepts being applied to clinical cardiology for both practical and exploratory causes. A recent area of interest in biomedical signal analysis problems is the application of wavelet transforms.

The use of the wavelet transform has gained popularity in time-frequency analysis because of the flexibility it offers in the choice of analysing basis functions. This is particularly advantageous in the analysis of characteristic physiological signals
where wavelet basis functions can be chosen to highlight the idiosyncrasies of signal’s morphology.

Proposed in this dissertation is a novel application of the wavelet transform to the analysis of P wave morphology recorded in clinical cardiology settings. These characteristics are sensitive to subtle changes in the morphology of the P wave which is indicative of variations in the electrical wavefront propagation through the atria. This novel method of characterising the P wave morphology uses both temporal and frequency domain aspects of the ECG signal.

This novel technique was applied to the problems of classification of P wave morphology and estimation of physical atrial dimensions. The investigations presented in this dissertation show that this novel analysis method compares well against traditional ECG descriptors in identifying abnormal P wave morphology. Additionally, several wavelet transform characteristics of the P wave proposed here provided significantly better indications of physical atrial dimensions than traditional scalar descriptors of the ECG P wave. This result has great potential benefit for rural and regional cardiovascular healthcare in areas where access to more elaborate cardiac assessment technologies is limited or non-existent.
Statement of Originality

“This work has not previously been submitted for a degree or diploma in any university. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.”

Signature _______________________________________________________

Adrian Diery
Acknowledgments

I would like to state my gratitude to my supervisors, Dr David Rowlands, Dr. Tim Cutmore and Dr. Daniel James and my appreciation of their patience and encouragement throughout my candidature. Thankyou David, for allowing the occasional meeting to drift onto discussions about football, special relativity, quantum information and Japan, when, in the interests of preventative healthcare, a reprieve from cardiovascular complications was required. Danny and Tim, thanks for the sustained optimism.

Sincere thanks to Dr. Roess Pascoe, Ruth Evans and Tony Foreshaw at Hearts 1st Cardiology Clinic for their initial interest in and support of this project. I would especially like thank to Tony Foreshaw and Greg Belous for their valuable time, knowledge and assistance in gathering experimental data. Thanks Tony, for answering any cardiology queries I had.

Warm thanks and much appreciation to the people I have shared labs and thoughts with over the past few years. Andra Wuertenberg, for starting the insanity; Jill Harris, for maintaining the dignity; Aidan Cameron, for the robotic fishing expeditions, cultural teachings and philosophical discussion in the common room; Meredith McHugh, for the coffee, dancing (especially the dance of the dyadic wavelet) and assistance in evading bears; Michael Fraser; for the perspective; Erin McCartney, for the postcards, coffee soon!

Finally, I would like to thank my friends and family for their continued support, for which I am grateful. Mi dispiace per anni degli allontanamento. Grazie mio fratello, apprezzo infinitamente la pazienza e l'appoggio che mi stai dimostrando.

愛 奈 あ
を 穂 り
込 子 が
め と
て う

iv
Table of Contents

Introduction 1

1.1 Research Topic ................................................. 3
1.2 Publications .................................................. 4
1.3 Thesis Outline ................................................ 5
1.4 References: .................................................. 7

The Cardiac System and Electrocardiology 8

2.1 Cardiovascular Physiology ........................................ 9
  2.1.1 Mechanical Cycle ........................................ 10
  2.1.2 Cardiac Electrical Activity ............................... 11
  2.1.3 Cardiac Action Potentials ................................. 11
  2.1.4 Conduction Path ........................................... 13
2.2 The Electrocardiogram .......................................... 15
2.3 Recording ...................................................... 17
  2.3.1 Lead Systems ............................................. 17
  2.3.1.1 12 Lead .................................................. 17
  2.3.1.2 Frank Lead (XYZ) ....................................... 19
  2.3.1.3 Holter ECG .............................................. 20
  2.3.1.4 Alternative Leads for P wave Emphasis .......... 20
  2.3.2 Conditions and Noise Sources ....................... 20
    2.3.2.1 Muscle Noise ........................................ 21
    2.3.2.2 Baseline wander ..................................... 21
    2.3.2.3 Mains noise .......................................... 21
    2.3.2.4 Movement Artefact ................................. 21
  2.3.3 Display ..................................................... 21
2.4 Interpretation of the ECG ......................................... 22
  2.4.1 Rhythm ..................................................... 23
    2.4.1.1 P wave ................................................ 23
    2.4.1.2 PR Interval ........................................... 24
    2.4.1.3 QRS Complex ....................................... 24
    2.4.1.4 ST and QTc segments .............................. 24
    2.4.1.5 T wave ................................................ 25
2.5 Atrial Disorders ................................................. 25
  2.5.1 Atrial Fibrillation and Atrial Flutter ................. 25
  2.5.2 Atrial Enlargement ....................................... 26
  2.5.3 Inter-Atrial Block ....................................... 27
  2.5.4 Effects of Aging ......................................... 27
2.6 Summary & Discussion .......................................... 28
2.7 References: .................................................. 29

Signal Analysis Theory And Application 31

3.1 Fourier analysis ................................................ 31
  3.1.1 Fourier Transform ....................................... 33
  3.1.2 Short-Time Fourier Transform ...................... 34
  3.1.2.1 Limitations ........................................... 36
3.2 Wavelets ........................................................ 37
  3.2.1.1 Admissibility criteria ............................... 37
  3.2.2 Wavelet Properties ..................................... 38
# Table of Contents

5.1.1.1 Cardiological Measures of Abnormal Atrial Conduction.......................................122
5.1.1.2 Experimental Data........................................................................................................126
5.1.2 Analysis Process...........................................................................................................127
5.1.2.1 P wave detection ............................................................................................................128
5.1.2.2 Wavelet Analysis ............................................................................................................129
5.1.2.3 Discriminant Characteristics........................................................................................133
5.1.3 Factor Analysis.............................................................................................................134
5.1.4 Classification ................................................................................................................135
5.1.5 Procedures and measurements .................................................................................137
5.1.5.1 Cardiological Analysis...................................................................................................137

5.2 Experiment 1 – Classification of Median P wave Characteristics  .........................139
5.2.1 Lead Characteristic Models........................................................................................140
5.2.2 II-V1 Lead Model........................................................................................................140
5.2.3 aVF-V2-V5 Model.......................................................................................................144
5.2.4 XYZ Model..................................................................................................................147
5.2.5 Factor Analysis Model ................................................................................................150
5.2.6 Summary of Results for Experiment 1.................................................................153

5.3 Experiment 2 – Classification of Signal Averaged P wave ......................................154
5.3.1 Modifications to Experiment.....................................................................................154
5.3.2 II-V1 model..................................................................................................................155
5.3.3 aVF-V2-V5 model.......................................................................................................157
5.3.4 XYZ Model..................................................................................................................159
5.3.5 Factor Analysis.............................................................................................................161
5.3.6 Summary of Results for Experiment 2 .................................................................163

5.4 Summary & Discussion.............................................................................................164

5.5 References: ............................................................................................................166

Conclusions and Summary .........................................................................................169

6.1 ECG Based Estimation of Atrial Dimensions ........................................................169

6.2 Classification of Abnormal P Wave Morphology..................................................170

6.3 Future Work ..............................................................................................................171

Appendix A Ethics Approval ..........................................................................................173

Appendix B Sample MATLAB Code ............................................................................175
# Table of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The Heart (figure modified from [1])</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>Cardiac Activation Sequence</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>The lead II ECG and its Characteristic Waves</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>the Cardiac electrical activation sequence and contributions to the ECG [1]</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>Planes of the 12 Lead ECG [1]</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>Conventional ECG Display</td>
<td>22</td>
</tr>
<tr>
<td>9</td>
<td>Normal Distribution</td>
<td>44</td>
</tr>
<tr>
<td>10</td>
<td>Inter Quartile Range of a Non-Symmetric Distribution</td>
<td>45</td>
</tr>
<tr>
<td>11</td>
<td>Classification of two normal populations</td>
<td>49</td>
</tr>
<tr>
<td>12</td>
<td>McCulloch and Pitts neuron model</td>
<td>51</td>
</tr>
<tr>
<td>13</td>
<td>4th order Butterworth IIR filter response</td>
<td>61</td>
</tr>
<tr>
<td>14</td>
<td>Baseline Removal from a Clinical ECG Recording</td>
<td>61</td>
</tr>
<tr>
<td>15</td>
<td>Aligned and Averaged P waves from a Noisy ECG</td>
<td>63</td>
</tr>
<tr>
<td>16</td>
<td>Averaged V1 ECG Cycle Produced by Cardioview</td>
<td>64</td>
</tr>
<tr>
<td>17</td>
<td>Aligned P waves from a Healthy Recording</td>
<td>67</td>
</tr>
<tr>
<td>18</td>
<td>Fourier analysis of healthy P waves in Figure 17</td>
<td>67</td>
</tr>
<tr>
<td>19</td>
<td>Energy and Phase Spectrums for P waves in Figure 17</td>
<td>68</td>
</tr>
<tr>
<td>20</td>
<td>Original ECG</td>
<td>69</td>
</tr>
<tr>
<td>21</td>
<td>Superimposition of original and exaggerated ECG</td>
<td>71</td>
</tr>
<tr>
<td>22</td>
<td>Superimposition of low amplitude P waves and exaggerated P waves</td>
<td>72</td>
</tr>
<tr>
<td>23</td>
<td>P wave Detection in Leads II and V1</td>
<td>73</td>
</tr>
<tr>
<td>24</td>
<td>ECG exhibiting a normal sinus rhythm</td>
<td>74</td>
</tr>
<tr>
<td>25</td>
<td>Wavelet energy distributions for each Limb Leads of Normal Sinus Rhythm ECG</td>
<td>75</td>
</tr>
<tr>
<td>26</td>
<td>Lead average wavelet energy distributions for distributions in Figure 25</td>
<td>75</td>
</tr>
<tr>
<td>27</td>
<td>ECG Exhibiting Changes in Atrial Conduction</td>
<td>76</td>
</tr>
<tr>
<td>28</td>
<td>P wave analysis of ECG shown in Figure 27</td>
<td>77</td>
</tr>
<tr>
<td>29</td>
<td>Lead averaged energy distribution for atrial arrhythm</td>
<td>77</td>
</tr>
<tr>
<td>30</td>
<td>Maximum Correlation between Basis Function and Lead II P waves</td>
<td>81</td>
</tr>
<tr>
<td>31</td>
<td>Maximum Correlation between Basis Functions and Lead V1 P waves</td>
<td>81</td>
</tr>
<tr>
<td>32</td>
<td>Peak Values in Lead II vs Peak Values in Lead V1</td>
<td>82</td>
</tr>
<tr>
<td>33</td>
<td>Anti-symmetric Function and its Fourier Transform</td>
<td>83</td>
</tr>
<tr>
<td>34</td>
<td>Central Frequency of 1st Order DOG</td>
<td>84</td>
</tr>
<tr>
<td>35</td>
<td>Central Frequency of 2nd Order DOG wavelet</td>
<td>85</td>
</tr>
<tr>
<td>36</td>
<td>Spectrogram of Anti-symmetric Function</td>
<td>86</td>
</tr>
<tr>
<td>37</td>
<td>Spectrogram of Symmetric Function</td>
<td>87</td>
</tr>
<tr>
<td>38</td>
<td>Comparison of Frequency Analyses</td>
<td>88</td>
</tr>
<tr>
<td>39</td>
<td>QV and IQR vs. Window Duration</td>
<td>89</td>
</tr>
<tr>
<td>40</td>
<td>QV and IQR vs. Frequency</td>
<td>90</td>
</tr>
<tr>
<td>41</td>
<td>Truncated Gaussian Pulses</td>
<td>91</td>
</tr>
<tr>
<td>42</td>
<td>IQR and QV vs. Standard Deviation</td>
<td>91</td>
</tr>
<tr>
<td>43</td>
<td>QV and IQR vs. Area under the Curves in Figure 41</td>
<td>92</td>
</tr>
<tr>
<td>44</td>
<td>Normal Averaged V1 P wave</td>
<td>93</td>
</tr>
<tr>
<td>45</td>
<td>LAE Averaged V1 P wave</td>
<td>93</td>
</tr>
<tr>
<td>46</td>
<td>Healthy Subject No. 1, Normal Sinus Rhythm, no atrial abnormality</td>
<td>96</td>
</tr>
<tr>
<td>47</td>
<td>Healthy Subject No. 2</td>
<td>96</td>
</tr>
<tr>
<td>48</td>
<td>Healthy Subject No. 3</td>
<td>97</td>
</tr>
</tbody>
</table>
Figure 49 Patient with LAE and atrial ectopic beats

Figure 50 Patient with LAE in sinus rhythm

Figure 51 Patient with Paroxysmal Atrial Fibrillation during Sinus Rhythm

Figure 52 Patient with Paroxysmal Atrial Fibrillation, during Sinus Rhythm

Figure 53 4 Chamber Apical View 2D Transthoracic Echocardiogram (TTE)

Figure 54 ECG P wave measures vs Left Atrial Diameter

Figure 55 ECG P wave measures vs Left Atrial Area

Figure 56 ECG P Wave Measures vs Right Atrial Area

Figure 57 Significant Z lead LAD Correlates

Figure 58 Significant Z lead LAA Correlates

Figure 59 Significant Z lead RAA Correlates

Figure 60 Significant X lead Echo Measurement Correlates

Figure 61 (a) shows normal atrial conduction as opposed to the enlarged left atrial depolarisation in (b) and (c)

Figure 62 2nd Order Gaussian Wavelet Base

Figure 63 Wavelet Analysis of Normal P wave

Figure 64 Wavelet Analysis of abnormal P wave

Figure 65 Distance Scores with Standard Error for II-V1 Model Averaged over 100 Classifications

Figure 66 Average Classification Performance over 100 runs

Figure 67 Distance Scores with Standard Error aVF-V2-V5 Model Averaged over 100 Classifications

Figure 68 Average Classification Performance over 100 runs

Figure 69 Average Distance Scores with Standard Error for XYZ Model

Figure 70 Average Classification Performance vs Number of runs

Figure 71 Distances Scores and Standard Errors for Factor Analysis Model Averaged over 100 runs

Figure 72 Average Classification Performance vs Number of runs

Figure 73 Classification using Signal Averaged II-V1 model

Figure 74 Average Classification Performance for II-V1 model

Figure 75 Classification using Signal Averaged aVF-V2-V5 model

Figure 76 Average Classification Performance for aVF-V2-V5 model

Figure 77 Classification using Signal Averaged XYZ model

Figure 78 Average Classification Performance for XYZ Model

Figure 79 Standard Errors of Factor Analysis Classification

Figure 80 Average Classification Performance for Factor Analysis Model
Glossary of Terms

Anterior ................................ positioned in front of, the front surface of a cavity
Arrhythmia .......................... abnormal condition or rhythm
Atrium .............................. the two smaller chambers of the heart situated above the
ventricles
AV Block ............................ a block causing disruption to the electrical conduction
between atria and ventricles
Biphasic .............................. comprising of two phases or components; a waveform having
two distinct deflections
Cardiac Cycle ...................... a heartbeat; a complete instance of electrical and mechanical
cardiac function beginning with atrial contraction and
completing with relaxation of the ventricles and electrical
recovery of myocardial cells
Epicardium .......................... a thin layer of cells on the outer surface of the heart
Endocardium ...................... a thin membrane of cells lining the interior of the heart
Hypertrophy ....................... an increase in muscle mass, most commonly occurring in the
ventricles in the presence of systolic overload
Depolarisation .................... the phase during which muscular cells return to their resting
potential
Diastole ............................. the resting period of electrical and mechanical cardiac
function
Dilatation ............................ the condition or process of pathological enlargement of a
cavity
Dilation ............................... see Dilatation
Frontal Plane ....................... the vertical plane running across the thorax, perpendicular to
sagittal and transverse planes
Infarction .......................... necrosis of tissue as a result of oxygen depletion and
disrupted circulation to the tissue
Inferior .............................. positioned lower than or beneath; the base or lower surface of
a cavity or mass
Ischaemia ......................... condition in which cardiac tissue is starved of blood
Lateral ............................... orientated in the left-right direction
Manubrium ........................ flat, triangular shaped segment of bone at the uppermost
section of the sternum
Morphology......................... characteristic shape of an event or waveform
Monophasic......................... having only one phase; a waveform with a single positive or negative deflection
Myocardium ........................ the middle layer of cardiac cells, the majority of muscle mass in the cardiac wall
Oesophageal....................... positioned near the oesophagus, an oesophageal electrode is positioned inside the throat
P mitrale........................... a P wave morphology displaying two separate peaks of the same polarity, a notched or bifid P wave
Posterior......................... positioned behind or towards the back of; the rear surface of a cavity or mass of tissue
Precordial........................ positioned on the surface of the thorax (chest) over the heart
Sagittal Plane..................... the inferior-anterior vertical plane
Senescence....................... a state of decline; a state of tissue degeneration associated with aging
Septum............................. tissue dividing two cavities, the inter-ventricular septum divides the ventricles; the inter-atria septum divides the left and right atria
Sino-atrial Node ................. a complex of cells which initiate depolarisation in the cardiac tissue
Superior........................... positioned above, the top surface of a cavity
Systole............................. period of muscular contraction, atrial systole or ventricular systole; refers to contraction of the left ventricle as a generalisation.
Systolic overload............... a state in which there is excessive demand on output from the cardiac cycle
Ventricle ......................... the larger chambers of the heart which actively eject blood
### List of Symbols

- **t**: Continuous time
- **f**: Frequency in Hertz; Also used to denote a function
- **f_s**: Sampling frequency
- **f_c**: Central frequency of a function
- **f(x)**: Function of a variable; **x**
- **x(t)**: Function of time
- **n**: Positive integer value
- **c_n**: nth Fourier series coefficient
- **ω**: Frequency radians = 2πf
- **τ**: Temporal shift
- **T**: Temporal period
- **π**: Circular constant;
- **π_n**: Classification group
- **e**: Natural logarithmic base; Also used to denote eigenvectors
- **e_i**: ith eigenvector
- **λ_i**: ith eigenvalue
- **j**: Imaginary number = \(\sqrt{-1}\)
- **X(ω)**: Denotes the Fourier transform of **x(t)**
- **k**: Discrete frequency
- **m**: Discrete temporal shift
- **N**: Length of finite data series
- **g_p(t)**: Normalised Gaussian function
- **h(t)**: Filter coefficient
- **H{x}**: Denotes a system transform of a variable; **x**
- **σ**: Standard deviation
- **ψ**: Variance vector
- **ψ_i**: ith element in a variance vector
- **a**: Scaling index of wavelet function, unless otherwise defined
- **b**: Temporal index of wavelet function, unless otherwise defined
- **ψ(t)**: Wavelet basis function; Mother wavelet
- **ψ_{a,b}(t)**: Wavelet scaling function; Daughter wavelet
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi(\omega)$</td>
<td>Fourier transform of wavelet basis function</td>
</tr>
<tr>
<td>$C_\phi$</td>
<td>Wavelet reconstruction constant</td>
</tr>
<tr>
<td>$C(a,b)$</td>
<td>Wavelet transform coefficients</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Wavelet basis function normalisation constant</td>
</tr>
<tr>
<td>$r$</td>
<td>Correlation value</td>
</tr>
<tr>
<td>$R$</td>
<td>Correlation value; Also used to denote classification regions</td>
</tr>
<tr>
<td>$R_n$</td>
<td>nth classification region</td>
</tr>
<tr>
<td>$p$</td>
<td>Significance value</td>
</tr>
<tr>
<td>$P(x)$</td>
<td>Probability density function of a variable; $x$</td>
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<tr>
<td>$E(x)$</td>
<td>Expectation of a variable; $x$</td>
</tr>
<tr>
<td>$\text{Var}(x)$</td>
<td>Variance of a variable; $x$</td>
</tr>
<tr>
<td>$\text{Cov}(x)$</td>
<td>Covariance of a variable; $x$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Vector of means</td>
</tr>
<tr>
<td>$L$</td>
<td>Factor loading matrix</td>
</tr>
<tr>
<td>$F$</td>
<td>Vector of common factors</td>
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<tr>
<td>$\varepsilon$</td>
<td>Vector of specific factors; Prediction error</td>
</tr>
<tr>
<td>$d$</td>
<td>Group allocation threshold</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Decision threshold</td>
</tr>
<tr>
<td>$SS$</td>
<td>Sum of squares</td>
</tr>
<tr>
<td>$S$</td>
<td>Sample covariance matrix</td>
</tr>
<tr>
<td>$S_j$</td>
<td>Joint covariance matrix</td>
</tr>
<tr>
<td>$J$</td>
<td>Group separation objective</td>
</tr>
<tr>
<td>$W$</td>
<td>Weighting vector</td>
</tr>
</tbody>
</table>
Chapter 1

INTRODUCTION

According to the most recent figures published by the Australian Bureau of Statistics and the Department of Health, cardiovascular or heart disease remains the leading cause of death in Australia, accounting for around 38% of all deaths in 2002 [1][2] and 34% of all deaths in 2006 [3]. These rates are consistent with those of other western developed countries such as New Zealand, the United States of America, the United Kingdom of Britain and the Scandinavian nations [4]; Table 1 lists the percentages of deaths caused by cardiovascular disease (CVD) in these countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>% of all Deaths caused by CVD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
</tr>
<tr>
<td>Australia</td>
<td>35.5</td>
</tr>
<tr>
<td>New Zealand</td>
<td>38.7</td>
</tr>
<tr>
<td>UK</td>
<td>39.2</td>
</tr>
<tr>
<td>USA</td>
<td>37.2</td>
</tr>
<tr>
<td>Denmark</td>
<td>35.1</td>
</tr>
<tr>
<td>Iceland</td>
<td>40.7</td>
</tr>
<tr>
<td>Norway</td>
<td>39.5</td>
</tr>
<tr>
<td>Finland</td>
<td>40.3</td>
</tr>
<tr>
<td>Sweden</td>
<td>45</td>
</tr>
</tbody>
</table>

The major contributing conditions under the umbrella of the CVD figures in these countries are ischaemic heart disease and cerebro-vascular conditions (stroke). Both groups of conditions arise from the restriction of blood flow. Ischaemic heart disease is the result of restricted blood supply to the cardiac tissue whereas strokes occur as a result of restricted blood supply to the brain. This restriction is most often caused by the constriction of arteries by atherosclerotic plaque and is exacerbated by blood clots. Risk factors associated with this condition include smoking, obesity, high blood pressure, high cholesterol and diabetes [5]. Mental stress and depression have also been cited as risk factors [6]. These factors impact upon the health of the heart and circulatory system.
Precursory conditions in the development of CVD can be observed as abnormalities in atrial function. These abnormalities may be the result of structural remodelling of the atria or changes in electrophysiological properties of the cardiac tissue within the atria.

The direct cost of CVD to the health system in Australia in 2004 was estimated at $7.6 billion. Associated costs of loss of productivity due to premature death and lowered employment rates plus the costs of care push the total financial costs to an estimated $14.2 billion which is equivalent to 1.7% of Australia’s gross domestic product. The associated costs of suffering and premature death due to CVD in Australia have been valued at $94 billion. In 2004, CVD affected 1 in 6 people with estimates predicating this rate to rise to 1 in 4 within 50 years. [7]

Specific to Australia are the issues of indigenous health and rural healthcare. The mortality rate amongst Aboriginal and Torres Strait Islander people is 2.6 times greater than the rest of the population [8]. The vast nature of the Australian continent and spread of rural populations prevents ready access to expensive medical examinations and procedures in metropolitan areas. As a consequence of these issues and the direct and indirect costs of CVD, cardiac health is a large area of interest in medical research and has spawned a great deal of research into preventative, medicinal and technological advances both in Australia and abroad. One such avenue of research is in improving cardiovascular diagnostics within the field of cardiology.

There exists today, several popular tools used extensively in cardiology practices which provide non-invasive observations of cardiac functioning. The least expensive of which is the electrocardiogram (ECG). The ECG is a recording of the electrical energy generated by the cardiac cycle as observed at the epidermis. One of the greatest advents in electrocardiology was the advent of digital ECG recording. The use of digital technology in ECG recordings provided a means of permanently storing the ECG. The ability to record and store ECG recordings en masse made the accumulation of pathological databases possible. Consequently, the creation of databases, like the MIT-BIH database [9], has aided the development of ECG processing techniques, such as compression schemes [10]. It also improved the diagnostic legibility of the ECG traces [11] making detailed scrutiny of the ECG component waveforms possible and providing the means for automated ECG analysis software to be developed.

One aspect of the ECG which has not received proportionate investigation, until recently, is the morphology of the P wave. The P wave is the only representation of atrial
electrical function readily discernible in the ECG. The early indicators provided by the state of atrial function, and therefore the information contained in the P wave, are of significant interest in the search for cost effective means of combating CVD. The reasons for the comparative lack of attention paid to the P wave are several. Most obvious are the factors of priority and clarity. Firstly, the left ventricle is the major chamber of the heart, containing the majority of muscle mass and supplying blood to the circulatory system. Its associated waveforms have therefore been the focus of the majority research conducted in the electrocardiology field. Secondly, P waves have comparatively low amplitudes and lack the dramatic characteristic features exhibited by QRS complexes and T waves which are readily interpreted as indicators of ventricular regularity or otherwise. The advent of higher resolution ECG recordings has significantly benefited the study of the P wave. It has provided more reliable and accurate means of determining P wave duration and associated measures of P wave dispersion and P terminal phase values which are indicative of atrial functioning. The last decade has seen the development of various robust approaches towards the problem of P wave detection and more recently, P wave morphological analysis.

1.1 Research Topic

The processing of cardiac signals has been a popular avenue of research into cardiac healthcare for several decades [12]. Automated signal analysis, statistical analysis and event detection have all been thoroughly researched and many developments have found their way into practical clinical usage. The ECG, echocardiogram and magnetic resonance imaging recordings of cardiac functioning have all been studied, analysed and diagnosed by various signal processing techniques and combined approaches. Primarily cardiological research focuses on the cardiac cycle as a whole or on the condition of ventricular activity, which has specific representation in the ECG. This is understandable as the majority of cardiac tissue and arrhythmias are associated with the ventricular mass. However, it has meant that atrial functioning is often overlooked. This is especially the case for the ECG, where atrial function is represented by a low amplitude waveform in which subtle changes in function can be difficult to observe.

With the exception of atrial fibrillation (AF), atrial conditions have not received a proportional amount of attention either in cardiological or biomedical literature. This lack of investigation into improving the analysis of atrial electrical function was the motivation for this work. Initial curiosity was raised by the Cardiology team at a private hospital from
observations of a large number of elderly inpatients. Observations noted a degeneration of the electrocardiogram signal in these patients which was attributed to electrophysiological changes in cardiac tissue and physical changes in the orientation of the heart [13]. This degeneration in the conductive properties of the aging myocardium made visual analysis of the atrial function via the ECG, extremely challenging.

Current ECG markers used to assess the P wave are widely known to be unspecific to the presence of abnormality. Despite these inadequacies, these markers continue to be used in clinical practice because of the inexpensive nature of the ECG as a screening and diagnostic test. The cardiological team were also interested in the importance of P wave morphology, a review of the literature showed that P wave morphology was becoming an increasingly popular avenue of interest within electrocardiology. The cardiology clinic were primarily interested in being able to study the P wave better by enhancing its presence in the ECG and the hypothesis that there maybe previously unstudied indicators contained within the atrial sequence.

The aim of this thesis was to investigate and formulate novel methods of analysing the P waves which could provide useful diagnostic information. This led to the development of a novel method of analysing the P wave using the wavelet transform. This analysis method produced several measures which could be used to characterise the P wave. One novel application of this analysis method provided a means of reliably estimating the physical dimensions of the atria from the clinical ECG. Another application makes use of the wavelet transform measures to characterise and classify atrial electrical activity.

1.2 Publications

The work undertaken in this thesis, developing and refining the P wave analysis technique had led to several publications. The early research and review work produced a peer review conference paper entitled: “Nonlinear Processing Techniques of the Electrocardiogram”. This paper appeared in the proceedings of the 2003 ANZIIS conference which was held in Sydney in November of 2003.

Developmental work on the wavelet based analysis of the P wave developed in this thesis produced the following peer reviewed journal paper:

The experimental results of the P wave classification produced during the course of this candidature have been reported in a journal submission and is under review:


1.3 Thesis Outline

The significance of the field of research encompassing the topic explored in this thesis has been covered in the introduction of this chapter. The theory and methods behind the practice and research of this field are presented in the following chapters. An introduction to cardiology and signal processing is presented in chapters 2 and 3 respectively. The chapter summarising cardiology covers the topics of the circulatory system, the theory and practice electrocardiography and atrial arrhythmia. The physiology of the heart and theory of cardiology is covered with special attention to the ECG. A brief overview of the functioning of the heart and the various components of the cardiac cycle, both mechanical and electrical, is given to provide a background for the reader. A more detailed explanation of the atrial activity and function is also provided. The chapter covering signal processing theory presents an introduction to time-frequency analysis techniques, with emphasis on the wavelet transform. Chapter 3 outlines standard signal processing techniques for time series analysis as well as an introduction to wavelet processing and its applications to the analysis of ECG signals and more general applications. Current techniques used for detecting and processing the P wave are also discussed.

In Chapter 4 the development of novel wavelet energy based method of analysing and characterising atrial electrical activity is presented. This method uses the wavelet transform of each P wave to derive information which characterises the atrial electrical activity present within an ECG. Information can be gathered from the energy distributions and their variation over time. Descriptions and justification of the steps involved in the developing the method and choosing the characteristic measures are presented. This chapter also contains an experimental section. In this section the wavelet analysis method
is applied to the problem of estimating physical dimensions of the atrium measured by two-dimensional echocardiogram.

Chapter 5 presents further experimental applications of the method developed in the previous chapter. The wavelet characteristics of the P wave are implemented to the problem of classification. Classification is performed using the wavelet characteristics as inputs to a linear discriminant analysis. The characteristics are derived from several leads from the standard ECG and the derived orthogonal leads. The performance of each classification approach is assessed and compared against traditional ECG measures of atrial activity. A discussion of the relative performance of each classification is presented in the conclusions to this chapter.

Finally a summary and review of the work and results presented in this thesis is covered in Chapter 6. This final chapter contains a discussion of possible further work to be undertaken to further develop and improve the methods presented in this thesis.
1.4 References:


2 Australian Institute of Health and Welfare (AIHW), 2004, Heart, stroke and vascular diseases – Australian facts 2004, AIHW, Cat. No. CVD 27, Canberra: AIHW and National Heart Foundation of Australia (Cardiovascular Disease Series No. 22).


8 Australian Institute of Health and Welfare (AIHW), "Heart, stroke and vascular diseases – Australian facts 2004", Canberra, AIHW and National Heart Foundation of Australia, AIHW Cat No CVD 27, (Cardiovascular Disease Series No. 22), 2004


Chapter 2

THE CARDIAC SYSTEM AND ELECTROCARDIOLOGY

The study of the heart’s functioning and complications has been an area of investigation since the early 1800s. Due to the importance of the heart and the high incidence of heart disease cardiology is an increasingly significant field in medicine. Significant diagnostic advances have included the invention of the electrocardiogram (ECG) and ultrasound analysis. The advent of the electrocardiogram provided a means of studying the conduction and temporal aspects within the cardiac cycle. This greatly improved the understanding of normal and abnormal heart function and the field of diagnostic cardiology.

Modern Cardiology has changed little in theory and practice since Eindhoven’s first experiments at the turn of the century. Originally devised by Waller and refined by Eindhoven [1], the ECG has proven to be an invaluably important and inexpensive tool to the field of cardiology. The most significant changes in modern cardiology have been the digitisation of the electrocardiogram and the advent of magnetic resonance imaging (MRI) and the echocardiograph (ultrasound). Digitisation of the ECG has allowed permanent storage allowing long term patient records to be kept and automated diagnostic tools to be developed and improved. Recent fields of interest in electrocardiology have included the study of atrial dysfunction, specifically that of atrial fibrillation and the significance of precursory or related conditions such as interatrial block and atrial enlargement. Detailed analysis of atrial electrical functioning in clinical ECG recordings has been limited by the atrium’s innate electrical characteristics. Atrial activity produces relatively low amplitude signals in the electrocardiogram and traditional methods of display can mask underlying processes.

Before attempting to perform any analysis an understanding of both the electrical and mechanical functioning of the heart is required as well as its physical and electrophysiological structures. An overview of basic concepts of cardiology and the
electrocardiogram are presented in this chapter. Additionally, a brief focus on atrial function and associated disorders is also presented.

2.1 Cardiovascular Physiology

The circulatory system is a closed loop with a positive mean pressure and the heart is the pump within this system, responsible for circulating blood around the body. The heart is a hollow organ consisting of four chambers; the left and right atria and left and right ventricles. Figure 1 shows the four chambers of the heart. The respective atria and ventricle pairs form separate pumps, effectively the two pumps are working in series. The left side of the heart controls the systemic circulation to the arteries, while the right side controls the pulmonary circulation to the lungs. The left atrium receives oxygenated blood from the lungs via the pulmonary veins. The blood then flows to the left ventricle via the mitral valve. The left ventricle pumps via the aortic valve into the aorta and arterial system, which distributes the blood around the body before returning to the right atria. The right atrium receives deoxygenated blood from the systemic circulation via the superior vena cava and inferior vena cava which then flows into the right ventricle via the tricuspid valve. Blood from the right ventricle then enters the pulmonary circulation, via the pulmonary valve, where it is carried to the lungs for re-oxygenation.

The left side of the heart contains the majority of muscle mass as the systemic circulation is much vaster in volume and distance than the pulmonary system and requires greater pumping power. The walls of the heart consist of muscle tissue which facilitates electrical conduction to control the muscular contraction of the chambers. The cardiac muscle mass consists of three layers, a thin single cell layer interior lining called the endocardium; the thick fibrous mass called the myocardium and an outer thin layer called the epicardium. The bulk of the cardiac muscle is contained in the myocardium, muscle tissue in this layer responds to electrical signals with muscular contractions. Damage to this layer of muscle can therefore be critical to the condition of the heart.

