

Enhanced Motor Imagery Classification using Riemannian Tangent Space Mapping

Shiu Kumar*, Kabir Mamun, and Alok Sharma

Abstract - Background: Classification of electroencephalography (EEG) signals for motor imagery based brain computer interface (MI-BCI) is an exigent task and common spatial pattern (CSP) has been extensively explored for this purpose. In this work, we focused on developing a new framework for classification of EEG signals for MI-BCI.

New Method: We propose a single band CSP framework for MI-BCI that utilizes the concept of tangent space mapping (TSM) in the manifold of covariance matrices. The proposed method is named CSP-TSM. Spatial filtering is performed on the bandpass filtered MI EEG signal. Riemannian tangent space is utilized for extracting features from the spatial filtered signal. The TSM features are then fused with the CSP variance based features and feature selection is performed using Lasso. Linear discriminant analysis (LDA) is then applied to the selected features and finally classification is done using support vector machine (SVM) classifier.

Results: The proposed framework gives improved performance for MI EEG signal classification in comparison with several state-of-the-art-methods methods.

Comparison with existing methods: Experiments conducted shows that the proposed framework reduces the overall classification error rate for MI-BCI by 3.16% and 5.10% (for BCI Competition III dataset IVa and BCI Competition IV Dataset I, respectively) compared to the conventional CSP method under the same experimental settings.

Conclusion: The proposed CSP-TSM method outperforms all state-of-the-art methods and the computational complexity is less compared to that of TSM method. Our proposed CSP-TSM framework can be potentially used for developing improved MI-BCI systems.

Index Terms—Brain Computer Interface (BCI); Common Spatial Pattern (CSP); Electroencephalography (EEG); Motor Imagery (MI); Riemannian Distance, Tangent Space Mapping (TSM).

I. INTRODUCTION

IN a brain computer interface (BCI) system, direct communication takes place between the brain and the external devices without the involvement of peripheral nerves and muscles in order to allow humans to interact with their surroundings [1]. The electrical control signals generated by

the brain activity are measured using the electroencephalogram (EEG) method. Motivated by the aim of re-establishing independence and reducing social exclusion for people with disabilities, BCI research has gained a vast range of interest in this field. BCI has numerous emerging applications such as applications for communication control [2, 3], environment control [4], movement control [5, 6] and neuro-rehabilitation [7-9]. Researchers have also concentrated on developing BCI systems such as P300-based word typing system [10], EEG controlled mouse [11], brain wave controlled robots [12] and brain wave controlled wheelchair for the disabled [13]. EEG signal has also been used for biometric identification [14]. The major focus and emphasis of BCI research is in the field of biomedical engineering [15-19].

Generally, in the realization of a BCI system the following stages are involved: signal acquisition, preprocessing, feature extraction, signal classification, and control interface. Several methods are used for measuring and studying the brain waves such as EEG, magnetoencephalogram (MEG), positron emission tomography (PET) and single photon emission computed tomography (SPECT). However, due to the facts that EEG is non-invasive, has relatively low cost, supports real time analysis and is portable makes it the most suitable method for BCI systems [20, 21]. With technological advancements, devices having low cost and reduced complexity such as Neurosky Mindwave [22] and Emotiv EPOC/EPOC+ headset [23] have been developed, which are used in BCI research and applications.

Several techniques have been proposed with respect to different aspects of the problem, the three major aspects being filter band selection, spatial filter estimation and feature extraction for optimal BCI performance. The conventional CSP method utilized a single filter band. However, various methods such as common spatio-spectral pattern (CSSP) [24], common sparse spectral spatial pattern (CSSSP) [25], sub-band CSP (SBCSP) [26], filter band CSP (FBCSP) [27], discriminative filter band CSP (DFBCSP) [28], sparse filter band CSP (SFBCSP) [29] and filter band selection using binary particle swarm optimization (BPSO) [30] have been proposed. These methods have been proposed due to the fact that the filter band that gives maximum discrimination between different tasks for optimal classification varies from subject to subject i.e. dependent on the users. CSP is applied to the signals combined with its time delayed signals to obtain the finite impulse response filter (FIR) coefficients in CSSP (limited to single delay) and CSSSP. For each channel, a

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different spectral pattern is found by CSSP whereas CSSSP finds a spectral pattern that is common to all channels. In SBCSP and FBCSP methods, the signal is decomposed into sub-bands using multiple filter banks having different frequency passband. CSP is then applied to each of the sub-bands separately. Recursive band elimination using support vector machine (SVM) is performed for SBCSP method in order to obtain the top ten sub-bands from the 24 sub-bands (each having bandwidth of 4 Hz) with better discriminating power. The selected sub-bands are then used for feature extraction and classification (using SVM). On the other hand, the FBCSP method utilizes all the sub-bands (having a bandwidth of 4 Hz in the frequency range of 4-40 Hz). However, feature selection using various techniques has been employed to obtain the features that give optimal discrimination resulting in better classification. DFBCSP selects the best four sub-bands from 12 filter banks (each with bandwidth of 4 Hz) using the fisher ratio calculated using single channel C3, which provides a measure of the discrimination between the two MI tasks. CSP features are then extracted separately from these sub-bands and SVM classifier is used for classification.

