Hedge Fund Style Analysis with the Gap Statistic

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Abstract

We propose a new approach to investment style analysis by classifying the hedge fund universe with the Tibshirani, Walther and Hastie (2001) Gap Statistic. This study finds the statistical presence of only three broad hedge fund investment styles for the period 1994 to 2001. The investment styles can be best described as: quasi-long-equity; non-directional; and, global-directional. We validate the findings of the Gap Statistic by passively replicating the systematic returns of these three investment styles with traditional asset classes.

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1. INTRODUCTION

One of the fastest areas of growth in funds management is the global hedge fund industry where the U.S. Securities and Exchange Commission (2003) estimate its size at approximately $600 to $650 billion in funds under management in the United States alone.[1] These specialist fund managers provide a wide range of investment styles and strategies to investors, or do they? The analysis of hedge fund investment style is an emerging issue given its importance to a plan sponsor's asset allocation process. The objective of this study is to ask the fundamental question of, how many different hedge fund investment styles are truly available to investors? This paper analyzes returns with a new classification technique known as the Gap Statistic and finds the presence of only three broad investment styles in the hedge fund industry. We examine the economic rationale behind these investment styles and reveal the investment philosophies that dominate the hedge fund universe. The three hedge fund investment styles can be best described as (i) quasi-long-equities, (ii) non-directional and (iii) global-directional. Furthermore, we demonstrate that the systematic returns of these three investment styles can be passively replicated with traditional asset classes.

In practice, the classification of hedge fund investment styles are determined by market participants who construct their own peer-group based classification methods. This means that there are neither generally accepted groupings of hedge funds nor methods available to accurately group these fund managers. For instance, Tremont TASS Europe Ltd (hereafter TASS) classify hedge funds into 11 broad categories, the Hennessee Group classify the industry into 23 different investment styles, while Hedge Fund Research, Inc. (HFR) has 30 classifications. Despite market participants developing their own hedge fund style categories, the debate over investment styles widens further when the academic literature is considered. While there are specific investment style studies in the hedge

[1] The United States Securities Exchange Commission (S.E.C.) (2003) expects the hedge fund industry to grow to over $1 trillion in the next five to ten years. As at 31 December 2002, the S.E.C.(2003) valued the U.S. stock market at $11.8 trillion, making the size of the hedge fund industry in the U.S. at approximately 5% of the value of the U.S. stockmarket.

fund literature, there is almost no investment style analysis of the hedge fund universe.[2] The recent work of Brown and Goetzmann (2003) remains the only industry-wide study. They provide evidence of eight hedge fund investment styles. These divergent views on hedge fund investment styles remains an unresolved issue which this study seeks to address.

Style analysis has a long tradition in industry and has been researched in the early performance studies including Carlson (1970), Elton and Gruber (1970, 1971) and McDonald (1974). However, the importance of style analysis was not fully appreciated until the seminal work of Sharpe (1992). The Sharpe (1992) framework employs a constrained OLS regression which relates funds with their factor loadings in order to determine style attributes. The Sharpe (1992) model remains the dominant investment style framework which assumes full knowledge of the risk factors and unconditional linearity in the relationship between portfolio returns and the common style factor loadings.

Although Sharpe (1992) remains the benchmark investment style framework, it is must be operationalised under restrictive conditions which are not accommodative towards hedge fund style analysis. The Sharpe (1992) model assumes full knowledge of the risk factors, returns must exhibit normality and its linear-based framework may not adequately model portfolio dynamics. The Sharpe (1992) model has motivated researchers to develop competing frameworks. Lo (2001) and Fung and Hsieh (2002b) inform us that returns which depart from normality, fund dynamics and returns which are not easily related to independent risk factors are inherent in hedge fund returns. An alternative investment style model which competes with Sharpe (1992) is the principal component analysis (PCA) framework from Fung and Hsieh (1997a, 2000). The unique feature of the PCA framework is the estimation of investment styles without the knowledge of factor loadings. Although PCA is a viable alternative to Sharpe (1992),

[2] Various style-specific studies include Fung and Hsieh (1997a, 2001, 2002a) who examine trend following CTAs, Mitchell and Pulvino (2001) evaluate a hedge fund investment strategy referred to as merger or risk arbitrage, Fung and Hsieh (2002c) and Kao (2002) investigate the fixed income hedge fund style.

studies by Brown (1989) and MacKinlay (1995) demonstrate the inference problems encountered by researchers when identifying the accurate number of principal components.

In another approach, Brown and Goetzmannn (1997, 2003) and Brown, Goetzmann, Hiraki, Otsuki and Shiraishi (2001) employ a *k*-means hard cluster analysis with a generalised least squares procedure in order to adjust for a fund's time-varying variance or heteroskedasticity. The Brown and Goetzmannn (1997, 2003) model (hereafter 'BG') has three characteristics which makes it more conducive for hedge funds than the Sharpe (1992) approach. First, it is able to relax the assumption of normally distributed returns. Second, the estimation of factor loadings (or style attributes) is not required, and, finally, factor loadings can be time-varying.[3] The BG model employs the conventional likelihood ratio (LR) test statistic to estimate the number of investment styles.

A key element in all investment style frameworks is the method by which funds are classified into their respective style attributes. The classification methodology therefore becomes the primary determinant of an investment style analysis. In this study we show that the BG model can be extended with a new classification test statistic which provides new insights in the appropriate grouping of hedge funds. Recently, a new test statistic known as the Tibshirani, Walther and Hastie (2001) Gap Statistic (hereafter 'Gap Statistic') has been developed which is specifically designed to estimate the optimal number of groups in a dataset. Tibshirani *et. al.*, (2001) demonstrate the robustness of the Gap Statistic against other classification statistics and therefore makes it an ideal candidate to address the issues in investment style analysis. This study adds a new

[3] The contribution of the Brown and Goetzmann (1997) model is important as it expresses the classic modern asset pricing model in the form of a modified *k*-means hard cluster analysis procedure. Modern asset pricing models take the mathematical form that returns are equal to a conditional (group) expected return plus an idiosyncratic error. The work of Brown and Goetzmann (1997) employs a standard *k*-means cluster analysis on monthly return data, which inherently has the characteristics and styles of each fund embedded in the monthly returns. The *k*-means cluster analysis classifies data using cross-sectional attributes of the data, and when the procedure is restricted to cluster solely on returns as the attribute, the Brown and Goetzmann (1997) model interprets this *k*-means cluster analysis as grouping return data based on conditional group mean returns, which is consistent with standard modern asset pricing theory. By considering this new mathematical framework, Brown and Goetzmann (1997) have developed an investment style analysis tool which is useful when the risk factors are not fully identified.

dimension to the literature by considering this newly developed classification test statistic.[4]

This paper finds the statistical presence of three broad investment styles in the hedge fund universe over the long-term. We re-classify the individual funds into their respective hedge fund styles and we find that the systematic returns of the three investment style portfolios can be passively replicated with traditional asset classes. The first hedge fund style consists of funds who are effectively long global equities and those who earn the small firm and momentum risk premia. The second investment style consists of funds whose returns are determined by *relative-value* investment decisions. That is, these funds are generally not exposed to directional-risk, but rather, derive their return and risk characteristics from the relative relationship of two assets or more. The third hedge fund investment style consists of fund managers who develop dynamic long/short directional positions across the four global investment classes of bonds, equities, commodities and currencies. Furthermore, we estimate that 42 to 83 percent of the variation of the systematic returns of these three hedge fund investment styles can be passively replicated with traditional asset classes.

