Teaching Number by Building on Students’ Strengths: An Investigation in Remote Australian Schools

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Abstract:
It is widely accepted that teaching should build on what students already know and understand. In this study, 42 Aboriginal children in Grades 3 to 6 were posed a series of related questions on addition and subtraction. The question items were posed in a one-on-one interview situation both in the context of money and with just numbers. The data revealed an alignment between the students’ capacity to work with money and to work with numbers. This implies that money may be a useful context for beginning the development of number understanding and fluency in remote Indigenous contexts.

If you want to teach, first find out what the learners know

In a visit to a remote school serving an Aboriginal community as part of the Mathematics in The Kimberley (MiTK) project, the first named author had asked the secondary level class to offer him advice on what he should teach the prospective teachers with whom he works. The first student to respond asked, “What are your mob good at?” We are inferring that this student, at least, was keen to know more about the people to whom he might offer advice. We suspect that mathematics teachers would do well to adopt the same orientation.

It goes without saying, regardless of the educational context, that teaching should build on what students already know. Hattie and Timperley (2007), for example, reviewed a large range of studies on the characteristics of effective classrooms. They found that feedback was one of the main influences on student achievement, and the key elements were “where am I going?”, “how am I going?”, and “where am I going to next?”. To provide this sort of feedback, it is clear that teachers need to have an understanding of what mathematics the students know and can do. Similarly, Tzur (2008) argued that instruction should begin with what the students already know and are confident with, and then move to content that is unfamiliar, rather than what he claims is the common approach of starting with unfamiliar content. The clear implication is that it is best if instructional decisions are informed and grounded in what the students know and where they are coming from.

Of course, beginning instruction from the students’ current capabilities and experiences is broader than just their mathematical knowledge and skills (Grootenboer & Zevenbergen, 2008). Teachers also need to be cognizant of their students’ mathematical identities – their knowledge, skills, attitudes, dispositions, beliefs, and prior mathematical experiences. Also, if teachers understand something of the personalities and general interests of the students, and their lives beyond school, then they are more likely to be able to provide learning experiences than are more relevant and accessible to the students (Palmer, 1993).

This also forms a consistent theme in recommendations for teaching Indigenous students. Stanton (1994), for example, argued that the curriculum and pedagogies for
Indigenous students could incorporate “both ways” or “common ground” approaches, and that this applies to school policy, management, structure, curriculum and pedagogy. In the case of mathematics, he suggested that the curriculum should: be negotiated; build on aspects of traditional culture; incorporate technology; recognize the interfaces with language; and utilize contexts. Similarly, Frigo, Corrigan, Adams, Hughes, Stephens, and Woods (2003), in reporting a study of schools with high proportions of Indigenous students, listed among key elements of effective numeracy teaching as teaching skills in real life contexts, and building on what the students know. The theme of understanding the students’ mathematical knowledge, capabilities and identities especially in contexts where the teachers and students are from different cultures, is a prominent and recurring one.

**But how is it done**

Like much else in education, this is easier said than done. There are two obvious dimensions to connecting prospective learning with retrospective experience: the first is to identify ideas with which students are familiar and build on those; the second is to create connections between ideas that are fundamental to a modern mathematics curriculum and students’ prior experience. In exploring the former of these in Indigenous contexts, it is possible, for example, to examine Indigenous languages to check mathematical ideas that are present in the culture. For example, in a publication that is both a language guide and dictionary for the Nyikina language (*Jarlimadangar Burru* Aboriginal Corporation, 2003), one of the languages in the Kimberley region, there is a developed lexicon for place and direction words. For example, there are 18 different words for describing location, as well as four different expressions for each of the four compass directions, making a total of 34 distinct terms that can be used. It would be possible to use this language as a starting point for an exploration of modern mathematical concepts associated with location, direction, map reading, networks, and possibly even co-ordinate geometry. Having said that, it is important to note that few of these students actually speak the Nyikina language as most speak Kriol, so it is not as simple as using these directional words. However, the language does reveal a cultural heritage that is rich with concepts of location and direction, and this is an integral part of the cultural life-world of the students.

In considering the latter dimension, it seems that both traditionally, as reflected in the language, and currently, in terms of the limited use of quantities in their everyday lives, number activities tend to be remote from students’ experience. To overcome this, we are exploring ways of building connections between money, with which the students have some familiarity, and more abstract number ideas. This generally involves making connections to realistic contexts with which the students are familiar.

This creates its own challenges. We note the advice of Bransford, Brown, and Cocking (1999) that real-life contexts can be confusing and increase cognitive load. We also note that, in a comprehensive review of the national testing system in the United Kingdom, Cooper and Dunne (1998) found that contextualising mathematics tasks created particular difficulties for low socio-economic status (SES) students, so much so that they performed significantly poorer than their middle-class peers, whereas performance on decontextualised tasks was equivalent. Likewise, Lubienski (2000), monitoring the implementation of a curriculum program and materials based on open-ended contextualised problems, found that pupils who preferred the contextualised trial materials and found them easier all had high SES backgrounds, while most pupils who preferred closed, context free tasks were low SES. In fact, many of the low SES pupils claimed to be worse off with contextualised problems, and none found the contextualised materials easier. This is a complex issue and it is not clear whether diminished performance was due to contextualization per se, or due to other factors like the particular contexts being unfamiliar and alienating for students in low SES communities.
The challenge for teachers is to find ways to build on students’ experience without increasing the complexity of the formulation of tasks, and the cognitive load that multi-step problems create. As highlighted previously, this means using contexts that are familiar and inviting to the Aboriginal learners. This study is an attempt to explore this further.