Recently, a SFBCSP [29] method that also uses multiple filter bands is proposed, which optimizes the sparse patterns. CSP on multiple overlapping frequency bands is performed, and significant CSP features are then selected using supervised technique. SVM classifier is then used for MI classification using the selected features. **Over the years, Bayesian learning has also gained increased attention and it has been used for feature selection in various applications [31]. In [32], a sparse Bayesian learning of filter banks (SBLFB) approach has been proposed for obtaining sparse features that are used for classification using the SVM classifier. The EEG signals were decomposed into multiple sub-bands, CSP features were extracted and significant features were selected using Bayesian learning. Furthermore, in [30], a frequency band selection using BPSO is presented. Unlike the DFBCSP method, the BPSO selects either one or two best sub-bands producing optimal results. However, the BPSO algorithm is computationally expensive and takes a huge amount of time due to which the authors have only used selected (25 and 14) channels of data for processing. These methods recommend that feature extraction using sub-band decomposition can improve the classification accuracy of MI-BCI.**

The measured EEG signals contain measurement noise and spontaneous components related to other brain activities that degrade the signal-to-noise ratio (SNR). Therefore, spatial filtering is usually performed on the EEG signal in order to discriminate between the overlapping activities and improve the SNR. In spatial filtering, the multiple channel EEG signals are linearly combined so that the unwanted sources are obscured and those sources that are of interest are enhanced. To this end, common spatial pattern (CSP) has been widely explored [33] and used in BCI research. Several methods have been proposed to optimize the temporal filter parameters for better classification in order to prevent time consuming

manual tuning. In [34], an iterative spatio-spectral patterns learning (ISSPL) method is proposed. In this method, the classifier and spectral filters are parameterized simultaneously to enhance the classification performance. A weighted averaging method [35] has also been proposed for estimating the covariance matrix for CSP. The number of training samples available for MI tasks is usually small and can result in the deterioration of the classification accuracy. To tackle this problem, a regularized CSP (R-CSP) with aggregation has been proposed by Lu et al. [36]. In this method, two parameters (β , γ) regularize the estimation of the covariance matrix with the aim of reducing the estimation variance and bias. In the regularization process, EEG samples from other subjects (other than the subject of interest) are utilized. A divergence based framework for CSP that also utilizes information from other subjects is presented in [37].

In this study, we focus on the aspect of extracting features from single frequency band that would further improve the performance of BCI system. Barachant *et al.* [38] proposed multiclass BCI classification by Riemannian geometry, showing that it improves the MI classification accuracy. However, they proposed to extract the spatial information directly from the bandpass filtered EEG signal and as the number of channels increases, it requires higher computational complexity. To this end, we propose to utilize Riemannian tangent space features extracted from the spatial filtered EEG signal, which leads to an improved MI classification accuracy and a reduction in the computational complexity of the proposed method. Riemannian tangent space features are combined with the CSP variance based features and feature selection is performed. LDA [39] reduces the dimensionality of the selected features and SVM classifier with linear kernel is then used for classification. The proposed method is validated on **two** publically available EEG dataset (BCI Competition III dataset IVa **and** BCI Competition IV dataset I), and compared with other related literature such as CSP, FBCSP, DFBCSP, SFBCSP **and** SBLFB. Experimental results show that the proposed method enhances the MI classification accuracy.

The rest of the paper is organized as follows. Section II presents the CSP method followed by the proposed CSP-TSM framework for MI classification. The results and discussion are presented in Section III and IV, respectively, while Section V draws the conclusion and gives insight of some future works.

II. CSP-TSM FOR MI CLASSIFICATION

Because existing competing methods such as FBCSP, DFBCSP, SFBCSP **and** SBLFB uses multiple frequency bands, the computational complexity of the system is higher in comparison with that of if a single frequency band is utilized. The proposed CSP-TSM method utilizes a single CSP based filter such as the CSSP, CSSSP, and SPEC-CSP [40] methods. This paper shows that a single frequency band relating to the brain activities associated with MI-BCI tasks can be sufficient to obtain optimal results.

In this section, the CSP-TSM algorithm for MI

classification is presented. The single bandpass filtered EEG signal is transformed using the spatial filter, obtaining a lower dimensional signal. Features are then extracted from the spatial filtered signal using tangent space mapping of the Riemannian distance. The features obtained after performing TSM are then fused with the CSP variance based features. Regression analysis using least absolute shrinkage and selection operator (Lasso) is used for feature selection in order to obtain the significant features. LDA is applied to the selected features to reduce the dimensionality of the features. SVM with linear kernel is then implemented for EEG signal classification of MI tasks.

A. Feature Extraction using CSP

The CSP technique that was first used for detection of abnormalities using EEG signal [41] has gained a wide attention in the field of MI-BCI for feature extraction. CSP is commonly used for transforming the signal to a new time series that has maximum discrimination between different MI tasks. Consider the two class problem having EEG samples $x_{n,1}$ and $x_{n,2} \in R^{c \times T}$, where n denotes the n -th trial, c is the number of channels and T is the number of sample points. The samples are bandpass filtered and it is assumed that the samples have been centered. The normalized spatial covariance matrix Σ of class l is given by (1), where N_l is the number of trials in class l .

$$\Sigma_l = \frac{1}{N_l} \sum_{n=1}^{N_l} \frac{X_{n,l} X_{n,l}^T}{\text{trace}(X_{n,l} X_{n,l}^T)} \quad (1)$$

In CSP, the function in (2) is maximized in order to get maximum separability between the variance of the two classes, where $w \in R^c$ is the spatial filter and $\|\cdot\|_2$ denotes the l_2 norm.

$$\max_w J(w) = \frac{w^T \Sigma_1 w}{w^T \Sigma_2 w} \quad \text{s.t.} \quad \|w\|_2 = 1 \quad (2)$$

Solving the generalized eigenvalue problem in (3) will result in the maximization of the Rayleigh quotient $J(w)$.

$$\Sigma_1 w = \lambda \Sigma_2 w \quad (3)$$

The spatial filter matrix W is then obtained using (4), where P is the whitening transformation matrix and U is matrix containing the eigenvectors. In this paper, it is assumed that the eigenvalues and their corresponding eigenvectors are arranged in descending order.

$$W = U^T P \quad (4)$$

The CSP spatial filter W_{csp} is then formed by selecting the first and last m columns of W . The transformation Z of a given EEG sample X is thus obtained using (5).

$$Z = W_{csp}^T X \quad (5)$$

The variance based CSP feature vector is then formed as $F_{CSP} = [F_1, F_2, \dots, F_{2m}]$, where F_i is given by (6) with $\text{var}(Z_m)$ denoting the variance of m -th row of Z .

$$F_i = \log \left(\frac{\text{var}(Z_i)}{\sum_{n=1}^{2m} \text{var}(Z_n)} \right) \quad (6)$$