This paper makes a number of contributions to the literature. First, we perform an investment style analysis on the hedge fund universe where little research has been undertaken. Second, this study extends the BG model by substituting the LR test statistic with the more effective Gap Statistic. This study is the first to consider the Gap Statistic on a sample of hedge funds. Third, the introduction of the Gap Statistic in the investment style literature provides researchers with a new tool specifically designed to determine the optimal number of groups in a dataset. Fourth, the issue of survivorship is well

[4] Of the three investment style models, the BG model seems most likely to accommodate the Gap Statistic. The BG model employs the conventional Quandt (1960) likelihood ratio (LR) test statistic to estimate the optimal number of investment styles, however, there are limitations with this test statistic. Specifically, the LR test assumes that returns are normally distributed and there is ambiguity in the appropriate degrees of freedom in such a test. The inherent limitations of the LR test motivates this study by extending the BG model with the Gap Statistic in order to more accurately estimate the number of hedge fund investment styles. The only known study to employ the Gap Statistic in a finance setting is Lajbcygier and Ong (2003) who considered the BG model with the Gap Statistic on Japanese mutual funds. This paper is the first to employ this framework to estimate the optimal number of hedge fund investment styles.

documented in the performance literature, however, it has been neglected in the area of investment style analysis. This study reports the impact of non-survivors on an investment style analysis for the first time. Fifth, the modelling of trend following CTA returns has been problematic in the literature due to the dynamic nature of their long/short directional positions. Studies such as Fung and Hsieh (2001) have proposed lookback option strategies to model the dynamics of these fund managers. This paper contributes to the literature by proposing a naïve moving average investment strategy which explains the dynamic investment behaviour of these fund mangers. We demonstrate its explanatory power and ease of use across equities, bonds, currencies and commodities without having to resort to an option-based framework. Finally, an important aspect of this study demonstrates that the systematic returns of these three hedge fund investment styles can be passively replicated with traditional asset classes.

The format of the paper is as follows. Section 2 describes the data and methodology, Section 3 provides the results of the Gap Statistic, Section 4 validates the Gap Statistic investment styles, Section 5 outlines the passive replication of these styles, while Section 6 provides a summary of the conclusions.

2. DATA AND METHODOLOGY

This study employs the TASS dataset of individual hedge fund survivors and non-survivors for the period January 1994 to August 2001, consisting of ninety-two monthly observations. This paper follows Fung and Hsieh (2000) and Liang (2000, 2001) by analysing hedge fund returns from January 1994 onwards in order to incorporate hedge fund survivorship bias.[5] A total of 3,012 hedge funds were available for analysis consisting of 1,836 survivors and 1,176 non-survivors. Within the cohort of survivors, 371 hedge funds possessed a complete performance history spanning the full ninety-two month sample.

The TASS dataset is similar to other fund databases whereby fund performance histories commence and cease at various points in time. This feature of the dataset reflects the normal life cycle of funds in the mutual and hedge fund industries. The issue of hedge fund survivors and non-survivors entering and exiting the dataset is not a trivial matter as it may cause serious problems and biases when employing various methods of analysis.[6] Addressing these data and computational issues are important considerations. Some researchers elect to make assumptions and construct hypothetical return data where there are no available performance figures. The decision to create synthetic data for these funds would require the modeling of the source and the shape of hedge funds returns to various risk factors and asset classes (such as stockmarket and bond proxies). The work of Lo (2001) and Fung and Hsieh (2002b) suggest that the use of standard econometrics to model hedge fund returns may be regarded as dubious and controversial, at best. If at all, the introduction of synthetic returns into the dataset would itself impose ‘survivability’ into the data, which therefore would create a new form of bias. This study actively confronts these data issues by making no assumptions or modifications to the original data.

[5] See Liang (2000) and Fung and Hsieh (2000) for comprehensive reviews on why hedge funds cease to report.

[6] An example of this is conventional matrix algebra which relies on a complete dataset to be available in order for the computation to be performed.

The BG Generalised Style Classification model is employed in this study as we are interested in quantifying the number of hedge fund investment styles without imposing restrictive assumptions.[7] Recall that the BG model is a *k*-means hard cluster analysis which clusters on monthly returns and has been modified as a generalised least squares (GLS) procedure in order to account for time-varying and fund-specific residual return variance. The GLS procedure accounts for heteroskedasticity by scaling the data observations by the inverse of the estimated standard deviation. The GLS methodology also reduces the impact that outliers may have on the classification algorithm thereby improving the results of the cluster analysis. The original BG model employs the LR test as the important test statistic which estimates the number of hedge fund styles. In this study, we estimate the number of groups with the BG model, however, we substitute the LR test with the Gap Statistic.[8]

The Gap Statistic's method in estimating the optimal number of groups in a dataset has intuitive appeal. While the euclidean distance in a conventional cluster analysis measures the distances of each observation to its cluster mean, the Gap Statistic differs as it measures the distance between each individual observation within each cluster. This calculation is referred to as the pooled within-cluster sum of squares known as W for k groups, resulting in $\log(W_k)$. The Gap Statistic then mathematically compares $\log(W_k)$ against its expectation under a null reference distribution of the data. The expectation under a null reference distribution is calculated by generating B reference samples from the original data and estimating the pooled within-cluster sum of squares of each sample W^*_{kb}, and calculating its expected value $(1/B)\sum\log(W^*_{kb})$. The Gap Statistic therefore becomes $Gap(k) = (1/B)\sum\log(W^*_{kb}) - \log(W_k)$. After an adjustment for sampling and simulation error, s_k, the Gap Statistic estimates the optimal number of

[7] Refer to Brown and Goetzmann (1997) for a full specification of the Generalised Style Classification (GSC) model.

[8] Although Ben-Hur, Elisseeff and Guyon (2002) acknowledge the robustness of the Gap Statistic, they identify its sum squared distance criterion as its weakness as this numerical technique makes it biased towards compact clusters or groups rather than sparse data relationships.

groups in a dataset as $\hat{k}$ = smallest k such that $Gap(k) \geq Gap(k+1) - s_{k+1}$. The Gap Statistic thus estimates the statistical benefit of increasing the number of groups while $Gap(k) < Gap(k+1) - s_{k+1}$. The appendix provides a technical specification of the Gap Statistic.

3. RESULTS

The results are structured to consider the following issues. First, what is the impact of survivorship bias when estimating the number of hedge fund styles? Second, how many hedge fund investment styles are there in the long-term and are there any short-term dynamics? To address these considerations, the estimates of the Gap Statistic are calculated on two subsets of the original data. The first subset of data consists of the full sample of survivors and non-survivors so that the impact of survivorship bias can be quantified. The second subset of data consists of the 371 hedge fund survivors who have a complete performance history over the full sample period. The second subset of data is employed in order to calculate the Gap Statistic over the long-term.

A. Full Sample

Table 1 reports summary statistics on the full sample of survivors and non-survivors.[9] The results provide well-known but nevertheless striking features of hedge fund returns. Table 1 shows that the distributions of hedge fund returns are wide and varied with some funds exhibiting negative skewness and severe excess kurtosis. The Jarque-Bera tests reject the assumption of normality across the majority of the TASS investment style categories. The non-normality of returns is a key feature of hedge funds which is consistent with previous research from Lo (2001) and Lochoff (2002).

[9] Refer to the works of Edwards and Park (1996), Liang (2000), Fung and Hsieh (2000) and Edwards and Caglayan (2001) for detailed analyses of the variety of hedge fund biases being, survivorship, instant history and multiperiod sampling bias.