In addition to the muscular tissue, there is a network of conduction fibres which act as conduction pathways between sections of the heart. The bundle of His and bundle branches facilitate electrical conduction from the atria to the base of the ventricles. The conduction delay created by these fibres is essential to producing efficient pumping. The Purkinje fibres distribute the electrical impulses throughout the ventricles.
Figure 1 The Heart (figure modified from [1])

The cardiac cycle relies upon electrical excitation and depolarisation of these cardiac cells to produce muscular contractions in each chamber of the heart. This period of muscular contraction is called systole. Diastole is the period when the cardiac cells repolarise and the cardiac muscles relax, allowing the chambers to passively expand. Systolic action between the atria and ventricles is staggered so the atria can sufficiently empty before ventricular contraction.

2.1.1 Mechanical Cycle

At the beginning of atrial diastole the ventricles are empty and the atrioventricular valves are closed. The valve connecting the right atrium and ventricle is the tricuspid valve, the valve connecting left atrium and ventricle is the bicuspid or mitral valve. These valves are shown in Figure 1 at the base of the atria. During diastole the atria passively fill and expand, causing the valves to open and allow blood to enter the ventricles. The ventricles also passively fill during diastole due to the positive pressure of the closed loop circulation. During atrial systole, the atria do not fully contract. This allows venous blood flow to continue to pass through the contracted atria and into the ventricles as blood continuously flows into the atria. The venous flow provides the majority of filling pressure to the ventricles with atrial contraction contributing a small but significant amount of the total systolic volume. As the ventricles fill, the ventricular pressure becomes greater than the pressure in the atria. This causes the atrio-ventricular valves to close before ventricular
systole takes place. During ventricular systole the pressure in the ventricles is greater than that in the aorta and pulmonary arteries, which causes the aortic and pulmonary valves to open. Blood can then flow from the left and right ventricles, respectively. The ventricular valves close at the completion of the ventricular systolic cycle and remain closed until the next systolic cycle.

Essentially the atria act as a buffer to regulate the continuous venous flow into the ventricles which are producing an intermittent outflow. This regulation maximises the pumping efficiency of the heart [2]. In normal conditions, the ventricles do not completely fill with blood, there is an excess of diastolic capacity. Both ventricles have approximately equal volume. However, the left ventricle contains the majority of the cardiac muscle mass as it is required to pump oxygenated blood around the body.

2.1.2 Cardiac Electrical Activity

The pumping mechanism of the heart is facilitated by electrical stimuli which cause the muscles in the cardiac mass to contract. Cardiac muscle tissue consists mainly of myocardial cells. These cells differ from skeletal muscle cell as they have longer depolarisation periods and slower intercellular propagation. The prolonged depolarisation is due to the slower calcium channels of the myocardial tissue. Proteins within the myocardial cells which react to electrical stimuli are responsible for this contractile nature. The pacemaker cells are non contractile, but leak negative ions so that the cells accumulate a potential. The pacemaker cells produce an electrical impulse when their potential breaches a threshold. The impulse conducts into nearby myocardial cells and throughout the whole cardiac mass. This allows positive ions to breach the cell membrane, altering the negative resting potential of the myocardial cells. The depolarisation of the myocardial cells causes them to shorten and contract; this is the mechanism of the systolic period of the respective chamber. The depolarising charge is conducted onto neighbouring cells to form a wavefront of propagation. When the positive ions are ejected the myocardial cells relax and lengthen allowing the diastolic expansion of the chambers.

2.1.3 Cardiac Action Potentials

The cardiac cells consist of several chemical channels which enable the conduction process and produce the two types of cardiac action potential [1]. The bulk of cardiac tissue, that which is found in the atria, ventricles and conducting fibres (Purkinje and Bundle of His), exhibit fast response. These fast response tissues contain both slow
calcium and fast sodium channels. The fast response sodium channels are common to both cardiac muscle cells and skeletal muscle cells. The slow response calcium channels are unique to the myocardial cell. The slow calcium channels prolong the depolarisation period of the myocardial cells and ensure maximal contraction of the cell. The pacemaker cells in the sino-atrial node, atrioventricular node and Bundle of His are slow response tissue, they do not contain the fast sodium channels [3].

Each cardiac action potential consists of four phases. These phases are shown for a fast response action potential in Figure 2. From the initial polarised state there is a positive depolarising deflection which is labelled Phase 0. This rapid depolarisation is due to the entry of sodium ions. Following the depolarisation, slight repolarisation may occur and if present it is labelled Phase 1. This corresponds to the exodus of potassium ions. The plateau of Phase 2 is created by the intake of calcium ions counteracting the outflow of potassium. Phase 3 terminates at the initial resting potential as potassium flow stops and sodium exits the cell. Phase 4 is the polarised, or resting, period between the end of repolarisation and the onset of depolarisation where potassium ions re-enter the cell. [4]
The slow response action potential present in the sino-atrial (SA) and atrioventricular (AV) nodal tissue is shown in Figure 3. Unlike the myocardial tissue, the pacemaker tissue is non contractile and self exciting. The self excitation of the pacemaker cells is due to a constant ion leak which means the cells have no true resting potential. Instead, they exhibit a gradual increase in potential during phase 4, also known as the pacemaker potential. The long depolarisation in phase 0 is due to slow the influx of calcium ions which is initiated once the pacemaker potential has breached an internal threshold, typically around -40 to -30 mV. The repolarisation in phase 3 occurs as calcium channels close and potassium ions flow out of the cell. Repolarisation of the cell is complete when the potential reaches approximately -60mV.

2.1.4 Conduction Path

The conduction path of the cardiac cycle is characterised by several events as shown in the flowchart of Figure 4. The initial pacemaker pulse is generated by a mass of self exciting tissue situated in the right atrium labelled the sino-atrial (SA) node. The initial impulse conducts into surrounding cells and forms a wavefront which propagates through the right atrium into the left atrium. The wavefront continues to propagate simultaneously through the atria until it reaches AV node situated at the inferior septal wall of the right atrium. The AV node produces a pulse which propagates through the common bundle of His into the left and right bundled branches.
These branches distribute the electrical impulse to the respective ventricular walls via the Purkinje fibres. The frequency of the cardiac cycle is regulated by the SA node. The cardiac cycle is initiated by the SA node at an excitation rate around 70 pulses per minute. The SA node is the heart’s primary pacemaker generally situated on the superior wall of the right atrium. Despite the term “node” it is essentially a complex of self exciting cell such that the initial activation in the cardiac cycle can be formed at alternative sites in the atrium [5] [6] [7]. Alternatively, initial impulse formation can occur at the AV node as is the case during atrial dysfunction such as atrial fibrillation. It has been suggested that the initial location of the P wave impulse is influenced by non-pathological factors such as heart rate [7]. After initial pacemaker excitation there is slight delay between right and left atrial systole because of the electrical conduction delay through the septal tissue.

Figure 4 Cardiac Activation Sequence
Conduction through the atria occurs solely through the myocardium of the right and left atria. For the impulse to propagate to the ventricles it must pass through the AV node and the conduction bundles.

The ventricles are electrically excited by the conduction path through the AV node, bundle of His, bundle branches, Purkinje fibres, endocardium and finally to the epicardium. Conduction from the atria is delayed through the AV node to allow the atria to complete their systolic cycle before the ventricles begin contracting. The AV pulse is propagated through the bundle of His and Purkinje fibres which disperse towards the apex of the ventricles. This causes the ventricles to contract from the apex towards their respective outlet valves, thereby ensuring that all blood in the ventricles is ejected. The AV node is also a self exciting pacemaker which can create 50 pulses per minute. However, the more frequent impulse originating from the SA node effectively controls the AV node. The AV node will revert to its intrinsic activation frequency if the SA node should fail or conduction from the atria becomes blocked [3]. The electrical impulses propagating through the cardiac mass are conducted to the surface of the epidermis where they can be observed by extracellular methods such as the ECG.

2.2 The Electrocardiogram

The ECG provides a time series recording of the electrical activity of the heart as seen at the surface of the epidermis. The origin of the ECG lies principally with Augustus Desiré Waller and Willem Einthoven. Waller was the first to record a human electrocardiogram in 1887 using a mercury capillary electrometer arrangement [8]. Einthoven later improved upon the instrumentation used to record the ECG and introduced the basic nomenclature used to describe the characteristics of the ECG. He also proposed a set of bipolar limb leads (I, II and III) which capture the cardiac electrical activity in the frontal plane. The main characteristic waves in the ECG are the P wave, the QRS complex, the T wave. In addition to these, several intervals between characteristics waves, such as the PR and ST segments, are measured. These characteristics are marked on a single ECG beat in Figure 5.
The P wave is representative of the initial sino-atrial pulse and its subsequent conduction through the right and left atria and the excitation of the atrio-ventricular node. The QRS complex is representative of Ventricular depolarisation whilst the T wave represents the repolarisation of the Ventricles. The contributions that each segment makes to the ECG can be seen in Figure 6. As previously mentioned, the ventricles consist the majority of the muscle mass of the heart and this is reflected by their contribution to the electrical to the ECG.
2.3 Recording

Currently most clinical ECGs are recorded using either a dedicated electrocardiograph or a peripheral attachment to a computer. Recordings can then be reproduced onto a thermal paper print or stored digitally for future reference. The advent of digital recording and storage of the ECG has proved advantageous in several respects. The use of digital ECG recordings has been shown to provide considerable improvements in diagnostic legibility of the ECG traces [9]. Additionally, offline noise reduction can be applied to the digitally recorded ECG signal. Storage of ECG has also made it possible to create common databases for the collation and dissemination of ECG data [8]. Digital ECG recordings can vary in sample rate from 250-1000Hz and precisions of 8-16 bits. All the ECG recordings used in this study were recorded using either the Micromedical™ Cardioview© system or the SynAmps™ system. Both essentially comprise of a multiple acquisition channels which feed into an amplifier and Analogue to Digital converters. The signals can then be digitally stored or printed. The Cardioview package records the ECG at a sampling frequency of 1000Hz which is then downsampled to 500Hz at a precision of 16 bits. The leads are connected via adhesive pads to the skin of a patient, conduction is from the epidermis to the lead is through a conductive gel compound. Recording points on the patient should be properly prepared to remove oil and acid from the surface so as to improve conductivity.

2.3.1 Lead Systems

There are several accepted configuration for recording the ECG, the most common clinical practice is to record the limb leads or the 12 lead ECG. Additionally there are configurations which have been developed for vectorcardiography and mobile ECG recording.

2.3.1.1 12 Lead

In practical clinical situations the ECG is the most commonly recorded using the 12 lead configuration. The 12 lead ECG records the cardiac electrical activity in three dimensions. These dimensions can be approximately described by the planes in Figure 7. The limb leads represent the frontal or anterior plane which covers the front of the torso. The precordial leads V1 to V6, shown in, represent the transverse plane which cuts horizontally through the torso and provides an optimal view of the left ventricle. Additionally leads aVF and Lead V2 are considered to be perpendicular and represent the sagittal plane which is perpendicular to both the frontal and transverse planes [1].
positioning includes 6 limb leads, which can either be individually recorded or derived from two or three limb leads, and 6 chest leads. The original limb lead set of I, II and III proposed by Einthoven are bipolar leads. They require a positive and negative pair of electrodes to measure change in potential. The positive electrodes are arranged on the left inferior side of the body as this is the main direction of propagation from the atria and ventricles. This ensures that the ECG is majorly positive in these leads. Electrodes are placed on the hands and feet, the electrode on the right foot is used as an electrical reference for each bipolar lead. Digital ECG recorder reduce input bandwidth by recording only two limb leads, typically I and II, and derive the remaining leads mathematically using the Eindhoven’s triangle relationships. These simple summations are shown in 2.3-1.

\[
III = I - II \\
aVR = - \frac{1}{2} (I + II) \\
aVL = I - \frac{1}{2} II \\
aVF = II - \frac{1}{2} I \\
aVR + aVL + aVF = 0
\]

The leads aVR, aVL and aVF are augmented unipolar V leads created with the limb lead electrodes. Unipolar leads measure potential between one positive electrode and the summation of several other electrodes which form a central reference terminal [1]. The central reference terminal is created by combining the three limb leads through a resistive bridge. This reference point is used as the negative pole with an exploring lead making the positive pole. When the augmented V leads are actually recorded, the central terminal is modified by decoupling the exploring limb lead from the reference terminal. Hence combining the electrodes from the left and right arms and comparing them to the electrode on the foot produces the aVF lead. The V leads, V1-V6, are unipolar leads recorded using the central terminal and an extra exploring electrode. These electrodes are placed around the chest to form a horizontal plane.
2.3.1.2 Frank Lead (XYZ)

The Frank lead system is a combination of 10 electrodes which provides the optimal lead combination for vectorcardiography. The electrodes are combined to produce 3 orthogonal vector leads; X, Y and Z. Leads X and Y describe the frontal plane and leads Y and Z describe the sagittal plane. This lead system allows the propagation of conduction through the heart to be viewed as a vector wavefront and has superseded vectorcardiography [1]. This can greatly enhance the information provided by the MRI or ultrasound images which show the mechanical movement of the heart. Typically the XYZ lead configuration is used with high resolution signal averaged ECGs in non practical cardiology applications because of the long recording time required.
2.3.1.3 Holter ECG

The Holter is a standalone portable ECG recording device that requires two points of contact on a patient. These points are generally on the front and back of the torso such that they create a recording axis through the heart, the short axis. This provides a good perspective of the left ventricle and the stages of right and left atrial depolarisation similar to lead V1. The Holter was designed to provide constant 24hr monitoring of patients with intermittent arrhythmia such as paroxysmal atrial fibrillation [1]. Longer recordings of the ECG are advantageous in studying the transient nature and observing any early onset indicators of such conditions. More recent implementations have made use of a modified electrode placement to provide several unipolar leads in the V5 and modified aVF directions. These leads are created using alternate placements of the limb electrodes.

2.3.1.4 Alternative Leads for P wave Emphasis

Often the electrical signal produced by the atria is relatively small in comparison to those originating from the ventricles. In some cases the P wave may be indiscernible in the standard leads. It would therefore be desirable to use an alternative placement of electrodes to better visualise the P wave. One such option is the oesophageal lead, which is inserted down the throat of a patient to provide a better representation of the atrial activity similar. This also effectively removes the baseline drift associated in electrical potentials recorded at the epidermal surface. This lead is rarely used in clinical practice because of the considerable discomfort to the patient. Another option is the bipolar S5 lead in which the positive electrode is positioned inferior to the V1 electrode at the fifth intercostal space near the sternum. The negative electrode is placed at the top intersection of the sternum, the manubrium. This electrode can greatly improve the visibility of the P wave. Both options are rarely used in daily clinical practice. Ultimately the lead configuration used is a matter of practicality and necessity, however all lead configurations are susceptible to the same interferences from muscular activity and external noise sources [10].

2.3.2 Conditions and Noise Sources

There are several types of additive noise which may be contained within an electrocardiogram which may affect the quality and overall diagnostic legibility of the recorded ECG and need to be considered and minimised during recording or before diagnosis.
2.3.2.1 Muscle Noise

Just as the heart produces electrical impulses so do other muscles in the body, these electrical impulses can also be detected by the ECG leads. Unlike the Cardiac muscles other muscles do not have a regulated cycle and therefore impulses generated by these muscles will not be represented as characteristic waveforms. This noise is usually of a low amplitude and high frequency and can be either sporadic or consistent throughout the recording.

2.3.2.2 Baseline wander

This noise is usually manifested as a very low frequency component throughout the entire recording. Whilst a patient is being monitored their body retains an electrical potential in relation to the surrounding environment, this potential may vary in time with slight environmental changes or by random fluctuations in the surface potential of a patient.

2.3.2.3 Mains noise

This noise is caused by interference from mains power sources being induced onto the recording leads of the ECG which introduces a sinusoidal component into the recording. In Australia this component is at a frequency of 50Hz. The interference created by this noise may either be sporadic in the case where a notch filter is employed in the recording process or consistent throughout the recording when a notch filter is not present.

2.3.2.4 Movement Artefact

A patient provides a small DC offset to the recorded ECG; this offset is common to the voltages at each lead and is of such small amplitude that it creates no great interference. However when a patient moves, either entirely or just a limb, there is a sharp offset created which is manifested in the ECG as a large discontinuity followed by a gradual return to equilibrium. This gradual return is usually a result of filtering during the recording process where low and high pass filters can be implemented to reduce both muscle noise and baseline removal.

2.3.3 Display

Conventionally the 12 Lead ECG is displayed as four columns each containing three leads; the first column contains leads I, II and III; the second column aVR, aVL,
aVF, the third column V1, V2, V3 and finally V4, V5 and V6 in the fourth column. Alternatively the Leads maybe arranged into two columns signifying the two different planar lead groups. The frontal plane leads are aligned top to bottom Leads aVL, I, -aVR, II, aVF, III which represents the orderly sequence of the leads in a clockwise direction. The transverse plan leads are arranged V1 through V6 which represents the orderly direction of the leads starting from the right of the patient’s sternum through to the side of their torso. This is sometimes referred to as the Swedish orderly representation. [10]

Figure 8 Conventional ECG Display

2.4 Interpretation of the ECG

Manual diagnosis is performed by a Cardiologist who examines the ECG recording and makes an educated interpretation of the waveform. The diagnosis of an ECG recording is made by observing the duration and morphology of the characteristic waves and intervals and any subsequent changes in these characteristics. Nine features are listed by Wagner [10] for systematic evaluation of the ECG. These features include morphologies of the P, QRS, T and U waveforms and several intervals which are also of diagnostic importance. These intervals include the R-R interval, P-R interval, ST segment and the corrected QT (QTc) interval. With the sheer weight of research into automated analysis there has been an increase in its popularity in cardiology. However, it is used only as a preliminary step before manual diagnosis, usually to determine heart rate and segment length. Various software packages offer automated analysis and employ different techniques to determine an ECG’s characteristics; however each package requires a heuristic set to provide a diagnosis. Micromedical provides an automated analysis package with their Cardioview© system that contains predefined heuristic sets for the diagnosis of recordings, however this diagnosis is usually considered as suggestive by the practicing
The simplest measures of the cardiac cycle to assess are those which describe rate and regularity.

### 2.4.1 Rhythm

Heart rate is defined by the R-R interval; this is the interval between successive R waves in a designated rhythm strip lead. A healthy heart rate being in the area of 60-100 beats per minute (bpm) and displays a regular pattern. The rate of the cardiac cycle is governed by the autonomous nervous system, of which the SA node is a part. This system controls involuntary actions in the cardiac muscle and glandular tissue throughout the body. Minor variations can be introduced to a resting cardiac cycle by the relative respiratory cycle. Typically an average of the R-R interval is calculated for a period of 10s in practical situations. A regular rhythm will not have significant variation in rate. A regular cardiac cycle should display a 1:1 ratio of atrial and ventricular events, such that every P wave precedes one QRS complex and T wave.

#### 2.4.1.1 P wave

The P-wave represents the depolarisation of the atrium and is most prevalent in Lead II which represents the long axis view of the heart. The accurate identification and analysis of the P wave is of vital importance in early identification and diagnosis of many cardiac arrhythmias [11]. The P wave and PR interval are indicative of atrial fibrillation and other conduction problems in the atrium [10]. Abnormal P wave morphology is also indicative of atrial enlargement and congenital heart disease. The P wave can also become less pronounced with age and therefore more difficult to read and diagnose which can make identification of the P wave, and consequently, diagnosis extremely difficult. The normal P wave should be a smooth contour from onset to offset regardless of whether it is mono or biphasic. The P wave duration is generally less than 0.12 seconds and should not exceed 0.2 seconds. The amplitude of the P wave should be less than 0.2 mV in the limb leads and less than 0.1mV in the precordial leads. The P wave is generally monophasic in all leads except V1, which represents the short axis of the heart such that left and right atrium contributions appear as opposite polarities in the ECG. The general axis of the P wave should be between 0° and 75° where lead I is on the 0° axis in the limb, or frontal, plane. [10] The morphology of the P wave can be indicative of several cardiac diseases such as atrial enlargement, stenosis of the cardiac valves and constrictive pericarditis [12]. P wave morphology can change with age, usually P wave duration lengthens [13] [14] and P wave dispersion also increases [15]. Due to its relatively low amplitude, close proximity to
the QRS complex and subtle changes in morphology, the P-wave presents a challenging problem in terms of identification, extraction or enhancement.

2.4.1.2 PR Interval

The PR interval is defined as the time from atrial excitation to the onset of ventricular excitation. This is represented as the trace from the beginning of the P wave through to the onset of the QRS complex. This period encompasses the propagation of the initial sino-atrial impulse through the atrial myocardium to the AV node and along the bundled branches to the Purkinje fibres. The slow conduction through the AV node represents a significant portion of the PR interval. This means that the PR interval responds to changes in heart rate, shortening as heart rate increases and lengthening as heart rate declines. The PR interval increases with age, in an adult heart the normal PR interval should be between 0.14 and 0.21 seconds in length. The PR interval is also used in conjunction with the P wave duration to indicate the presence of enlargement of either atria [16] and mitral stenosis [17].

2.4.1.3 QRS Complex

The QRS complex is the most prominent feature of the ECG and represents the depolarisation of the ventricles. The contour of the QRS complex is rather peaked and each peak has its own label. The label Q is applied to the first negative deflection, R is the first positive deflection and S is the deflection after the R wave. The prominence of each deflection is lead dependent and the QRS complex as a whole can exhibit wide amplitude variation in normal operation. Generally minimum amplitudes below 0.5mV in the limb leads and 1mV in the precordial chest leads indicate an abnormally low QRS complex [10]. Due to the prominence of the QRS complex and the importance of the process it represents, there are many rules and characteristics used to diagnose it. The general normal limits of duration for the QRS are between 0.07 and 0.11 seconds.

2.4.1.4 ST and QTc segments

The ST segment is the label applied to the iso-electric segment between the final deflection of the QRS complex and the onset of the T wave. During this time the ventricular mass remains depolarised and contracted until the gradual onset of repolarisation and diastole. Severe elevation of the ST segment can be indicative of a pathological condition in the ventricles. However, elevation of the ST segment can also occur in young healthy individuals [10]. The QTc segment is the corrected QT segment which is defined as the period beginning at the onset of the first deflection of the QRS
complex and ending at the onset of the T wave, this effectively covers all ventricular activity. The correction of the QT interval takes into account the heart rate described by the RR interval which is the time between R waves.

2.4.1.5 T wave

The T wave represents the repolarisation of the ventricles after the depolarisation represented by the QRS complex. The T wave morphology is smooth with a slow onset and more rapid offset. Because of this the T wave is not a major focus of diagnosis on its own but takes on greater significance when considered as part of the QT interval. Normal limits of the T wave amplitude, like the QRS, have a wide range and amplitudes are typically higher in males. Like the P wave and QRS, the amplitude of the T decreases with age. [10] A summary of the normal limits of the main waves within the ECG is presented in Table 2.

<table>
<thead>
<tr>
<th>Table 2 Normal ECG Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NORMAL LIMITS</strong></td>
</tr>
<tr>
<td>Heart rate</td>
</tr>
<tr>
<td>P wave</td>
</tr>
<tr>
<td>PR interval</td>
</tr>
<tr>
<td>QRS complex</td>
</tr>
</tbody>
</table>

2.5 Atrial Disorders

Analysis of cardiac disorders and disease has been primarily fixated with left ventricular conditions. There are several anomalies associated with the atrium. The most significant are the family of atrial tachyarrhythmia which include atrial flutter and atrial fibrillation. These conditions have increased prevalence in the presence of structural heart disease such as atrial and ventricular hypertrophy.

2.5.1 Atrial Fibrillation and Atrial Flutter

Atrial fibrillation (AF) is an erratic firing of the atrial cycle caused by re-entrant waves of propagation. This manifests in surface ECG recordings as an absence of characteristic P waves and creates a total disruption to normal atrial electromechanical functioning. The electrical signature of AF may be indiscernible from muscular noise in many cases. There are several contributing factors to the development of AF. Structural
changes to the physical dimensions of the atria and electrophysiology of its tissue fibres have significant in the eventuation of AF. Chronic AF can exist in several states, most commonly termed paroxysmal, persistent and permanent. The term paroxysmal is used to describe an intermittent reoccurrence of this behaviour with sudden onset and offset. This produces spasmodic atrial behaviour which can lead to inefficiencies in the ventricular outflow. Most commonly this results in blood clots, thrombus, forming in the atria which restrict blood flow in the circulatory system. Restriction of blood supply to the heart or brain can result in ischemia or stroke. Paroxysmal AF may degenerate into persistent or permanent AF, thereby increasing the likelihood of stroke [4]. The related condition of atrial flutter is also principally due to re-entrant circuitry. Atrial flutter presents as a characteristic continuous oscillation bearing no relation to ventricular activity. The cause of atrial flutter is due to a re-entrant mechanism located in the right atrium [18]. The occurrence of atrial tachycardia has been linked to precursory atrial conditions such as structural change in the atria and electromechanical dysfunction [19] [20] [21].

2.5.2 Atrial Enlargement

Structural changes to the atria are usually due to chronic haemodynamic demand or an increased resistance to atrial outflow. The left atrium is especially susceptible to enlargement as it is part of the systemic circulation, buffering the major cardiac chamber, the left ventricle, which pumps oxygenated blood around the body. Increase in demand upon the left side of the heart can occur in healthy persons such as athletes, whose cardiovascular systems are subject to considerable exertion [22]. Alternatively, pathological conditions such as weak atrial contraction can result in dilatation of the atria as a means of compensating for poor systolic action [4]. Resistance to atrial outflow is associated with mitral stenosis, where the mitral valve becomes narrower, thereby restricting the flow of blood from the atrium into the ventricle. This can lead to thickening of the atrial walls, hypertrophy, to increase atrial pressure and maintain adequate atrial systole flow [17] [23].

ECG markers of left atrial enlargement include:

- Lengthening of the P wave’s duration [10]
- A significant negative deflection of the P wave in lead V1 [10]
- Bifid morphology of the P wave in the limb leads [12]
• P/PR ratio greater than 1.6 [16]

In contrast, the enlargement of the right atrium is less prevalent and more likely to be associated with congenital malformation [24] [25].

ECG markers for right atrial enlargement include:
• Peaked positive deflection in P wave of lead V1 [10]
• Early peak in P wave of lead II [12]
• P/PR ratio less than 1 [16]

Prolongation of the P wave is not exclusively symptomatic of left atrial enlargement. Pathological conduction delay between the two atria can also result in lengthened P wave durations.

2.5.3 Inter-Atrial Block

The most common conduction problem associated with the atrium is a delay or block of conduction from the right to left atria called inter-atrial block (IAB) [26]. This delay can manifest as a notching of the P wave (bifid) in several leads and often a lengthening of its duration. This bifid morphology of the P wave is a result of asynchronous conduction in the right and left atria. This can be due to either IAB [7] [27] or initial impulse formation occurring at alternative sites in the right atria which results in opposing wavefronts in the two atria [6] [28]. A significant correlation exists between the coexistence of IAB and left atrial enlargement resulting in left atrial electromechanical dysfunction [29]. Several studies have suggested that IAB could be linked to more severe conditions such as atrial flutter, atrial fibrillation and embolic stroke [30] [31]. The initial cause of IAB is not fully understood, this is reflected in the limited authorship of literature on the condition.

2.5.4 Effects of Aging

Changes in atrial conduction patterns and electrical activity may not necessarily be pathological. The effect of senescence in the cardiac mass can produce several physiological changes which affect atrial function and alter electrical conduction in the myocardium [14] [23]. Electrophysiological changes related to aging of the myocardium
can be observed as reduction in voltage potentials [32]. Aging causes tissue to lose elasticity or stiffen. Subsequently, the contribution of atrial contraction to ventricular filling is increased due to stiffening of the cardiac muscles impeding relaxation of ventricles. This in turn impedes passive filling which occurs when the atria and ventricles are both relaxed. The initial impulse formation in the sino-atrial node is also affected by aging [33]. Between the ages of 20 and 75 the number of sino-atrial cells can be reduced by up to 80% [34]. This can lead to the increase in prevalence of pathological conduction conditions such as IAB in aging patients [35]. Conversely, it can explain increases in healthy P wave duration, which have been shown to have significant positive correlation with increases in age [14].

2.6 Summary & Discussion

The electrocardiogram has been a fundamentally important tool in the field of cardiology since its invention and continues to be heavily relied upon despite the advance of more advanced techniques of cardiac imaging. The ECG is an inexpensive and simple cardiological procedure which effectively represents the electrical activity of the cardiac mass. Aging in the cardiac mass can alter electrophysiology, thereby increasing normal and pathological conduction delay in myocardial tissue. This can reduce the prominence of ECG characteristics, and in the case of the relatively small amplitude P wave, make detailed analysis difficult.
References:


Chapter 3

Signal Analysis Theory and Application

The ECG has been heavily scrutinised in the field of signal processing for several decades. Many applications of signal processing techniques have been discovered and developed for the ECG in all its forms, such as automated heartbeat and feature detection, preliminary diagnostic software and compression algorithms for ambulatory ECG. Applications such as these are based on several signal processing techniques which operate on the ECG in its original time series form data or transform the ECG. In this chapter, methods of signal analysis, statistical analysis, classification and discrimination which are used in the processing of physiological signals, specifically the ECG, will be presented. Transform techniques are particularly useful in signal analysis as they provide a different perspective of the signal being observed.

The approaches developed in this research have relied upon a particular transform technique and related processes called wavelet processing. This is a relatively recent field in mathematics and signal processing though similar time-frequency transforms have existed for sometime. The most notable and integral is the Fourier transform, a brief summary of these transform techniques is presented in this chapter as well as several other signal processing techniques which have been applied to the processing of ECG signals. In particular, current and past methods of classifying the ECG P wave are discussed. Attention is also paid to outlining the advantages and flexibility of wavelet processing as a means of signal analysis. To appreciate the advantages of wavelet transforms to the application of signal processing, it is first necessary to look at the Fourier analysis and its set of transforms.

3.1 Fourier analysis

Fourier analysis was originally suggested in 1822 by Fourier in his publication “Théorie Analytique de la Chaleur (The Analytic Theory of Heat)” where Fourier applied the
process to solve heat equations in the form of a trigonometric series; Fourier series. It has since, become a fundamental concept in many fields, especially in the area of signal analysis. Essentially, Fourier analysis transforms a signal or function from the time, or spatial, domain into the frequency, or spectral, domain. This produces several benefits for the analyst. Transforming the original function, or signal, to another domain means that new information and observations can be derived from it. The transform also allows complex time domain operations, such as convolution, to be computed as simpler multiplications in the frequency domain. [1] Central to the concept of Fourier analysis is the idea that a function, \( f(t) \), which is periodic, over \( T \), can be approximated by a weighted sum of sinusoidal and co-sinusoidal functions. The coefficients, \( c_n \), of this series form the function’s Fourier series. The periodic function can then be represented as such:

\[
f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\omega_n t}
\]  

(3.1-1)

Where \( \omega_n \) is the \( n \)th harmonic given by:

\[
\omega_n = 2\pi \frac{n}{T}
\]  

(3.1-2)

The complex term, \( e^{j\omega t} \), represents the sum of a co-sinusoid and sinusoid given by Euler’s identity:

\[
e^{j\omega_n t} = \cos(\omega_n t) + j \sin(\omega_n t)
\]  

(3.1-3)

The coefficients of the Fourier series are derived by:

\[
c_n = \frac{1}{T} \int_{0}^{T} f(t) e^{-j\omega_n t} dt
\]  

(3.1-4)

The sinusoidal functions have a finite linewidth (frequency) in the frequency domain. Therefore, each Fourier series coefficient in 3.1-4 corresponds to a discrete frequency, \( \omega_n \). The complex series coefficients provide amplitude and phase information about the original function at each of these frequencies. In the field of signal processing, the functions being analysed are predominantly continuous and non-periodic. Analysis of these signals is performed via the Fourier Transform.
3.1.1 Fourier Transform

The Fourier transform is used extensively in the signal processing field as a means of decomposing continuous time domain functions or finite energy signals, \( x(t) \), to their respective frequency domain, \( X(\omega) \). The transform is essentially an extension of the Fourier series, such that aperiodic signals can be considered as periodic, with a period approaching infinity. The Fourier transform is defined as:

\[
X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt
\]  

(3.1-5)

This form of the transform is also referred to as the radial Fourier transform. The Fourier transform has several desirable characteristics which have made it invaluable to the signal processing field. The sinusoidal basis function provides fixed time-frequency localisation, which means the transform is linear time invariant [2]. The finite line width of the sinusoid also allows for continuity of the decomposed frequency spectrum.

3.1.1.1 Fourier Transform Properties

There are several properties of the Fourier transform which are beneficial to signal analysis applications, the most relevant of these features are briefly discussed here. As previously mentioned, an attractive feature of Fourier analysis is the fact that complex convolution operations in one domain can be computed as simpler multiplication operations in the other domain. This is expressed mathematically as:

\[
x(t) \ast y(t) \Leftrightarrow X(\omega) \times Y(\omega)
\]

\[
X(\omega) \ast Y(\omega) \Leftrightarrow x(t) \times y(t)
\]  

(3.1-6)

Where the uppercase symbols indicate the frequency domain representation and the lowercase symbols are the relative time domain components.

3.1.1.2 Discrete Time Fourier Transform

In the digital signal processing world it is necessary to modify the continuous transforms to practically implement them. The most obvious constraint is the reality that a digital system cannot operate upon a continuous time function. The continuous time function, \( x(t) \), must be sampled at discrete intervals, \( T \). The sampling frequency, \( f_s \), is inversely proportional to the sampling period.
In practice the sampling frequency is determined by physical constraints of the codec interface and the nature of the signal to be sampled. Physiological signals such as the ECG require relatively low sampling rates when compared to other signals such as speech or music. Most modern digital ECG systems have sampling rates greater than or equal to 500Hz, this is sufficient to satisfy the Nyquist criterion for the diagnostic information within the ECG. Sampling the function means it is no longer continuous and the integral of the transform in (3.1-5) can be replaced by a summation:

\[
X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n/f_s},
\]

(3.1-8)

From (3.1-9), it is apparent that the frequency domain is now bounded by the upper limit of the sampling frequency. However, the frequency spectrum is still continuous; this is not a practical result for digital implementation of the transform.

### 3.1.1.1.3 Discrete Fourier Transform

An approximation of the DTFT can be made by generating Fourier coefficients at discrete frequency intervals, \( k \), to form a subset of the continuous frequency spectrum. Because this transform is now discrete in respect to both time and frequency it is labelled the Discrete Fourier transform (DFT) and is defined as:

\[
X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi \frac{k}{N}n}
\]

(3.1-9)

The main drawback with the Fourier transform is its insensitivity to localised temporal events.