B. Single Band CSP with TSM (CSP-TSM)

The CSP-TSM framework utilizes the theory of Riemannian geometry in the manifold of covariance matrices [38] and uses the tangent space vectors as features. Fig. 1 illustrates the proposed CSP-TSM framework that utilizes a single frequency band. The projected data $\hat{X} \in R^{2m \times T \times N}$ obtained after spatial filtering using (5) is utilized for further processing, where N is the total number of trials. The normalized covariance matrix $\Sigma_i \in R^{2m \times 2m}$ for each projected sample \hat{X}_i is computed using (7), where Σ_i represents the sample covariance matrix (SCM) of the spatial filtered data for the i -th sample.

$$\Sigma_i = \frac{\hat{X}_i \hat{X}_i^T}{\text{trace}(\hat{X}_i \hat{X}_i^T)} \quad (7)$$

Consider $\Gamma(t) : [0,1] \rightarrow \Sigma(n) \in R^{2m \times 2m}$ to be any differentiable path between $\Gamma(0) = \Sigma_1$ and $\Gamma(1) = \Sigma_2$, where $\Sigma(n)$ is symmetric positive-definite (SPD) matrices (the space of which is a differentiable Riemannian manifold (M)). Equation (8) gives the length of $\Gamma(t)$. The geodesic is the curve with minimum length that connects the two points on the manifold. The length of this curve gives the Riemannian distance between the two points.

$$L(\Gamma(t)) = \int_0^1 \|\Gamma(t)\|_{\Gamma(t)} dt \quad (8)$$

Classification algorithms such as SVM, LDA and Neural Networks cannot be implemented directly in the Riemannian manifold as they are based on projections into hyper-planes. Thus each of the SCM is mapped into the tangent space located at the Riemannian mean Σ of the whole set of SCM matrices. The Riemannian mean is given by (9), where $\mathcal{R}(\cdot)$ operator denotes computation of Riemannian mean and $i = 1, \dots, N$. Due to the fact that there is no closed-form expression for computing the Riemannian mean, an efficient iterative algorithm given in [42] has been adopted.

$$\Sigma = \mathcal{R}(\Sigma_i) = \arg \min_{\Sigma \in \Sigma(n)} \sum_{i=1}^N \delta_R^2(\Sigma, \Sigma_i) \quad (9)$$

The Riemannian distance δ_R can be obtained using (10), where $\text{upper}(\cdot)$ means taking the upper triangular part of the symmetric matrix, vectorizing it and applying $\sqrt{2}$ weight to the out-of-diagonal elements [43], the logarithmic mapping $\text{Log}_\Sigma(\Sigma_i)$ is given by (11) and s_i is a vector of the normalized tangent space.

$$\begin{aligned} \delta_R(\Sigma, \Sigma_i) &= \|\text{Log}_\Sigma(\Sigma_i)\|_\Sigma \\ &= \|\text{upper}(\Sigma^{-1/2} \text{Log}_\Sigma(\Sigma_i) \Sigma^{-1/2})\|_2 = \|s_i\|_2 \end{aligned} \quad (10)$$

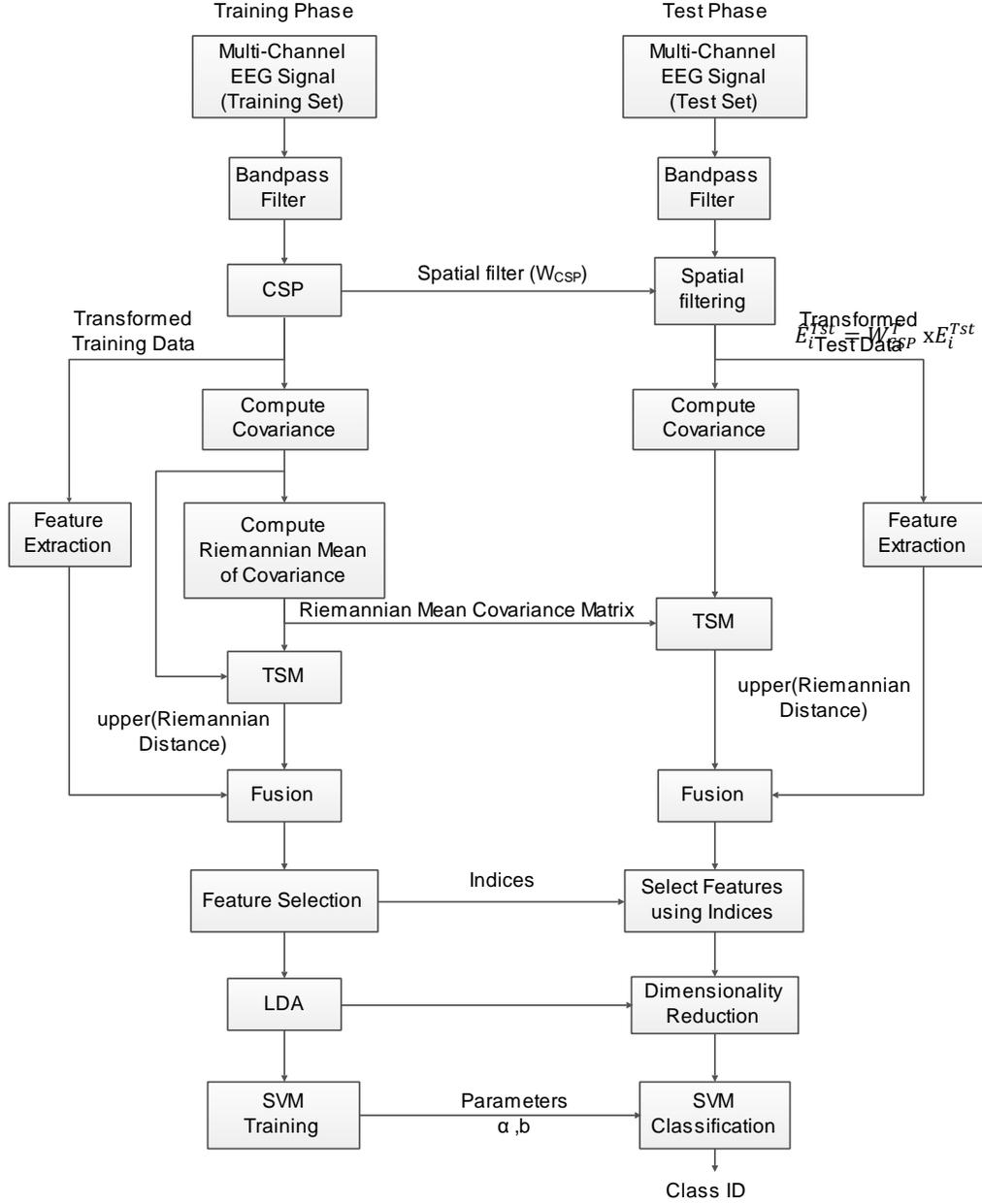


Fig. 1. Illustration of the proposed single band CSP-TSM framework.