TABLE 1

Descriptive Statistics (Survivors and Non-survivors)

Categories	Mean	Max.	Min.	Std. Dev.	Skew.	Kurtosis	J- Bera
All Funds (inc FOFs)	0.96	6.63	-5.11	1.87	0.26	4.01	4.898
All Funds (ex FOFs)	1.04	6.84	-5.31	1.92	0.21	4.06	4.959
Convertible Arbitrage	0.99	3.80	-3.90	1.27	-0.93	4.76	25.145**
Dedicated Short Bias	0.40	22.48	-12.10	5.60	0.69	4.51	16.014**
Emerging Markets	0.67	12.09	-21.61	5.03	-0.77	6.02	44.140**
Equity Market Neutral	1.09	3.38	-1.03	0.84	0.01	2.99	0.002
Event Driven	1.06	4.27	-6.90	1.41	-1.81	12.39	388.332**
Fixed Income Arbitrage	0.79	2.67	-6.29	1.10	-3.34	21.02	1415.385**
Fund of Funds (i.e. FOFs)	0.64	5.63	-4.16	1.73	0.34	3.59	3.132
Global Macro	0.75	7.11	-3.79	2.09	1.02	4.35	22.838**
Long/Short Equity Hedge	1.51	11.55	-8.52	3.11	0.25	4.56	10.297**
Managed Futures	0.64	7.79	-4.05	2.43	0.34	2.86	1.884
Other	0.74	6.66	-6.57	2.29	-0.15	3.47	1.186

This table presents the descriptive statistics of hedge fund returns from monthly observations from 1994-2001 and classified in their TASS Tremont investment style classifications. The sample consists of 3,012 funds consisting of 1,836 survivors and 1,176 non-survivors. The final column is the Jarque-Bera hypothesis test statistic for normality. * and ** denote statistical significance at the 5% and 1% levels, respectively.

We proceed to estimate the Gap Statistic on the full sample of survivors and non-survivors. Calculations on this dataset is limited because the Gap Statistic requires a complete data matrix. That is, the Gap Statistic requires a full performance history of each fund in its calculation. To circumvent the issue of funds entering and exiting the database at various time intervals, the length of the tests were restricted to three year time horizons so that both surviving and non-surviving funds that possessed a full three year performance history were included in the calculation.[10] The estimate of the Gap Statistic on the full sample is presented in Table 2. The results show that the Gap Statistic persistently estimates between one and four hedge fund investment styles over the various time periods. Although the results vary for each time period, the Gap Statistic consistently estimates four hedge fund styles or less, which are persistently lower than the previous findings in Brown and Goetzmann (2003).

[10] As the Gap Statistic calculation cannot accept funds with missing observations, this test excludes hedge fund survivors and non-survivors who may have had a partial performance history over the various test periods. While this study attempts to incorporate survivorship effects by including hedge fund non-survivors, a residual level of bias still exists in the calculation.

TABLE 2

Gap Statistic (Survivors and Non-survivors)

		Estimated Number of Groups (Investment Styles)						
Period	Funds	1	2	3	4	5	6	7
1994-1996	665	0%	0%	17.3%	82.7%	0%	0%	0%
1995-1997	827	0%	99.5%	0.5%	0%	0%	0%	0%
1996-1998	963	98.1%	1.9%	0%	0%	0%	0%	0%
1997-1999	1,092	0%	7.1%	0%	92.9%	0%	0%	0%
1998-2000	1,204	0%	0%	90.2%	7.8%	0%	0%	0%
1999-2001	1,296	0%	22.4%	0%	77.6%	0%	0%	0%

This table reports the Gap Statistic estimates on the dataset containing survivors and non-survivors across various three year time periods. The calculations of the Gap Statistic are based on 10 reference sets and 1,000 trials. The dominant Gap Statistic estimate for each time period is highlighted. The second column reports the number of surviving and non-surviving funds available for each Gap Statistic estimate.

While the inclusion of non-survivors in the Gap Statistic minimises the impact of survivorship bias, it also limits the calculation to short time intervals. The Gap Statistic requires a full performance history for each fund and this is problematic considering that funds begin and terminate at various time intervals. In order to calculate the Gap Statistic over the long-term, we consider the 371 survivors that possess a full performance history for the 1994-2001 period.

B. Survivors Only

To estimate the Gap Statistic over the long-term, we must employ funds with a complete performance history, however, this introduces data biases. To calculate the long-term Gap Statistic, there is a tradeoff by introducing survivorship bias into the analysis. However, this long-term Gap Statistic estimate provides investors with pertinent information content regarding the types of ‘surviving’ investment styles available in the hedge fund universe. The summary statistics of the sample of funds with a complete performance history are presented in Table 3.

TABLE 3

Descriptive Statistics (Funds with Complete History)

Categories	Mean	Max.	Min.	Std. Dev.	Skew.	Kurtosis	J-Bera
All Funds (inc FOFs)	0.95	6.12	-5.36	1.88	0.09	3.68	1.917
All Funds (ex FOFs)	1.02	6.53	-5.71	1.96	0.06	3.76	2.268
Convertible Arbitrage	0.80	3.13	-3.89	1.38	-0.80	3.65	11.369**
Dedicated Short Bias	0.25	23.56	-15.02	5.41	0.67	6.03	41.934**
Emerging Markets	0.26	16.02	-27.71	6.28	-0.66	6.38	50.298**
Equity Market Neutral	1.11	4.57	-2.08	1.45	0.07	2.56	0.824
Event Driven	1.06	4.28	-7.27	1.43	-2.07	13.94	524.721**
Fixed Income Arbitrage	0.64	7.99	-3.06	1.24	2.06	16.19	731.607**
Fund of Funds (i.e. FOFs)	0.73	5.46	-4.29	1.73	0.21	3.44	1.441
Global Macro	0.87	9.87	-2.82	2.19	1.28	5.15	42.818**
Long/Short Equity Hedge	1.37	11.70	-8.77	3.16	0.22	4.42	8.464*
Managed Futures	0.80	7.66	-5.86	3.17	0.04	2.42	1.334
Other	0.98	4.32	-4.77	1.46	-1.06	5.74	45.885**

This table presents the descriptive statistics of hedge fund funds from monthly observations from 1994-2001 and classified in their TASS Tremont investment style classifications. The sample consists of 371 funds that possess a complete performance history for the full sample period. The final column is the Jarque-Bera hypothesis test statistic for normality. * and ** denote statistical significance at the 5% and 1% levels, respectively.

As expected, the descriptive statistics of these funds are consistent and similar with the previous results of the full sample. Once again, the skewness, kurtosis and Jarque-Bera statistics of the survivors suggest that a majority of hedge fund returns are not normally distributed. The similarities of Tables 1 and 3 suggest that survivorship bias does not influence the non-normality characteristics of hedge fund returns.

The estimates of the Gap Statistic on the funds with a complete performance history are presented in Table 4. The results show that over short-term time horizons, the Gap Statistic calculates the presence of up to five investment styles. The marginal difference in Table 2 and 4 demonstrates the small upward bias in the estimation of the Gap Statistic when one excludes non-survivors from the analysis.

TABLE 4

Gap Statistic (Funds with Complete History)

		Estimated Number of Groups (Investment Styles)						
Period	Funds	1	2	3	4	5	6	7
1994-1996	371	0%	0%	98.0%	2.0%	0%	0%	0%
1995-1997	371	0%	0%	92.8%	7.2%	0%	0%	0%
1996-1998	371	0%	17.6%	8.8%	29.7%	44.0%	0%	0%
1997-1999	371	0%	91.5%	0%	8.5%	0%	0%	0%
1998-2000	371	0%	0%	16.2%	83.8%	0%	0%	0%
1999-2001	371	0%	0%	99.6%	0.4%	0%	0%	0%
1994-2001	371	0%	0%	84.0%	0.0%	0%	10.9%	5.1%

This table reports the Gap Statistic estimates on the dataset containing all funds with a complete performance history. The calculations of the Gap Statistic are based on 10 reference sets and 1,000 trials. The Gap Statistic was estimated on various three year time periods and also for the full 1994-2001 sample period. The dominant Gap Statistic estimate for each time period is highlighted. The second column reports the number of funds available for each Gap Statistic estimate.