### 3.1.2 Short-Time Fourier Transform

The Short-Time Fourier Transform (STFT), often referred to as the Windowed Fourier Transform (WFT), differs from the Fourier transform by incorporating a windowing function to allow for temporal localisation in the transform. This allows the frequency spectrum to be evaluated over short overlapping windows of the input signal. The STFT was originally presented by Gabor, who incorporated a Gaussian windowing function into the Fourier Transform [3].
\[ X(\tau, \omega) = \int_{-\infty}^{\infty} x(t)w(t-\tau)e^{-j\omega t} \, dt \quad (3.1-10) \]

Where \( \tau \) denotes temporal location and \( \omega \) frequency at which the transform is being evaluated in the time-frequency plane. The STFT is therefore a generalisation of the Gabor transform as it allows for any window function, real or complex, which satisfies the condition:

\[ W(0) = \int_{-\infty}^{\infty} w(t) \, dt \neq 0 \quad (3.1-11) \]

Where \( W \) denotes the Fourier transform of \( w(t) \). This implies that the windowing function has a nonzero component at the zero frequency. Essentially the window function is a lowpass filter. The function, \( w(t) \), can be as simple as a moving average square window, triangular window, or more commonly a Hamming, Blackman or, as previously mentioned, the Gaussian function. Each windowing function has specific frequency spectrum characteristics, defined by several parameters. Parameters such as sidelobe amplitude and mainlobe width determine the frequency masking and resolution properties of the windowing function. Of greater importance to the continuous analysis problem is the windowing function’s time-frequency localisation ability. This quality is quantified by the time-frequency product of the window’s time and frequency RMS radii, \( \Delta_w \) and \( \Delta_W \), respectively, and the window’s focal centre. Symmetric windows are centred at 0 in the time and frequency domains; \( w^* = W^* = 0 \). The time-frequency product is bounded by the Heisenberg uncertainty principle which gives:

\[ \Delta_w \Delta_W \geq \frac{1}{2} \quad (3.1-12) \]

The equality of (3.1-12) has been shown only to exist for functions of the form:

\[ f(t) = a e^{-bt^2} \quad (3.1-13) \]

Where \( b \) may be any real number and the coefficient \( a \) may be a scaling term. This is the basic form of the Gaussian pulse of (3.1-14). In this form the term \( \sigma^2 \) determines the spread of the pulse. The scaling term in front of the Gaussian exponential ensures that the area under the curve of the pulse is equal to one.
\[ g_p(t) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{t^2}{2\sigma^2}} \]

\[ \int_{-\infty}^{\infty} g_p(t) dt = 1 \]  

(3.1-14)

The STFT can also be evaluated as a discrete transform in discrete time, \( m \), and frequency, \( \omega \):

\[ X(m, \omega) = \sum_{n=-\infty}^{\infty} x(n) w(n - m) e^{-j\omega n} \]  

(3.1-15)

The discrete form of the STFT preserves the linear time invariance of the continuous form of the transform. Any time shift in the original signal produces an equivalent shift in the STFT result with a proportional shift in phase. Similarly, any frequency shift in the original signal, such as the modulation of a carrier frequency, will produce an equivalent shift in the STFT with no phase change. Therefore the STFT is a linear time-frequency transform [1].

### 3.1.2.1 Limitations

The choice of window function ultimately dictates the joint time-frequency resolution. It has been shown that the Gaussian window function provides the optimal performance in terms of joint resolution [4]. Ultimately the choice of window function is dependent upon the application. For example, triangular functions will provide better emphasis for event detection applications, as they emphasise features located in the centre of the window while greatly diminishing surrounding observations.

Limitations to this transform do exist. The most important limitation has already been touched upon. Although the STFT does provide time localised frequency information, the fixed window length constraint does present another problem, which is the trade-off between time and frequency localisation. Good temporal resolution is achieved by use of a narrow window; this effectively blurs low frequency components of the spectrum. Increasing the window width provides better localisation of lower frequencies while sacrificing temporal and high frequency resolution.

A secondary limitation arises from a discrepancy between Fourier’s original theory and its practical result. The Fourier series showed that a continuous periodic function could be well approximated by a weighted sum of sinusoids. However, at any
discontinuities in the function, the Fourier series representation produces a harmonic ringing known as the Gibb’s phenomenon. The search for a more flexible approach to continuous spectral analysis lead to the development of the wavelet transforms.

### 3.2 Wavelets

The area of Wavelets and their application in the field of signal processing has flourished since their popularisation in the late 1980s, as can be judged by the weight of publications in the area. Wavelet analysis has become popular in recent years as an effective means of analysing localised power and frequency components of time series data. By overcoming the fixed window length constraint, inherent in the STFT, wavelet analysis produces superior time-scale resolution and allows for tailored analysis parameters [5] [6]. The fixed window constraint of the STFT is overcome by using the dilation and translation of wavelet function basis to perform multi-resolution analysis. A basis function, \( \psi(t) \), can be dilated and translated by the parameters \( a \) and \( b \) respectively.

\[
\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi \left( \frac{t - b}{a} \right) 
\]

(3.2-1)

The equation in 3.2-1 shows the scaling of the mother wavelet to produce a daughter wavelet, \( \psi_{a,b}(t) \). The term in front of the mother wavelet normalises the daughter wavelet so that it has unit energy. The mother wavelet can be any number of finite functions which satisfy the admissibility criteria.

#### 3.2.1.1 Admissibility criteria

An acceptable function basis must have a zero mean, which implies that, unlike the STFT, a wavelet function base must be zero at \( \omega = 0 \). Therefore the basis function behaves like a bandpass filter. The admissibility condition is expressed in (3.2-2). This also ensures that the integral of 3.2-3 is finite.

\[
\Psi(0) = \int \psi(t).dt = 0 
\]

(3.2-2)

\[
C_{\Psi} = \int \frac{|\Psi(\omega)|^2}{|\omega|} d\omega \left[ ^{\infty}_{-\infty} \right] 
\]

(3.2-3)

Where \( \Psi(\omega) \) denotes the Fourier transform of the wavelet basis. The finite term, \( C_{\Psi} \), is important for the reconstruction of the original signal using the inverse wavelet
transform. Given these admissibility conditions a range of basis functions can be defined, these functions can then be categorised by their characteristics.

### 3.2.2 Wavelet Properties

There are several characteristics by which a wavelet basis function can be characterised the most commonly referred to are orthogonality, compact support, symmetry and regularity. Orthogonality is an important fundamental concept in linear algebra and signal analysis. The inner product of two discrete signals is essentially an indicator of how alike the signals are. The inner product of a discrete signal, $x$, with itself is defined as:

$$\langle x, x \rangle > 0$$

(3.2.4)

Signals that are completely dissimilar will produce a zero inner product and are said to be orthogonal:

$$\langle x, v \rangle = 0$$

$$\therefore x \perp v$$

(3.2.5)

Orthogonality in wavelet function bases is important for applications requiring efficient data representation, as is the compact support characteristic which allows for perfect reconstruction of a decomposed signal. The compact support of a wavelet basis function is analogous to the finite impulse response of a filter. Compact support indicates that all energy from an impulse input into a wavelet transform will be contained within a finite envelope. Orthogonal wavelet bases are associated with the dyadic wavelet transform which decomposes time series data into scales of non-overlapping frequency bands. Several well defined families of wavelet functions exist, and the choice of which family of functions to use is dependent upon the application. In feature detection or analysis applications it is desirable to use a basis function which is highly correlated to the features being analysed.

### 3.2.3 Continuous Wavelet Transform

The continuous wavelet transform (CWT) provides a continuous time-frequency output for a continuous analogue signal, $x(t)$, and is defined as:
\[ C(a,b) = \int_{-\infty}^{\infty} x(t)\psi_{a,b}^*(t)dt \]  

(3.2-6)

Where \( C(a,b) \) is the wavelet transform coefficients at scale \( a \), and time \( b \). The term, \( \psi^* \), represents the complex conjugate of the basis function. The CWT produces coefficients at a continuous set of frequency and time intervals, when applied to discrete signals this becomes an approximation of the CWT as the time intervals are dependent upon the frequency at which the signal has been sampled. Equation 3.2-6 shows that the CWT is essentially a convolution of the original signal, \( x(t) \), with the conjugate of the daughter wavelets at each scale. As with the Fourier transform, an inverse equation also exists. The inverse Wavelet transform is defined as:

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} C(a,b)\psi_{a,b}(t)\frac{da.db}{a^2} \]  

(3.2-7)

The term \( C_\psi \) is the integral in 3.2-3 and is dependent upon the wavelet basis used in the transform. The choice of mother wavelet also determines the relationship between wavelet transform scale and Fourier frequencies.

\[ f_a = \frac{f_c.f_s}{a} \]  

(3.2-8)

This relationship is approximated by 3.2-8 [7], where \( f_s \) is the sampling frequency, \( f_c \) is the central frequency of the wavelet basis function and \( f_a \) is the relative Fourier frequency of the wavelet scale, \( a \).

### 3.2.4 Discrete Wavelet Transform

The Discrete Wavelet Transform (DWT) is a variant of the CWT where coefficients are computed at discrete non-overlapping time and frequency intervals. Most commonly dyadic or half band frequency intervals are used to decompose a signal into frequency octaves. The input signal is filtered by a dilation of the analysis filter \( h_a(n) \) derived from the wavelet base and the resultant signal can then be downsampled and the process repeated. In this way the dyadic wavelet transform is similar to the subband coding process.
Each band of filtered and downsampled coefficients is referred to as the approximation coefficients, \( y_i(n) \). The set of residual coefficients created by the lowpass filter on the final level of decomposition are referred to as the detail coefficients. These lowest level coefficients are not downsampled so that all information required to reconstruct the original signal is retained.

### 3.2.4.1 Limitations

The DWT is attractive for signal analysis applications because the reduction of complexity of computation and redundant information compared to the CWT. These advantages are gained by computing the transform at octave scales and down sampling the residual coefficients after each level of transform. The latter process introduces shift variance into the DWT. This means that any time shift in the input signal will not translate into a time shift in the output coefficients but rather a different set of coefficients.

\[
\begin{align*}
y(n) &= H\{x(n)\} \\
\tilde{x}(n) &= x(n + m) \\
\tilde{y}(n) &= H\{\tilde{x}(n)\} = y(n + m)
\end{align*}
\]

This is a main detractor in the use of the DWT for some analysis purposes such as event detection. The simplest method of overcoming this shortfall is to discard the down sampling procedure from the DWT. This means detail coefficients of the transform are retained at every decomposition level. Boundary effects have been discussed in relation to the Fourier transform and the use of windowing functions in the short time Fourier transform. The problem arises from the extremities of finite data series. These extremities represent sudden onsets and offsets in the analysis process which introduce a broad spectrum noise component into the frequency domain. Because the CWT uses dilations of the wavelet basis function, the influence of boundary effects is dependent upon the scale of wavelet dilation. At higher scales the analysing wavelet function has a much longer duration and the boundary effects are therefore present in more coefficients around the extremities of the signal. Consideration of these effects is required when analysing finite data records.
3.2.5 Applications

Wavelet processing has numerous applications in the field of signal processing. As discussed earlier, it provides a superior method of continuous spectral analysis. This has seen wavelet analysis applied to several forms of time series data. The flexibility of the CWT’s time-frequency localisation is perfectly suited to the analysis of weather oscillation patterns [8]. Where wavelet analysis can highlight short, mid or long-term patterns in daily weather observations recorded over a period of decades. Wavelet processing has also been extensively applied to the EEG and ECG [9] using a variety of approaches. Wavelet energy spectrum and entropy are popular measures used in wavelet based analysis [10]. The wavelet energy spectrum has previously been used in analysis applications for time series data such as seismic recordings, satellite phase delay data for atmospheric and ionospheric analysis [11]. In most cases the wavelet energy spectrum is then analysed for maximum correlations and trends in these correlations. Additionally wavelet processing may be used to enhance or improve the visualisation of signals, especially in the realm of physiological signals. Because of the flexible time-frequency localisation properties, wavelet transforms can be applied to problems such as separation of foetal and maternal ECG signals [12] and the de-noising of biomedical signals [13]. Both problems are addressed by sufficiently isolating the processes to a subset of non-overlapping wavelet scales. In the case of foetal ECG extraction, the subset of wavelet scales containing the majority of the foetal ECG’s energy can be reconstructed independently of the original ECG. Similarly, for the problem of de-noising, significant components of the original signal can be well separated from noise by selectively suppressing wavelet coefficients which do not correspond to them.

3.3 Multivariate Analysis

The scientific approach to understanding a process’ behaviour relies upon observing a number of variables derived from the process. The interdependencies between these variables and trends in their relationships can also provide insightful information about the original processes. Identifying significant variables and quantifying their interactions is the objective of multivariate analysis. Multivariate analysis has applications in many fields, such as medical analysis, psychological research, marketing and forecasting. The importance of multivariate analysis to this work is to identify significant measures derived from the wavelet transforms of the P wave.
3.3.1 Descriptive Statistics

It is often impractical or impossible to observe an entire population within the confines of a scientific study. Therefore it is important to choose robust measures which describe the sample populations used. Descriptive statistics aim to summarise sample data such that a generalised description of the processes may be obtained. Common descriptive statistics look at the tendency of a data series, such as its expected value or mean, or at its variability. Analysis of variability is particularly useful when comparing the same characteristic of differing processes or functions.

3.3.1.1 Mean and Expected Values

The mean of a random variable, or observations of process, is an indication of the central tendency of the variable or process. The mean of a series of \( N \) observations of \( x \) can be defined as:

\[
\bar{x} = \frac{1}{N} \sum_{i} x_i
\]

(3.3-1)

If \( x \) is a discrete random variable, with a certain probability density function \( P(x) \), its mean is the expectation, or expected value, of \( x \):

\[
E(x) = \sum_{i} x_i P(x)
\]

(3.3-2)

For a uniformly distributed variable, the expectation is equivalent to the mean.

3.3.1.2 Variance and Standard Deviation

Variance is a measure of a variable’s dispersion and deviation from its expected value. The variance of a random variable, \( x \), is defined:

\[
Var(x) = \frac{1}{N} \sum_{i} x_i^2 - \left( \frac{1}{N} \sum_{i} x_i \right)^2
\]

(3.3-3)

\[
Var(x) = E(x^2) - (E(x))^2
\]

Standard Deviation, \( \sigma_x \), is another form of expressing a variable’s dispersion. It is the positive square root of the variance of a random variable.
\[
\sigma_x = \sqrt{\frac{1}{N} \sum x_i^2 - \left( \frac{1}{N} \sum x_i \right)^2}
\]

(3.3-4)

\[
\sigma_x = \sqrt{Var(x)}
\]

Both these measures of variability work best for variables which have a symmetric distribution around an expected value.

3.3.1.3 Correlation

The correlation coefficient gives an indication of the relationship between two variables. Given \( n \) observations of two independent variables \( x \) and \( y \), the Pearson correlation coefficient is calculated from the sum of squares of each variable.

\[
\begin{align*}
SS_{xx} &= \sum x^2 - n \bar{x}^2 \\
SS_{yy} &= \sum y^2 - n \bar{y}^2 \\
SS_{xy} &= \sum x^2 y^2 - n \bar{x} \bar{y}
\end{align*}
\]

(3.3-5)

These values represent un-normalised variance and covariance values of the variables \( x \) and \( y \) [14]. The correlation coefficient, \( r^2 \) (or \( R \)), is given by:

\[
r^2 = \frac{SS_{xy}^2}{SS_{xx} SS_{yy}}
\]

(3.3-6)

The correlation coefficient gives a measure of the strength of a linear regression fit between the observations of \( x \) and \( y \), of the form:

\[
\begin{align*}
y &= a + bx \\
x &= a + b'y
\end{align*}
\]

(3.3-7)

\[
r^2 \equiv bb'
\]

Where \( b \) and \( b' \) are regression coefficients and \( a \) is a constant. The correlations coefficient therefore describes the proportion of variance of explained by the regression fit. Pearson’s correlation method relies upon the assumption of a normal distribution of the observed variables.
3.3.2 Probability Distribution

The goal of analysing a sample population is to find a generalised model or set of observations which will fit the entire population with a reasonable level of confidence. Probabilistic models offer simplicity of construction and good generalisation characteristics.

3.3.2.1 Normal Theory

The normal distribution is a relatively robust estimation of most random processes. Central limit theory suggests that if the original process does not strictly adhere to a normal distribution, a sample of the process should tend towards a normal distribution [15]. For sample populations, the normal distribution, shown in Figure 9, is often more useful in identifying outliers, rather than verifying that the sample adheres to distribution’s probability density [15]. Nevertheless, the normal distribution is integral to the technique of linear discriminant analysis, discussed later in this chapter.

\[
P(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]

(3.3-8)

![Figure 9 Normal Distribution](image-url)
3.3.2.2 Quartiles

The inter-quartile range (IQR) is a measure used to quantify the dispersion of data, especially useful for the case of non-uniform distributions. The IQR is defined as the range between the 25\textsuperscript{th} and 75\textsuperscript{th} percentiles, or first and third quartiles, and therefore contains the central 50\% of the original data. The quartiles are calculated from a cumulative frequency distribution of the observed variable. The second quartile, \( Q2 \), is the median point of the distribution.

\[
IQR = Q3 - Q1 \\
QV = \frac{Q3 - Q1}{Q3 + Q1}
\]

(3.3-9)  
(3.3-10)

Figure 10 displays a non-uniform distribution with its first, second and third quartiles marked as \( Q1 \), \( Q2 \) and \( Q3 \) respectively.

These individual descriptors provide information about a single observation of a process. To derive a meaningful interpretation of a series of observations requires a multivariate analysis which can identify significant relationships between variables and reduce the amount of information needed to describe the process.
3.3.3 Factor Analysis

There are several multivariate techniques which aim to reduce the order of a large set of observations. Factor analysis examines the relationships between random variables observed for a sample population of individuals within a specific group. The aim is to describe a process in terms of a set of unobserved factors derived by analysing the covariance relationships between the observed variables.

3.3.3.1 Common factor model

The common factor method hypothesises that the initial vector, $X$, of observed variables, with a dimensionality of $p$, can be efficiently represented by a linear combination of unobserved common factors, $F$, and additional errors, $\varepsilon$, of order $m$. This model is expressed in matrix form as:

$$X = \mu + L \cdot F + \varepsilon$$

(3.3-11)

The vector $\mu$ is the mean vector and $\Sigma$ is the covariance matrix of $X$. The matrix $L$ contains the factor loadings which weight the factors in the linear combination. The elements of vector $\varepsilon$ are also referred to as the specific factors, as they tend to be uniquely related to their respective variables in $X$. For the factor model to be orthogonal the covariance must have the following structure:

$$Cov(X) = \Sigma = LL' + \Psi$$

(3.3-12)

The total variance of $X$ is explained by the variance of the common factors and the specific factors. The variance explained by the common factors is the communality, $LL'$. The remaining variance attributed to the specific factors is the uniqueness, $\Psi$. The total variance of each variable in $X$ can be expressed as:

$$Var(X_i) = \sigma^2_{i,i} = l_{i,1}^2 + l_{i,2}^2 + \ldots + l_{i,m}^2 + \psi_i \quad i = 1, 2, \ldots, p$$

(3.3-13)

Where, $\psi_i$ is the specific variance of the $i$th variable. A factor analysis of $X$ will only be useful if the variables are related. This is ascertained by the off-diagonal elements of the covariance matrix with larger values indicating that the variables are related. The initial step in constructing the factor analysis is the estimation of the factor loadings. There
are several methods to estimate the factors and factor loadings. Principal component analysis is a popular technique for the solution of this common factor model.

### 3.3.3.2 Principal Component Method

The principal component analysis returns the same number, \( p \), of components as the original variables of \( X \). This means that the original data can be restored from the derived components. The method is initialised by the factoring of the covariance matrix in terms of its eigenvalue-eigenvector pairs, \((\lambda_i, e_i)\).

\[
\Sigma = \lambda_1 e'_1 + \lambda_2 e'_2 + \ldots + \lambda_p e'_p + \psi_i
\]

\[
= \begin{bmatrix}
\sqrt{\lambda_1} e'_1 \\
\sqrt{\lambda_2} e'_2 \\
\vdots \\
\sqrt{\lambda_p} e'_p
\end{bmatrix}
\begin{bmatrix}
\sqrt{\lambda_1} \\
0 \\
\vdots \\
0
\end{bmatrix}
+ \begin{bmatrix}
\psi_1 & 0 & \ldots & 0 \\
0 & \psi_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \psi_p
\end{bmatrix}
\]

The pairs are ordered by size of the eigenvalues such that the first component describes as much of the variability of the original data, with each successive component describe as much of the residual variability as possible. In practice the total population covariance matrix, \( \Sigma \), is unknown and therefore approximated by the sample covariance matrix \( S \). Ultimately the goal is to derive a smaller number, \( m \), of factors which explain a appropriate amount of the original data’s variance. The first \( m \) factor loadings for the sample covariance matrix are estimated by:

\[
\hat{L} = \begin{bmatrix}
\sqrt{\hat{\lambda}_1} \hat{e}_1 \\
\sqrt{\hat{\lambda}_2} \hat{e}_2 \\
\vdots \\
\sqrt{\hat{\lambda}_m} \hat{e}_m
\end{bmatrix}
\]

The specific variances for each variable are estimated by:

\[
\hat{\psi}_i = s_{i,i} - \sum_{j=1}^{m} l_{i,j}^2
\]

The diagonal elements of \( S \) give the sample variance. The total sample variance is the sum of these elements. The selection of the number of factors to retain from the solution is either determined by prior conditions or by a total variance condition. The eigenvalues of the estimated factor loadings describe the respective factor’s contribution to the total sample variance. Therefore the number of factors can be determined by their
cumulative contribution to the total sample variance. The principal component analysis method is a useful data reduction method in its own right.

### 3.3.3.3 Singular Value Decomposition

Singular Value Decomposition (SVD) is a data reduction technique similar to PCA. It is a mathematical transform that can be used to efficiently represent a matrix. This allows for the efficient coding of images by means of lossy compression. The SVD of a matrix \( A \) is defined as:

\[
A = USV^T \quad (3.3-17)
\]

\( S \) is a diagonal matrix whose leading diagonal elements are known as the singular values. The matrices \( U \) and \( V \) are the left and right single value matrices. Singular value decomposition is used in signal processing applications primarily such as noise removal [16] and image compression [17]. The SVD is also employed in the derivation of system models [18].

### 3.4 Discrimination and Classification

The processes of discrimination and classification are multivariate techniques which aim to separate observations of random variables into distinct groups and allocate new observations of these variables into the defined groups [19]. Specifically, discrimination attempts to describe the differentiating features of distinct groups and to identify discriminating variables which maximally separate the groups. Classification aims to sort cases into predefined groups thereby deriving a rule or function which can allocate new cases. The simplest case, for both classification and discrimination, is the problem of defining two groups using a linear boundary.

#### 3.4.1 Linear Classification of Two Groups

For \( N \) observations of a random variable, \( x \), observed for two groups, \( \pi_1 \) and \( \pi_2 \), with equal probabilities of occurrence we define two regions, \( R_1 \) and \( R_2 \). Observations of \( x \) falling within \( R_1 \) are classified as belonging to group \( \pi_1 \) and conversely, observations falling within \( R_2 \) are classified as belonging to \( \pi_2 \). If the two groups have overlapping distributions there will be a level of misclassification. The probability functions in 3.3-15 to 3.3-18 give the definitions of correct and incorrect classifications for both groups.
\[ P(x \in R_1 | \pi_1) = P(1|1) = \int_{R_1} f_1(x)dx \]  

(3.4-1)

\[ P(x \in R_1 | \pi_2) = P(1|2) = \int_{R_1} f_2(x)dx \]  

(3.4-2)

\[ P(x \in R_2 | \pi_1) = P(2|1) = \int_{R_2} f_1(x)dx \]  

(3.4-3)

\[ P(x \in R_2 | \pi_2) = P(2|2) = \int_{R_2} f_2(x)dx \]  

(3.4-4)

The classification allocation in (3.3-19) relies upon the probability density functions, \( f_1(x) \) and \( f_2(x) \), of groups, \( \pi_1 \) and \( \pi_2 \) respectively. Therefore, an observation is classified to the group which it has the highest probability of belonging to.

\[ d = \frac{f_1(x)}{f_2(x)} \]

\[ x \in \pi_1 \text{ if } d \geq 1 \]

\[ x \in \pi_2 \text{ if } d < 1 \]  

(3.4-5)

As discussed earlier, the normal distribution function is an acceptable choice of model for sample populations. By assuming the original groups to be normal populations, with means of \( \mu_1 \) and \( \mu_2 \), the intersection of the two distributions defines the regions \( R_1 \) and \( R_2 \).

![Figure 11 Classification of two normal populations](image-url)
Figure 11 shows two normally distributed populations. The decision boundary, \( \theta \), between the two populations is equal to \( x \) where \( f_1(x) \) and \( f_2(x) \) are equal. The case of two normal populations with equal covariance matrices is worth considering as it has parallels to the Fisher’s linear discriminant process. For this case the sample covariance matrices, \( S_1 \) and \( S_2 \), are replaced by a joint covariance matrix, \( S_J \):

\[
S_J = \left[ \frac{n_1 - 1}{(n_1 - 1) + (n_2 - 1)} \right] S_1 + \left[ \frac{n_2 - 1}{(n_1 - 1) + (n_2 - 1)} \right] S_2
\]

(3.4-6)

Which produces the following allocation rule:

\[
x \in \pi_1 \quad \text{if} \quad (\mu_1 - \mu_2)^T S_J^{-1} x - \frac{1}{2} (\mu_1 - \mu_2)^T S_J^{-1} (\mu_1 + \mu_2) \geq 0
\]

(3.4-7)

\[
x \in \pi_2 \quad \text{otherwise}
\]

3.4.1.1 Fisher’s Linear Discrimination

Fisher’s method is advantageous in the case of non-normal populations as it does not make assumptions about the group distributions. However, it does assume that the covariance matrices of the two groups are equal [19]. Fisher’s linear discriminant analysis (LDA) separates cases of different classes by maximising the distance between groups while minimizing the within-group distribution. This is achieved by transforming the multivariate observations in \( X \), containing \( N \) observations and a dimension of \( p \), into a univariate observation, \( y \). The new observation is a linear combination of the vector elements in \( x \), with coefficients \( a \). The separation, \( J \), between the groups is defined as the distance, in standard deviations, \( \sigma_y \), between the means.

\[
J = \frac{|\overline{y}_1 - \overline{y}_2|}{\sigma_y}
\]

(3.4-8)

The separation objective is to maximise \( J \), this is achieved by selecting a suitable linear combination of the variables.

\[
y = a^T X = (\overline{X}_1 - \overline{X}_2)^T S_J X
\]

(3.4-9)

The sample variance is given by:
The discriminant and classification methods described here produce computationally simple linear equations. The linear model also implies stable generalisation properties, unlike quadratic or higher order classification methods which can be susceptible to the effect of outliers in small sample populations. Linear discriminators are similar in concept to the perceptron model used in artificial neural networks. Both employ a linear equation to produce a score for a given input. In this way artificial neural networks may be used for the problem of discrimination and classification.

3.4.1.2 Artificial Neural networks

Artificial Neural Networks (ANN) mimics the biological neural process, such as the brain, by modelling the neurological information processing processes by means of a mathematical paradigm [20]. The building block of any ANN is the neuron; each neuron has two functions, the activation function and the network function.

The network function may not be either a linear summation or non-linear function; alternatively the network function can be the product of all the weighted inputs. The network function produces a weighted combination \( (u) \) of the inputs to the neuron, \( (y_1, y_2, \ldots, y_N) \). The \( \theta \) input to the network function is the bias or threshold.

\[
u = \sum_{j=1}^{N} Y_j W_j + \theta
\]  

(3.4-11)
The activation function is then either a linear or non-linear function which determines the neuron output.

\[ a = f(u) \]  \hspace{1cm} (3.4-12)

The actual function employed by the activation function can be graded slopes, thresholds or linear slopes; each has advantages for specific applications. The application of artificial neural networks in pattern recognition, the main application for them in the ECG problem is in ECG characteristics identification. The simplest method of implementing a neural network for pattern recognition is to make the weighting vector, \( W = \{w_1, w_2, \ldots, w_N\} \), indicative of the pattern that is being tested for. In this manner, if the input data series, \( X = \{x_1, x_2, \ldots, x_N\} \), is similar to the weighting vector then their inner product should exceed an appropriately chosen threshold, \(-\theta\). The performance of ANN for classification and discrimination is similar to that of the linear discrimination methods [19]. A neural network with no hidden layer essentially applies a linear operation upon the input vector to produce an output. Both, neural network and linear discriminant, classification schemes require a descriptive set of input features to classify cases into predefined groups. These features or characteristics are derived through pre-processing of the source data. In the work presented here, a novel application of the wavelet transform is used to provide a set of descriptive characteristics to a linear discriminant analysis.

3.5 Processing of the ECG

The processing of biomedical signals is an established field within signal processing. In particular, electrocardiology and the ECG have been the focus of extensive research and development, which have improved diagnostic capabilities considerably. This improvement is due to developments in several aspects of electrocardiology, quantified in [21] as:

- Increasing the informativity\(^1\) [sic] of the initial data
- Improving the quality of the mathematical processing of data
- Increasing the general amount of electrocardiographic information and intensity of its exchange on the basis of computerized data bases and telecommunications

\(^1\) This term is, understood to be, synonymous with the phrase := potential to inform.
• Ensuring the most harmless, comfortable, technically simple and sufficiently cheap electrocardiographic measurement procedures

The digitisation of the ECG has provided the platform for many of these developments to take place. The ability to record and store ECG recordings en masse has made the accumulation of databases possible. Consequently, the creation of databases, like the MIT-BIH database, has aided the development of ECG processing techniques, such as compression schemes [22]. The advent of faster and more efficient processors has aided in several of these points. Advances in processor technology has provided higher analogue to digital conversion rates with increased precision. Effectively, this means increasing the bandwidth and information content of the recorded ECG. The analysis of the P wave is a recent area that has benefited greatly from the advent of computers in electrocardiology.

### 3.5.1 Software Analysis of the P Wave

Detection is the primary problem in the analysis of the P wave. As detailed in the previous chapter, the P wave represents atrial depolarisation and is often a low amplitude component within the ECG. The effects of background noise and the diminishing effects of aging can compound the difficulty of analysing the P wave. The use of oesophageal leads to provide better perspective of atrial depolarisation has been, all but abandoned in clinical practice in the interests of minimising patient discomfort and simplicity of recording. The advent of higher resolution ECG recordings has significantly benefited cardiologists. Providing a more reliable and accurate means of determining P wave duration and associated measures of P wave dispersion and P terminal phase values. The discrete nature of digital representation also reduces the effects of intra-observer variability [23]. Digitisation of the ECG has also provided the means for better automated analysis methods.

The last decade has seen the development of various robust approaches towards the problem of P wave detection. The majority of approaches rely on the successful detection of the QRS complex as a landmark on which to base tertiary detection procedures for the P wave [24] [25] [26] [18] [27]. Other approaches, based on adaptive filtering, have circumvented the reliance on QRS detection to accurately define the extremities of the P wave [28].
Successful detection allows feature extraction of the P wave to take place. The previous chapter detailed the features most often derived from the P wave to describe the state of atrial electrical activity. Measures such as:

- P duration
- P/PR segment ratio
- P terminal phase voltage or RMS value in lead V1
- Approximate area of the P wave in lead II
- P dispersion

Have proven to be reasonably reliable as diagnostic indicators with sensitivities and specificities in excess of 70% [29] [30] [31] [32]. These measures are limited by the two dimensional representation of the P wave, time and amplitude, and are susceptible to the effects of noise as discussed earlier. This has lead to the growing interest in the analysis of P wave morphology [33]. The majority of work published in the literature has been focused on P wave feature extraction for patients prone to atrial fibrillation [24] (paroxysmal atrial fibrillation) [18] or atrial activity during atrial fibrillation [34].

The techniques which have been investigated in the analysis of the P wave morphology have followed a similar approach. A generalised model of the P wave in sinus rhythm is generated by several means and individual cases of normal and abnormal morphologies are compared against it. System models generated from state-space system approximations of atrial activity have shown to be effective in the classification of normal and abnormal P wave morphology, significantly outperforming Fourier based methods of analysis [18]. Artificial neural networks have also been applied to the classification of P wave morphology.

ANN approaches rely upon suitable activation or basis functions to compare input vectors against. This flexibility of the ANN approach is that it allows several basis functions to be used in the comparison stage so that the P wave can be compared against several morphologies [27]. However, the use of wavelets in the problem of P wave morphological analysis has not been extensively investigated. The correlation of basis functions with the P wave would appear to suit the application of wavelet analysis as an approach to morphological classification. Therefore, developing and applying novel wavelet techniques for analysing the P wave.
3.6 Summary & Discussion

A number of signal processing and statistical analysis methods which are employed in the process of ECG, and more significantly, P wave analysis, have been presented here. The superiority of wavelet analysis over Fourier techniques in the field of time-frequency analysis has been covered and the application of wavelet processing to the analysis of physiological signals, in particular, the ECG has been touched upon. A brief review of current work on the processing of the P wave has been included to support the approach of using wavelet analysis for the classification of P wave morphology. The following chapters will detail the techniques developed to analyse and classify the P wave which form the novel work of this thesis.
3.7 References:


Chapter 4

NOVEL WAVELET ANALYSIS OF CLINICAL ECG P WAVES

The study of heart function in practical medicine has long been dominated by the analysis of left ventricular activity. This focus is dictated by the importance of the left ventricular action to the cardiovascular system. However, an increased appreciation of the atrial contribution to left ventricular function has resulted in renewed investigation of atrial activity. This is most evident for the case of atrial fibrillation which can lead to serious complications of the cardiovascular system [1]. The P wave is a relatively small amplitude component of the ECG and can therefore be prone to distortions from noise and masking from T waves. Enhancing a signal’s characteristics for analytical purposes can be pursued through more than one approach. Enhancement can be achieved by reducing the effect of extraneous components which mask or distort desirable information in a signal. Alternatively, analysis can be enhanced by extracting salient diagnostic features of a signal. Typical characteristics used in diagnosis of the P wave and atrial electrical activity are duration, dispersion, morphology and amplitude. These characteristics can be difficult to discern in clinical ECG cases for a number of reasons. The P wave is a relatively low amplitude feature of the ECG which can be heavily affected by noise. The majority of cardiac conditions are observed in aging subjects, physiological changes due to aging can result in reduced electrical potentials in cardiac tissue. Additionally, these traditional characteristics of the P wave are limited in their ability to describe physical properties of the atria. The assessment of morphology is an objective consideration and therefore prone to discrepancies arising from individual assessors. Duration and amplitude measures have been shown to correlate with left atrial diameter but none have been shown to be good indicators of left atrial area. Other, more expensive methods of atrial analysis are used for such measurements most effective are the magnetic resonance and ultrasound imaging techniques. Both methods provide a view of the physical properties and mechanical function of internal physiology. Ultrasound technology is used in a number of medical
applications including cardiac analysis. It can provide accurate measurements of left atrial
dimensions but has a relatively low resolution and is limited to 2D imaging. MRI scans
provide higher resolution images of the biological tissue and are widely acknowledged as
the most accurate method of medical imaging. They can also produce 3D renderings of
internal physiology. However, both technologies are much more expensive than the ECG.
The cost of MRI technologies is prohibitive to its everyday clinical use and is only available
in specialised clinics and research centres. Similarly, ultrasound imaging is primarily used in
specialised cardiology clinics and is not readily available to general practitioners.