$$\text{Log}_{\Sigma}(\Sigma_i) = \Sigma^{1/2} \log(\Sigma^{-1/2} \Sigma_i \Sigma^{-1/2}) \Sigma^{1/2} \quad (11)$$

Therefore, each of the SCM Σ_i is mapped into the tangent space resulting in a set of $2m(2m+1)/2$ dimensional vectors S_i . These vectors S_i are used as the tangent space features in our proposed framework.

C. Feature Selection and Classification

The variance based CSP features F_{CSP} are concatenated with the tangent space features S_i to form a $2m^2 + 3m$ dimensional feature vector F . However, the success of any MI-BCI depends largely on the extracted features. A good selection of features will result in an improvement in the classification performance of MI tasks for BCI applications. Thus we employ feature selection using sparse regression model known

as Lasso estimate [44] given by (12) in order to obtain significant features, where u is the sparse vector that is to be learned, λ is the nonnegative regularization parameter that controls the sparsity of u , and $y \in R^N$ is the vector consisting of the target class labels.

$$u = \arg \min_u \frac{1}{2} \|Fu - y\|_2^2 + \lambda \|u\|_1 \quad (12)$$

The expression in (12) is an optimization problem and for solving this the coordinate descent algorithm [45] has been adopted. The optimized sparse vector u obtained is used for selecting the features. Top d features in F corresponding to the largest d values in u are selected to form an optimized feature set F_{sel} containing the significant features. LDA is applied to further reduce the dimensionality of the optimized

$$\Sigma_i \in R^{2m \times 2m}$$

feature set using (13), where w_{LDA} is the LDA transformation matrix obtained using (14), in which μ_l is the mean of class l , and S_B and S_W are the between-class and within-class scatter matrices, respectively.

$$F_{LDA} = w_{LDA}^T F_{sel} \quad (13)$$

$$w_{FDA} = \arg \max_w \left(\frac{w^T S_B w}{w^T S_W w} \right) = S_W^{-1} (\mu_1 - \mu_2) \quad (14)$$

The features F_{LDA} obtained after applying LDA are used for training the SVM classifier using a linear kernel. The trained SVM classifier is then used to classify the test samples.

III. RESULTS

A. Description of Data

The BCI Competition III dataset IVa [46] and BCI Competition IV dataset I [47], available publicly have been used for evaluating the proposed method. The BCI Competition III dataset IVa contains the EEG signals recorded from five subjects labeled *aa*, *al*, *av*, *aw*, and *ay* performing two different (right hand and left foot) MI tasks. 118 channels at positions of the extended international 10/20 system [48] were used for measuring the EEG signal, sampled at a rate of 1000 Hz. The dataset contains a total of 280 trials for each subject with equal number of trials for each task. The data provided was bandpass filtered with passband of 0.05-200 Hz. The down-sampled data at 100 Hz is used. A detail description of the dataset can be found at <http://www.bbc.de/competition/iii/>.

The BCI Competition IV dataset I contains 59 channels of real long-term EEG signals recorded for left hand, and right hand MI tasks acquired from 7 healthy subjects (named *a* to *g*). The down sampled signal at 100 Hz has been used, which contains 200 trials for each subject with equal number of each type of MI tasks. A detail description of the dataset can be found online at [<http://www.bbc.de/competition/iv/>].

B. Experimental Setup

In this study, data between 0.5 seconds and 2.5 seconds (i.e. 200 sample points) after the visual cue has been extracted for processing [49]. The data was further bandpass filtered using a

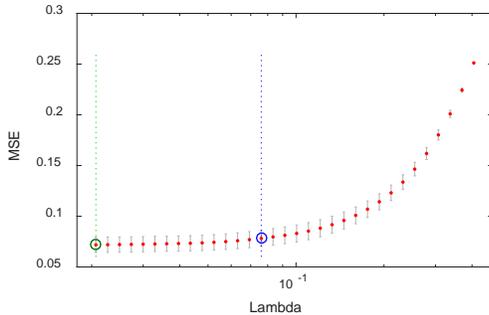


Fig. 2. Cross-validated MSE of Lasso fit for one of the experimental runs for subject *aa*.

butterworth bandpass filter having passband of 7-30 Hz. The parameter m used was also chosen out of $\{1, \dots, 10\}$ in advance. The value of $m = 3$ has been adopted as it gave the maximum classification accuracy for all subjects.

The parameter λ in (12) was chosen out of $\{0.01, \dots, 0.99\}$ using a 10-fold cross validation. Fig. 2 presents an example of the effect of varying λ on the mean squared error for subject *aa*. The value of λ selected can be any value between the points circled in blue and green. Similar result was obtained for all other subjects and thus the value of $\lambda = 0.15$ have been used.