The long-term Gap Statistic for the full 1994-2001 period is reported in the last row of Table 4. The result shows that 84% of the 1,000 Gap Statistic simulations calculated the statistical presence of three hedge fund investment styles over the long-term. This statistical finding of only three hedge fund styles over the long-term is significant as it differs to Brown and Goetzmann's (2003) finding of eight styles and is in stark contrast to current industry practice. It is noteworthy that the Gap Statistic for the full 1994-2001 period finds the presence of six and seven hedge fund investment styles at times, however, these simulations are not statistically significant. These insignificant estimates of six and seven styles partially explain the results from Brown and Goetzmann (2003) who employ the LR test statistic which tends to over-estimate the number of groups, thus causing an upward bias in the number of investment styles.

TABLE 5

Cross-Tabulation of Styles and Correlations

	Total	Style 1	Style 2	Style 3
Panel A: Cross-Tabulation with TASS Styles				
Convertible Arbitrage	13	2	11	0
Dedicated Short Bias	6	0	5	1
Emerging Markets	20	17	3	0
Equity Market Neutral	7	1	6	0
Event Driven	60	11	49	0
Fixed Income Arbitrage	6	0	6	0
Fund of Funds (FOFs)	91	41	36	14
Global Macro	15	3	7	5
Long/Short Equity Hedge	102	76	25	1
Managed Futures	50	9	8	33
Other	1	0	1	0
Total Number of Funds	371	160	157	54
Panel B: Correlation to the S&P500 All Return Index				
All Months – Mean Correl.		0.461	0.156	-0.055
All Months – 95% Percentile		0.655	0.498	0.148
All Months – 5% Percentile		0.254	-0.331	-0.194
Up Months – Mean Correl.		0.233	0.045	-0.020
Up Months – 95% Percentile		0.440	0.317	0.137
Up Months – 5% Percentile		-0.033	-0.234	-0.188
Down Months – Mean Correl.		0.389	0.148	-0.128
Down Months – 95% Percentile		0.632	0.501	0.208
Down Months – 5% Percentile		0.074	-0.369	-0.374
Panel C: Correlation to Lehman Bros. U.S. Gov. Long Term Bond Index				
All Months – Mean Correl.		0.023	0.031	0.201
All Months – 95% Percentile		0.199	0.210	0.348
All Months – 5% Percentile		-0.125	-0.146	-0.003
Up Months – Mean Correl.		-0.044	0.031	0.195
Up Months – 95% Percentile		0.135	0.210	0.436
Up Months – 5% Percentile		-0.224	-0.146	-0.106
Down Months – Mean Correl.		0.196	0.119	0.157
Down Months – 95% Percentile		0.400	0.377	0.333
Down Months – 5% Percentile		-0.051	-0.147	-0.072

Panel A presents the number of funds classified in the three Gap Statistic investment styles versus their original TASS investment style classifications. Panel B reports the distribution of the correlation coefficient between funds and the S&P500 All Return Index while Panel C reports the distribution in the correlation coefficient between funds and the Lehman Brothers U.S. Government Long Term Bond Index. The reported correlation coefficient is the mean, 5^{th} and 95^{th} percentile correlation coefficient of the individual funds against these traditional asset classes for all, up and down months.

C. Fund Behaviour Based on Gap Statistic Style Classifications

Table 5 provides a preliminary examination of fund behaviour based on their long-term Gap Statistic classifications. Panel A of Table 5 cross-tabulates the funds in their original TASS groupings with their associated Gap Statistic style categories. Panel A shows that style 1 is dominated with Equity Long/Short funds which suggests that some equity factor may determine the returns of this investment style. A striking feature of Panel A is that a large proportion of the sample (ie. 160 of the 371 funds) are classified in this investment style. The style 2 category contains a large proportion of relative-value or non-directional style funds.[11] Finally, style 3 is dominated by the funds in the managed futures TASS classification.

Panel B of Table 5 reports the correlation behaviour of the individual funds in the three styles against the stockmarket proxy of the Standard and Poors 500 All Return Index. The funds in style 1 demonstrate a strong and positive correlation to US equities which reflects the dominance of Equity Long/Short funds in this style classification. The funds in style 2 report little or no correlation to US equities which is expected considering that many relative-value or non-directional funds were classified in this group. The funds in style 3 report little or no correlation to US equities, however, the negative correlation coefficients for all, up and down months reveals an interesting dynamic reflecting the dynamic long and short directional investment behaviour of managed futures funds.[12] Panel C of Table 5 is similar to Panel B and reports the correlation of funds to a bond proxy. The funds in the sample are generally not correlated with US government bond returns.

[11] Gatev, Goetzmann and Rouwenhorst (1999), Mitchell and Pulvino (2001), Lo (2001), Kao (2002) and Fung and Hsieh (2002c) inform us that these investment styles possess non-directional characteristics as they tend to hold a combination of long/short positions of similarly related assets. See Shleifer and Vishy (1997) for the theoretical rationale of the inherent risks involved in arbitrage. For an empirical example, refer to Mitchell and Pulvino (2001) who examine the return and risks in merger arbitrage.

[12] This interesting relationship with the S&P500 is consistent with similar characteristics in trend followers found in Edwards and Liew (1999) and Fung and Hsieh (1997a, 2002a). Elton, Gruber and Rentzler (1987) and Fung and Hsieh (1997b) inform us that these fund managers employ trend-following or momentum based strategies which are directional in nature and tend to exhibit reasonable volatility in returns.

Overall, Table 5 shows that funds in style 1 are dominated by Equity Long/Short funds which possess a positive association with equities. The funds in style 2 are relative-value style funds which tend to exhibit little or no association with stocks and bonds. Finally, the funds in style 3 also exhibit little or no correlation with stocks and bonds, however, they possess a small but negative association with US stock returns which will be explored further in the proceeding sections.

4. VALIDATION OF GAP STATISTIC HEDGE FUND STYLES

The economic rationale of the three Gap Statistic categories must be examined to ensure that these investment styles are meaningful to investors. To consider these categories in a conventional finance framework, three style index portfolio returns are constructed. We eliminate the idiosyncratic risk of individual hedge funds by forming the systematic returns of these three investment styles. The equal weighted returns of each classification group is calculated to form the three investment style portfolio returns. These three investment style index portfolios form the basis of much of the analysis for the remainder of the paper. The three style portfolio indices are presented in Figure 1 and their summary statistics are reported in Table 6.

Figure 1. Performance of the Gap Statistic Style Indexes, 1994-2001.

(Initial Wealth 1/1994 = $1,000)

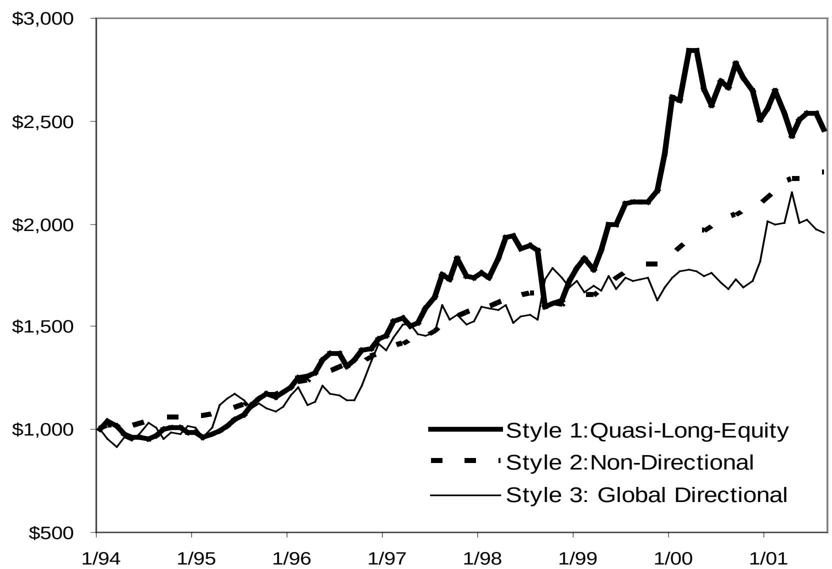

TABLE 6

Summary Statistics of Style Index Portfolio Returns

Style	Mean	Max.	Min.	S.D.	Skew.	Kurt.	Jarque Bera	Sharpe	First Auto.	95% VaR	95% CVaR
1	1.03	11.70	-14.47	3.70	-0.483	5.598	29.45**	0.585	0.144	-4.40	-7.25
2	0.90	2.41	-2.32	0.71	-0.984	6.191	53.88**	2.372	0.395**	-0.25	-0.82
3	0.85	12.82	-7.45	4.02	0.534	3.291	4.70	0.378	-0.026	-4.97	-6.27