In this chapter wavelet processing techniques are used to extract novel diagnostic
characteristics from clinical P wave data. The continuous wavelet transform is used to
construct the P wave’s energy spectrum from which several objective characteristics can be
derived. These novel characteristics describe inherent aspects of the P wave morphology
and are sensitive to its variations. This provides a new perspective for the clinical analysis
of atrial electrical activity and specific atrial conditions. Several of the novel characteristics
show promise as indicators of physical properties of the atria. A brief overview of ECG
processing is required to understand how these novel characteristics may be beneficial to
clinical cardiology.

4.1 Processing the ECG

The digitisation of the ECG signal is advantageous to analysis procedures, as it
allows for the application of numerical procedures which assist the extraction of
quantitative and diagnostic information. Procedures which enhance signal characteristics,
such as noise reduction, are useful for applications where signal to noise ratios are low.
Such enhancements procedures provide an initial process stage which assists analysis of the
data. Enhancement can be used to improve the visualisation of recorded data for
observation by clinicians, or improve the performance of numerical analysis algorithms.
Digital recording and storage of physiological signals like the ECG allows larger amounts
of data to be stored and analysed. Signal measurement tools which identify and quantify
signal characteristics automatically, expedite the analysis of large amounts of data and can
significantly reduce the effects of human subjectivity and error. Many physiological signal
measurement and analysis tools employ signal transforms, such as the Fourier and wavelet
transforms in Chapter 3, to isolate signal features or provide new perspectives on the
original data.
4.1.1 Noise Reduction

The nature of the signal source and the interface needed to record it, make the ECG highly susceptible to noise. The noise component in ECG signals generally arises from skeletal muscle potentials, electronic noise in recording equipment, power line noise (50Hz in Australia and the UK) and poor conductance between skin and electrodes. Noise reduction can be achieved by isolating the contribution of a noise source and excluding it from analysis, or by reducing the effect of the noise upon the signal.

4.1.1.1 Filters

Clinical ECG recordings undergo several stages of filtering in the attempt to reduce noise. Each noise source resides in a characteristic frequency band. Poor conductance between skin and electrode creates slowly varying potentials which manifests as baseline wander in the ECG. Electrical interference produces sine wave ripples throughout the recording. Muscle noise is generally higher frequency and appears as a random noise component. Baseline wander can be reduced in real time by the use of analogue high pass filters in the recording amplifiers. Typically, the analogue filters have a cut-off frequency of 0.05Hz. This effectively reduces baseline wander but does not remove it entirely. Offline filtering can effectively extract baseline components which are then removed from the original signal. The most common low pass filter used for this application is a low order infinite impulse response (IIR) filter such as a Butterworth filter. Magnitude and phase responses for a 4th order Butterworth filter used by the Cardioview digital ECG interface are shown in Figure 13. IIR filters offer good transition band characteristics at low coefficient orders which make them efficient to implement. The effect of their non-linear phase response is negligible over the frequency range of the baseline signal. The stages of the baseline reduction process are displayed in Figure 14. The original ECG is passed through the lowpass IIR filter to extract the baseline wander shown in middle plot of Figure 14. The extracted baseline is then subtracted from the original ECG to produce the corrected signal shown in the plot at the bottom of Figure 14.
Linear lowpass filtering can also be effective in reducing the effect of high frequency noise introduced by skeletal muscle potentials. The cutoff frequency must be carefully selected to minimise the distortion of diagnostically important features of the ECG while still reducing extraneous noise components. Adaptive filtering techniques provide superior noise removal performance in this regard and are well suited to processing quasi periodic signals such as the ECG. Such techniques are limited in their
ability to preserve or enhance finer details of the low amplitude P wave as they rely on initial assumptions about the signal properties.

4.1.1.2 Signal Averaging

Averaging a signal complex or waveform is an effective way of reducing the effect of noise which does not rely on assumptions about a signal’s frequency properties. A noisy signal, $y(n)$, can be expressed as the sum of two time varying components, the signal of interest, $x(n)$, and a background noise component, $u(n)$.

$$y(n) = x(n) + u(n)$$  \hspace{1cm} (4.1-1)

Signal averaging makes the assumption that the background noise and the signal of interest are uncorrelated [2]. Therefore, averaging M number of instances of a waveform reduces the effect of noise by consolidating the recurring characteristic features of the waveform while reducing the contribution of the uncorrelated components.

$$y_{\text{average}}(n) = \frac{1}{M} \sum_{i=1}^{M} y_i(n)$$  \hspace{1cm} (4.1-2)

The improvement in the signal to noise (SNR), gained from signal averaging, can be approximated by the equation:

$$\text{SNR}_{\text{improvement}} = \frac{1}{\sqrt{M}} \%$$  \hspace{1cm} (4.1-3)

The traces in Figure 15 show a set of P waves extracted from a single 10 second epoch of a clinical ECG. In the plots at the top of Figure 15 the P waves from leads II and V1 have been aligned to the onset of the P wave in lead II. The plots below them show the respective averaged traces. The improvement in visibility of the averaged P wave trace is obvious. The erratic high frequency variations have been dramatically reduced. The 50 Hz modulation has also been reduced, but to a lesser extent. This is because the 50Hz signal is a periodic signal and may therefore have some correlation between P wave instances. Many commercial ECG packages, such as MicroMedical’s Cardioview™, employ a beat averaging scheme to provide a succinct summary of an ECG in a magnified display. Beat averaging improves the performance of the automated measurement procedures.
Figure 15 Aligned and Averaged P waves from a Noisy ECG

Figure 16 shows an averaged ECG cycle produced by Cardioview™. The durations, onsets and offsets of the ECG’s characteristic points have been detected and marked by the package’s integrated algorithms. The averaged beat is created from a 10 second recording of a 12 lead ECG; averaged cycles are produced for each lead. Beat averaging in short clinical recordings is performed primarily to summarise the information of the ECG and reduce the effects of respiration. This approach to noise reduction is exploited by high resolution ECG recorders to produce the signal averaged ECG (SAECG). Typically, the recording length of the SAECG is around 10 minutes, much longer than ECG recordings in clinical settings. This produces large number of cardiac cycles which can be aligned and averaged to produce a statistically significant average of the waveform. The sampling rate of such recorders is 1-2 kHz, much higher than most clinical ECG recordings. The SAECG is also recorded in the modified frank lead format, using 3 leads to represent the ECG in X, Y and Z directions. The combination of reduced noise and a higher sampling rate mean that higher frequency analysis of the ECG can be carried out. Analysis of the P wave SAECG in frequency bands of 200-160Hz, 150-100Hz and 90-50Hz has been shown to be of prognostic significance for patients with paroxysmal
atrial fibrillation [3]. Because of this, the SAECG has become an area of interest in cardiology [4] and biomedical signal processing, specifically, the signal averaged P wave [5] [6] [7]. Its use in practical clinical cardiology, however, is not widespread, particularly in Australia.

The clinical ECG is predominantly recorded in 12 leads at a sampling rate of 500Hz for short periods of time, usually less than a minute. This sets constraints on the original information content of the ECG. The challenge is therefore, to find better means by which to analyse the clinical ECG. This can be achieved by using new perspectives on the data to provide more information to the clinical cardiologist. This is the aim of signal transform procedures. Underlying characteristics or processes may be identified by transforming a time varying signal, such as the ECG, into another domain or form. Time-frequency transforms provide a means of analysing a signal's frequency characteristics. The most fundamental time-frequency transform is the Fourier transform described in Chapter 3.
4.1.2 Feature Detection

The automated measurement of ECG characteristics, like the P wave, depends heavily upon the accurate detection of features within the ECG. Because of this, much of the early work carried out on digital processing of the ECG focused on event detection. Feature detection in the ECG has relied upon the accurate detection of the prominent QRS complex with many techniques being applied to this problem. Early detection algorithms focused on minimising computational loads, generational improvements in processing technology have seen this consideration be superseded by detection accuracy. A good summary of QRS detection methods is presented by Köhler in [8]. Wavelet processing methods have proven to be good approaches to this problem. The multi-resolution analysis properties of wavelet analysis provide the added advantage of multiple feature detection [9]. This means wavelet based QRS detection methods can be expanded to locate P and T waves and their onset and offsets [10]. However, the prominence of the QRS complex presents a problem for multi-resolution analysis of the P wave. The amplitude and morphology of the QRS complex have made it an excellent example for the use of multi-resolution analysis for diagnostic purposes [11] [12]. At higher scales of wavelet analysis, the wavelet basis function is widely dilated. This means that wide band characteristics can influence the magnitude of wavelet coefficients of adjacent characteristics. This is the case of the QRS complex and the smaller amplitude P wave when observing low frequency scales of the continuous wavelet transform. Consideration should be paid to this effect when considering wavelet function basis and the range of wavelet scales to analyse.
4.2 P wave analysis

In chapter 3, the various forms of the wavelet transform and its flexibility in time
frequency analysis was discussed. The time-frequency localisation and frequency scaling
qualities of the continuous wavelet transform suggest it would be well suited to
morphology analysis of signal characteristics, such as the P wave. Variations in
morphology would produce varying correlations in wavelet coefficients. The wavelet
energy density spectrum can provide an intuitive representation of a signal’s characteristics
from which variations in the signal can be easily observed. Deriving quantifiable
characteristics from energy distribution provides an objective means of analysing the P
wave which removes the variability of inter-observer measurements and is easily
reproducible. The CWT is applied to the ECG signal, the energy spectrum for each P
wave time window is then calculated and time averaged. By applying the CWT to the
entire ECG file and then calculating the energy spectrum for each P wave window the
boundary effects of the finite time series data are effectively eliminated. The boundary
effects will still be present in high wavelet scales corresponding to extremely low
frequencies below the frequency range of interest to the P wave analysis. Lower frequency
scales within the localities of the P wave may also contain contributions from neighbouring
QRS complexes. These contributions usually occur below the frequency range of interest.
Though in cases where the PR interval is short the QRS may affect the P wave’s region of
interest in the wavelet transform. Figure 17 shows the P waves from the recording of a
healthy patient aligned to the onset of atrial depolarisation. The P waves in Figure 17 have
not been corrected for DC offset for presentation purposes. Fourier analysis of such
typical P waves found that the majority of the P wave’s energy is contained within a band
from 3 – 15 Hz. Figure 18 shows that the frequency band containing the majority of the P
waves’ energy corresponds to wavelet scales of 11 – 50 for the chosen Gaussian derivative
function.
Figure 17 Aligned P waves from a Healthy Recording

Figure 18 Fourier analysis of healthy P waves in Figure 17
This is illustrated in Figure 19 which shows the time averaged wavelet energy spectrums for the P waves in Figure 17.

\[
\frac{1}{E} \frac{1}{N_p} \sum_{b} |C(a,b)|
\]  

(4.2-1)

The wavelet scale axis in Figure 19 represents an incremental dilation of the wavelet function basis. The relationship between frequency and wavelet scale is nonlinear. The low frequency component of the spectrum becomes more detailed. This is particularly advantageous to the analysis of the P wave as subtle changes in duration and morphology will be highlighted. The distributions in Figure 19 have distinct peaks or dominant frequencies. These dominant frequencies of the P wave are heavily related to its morphological features such as duration and deflections.

### 4.2.1 P wave Detection

The P wave detection method employed in this work, relies upon the initial location of QRS complexes. A continuous wavelet transform was computed at several scales for the original ECG signal, an example is shown in Figure 20. The singularities of
the QRS complex create local maxima across several selected scales of wavelet coefficients. A Mexican hat basis function, essentially a 2nd order derivative of the Gaussian function, was employed in a continuous wavelet transform. Coefficients were computed for a set of scales \(\{1, 5, 10, 14\}\) using 4.2-1. This set of wavelet dilation scales have corresponding central frequencies of \(\{25, 12.5, 8.93, 6.94\}\) Hz, which covers the dominant frequencies of the ECG component waveforms. The QRS complexes can be identified by correlating the local maxima across the scales [13]. The scales from 4.2-1 were used to locate the QRS complexes.

\[
C(a,b) = \int_{-\infty}^{\infty} x(t) \psi^*_{a,b}(t) \, dt \quad a = \{1,5,10,14\} \tag{4.2-2}
\]

Figure 20 Original ECG

Cross multiplying these scales of wavelet coefficients, 4.2-2, emphasises the high frequency components of the QRS complexes. This discriminates between QRS complexes and broader T waves which are produced from the same muscle mass and also have high amplitudes.
The coefficients corresponding to the QRS complexes are quite prominent, the location of the peaks can be determined using a peak picking algorithm. A hard threshold is applied to the signal, \(x_{\text{QRS}}\), to improve the performance of the peak picking. The threshold is obtained from the maximum value of the first second of the wavelet coefficients. This maximum value corresponds to the QRS complex contained within the first 1 second frame of the ECG. This condition was adequate for the majority of ECG recordings. In one case where the subject exhibited extreme bradycardia, a longer initial frame of 1.5 seconds was required to identify the first QRS complex. The threshold value is set to 0.4 of this maximum value and updated using the values of successively detected QRS peaks [13]. Coefficients above the threshold will indicate the locality of the QRS complexes.

\[
x_{\text{thresh}}(b) = \begin{cases} 
  x_{\text{QRS}}(b) & \text{if } x_{\text{QRS}}(b) > \text{threshold} \\
  0 & \text{if } x_{\text{QRS}}(b) \leq \text{threshold} 
\end{cases} \tag{4.2-4}
\]

Once the QRS locations are known the P waves can be located by searching in the vicinity preceding each QRS complex. This approach does not take into account 2\textsuperscript{nd} or higher degree AV block which can cause dissociation between the P wave and QRS complex. However, no such subjects were included in the data sets used here. To aide in the location of the P wave a smoothed approximation of the ECG signal is derived from a set of scales derived using a 1\textsuperscript{st} order Gaussian derivative basis function, 4.2-4. The central frequencies of these scales cover a range from 25-833Hz.

\[
C(a,b) = \int_{-\infty}^{\infty} x(t) \psi_{a,b}^* (t) dt \quad a = \{4,5,\ldots,12\} \tag{4.2-5}
\]

These scales are used to reconstruct an exaggerated form of the original ECG in which the low frequency components of the ECG are emphasised. The smoothed ECG, \(x_{\text{smoothed}}\), is created by taking an ensemble sum across the levels of coefficients and approximating the integral with respect to \(b\), using a cumulative sum as:

\[
x_{\text{smoothed}}(k) = \sum_{a=4}^{12} \sum_{b=1}^{k} C(a,b) \quad a = \{4,5,\ldots,12\} \tag{4.2-6}
\]
Due to the distortion inherent in the reconstruction process this exaggerated ECG cannot be used for diagnosing morphology or amplitude of the characteristic waves, only information relating to temporal locations can be obtained. It is clearly visible in Figure 21 that the Q waves have been effectively masked by the emphasis of the lower frequency components of the ECG signal. However, because the wavelet basis is symmetric, the temporal position of the P and T waves peaks and QRS complex peaks have not been affected.

Figure 21 Superimposition of original and exaggerated ECG

By emphasising the lower frequency components, it has become easier to identify the peak position of the P wave and to identify an onset point where the P wave deviates from the iso-electric segment following the U wave. The low amplitude U wave is also significantly emphasised in the reconstructed signal. A second example of this exaggeration is shown in Figure 22 where the P waves are of much lower amplitude than those in Figure 21. The noise component is of a similar magnitude to the P wave. The morphology of the P wave is therefore effectively masked. The reconstruction process significantly enhances the presence of the P wave such that their onset and offset points can be clearly seen. The peaks of each P wave are also now visible as well as the variations in morphology between successive P waves. A search window of 50-250ms prior to each QRS complex was processed to identify the P wave peak in the exaggerated ECG.
Though similar studies have used longer timeframes [14], this window was found to be sufficient to account for the normal PR interval duration of 140-210ms and the normal maximum Q wave duration of 30ms [10]. A peak picking algorithm is used to identify the peak position of the P wave and its onset, these positions can then be translated back to the original ECG signal. The peak picking algorithm uses the conditions:

\[ x(n-1) \leq x(n) \leq x(n+1) \] (4.2-7)

Where the signal \( x \) is the smoothed ECG produced in 4.2-5. Onset and offset limits for each P wave were determined to be minima surrounding the P wave peak. The duration of the P wave was derived from the rhythm strip of lead II which typically produces the longest duration over all the 12 leads. The duration of the P wave in lead II was derived from this process and served as the standardised measure of the P wave’s duration. The onset and offset of the P wave were also determined from this lead using a combination of minima and slope detection in an area surrounding the peak of the P wave.

\[ \text{Figure 22 Superimposition of low amplitude P waves and exaggerated P waves} \]
4.2.2 Normal Atrial Electrical Activity

Analysis of atrial electrical activity, by the use of cardiac electrical mapping, has shown that abnormal activity can be characterised by variations in conduction path [15]. These subtle variations are represented in the surface ECG but may not be readily apparent in a short clinical recording. The ECG signal in Figure 24 is a lead II recording which exhibits normal sinus rhythm. The activation sequence is regular, with the P waves being followed by a QRS complex and T wave in a quasi-periodic manner. The P waves in this signal are quite prominent and easily identified.

The high amplitude of the waves and absence of any noise also makes it easy to examine their morphology. The limb leads were chosen to observe any variations in energy distributions. The limb leads describe the electrical activity in the frontal plane. They offer discrete perspectives of the propagation wavefront in this plane at 30° intervals. The generalised P wave morphology in each lead is subtly different and will produce differing energy distributions. Therefore variations in the atrial conduction path should be more
apparent by observing multiple leads. Figure 25 shows the wavelet energy distribution of all P waves in each limb lead. The energy distribution of the P waves remains fairly uniform throughout the recording.

![Figure 24 ECG exhibiting a normal sinus rhythm](image)

Each lead in Figure 25 exhibits a low variance in the energy distribution pattern which is expected from a healthy conduction cycle. Lead aVL exhibits a different pattern, this is due to the different P wave morphology existent in this lead. The P wave in lead aVL is quite low in amplitude, therefore the high frequency noise component becomes more pronounced in the energy distribution.

This difference in energy distribution is evident in Figure 26 which shows the average distribution of each lead. Each average energy distribution in Figure 26 is contained within scales 10 to 50. This example demonstrates that regularity could be an important feature of normal atrial activity. It has previously been reported that P wave variability is an important identifier in cases of atrial fibrillation [16].
Figure 25: Wavelet energy distributions for each Limb Leads of Normal Sinus Rhythm ECG

Figure 26: Lead average wavelet energy distributions for distributions in Figure 25
4.2.3 Abnormal Atrial Activity

An example of abnormal atrial activity is shown in the ECG in Figure 27. The ECG exhibits a conduction block and the morphology of the P waves is difficult to discern. By calculating the wavelet energy distributions for this signal it becomes evident that the P waves in this recording vary considerably. These variations are highlighted in Figure 28 by the large variations in energy distribution across all limb leads. This indicates that the atrial conduction direction in the limb lead plane changes significantly over time.

![ECG Exhibiting Changes in Atrial Conduction](image)

Variations in atrial activation sequence can be observed in normal patients as well as patients exhibiting atypical surface P waves such as notched P waves suggesting LAE or inter atrial conduction delays [17]. Variations in atrial conduction direction can often be the case in non-sinus atrial rhythm or retrograde atrial activation. This occurs when the SA node fails and activation results from the self excitation of the AV node [18]. Respiration can also produce changes in the position of the heart which will manifest as alterations in the recorded surface ECG [19]. Heart rate is directly affected by the inspiration phase of the respiratory cycle.
Figure 28 P wave analysis of ECG shown in Figure 27

Figure 29 Lead averaged energy distribution for atrial arrhythmia
4.2.4 Energy Density Spectrum Analysis

The energy density spectrum can reveal characteristics of a signal which are not evident in the original time series. Analysis of the wavelet energy spectrum has been applied extensively to meteorological [20] [21] and physiological data such as blood flow [22] and the ECG [11] [23]. The common feature of these signals is that they contain significant information in lower frequency ranges. Daily meteorological observations can contain weather patterns and oscillations with cycles spanning several years. Physiological recordings sampled at 500Hz or higher observe mechanisms occurring at a few cycles per second or lower. The superior low frequency resolution of the wavelet transform means it is an analysis method sensitive to patterns and variations in low frequency information. The wavelet energy density spectrum is derived from the transform coefficients with (4.2-1).

\[ E_{a,b} = \left| C_{a,b} \right|^2 \]  

(4.2-8)

The energy density spectrum can be presented visually as a spectrogram or scalogram [24]. The energy density at discrete time and scale locations is typically described by a colour map which contrasts high and low density levels. This provides an intuitive representation where highlights in the scalogram represent regions of interest. The axes of the spectrogram represent temporal and scale locations. Areas of high energy density are located at points of correspondence between the wavelet scaling function and the relative area of the signal being analysed. To produce a qualitative analysis a set of descriptive characteristics must be derived from the energy spectrum.

4.2.4.1 Wavelet Features

Several features of the energy density spectrum were selected to characterise the P wave. These features are:

- The maximum, or peak, value of the energy density. This value is determined by the amplitude of the dominant frequency in the original signal.
- The scale of the dominant frequency is determined by the morphology and duration of the original signal.
- The median scale of the energy distribution.
- The interquartile range (IQR) quantifies the dispersion of signal’s peak energy across the frequency spectrum as a frequency range.
- The quartile variation (QV) standardises the IQR as a percentage of the entire frequency spectrum being analysed.

The choice of wavelet function will affect the properties of the energy density spectrum as well as frequency range and resolution.

### 4.2.4.2 Choice of Wavelet Basis Function

Any number of functions can satisfy the wavelet admissibility criteria discussed in chapter 3 [25]. Each function presents its own time and frequency characteristics, such as the time-frequency product which determines the support of the function in both dimensions. Physical characteristics, such as symmetry, are also important considerations in choosing a basis function. In most applications, the choice of wavelet function usually bares a physical similarity to the pattern being sought after. If a wavelet function closely matches a particular characteristic of a signal there will be a high correlation between the two, which will manifest as a large value in the wavelet transform coefficients. Conversely, uncorrelated segments will produce low value wavelet coefficients. These properties are exploited by compression and pattern detection schemes [25]. Symmetrical functions are advantageous to edge detection, whereas asymmetric functions pattern detection. The choice of a basis function is therefore dictated by the application. The wavelet transform can be rewritten as a convolution operation between the signal and the wavelet scaling function.

\[
C(a, b) = x(t) * \psi_{a,b}^*(t)
\]  

(4.2-9)

The transform produces a coefficient which describes the correlation between the original signal, \(x(t)\), and the conjugate of the wavelet scaling function, \(\psi^*\), at temporal, \(b\), and scale locations, \(a\). The scales of the wavelet transform are logarithmically proportional to frequency, with higher value scales representing lower frequencies. Each scale in a wavelet transform corresponds to a frequency band determined by the bandwidth of the mother wavelet and the sampling rate of the signal. The frequency bands of the scales are spread around a central frequency dependent upon the mother wavelet used. The central frequency and bandwidth of the mother wavelet determine the frequency response of the wavelet transform [25]. Using the wavelet transform for analysis of signal morphology is similar to RMS template matching schemes, where high correspondence between templates and signals result in low RMS error values. The advantage of wavelet based pattern detection over RMS template matching is the scalability of the wavelet basis
function, which allows for morphological changes in the characteristics patterns. The scale at which maximum wavelet coefficients occur will therefore reflect variation in the morphology of the P waves being analysed.

4.2.4.2.1 Maximal Wavelet Coefficients

The leads II and V1 provide distinct P wave morphologies. Lead II presents the long axis view of atrial conduction and therefore generally contains the highest amplitude P waves which are positive and monophasic. Lead V1 provides a cross atrial view of the atria which produces a characteristic biphasic shape. These general morphologies can still be observed with localised alterations in the presence of abnormal atrial conditions, with the exceptions of atrial flutter and fibrillation which disrupt or remodel orthodox atrial activity. An assessment of the correspondence between these P wave morphologies and several wavelet basis functions was performed using normal and abnormal subject sets. The abnormal set comprised of 14 patients with abnormal P waves resulting from left atrial conditions. Such abnormalities typically result in significantly longer P wave durations in lead II and larger P terminal forces in lead V1 [26] [27]. The maximum coefficient values produced by the wavelet transform using the selection of wavelet functions on the set of lead II P waves are shown in Figure 30. The maximal wavelet coefficient values of the same sets of wavelet functions on the P waves in lead V1 are shown in Figure 31. The vertical axis of each figure represents the square root of the absolute maximum value of the coefficient produced between the wavelet basis and the P wave:

\[ y = \sqrt{\max|C(a,b)|} \quad (4.2-10) \]

Each subject’s successive P waves were analysed and their maximum coefficient values were averaged. Group averages were created from the maximum wavelet coefficients of all subjects within each group. Figure 30 and Figure 31 indicate that the abnormal group had a higher average maximum correspondence with all investigated basis functions.
Essentially the wavelet transform is a series of convolutions of a function and a signal of interest. The instantaneous value of this convolution is dependent upon the magnitude of the signal and its correspondence to the basis function of the transform. It has previously been shown that the area under the trace of the P wave in lead II provides a good estimate of left atrial diameter [28]. The broader P waves of the abnormal group therefore tend to maximally correlate with larger dilations of the wavelet basis functions.
and produce a higher maximum wavelet coefficient. The Gaussian derivative functions collectively have much higher coefficient values in lead II than the other wavelet families. Most of the basis functions performed much better in lead V1 with the Coiflet family performing quite similarly to the Gaussian functions. This is most likely due to the biphasic nature of the P wave in lead V1 which produces better correlations with the asymmetric functions.

The scatterplot in Figure 32 shows the maximum wavelet values in lead II versus those in lead V1 for the two groups. The results in Figure 30 and Figure 31 suggest that the Gaussian derivative functions correspond well to the P wave. A significant bi-variate correlation (0.551, p=0.01) between the maximum value of the wavelet transform for P waves in lead V1 and group membership was found. The maximum coefficient in lead V1 was also found to be significantly correlated with the P terminal force measure in lead V1 (-0.69, p<0.001). This result suggests that the maximum value of a P wave's wavelet transform could be a useful to discriminate between normal and abnormal P waves produced by left atrial enlargement and should be investigated further. However, the maximum wavelet transform value alone does not provide a complete description of the P wave’s characteristics. The energy distributions of signals from a particular class can vary depending upon signal idiosyncrasies. The continuous wavelet transform offers many avenues for analysing and quantifying a signal’s energy distribution.
4.2.4.2.2 Quantifying Scale Energy Distributions

As previously mentioned in section 4.2.4.2, the energy distribution and frequency resolution of the wavelet transform are dependent upon the basis function employed. Figure 33 shows an anti-symmetric function derived from opposing Gaussian pulses and its Fourier transform and normalised to contain unit energy.

![Figure 33 Anti-symmetric Function and its Fourier Transform](image)

The central frequency of the time domain function in Figure 33 is determined by the Fourier transform to be 4.9751 Hz. The basis function of the Fourier transform is fixed as a sinusoidal function while the wavelet transform allows for any function basis which meets the admissibility criteria. Each basis function will have its own intrinsic frequency properties which will alter the energy spectrum. Here, we are considering the first and second order derivatives of the Gaussian function as basis functions, given their performance in section 4.2.4.2.1. The first order derivative of the Gaussian function is given by Equation (4.2-3).

$$\psi(t) = t\exp\left(-\frac{t^2}{2}\right)$$

This wavelet function and its associated central frequency are shown in Figure 34. The blue trace shows the basis function, the red trace shows the sinusoid which approximates the basis’ central frequency. The central frequency is determined by finding the frequency of the sinusoid which maximises the wavelet energy spectrum [29]. The normalised central frequency of the first order derivative is 0.2. Unlike the sinusoid, the Gaussian function contains a range of frequency components spread around the central frequency.
The second order derivative of the Gaussian function is sometimes referred to as the Mexican hat wavelet function; it is defined by in (4.2-4). [25]

\[ \psi(t) = \beta(1-t^2)\exp\left( -\frac{t^2}{2} \right) \]

\[ \beta = \frac{2}{\pi^{1/4} \sqrt{3}} \]

(4.2-12)

The term \( \beta \) is the normalising constant for the wavelet base required to normalise the energy of the basis function such the scaling functions have unit energy.

\[ \int_{-\infty}^{\infty} |\psi(t)|^2 \, dt = 1 \]

(4.2-13)

The basis function in 4.2-10 has a normalised central frequency of 0.25; this is the dominant frequency of the wavelet basis function. The MATLAB© wavelet toolbox uses a slight variation of 4.2-10 to create the Gaussian wavelet functions. MATLAB’s 2\textsuperscript{nd} order derivative of Gaussian basis function has a normalised central frequency of 0.3. This function is shown in Figure 35.
The blue trace in Figure 35 shows the second derivative of Gaussian wavelet function and the red traces is the sinusoid corresponding to its approximate central frequency. The effect of the wavelet basis function on the frequency resolution of the energy density distribution can be easily observed in a comparison of spectrograms derived using differing wavelet bases.

Figure 36 and Figure 37 show the spectrograms of the waveforms in Figure 33 derived using the 1\textsuperscript{st} and 2\textsuperscript{nd} Gaussian derivative wavelet bases. The anti-symmetric waveforms in Figure 36 are similar to the 1\textsuperscript{st} Gaussian derivative's biphasic shape and therefore produce a single maximum located centrally within the waveforms' temporal limits. Whereas the symmetric function in Figure 37 is produces two local maxima when processed using the 1\textsuperscript{st} Gaussian derivative. The spectrograms have been normalised such that the minimum and maximum values of the energy density distribution are 0 and 1 respectively. The area of high energy density is represented by red hues and areas of low energy density are represented by the blue hues in the spectrogram.

There are fewer scales describing high frequencies than there are describing the low frequencies. The higher frequency pulse produces an energy spectrum that spans a wider frequency range than the lower frequency pulse. It therefore has a greater IQR than the lower frequency pulse. Here the IQR is described in Hertz (Hz). The QV describes the percentage of the total observed frequency spectrum which the IQR spans.
\[ QV = \frac{IQR}{Q3 + Q1} \] (4.2-14)

The lengthening of the waveform's duration produces a decrease in the dominant frequency and associated changes in the values of IQR and QV.

Figure 36 Spectrogram of Anti-symmetric Function
4.2.4.2.3 **IQR and QV Sensitivity**

To demonstrate the behaviour of the IQR and QV measures, they were calculated for a range of known signals. The signal chosen was a truncated Gaussian window function; this function was used. Truncation was performed to ensure the window was finite with minimal discontinuities at its extremities. The windowing function was generated using a simulated sampling frequency of 500Hz, which is equivalent to the sampling frequency of the ECG recordings. The length of the window was incremented from 40 ms to 160 ms. The limited Fourier transform low frequency resolution is apparent when compared to that of the continuous wavelet transform in Figure 38.
The Fourier transform is quite coarse and therefore insensitive to subtle variations in the duration of the window function. The wavelet transforms provide a more sensitive estimate of the dominant frequency of the window function. Of the two wavelet basis, the second derivative Gaussian adheres closest to the Fourier analysis approximation. The energy distributions created by the analysis of the window function using each wavelet basis were measured with the IQR and QV measurements. The plots in Figure 39 show the measures of QV and IQR versus the window function’s duration.
The 2\textsuperscript{nd} derivative Gaussian basis provides a greater dynamic range for the QV and IQR measures than the 1\textsuperscript{st} derivative basis for the given range of window durations. The dynamic ranges of the QV and IQR measurements derived using both bases are quantified in Table 3. The larger dynamic range of the 2\textsuperscript{nd} derivative basis essentially implies that it is more sensitive to changes in duration for the range shown in Figure 39.

Table 3 Range of QV and IQR measurements in Figure 39

<table>
<thead>
<tr>
<th>Basis</th>
<th>QV</th>
<th>IQR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaus1</td>
<td>14.52%</td>
<td>2.87Hz</td>
</tr>
<tr>
<td>Gaus2</td>
<td>17.12%</td>
<td>3.63Hz</td>
</tr>
</tbody>
</table>

The relationship of IQR and QV, measured with the 2\textsuperscript{nd} derivative basis, with respect to the dominant frequency of the window are shown in Figure 40. The IQR measure has an approximately linear relationship with the dominant frequency. However, as the QV measure is maximally bounded it shows an approximate quadratic relationship tending towards an upper limit for the measured pulses. The lower limit of both measures is determined by the minimum frequency resolution of the wavelet transform.
A waveform which is not confined to a discrete frequency is therefore not adequately described by a scalar quantity such as its duration. The frequency spectrum of such cases is useful in efficiently describing such cases. The area under a Gaussian window of constant width can be controlled by the standard deviation term. The truncated Gaussian window function was generated for a range of standard deviation between 0.25 and 2 at a simulated sampling frequency of 500Hz. Figure 41 shows the truncated window functions generated for 3 standard deviation values. IQR and QV were measured from the wavelet energy distributions of the truncated functions. These measurements are displayed in the plots of Figure 42.
Figure 41 Truncated Gaussian Pulses

Figure 42 IQR and QV vs. Standard Deviation
The IQR and QV measures are most sensitive to the changes in the shape of the curve for standard deviations less than 1. The plots in Figure 43 show the relationship between the area under the curve for the pulses and the measures of QV and IQR. The behaviour of the IQR and QV measurements in the previous examples can be generalised as such:

- IQR and QV increase as dominant frequency increases
- IQR and QV decrease as area under the curve increases

Alterations to the morphology of the P wave indicate a disruption to normal electrical conduction. Lengthening and broadening of the P wave are common indicators of abnormal atrial conduction.