C. Analysis of the CSP-TSM framework with TSM and CSP methods

Sparse regression has been exploited in this study and a properly determined regularization parameter λ leads to an optimal learned sparse vector u from (12). The sparse vector u

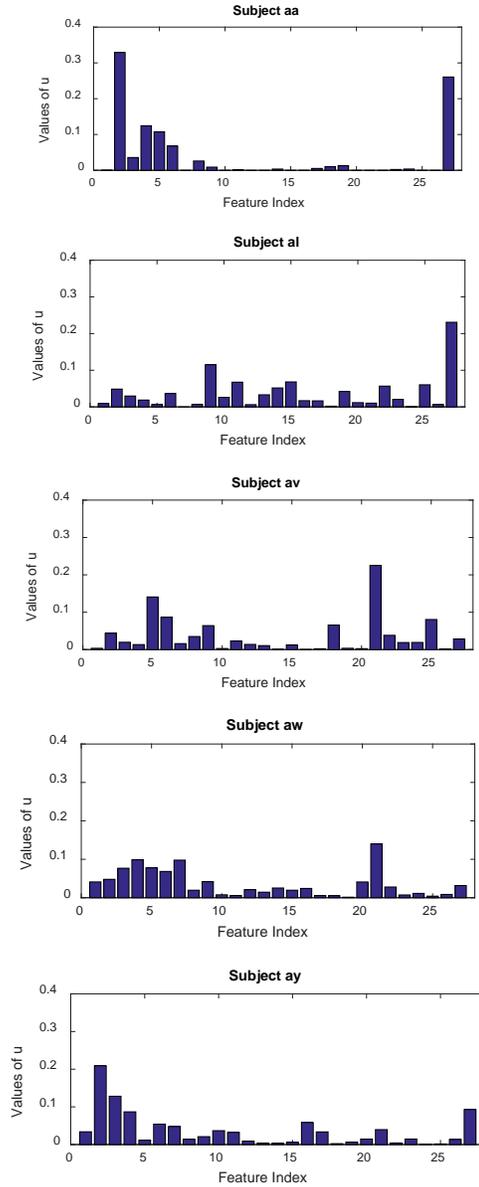


Fig. 3. Example of sparse vectors learned by CSP-TSM method for each of the five subjects in BCI Competition III dataset IVa.

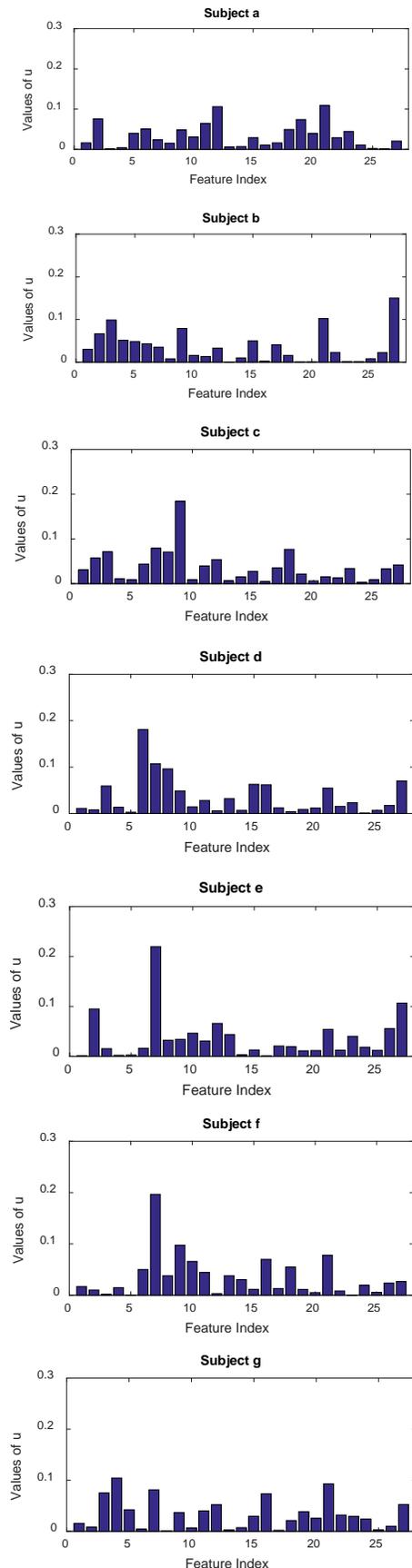


Fig. 4. Example of sparse vectors learned by CSP-TSM method for each of the seven subjects in BCI Competition IV dataset I. is utilized for selecting the significant features from the

variance based CSP features and TSM features obtained from spatially filtered data.

An example of the sparse vectors learned by the proposed CSP-TSM framework for each subject in the BCI Competition III dataset IVa and BCI Competition IV dataset I are presented in Fig. 3 and Fig. 4, respectively. The feature indices from 1 to 6 corresponds to the variance based CSP features while the indices from 7 to 27 corresponds to the 21 TSM features obtained from the spatially filtered data. The significant features appeared at both the types of features used in this study i.e. the CSP variance based features and TSM features. It can be noted from Fig. 3 that the most significant feature for subjects *aa* and *ay* were from the variance based CSP features. On the other hand, for subjects *al*, *av* and *aw* the most significant feature was from the TSM features. In Fig. 4, the most significant feature for subjects *d* and *g* was from variance based CSP features while for subjects *a*, *b*, *c*, *e* and *f* the most significant feature was from TSM features. The significant features selected were different for each of the subjects, which is in agreement with the related literatures [28, 30, 32, 50] (can be due to different skull size, skin thickness and due to the fact that the way subjects think about the same task differs from subject to subject). This indicates the reason for using both types of features and explains the need for feature selection in our proposed framework for enhanced MI classification accuracy.