This table presents the summary statistics of the style index portfolio returns. The first column indicates the style index portfolio. The second to the sixth columns are the four moments of the distribution of monthly returns. Jarque-Bera denotes the hypothesis test statistic for normality. The column heading Sharpe is the annualised Sharpe Ratio of the monthly returns. The column heading First Auto. reports the first-order autocorrelation. The 95% VaR denotes the historical 95% Value-at-Risk of the monthly returns. The 95% CVaR reports the historical 95% Conditional Value-at-Risk of the monthly returns. ** denotes statistical significance at the 1% level.

Table 6 shows that the portfolio returns of styles 1 and 3 have considerable variance in comparison to their mean. Styles 1 and 2 possess negative third moments in their return distributions which differs to style 3. The Jarque-Bera test for normality is rejected for style 1 and 2 due to the negative third moments and significant excess kurtosis. Style 2 generates 90 percent of the returns of style 1 with only a fraction of the volatility. This translates to an attractive sharpe ratio for the style 2 index portfolio in comparison to styles 1 and 3. Table 6 also reports the historical 95% VaR and 95% CVaR of style 2 at only a fraction of those of the style 1 and 3 portfolios.[13] The impressive statistics for style 2 must be tempered with the inherent risks in hedge fund returns which are located in the third and fourth moments of the return distributions.[14] Another striking feature of the style 2 index portfolio is the strong first-order autocorrelation of returns relative to the other style portfolios. Conversely, the style 3 portfolio exhibits a positive third moment and a relatively normal distribution of returns. Overall, the summary statistics in Table 6 provides evidence to suggest that the systematic returns of these three style portfolios are indeed different.

[13] The skewed distributions of the three style index portfolio returns makes the CVaR more conducive than VaR as a risk metric.

[14] Refer to Geman and Kharoubi (2003) for a discussion of hedge fund risks and the third and fourth moments of the return distribution.

A. Evaluation of Funds and Associated Styles

As a form of validation, we examine the individual funds in each style to ensure that they have been classified in a way which is consistent with modern portfolio theory. We calculate the correlation coefficient of each fund with their respective style index portfolio return. We would expect a high correlation coefficient between the funds and their respective style index returns. Funds with a low correlation with its style portfolio returns would indicate that they may have been misclassified by the Gap Statistic. Table 7 reports the distribution of correlation coefficients between the funds and their respective style index portfolio returns.

TABLE 7
Distribution of Correlations between Funds and Style Portfolio

Style	Min.	5^{th}	10^{th}	25^{th}	50^{th}	75^{th}	90^{th}	95^{th}	Max.
1	0.280	0.396	0.456	0.552	0.670	0.777	0.836	0.854	0.918
2	-0.257	-0.078^	0.060	0.219	0.382	0.519	0.647	0.686	0.787
3	0.269	0.304	0.364	0.554	0.712	0.799	0.878	0.893	0.905

This table reports the percentile distribution of correlation coefficients between the fund returns with their associated style index returns across various percentile intervals. ^ denotes that eleven funds in style 2 at or below the sixth percentile exhibit correlations of less than zero to their respective style index returns.

Table 7 shows that the correlations in styles 1 and 3 are very high. Table 7 also reports a less pronounced correlation of funds in style 2 which is a surprising finding. The funds in style 2 exhibit a lower correlation to their respective style index across all reported percentiles in comparison to funds in styles 1 and 3. Second, there are 11 (ie. 7 percent) out of the 157 funds in style 2 that possess a negative correlation with their style 2 index portfolio, although they are located at the sixth percentile and lower in the distribution.[15] The effect of a broad-based estimate of three hedge fund styles must result in a small number of funds with a negative correlation (ie. 2.96% of the entire sample). One would expect to find a small number of funds that may not fit these three broad investment styles. Overall, Table 7 illustrates the merit of the Gap Statistic's precision in accurately classifying these funds into three investment styles.

[15] These 11 funds have the following TASS Tremont investment styles: 4 fund of funds, 3 long-short equity hedge, 2 event-driven, 1 convertible arbitrage and 1 dedicated short bias. The Gap Statistic has determined that it is not statistically viable to increase the number of investment styles to accommodate these 11 funds which represent 2.96% of the 371 fund sample.

B. Correlation Between Style Index Portfolio Returns

To further examine the validation of the three investment style portfolios, Table 8 presents the correlations and equality tests of the three style index portfolio returns. A perfect classification scheme would form three style portfolio returns that are uncorrelated. Table 8 reports weak correlation between the style index portfolio with the exception of style 1 and 2. Despite the high correlation between style 1 and 2, the equality test in Panel C indicates that style 1 and 2 exhibit statistically significant variances. Overall, the statistically significant correlation between styles 1 and 2 is a surprising result which contradicts the findings in Tables 6 and 7. Although correlation measures the association between returns, it does not consider causation which is examined in the following section.

TABLE 8

Style Portfolio Correlations and Equality Tests

Panel A: Pearson Correlations				
ρ (Pearson)		Style 1	Style 2	Style 3
	Style 1	1		
	Style 2	0.604**	1	
	Style 3	-0.056	0.193	1
Panel B: Spearman Rank Correlations				
ρ (Spearman)		Style 1	Style 2	Style 3
	Style 1	1		
	Style 2	0.524**	1	
	Style 3	0.078	0.345**	1
Panel C: Equality Tests				
	Styles	Mean	Median	Variance
	1-2	0.858	0.685	0.000**
	1-3	0.730	0.419	0.555
	2-3	0.766	0.389	0.000**

Panel A presents the Pearson correlation coefficients of the returns of the three Gap Statistic style portfolios. Panel B provides the Spearman rank-correlation coefficients. The test performed on Panel A and B is the two-tail hypothesis test of the correlation is not zero. Bonferroni corrected critical values were employed in order to mitigate the effect of a Type I error when testing across three investment styles. Panel C reports the p-values of the equality test in means, median and variance of the style index portfolio returns. The p-values reported for the mean difference is the parametric two-tailed test for equality of means. The p-values reported for the median difference is the nonparametric Wilcoxon Signed Rank two-tailed test for equality of medians. The p-values reported for the variance difference is the two-tailed F-test for equality of variances. * and ** denote statistical significance at the 95% and 99% confidence levels.