4.2.4.3 Case Comparison

A common atrial abnormality associated with several pathological conditions is the enlargement of the left atrium. This structural change to the atrial chamber affects the path of electrical conduction, altering the morphology of the P wave by increasing the area of propagation.
Figure 44 Normal Averaged V1 P wave

Figure 45 LAE Averaged V1 P wave
The previous figures, Figure 45 and Figure 44 show the spectrograms and related distributions of the averaged V1 lead P waves of two subjects. The first example shows a normal averaged P wave recorded in lead V1. The differences between the P waves in Figure 44 and Figure 45 are apparent from the wavelet characteristics and the cardiological measures in Table 4. This indicates that the wavelet characteristics could be useful metrics in assessing the condition of atrial conduction represented by the P wave. A more comprehensive assessment of the wavelet characteristics’ discriminating potential is required.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Normal</th>
<th>LAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>96 ms</td>
<td>136 ms</td>
</tr>
<tr>
<td>PV1</td>
<td>-3.02 mV ms</td>
<td>-5.03 mV ms</td>
</tr>
<tr>
<td>QV</td>
<td>33.3 %</td>
<td>28.8 %</td>
</tr>
<tr>
<td>IQR</td>
<td>4.83 Hz</td>
<td>3.45 Hz</td>
</tr>
<tr>
<td>Peak Frequency</td>
<td>9.18 Hz</td>
<td>6.25 Hz</td>
</tr>
<tr>
<td>Peak Value</td>
<td>24.00</td>
<td>53.62</td>
</tr>
<tr>
<td>RMS Value</td>
<td>379.45</td>
<td>480</td>
</tr>
</tbody>
</table>

4.2.4.4 Serial P Wave Variability

The preceding sections have shown that variations in atrial electrical activity can be detected and observed using the wavelet energy spectrum of multiple leads. In section 4.2.1 variations in the energy distributions of successive P waves were observed in both the normal subject and the abnormal subject. These variations are the result of serial changes in the morphology of the P wave and were more pronounced in the abnormal subject. The presence of serial changes in P wave morphology has previously been noted in longitudinal ECG studies of subjects who had suffered tissue damage to the myocardium [30]. These changes were observed in the P terminal force in lead V1 between three ECG recordings of each individual recorded over a period of 6 months. The changes in morphology were as a result of atrial overload caused by the damage to myocardial tissue. Natural variations in serial recordings of P waves from healthy subjects have also been
reported [31]. Variations in the P wave duration, P area and the PR interval were observed in averaged ECG complexes taken from 10 second ambulatory ECG recordings recorded at 2 minute intervals over a 24 hour period. The variations in P wave morphological parameters were found to adhere to a circadian pattern. The sensitive analyses required to reliably detect such subtle variations over a 24 hour period has become possible with the advent of digital recording and processing technologies [32]. Digital recording of the ECG allows processing techniques such as wavelet analysis to be applied to large amounts of data. Such analysis eliminates subjective observer errors associated with manual analysis of large data records or multiple recordings [33] as the wavelet analysis uses a constant basis function across all data records. Subtle differences between successive P waves will produce varied correlations with a respective wavelet scaling function. These differences will be represented in the wavelet transform coefficients for a range of scaling functions which cover majority of the P wave’s energy spectrum. The examples in section 4.2.4.2.1 indicate that the peak value of the energy distribution and the scale at which the peak is located are distinctive markers of the variation or regularity of successive P waves. The peak energy distribution values and their corresponding frequencies were derived for successive P wave in several subjects with various atrial conditions. Each subject was recorded for one minute and their P waves in lead II were analysed. The lead II was chosen as it provides the long axis view of cardiac electrical activity. The P waves in lead II are therefore generally the longest in duration and totally positive. The x axis shows the peak value of the energy distribution. The y axis represents the dominant frequency of the P wave in lead II, the axis is measured in Hertz. The first three subjects shown in Figure 46, Figure 47 and Figure 48 had normal sinus rhythm with normal P wave morphology. These subjects were all classified as normal with no signs of abnormality and no history of heart disease or hypertension. The scatter plots in Figure 46, Figure 47 and Figure 48 exhibit a tight clustering of P waves in frequency and peak energy value. This indicates that there is little variation morphology and duration between successive P waves recorded in lead II for these subjects. Small variations in the RMS energy may be attributed to the effects of respiration which alters the position of the heart [34]. The effects of respiration have been shown to affect the amplitude of the ECG waves. Inhalation changes the position of the heart, moving the cardiac mass closer to the surface of the chest, thereby decreasing the impedance of the electrical conduction path from the epicardium to the electrodes at the epidermis.
Figure 46 Healthy Subject No. 1, Normal Sinus Rhythm, no atrial abnormality

Figure 47 Healthy Subject No. 2
Figure 48 Healthy Subject No. 3

Figure 49 Patient with LAE and atrial ectopic beats
The previous two plots are taken from subjects exhibiting abnormal atrial conduction. The plots in Figure 49 and Figure 50 are taken from subjects with enlarged left atria. In conjunction to an enlarged atrium, the subject in Figure 49 also exhibited atrial ectopic beats. Ectopic, or extrasystole, beats are beats which occur superfluous to sinus rhythm. These beats are the result of impulse formation which occurs either in an alternate position to the sino-atrial node. The extrasystolic beats produce a dramatic change in the dominant frequency. This banding of the dominant frequencies indicates a change in conduction path observed by lead II. The subject in

The following two plots are also taken from subjects exhibiting enlarged left atria and have a history of paroxysmal atrial fibrillation (PAF). These plots display the variation between successive P waves recorded for this subject. The variation in energy and frequency shows a curiously differing relationship to that of all the previous subjects. In the previous plots of Figures 46-50 there has been a moderate inverse relationship between RMS energy and frequency, with the exception of the AEB subject in Figure 49. However, in Figure 51 and Figure 52 the relationship between frequency and RMS energy is positive. It is not possible to state the significance of this difference from the results here, however further investigation of the relationship between frequency and RMS energy variations of the P wave in the presence of PAF are warranted.
Figure 51 Patient with Paroxysmal Atrial Fibrillation during Sinus Rhythm

Figure 52 Patient with Paroxysmal Atrial Fibrillation, during Sinus Rhythm
The previous plots show considerable difference in median magnitude of RMS values between the subjects. These differences may be explained by individual differences in conductivity of the myocardium, pericardial tissue, lungs and blood. The effects each of these parameters had on surface electrical potentials were shown in [35] using eccentric spherical models as well as the effects of fatty tissue and dilation of the heart chambers. The same study showed that dilation of the chambers augmented the conduction distribution, effectively lengthening the waveforms of the ECG. Variance in the dominant frequency of the P waves of normal and abnormal subjects may also be explained by alternation in the electrical conduction path through the atria which can occur during sinus rhythm [15], or in the presence of multifocal ectopic beats which are initiated from various excitation sites around the right atria [36] [37]. This can subtly alter the morphology of the P wave or have more significant effects which are readily observed, such as inversion of the waveform. Alterations in conduction path have been linked to atrial fibrillation where re-entrant waves cause [38] [39]. The variance in the P wave parameters might not be a robust means of classifying the state of atrial conduction. But, these results demonstrate the sensitivity of the wavelet energy spectrum to changes in the P wave morphology. The results also highlight the need to consider and appropriately address the variation of the P wave in any attempt to analyse its morphology.
4.3 Approximating Left Atrial Dimensions from the Clinical ECG P Wave

There are several conditions which cause changes to the structural dimensions of the left atria to occur. Most commonly dilatation or hypertrophy of the atrial chambers is in response to increased demand upon the cardiac system. This can be caused by pathological conditions such as weakened systolic function and mitral stenosis [40]; or by non-pathologic conditions such as prolonged exertive exercise in elite athletes [41]. This structural change to the left atria can have significant effects on the overall cardiac function. Assessment of atrial dimensions has been shown to be an effective measure of general atrial function and a predictor of cardiovascular events such as atrial fibrillation and stroke [42][43][44]. In everyday clinical cardiological practice the simplest and most reliable form of non-invasive assessment of atrial structure is the echocardiogram. Early “M mode” echocardiography allowed for the instantaneous measurement of only one dimension of the atria; typically the parasternal long-axis which measures the anterior-posterior diameter [45]. Several electrocardiographic measures such as prolonged duration, P/PR ration, P mitrale (bifid) morphologies and large terminal forces in the P wave of lead V1 (PV1) have been proposed as indicators of left atrial enlargement and shown to compare with varying reliability, in terms of sensitivity and specificity, to echocardiographic measurements from M mode echocardiograms [46]. This method is now superseded in clinical practice by 2D echocardiography which provides more accurate means of assessing chamber dimensions and allows for estimation of atrial volume [47]. Recent studies using 2D echocardiograms and cardiovascular magnetic resonance imaging technologies have suggested that the related ECG markers of PV1, bifid morphologies, and prolonged duration are inadequate in determining enlargement of the left atria [27][48]. When compared to atrial measurements made using 2D echocardiograms and CMR imaging the duration of the P wave duration in lead II was found to be reasonable sensitive but nonspecific. Conversely, the measures of PV1 and P mitrale were found to be quite specific to absence, yet insensitive to the presence, of enlarged left atria. Despite these findings, these ECG parameters are still routinely used in clinical practice. The comparatively inexpensive nature of ECG recording equipment and procedures and the importance of identifying left atrial conditions have made it desirable to find a reliable method of estimating left atrial size using the ECG. More recent methods of estimating single LA dimensions from ECG parameters have been shown to be reliable. A significant formulaic correlation has been achieved between the duration of the widest P wave in a 12
lead ECG and the parasternal long axis dimension in a 2D transthoracic echocardiogram (TTE) for cases exhibiting enlarged left atria \((r=0.662)\) [49]. Similarly, the P wave area in lead II of patients with mitral stenosis has been shown to correlate well with dilated left atrial diameters measured in 2D TTE \((r=0.739)\) [50]. However, atria do not enlarge proportionally in all dimensions and therefore more informative indicators of atrial function require measurement in more than one dimension [47] [51]. Atrial area is a proven measurement integral to the estimation of atrial volume and is used in clinical practice for measurement of both atria [45] [51]. However, to the author’s knowledge, no reliable means of estimating atrial area from the ECG has previously been proposed. It has been shown in section 4.2.4.2 that the morphology of a wavelet function basis dictates its frequency characteristics. It is then reasonable to hypothesise that by fixing the morphology of the wavelet basis, any variations in morphology of the signal being analysed will be reflected in the frequency information provided by the wavelet transform. Such a method may be achieved by the implementation of the wavelet analysis procedure and measures presented in the previous sections. Therefore it may be possible to estimate the physical dimensions of the left atrium by the application of a wavelet transform to the P wave in several leads.

### 4.3.1 Procedure

Archived echocardiogram reports and digital ECG recordings were collated for 27 subjects exhibiting enlarged left atria (Mean Age = 78.26 ± 8.6 years, 20 males) during sinus rhythm. Each subject had a clinical 12 lead ECG and echocardiogram recorded within 14 days. The ECG recordings are 10 seconds in length, recorded using Micromedical’s ECG PC link TM, with a precision of 16 bits at a sampling rate of 1 kHz which is downsampled to 500Hz for storage. The subjects exhibited echocardiographic characteristics of enlarged left atria with left atrial areas greater than 20 cm\(^2\) [45]. One subject was a borderline case with a left atrial area (LAA) of 19.7 cm\(^2\). Three of the subjects had bradycardia rhythms and five of the subjects exhibited 1\(^{st}\) degree atrioventricular conduction block. Measurements of left atrial diameter (LAD) and right atrial area (RAA) were also recorded, but not for all subjects. The measurements of left atrial area were made at the end of ventricular systole when the atria are fully dilated [45]. The echocardiographic measurements were not adjusted using the Body Surface Area index as it is not a unanimously supported measure amongst cardiologists and is not clinical procedure at Hearts 1\(^{st}\).
Table 5 Echocardiographic and Electrocardiographic Measurements of Sample

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>27</td>
<td>58</td>
<td>97</td>
<td>78.26</td>
<td>8.64</td>
</tr>
<tr>
<td>LAA (cm$^2$)</td>
<td>27</td>
<td>19.7</td>
<td>35.8</td>
<td>28.58</td>
<td>4.37</td>
</tr>
<tr>
<td>LA Diameter (cm)</td>
<td>17</td>
<td>3.43</td>
<td>5.91</td>
<td>4.74</td>
<td>0.65</td>
</tr>
<tr>
<td>RAA (cm$^2$)</td>
<td>19</td>
<td>12.70</td>
<td>27.00</td>
<td>21.46</td>
<td>3.58</td>
</tr>
<tr>
<td>P Duration ms</td>
<td>27</td>
<td>98</td>
<td>158</td>
<td>125.44</td>
<td>12.80</td>
</tr>
<tr>
<td>PII Area mV.ms</td>
<td>27</td>
<td>.92</td>
<td>6.68</td>
<td>3.32</td>
<td>1.73</td>
</tr>
<tr>
<td>P/PR</td>
<td>27</td>
<td>.81</td>
<td>11.61</td>
<td>4.92</td>
<td>2.74</td>
</tr>
<tr>
<td>PV1 mV.ms</td>
<td>27</td>
<td>-11.84</td>
<td>0.00</td>
<td>-4.41</td>
<td>2.71</td>
</tr>
</tbody>
</table>

The measurements of LAA and RAA were made from 4 chamber apical views of 2 dimensional TTE in accordance with clinical procedure recommendations made by the American Society of Echocardiography [52]. An example of this view is shown in Figure 53. The TTE image presents a symmetric view of the two sides of the heart along the atrio-ventricular plane [53].

Figure 53 4 Chamber Apical View 2D Transthoracic Echocardiogram (TTE) [54]
The atrio-ventricular plane closely corresponds to the frontal plane of the chest leads. The Y axis of the atrio-ventricular plane lies along the long axis of the heart corresponding with lead II. The frontal plane is defined by a vertical Y axis parallel with the sternum and a horizontal X axis across the torso. In the ECG these axes are approximated by leads I and V5 in the X direction and lead aVF in the Y direction. The chest leads, V1 and V2, are perpendicular to the frontal plane and run approximately along the Z axis. This provides the short axis view of the atria which is not explicitly portrayed in the TTE image. However, it was hypothesised that ECG leads in this direction would be most indicative of the left atrial area measured in the TTE.

An orthogonal lead set which contains leads in the X Y and Z directions can be derived from the 12 lead ECG using the inverse Dower transform \[ (55) \]. The original Dower transform was derived to convert VCG leads into the 8 independent leads of the 12 lead ECG \[ (56) \]. This transformation is a matrix operation using the following matrix \[ (57) \]:

\[
D = \begin{pmatrix}
-0.515 & 0.157 & -0.917 \\
0.044 & 0.164 & -1.387 \\
0.882 & 0.098 & -1.277 \\
1.213 & 0.127 & -0.601 \\
1.125 & 0.127 & -0.086 \\
0.831 & 0.076 & 0.230 \\
0.632 & -0.235 & 0.059 \\
0.235 & 1.066 & -0.132
\end{pmatrix}
\]

\[ (4.3-1) \]

The \( D \) matrix is not square and therefore a pseudo inverse of the matrix must be used for the inverse operation. The pseudo inverse of \( D \) is proposed in \[ (57) \] as:

\[
D' = \left( D^T D \right)^{-1} D^T
\]

\[
D' = \begin{pmatrix}
-0.172 & -0.074 & 0.122 & 0.231 & 0.239 & 0.194 & 0.156 & -0.010 \\
0.057 & -0.019 & -0.106 & -0.022 & 0.041 & 0.048 & -0.227 & 0.887 \\
-0.229 & -0.310 & -0.246 & -0.063 & 0.055 & 0.108 & 0.022 & 0.102
\end{pmatrix}
\]

\[ (4.3-2) \]

The independent leads of the 12 lead ECG, V1-V6 and limb leads I and II, are arranged into the \( S \) matrix:
The orthogonal X, Y and Z are derived using the matrix multiplication:

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = D'S
\]  

(4.3-4)

The X and Y directions of the ECG leads do not precisely align with the long and short axes of the atria as provided by the TTE. The long axis of the TTE 4 chamber apical view is most closely aligned with lead II from the 12 lead ECG. The derived leads X and Y are highly correlated with the original 12 lead ECG leads V5 and aVF respectively. The lead Z runs opposite in direction to the V1 and V2 chest leads. Both sets are used here to assess their ability to estimate the left atrial area both individually and as combined 2D measurements and to assess the benefit of the Dower transform in this application. Several measures from lead II are also used to determine their ability to indicate left atrial area.

### 4.3.2 Analysis

Analysis was performed on an averaged P wave from each subject. An averaged P wave signal was produced from each 10 second clinical ECG recording using the averaging method detailed in section 4.1.1.2. The QRS complexes were suppressed in the averaged signal by a similar method. An averaged QRS complex was derived and subtracted from the detected complexes in the recording. The averaged P waves were processed using the approach outlined in section 4.2.4.4 to obtain their wavelet energy density spectrums. The wavelet transform was performed on the averaged P wave from the leads representing the X, Y and Z directions. The wavelet measures listed in section 4.2.4.1 were calculated for leads V2, V5 and aVF and their derived orthogonal equivalents, X and Y. These measurements quantify the magnitude and spread of the energy distributions of the P waves recorded in each lead. The measure of total energy, \( E_{total} \), describes the root mean squared average of the energy contained in the average P wave over the scales of the wavelet transform. The measures of peak value, \( E_{max} \), and peak frequency, \( f_{peak} \), describe
the value and frequency location of the P wave’s energy spectrum maximum. These values are derived from the peak distribution. The peak distribution is taken from the wavelet energy density spectrum, given by Equation (4.21), by retaining the maximum value of the distribution at each scale in the temporal vicinity of the P wave. The extremities of the search window were defined manually as the onset and offset of the P wave in the time domain.

\[ F(E_{\text{peak}}) = \sum_{f \in \mathbb{H}} \max \left[ E_{f,b}^2 \right] \quad t_1 \leq b \leq t_2 \]  

(4.3-5)

Where \( E_{\text{peak}} \) is the peak wavelet value at each frequency, \( f \), corresponding to each scale of the wavelet transform. The frequencies were calculated using 3.2-8. The peak value of the energy spectrum is the maximum value of the peak distribution:

\[ \max(E_{\text{peak}}) \]  

(4.3-6)

The discrete frequency at which this maximum occurs is the peak frequency. The IQR was calculated from the cumulative frequency distribution from the peak wavelet energy distribution. The inter-quartile range is the range of the distribution between the first and third quartiles which describes the spread of the middle 50% of the distribution.

\[ IQR = Q3 - Q1 \]  

(4.3-7)

The first and third quartiles, \( Q1 \) and \( Q3 \), correspond to the cumulative probabilities of 0.25 and 0.75 respectively. The quartiles are approximated from the cumulative frequency distribution using a minimum error criterion.

\[ Q1 = i, \ \min\left[0.25 - F(E_i)\right] \]  

(4.3-8)

\[ Q3 = i, \ \min\left[0.75 - F(E_i)\right] \]  

(4.3-9)

The second quartile is also the statistical median of the distribution. This frequency at which this median was located was also kept as a descriptive characteristic, \( f_{\text{median}} \). These wavelet characteristics were compared against a set of ECG measures commonly used to indicate enlargement of the left atria. The ECG measures were P wave duration in lead II, P terminal force (PV1) and the ratio of the P wave duration to the PR segment. Pearson’s method of bivariate correlation analysis was used to produce a correlation value, \( R \), and a one-tailed significance value, \( p \).
4.3.3 Results

Correlation analysis was performed to quantify any linear relationships between the echocardiographic measurements and the independent variables derived from the ECG. Linear regression analyses of significant relationships were performed using the MATLAB fitting toolbox. The linear relationships were obtained using the method of least squares [58]. RMS and the norm of residual values are given by (4.3-10) and (4.3-11) respectively, where $A$ is the left atrial area in cm$^2$ and $y$ is the estimated area using (4.3-8).

\[
RMS = \sqrt{\frac{\sum_{n=1}^{N} A_n^2 - y_n^2}{\sum_{n=1}^{N} A_n^2}} \quad (4.3-10)
\]

\[
\text{Norm} = \sqrt{\frac{\sum_{n=1}^{N} A_n^2 - y_n^2}{\sum_{n=1}^{N} A_n^2}} \quad (4.3-11)
\]

A summary of the correlation analysis for the cardiological measures from the ECG is presented in Table 6. The age of subjects in this study was found to be significantly correlated with right atrial area ($R = -0.553$). Of the ECG measurements, only the P terminal force, PV1, produced a significant correlation with any of the echocardiographic measurements. PV1 was found to be significantly correlated with the left atrial diameter ($R = -0.489$) and right atrial area ($R = -0.446$).

<table>
<thead>
<tr>
<th></th>
<th>LAD N = 16</th>
<th>LAD N = 16</th>
<th>LAA N = 27</th>
<th>LAA N = 27</th>
<th>RAA N = 18</th>
<th>RAA N = 18</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R</td>
<td>p</td>
<td>R</td>
<td>p</td>
<td>R</td>
<td>p</td>
</tr>
<tr>
<td>Age</td>
<td>-0.352</td>
<td>0.091</td>
<td>-0.223</td>
<td>0.132</td>
<td>-0.553 **</td>
<td>0.009</td>
</tr>
<tr>
<td>Sex</td>
<td>0.236</td>
<td>0.190</td>
<td>-0.083</td>
<td>0.368</td>
<td>0.052</td>
<td>0.419</td>
</tr>
<tr>
<td>P Duration ms</td>
<td>0.045</td>
<td>0.434</td>
<td>0.057</td>
<td>0.388</td>
<td>0.149</td>
<td>0.278</td>
</tr>
<tr>
<td>PII area mV.ms</td>
<td>0.317</td>
<td>0.116</td>
<td>0.092</td>
<td>0.325</td>
<td>0.398</td>
<td>0.051</td>
</tr>
<tr>
<td>P/PR</td>
<td>-0.320</td>
<td>0.113</td>
<td>-0.039</td>
<td>0.423</td>
<td>-0.156</td>
<td>0.268</td>
</tr>
<tr>
<td>PV1 mV.ms</td>
<td>-0.489 *</td>
<td>0.027</td>
<td>-0.201</td>
<td>0.157</td>
<td>-0.446 *</td>
<td>0.032</td>
</tr>
</tbody>
</table>
The plots in Figure 54 show the relationships between the ECG measures and LAD. Because PV1 is a negative quantity, the negative correlation indicates that the left atrial diameter and right atrial area increase as the absolute value of PV1 increases. This trend can be seen in the lower right hand plot of Figure 54.

*Figure 54 ECG P wave measures vs Left Atrial Diameter*
Figure 55 ECG P wave measures vs Left Atrial Area

Figure 56 ECG P Wave Measures vs Right Atrial Area
Table 7 Correlations between Wavelet Characteristics and Atrial Dimensions

<table>
<thead>
<tr>
<th>CHARACTERISTIC</th>
<th>LAD (N = 16)</th>
<th>LAA (N = 27)</th>
<th>RAA (N=18)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R</td>
<td>P</td>
<td>R</td>
</tr>
<tr>
<td>IQR X</td>
<td>-0.364</td>
<td>0.083</td>
<td>0.035</td>
</tr>
<tr>
<td>IQR Y</td>
<td>-0.091</td>
<td>0.369</td>
<td>-0.181</td>
</tr>
<tr>
<td>IQR Z</td>
<td>-0.235</td>
<td>0.191</td>
<td>-0.439</td>
</tr>
<tr>
<td>f_{median} X</td>
<td>-0.027</td>
<td>0.460</td>
<td>0.063</td>
</tr>
<tr>
<td>f_{median} Y</td>
<td>-0.165</td>
<td>0.270</td>
<td>-0.202</td>
</tr>
<tr>
<td>f_{median} Z</td>
<td>-0.609</td>
<td>** 0.006</td>
<td>-0.535</td>
</tr>
<tr>
<td>f_{peak} X</td>
<td>-0.168</td>
<td>0.267</td>
<td>-0.115</td>
</tr>
<tr>
<td>f_{peak} Y</td>
<td>-0.048</td>
<td>0.429</td>
<td>-0.321</td>
</tr>
<tr>
<td>f_{peak} Z</td>
<td>-0.679</td>
<td>** 0.002</td>
<td>-0.575</td>
</tr>
<tr>
<td>E_{max} X</td>
<td>0.431</td>
<td>* 0.048</td>
<td>0.337</td>
</tr>
<tr>
<td>E_{max} Y</td>
<td>0.052</td>
<td>0.453</td>
<td>0.160</td>
</tr>
<tr>
<td>E_{max} Z</td>
<td>0.558</td>
<td>* 0.012</td>
<td>0.404</td>
</tr>
<tr>
<td>QV X</td>
<td>-0.407</td>
<td>0.059</td>
<td>0.003</td>
</tr>
<tr>
<td>QV Y</td>
<td>-0.111</td>
<td>0.342</td>
<td>-0.162</td>
</tr>
<tr>
<td>QV Z</td>
<td>0.067</td>
<td>0.403</td>
<td>-0.122</td>
</tr>
<tr>
<td>E_{total} X</td>
<td>0.374</td>
<td>0.077</td>
<td>0.272</td>
</tr>
<tr>
<td>E_{total} Y</td>
<td>0.013</td>
<td>0.481</td>
<td>0.130</td>
</tr>
<tr>
<td>E_{total} Z</td>
<td>0.597</td>
<td>** 0.007</td>
<td>0.287</td>
</tr>
</tbody>
</table>

Figure 57 Significant Z lead LAD Correlates
Figure 58 Significant Z lead LAA Correlates

Figure 59 Significant Z lead RAA Correlates
4.3.4 Discussion of Results

There are inherent limitations on the accuracy of any ECG based method of approximating atrial size. As described in section 4.2.4.4, variations in the ECG can be produced by natural fluctuations in the position of the heart and other factors which are unrelated to chamber size. A limitation to this novel method of estimating the left atrial area could be presented by damage to the myocardial tissue. However, such damage would alter the morphology of the P wave observed in the surface ECG and be independently indicative of an abnormal condition within the cardiac mass. A thorough investigation of this approach using a large subject sample is warranted to further assess the viability of this measure’s ability to indicate the cross sectional area of the left atrium.

Figure 60 Significant X lead Echo Measurement Correlates
4.4  Summary & Discussion

In his review of biomedical engineering over the past 50 years L. Titomir [32] states that any advances in the processing of biomedical signals should result in increased informativity [sic] of the original data. That is, to warrant the increased complexity of analysis, any additional processing of the original data should yield more diagnostically important information than would otherwise be available. Such is the case with wavelet processing techniques which have gained in popularity in the past decade and have been rapidly applied to many signal analysis problems. Many applications have occurred in the field of biomedical signals, where RMS power analysis of the discrete wavelet transform energy spectrum has proven effective in detecting anomalous conditions or highlighting changes which occur in relation to stimuli or pathological condition.

In this chapter a novel application of the continuous wavelet transform to the analysis of the electrocardiographic P wave has been presented and detailed. This method makes use of the continuous wavelet transform (CWT) with a derivative of Gaussian basis function and its associated energy spectrum to characterise the electrical activity generated by the atria. The use of the CWT increases the number of operations and the complexity of the analysis when compared to the DWT, however, it also increases the accuracy or resolution of energy distribution which is exploited by the method presented here. The second order derivative of Gaussian was found to produce the best correlation to the P wave in lead II of the standard 12 lead ECG, the symmetric nature of the basis function was also found to be advantageous in analysing the P waves contained in lead V1.

The superior low frequency resolution of the wavelet energy spectrum, compared to the windowed Fourier transform, is advantageous to the analysis of the ECG and more specifically the P wave. ECG signals are sampled at relatively low rates (250-1000Hz), therefore subtle changes in duration and morphology of the P wave will occur at low frequencies which are highlighted by the wavelet energy spectrum, as shown in section 4.2.4.2.3. Changes in morphology and duration were quantified by a set of descriptive measures which describe the energy distributions of the P wave produced by the wavelet transform. These measures are related to traditional ECG characteristics such as duration, amplitude, terminal force and area of the P wave and were found to be sensitive to differences in P wave morphology resulting from abnormal conduction conditions. The advantage of the wavelet measures over the traditional measures is evident in that the wavelet measures provide an objective means of characterising the P wave and effectively
eliminate inter-observer discrepancies and observer variations between multiple data records.

Of most interest is the observation in section 4.3 that several wavelet transform characteristics of the P wave provided a good indication of physical atrial dimensions measured by clinical 2 dimensional 4 chamber apical view echocardiograms. The characteristics were produced from the wavelet transform of signal averaged P waves in a derived orthogonal ECG lead set and the relative leads from the standard 12 Lead ECG. It was observed that the measures derived from the P waves in the Z lead orthogonal to the echocardiogram plane produced the highest correlation to left atrial area. The measures which characterised the dominant frequency and median point of the P wave energy spectrum were observed to correlate highly to a linear relationship with left atrial diameter and area.

The peak frequency showed high correlation with left atrial diameter (R=-0.679) and left atrial area (R=-0.575). Therefore, the lower the dominant frequency of the P wave, the larger the diameter and size of the left atria. This result is analogous to the theory of the P terminal force which takes into account the duration and amplitude of the negative phase of the P wave in lead V1. However, instead of relying upon an approximation of area under the curve from scalar measurements, the peak wavelet frequency matches the shape of the P wave objectively using a consistent reference, the basis function. The median frequency, which indicates the midpoint of the P waves energy-frequency distribution produced similar correlations with left atrial diameter (R=-0.609) and left atrial area (R=-0.535).

The right atrial area produced high correlations with the total RMS (R=0.593) and peak values (R=0.681) of the P wave’s energy spectrum observed in lead Z. This indicates that higher energy values are observed in P waves as the right atria increases in area. Dilation of cardiac chambers generally increases conduction time, which, in the case of the right atria, means that electrical activity in the right atria will overlap increasing overlap with left atrial activity producing a larger cumulative surface electrical potential in the Z direction.

Left atrial functioning is important to the overall state of the cardiac cycle and is indicative of several serious complications to heart functioning. Currently, ECG indicators of left atrial dimension are unreliable, providing either satisfactory sensitivity with poor specificity or vice versa. The most accurate means of assessing left atrial dimensions are
obtained by echocardiogram or magnetic resonance imaging (MRI), both of which are significantly more expensive than the ECG and consequently not as widely accessible. A larger scale and more comprehensive study would be required to determine the accuracy wavelet energy distribution characteristics in indicating left atrial area and the necessity of the extra complexity of deriving the orthogonal lead set. An accurate method of determining the left atrial area from electrocardiograms would be of considerable benefit to the wider community. Such a cost effective tool would be directly beneficial to rural and remote healthcare in countries like Australia.

There are several avenues for future work which could be pursued. The development of this novel method of P wave analysis would require larger scale studies to assess its reliability. Refinements to the method could be focused on the definition of a customised wavelet basis function for the initial derivation of the wavelet energy spectrum. Additionally, differing wavelet basis functions could be used to analyse the P waves in each lead utilised in the analysis as the P wave morphology varies considerably across the 12 leads. As presented here, the method has proven to be sensitive enough to autonomously identify changes to the morphology of the P wave. The method could therefore be capable of identifying underlying properties of the atrial electrical activity and may be viable for clinical diagnostic purposes, such as automated classification of the P wave. This application will be investigated in the following chapter.
4.5 References


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42 C.P. Appleton, J.M. Galloway, M.S. Gonzalez, M. Gaballa and M.A. Basnignt, “Estimation of left ventricular filling pressures using two-dimensional and Doppler echocardiography in adult patients with cardiac disease: additional value of analyzing left atrial size, left atrial ejection fraction and the difference in duration of


Chapter 5

Characterisation of Abnormal Atrial Electrical Activity

A method of analysing clinical ECG P waves using the continuous wavelet transform was developed and demonstrated. The method was shown to be sensitive to subtle changes in the morphology of the P wave and the extracted parameters could succinctly describe the P wave. The descriptive potential of the wavelet analysis method is further investigated in this chapter by applying it to the problem of classification. Manual analysis of the P wave provides several descriptive parameters which can be used to indicate abnormality. Abnormalities in the P wave may be produced by structural changes, such as left atrial enlargement (LAE), or by complications to the conduction path, such as inter-atrial block (IAB). These conditions can occur in conjunction with one another [1], but they are also associated with several other complications and pathologies. LAE has been significantly linked to a host of pathological conditions and complications such as long term and permanent atrial fibrillation [2] [3], atrial thrombus and embolic stroke. IAB has been shown to be significantly prevalent in patients who suffer embolic strokes [4].

Over the past 4 decades, several electrocardiographic indicators of LAE have been derived and used in clinical practice. In recent years these indicators have proven to be unreliable and nonspecific to LAE [5][6][7] by the use of echocardiograms, which allows accurate measurements of the heart’s chambers, and magnetic resonance imaging (MRI), which provides even greater resolution imaging of the cardiac mass. Thresholds of electrocardiographic markers, such as minimum duration, have been proposed to indicate the presence of IAB [8] [9] [10]. These thresholds have conflicted with traditional normal limits of the P wave effectively further blurring the line between what is considered a normal and abnormal P wave.
5.1 P Wave Classification

Due to the comparative costs and availability of the technologies, the ECG is still the most common method of analysis in cardiology clinics. Despite their limited specificity to LAE, ECG indicators of LAE are still used in everyday clinical practice to identify abnormal atrial conduction so that patients to more extensive testing or treatment. Because of this, continued research is focused on the analysis of the ECG. Analysis of the P wave morphology has gained interest in the past decade as a means of addressing the ambiguity of traditional P wave ECG markers[11], specifically in the analysis of paroxysmal atrial fibrillation (PAF), where analysis of the P wave in sinus rhythm can give indications of the onset of atrial fibrillation [3]. In the previous chapter, a novel application of the wavelet transform to the analysis of the P wave was presented. From the results of this application, it was inferred that the condition of the P wave conduction may be characterised by its time-frequency energy distributions. The characteristics are derived from the wavelet transform coefficients of the P wave and employed in a classification model. Several classification models are created using different lead sets and compared. The performance of the classification models is compared against the clinical electrocardiographic P wave markers. The wavelet characteristics are also tested for correlation with the P wave duration, which is a universally accepted indication of the normality of atrial conduction [12] [13] [14].