To compare the performance of the CSP-TSM framework with the TSM and conventional CSP algorithm, we have evaluated the classification performance of these methods using selected 25 channels of data (from BCI Competition III dataset IVa) as used in [30]. This is because the TSM algorithm could not be run on the desktop personal computer (Matlab ® on windows 7 with 6 GB Ram operating at 3.3 GHz) using all 118 channels of EEG signal as Matlab becomes irresponsive and the computer freezes. The three algorithms were run simultaneously so that all the classifiers were built using the same set of training data and the results evaluated exactly on the same set of test data for all the methods. The results obtained are depicted in Fig. 5. For all subjects except for subject *aw*, the CSP-TSM method performs better while for subject *aw*, the best performance was obtained using TSM method. It should be noted that on

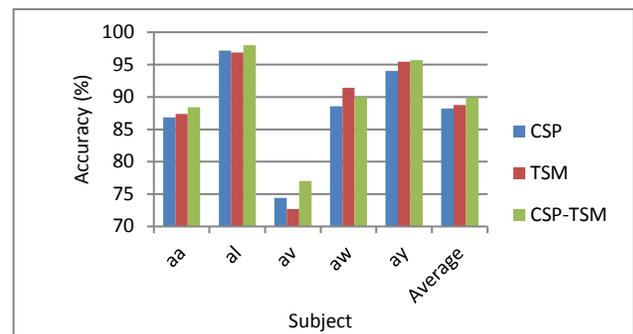


Fig. 5. 10x10-fold cross validation accuracies for conventional CSP, TSM and CSP-TSM methods using selected (25) channels for BCI Competition III dataset IVa.

average the CSP-TSM method performs better than both the conventional CSP and TSM methods by 1.73% and 1.17%, respectively.

D. Analysis of the CSP-TSM framework with other competing methods

In this section, the CSP-TSM framework is evaluated against other competing methods. Table I and Table II shows the 10x10-fold cross validation classification error rates of the proposed CSP-TSM framework along with the conventional CSP, FBCSP, DFBCSP, SFBCSP and SBLFB methods for the two datasets. All the methods are evaluated under the same experimental conditions using all channels of EEG data. For the conventional CSP, a wide filter band of 7-30 Hz was adopted while for FBCSP, DFBCSP and SFBCSP 17 filter bands (sub-bands) with a bandwidth of 4 Hz overlapping each other at a rate of 2 Hz was adopted from [28]. The regularization parameter λ for SFBCSP was determined using the same procedure as mentioned earlier for CSP-TSM framework. All methods employ SVM as the classifier. **In comparison with other competing methods; conventional CSP, FBCSP, DFBCSP, SFBCSP and SBLFB, the proposed CSP-**

TSM framework offers average error rate reductions of 3.16%, 1.76%, 0.63%, 4.10% and 3.42%, respectively for BCI Competition III dataset IVa and 5.10%, 4.10%, 3.80%, 8.10% and 1.50%, respectively for BCI Competition IV dataset I. Our method outperforms all other state-of-the-art methods on average while also obtaining the lowest classification error rate for 3 out of 5 subjects (*av*, *aw* and *ay*) for BCI Competition III dataset IVa and 5 out of 7 subjects (*a*, *b*, *c*, *e* and *f*) for BCI Competition IV dataset I. Our proposed method performed best for subject *al* of BCI Competition III dataset IVa and subject *g* of BCI Competition IV dataset I, having error rate of 2.14% and 7.8%, respectively. On the other hand, as expected, the worst performance was noted for subject *av* and subject *b* with error rate of 24.90% and 40.90%, respectively (consistent with results of other methods as it contains low quality signals).

IV. DISCUSSION

All the body activities including motor and muscle movement are controlled by the brain, which is an essential part of the human body. The message is firstly constructed in the brain for every communication that is initiated. The human brain contains over 100 billion neurons [51]. These neurons communicate with each other producing different pattern of electrical signals (generated due to electromagnetic activities inside the brain) for different thoughts [52]. The function of a BCI system is to capture the EEG signal and decode them for different brain activities thus enabling a direct channel of communication between the brain and the external devices without the need for any muscular movement [53]. Neural network have been widely used in various applications [54, 55] and has also been used for EEG signal classification [56-58]. However, methods that use CSP [25-28, 50, 59-61] have shown to be effective in MI EEG signal classification. Although EEG signals of larger fragments are highly non-Gaussian, up to 90 percent of short fragments of 4 seconds can be considered as Gaussian [62, 63]. In this research, we have used 2 second segments and therefore the EEG signals are assumed to be Gaussian. This is shown by the fact that the CSP method is able to perform well in MI EEG signal classification. Thus, we have combined the power of CSP and TSM to improve the performance of MI EEG signal classification. We have employed Lasso estimate for feature selection, however, other methods of feature selection [64] could also be employed.

Research is also being carried out to improve the CSP algorithm. In [65], a neural network-based optimal spatial filter design method has been proposed. The spatial filter and the classifier are trained simultaneously using a neural network named spatial filter network (SFN). Methods have also been proposed to optimize the classifier performance [57]. The performance of CSP largely depends on the frequency bands that are selected and as such research still continues in finding the best frequency bands. In [32], a sparse Bayesian learning approach for filter band selection (SBLFB) is proposed to find significant features. Binary particle swarm optimization has been proposed for selecting the filter bands that contain the most significant features [30], while a subject based feature extraction method has been proposed that uses

TABLE I
10X10-FOLD CROSS VALIDATION CLASSIFICATION ERROR RATE (%) OF CSP, FBCSP, DFBCSP, SFBCSP, SBLFB AND CSP-TSM METHODS, RESPECTIVELY, USING BCI COMPETITION III DATASET IVa. FOR EACH SUBJECT THE LOWEST ERROR IS MARKED IN BOLDFACE.

Method	Subject					Average
	<i>aa</i>	<i>al</i>	<i>av</i>	<i>aw</i>	<i>ay</i>	
CSP	21.00 ± 5.31	3.86 ± 3.93	28.29 ± 7.46	10.36 ± 5.10	3.86 ± 4.11	13.47 ± 5.18
FBCSP	17.14 ± 8.19	1.29 ± 1.88	30.36 ± 8.23	6.50 ± 4.55	5.07 ± 4.68	12.07 ± 5.51
DFBCSP	9.64 ± 5.01	1.00 ± 1.91	31.21 ± 8.92	4.64 ± 4.75	8.21 ± 5.06	10.94 ± 5.13
SFBCSP	18.43 ± 8.26	1.64 ± 1.36	29.93 ± 6.44	9.29 ± 8.58	12.79 ± 5.96	14.41 ± 5.57
SBLFB	18.71 ± 7.45	1.36 ± 2.03	29.64 ± 9.98	6.57 ± 4.47	12.36 ± 7.22	13.73 ± 6.23
CSP-TSM (Proposed)	16.79 ± 6.29	2.14 ± 3.53	24.90 ± 9.10	4.53 ± 2.80	3.21 ± 2.53	10.31 ± 4.85

TABLE II
10X10-FOLD CROSS VALIDATION CLASSIFICATION ERROR RATE (%) OF CSP, FBCSP, DFBCSP, SFBCSP, SBLFB AND CSP-TSM METHODS, RESPECTIVELY, USING BCI COMPETITION IV DATASET I. FOR EACH SUBJECT THE LOWEST ERROR IS MARKED IN BOLDFACE.