TABLE 9

Regression: Funds on Three-Factor Style Index Model

Dependent Variables			Independent Variables							
Funds	No.	Mean Intercept	Mean Style 1 Coeff.	Mean Style 2 Coeff.	Mean Style 3 Coeff.	Sig./Insig. Intercept	Sig/Insig. Style 1 Coeff.	Sig/Insig. Style 2 Coeff.	Sig/Insig. Style 3 Coeff.	Mean Adj. R^2
Panel A: Gap Statistic Styles										
Style 1	160	-0.14%	1.02	-0.01	-0.00	13/147	159/1	74/86	35/125	0.78
Style 2	157	-0.03%	0.00	1.00	0.00	15/142	81/76	153/4	57/100	0.90
Style 3	54	-0.17%	0.01	0.00	0.99	3/51	9/45	15/39	54/0	0.76
All Funds	371	-0.10%	0.44	0.42	0.14	31/340	249/122	242/129	146/225	0.83
Panel B: TASS Styles										
Conv.Arb	13	-1.67%	0.20	0.81	-0.04	5/8	6/7	12/1	4/9	0.93
Ded. Sh.Bias	6	1.94%	-1.33	2.16	0.23	0/6	6/0	6/0	1/5	0.74
Em. Mkts.	20	2.40%	1.39	-0.09	-0.21	1/19	18/2	9/11	6/14	0.69
Eq. Mkt. Ntrl.	7	0.70%	0.07	0.96	-0.02	0/7	4/3	6/1	0/7	0.84
Ev. Driven	60	0.28%	0.22	0.87	-0.09	3/57	31/29	55/5	28/32	0.92
F.I.Arb	6	-0.82%	-0.11	1.13	-0.04	2/4	4/2	6/0	2/4	0.89
FOFs	91	-0.40%	0.34	0.47	0.18	11/80	66/25	67/24	46/45	0.91
G.Macro	15	-0.52%	0.16	0.48	0.35	0/15	5/10	9/6	9/6	0.81
LSEQ	102	-0.32%	0.79	0.17	0.02	8/94	88/14	51/51	13/89	0.76
MFutures	50	-0.31%	0.27	0.00	0.73	1/49	20/30	20/30	37/13	0.74
Other	1	1.12%	-0.18	1.25	-0.05	0/1	1/0	1/0	0/1	0.96

This table presents the summary of various OLS multi-factor regressions. The dependent variable in these regressions are the excess returns of each fund as classified in their respective investment styles. The three independent variables are the time series of monthly excess returns associated with (1) style 1 index portfolio, (2) style 2 index portfolio, and (3) the style 3 index portfolio. The mean value of the intercept and the coefficients of all regressions are reported. The second column denotes the number of funds in each category. The columns with the headings Sig./Insig. denote the number of funds with a statistically significant or insignificant (at the 5% level) regression coefficient for each variable. The mean adjusted coefficient of determination of all regressions is reported in the final column of this table.

C. Explanatory Power of Style Index Portfolio Returns

We examine the explanatory power of the systematic returns of the style index portfolios and the way they jointly describe the returns of the hedge fund universe. One would expect that the systematic returns of these three investment styles would capture a large proportion of the cross-sectional variation in hedge fund returns. We regress the returns of individual funds against the systematic returns of the style index portfolios to form the following three-factor model.

$$R_i - R_F = \alpha_i + \beta_{i1}(R_1 - R_F) + \beta_{i2}(R_2 - R_F) + \beta_{i3}(R_3 - R_F) + e_i$$

In this model, R_i is the return on the individual hedge fund, R_F is the risk-free rate of return derived from the 1 month T-Bill rate from Ibbotson and Associates, R_1 is the style 1 portfolio return, R_2 is the style 2 portfolio return, R_3 is the style 3 portfolio return, β_i is the sensitivity of fund i to the relevant style index, and e_i is the residual risk. To address the multicollinearity concerns from the high correlation between style 1 and 2, the style 2 index returns in his model are made orthogonal to the style 1 index returns.[16] The orthogonalization captures the marginal effect of style 2 over and above the factor loading of the style 1 portfolio return.

Panel A of Table 9 demonstrates that the three-factor style model has high explanatory power for funds when classified under the Gap Statistic. Panel B of Table 9 demonstrates that the three Gap Statistic investment style portfolio returns have high explanatory power when funds are grouped in their original TASS classifications, however, the statistically significant beta coefficients are more dispersed and are not concentrated as in Panel A. Overall, these findings suggest that the systematic returns of the three Gap Statistic investment styles captures the cross-sectional variation of fund returns in the hedge fund universe. These findings provide the motivation to consider the opportunity of passively replicating these systematic returns which is examined in a subsequent section of this study.

[16] Refer to Elton, Gruber, Das and Hlavka (1993) and Chapman and Pearson (2000) for excellent treatments of orthogonalization in finance settings.

D. Evaluation of Style Consistency

Although the three-factor style model exhibits strong explanatory power, we need to consider the consistency of these three investment styles. Table 10 reports the mean R^2_{adj} values of the fund regressions and their associated style index portfolio return segregated across various time horizons. The results show that the style portfolio returns can effectively explain the cross-sectional variance of the respective individual funds over short-term periods and for the full 1994-2001 period also.

TABLE 10

Cross-Sectional Variance

	Mean Adj. R Squared		
Time Period	Style 1	Style 2	Style 3
1994-1996	0.498	0.639	0.582
1995-1997	0.510	0.797	0.622
1996-1998	0.519	0.794	0.694
1997-1999	0.489	0.810	0.742
1998-2000	0.618	0.765	0.794
1999-2001	0.619	0.668	0.798
1994-2001	0.565	0.741	0.707

This table reports the mean adjusted coefficient of determination of the individual regressions of funds against their respective style index. The time periods are the same as those employed to calculate the Gap Statistic. The last row of this Table reports the cross sectional variance for the full 1994-2001 sample period.

As a second test of style consistency, we follow Gallo and Lockwood (1999) and employ the Chow (1960) test to detect investment style drift between funds and their associated investment styles. The Chow (1960) test examines the stability of regression coefficients over two distinct time periods. Funds with statistically significant p-values under a Chow (1960) test possess regression coefficients that are statistically different, which suggests the evidence of changing investment style with respect to its associated style portfolio returns. The Chow (1960) test is estimated under a single-factor model. We perform the Chow (1960) tests on the funds in their Gap Statistic classification scheme in Panel A of Table 11 versus their original TASS style classifications in Panel B.

Panel A of Table 11 reports that 10.78 per cent of funds were found to exhibit style inconsistency when classified in the three Gap Statistic classifications. As a comparison, Panel B reports 10.24 per cent of funds to exhibit style inconsistency when regressed against their TASS peer-group equal weighted portfolio returns. The results in Table 11 provide compelling evidence that little or no loss of style consistency exists when re-classifying the hedge fund universe from eleven TASS styles to three Gap Statistic styles.[17]

TABLE 11

Chow Test

			Chow Test		
Funds	No.	Independent Variable	No. Insignif.	No. Signif.	%
Panel A:					
Style 1	160	Style 1 Portfolio	141	19	
Style 2	157	Style 2 Portfolio	142	15	
Style 3	54	Style 3 Portfolio	48	6	
			331	40	10.78%
Panel B:					
Convertible Arbitrage	13	EW. Conv. Arb.	12	1	
Dedicated Short Bias	6	EW. Ded. Sh. Bias	5	1	
Emerging Markets	20	EW. Em. Mkts	20	0	
Equity Market Neutral	7	EW. Eq. Mkt. Ntrl.	6	1	
Event Driven	60	EW. Ev. Driven	55	5	
Fixed Income Arbitrage	6	EW. F.I. Arb.	5	1	
Fund of Funds	91	EW. FOFs	74	17	
Global Macro	15	EW. G. Macro	14	1	
Long/Short Equity Hedge	102	EW. LS. Eq. Hdg.	93	9	
Managed Futures	50	EW. M.Fut.	48	2	
Other	1	EW. Other	1	0	
			333	38	10.24%

This table reports the Chow (1960) breakpoint test as a form of style consistency test. The test was performed by dividing the sample into two periods of 46 months. EW denotes the equal weighted portfolio of funds within each TASS style classification. The penultimate and final columns report the number of funds with statistically insignificant vs. significant p-values at the 5% level, respectively.

[17] We also interpret this as evidence to indicate that hedge funds, as a group, generally do not attempt to engage in strategic investment style drift.