5.1.1 Abnormal Atrial Conduction

Abnormal atrial conduction may result from either damage or structural change to the cardiac tissue in the atrium or irregularities in the initial activation pulse or subsequent conduction throughout the atria. The ECG waveforms represent the collective potential of the cardiac tissue, therefore any change conditional or structural changes to the cardiac tissue will be reflected in the ECG. The P wave directly represents the atrial electrical activity, beginning with the initial pulse created by the sino-atrial node and its conduction path through the right and left atria, successively. The duration of the P wave can be affected by several pathologies. In the case of LAE, the abnormal conduction patterns are due to the increased propagation distance created by atrial hypertrophy or dilation. A similar prolongation of the P wave can be observed in the presence of IAB, though the cause of the P wave augmentation is delayed conduction between the two atria [10]. Damage to the myocardial tissue results from restricted supply of blood and oxygen, this leads to necrosis, or infarction, of the tissue [15]. This damaged or dead tissue impedes
electrical conduction and will also create alterations in the P wave’s appearance. The standard 12 lead ECG offers discrete views of the cardiac electrical activity in three dimensions. Each lead is essentially a conduction vector describing the electrical potential in a discrete direction. The limb leads (I-III, aVF, aVL, aVR) representing the frontal plane are evenly spaced 30 degrees apart, the differences in the P wave waveform between these leads can be observed as a gradual change. Similarly, the precordial, or chest, leads provide discrete views perpendicular to the frontal plane. Significant differences in P wave morphology can be observed between leads which are orthogonal to each other. This is shown in Figure 61, the P wave in Figure 61 (a) shows normal atrial conduction in three leads selected from the 12 lead ECG. The P waves in Figure 61 (b) and (c) are taken from two recordings indicating LAE. The P wave in Figure 61 (b) exhibits bifid patterns, which are a result of asynchronous depolarisation in the left and right atria [16]. The positive and negative deflections in leads V2 indicate conduction through the right and left atria respectively. In Figure 61 (a), the opposing deflections in V2 have similar amplitudes with a smooth transition between them. The respective P waves in (b) and (c) display a change in slope between the two deflections which indicates a delay in the propagation of the conduction wavefront between the atria. This asynchronous conduction can be due to increased propagation distance in the case of Left Atrial Enlargement (LAE) [15] or a pathological delay in conduction between the atria, as is the case of Inter Atrial Block (IAB) [10].

Figure 61 (a) shows normal atrial conduction as opposed to the enlarged left atrial depolarisation in (b) and (c)
5.1.1.1 Cardiological Measures of Abnormal Atrial Conduction

Though the standard clinical ECG recordings use 12 leads, the most commonly used ECG indicators of atrial abnormality are derived from the waveforms in lead V1 and lead II. This is due to the perspectives offered by these leads. The opposing waves of the atria are most prominent in lead V1 which results in a biphasic waveform in the P wave, accentuating the depolarisation of the separate atria and any asynchronicity. The waveform in lead II is generally the highest amplitude and widest P wave as it describes the long axis view of the heart and is perpendicular to lead V1. Though alterations in atrial conduction can be the result of structural remodelling the ECG does not provide direct observation of the cardiac structure. The echocardiogram provides a non-invasive means of assessing the structural dimensions of the heart using ultrasonics [17]. There are two prevailing methods of echocardiogram, M mode which uses a single beam and the 2D method. The latter has been shown to be the more accurate and informative method of measurement and is now the standard [18]. The multiple beam approach offers several possible views of the cardiac chambers. The standard view for measurement of atrial dimensions is the 4 chamber apical view [18]. The left atrial is considered to be enlarged if its transverse dimension is in excess of 4.0 mm or the ratio of the dimensions of the left atrium to aortic root is greater than 1.17 [19]. This dimension is also reported with respect to the body surface area (BDA). In this case, the normal range is reported to be between 1.2-2.0cm/m² [19].

The most fundamental and robust measure for analysis of the P wave is its duration. This denotes the time in milliseconds between the initial excitation of the sino-atrial node (onset) and the completion of depolarisation in the left atria (offset). Typically, both IAB and LAE result in prolonged P wave duration [1]. There are varying definitions of how long a P wave must be before it is considered prolonged. Normal duration is considered to be less than 120ms by most textbook standards. Thresholds for the consideration of LAE vary, with some sources specifying that durations greater than 110ms [10] [20] or 120ms [21] are abnormal. P waves of normal duration can be observed in the presence of IAB [4] and more frequently in LAE [21]. However, it is assumed that the presence of LAE can be ruled out completely for P wave durations less than 105ms [22]. Table 8 lists the most common electrocardiographic criteria used by cardiologists to indicate LAE.
Table 8 ECG P Wave Criteria Suggesting LAE

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Sensitivity</th>
<th>Specificity</th>
</tr>
</thead>
<tbody>
<tr>
<td>P wave duration &gt;105ms [22]</td>
<td>100%</td>
<td>83%</td>
</tr>
<tr>
<td>P wave duration &gt;120ms [21]</td>
<td>4%</td>
<td>99.5%</td>
</tr>
<tr>
<td>P wave duration &gt;110ms [20]</td>
<td>33%</td>
<td>88%</td>
</tr>
<tr>
<td>P terminal force V1 &lt;-4mV ms [22]</td>
<td>75%</td>
<td>83%</td>
</tr>
<tr>
<td>P terminal force V1 &lt;-0.03 mm sec [21]</td>
<td>11%</td>
<td>86%</td>
</tr>
<tr>
<td>P terminal force V1 &lt;-0.04mm sec [20]</td>
<td>69%</td>
<td>93%</td>
</tr>
<tr>
<td>Negative phase of P V1 &gt;40 ms [20]</td>
<td>83%</td>
<td>80%</td>
</tr>
<tr>
<td>Negative phase of P V1 ≥1nm [20]</td>
<td>60%</td>
<td>93%</td>
</tr>
<tr>
<td>Notched P inter-peak interval &gt;40 ms [20]</td>
<td>15%</td>
<td>100%</td>
</tr>
<tr>
<td>P wave duration/PR duration² &gt;1.6 [22]</td>
<td>58%</td>
<td>89%</td>
</tr>
<tr>
<td>Negative phase of PV1/PR duration &gt;1.0 [21]</td>
<td>6%</td>
<td>99%</td>
</tr>
<tr>
<td>P wave duration/PR duration &gt;1.6 [20]</td>
<td>31%</td>
<td>64%</td>
</tr>
<tr>
<td>P duration, PV1 &amp; PV1/PR ratio [21]</td>
<td>59%</td>
<td>80%</td>
</tr>
<tr>
<td>P wave area in II &gt;= 4ms mV [23]</td>
<td>85.8%</td>
<td>93.7%</td>
</tr>
</tbody>
</table>

The amplitudes of the P wave deflections in various leads are also used as an indicator of the presence of LAE. Leads V1 or V2 contain several criteria which can indicate the presence of LAE. The negative, or terminal, phase of the P wave in lead V1 corresponds to the left atrial excitation period. Enlargement of the left atria will affect the duration of this phase and also increase the sum of its propagation wavefront recorded at the epidermis.

The P terminal force in V1, often written as PV1 or referred to as the Morris index, is defined as the product of duration and amplitude of the P wave’s negative phase. It was first suggested as a highly specific indicator for the presence of general left-sided valvular lesions [24]. The value of PV1 is also indicative of the severity of these conditions once their presence has been confirmed. The threshold of 0.04mm.sec (4mV.ms) is commonly used as an indicator for the specific presence of LAE. Both [21] and [20] concluded that combination of two or more of the measures did not significantly improve both sensitivities and specificity. However, the combination of the three criteria investigated in [21] elicits degradation (a 19% drop) in specificity for a significant improvement in sensitivity, from 11% to 59%.

The ratio of the P wave duration over the PR segment duration (P/PR) was first proposed in [25] as a general indicator of both left and right atrial size. A ratio of less than 1.0 was shown to indicate right atrial enlargement. A ratio of greater than 1.6 indicated left atrial enlargement. The limits reflect the alterations to the atrial conduction path caused by enlargement of either atrium. Enlargement of the right atrium affects the conduction time between the sino-atrial node and the atrio-ventricular node, this effectively increases the

² The cut-off for this criterion was not explicitly mentioned in [22] but it is assumed to be that of [25].
PR duration. It does not affect the overall duration of the P wave, as prolonged right atrial conduction occurs simultaneously with normal left atrial conduction. Therefore, only the denominator of the ratio is affected by right atrial enlargement. Left atrial enlargement does not affect the inter-nodal conduction time, but the increased conduction path through the left atria extends the terminal phase of the P wave, thereby increasing the overall P wave duration. The increase in duration therefore affects the numerator of the ratio.

The P wave area is defined as half the duration multiplied by the maximum amplitude of the P wave in lead II [23]. This ECG measure was shown to be more sensitive to atrial enlargement than P duration or P mitrale. It was also shown to have a significant correlation to the left atrial diameter. A study which compared 4 of the ECG P wave measures to 2D echocardiographically derived left atrial volumes found the measures of P duration and P wave to be significantly but modestly correlated to left atrial volume [7]. The same study also showed that the P wave measures of P duration, PV1 and P/PR segment were highly specific but insensitive to left atrial enlargement, while P area was more sensitive but poorly specific. Similar results were found in a study comparing ECG measures of atrial enlargement to measurements made using magnetic resonance imaging [5]. Conduction delay between the atria, such as IAB, is another cause of conduction abnormality. This condition has a similar effect to enlargement of the left atria upon the P wave, as it increases the time required for the atrial conduction wavefront to propagate from right to left atria. This results in an overall prolongation of the P wave and an associated change in morphology. Table 9 several proposed ECG markers of IAB.

<table>
<thead>
<tr>
<th>CRITERION</th>
<th>SENSITIVITY</th>
<th>SPECIFICITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>P wave duration &gt;110ms</td>
<td>100%</td>
<td>88.9%</td>
</tr>
<tr>
<td>P wave duration &gt;130ms</td>
<td>64%</td>
<td>100%</td>
</tr>
<tr>
<td>“notched” P waves</td>
<td>75%</td>
<td>94%</td>
</tr>
<tr>
<td>“dome-spike” P morphology</td>
<td>96%</td>
<td>70%</td>
</tr>
</tbody>
</table>

A secondary temporal measure which is not common in clinical practice but which has gained prominence in analytical studies is P wave dispersion. Dispersion of P wave duration across the 12 leads was not examined as this has been extensively investigated, specifically for the case of atrial fibrillation where multiple wavefronts in the atrium [26] result in significant differences of P wave duration [27]. Other experimental measures often used in diagnostic assessment of the P wave include the dispersion of atrial conduction and the use of P wave signal averaged ECG. Both have been shown to be reliable clinical predictor, primarily in the presence of atrial fibrillation [2], [28], [29], [30].
Subsequent studies of left atrial dimensions using imaging technologies have shown that the ECG measures in Table 8 have poor sensitivity and correlation to left atrial enlargement. However, the ECG measures of duration of the P wave and the P area were found to have some significant but modest correlation to the left atrial volume [7]. Further ambiguity can be observed in the crossover of acceptable normal P wave durations in Table 8 and Table 9. Recent suggestions in the cardiology literature have suggested that a change in terminology is required to reflect these limitations of the ECG markers [5] [10].

A contributing factor to the inaccuracy of P wave parameters in the past was the manual analysis of ECG poor resolution paper traces. Manual analysis is also prone to inter-observer variations and observation variations across a set of recordings [27]. The advent of digital ECG recording and software analysis has provided great improvements in this respect.

5.1.1.1 Automated Analysis

As automated analysis of the ECG has evolved from experimental development into clinical practice, time-frequency analysis techniques have provided a significant addition to analytic tools used by cardiologists [31]. Various approaches have been applied to classification of the ECG and specifically the P wave. Classification of the P wave based on morphology has previously been approached using artificial neural networks [32], system modelling [33] on both standard 12 lead and XYZ lead configurations. The use of wavelets for analysis and classification of biomedical signals such as the ECG is well documented [34] [35]. Primarily the work involving wavelet analysis of atrial activity has centred on the extraction of atrial flutter and the analysis of atrial fibrillation [34]. Wavelet analysis of P wave morphology has also shown promise in prediction of atrial fibrillation in post coronary patients [36]. Everyday clinical practice and most ECG research studies, use the 12-lead ECG configuration or one of its simpler derivations; although some recent studies concerned with morphological analysis of the P wave have used the Frank lead configuration [36] or a derived version of it [33]. The present study chose to analyse the P wave using configurations of the 12 lead ECG because of their familiarity to clinical cardiologists. The leads II and V1 were chosen as they are the most heavily scrutinised for the case of LAE and IAB, as can be seen from the measures detailed in Table 8. Studies have also shown that the P wave is most prominent in these leads of the 12 lead ECG [37]. Leads aVF, V2 and V5 were chosen as a succinct representation of the frontal, sagittal and transverse planes of the ECG [38]. These planes are considered to be approximately orthogonal in the 12 lead ECG. An accurate orthogonal representation of the heart vector
requires a set of corrected vectocardiographic leads such as the Frank lead system [37]. These leads roughly correspond to those of the XYZ of the Frank lead system. Lead V5 corresponds to the X direction, Lead aVF to the Y direction and Lead V2 is opposite in polarity to the Z direction of the Frank lead system. As a comparison the reconstructed XYZ lead set was also derived from the 12 lead ECG using the inverse Dower transform [39].

5.1.1.2 Experimental Data

The arrhythmic set contained thirty-two participants (age 77.6±10.9 years) who were identified as having abnormal atrial conduction using the ECG measures of P wave duration in lead II and the ECG indicators of lead V1. Several of the subjects were verified by the use of echocardiograms. All the subjects had a documented history of atrial conditions. In addition to this, 4 subjects displayed possible inferior infarction, 8 displayed sinus bradycardia, 8 displayed 1st degree AV block and 1 subject displayed bi-atrial hypertrophy. Initially the control group consisted of 10 participants. The criteria for the control group required subjects to be over 50 years of age with no history of heart disease or hypertension. All control group recordings were screened by practicing cardiologists. Subsequently, one of the control subjects was identified as exhibiting LAE characteristics and was removed from the control group. Two other subjects were identified as having abnormal P waves linked to sinus bradycardia and were also removed from the control group. The control group therefore consisted of 7 subjects (56.43±4.47 years). The screening of control participants and data collection received clearance by the Griffith University ethics council prior to the commencement of the study. (Ethical clearance protocol is contained in Appendix A). It can be noted that there is a difference between the average ages of the groups and aging has been attributed to several changes in atrial functioning [40]. However, age was not found to be a confounding factor in this analysis. This result agrees with the findings of [41] where the prevalence of IAB was found to drastically increase from 32% to 50% between the ages of 40 and 50 in a general hospital population. However, IAB was not found to increase significantly after the age of 60, where prevalence remained at 59%. Therefore, the results here can be expected to extrapolate well if a more aged normal group were used. Obtaining a sample data set with no cardiac or related pathology is a significant challenge to researchers and was outside the means of data collection for this thesis. There is also a discrepancy in the sizes of the

3 This is consistent with the nomenclature of [33] which differs from that of [38].
normal and abnormal training set, though it has previously been shown that relative set sizes do not need to be equivalent in discriminant analysis [50].

All data was recorded at a private cardiology clinic by cardiac technicians using the standard 12 Lead ECG configuration. The recordings were made using the Micromedical Cardioview ECG->PC link™. This interface acquires the ECG signals with 16 bit precision at a sampling rate of 1000Hz compressed to 500Hz using an averaging and peak picking algorithm. The amplifier input range is +/- 6mV with a 3dB bandwidth of 0.05-175Hz. All recordings were made with permanent 1st order high and low pass filters at 0.05Hz and 170Hz, respectively. This system is integrated with the Cardioview 3000™ [18] software which provides several user selected filters to reduce the appearance of noise during diagnosis. These filters are not applied to the archived data. ECG data was recorded in 10 second epochs as per clinical practice. Six epochs were recorded for each control participant and 14 of the participants exhibiting bifid P waves. These subjects were used in the test set to produce the classification models in 5.2. One, 10 second, epoch was recorded for the remaining participants in the LAE group. These subjects were used in the cross validation set of the classification models. A single epoch from each subject was used to derive an averaged P wave for the analysis in 5.3.

5.1.2 Analysis Process

In chapter 3, several methods devised to analyse the P wave were mentioned. Each method followed the basic steps of event detection, derivation of characteristic model and testing, or classification.

The analysis process employed in this work follows similar stages.

- P wave detection – To identify the location of the P wave and the region of interest for the analysis stage.

- Wavelet analysis – For energy spectrum analysis and derivation of characteristics

Several methods were used to identify appropriate wavelet characteristics for the classification stage.

- Orthogonal lead sets – To characterise atrial conduction in a set of clinical leads which efficiently represent the frontal, sagittal and transverse planes of the 12 lead ECG.
• Clinical diagnostic lead groups – To characterise atrial conduction in the leads in which the P wave is most often analysed; leads II and V1.

• Data reduction and factor analysis – To reduce the number of characteristics required for classification by deriving representative factors from the original characteristics.

• Classification – To assess the classifying capabilities of the wavelet characteristics.

To construct a characterisation model, it is necessary to define a consistent region of interest for all the P wave cases.

5.1.2.1 P wave detection

P waves were identified by a standard approach involving automated multi-scale event detection to determine the location of the QRS complex as a landmark to search for the P wave [42]. The P wave detection approach has previously been detailed in Chapter 4. Pre-processing of the ECG involved the removal of baseline wander. Baselines were extracted using a two step process. A 4th order lowpass Butterworth filter with a 3dB cutoff at 0.5Hz was used to extract the low frequency baseline component from the original signal. This allows the baseline signal to be subtracted from the original signal. The baseline removal step is performed primarily to enhance the display of the final ECG, it is also important in the signal averaging of the P wave which will be discussed later.

The QRS complexes were isolated by wavelet decomposition and applying a hard threshold to several selected scales of wavelet coefficients. A Mexican hat basis function, essentially a 2nd order derivative of the Gaussian function, was employed in a continuous wavelet transform to produce coefficients at scales with corresponding central frequencies of \{25, 12.5, 8.93, 6.94\} Hz. Cross multiplying these scales of wavelet coefficients emphasises the high frequency components of the QRS complexes. This discriminates between QRS complexes and broader T waves which are produced from the same muscle mass and also have high amplitudes. A search window of 50-250ms prior to each QRS complex was processed to identify the P wave peak. Though similar studies have used longer timeframes [33], this window was found to be sufficient to account for the normal PR interval duration of 140-210ms and the normal maximum Q wave duration of 30ms [15]. The duration of the P wave was derived from the rhythm strip of lead II which typically produces the longest duration over all the 12 leads. The duration of the P wave in lead II was derived from this process and served as the standardised measure of the P
wave’s duration. The onset and offset of the P wave were also determined from this lead using a combination of minima and slope detection in an area surrounding the peak of the P wave.

### 5.1.2.2 Wavelet Analysis

As previously mentioned in Chapter 4, the wavelet transform allows the frequency content of a continuous signal to be analysed with more flexible temporal localisation compared to the Fourier transform [43]. The application of the wavelet spectrum methods has been shown to be effective in the multi-frequency analysis of other physiological signals such as the EEG [44]. Wavelet analysis of the ECG can provide information about the amplitude, scaling and location of its characteristic points. The chosen wavelet for this application was the 2nd order derivative of the Gaussian function, which was found to provide the highest correlation with the healthy P wave [45]. This is fortuitous, as the Gaussian wavelet bases also provide the optimal time-frequency localisation [46]. The 2nd order derivative wavelet basis function is defined as:

\[
\psi(t) = B_0 (1 - t^2) \exp(-\frac{t^2}{2})
\]

(5.1-1)

The constant, \(B_0\), is the normalising coefficient for the wavelet base required to normalise the energy of the basis function. An un-normalised form of the basis function is displayed in Figure 62. The continuous wavelet transform of 5.3-2 is performed for over a continuous range of scales which contain the P wave information.

\[
C(a,b) = \int_{-\infty}^{\infty} x(t) \psi^*_{a,b}(t) \, dt
\]

(5.1-2)
The set of wavelet coefficients and the wavelet energy spectrum were then used to derive the following characterizations of the P wave: The wavelet energy spectrum is given by 4.5-3; this is often referred to as the scalogram [43].

\[ E(a, b) = |C\{a, b\}|^2 \]  

The equation in 4.5-3 was modified by limiting the energy density in the time axis to the extremities of each P wave. The energy spectrum is computed for each P wave and averaged across its duration, \( \tau \). The total energy, \( E_p(a) \), contained in each scale, \( a \), of wavelet coefficients for a P wave was defined as:

\[ E_p(a) = \frac{1}{\tau} \sum_{\tau=p}^{\tau} |C\{a, b\}|^2 \]  

Where the total energy of the P wave at each level was defined as the sum of all energy at all locations between the onset, \( p' \), and offset, \( p'' \), of the P wave. The wavelet coefficients are then processed to derive several energy, frequency and variance measures of the P wave. The energy distribution trends and measures, such as total energy, central frequency of distribution, and frequency of maximum energy, are derived from the wavelet energy spectrum. In deciding on the frequency band to investigate this study was guided by previous work [45], which is detailed in the previous chapter. The wavelet transform scales which correspond to the frequency band 2.5Hz to 25Hz were found to be highly
correlated to the overall morphology of the P wave. The high correlation values indicate that the majority of the P wave energy is contained within this band. These results correspond to those of [47] which found the energy of atrial activity during atrial fibrillation was confined to the range of 3Hz to 12Hz. This type of atrial activity is often referred to as F waves. In the presence of atrial flutter, the frequencies around 12Hz are representative of higher order harmonics of the F waves. In regular atrial depolarisation, significant details of P wave morphology can be observed in higher frequencies [36]. Retention of higher frequency scales is therefore beneficial to the application of P wave morphology analysis. Frequency bands of coefficients lower than 2.5Hz were also observed to be significantly affected by adjacent QRS complexes and were therefore discarded from analysis.

The scalogram and associated plots for the averaged P wave, recorded in lead V1 of a control subject, is shown in Figure 63. The same plots for an abnormal subject exhibiting severe LAE are shown in Figure 64. The colours of the scalogram in the top left corners of Figure 63 and Figure 64 indicate the magnitude of the correlation between the wavelet basis function and the P wave. The scalograms have been normalised such that the darkest red colour corresponds to a value of 1 and the deep blue corresponds to a values approaching 0. The scalograms show two areas of high energy concentration. These areas correspond to the positive and negative deflections of the averaged P wave, shown in the lower right hand plots. As previously mentioned, the positive and negative deflection of the P wave in lead V1 correspond to conduction through the right and left atria respectively. Any difference in amplitude, and therefore energy content, between the positive and negative deflections is accentuated by the symmetric shape of the wavelet basis function.
Figure 63 Wavelet Analysis of Normal P wave

Figure 64 Wavelet Analysis of abnormal P wave
The difference between the two areas of energy concentration is shown in the lower left hand plots. This plot shows the temporal profile of the scalogram at the scale of maximum correlation. The scalogram of Figure 64 highlights the disparity between the positive and negative deflections. The upper right hand plot shows the frequency profile of the scalogram, this indicates the maximum correlation of the P wave at each of the wavelet transform scales. The difference in scale of maximum correlation between the normal P wave and the abnormal P wave is obvious. The spread of the energy distributions of the normal are also markedly different. The abnormal case exhibits a broader, more dispersed distribution than the normal case. A set of characteristics was devised to quantify these observations.

5.1.2.3 Discriminant Characteristics

A total of six characteristics were derived from the wavelet transforms of each lead used in the various models. The measures which are derived from the wavelet transform coefficients relate to the morphology of the original P wave and succinctly describe its wavelet energy spectrum. The measures derived from the wavelet energy spectrum, defined 5.3-3, were:

- Total energy – $E_{total}$
- Peak Frequency – $f_{peak}$
- Peak Value – labelled as $E_{max}$
- Interquartile Range – $IQR$
- Quartile Variation – $QV$
- Median Frequency – $f_{median}$

The measure for total energy was defined as the sum of energy contained within all scales for the duration of the P wave as defined in 5.3-5. The sum of the energy in all scales of the transform is normalised by the number of scales of wavelet dilation, $k$.

$$E = \frac{1}{k} \sum_a E_p(a)$$

(5.1-5)

The peak value, $E_{max}$, characteristic corresponds to the maximum value of $E_p(a)$, produced by the wavelet transform within the limits of the P wave. The peak frequency, $f_{peak}$, is the central frequency of the wavelet scale at which this maximum occurs.
Three variables describing the shape of the P wave peak energy distribution were also derived. Measures using the quartiles of the distribution were favoured over mean and deviations because of the non uniform intervals between wavelet scales and the asymmetry of the distributions produced. A distribution can be divided into 4 evenly sized sets called quartiles. A quartile represents 25% of the total distribution of an observation. In a non-uniform or asymmetrical distribution, the range of each quartile will be unique and can be quantified by these measures. The dispersion of the energy throughout the wavelet scales was quantified by the inter-quartile range and the quartile variation measures. These measures are common in variance analysis as they can describe irregular distributions more accurately than measures such as standard deviation and variance. In this application these measures quantify the spread of the P wave’s energy distribution which gives an indirect indication of the initial P wave morphology. The inter-quartile range, $IQR$, and quartile variation, $QV$, defined, respectively as:

\[
IQR = Q3 - Q1
\]

\[
QV = \frac{Q3 - Q1}{Q3 + Q1}
\]

The first, $Q1$, and third, $Q3$, quartiles are median points between the extremes of each P wave’s energy distribution and its statistical median. The inter-quartile range describes the dispersion of the central 50% of the total energy of the P wave across the wavelet frequency spectrum. The $QV$ expresses the $IQR$ as a proportion of the whole range of the energy distribution. The quartile variation gives an indication of the inter-quartile range’s spread relative to the complete range of interest. The second quartile is the statistical median; the central frequency of the wavelet scale corresponding to the median was also used as a characteristic, $f_{median}$, as it is a reference point for the $IQR$ and $QV$ measures. These characteristics derived from each set of leads were entered into the linear discriminant model. The characteristics were also processed using factor analysis and correlation analysis to reduce the order of the classifying models.

### 5.1.3 Factor Analysis

The term factor analysis refers to a group of processes which are used to analyse relationships and trends in an observed set of random variables for a given population [48]. The population usually sample population representing a specific group. Here the groups are defined as normal and abnormal atrial conduction caused by enlarged left atrial size.
The observations are a set of random variables whose observations vary for each member in the sample population. This is achieved by examining weighted combinations of the initial variables, \( \mathbf{X} \), to find a tertiary set of independent combinations which describe as much of the original set with minimal information loss. This results in the common factor model of (5.3-7). [49]

\[
\mathbf{X} = \mu + \mathbf{L} \cdot \mathbf{F} + \mathbf{\epsilon}
\]

The terms \( \mu \) and \( \mathbf{\epsilon} \) represent the mean vector of \( \mathbf{X} \) and the specific variances associated which each characteristic in \( \mathbf{X} \). The factor loadings, \( \mathbf{L} \), describe the contribution of each characteristic in \( \mathbf{X} \) to the common factors in \( \mathbf{F} \). The methods used to in the derivation of the factors, are outlined in chapter 3. In this work the sample correlation matrix was used in the principal component method derivation of the factor loadings. The selection of the number of factors was based on the eigenvalues produced by the principal component method analysis of the sample covariance matrix. Factors with an eigenvalue greater than 1 were retained for the factor model. Accordingly, the same numbers of factors were retained from the respective factor analyses using the correlation matrix. Factor rotation was performed using the normal varimax transformation. The varimax transformation attempts to minimise the number of high loadings on each factor by more evenly distributing the total variance. This makes interpretation of the factors simpler and reduces the likelihood of a generalised factor being produced [48]. The various common factor models were also assessed using the linear discriminant analysis applied to the original variables.

5.1.4 Classification

The classification of group membership of the data sets was performed using linear discriminant analysis (LDA) method detailed in Chapter 3. The aim of LDA is to optimally assign group membership using predictors to maximise the variance between groups and minimise the variance within groups [50]. LDA is a good initial choice for a classifier because of its relative simplicity and good overall classifying generalization with small sample sizes [51]. The selection of a nonlinear discriminator is usually made after a linear discrimination has not performed well. Non-linear discriminators, such as a neural network with hidden layers, have a tendency to overfit the classification during the training phase. This risk of over fitting to idiosyncrasies within the data is greater with small data
sets and can produce poor generalisation performance [52]. In this work, a linear discrimination was shown to perform the classification procedure adequately without the extra complexity of nonlinear or quadratic functions. The characteristics for each case in the training sets were arranged into a feature vector, \( X \). These vectors were used to derive a linear discriminant equation, 5.3-9, using the SPSS® v13.0 software [53].

\[
X_k = \begin{bmatrix}
  x_{k,1} & x_{k,2} & x_{k,3} & x_{k,4} & x_{k,5} & x_{k,6}
\end{bmatrix}^T \quad k \in K
\]  

(5.1-9)

\[
D_n = \sum_{m=1}^{M} h_m x_{n,m} \quad 1 \leq n \leq N
\]  

(5.1-10)

The set of canonical coefficients, \( h \), of the linear discriminant equation were applied to the feature vectors of each case, \( n \), in the training set to produce a distance score, \( D_n \). The discriminating threshold, \( \theta \), is set accordingly based on the distributions of distance scores derived from the training set. The canonical coefficients, \( h_m \), were standardized using the means, \( \langle x_m \rangle \), and standard deviations, \( \sigma_m \), of the respective characteristics from the training set as follows:

\[
\bar{x}_m = \sum_{n=1}^{N} x_{n,m} \quad 1 \leq m \leq M
\]  

(5.1-11)

\[
\sigma_m = \sqrt{\frac{1}{N-1} \sum_{n=1}^{N} (V_{n,m} - \bar{V}_m)^2} \quad 1 \leq m \leq M
\]  

(5.1-12)

\[
\hat{h}_m = \frac{h_m - \langle \bar{V}_m \rangle}{\sigma_m}
\]  

(5.1-13)

The standardised coefficients are useful in assessing the influence the individual variables have on the classification and the distance score produced. Reducing each subject’s set of P waves to a single distance score requires an approach to deal with the variance observed between successive P waves. Two approaches were employed for this purpose. In the first experiment, distance scores were calculated for each subject using a representative median vector of P wave parameters. Each subject contributed a set of characteristic vectors, each vector representing an individual P wave. The representative vectors of each subject contain the median values of the relative characteristics in the subject’s characteristic vectors. The discriminating threshold was calculated from the distribution of the distance scores produced by the median vectors in the training set. In
this experiment, distance scores were also produced for all the P wave characteristic vector sets for each subject to observe the effect of the individual P waves’ variance. Classification of the subject was based upon the score of the median vector. In the second experiment, the P waves of each subject were signal averaged to produce one waveform. The representative vector was derived from this averaged P wave and used in the linear discriminant analysis. The threshold was calculated using the distance scores of the training set and applied to the cross validation set in the same manner as experiment 1. The performance of the classification models was quantified using the sensitivity and specificity measures achieved for the training and cross validation sets. Only the sensitivity measure could be reported for the cross validation set as it consisted solely of abnormal subjects. Cross validation was performed using 100 random resamplings of the design set.

### 5.1.5 Procedures and measurements

The measures of sensitivity and specificity are defined as the percentages of correctly classified abnormal and normal subjects, respectively. These measures were calculated from the subjects’ average distance score obtained from the linear discriminant equation. The classification models were compared against the performance of the cardiological measures, detailed in Table 8, for the set of experimental data.

#### 5.1.5.1 Cardiological Analysis

A classification of the data was performed using the averaged P wave duration of each patient obtained from the Cardioview© software. The software produces an averaged beat for each 10 second epoch and measurements of this averaged beat are provided to the clinician. As stated previously in section 5.1.1.1, there are several criteria used to diagnose LAE or IAB and are more generally used to indicate atrial abnormality. The simplest criterion is the duration of the P wave. The mean and standard deviations of P wave durations for each group are shown in Table 10.

<table>
<thead>
<tr>
<th>GROUP</th>
<th>P DURATION (MS)</th>
<th>P/PR</th>
<th>PR1 MV MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>NORMAL</td>
<td>105 ± 10.32</td>
<td>2.64 ± 2.44</td>
<td>-3.97 ± 1.74</td>
</tr>
<tr>
<td>ABNORMAL</td>
<td>123 ± 11.83</td>
<td>2.31 ± 0.97</td>
<td>-5.43 ± 2.29</td>
</tr>
</tbody>
</table>
Table 11 Classification by Cardiological measures

<table>
<thead>
<tr>
<th>CLASSIFIER</th>
<th>ABNORMAL</th>
<th>NORMAL</th>
<th>SENSITIVITY</th>
<th>SPECIFICITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration &gt;110ms</td>
<td>27</td>
<td>3</td>
<td>84%</td>
<td>57%</td>
</tr>
<tr>
<td>Duration &gt;120ms</td>
<td>16</td>
<td>0</td>
<td>50%</td>
<td>100%</td>
</tr>
<tr>
<td>P/PR &gt; 1.6</td>
<td>22</td>
<td>4</td>
<td>69%</td>
<td>43%</td>
</tr>
<tr>
<td>PV1 &lt; -4mV ms</td>
<td>22</td>
<td>3</td>
<td>69%</td>
<td>57%</td>
</tr>
<tr>
<td>Notch &gt; 40ms</td>
<td>9</td>
<td>0</td>
<td>28%</td>
<td>100%</td>
</tr>
<tr>
<td>Either Prolonged P, P/PR or PV1</td>
<td>31</td>
<td>5</td>
<td>97%</td>
<td>29%</td>
</tr>
<tr>
<td>Prolonged P, P/PR and PV1</td>
<td>10</td>
<td>0</td>
<td>31%</td>
<td>100%</td>
</tr>
</tbody>
</table>

The results of the classification by the ECG of abnormality are displayed in Table 11. The differing duration criteria classify the participants produce similar specificity results to those stated in Table 8. Duration of greater than 110ms correctly identifies 4 of the control subjects and 30 of the abnormal group (Sp = 57%, Sn = 84%). Duration of greater than 120 ms (Sp = 100%, Sn = 50%). The cardiological measures were also employed in a linear discriminant approach to assess how well they separated the experimental data set. The LDA was run 100 times randomly sampling the abnormal subjects into training and cross validation sets. Using a single variable as input to the LDA reduces the problem to selecting a threshold value which optimally separates the two groups.