Method	Subject							Average
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	
CSP	13.2 ± 8.1	42.8 ± 12.3	43.7 ± 11.3	22.4 ± 8.8	18.0 ± 9.7	22.5 ± 10.8	7.10 ± 5.1	24.2 ± 9.4
FBCSP	19.1 ± 9.4	44.7 ± 11.3	35.7 ± 9.6	22.2 ± 9.0	14.0 ± 9.1	19.6 ± 8.6	6.9 ± 6.6	23.2 ± 9.1
DFBCSP	16.8 ± 7.8	42.9 ± 9.7	35.2 ± 8.5	23.5 ± 8.4	18.3 ± 8.8	14.3 ± 8.6	9.0 ± 5.1	22.9 ± 8.1
SFBCSP	17.4 ± 5.9	45.3 ± 6.6	43.0 ± 11.6	29.5 ± 10.1	24.7 ± 10.3	20.9 ± 6.5	9.7 ± 5.0	27.2 ± 8.0
SBLFB	19.1 ± 9.7	41.5 ± 11.1	33.2 ± 12.5	11.5 ± 7.9	11.6 ± 6.9	21.2 ± 9.0	5.9 ± 5.4	20.6 ± 9.4
CSP-TSM (Proposed)	11.9 ± 6.6	40.9 ± 10.3	32.1 ± 8.7	15.7 ± 7.9	9.8 ± 5.8	14.1 ± 7.1	7.8 ± 5.7	19.1 ± 7.5

wavelet packet decomposition (WPD) together with CSP [66]. Therefore, implementing the proposed method with multiple

selected filter banks may further improve the classification performance of the system at the cost of higher computational time.

A. Comparison of feature distribution

To further compare the performance of the CSP-TSM framework with the TSM and conventional CSP algorithm, we have analyzed the distribution of the extracted features for each of the methods. The distributions of the two most significant features obtained by CSP, TSM and CSP-TSM methods respectively from subject *al* is depicted in Fig. 6. If we draw an imaginary non-linear line separating the two clusters, the minimum number of misclassified samples obtained for CSP, TSM, CSP-TSM and SBLFB methods from Fig. 6 would be 16, 14, 12 and 16, respectively. Thus it can be seen that CSP-TSM method achieved the maximum discriminability of features. This explains the fact that the CSP-TSM method outperforms the CSP and TSM methods. The dataset used is from BCI Competition III and thus it may or may not contain outliers. However, the overlap in the feature points of CSP and TSM are in agreement with the related literatures. It is shown in [38] that using Riemannian distance to Riemannian mean as used by TSM is more effective in providing relevant information about the class membership than using Euclidean distance to its mean as used by CSP. This explains the fact that the CSP-TSM approach is able to attain a better separation as it uses the properties of both CSP and TSM. It can be noted from Fig. 6 that some of the feature points from one class appear to completely resemble that of another class. This can be due to poor quality of the signal recorded or possibility of an error made during recording, for example the subject was actually performing class 1 MI task during recording of class 2 MI task. Supervised methods of training adopted in this research cannot

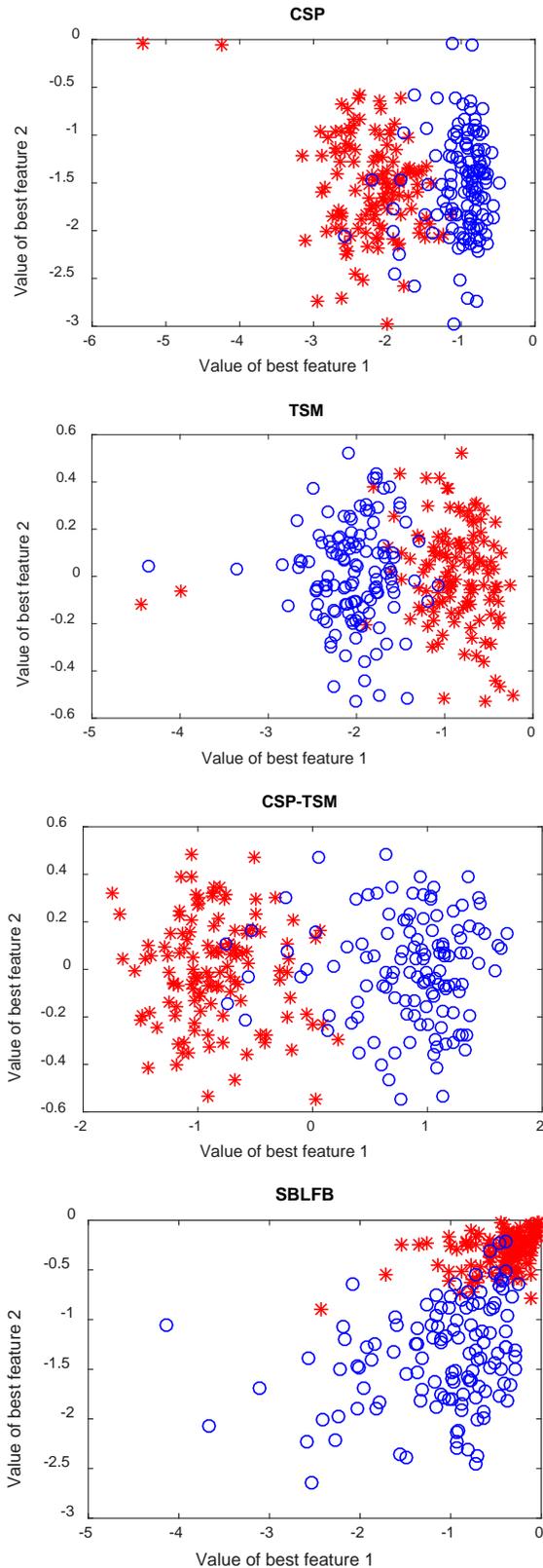


Fig. 6. Distribution of two most significant features obtained by CSP, TSM, and CSP-TSM, respectively, from subject *al* of BCI Competition III dataset IVa.