5. PASSIVE REPLICATION OF STYLE PORTFOLIO RETURNS

The final analysis of this study is the passive replication of these three investment styles with traditional asset classes. If these three investment style portfolios can be replicated then this must surely provide a meaningful validation of their very existence. To passively replicate these three styles, we exploit the previous findings in Agarwal and Naik (2004), Capocci and Hubner (2004) and Fung and Hsieh (2004) and the information content in Tables 5 and 6. The systematic returns of style 1 tends to be positively correlated with equities, style 2 exhibits low volatility and tends to be exposed to relative risks, and style 3 may possess directional exposures across global investments. We employ a parsimonious approach to the following sections which report the significant risk factors that capture the variation of returns of these three style portfolio returns.

A. Style 1: Quasi-Long-Equity

The correlations and style classifications in Table 5 suggest that these funds are strongly associated with equities. We construct a three-factor style model based on equity risk factors in the following form.

$$R_t - R_{Ft} = \alpha + \beta_1 (MACWEI_t - R_{Ft}) + \beta_2 SMB_t + \beta_3 UMD_t + e_t$$

where R_t is the style 1 portfolio return, R_{Ft} is the risk-free 1 month T-Bill return earned at month t, MACWEI is the return on the MSCI All Country World Equity Index, SMB is the Fama and French (1993) value-weighted zero investment mimicking portfolios for size, and UMD is the Fama-French based Carhart (1997) value weighted, zero investment, factor mimicking portfolio for one-year return momentum.[18]

[18] The Fama-French (1993) HML mimicking portfolio for book-to-market equity (‘high minus low’) was found to be statistically insignificant.

Table 12

Passive Replication of Style Portfolios

	Intercept	MACWEI	SMB	UMD	FFSVMLG	LHYMWA	VIX	DJAIG	CWBI	SP	DX	Adj R-Sq
Panel A: Style 1 three-factor model												
Style 1	0.29%	0.73**	0.39**	0.11**								0.84
	(0.00)	(0.04)	(0.04)	(0.03)								
Panel B: Style 2 five-factor model												
Style 2	0.41%**	0.09**			0.06**	0.10**	0.01**	0.08**				0.61
	(0.00)	(0.01)			(0.01)	(0.03)	(0.00)	(0.01)				
Panel C: Style 3 four-factor model												
Style 3	0.22%							0.31**	1.68**	0.21**	0.30**	0.40
	(0.00)							(0.90)	(0.38)	(0.06)	(0.08)	
Panel D: Combined model												
Style 1	0.17%	0.67**	0.26**	0.11**	0.05	0.47**	0.00	0.14**	-0.01	0.06	-0.05	0.91
	(0.00)	(0.04)	(0.04)	(0.02)	(0.04)	(0.06)	(0.01)	(0.04)	(0.18)	(0.03)	(0.03)	
Style 2	0.38**	0.10**	-0.01	0.02	0.06**	0.11**	0.01*	0.08**	0.07	-0.00	0.00	0.61
	(0.00)	(0.02)	(0.01)	(0.01)	(0.01)	(0.03)	(0.00)	(0.01)	(0.06)	(0.01)	(0.01)	
Style 3	0.23%	-0.14	0.08	0.01	-0.09	0.15	-0.00	0.30**	1.79**	0.23**	0.30**	0.37
	(0.00)	(0.14)	(0.11)	(0.04)	(0.08)	(0.15)	(0.03)	(0.09)	(0.38)	(0.07)	(0.10)	

This table presents the risk factors which are found to capture the variation of returns of the three Gap Statistic style portfolio returns. Panel A is the parsimonious three-factor model of the style 1 portfolio. Panel B describes the parsimonious five-factor model for the style 2 portfolio. Panel C is the parsimonious four-factor model for the style 3 portfolio. Panel D reports the regression output of the combined risk factors for each style portfolio. The independent risk factors are the excess return on the MSCI All Country World Equity index (MACWEI), Fama-French small-minis big portfolio (SMB) and Fama-French Carhart (1997) 12 month return momentum risk factor (UMD), Fama-French small high book-to-market minus large cap low book-to-market portfolio (FFSVMLG), excess returns of the Lehman Global High Yield index minus the excess returns of the Lehman Global Aggregate Index (LHYMWA), log change in the VIX volatility index (VIX), the excess returns of the Dow Jones AIG Commodity Total Return index (DJAIG), the excess return from the daily 50 period moving average investment strategy in the Citigroup World Bond index (CWBI), S&P500 All Return index (SP) and the US Dollar index (DX). Newey-West (1987) heteroscedasticity and autocorrelation adjusted standard errors are in parentheses. The final column reports the adjusted r-square for each regression. * and ** denote statistical significance at the 5% and 1% levels, respectively.

The regression results in Panel A of Table 12 demonstrate that the style 1 portfolio returns can be replicated with the excess returns from the MSCI World Equity Index (MACWEI) and the Fama-French (1993, 1996) small firm (SMB) and momentum (UMD) zero-cost portfolios. In particular, the excess returns from the MACWEI exhibits a very high factor loading. In tests not reported here, the MACWEI risk factor as a single-factor model explains 65 per cent of the variation in returns of style 1. The R^2_{adj} of 0.84 reported in Panel A demonstrates the high explanatory power of this equity based style model and suggests that nearly half of the hedge funds in this sample can be explained by these common equity risk factors. This strong and positive relationship with stocks is consistent with similar findings from Asness, Krail and Liew (2001), Fung and Hsieh (2002a, 2004) and Agarwal and Naik (2004). A closer inspection of the mean and median management fees of funds classified in this style are 1.16 and 1.00 per cent, while the mean and median incentive fees are 12.81 and 15.00 per cent, respectively.[19] Plan sponsors allocating capital to this hedge fund style must consider whether the fees charged by these funds can be justified, given that a similar exposure can be passively replicated with well known equity risk factors. Due to the strong explanatory power of the equity risk factors, the style 1 portfolio can be referred to as 'Quasi-Long-Equity'.

B. Style 2: Non-Directional

To examine the non-directional nature of Style 2, we develop risk factors which are based on relative-risks. We construct the following five-factor model.

$$R_t - R_{Ft} = \alpha + \beta_1(MACWEI_t - R_{Ft}) + \beta_2(FFSVMLG_t - R_{Ft}) + \beta_3(LHYMWA_t - R_{Ft}) + \beta_4 \ln chgVIX_t + \beta_5(DJAIGTR_t - R_{Ft}) + e_t$$

Panel B of Table 12 reports the regression results which reveal significant factor loadings (at the 1 per cent level) towards a relative long position in the Fama-French small-cap/high book-to-market ratio portfolio and a short portfolio position in the large cap/low book-to-market ratio portfolio. These factor loadings reflect the

[19] Only 31 funds or 8.4% of the 371 hedge fund survivors disclosed a high water mark incentive fee structure.

underlying relative-risk exposure in equities which is consistent with similar findings in Capocci and Hubner (2004). Another relative-risk factor is the positive coefficient in the Lehman Global High Yield Index and the negative coefficient reported in the Lehman Global Treasury index (at the 1 per cent level). The style index captures the credit risk premium in global bonds. This finding is also consistent with similar results found in Capocci and Hubner (2004) and Fung and Hsieh (2002c).

Panel B of Table 12 also finds that the Dow Jones AIG Commodities index, the log change in the VIX volatility index and global equity risk factors are also significant. Although the systematic returns of this style index portfolio exhibits low volatility, there seems to be a small degree of directional-risk exposure inherent in this investment style. Overall, these risk factors estimate a R^2_{adj} of 0.61 which provides reasonable explanatory power. The statistically significant intercept term in this regression suggests that hedge funds in this style are able to earn high levels of alpha or there is an omitted explanatory variable which has not yet been identified. The latter of these two views seems most likely. This remains an area for future research.