Table 12 Performance of LDA using Cardiological Measures on Experimental Data Set

<table>
<thead>
<tr>
<th>MEASURES</th>
<th>TRAINING NORMAL N=7</th>
<th>ABNORMAL N=16</th>
<th>CROSS VALIDATION ABNORMAL N=16</th>
<th>TOTAL N = 39</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>71.71%</td>
<td>73.06%</td>
<td>74.63%</td>
<td>73.46%</td>
</tr>
<tr>
<td>All ECG Measures</td>
<td>74.00%</td>
<td>76.44%</td>
<td>74.00%</td>
<td>75.00%</td>
</tr>
<tr>
<td>PV1</td>
<td>69.43%</td>
<td>55.63%</td>
<td>55.19%</td>
<td>57.92%</td>
</tr>
<tr>
<td>P/PR</td>
<td>14.29%</td>
<td>61.25%</td>
<td>59.00%</td>
<td>51.90%</td>
</tr>
</tbody>
</table>

Of the 3 ECG indicators of atrial abnormality, duration performed the best cross validation classification of the experimental data set. The other indicators performed well as reasonably sensitive measures indicating the presence of conduction abnormality. The combination of all three measures into a linear discriminant analysis did not improve upon the cross validation results obtained using only the duration.
5.2 Experiment 1 – Classification of Median P wave Characteristics

The clinical recording of the ECG provides a 10 second rhythm strip for cardiologists to diagnose. This rhythm strip contains a number of cardiac cycles, each cycle containing a respective P wave representing a single event of atrial depolarisation. In this experiment, each individual P wave was processed by the aforementioned method to produce a set of characteristics. The median point for each characteristic was found for each subject.

\[ V_{n}^{lead} = \{ E_{total}, E_{max}, f_{peak}, f_{median}, IQR, QV \} \]  

(5.2-1)

The vector in 5.2-1 is generated for each lead selected for the characterisation model. The generalised feature matrix, \( X \), contains the \( N \) number of row vectors of each of the \( L \) number of leads. The matrix is arranged as:

\[
X = \begin{bmatrix}
V_{lead1}^{1} & V_{lead2}^{1} & \cdots & V_{leadL}^{1} \\
V_{lead1}^{2} & V_{lead2}^{2} & \cdots & V_{leadL}^{2} \\
\vdots & \vdots & \ddots & \vdots \\
V_{lead1}^{N} & V_{lead2}^{N} & \cdots & V_{leadL}^{N}
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
x_{1,1} & x_{1,2} & \cdots & x_{1,M} \\
x_{2,1} & x_{2,2} & \cdots & x_{2,M} \\
\vdots & \vdots & \ddots & \vdots \\
x_{N,1} & x_{N,2} & \cdots & x_{N,M}
\end{bmatrix}
\]  

(5.2-2)

\( M = 3 \times L \)

The feature matrix in the test set contained 16 of the abnormal subjects and all 7 of the normal subjects. The cross-validation group contained all the cases from the remaining 16 subjects in the abnormal group. The abnormal test and cross validation sets were selected randomly. The classification was repeated 100 times with a new random sampling of the subjects for each run. The same 100 random sampling allocations were used for each model. Group means were calculated for the groups in the test set. Group mean vectors are required to obtain the maximal separation.
5.2.1 Lead Characteristic Models

The initial classification models were created using the characteristics derived for two leads sets from the 12 lead ECG. The derived VCG leads were also used to create a classification model. Multiple lead models should provide an accurate means of characterising the atrial conduction represented by the P wave.

5.2.2 II-V1 Lead Model

This model employed the characteristics from leads II and V1. These leads are heavily relied upon in the diagnosis of atrial arrhythmia. Lead II provides the longest axis view of the atria and therefore the duration of the P wave is usually greatest in this lead. Typically, the P wave is also monophasic and positive. Lead V1 provides an axis through the atria which helps to visualise the separation of the right and left atrial conduction patterns. The characteristics matrix for this model is shown in 5.4-3.

\[
X^{II,V1} = \begin{bmatrix}
V_1^{II} & V_1^{V1} \\
V_2^{II} & V_2^{V1} \\
\vdots & \vdots \\
V_N^{II} & V_N^{V1}
\end{bmatrix}
\] (5.2-3)

The order of classifier will affect the accuracy of the classification and the generalisation performance of the classifier [51]. A higher dimensionality of classifier would produce a closer fit to the training set but increase the likelihood of the classifier being affected by idiosyncrasies within the training set. It is therefore desirable to have fewer variables as feature inputs to the classifier. The optimal number of features for the classification procedure, \(O\), is dependent upon the type of classifier, the sample size and the method of feature selection [51]. The characteristics were ranked using correlation analysis to determine which ones to use as feature in the LDA [54]. The significance of bi-variate correlations can be quantified by the Pearson correlation coefficient, \(R\), and the significance value, \(p\). The bi-variate correlations for group membership were calculated for each characteristic in the set of characteristics. When the features are highly correlated \((R \geq 0.5)\) the optimal number of features is proportional to the square root of the sample size, \(O = \sqrt{N}\). When the correlation is low \((R < 0.125)\) the number of features required is \(N-1\) [54] [55].
Table 13 Bivariate Correlations Between Group Membership and II-V1 Characteristics

<table>
<thead>
<tr>
<th>CHARACTERISTIC</th>
<th>CORRELATION R</th>
<th>SIGNIFICANCE P</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1 f_{median} **</td>
<td>-0.508</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>II f_{median} **</td>
<td>-0.434</td>
<td>0.003</td>
</tr>
<tr>
<td>II QV *</td>
<td>0.344</td>
<td>0.016</td>
</tr>
<tr>
<td>II E_{max} *</td>
<td>0.334</td>
<td>0.019</td>
</tr>
<tr>
<td>V1 E_{max} *</td>
<td>0.320</td>
<td>0.024</td>
</tr>
<tr>
<td>II E_{total}</td>
<td>-0.255</td>
<td>0.059</td>
</tr>
<tr>
<td>II IQR</td>
<td>0.232</td>
<td>0.077</td>
</tr>
<tr>
<td>V1 f_{peak}</td>
<td>-0.142</td>
<td>0.194</td>
</tr>
<tr>
<td>V1 IQR</td>
<td>-0.081</td>
<td>0.313</td>
</tr>
<tr>
<td>II f_{peak}</td>
<td>0.062</td>
<td>0.353</td>
</tr>
<tr>
<td>V1 E_{total}</td>
<td>-0.018</td>
<td>0.456</td>
</tr>
<tr>
<td>V1 QV</td>
<td>-0.013</td>
<td>0.469</td>
</tr>
</tbody>
</table>

The bivariate correlations between group membership (normal and abnormal) and the characteristics from leads II and V1 are shown in Table 13. The majority of characteristics produced moderate correlations. In experiments performed in [54] the optimal number of features for sample sizes between 10 and 20 subjects with slightly correlated features is between 5 and 8. The design sample size of 16 subjects was used in the training phase. A subset of 6 discriminating characteristics was determined to be sufficient to produce accurate generalisation. The sign of the correlation coefficients for the characteristics indicates the direction of correlation between the characteristic and group membership. A positive correlation indicates that a higher value for any particular characteristic is more likely to be associated with the abnormal group. The correlations of median frequency in leads II and V1 are both negatively correlated with group membership. This result indicates that the majority of the P wave’s energy is contained at higher frequencies in the normal group than it is in the abnormal group. The subset of characteristics is chosen from the rankings of the 12 characteristics in Table 13. The 6 highest ranked characteristics are arranged into the feature matrix used in the linear discrimination:

$$
\begin{pmatrix}
  f_{1\text{median}}^V & f_{1\text{median}}^I & QV_{1\text{median}}^I & E_{1\text{max}}^I & E_{1\text{max}}^V & E_{1\text{total}}^I \\
  f_{2\text{median}}^V & f_{2\text{median}}^I & QV_{2\text{median}}^I & E_{2\text{max}}^I & E_{2\text{max}}^V & E_{2\text{total}}^I \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  f_{N\text{median}}^V & f_{N\text{median}}^I & QV_{N\text{median}}^I & E_{N\text{max}}^I & E_{N\text{max}}^V & E_{N\text{total}}^I
\end{pmatrix}
$$

(5.2-4)
Figure 65 Distance Scores with Standard Error for II-V1
Model Averaged over 100 Classifications
The output of the LDA is a distance score for each subject. Distance scores are derived for each subject by applying a set of linear coefficients to each median characteristic vector. The averaged distance over the 100 classification runs are plotted in Figure 65. The classification of subjects is based upon these distance scores. The abnormal cases are indicated by the black markers, the red markers indicate the normal cases, and the blue line indicates the discrimination threshold. The vertical blue lines on the abnormal cases indicate the standard error for each averaged distance score. The cumulative performance of the classifier is shown in Figure 66.

The rate of correct classification is shown against the number of trials. The red line indicates the rate of correctly classified normal subjects during the training phase. The black line indicates the rate of correctly classified abnormal subjects during the training phase. The dashed line indicates the rate of correctly classified abnormal subjects during the cross validation phase. As the number of trials increases, an average trend can be seen. This model performed well in the training phase. The linear discriminator derived for the model separated the training groups extremely well with a correct classification rate greater than 80%. The model also performed well on cross validation. These results translate into the sensitivities and specificity listed in Table 18 in the results section.
5.2.3 aVF-V2-V5 Model

This model was created using the leads of aVF, V2 and V5. The aim was to characterise the P wave in the 3 conduction planes of the 12 lead ECG. The added lead, and dimension, should provide a more comprehensive characterisation of the P wave conduction than the previous model. A total of 18 characteristics are generated in this model. The characteristic matrix of this model is shown in 5.4-5.

$$X^{aVF, V2, V5} = \begin{bmatrix}
V_1^{aVF} & V_1^{V2} & V_1^{V5} \\
V_2^{aVF} & V_2^{V2} & V_2^{V5} \\
\vdots & \vdots & \vdots \\
V_N^{aVF} & V_N^{V2} & V_N^{V5}
\end{bmatrix}$$  \hspace{1cm} (5.2-5)

As in the previous model, a subset of 6 characteristics is chosen from the original set of characteristics. The correlations between group membership and the characteristics from leads aVF, V2 and V5 are listed in order of significance in Table 14.

<table>
<thead>
<tr>
<th>CHARACTERISTIC</th>
<th>R</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>V5 f\text{median}**</td>
<td>-0.457</td>
<td>0.0015</td>
</tr>
<tr>
<td>V2 f\text{median}**</td>
<td>-0.375</td>
<td>0.0095</td>
</tr>
<tr>
<td>aVF f\text{median}*</td>
<td>-0.316</td>
<td>0.025</td>
</tr>
<tr>
<td>aVF E\text{max} *</td>
<td>0.283</td>
<td>0.0405</td>
</tr>
<tr>
<td>V2 E\text{max} *</td>
<td>0.275</td>
<td>0.045</td>
</tr>
<tr>
<td>aVF QV</td>
<td>0.257</td>
<td>0.057</td>
</tr>
<tr>
<td>aVF f\text{peak}</td>
<td>0.254</td>
<td>0.0595</td>
</tr>
<tr>
<td>V5 E\text{max}</td>
<td>0.234</td>
<td>0.0755</td>
</tr>
<tr>
<td>V5 E\text{total}</td>
<td>-0.233</td>
<td>0.077</td>
</tr>
<tr>
<td>aVF IQR</td>
<td>0.195</td>
<td>0.1165</td>
</tr>
<tr>
<td>aVF E\text{total}</td>
<td>-0.128</td>
<td>0.2185</td>
</tr>
<tr>
<td>V5 IQR</td>
<td>-0.092</td>
<td>0.288</td>
</tr>
<tr>
<td>V2 f\text{peak}</td>
<td>0.082</td>
<td>0.311</td>
</tr>
<tr>
<td>V2 IQR</td>
<td>-0.079</td>
<td>0.316</td>
</tr>
<tr>
<td>V5 QV</td>
<td>-0.079</td>
<td>0.317</td>
</tr>
<tr>
<td>V2 E\text{total}</td>
<td>-0.068</td>
<td>0.3405</td>
</tr>
<tr>
<td>V2 QV</td>
<td>-0.015</td>
<td>0.4635</td>
</tr>
<tr>
<td>V5 f\text{peak}</td>
<td>-0.001</td>
<td>0.4975</td>
</tr>
</tbody>
</table>

The classification vector contains the first 6 characteristics listed in Table 14. The median frequency characteristic of each lead used produced a significant correlation. The
correlation value for each of the median frequency characteristics is negative. This relationship suggests that the abnormal cases have greater concentration of signal energy at lower frequencies than the normal group. A similar result was found in the correlation analysis of the previous model. These are arranged into the feature matrix in 5.4-6.

\[
\begin{pmatrix}
  f_{V5}^{N \text{median}} & f_{V2}^{N \text{median}} & f_{aVF}^{N \text{median}} & E_{max}^{aVF} & E_{max}^{V2} & E_{total}^{V5} \\
  f_{V5}^{1 \text{median}} & f_{V2}^{1 \text{median}} & f_{aVF}^{1 \text{median}} & E_{max}^{aVF} & E_{max}^{V2} & E_{total}^{V5} \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  f_{V5}^{N \text{median}} & f_{V2}^{N \text{median}} & f_{aVF}^{N \text{median}} & E_{max}^{aVF} & E_{max}^{V2} & E_{total}^{V5}
\end{pmatrix}
\]

(5.2-6)

The plot in Figure 67 shows the distance scores of each subject, averaged over the 100 classification runs. The vertical lines on the abnormal cases indicate the range of the standard error around the average distance score. The performance of this model is similar to that of the previous model. There is excellent separation of the training sets; >80%. The cross validation performance is lower but still reasonable; >70%. The exact sensitivity and specificity values are listed in Table 18 in the results section.
Figure 67 Distance Scores with Standard Error aVF-V2-V5 Model Averaged over 100 Classifications

Figure 68 Average Classification Performance over 100 runs
5.2.4 XYZ Model

The XYZ leads were derived from the 12 lead ECG leads using the inverse Dower transform [39]. This transform aims to create a simulated set of VCG leads. The transform effectively reduces the number of leads needed to represent the information contained in the 12 standard ECG leads. The transform is applied to 8 leads, the 6 unipolar precordial leads and 2 of the limb leads, leads I and II. These are the 8 leads recorded by most commercial ECG interfaces, the remaining limb leads are generated by the Einthoven identity. Like the aVF-V2-V5 model, the XYZ model will characterise the P wave in 3 dimensions. The characteristic matrix is then:

\[
X^{X,Y,Z} = \begin{bmatrix}
V_1^X & V_1^Y & V_1^Z \\
v_2^X & v_2^Y & v_2^Z \\
\vdots & \vdots & \vdots \\
V_N^X & V_N^Y & V_N^Z
\end{bmatrix}
\]

(5.2-7)

The correlations between each characteristic and group membership were calculated and are listed in Table 15

<table>
<thead>
<tr>
<th>CHARACTERISTIC</th>
<th>R</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y QQV *</td>
<td>0.339</td>
<td>0.018</td>
</tr>
<tr>
<td>Y IQR *</td>
<td>0.298</td>
<td>0.033</td>
</tr>
<tr>
<td>Y fmedian *</td>
<td>0.284</td>
<td>0.020</td>
</tr>
<tr>
<td>Y fpeak</td>
<td>0.155</td>
<td>0.173</td>
</tr>
<tr>
<td>X QQV</td>
<td>0.136</td>
<td>0.205</td>
</tr>
<tr>
<td>Z QQV</td>
<td>0.124</td>
<td>0.227</td>
</tr>
<tr>
<td>X IQR</td>
<td>0.115</td>
<td>0.244</td>
</tr>
<tr>
<td>Y E_{max}</td>
<td>-0.097</td>
<td>0.279</td>
</tr>
<tr>
<td>X fmedian</td>
<td>0.095</td>
<td>0.282</td>
</tr>
<tr>
<td>Z fpeak</td>
<td>0.089</td>
<td>0.296</td>
</tr>
<tr>
<td>Z IQR</td>
<td>0.086</td>
<td>0.301</td>
</tr>
<tr>
<td>X fpeak</td>
<td>-0.080</td>
<td>0.315</td>
</tr>
<tr>
<td>Z fmedian</td>
<td>0.071</td>
<td>0.335</td>
</tr>
<tr>
<td>X E_{max}</td>
<td>-0.052</td>
<td>0.376</td>
</tr>
<tr>
<td>Z E_{max}</td>
<td>0.039</td>
<td>0.407</td>
</tr>
<tr>
<td>Z E_{total}</td>
<td>-0.015</td>
<td>0.463</td>
</tr>
<tr>
<td>X E_{total}</td>
<td>-0.004</td>
<td>0.491</td>
</tr>
<tr>
<td>Y E_{total}</td>
<td>-0.001</td>
<td>0.499</td>
</tr>
</tbody>
</table>

The correlations of the XYZ characteristics do not precisely correspond with those of the relative characteristics from the standard leads listed in Table 14. Most notably, the
correlation directions for the median frequency characteristics of each lead are opposite to those of the previous models. That is, a positive correlation exists for the median frequency characteristics of the XYZ leads and group membership. However, only the median frequency in the Y lead shows a significant, yet moderate correlation. The characteristics derived from the Y direction lead dominated this model. The other characteristics which produce significant correlations are the related measures of quartile variation and inter-quartile range. The feature matrix for this model is:

\[
\begin{bmatrix}
QV_1^Y & IQR_1^Y & f_{1median}^Y & E_{1max}^Y & QV_1^X & QV_1^Z \\
QV_2^Y & IQR_2^Y & f_{2median}^Y & E_{2max}^Y & QV_2^X & QV_2^Z \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
QV_N^Y & IQR_N^Y & f_{Nmedian}^Y & E_{Nmax}^Y & QV_N^X & QV_N^Z 
\end{bmatrix}
\]  

(5.2-8)

Figure 69 shows the averaged distance scores with standard errors for the abnormal distance scores. The standard error of distance scores in Figure 69 is significantly larger for most subjects than the errors produced by the previous models. The cumulative classification performance for this model is shown in Figure 70. The cross validation performance was dramatically poorer than the previous two models in terms of correct classification and lower than anticipated. Initial separation of the training set was mixed. The classification of abnormal subjects in the training set was quite moderate, between 60 and 70%. Though classification of normal subjects was still high, >90%. The cross validation performance was much poorer, >50%. One possible explanation of this result is that too much variation is introduced into each variable over the sample size by using the 8 independent leads of the ECG in an orthogonal combination. It may be the case that a much larger sample size is required to observe the advantages of an XYZ lead wavelet classifying model. The models presented so far have employed characteristics from the chosen leads on the basis that the characterisation of the P wave in these leads would implicitly represent an underlying difference between the two groups. The relationships between the individual characteristics and the groups were not considered in these models. The previous models employed the characteristics in a linear combination which did not consider intrinsic relationships between the characteristics. Such relationships can be postulated by the use of factor analysis.
Figure 69 Average Distance Scores with Standard Error for XYZ Model

Figure 70 Average Classification Performance vs Number of runs
5.2.5 Factor Analysis Model

By accounting for the inter-correlations between the original characteristics, a significant amount of their variance can be explained by a smaller set of factors. Additionally, the factors attained may be indicative of underlying behaviour in the original data. A factor analysis of the abnormal group was performed using the correlation matrix method. All cases from the abnormal group were analysed using the principal component method to derive factor loadings. Factor scores were then calculated for all the cases in the data set using these factor loadings. Accordingly, the factor scores generated for the cases in the training set were used in the derivation of the linear discriminant model. This method yielded 9 factors from the set of standard lead characteristics. The amount of the total variance described by the correlation method (87.47%) and each factor’s contribution to the total variance is detailed in Table 16. The eigenvalues for the initial factors and the factors attained after rotation are also listed. The respective eigenvalues demonstrates how the percentage of total variance is more evenly distributed by the rotated solution. The significance values, \( p \), of each factor’s bi-variate group membership correlation, \( R \), are listed in Table 16. Two of the 9 derived factors were found to be significant at a level of 0.05. These were factor 2 and factor 6. The factor loadings of each of the original variables are listed in Table 17. An interpretation of the factors can be formulated from their composition. The shading in each factor column of Table 17 indicates the groupings of variables which have large loadings on each factor. The three most significantly correlated factors in order of significance are factors 2, 6 and 3.

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>INITIAL EIGENVALUES</th>
<th>ROTATED VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total % of Variance</td>
<td>Cumulative %</td>
</tr>
<tr>
<td>1</td>
<td>6.38 21.26 21.26</td>
<td>4.41 14.70</td>
</tr>
<tr>
<td>2</td>
<td>5.69 18.96 40.22</td>
<td>4.19 13.98</td>
</tr>
<tr>
<td>3</td>
<td>3.63 12.11 52.33</td>
<td>3.73 12.42</td>
</tr>
<tr>
<td>4</td>
<td>3.03 10.09 62.42</td>
<td>2.98 9.93</td>
</tr>
<tr>
<td>5</td>
<td>2.29 7.65 70.07</td>
<td>2.63 8.76</td>
</tr>
<tr>
<td>6</td>
<td>1.63 5.43 75.50</td>
<td>2.51 8.38</td>
</tr>
<tr>
<td>7</td>
<td>1.38 4.60 80.10</td>
<td>2.50 8.34</td>
</tr>
<tr>
<td>8</td>
<td>1.16 3.86 83.96</td>
<td>1.69 5.64</td>
</tr>
<tr>
<td>9</td>
<td>1.05 3.51 87.47</td>
<td>1.60 5.33</td>
</tr>
</tbody>
</table>
Table 17 Rotated Component Matrix

<table>
<thead>
<tr>
<th></th>
<th>COMPONENT</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<tbody>
<tr>
<td>V1 f_{peak}</td>
<td>.924</td>
<td>.115</td>
<td>.037</td>
<td>-.006</td>
<td>.234</td>
<td>.048</td>
<td>-.058</td>
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<td>V5 f_{peak}</td>
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<td>-.112</td>
<td>-.045</td>
<td>.215</td>
<td>-.071</td>
<td>-.127</td>
<td>.125</td>
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<tr>
<td>V5 IQR</td>
<td>.876</td>
<td>-.134</td>
<td>-.024</td>
<td>.224</td>
<td>-.036</td>
<td>.140</td>
<td>.093</td>
<td>.215</td>
<td>.161</td>
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<tr>
<td>V5 QV</td>
<td>.779</td>
<td>.054</td>
<td>-.053</td>
<td>.292</td>
<td>.070</td>
<td>-.001</td>
<td>.203</td>
<td>.286</td>
<td>.079</td>
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</tr>
<tr>
<td>II f_{median}</td>
<td>-.054</td>
<td>.903</td>
<td>.049</td>
<td>.064</td>
<td>-.012</td>
<td>-.206</td>
<td>-.083</td>
<td>.010</td>
<td>.038</td>
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<tr>
<td>aVF f_{median}</td>
<td>.185</td>
<td>.878</td>
<td>.034</td>
<td>-.102</td>
<td>.090</td>
<td>-.177</td>
<td>-.048</td>
<td>-.279</td>
<td>-.052</td>
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</tr>
<tr>
<td>V1 IQR</td>
<td>-.225</td>
<td>.790</td>
<td>.227</td>
<td>.116</td>
<td>.107</td>
<td>-.194</td>
<td>-.108</td>
<td>.342</td>
<td>.079</td>
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<tr>
<td>V2 f_{median}</td>
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<td>.011</td>
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<td>.534</td>
<td>-.278</td>
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<td>.043</td>
<td>-.206</td>
<td>.089</td>
<td>.378</td>
<td>.118</td>
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<tr>
<td>V1 E_{total}</td>
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<td>.456</td>
<td>-.153</td>
<td>-.453</td>
<td>-.208</td>
<td>.151</td>
<td>.405</td>
<td>-.038</td>
<td>.273</td>
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<tr>
<td>aVF QV</td>
<td>-.256</td>
<td>.015</td>
<td>.913</td>
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<td>.085</td>
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</tr>
<tr>
<td>aVF IQR</td>
<td>-.066</td>
<td>.043</td>
<td>.909</td>
<td>.126</td>
<td>-.004</td>
<td>-.032</td>
<td>-.002</td>
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<td>.075</td>
<td>.267</td>
<td>.065</td>
<td>.123</td>
<td>-.035</td>
<td>-.096</td>
<td></td>
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<tr>
<td>II QV</td>
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<td>.699</td>
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<td>-.102</td>
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<td>.253</td>
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<td>V2 QV</td>
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<td>.011</td>
<td>.111</td>
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<td>V2 IQR</td>
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<td>.021</td>
<td>-.887</td>
<td>.292</td>
<td>-.030</td>
<td>.017</td>
<td>.051</td>
<td>.014</td>
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<tr>
<td>V1 IQR</td>
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<td>.016</td>
<td>.161</td>
<td>.433</td>
<td>.802</td>
<td>-.153</td>
<td>-.092</td>
<td>.193</td>
<td>-.135</td>
<td></td>
</tr>
<tr>
<td>V1 QV</td>
<td>.078</td>
<td>.202</td>
<td>.131</td>
<td>.310</td>
<td>.790</td>
<td>-.226</td>
<td>-.140</td>
<td>.098</td>
<td>-.243</td>
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<tr>
<td>V2 f_{peak}</td>
<td>-.220</td>
<td>-.527</td>
<td>-.184</td>
<td>.334</td>
<td>.611</td>
<td>-.073</td>
<td>.017</td>
<td>-.072</td>
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</tr>
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<td>II f_{peak}</td>
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<td>-.024</td>
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<td>V1 E_{peak}</td>
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<td>.826</td>
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<td>.155</td>
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<td>.225</td>
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<td>.636</td>
<td>-.159</td>
<td>-.036</td>
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<td>aVF E_{peak}</td>
<td>.438</td>
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<td>-.392</td>
<td>.363</td>
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<td>.543</td>
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<td>-.051</td>
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<tr>
<td>V5 E_{total}</td>
<td>.025</td>
<td>-.017</td>
<td>.116</td>
<td>.069</td>
<td>.003</td>
<td>-.029</td>
<td>.906</td>
<td>-.098</td>
<td>.247</td>
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<tr>
<td>V5 E_{peak}</td>
<td>.115</td>
<td>-.321</td>
<td>.154</td>
<td>.053</td>
<td>-.034</td>
<td>.162</td>
<td>.801</td>
<td>-.032</td>
<td>.012</td>
<td></td>
</tr>
<tr>
<td>aVF E_{total}</td>
<td>-.055</td>
<td>.143</td>
<td>-.288</td>
<td>-.224</td>
<td>-.194</td>
<td>-.027</td>
<td>.681</td>
<td>.036</td>
<td>-.413</td>
<td></td>
</tr>
<tr>
<td>aVF f_{peak}</td>
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<td>-.021</td>
<td>.140</td>
<td>.143</td>
<td>-.106</td>
<td>-.117</td>
<td>.872</td>
<td>.117</td>
<td>-.117</td>
<td></td>
</tr>
<tr>
<td>V2 E_{total}</td>
<td>.226</td>
<td>.291</td>
<td>-.120</td>
<td>-.156</td>
<td>-.143</td>
<td>.098</td>
<td>.202</td>
<td>-.051</td>
<td>.772</td>
<td></td>
</tr>
</tbody>
</table>

The factor with highest group membership correlation is Factor 2. The loadings in Table 17 show that this factor is dominated by contributions from the median frequencies of all leads. This indicates that there is a moderate but significant trend in the energy distributions of the normal and abnormal groups as measured by the wavelet analysis procedure. The negative correlation value for Factor 2 and the positive loadings of the median characteristics within the factor, indicate that the relationship between median frequency and group membership is negative. This result is supports the results from the II-V1 and aVF-V2-V5 correlation analysis. It indicates that the majority of the P wave’s energy is contained at lower frequencies in the abnormal cases than the normal cases.
Figure 71 Distances Scores and Standard Errors for Factor Analysis Model Averaged over 100 runs

Figure 72 Average Classification Performance vs Number of runs
The averaged distance scores in Figure 71 produced by the factor analysis model have very small standard errors around the mean. The average performance of this model is also quite good, comparable to that of the II-V1 model. The cross validation performance of the model is close to 80%.

### 5.2.6 Summary of Results for Experiment 1

This experiment has shown that accurate detection of abnormal atrial electrical activity is possible using wavelet analysis. The summary of classification results in Table 18 shows that this method compares favourably to traditional scalar ECG measures used to assess the P wave.

<table>
<thead>
<tr>
<th>Classifier</th>
<th>Training Set</th>
<th>Cross Validation Set</th>
<th>Training</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal N=7</td>
<td>Abnormal N=16</td>
<td>Abnormal N=16</td>
<td></td>
</tr>
<tr>
<td>II V1</td>
<td>99.57</td>
<td>86.63</td>
<td>81.94</td>
<td>90.57</td>
</tr>
<tr>
<td>Factor Analysis</td>
<td>99.43</td>
<td>82.13</td>
<td>79.75</td>
<td>87.39</td>
</tr>
<tr>
<td>aVF V2 V5</td>
<td>96.43</td>
<td>85.56</td>
<td>72.38</td>
<td>89.57</td>
</tr>
<tr>
<td>XYZ</td>
<td>92.86</td>
<td>68.63</td>
<td>54.81</td>
<td>76.00</td>
</tr>
<tr>
<td>Duration</td>
<td>71.71</td>
<td>73.06</td>
<td>74.63</td>
<td>72.65</td>
</tr>
<tr>
<td>All Measures</td>
<td>74.00</td>
<td>76.44</td>
<td>74.00</td>
<td>75.70</td>
</tr>
<tr>
<td>PV1</td>
<td>14.29</td>
<td>61.25</td>
<td>59.00</td>
<td>46.96</td>
</tr>
<tr>
<td>P/PR</td>
<td>69.43</td>
<td>55.63</td>
<td>55.19</td>
<td>59.83</td>
</tr>
</tbody>
</table>

The best performance was achieved using the two lead model which utilised characteristics from leads II and V1. However, this classification model only made use of time-frequency information from lead V1. The model which utilised the factor analysis approach produced similar results for initial group separation and cross validation. This factor analysis model also performed better than the scalar ECG measures in both training and cross-validation classification. The correlation analysis used by the lead models highlighted the median frequency characteristic of the P wave to have some singular significance in terms of classifying potential. An extension of the trial would determine if the novel analysis method developed in this thesis could be used to determine the general state of P wave morphology, normal or abnormal, in a subject.
5.3 Experiment 2 – Classification of Signal Averaged P wave

The previous experiments demonstrated that the variance between successive P waves of each individual was too great for the sample size to produce a satisfactory result in the factor analysis procedure. High order classification models were required to produce good classification results. This approach was used in [33] to demonstrate the effectiveness of system modelling methods of P wave morphological classification.

5.3.1 Modifications to Experiment

The derivation of the averaged P wave was performed by first identifying the location of the P waves using the same method as the previous experiments. The QRS complexes were initially identified using a wavelet event detection algorithm. A time window preceding each QRS was analysed using a modified version of the wavelet event detector to determine the location of each P wave’s maxima. The P wave maxima provide relative alignment points for the averaging of the P waves. Predominantly lead II was used for the detection of the P waves, as the QRS complexes are distinguished and the P wave morphology is usually monophasic. Averaged P waves were derived for the XYZ, II, aVF, V1, V2 and V5 leads and their onset and offset were marked manually according to the longest duration in all leads. QRS complexes were also suppressed to reduce their effect on low frequency scales of the P wave’s coefficients. Additionally, the wavelet transform, and subsequent energy spectrum, were calculated for a fewer number of scales. A set of scales \{4, 8, 12, 20, 32, 48\} with approximate central frequencies of \{37.5, 18.75, 12.5, 7.5, 4.6875, 3.125\} Hz were chosen. This range of scales covered the frequency range of the P wave. The range of scales was reduced so frequencies below 3.125Hz were excluded from the energy spectrum to reduce the effects of depressed PR segments in the wavelet transform. The depressed PR segments introduce a DC component akin to the broad spectrum noise of an abrupt discontinuity in a time varying signal.

The models were constructed in the same manner as experiment 1. Characteristic vectors were derived from each signal averaged P wave lead. The characteristic vectors were arranged into the lead sets used in Experiment 1 to create the classifier models. Each classifier was run 100 times using random samplings of 14 abnormal cases in the training set. The remaining 14 abnormal cases were used in the test set. The performance of each classifier was averaged over the 100 classification runs. The selection of characteristics in each model in this experiment was performed in using the same criteria as the previous
experiment. The 6 characteristics with the highest group membership correlation were retained from the original set.

### 5.3.2 II-V1 model

The correlations in Table 19 indicate that the characteristics from the averaged P wave in lead V1 were most indicative of group membership. The correlation values for all the V1 characteristics are moderate and significant with $p < 0.05$. The only V1 characteristic which does not show a significant correlation is the QV measure.

<table>
<thead>
<tr>
<th>CHARACTERISTIC</th>
<th>R</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1 $E_{max}$</td>
<td>0.340</td>
<td>0.017</td>
</tr>
<tr>
<td>V1 $E_{total}$</td>
<td>0.330</td>
<td>0.020</td>
</tr>
<tr>
<td>V1 $f_{peak}$</td>
<td>-0.304</td>
<td>0.030</td>
</tr>
<tr>
<td>V1 $f_{median}$</td>
<td>-0.275</td>
<td>0.045</td>
</tr>
<tr>
<td>V1 IQR</td>
<td>-0.275</td>
<td>0.045</td>
</tr>
<tr>
<td>V1 QV</td>
<td>-0.245</td>
<td>0.067</td>
</tr>
<tr>
<td>II $E_{max}$</td>
<td>0.224</td>
<td>0.085</td>
</tr>
<tr>
<td>II $E_{total}$</td>
<td>0.179</td>
<td>0.137</td>
</tr>
<tr>
<td>II $f_{peak}$</td>
<td>0.088</td>
<td>0.297</td>
</tr>
<tr>
<td>II $f_{median}$</td>
<td>0.056</td>
<td>0.367</td>
</tr>
<tr>
<td>II IQR</td>
<td>0.028</td>
<td>0.433</td>
</tr>
<tr>
<td>II QV</td>
<td>-0.006</td>
<td>0.485</td>
</tr>
</tbody>
</table>

Figure 73 shows the average distance score for each subject with the standard error around the averaged indicated by the blue line. Figure 74 shows the cumulative correct classification of the factor analysis model over the 100 classification runs. The classification performance results for this model, and the following, are listed in Table 24.
Figure 73: Classification using Signal Averaged II-V1 model.