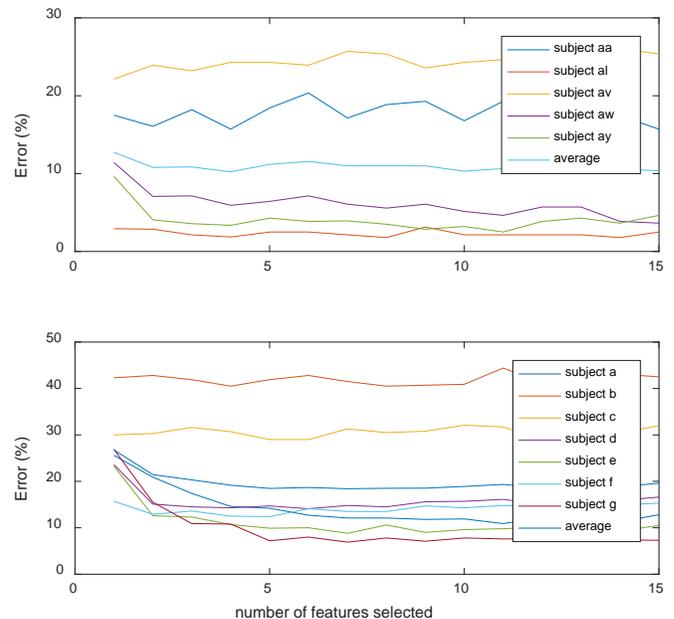


Fig. 7. Misclassification error rates using different number of selected features. Top: BCI Competition III dataset IVa. Bottom: BCI Competition IV dataset I)

overcome this problem. Therefore, use of unsupervised methods such as cluster methods [67-70] can be utilized to overcome this problem and may further improve the performance of the system.

B. Feature selection

The proposed CSP-TSM algorithm utilizes feature selection of the fused CSP variance based features and TSM (from spatially filtered data - CSP) features. We have employed Lasso for selecting the significant features as it has performed well in [29]. However, it requires the selection of the number of features to be selected. We have conducted experiments to select features in the range of 1 to 15 and the classification accuracies obtained for BCI Competition III dataset IVa and BCI Competition IV dataset I are shown in Fig. 7. From Fig. 7, it can be noted that using 10 selected features produced optimal results and therefore, the top 10 significant features have been used. LDA have been employed after feature selection as in our previous work [71], we have shown that using LDA improves the classification performance of the system. Other approaches such as using only the TSM features obtained from the spatially filtered data and applying principal component analysis (PCA) or LDA have also been evaluated. However, the proposed framework shown in Fig. 1 produced optimal results and thus has been adopted. It is left as future study to evaluate other feature selection methods such as mutual information based feature selection [27] and sparse Bayesian learning for feature selection [32], which have also performed well in selecting the most significant features.

C. Comparison of computational complexity

The authors in [38] proposed to use TSM technique, which directly depends on the covariance matrices of the EEG signal. The TSM method is an unsupervised process and has the ability to extract spatial information that is comparable to the state-of-the-art CSP. Because the TSM method directly depends on the covariance matrices, the computational complexity of the system increases drastically when signal from large number of EEG channels is used. In this case, a channel selection algorithm will be required in order to select the channels with most significant information. In this work, we have proposed to use CSP to reduce the dimensionality of the EEG signal and then apply TSM. In this way, the computational complexity of the system is reduced drastically.

To evaluate the computational complexity of our proposed method with that of TSM, the Big O Notation has been used. Big O Notation has been widely used for evaluating the computational complexity of an algorithm. The worst case scenario is specifically defined by Big O. It can be used to evaluate the execution time required by an algorithm. Table III shows the computational complexity of CSP, TSM, and the proposed CSP-TSM framework. It can be noted that the computational complexity of the TSM method will increase as a cubic function with respect to the number of channels of EEG signal used. On the other hand, for the CSP and proposed CSP-TSM method, the computational complexity would only increase linearly. Thus, it is evident that the proposed CSP-TSM method is computationally less expensive compared to the TSM method. We did not take into account the training

TABLE III
COMPLEXITY OF TSM AND CSP-TSM METHODS (WHERE n IS THE NUMBER OF CHANNELS).

Method	Complexity
CSP	$O(n)$
TSM	$O(n^3)$
CSP-TSM	$O(n)$

phase for evaluating the computational complexity of the methods as it can be performed offline.

Moreover, the state-of-the-art methods such as FBCSP, DFBCSP, SFBCSP and SBLFB utilize multiple frequency bands, whereas our method uses only a single frequency band for achieving optimal results. The use of a single frequency band compared to using multiple frequency bands makes our system computationally less expensive.

V. CONCLUSION

This paper presents a new framework for MI classification by utilizing two types of features; the variance based CSP features and the TSM features of the spatially filtered data. The proposed method selects the most significant features from these two types of features. LDA is applied to the selected features, which is then classified using the trained SVM classifier. The proposed method has enhanced the classification accuracy of BCI competition III dataset IVa and BCI competition IV dataset I, and has outperformed all other competing methods. We have also shown that the CSP-TSM framework enhances the performance of TSM method while also reducing the computational complexity of the TSM method. Future research should aim to test the proposed framework on a larger set of experimental data. Use of multiple bands with appropriate sub-bands such as that of FBCSP, DFBCSP, SFBCSP and SBLFB might further improve the performance of the proposed CSP-TSM framework. As future works, we will also investigate the use of weighted voting scheme for MI EEG signal classification using features from multiple filter banks.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest.

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