To address the strong correlations in Table 8, the style 2 portfolio returns were regressed against the risk factors identified for style 1 with a reported R^2_{adj} of 0.18. This test demonstrates that the systematic returns of style 2 are less reliant on common equity risk factors in comparison to style 1. Given the explanatory power of the relative-based risk factors, the style 2 portfolio is referred to as 'Non-Directional'.

C. Style 3: Directional

This investment style has been problematic for researchers in the past due to the dynamic long/short investment behaviour of trend following managed futures funds. Fung and Hsieh (2001, 2004) have recently overcome these issues by replicating these fund managers with option-based lookback straddle strategies. We take another approach to this problem by capturing the dynamic nature of these fund managers in a more simplified framework. We mimic the long/short dynamics of managed futures funds by introducing a simple moving average investment strategy across the major investment classes. We construct a four-factor style model which represents the risk factors of the four broad investment classes in the following form.

$$R_t - R_{Ft} = \alpha \;\; + \beta_1 (CWBI50p_t - R_{Ft}) + \beta_2 (SP50p_t - R_{Ft}) + \beta_3 (DX50p_t - R_{Ft})$$
$$+ \beta_4 (DJAIG_t - R_{Ft}) + e_t$$

where R_t is the style 3 portfolio return, CWBI is the mimicking portfolios of the Citigroup world bond index, SP is the S&P500 All Return Index, DX is the US Dollar Index and DJAIG is the Dow Jones AIG Commodity Index. To mimic the long/short dynamics, the 50 period moving average of the index values were constructed from *daily* data on the CBWI, SP and DX risk factors. Long positions were simulated when the index was above its moving average and short positions when the index value was below their moving average. The *daily* returns were then consolidated into *monthly* returns and then employed as the independent risk factors in the above OLS estimation.

The regression results in Panel C of Table 12 demonstrate that these four risk factors explain 40 per cent of the variation of returns of style 3.[20] Other results not reported shows that statistically similar results are achievable with moving averages ranging from 20 to 100 periods, thus demonstrating the robustness of this approach. This simple framework contributes to the literature by capturing the cross-variation of returns of managed futures funds without having to resort to option-based strategies.

[20] In a static traded (ie. low turnover) portfolio, the alpha and beta regression coefficients describe the behaviour of the underlying assets in the investment portfolio. In a dynamic investment portfolio, the alpha and beta coefficients describe the dynamics of the fund manager and not the underlying securities or assets in the portfolio.

6. CONCLUSION

The growing interest in hedge funds makes investment style analysis an important area of research for practitioners, academics and regulators. This paper contributes to a new dimension to the hedge fund investment style debate. This paper introduces a new approach to hedge fund style analysis by employing the recently developed Gap Statistic. The statistical evidence suggests that the hedge fund universe is broadly dominated by three investment styles. To evaluate the Gap Statistic classification, various tests were performed on the systematic returns of these investment styles to ensure that they were economically meaningful to investors. As a further method of validation, evidence exists that the systematic returns of these three investment styles can be passively replicated with traditional asset classes. We show that the first investment style can be readily replicated with long global equities exposure and the small firm and momentum risk premia. The second hedge fund investment style can be passively replicated with relative-risk exposures across various asset classes. Finally, the third investment style can be mimicked with a simple moving average investment strategy across global investment classes.

Our findings have several implications for the hedge fund industry. First, the statistical presence of three investment styles provides a deeper understanding of the common risk factors that determine hedge fund returns. Second, the results demonstrate that underneath the idiosyncratic layer of hedge fund returns, common risk factors can be identified by re-classifying funds with the Brown and Goetzmann (1997, 2003) model and the Gap Statistic. The ability to passively replicate the hedge fund investment styles brings into question the justification of high management fees. The consideration for future research lies in the mimicking of these three investment styles with traditional asset classes in a performance evaluation framework. We posit that plan sponsors must carefully consider whether the hedge fund returns in their investment portfolios can be passively replicated at low cost.

TECHNICAL APPENDIX

The Gap Statistic can be simply described as a test statistic derived from a bootstrapped grouping procedure. The Gap Statistic effectively measures the most probable within sum-of-square distances from a set of Monte Carlo samples which are derived from the original dataset. After the adjustment for the simulation and estimation error, the optimal number of clusters or groups is calculated.[21] The estimation of the Gap Statistic is operationalised by employing the following procedure;

Step 1: Group the observed data and vary the number of categories from k = 1, 2,..., K, thus generating the within-dispersion measures W_k, where k = 1, 2, …, K.[22]

Step 2: Generate B reference data sets, using either the uniform distribution or the singular variance decomposition (SVD) method, and then cluster each one giving within-dispersion measures W^*_{kb}, where b = 1, 2, …, B, and k = 1, 2, …, K. The Gap Statistic can be estimated as $Gap_n(k) = E^*_n\{\log(W_k)\} - \log(W_k)$ where E^*_n is the expectation under a sample of size n drawn from the reference distribution.

Step 3: Estimate $\bar{I}$ as the average pooled within-cluster sum of squares from B samples, sd_k as the standard deviation, and s_k as a form of standard error estimation.[23]

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[21] Refer to Tibshirani et al (2001) for a full specification of the Gap Statistic.

[22] Each observation in a dataset is denoted as i, let the distance between observation i and observation $i^{'}$ equate to $d_{ii^{'}}$. The squared Euclidean distance in the Gap Statistic can therefore be mathematically described as $d_{ii^{'}} = \sum_j (x_{ij} - x_{i'j})^2$. For a dataset grouped into k clusters, $C_r = C_1, C_2, ...C_k$, where C_r denotes the indices of observations in cluster r, and n_r refers to the number of observations in C_r, then the sum of the pairwise distances for all datapoints in cluster r is denoted as $D_r = \sum_{i,i' \in C_r} d_{ii'}$. Thus, the pooled within-cluster sum of squares around the cluster mean for cluster k is mathematically described as $W_k = \sum_{r=1}^{k} \frac{1}{2n_r} D_r$.

Step 4: The estimate $\hat{k}$ (i.e. the optimal estimated number of groups) will be the value maximising $\mathrm{Gap}_n(k)$ after the adjustment for the sampling distribution in $E_n^*\{\log(W_k)\}$. This means that the optimal estimated number of groups can be expressed as $\hat{k}$ = smallest k such that $Gap(k) \geq Gap(k+1) - s_{k+1}$.

The Gap Statistic provides two choices of reference distribution, namely, the uniform distribution and the singular value decomposition (SVD) method to derive a set of principal components of the data.[24] Although the uniform method has minor limitations, this study estimates the reference distribution by employing the more thorough SVD method. This study generates ten reference datasets for each Gap Statistic trial and 1,000 trials for each test to ensure its accuracy.[25]

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[23] Let $\bar{I} = (1/B)\sum_{b=1}^{B}\log(W_{kb}^*)$ be the average pooled within-cluster sum of squares, $sd_k = [(1/B)\sum_b \{\log(W_{kb}^*) - \bar{I}\}^2]^{1/2}$ is the standard deviation, and $s_k = sd_k\sqrt{(1+1/B)}$ is the standard error estimation.

[24] The uniform distribution approach draws samples uniformly over the range of funds for each time period and has the advantage that it is straight-forward and simple to employ, however, the uniform distribution may contain datapoints which are not representative of the original dataset. The second method of reference distribution for the Gap Statistic is the SVD method, which involves the generation of principal components of the data. The SVD method has the advantage of taking into account the shape of the data distribution and makes the procedure rotationally invariant. The advantage derived from a rotationally invariant procedure is that the sample datasets drawn from the SVD reference distribution method are more likely to replicate the distribution of the original dataset.

[25] The trials were limited to 1,000 trials for each test due to the large computational time involved in estimating each single trial.

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