Figure 74: Average Classification Performance for II-V1 model.
5.3.3 aVF-V2-V5 model

The correlations in Table 20 indicate that the characteristics from the averaged P wave in lead V2 were most indicative of group membership. This is similar to the result for the previous model where the characteristics from lead V1 showed high group membership correlations.

<table>
<thead>
<tr>
<th>CHARACTERISTIC</th>
<th>R</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.328</td>
<td>0.021</td>
</tr>
<tr>
<td>V2 E\text{total} *</td>
<td>0.324</td>
<td>0.022</td>
</tr>
<tr>
<td>V2 IQR</td>
<td>-0.262</td>
<td>0.054</td>
</tr>
<tr>
<td>V2 f\text{median}</td>
<td>-0.256</td>
<td>0.058</td>
</tr>
<tr>
<td>V2 QV</td>
<td>-0.245</td>
<td>0.067</td>
</tr>
<tr>
<td>aVF E\text{max}</td>
<td>0.229</td>
<td>0.080</td>
</tr>
<tr>
<td>V5 QV</td>
<td>-0.192</td>
<td>0.120</td>
</tr>
<tr>
<td>aVF E\text{total}</td>
<td>0.185</td>
<td>0.130</td>
</tr>
<tr>
<td>V2 f\text{peak}</td>
<td>-0.178</td>
<td>0.139</td>
</tr>
<tr>
<td>V5 IQR</td>
<td>-0.162</td>
<td>0.162</td>
</tr>
<tr>
<td>V5 f\text{median}</td>
<td>-0.090</td>
<td>0.292</td>
</tr>
<tr>
<td>V5 E\text{max}</td>
<td>0.075</td>
<td>0.326</td>
</tr>
<tr>
<td>aVF f\text{peak}</td>
<td>0.072</td>
<td>0.331</td>
</tr>
<tr>
<td>aVF IQR</td>
<td>0.050</td>
<td>0.381</td>
</tr>
<tr>
<td>aVF f\text{median}</td>
<td>0.048</td>
<td>0.385</td>
</tr>
<tr>
<td>V5 f\text{peak}</td>
<td>0.031</td>
<td>0.425</td>
</tr>
<tr>
<td>aVF QV</td>
<td>0.017</td>
<td>0.460</td>
</tr>
<tr>
<td>V5 E\text{total}</td>
<td>0.010</td>
<td>0.475</td>
</tr>
</tbody>
</table>

Figure 75 shows the average distance score for each subject with the standard error around the averaged indicated by the blue line. Figure 76 shows the cumulative correct classification of the factor analysis model over the 100 classification runs.
Figure 75 Classification using Signal Averaged aVF-V2-V5 model

Figure 76 Average Classification Performance for aVF-V2-V5 model
5.3.4 XYZ Model

The group membership correlations for each characteristic are listed in Table 21. The highest correlation values are associated with the characteristics in the Z lead. This agrees with the results from the previous experiment as leads V2 and Z are highly correlated with each other.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>R</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z IQR *</td>
<td>-0.350</td>
<td>0.015</td>
</tr>
<tr>
<td>Z E total *</td>
<td>0.306</td>
<td>0.029</td>
</tr>
<tr>
<td>Z QV *</td>
<td>-0.303</td>
<td>0.030</td>
</tr>
<tr>
<td>Z E max *</td>
<td>0.295</td>
<td>0.034</td>
</tr>
<tr>
<td>Z f peak *</td>
<td>-0.277</td>
<td>0.044</td>
</tr>
<tr>
<td>Z f median</td>
<td>-0.227</td>
<td>0.082</td>
</tr>
<tr>
<td>Y E max</td>
<td>0.223</td>
<td>0.087</td>
</tr>
<tr>
<td>Y E total</td>
<td>0.177</td>
<td>0.141</td>
</tr>
<tr>
<td>X E total</td>
<td>-0.098</td>
<td>0.275</td>
</tr>
<tr>
<td>Y f median</td>
<td>0.079</td>
<td>0.315</td>
</tr>
<tr>
<td>Y f peak</td>
<td>0.066</td>
<td>0.346</td>
</tr>
<tr>
<td>X QV</td>
<td>-0.056</td>
<td>0.368</td>
</tr>
<tr>
<td>Y IQR</td>
<td>0.048</td>
<td>0.385</td>
</tr>
<tr>
<td>X IQR</td>
<td>-0.034</td>
<td>0.418</td>
</tr>
<tr>
<td>X f peak</td>
<td>-0.034</td>
<td>0.418</td>
</tr>
<tr>
<td>Y QV</td>
<td>0.015</td>
<td>0.463</td>
</tr>
<tr>
<td>X f median</td>
<td>0.015</td>
<td>0.463</td>
</tr>
<tr>
<td>X E max</td>
<td>-0.011</td>
<td>0.474</td>
</tr>
</tbody>
</table>

Figure 77 shows the average distance score for each subject with the standard error around the averaged indicated by the blue line. Figure 78 shows the cumulative correct classification of the factor analysis model over the 100 classification runs.
Figure 77 Classification using Signal Averaged XYZ model

Figure 78 Average Classification Performance for XYZ Model
5.3.5 Factor Analysis

The factor analysis model as constructed using the same process as the respective model in experiment 1. The factor analysis summary is shown in Table 22. The three most highly correlated factors were used in this classification model.

**Table 22 Factor Analysis Summary**

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>INITIAL EIGENVALUES</th>
<th>ROTATED VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total % of Variance</td>
<td>Cumulative %</td>
</tr>
<tr>
<td>1</td>
<td>10.145</td>
<td>33.815</td>
</tr>
<tr>
<td>2</td>
<td>9.599</td>
<td>31.997</td>
</tr>
<tr>
<td>3</td>
<td>3.884</td>
<td>12.946</td>
</tr>
<tr>
<td>4</td>
<td>2.218</td>
<td>7.395</td>
</tr>
<tr>
<td>5</td>
<td>1.393</td>
<td>4.644</td>
</tr>
<tr>
<td>6</td>
<td>1.057</td>
<td>3.525</td>
</tr>
</tbody>
</table>

The composition of the top 3 factors is highlighted in Table 23. The two significant factors are comprised entirely of characteristics from leads V1 and V2, which are, spatially, closely related. The characteristics describing the amplitude of the P wave, \( E_{\text{total}} \) and \( E_{\text{max}} \), are contained in factor 4. The characteristics describing the frequency spectrum of the P wave, \( f_{\text{peak}} \), \( f_{\text{median}} \), QV and IQR, are contained in factor 2.

**Table 23 Composition of Significant Factors**

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>V2 ( f_{\text{peak}} )</td>
<td>.952</td>
<td>.034</td>
<td>-.147</td>
</tr>
<tr>
<td>V2 IQR</td>
<td>.921</td>
<td>.060</td>
<td>-.112</td>
</tr>
<tr>
<td>V2 ( f_{\text{median}} )</td>
<td>.882</td>
<td>-.148</td>
<td>-.196</td>
</tr>
<tr>
<td>V1 QV</td>
<td>.859</td>
<td>.187</td>
<td>-.045</td>
</tr>
<tr>
<td>V2 QV</td>
<td>.859</td>
<td>.187</td>
<td>-.045</td>
</tr>
<tr>
<td>V1 IQR</td>
<td>.650</td>
<td>.387</td>
<td>-.101</td>
</tr>
<tr>
<td>V1 ( f_{\text{median}} )</td>
<td>.644</td>
<td>.453</td>
<td>-.143</td>
</tr>
<tr>
<td>V1 ( f_{\text{peak}} )</td>
<td>.639</td>
<td>.368</td>
<td>-.102</td>
</tr>
<tr>
<td>II ( E_{\text{total}} )</td>
<td>.138</td>
<td>.940</td>
<td>.094</td>
</tr>
<tr>
<td>aVF ( E_{\text{total}} )</td>
<td>.151</td>
<td>.934</td>
<td>.071</td>
</tr>
<tr>
<td>aVF ( E_{\text{max}} )</td>
<td>.104</td>
<td>.932</td>
<td>.128</td>
</tr>
<tr>
<td>II ( E_{\text{max}} )</td>
<td>.089</td>
<td>.932</td>
<td>.156</td>
</tr>
<tr>
<td>V5 ( E_{\text{total}} )</td>
<td>-.028</td>
<td>.578</td>
<td>.475</td>
</tr>
<tr>
<td>V5 ( E_{\text{max}} )</td>
<td>-.055</td>
<td>.541</td>
<td>.508</td>
</tr>
<tr>
<td>V1 ( E_{\text{total}} )</td>
<td>-.092</td>
<td>.018</td>
<td>.959</td>
</tr>
<tr>
<td>V1 ( E_{\text{max}} )</td>
<td>-.120</td>
<td>.033</td>
<td>.951</td>
</tr>
<tr>
<td>V2 ( E_{\text{total}} )</td>
<td>-.171</td>
<td>.190</td>
<td>.912</td>
</tr>
<tr>
<td>V2 ( E_{\text{max}} )</td>
<td>-.207</td>
<td>.151</td>
<td>.904</td>
</tr>
</tbody>
</table>
Figure 79 Standard Errors of Factor Analysis Classification

Figure 80 Average Classification Performance for Factor Analysis Model

Figure 79 shows the average distance score for each subject with the standard error around the averaged indicated by the blue line. Figure 80 shows the cumulative correct classification of the factor analysis model over the 100 classification runs.

5.3.6 Summary of Results for Experiment 2

The results listed in Table 24 are the average performance for each classifier over the 20 classification runs. The classifiers derived from the average waveform did not improve upon the cross validation classification provided by the ECG measures, specifically the P wave duration. The classification model derived from the XYZ leads performed best out of the wavelet based classifiers, producing the best separation of the training sets and cross validation results. Overall, the performance of the wavelet based classification models in this experiment was worse than the respective models of the previous experiment.

Table 24 Average Classification Performance

<table>
<thead>
<tr>
<th>Classifier</th>
<th>Training Normal N=7</th>
<th>Abnormal N=16</th>
<th>Cross Validation Normal N=16</th>
<th>Abnormal N=16</th>
<th>Training N=23</th>
<th>Total N=39</th>
</tr>
</thead>
<tbody>
<tr>
<td>XYZ</td>
<td>84.86</td>
<td>80.06</td>
<td>68.56</td>
<td>81.52</td>
<td>76.21</td>
<td></td>
</tr>
<tr>
<td>FA</td>
<td>74.57</td>
<td>74.13</td>
<td>64.38</td>
<td>74.26</td>
<td>70.21</td>
<td></td>
</tr>
<tr>
<td>aVF V2 V5</td>
<td>83.43</td>
<td>76.31</td>
<td>64.06</td>
<td>78.48</td>
<td>72.56</td>
<td></td>
</tr>
<tr>
<td>II V1</td>
<td>77.29</td>
<td>71.69</td>
<td>56.81</td>
<td>73.39</td>
<td>66.59</td>
<td></td>
</tr>
<tr>
<td>Duration</td>
<td>71.71</td>
<td>73.06</td>
<td>74.63</td>
<td>72.65</td>
<td>73.46</td>
<td></td>
</tr>
<tr>
<td>ECG Measures</td>
<td>74.00</td>
<td>76.44</td>
<td>74.00</td>
<td>75.70</td>
<td>75.00</td>
<td></td>
</tr>
<tr>
<td>PV1</td>
<td>14.29</td>
<td>61.25</td>
<td>59.00</td>
<td>46.96</td>
<td>51.90</td>
<td></td>
</tr>
<tr>
<td>P/PR</td>
<td>69.43</td>
<td>55.63</td>
<td>55.19</td>
<td>59.83</td>
<td>57.92</td>
<td></td>
</tr>
</tbody>
</table>
5.4 Summary & Discussion

In this chapter, several methods of deriving classification models from wavelet coefficients of the ECG P wave have been presented. Essentially, classification relies upon the comparison of distinguishing features taken from two or more classes of a particular signal. In this thesis two classes of “normal” and “abnormal” P wave were considered. The comparable features of the P waves were generated from energy distributions computed using the continuous wavelet transform. These features were shown to represent changes in the P wave morphology in the chapter 4. This chapter demonstrates that wavelet analysis classification of atrial activity is possible using features of the P wave wavelet energy spectrum. This classification method compared favourably with the traditional ECG markers used to indicate atrial abnormality.

The recorded P wave can exhibit significant variation while a subject is being recorded. This variation creates an obstacle for morphological classification in clinical ECG recordings. In this chapter two approaches devised to overcome this problem were presented. The first created a representative set of characteristics for a subject’s P wave from the median points of each individual P wave. The second method used signal averaging to produce a single P wave. These methods are not commutative and the wavelet classifiers had noticeably lower performance classifying the signal averaged P waves than they did using the median vector approach.

This reduction in performance could be explained by two factors. Firstly, the application of signal averaging to a relatively small number of events may not generate an accurate average waveform. Typically ECG signal averaging is done over a period of several minutes to provide a statistically robust signal average by rejecting individual events which fall outside an arbitrary tolerance threshold. Clinical ECG recordings do not provide a large enough number of ECG cycles to allow for such an attrition rate. Therefore, including all recorded P waves into the signal average may have been detrimental to the morphology characterisation procedure. Secondly, the wavelet transform of the averaged P wave was performed at fewer scales than in experiment 1. Using a coarser resolution of the frequency spectrum diminishes the accuracy of the frequency measures, peak frequency, median frequency, IQR and QV. The measures describing the spread of the energy distribution, IQR and QV, would be most significantly affected by the number of scales used.
Using an automated analysis and classification technique ensures objective analysis and consistency across subjects. This is particularly advantageous in the diagnosis of low amplitude and dispersed P waves, which are commonly observed in elderly participants [13], where diagnosis of the ECG by visual inspection can be limited. The wavelet analysis approach presented here can uncover and quantify significant time-frequency information in such lower amplitude signals to provide more diagnostically useful information.

The most significant limitation to this study was the small number of control subjects. This made comparison of the ECG indicators’ and the wavelet classifiers’ sensitivity impossible. The number of participants used in this trial was modest (n=39); and future work is indicated to further validate the approach outlined here. The accurate diagnosis of a participant’s general atrial conduction may also possible using these indicators, though a larger scale study is required for verification. It would be beneficial to further assess the reliability of the characteristics used in this work by calculating them from the derived VCG and comparing the two sets. Further development could also focus on reducing the order of the classifier. The classification models presented in this chapter were of a relatively high order. Reducing the order of the wavelet classifier would produce better classifying generalisation. A reduction in the order of the wavelet classifier may be achieved by combining several of the energy spectrum features which were highly correlated, or by the choice of alternative measures of the dispersion of the P wave energy and frequency content.
5.5 References:


22 R. Chirife, G.S. Feitosa, W.S. Frankl, “Electrocardiographic detection of left atrial enlargement: Correlation of P wave with left atrial dimension by electrocardiography”, British Heart Journal, Vol 37, 1975, pp. 1281-1285


53 SPSS® V 13.0, SPSS© Inc., Chicago, IL., Software Package

54 J. Hua, Z. Xiong, J. Lowey, E. Suh, E.R. Dougherty, ”Optimal number of features as a function of sample size for various classification rules”, Bioinformatics, Vol. 21, no. 8, 2005, pp 1509–1515

Chapter 6

CONCLUSIONS AND SUMMARY

The focus of this thesis was the analysis of P wave morphology in clinical ECG recordings using the wavelet transform. A novel analysis process which employs the continuous wavelet transform was developed. The analysis method exploits the wavelet transform’s time-frequency spectrum to derive measures which characterise the time series P wave.

This novel analysis method was applied to two problems involving the P wave in clinical electrocardiograms. These problems were the use of the ECG P wave to estimate the physical dimensions of the atria and the classification of the P wave morphology. The method was compared against traditional scalar ECG measures and found to provide equal or superior performance.

6.1 ECG Based Estimation of Atrial Dimensions

The estimation of the atria’s physical dimensions was investigated in chapter 4. Left atrial functioning is important to the overall state of the cardiac cycle and is indicative of several serious complications to heart functioning. Currently, ECG indicators of left atrial dimension are unreliable, providing either satisfactory sensitivity with poor specificity or vice versa. The most accurate means of assessing left atrial dimensions are obtained by echocardiogram or magnetic resonance imaging (MRI), both of which are significantly more expensive than the ECG and consequently not as widely accessible.

In section 4.3 of chapter 4 several measures derived from the wavelet transform of the P wave were observed to provide a good estimation of physical atrial dimensions. The characteristics were produced from the wavelet transform of signal averaged P waves in a derived orthogonal ECG lead set and the relative leads from the standard 12 Lead ECG. It was observed that the measures derived from the P waves in the Z lead orthogonal to the echocardiogram plane produced the highest correlation to left atrial area. The measures which characterised the dominant frequency and median point of the P wave
energy spectrum were observed to significantly correlate (p<0.01) to a linear relationship with left atrial diameter and area. The peak frequency showed high correlation with left atrial diameter ($R=-0.679$) and left atrial area ($R=-0.575$). The median frequency, which indicates the midpoint of the P waves energy-frequency distribution produced similar correlations with left atrial diameter ($R=-0.609$) and left atrial area ($R=-0.535$). These results were significantly better than any correlations produced by traditional ECG measures such as P wave duration and P terminal force. Only P terminal force (PV1) produced a significant correlation ($R=-0.489$) for any measure of left atrial size, in this case it was the scalar quantity of left atrial diameter.

The right atrial area produced high correlations with the total RMS ($R=0.593$) and peak values ($R=0.681$) of the P wave’s energy spectrum observed in lead Z. Once again, this outperformed all the traditional ECG measures. Only PV1 produced a correlation of significance ($R=-0.446$).

### 6.2 Classification of Abnormal P Wave Morphology

Several classification models were derived from the wavelet transform characteristics of P waves in various ECG leads. Changes in morphology and duration were quantified by a set of descriptive measures which describe the energy distributions of the P wave produced by the wavelet transform. These measures are related to traditional ECG characteristics such as duration, amplitude, terminal force and area of the P wave and were found to be sensitive to differences in P wave morphology resulting from abnormal conduction conditions.

The classifier was trained to distinguish between two classes of “normal” and “abnormal” P waves. Two approaches of dealing with intra-subject variability of the P wave were presented. The first created a representative set of characteristics for a subject’s P wave from the median points of each individual P wave. The second method used signal averaging to produce a single P wave. These methods are not commutative and the wavelet classifiers had noticeably lower performance classifying the signal averaged P waves than they did using the median vector approach.

The best performance was observed for the classification model employing the median vector characteristics from leads II and V1. This model produced excellent separation of the training set over 100 classification runs, correctly classifying 99.57% of normal and 86.63% of abnormal subjects. It also performed well on cross validation, correctly classifying 81.94% of abnormal subjects. This classification method compared
favourably with the traditional ECG markers used to indicate atrial abnormality. The highest overall classification performance using traditional ECG measures was observed for the case when P duration, PV1 and P/PR ratio were used in an LDA configuration. For the same data set and random selections this model separated the training set by correctly classifying 74% of normal and 76.44% of abnormal subjects. The cross validation was 74% of the remaining abnormal subjects.

6.3 Future Work

The most significant limitation to this study was the small number of control subjects. This made comparison of the ECG indicators’ and the wavelet classifiers’ sensitivity impossible. The number of participants used in this trial was modest (n=39); and future work is indicated to further validate the approach outlined here. The accurate diagnosis of a participant’s general atrial conduction may also be possible using these indicators, though a larger scale study is required for verification.

Further development could also focus on improving the performance and reducing the complexity of the classifier. Reducing the order of the wavelet classifier would produce better classifying generalisation for larger unseen populations. A reduction in the order of the wavelet classifier may be achieved by combining several of the energy spectrum features which were highly correlated, or by the choice of alternative measures of the dispersion of the P wave energy and frequency content.

A larger scale and more comprehensive study would be required to determine the accuracy wavelet energy distribution characteristics in indicating left atrial area and the necessity of the extra complexity of deriving the orthogonal lead set. An accurate method of determining the left atrial area from electrocardiograms would be of considerable benefit to the wider community. Such a cost effective tool would be directly beneficial to rural and remote healthcare in countries like Australia. There are several avenues for future work which could be pursued. The development of this novel method of P wave analysis would require larger scale studies to assess its reliability. Refinements to the method could be focused on the definition of a customised wavelet basis function for the initial derivation of the wavelet energy spectrum. Additionally, differing wavelet basis functions could be used to analyse the P waves in each lead utilised in the analysis as the P wave morphology varies considerably across the 12 leads. The novel P wave analysis method presented in this thesis has proven to be sensitive enough to autonomously identify changes to the morphology of the P wave. The method could therefore be
capable of identifying underlying properties of the atrial electrical activity and may be viable for clinical diagnostic purposes, such as automated classification of the P wave.

This thesis has demonstrated novel techniques to analyse the P wave using the wavelet transform. The development of an accurate automated analysis and classification technique of the P wave has clear benefits. It ensures objective analysis and consistency across subjects. The novel technique presented here can uncover and quantify time-frequency information in the P wave to provide a practical diagnostic tool. The technique was employed to practical problems in electrocardiology and the results indicate the viability of the techniques.
Appendix A  Ethics Approval

The following page contains a scanned copy of an approval letter from the Griffith University Human Research Ethics Committee (HREC). Ethics approval was required for the collection of ECG signals from healthy
19 June 2009

TO WHOM IT MAY CONCERN

Griffith University Human Research Ethics Application – MEE/01/05/HREC

This is to confirm that Human Research Ethics Application MEE/01/05/HREC titled “Clearance for recording of Patient Electrocardiograms?” conducted by Adrian Diery, David Rowlands, Timothy Cutmore and Daniel James was approved by the Griffith University Human Research Ethics Committee (HREC) on 31 October 2005. The authorisation for this research was issued from 31 October 2005 to 7 April 2007.

The HREC is constituted and operates in accordance with the National Statement on Ethical Conduct in Research Involving Humans.

Please do not hesitate to contact me if you have any further queries about this matter.

Regards

Gary Allen
Manager, Research Ethics
Office for Research
Appendix B   Sample MATLAB Code

Analysis of Signal Averaged P Waves

close all
clear all
cd 'C:\Program Files\MATLAB\R2006b\work\LA area study';
[ecg,Ts,Fs,AmV,N,file,patient] = mlecg;
[VCG,ECG] = ecg2vcg(ecg,1,1);

x = ECG.II;
X = ECG.V5;

[P,Pon,Poff,Peak,T,xr] = Pwavedetection(x,500);
[PX,PonX,PoffX,PeakX,T,xr] = Pwavedetection(X,500);

% PLOT the baseline corrected signal with the P waves marked
display('plot');
figure(1);
t = (1:length(xr))/500;
subplot(2,1,1);
plot(t,x,'r'); xlabel('time seconds'); ylabel('amplitude mV');
hold on;
subplot(2,1,2);
plot(t,X,'r'); xlabel('time seconds'); ylabel('amplitude mV');
hold on;

subplot(2,1,1);
for p = 1:length(P)
    if p < length(Peak)
        line([Peak(p) Peak(p)]/500,[min(x)*1.2 max(x)*1.2]);
    end
    line([P(p) P(p)]/500,[min(x)*1.2 max(x)*1.2]);
    plot(Pon(p)/500,x(Pon(p)),'b^');
    plot(Poff(p)/500,x(Poff(p)),'bo');
end

subplot(2,1,2);
for p = 1:length(PX)
    if p < length(Peak)
        line([PeakX(p) PeakX(p)]/500,[min(x)*1.2 max(x)*1.2]);
    end
    line([PX(p) PX(p)]/500,[min(x)*1.2 max(x)*1.2]);
    plot(PonX(p)/500,X(PonX(p)),'b^');
    plot(PoffX(p)/500,X(PoffX(p)),'bo');
end

flag = input('use II or V5? 0 or 1 ');

if flag == 1
    Peak = PeakX;
P = PX;
Pon = PonX;
Poff = PoffX;
end
R = find(Peak > P(1));
P = Peak(R);
R = find(Peak <= 4975);
P = Peak(R);

pwaves = input('keep which beats? 
');
[val, ind] = max(Poff(pwaves) - Pon(pwaves));
p_dur = (Poff(pwaves) - Pon(pwaves)) * 2;
N = length(pwaves);
win = 180;

VCG = qrsremoval(VCG, PeakX, 500);
ECG = dow(VCG);

xP.V1 = zeros(N, win);
xP.aVF = zeros(N, win);
xP.V2 = zeros(N, win);
xP.V5 = zeros(N, win);
xP.X = zeros(N, win);
xP.Y = zeros(N, win);
xP.Z = zeros(N, win);
xP.II = zeros(N, win);
for p = 1:N
  i = pwaves(p);
  L = Poff(i) - Pon(i) + 1;
  pad = win - L - 40;
  xP.V1(p,:) = ECG.V1(P(i)-90:P(i)-89) - ECG.V1(P(i));
  xP.V1(p,:) = xP.V1(p,:) - ECG.V1(P(i));
  xP.aVF(p,:) = ECG.aVF(P(i)-90:P(i)-89) - ECG.aVF(P(i));
  xP.aVF(p,:) = xP.aVF(p,:) - ECG.aVF(P(i));
  xP.V2(p,:) = ECG.V2(P(i)-90:P(i)-89) - ECG.V2(P(i));
  xP.V2(p,:) = xP.V2(p,:) - ECG.V2(P(i));
  xP.V5(p,:) = ECG.V5(P(i)-90:P(i)-89) - ECG.V5(P(i));
  xP.V5(p,:) = xP.V5(p,:) - ECG.V5(P(i));
  xP.X(p,:) = VCG.X(P(i)-90:P(i)-89) - VCG.X(P(i));
  xP.X(p,:) = xP.X(p,:) - VCG.X(P(i));
  xP.Y(p,:) = VCG.Y(P(i)-90:P(i)-89) - VCG.Y(P(i));
  xP.Y(p,:) = xP.Y(p,:) - VCG.Y(P(i));
  xP.Z(p,:) = VCG.Z(P(i)-90:P(i)-89) - VCG.Z(P(i));
  xP.Z(p,:) = xP.Z(p,:) - VCG.Z(P(i));
  xP.II(p,:) = ECG.II(P(i)-90:P(i)-89) - ECG.II(P(i));
  xP.II(p,:) = xP.II(p,:) - ECG.II(P(i));
end

PV1 = mean(xP.V1); %PV1 = PV1 - mean(PV1);
vV1 = var(xP.V1);

PV2 = mean(xP.V2); %PV2 = PV2 - mean(PV2);
vV2 = var(xP.V2);

PV5 = mean(xP.V5); %PV5 = PV5 - mean(PV5);
vV5 = var(xP.V5);

PaVF = mean(xP.aVF); %PaVF = PaVF - mean(PaVF);
vaVF = var(xP.aVF);

PX = mean(xP.X); %PX = PX - mean(PX);
vX = var(xP.X);

PY = mean(xP.Y); %PX = PX - mean(PX);
vY = var(xP.Y);

PZ = mean(xP.Z); %PZ = PZ - mean(PZ);
vZ = var(xP.Z);

PII = mean(xP.II); %PZ = PZ - mean(PZ);
vII = var(xP.II);

PV1 = PV1 - PV1(21);
PV2 = PV2 - PV2(21);
PV5 = PV5 - PV5(21);
PaVF = PaVF - PaVF(21);
PX = PX - PX(21);
PY = PY - PY(21);
PZ = PZ - PZ(21);
PII = PII - PII(21);

mp = round(mean(p_dur)/2);

figure;
subplot(2,2,1);
plot(PII);
subplot(2,2,2);
plot(PV1);
subplot(2,2,3);
plot(PX); hold on;
plot(PY,'r');
plot(PZ,'k');
subplot(2,2,4);
plot(PV5);
axis square

cut = input('cut off trace at ');
L = length(cut:180);
decay = linspace(1,0,L);

PII(cut:end) = PII(cut);
PV1(cut:end) = PV1(cut);
PaVF(cut:end) = PaVF(cut);
PV2(cut:end) = PV2(cut);
PV5(cut:end) = PV5(cut);
PX(cut:end) = PX(cut);
PY(cut:end) = PY(cut);
PZ(cut:end) = PZ(cut);%:end).*decay.^4;
figure;
plot(PV1);
hold on; plot(PV5,'r');
plot(PV2,'k'); plot(PaVF,':');
grid;
pon = input('enter P onset ');
poff = input('enter P offset ');
P_dur = (poff-pon)*2;

PV1 = PV1 - PV1(pon);
PV2 = PV2 - PV2(pon);
PV5 = PV5 - PV5(pon);
PaVF = PaVF - PaVF(pon);
PX = PX - PX(pon);
PY = PY - PY(pon);
PZ = PZ - PZ(pon);
PII = PII - PII(pon);

[Ep,C2,t,f,pv,pf,IQR,QV,M] = averagePchar(PV1,pon,poff);
data(:,1) = [Ep; f; t'; pv; pf; P_dur;QV;IQR;M];
[Ep,C2,t,f,pv,pf,IQR,QV,M] = averagePchar(PV5,pon,poff);
data(:,2) = [Ep; f; t'; pv; pf; P_dur;QV;IQR;M];
[Ep,C2,t,f,pv,pf,IQR,QV,M] = averagePchar(PaVF,pon,poff);
data(:,3) = [Ep; f; t'; pv; pf; P_dur;QV;IQR;M];
[Ep,C2,t,f,pv,pf,IQR,QV,M] = averagePchar(PV2,pon,poff);
data(:,4) = [Ep; f; t'; pv; pf; P_dur;QV;IQR;M];
[Ep,C2,t,f,pv,pf,IQR,QV,M] = averagePchar(PX,pon,poff);
data(:,5) = [Ep; f; t'; pv; pf; P_dur;QV;IQR;M];
[Ep,C2,t,f,pv,pf,IQR,QV,M] = averagePchar(PY,pon,poff);
data(:,6) = [Ep; f; t'; pv; pf; P_dur;QV;IQR;M];
[Ep,C2,t,f,pv,pf,IQR,QV,M] = averagePchar(PZ,pon,poff);
data(:,7) = [Ep; f; t'; pv; pf; P_dur;QV;IQR;M];
[Ep,C2,t,f,pv,pf,IQR,QV,M] = averagePchar(PII,pon,poff);
data(:,8) = [Ep; f; t'; pv; pf; P_dur;QV;IQR;M];

fpath = 'C:\Program Files\MATLAB\R2006b\work\LA area study';
fsave = [fpath ' file(1:end-4)_averaged.dat'];
dlmwrite(fsave,data,['	']);

C.V1 = cwt(PV1,1:48,'gaus1');
E.V1 = C.V1.^2;
F1 = scal2frq(1:48,'gaus1',1/500);
F2 = scal2frq(1:48,'gaus2',1/500);
f = max(E.V1(:,(pon:poff)));
f = f/max(f);
[val,ind] = max(E.V1);
t = sum(C.V1.^2);
t = t/max(t);
figure;
subplot(2,2,1); imagesc(E.V1);
title('|C(a,b)|^2');
xlabel('time (b)'); ylabel('scale (a)');
subplot(2,2,2); plot(f,-1:-1:-48);
title('Scalewise Maxima');
xlabel('normalised magnitude $|C(a,b)|^2$');
ylabel('scale (a)');
subplot(2,2,3); plot(1:180,t);
title('Energy density for maximum scale');
xlabel('time (b)');
ylabel('normalised magnitude $|C(a,b)|^2$');
subplot(2,2,4); plot(PV1); axis square;
title('Averaged P wave in lead V1');
ylabel('Amplitude \mu V'); xlabel('time T=1/500 s');

C.X = cwt(PX,1:48,'gaus1');
C.Y = cwt(PY,1:48,'gaus1');
C.Z = cwt(PZ,1:48,'gaus1');
C.V5 = cwt(PV5,1:48,'gaus1');
C.aVF = cwt(PaVF,1:48,'gaus1');
C.V2 = cwt(PV2,1:48,'gaus1');

E.V5 = C.V5.^2;
fx = max(E.V5);
fx = fx/max(fx);
[val,ind] = max(E.V5);

E.aVF = C.aVF.^2;
fy = max(E.aVF);
fy = fy/max(fy);
[val,ind] = max(E.aVF);

C.C = cwt(C.X,1:48,'gaus1');
C.Y = cwt(C.Y,1:48,'gaus1');
C.Z = cwt(C.Z,1:48,'gaus1');
C.V5 = cwt(C.V5,1:48,'gaus1');
C.aVF = cwt(C.aVF,1:48,'gaus1');
C.V2 = cwt(C.V2,1:48,'gaus1');

E.V5 = C.V5.^2;
fx = max(E.V5);
fx = fx/max(fx);
[val,ind] = max(E.V5);

E.aVF = C.aVF.^2;
fy = max(E.aVF);
fy = fy/max(fy);
[val,ind] = max(E.aVF);
Derivation of Characteristics

function [Ep,C2,t,f,pv,pf,IQ,QVC,X] = averagePchar(x,pon,poff)

% Calculate maximum value of correlation
% scale of maximum correlation
% calculate energy and normalise it over duration and scales
C2 = cwt(x,1:48,'gaus2');
P = poff - pon;

% Normalise energy spectrum over duration and scales
E = (C2.^2);
Ep = sum(E([4 8 12 20 32 48],pon:poff))/P;
Ep = Ep/6;
[val, ind] = max(Ep);
Ep = sum(Ep);

E = E/48; % Total Energy
offset = ind+pon-1;

% frequency profile
f = E(:,offset);
[val,pf] = max(f); % Peak Scale
f = f/max(f); % Scale Profile

U = 1:48; [IQ,QVC,Q1,X,Q3,Q2] = interquart(f,U);

% time profile
t = E(pf,:);
pv = max(t); % Peak Value
t = t/pv; % Time Profile