The data reported below is an outcome of classroom observations of lessons in which we were seeking to develop generalisable number ideas such as partitioning and additive and multiplicative thinking. The overall project is based on a hypothetical pedagogical framework derived from the work of Boaler (2008) that was elaborated by Grootenboer (2009), Jorgensen (2009), and Sullivan (2009).

The data collection

The impetus for the data collection reported below came from one of the project teachers who suggested that she had noticed that the students were adept with using money at the school fete, more so than seemed evident in their classroom mathematics. In the course of developing some learning experiences associated with partitioning, Sullivan, Youdale, and Jorgensen, (2009) reported some activities that required the students to perceptually recognise amounts of money. We have a video of one group of junior secondary students who were able to recognise immediately and accurately an amount made up of around 14 separate coins in various denominations. This seems similar to what Sousa (2008) described as perceptual counting, in which very young students can identify the number of objects in a collection without counting one by one and without using particular patterns.

Yet later in that same lesson, Peter S. posed some exercises related to giving change. He said he had $1 in his pocket and, while jingling the coins, took out 15c. He then asked students to say how much was left in his pocket. No student gave a correct answer. This seemed to be something of an anomaly, and create the impetus for the data collection reported here.

The research questions were:

1. Are the students fluent with recognising money amounts, and if so, what sorts of amounts can they perceptually count?
2. What is the relationship between fluency with recognising money amounts and items involving equivalent addition tasks poses using only numbers?
3. What is the relationship between the coins and addition items, and equivalent subtraction items both involving coins and numbers?

We considered an interview as the most appropriate way to gain insights into what the students could do. This was partly so a student could answer without concern for what other class members might say, and partly so that the researcher could monitor the progress of the student and adapt the protocol accordingly. Also, the researchers were able to probe student responses to further understand their thinking as they worked through the items. The interviews were conducted in a quiet place separate from the rest of the class (e.g., in a separate room) so the student could focus on the tasks and questions at hand. The students were familiar with the researchers because we had spent some time in the classes before the interviews were undertaken, and during the interviews the researchers endeavoured to make the students feel comfortable, valued and relaxed so they could answer freely and honestly.

The students were in two remote Aboriginal community schools. One of us interviewed all of the students in grades 3, 4 and 5 in one school, and the other did the same in another school except that two grade 6 students were also interviewed. The data presented here are for all students from both schools together.

An interview protocol was developed that posed 10 sets of items, with four items in each set (see Table 1). For example, the first set of items was as follows:
• **Perceptual recognition of the value of a collection of coins**: Two 20c pieces were covered then shown for 2 seconds (“and 1 and 2”) and then covered. The students were asked “how much money is under my hand?”

• **Number addition**: Students were shown a card on which was written “20 + 20” and the question “what is 20 plus 20?” was posed.

• **Calculation of difference using money**: The researcher put out two 20c pieces on the table without the students seeing them, and uncovered one 20c piece. The question was posed “There is 40c on the table altogether. How much is still covered by my hand?”

• **Calculation of difference using number**: Students were shown a card on which was written “40 – ? = 20” and the question “what number do I take from 40 to get 20?” was posed. If this was not clear the first time, the question was asked another way.

In posing the items, the first item in each set was posed first. In other words, all the perceptual recognition of coins items were posed in sequence, and then the researcher went back to pose the number addition items in sequence, and so on. The items with cents became progressively more difficult, as did the items using dollar amounts. Once a student experienced difficulty with a sequence of items, we jumped to the start of the next sequence. For example, when a student was having difficulty with an items on the value of coins involving cents we would skip forward to the items involving $1 and $2 coins.

The items are shown in Table 1 below:

<table>
<thead>
<tr>
<th>Item</th>
<th>Coin Addition</th>
<th>Number Addition</th>
<th>Coin Difference</th>
<th>Number Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20c, 20c</td>
<td>20+20</td>
<td>40c - 20c</td>
<td>20 + ? = 40</td>
</tr>
<tr>
<td>2</td>
<td>20c, 20c, 10c</td>
<td>20+20+10</td>
<td>50c - 10c</td>
<td>60 + ? = 50</td>
</tr>
<tr>
<td>3</td>
<td>50c, 10c</td>
<td>50+10</td>
<td>60c - 10c</td>
<td>70 + ? = 60</td>
</tr>
<tr>
<td>4</td>
<td>50c, 20c, 10c</td>
<td>50+20+10</td>
<td>80c - 20c</td>
<td>60 + ? = 80</td>
</tr>
<tr>
<td>5</td>
<td>50c, 20c, 20c, 10c</td>
<td>50+20+20+10</td>
<td>$1 - 20c</td>
<td>80 + ? = 100</td>
</tr>
<tr>
<td>6</td>
<td>50c, 20c, 10c, 5c</td>
<td>50+20+10+10+5+5</td>
<td>$1 - 10c</td>
<td>90 + ? = 100</td>
</tr>
<tr>
<td>7</td>
<td>50c, 20c, 20c, 20c</td>
<td>50+20+20+20</td>
<td>$1.10 - 20c</td>
<td>90 + ? = 110</td>
</tr>
<tr>
<td>8</td>
<td>$2, $2, $2</td>
<td>2+2+2</td>
<td>$6 - $2</td>
<td>4 + ? = 6</td>
</tr>
<tr>
<td>9</td>
<td>$2, $2, $2, $1</td>
<td>2+2+2+1</td>
<td>$7 - $1</td>
<td>6 + ? = 7</td>
</tr>
<tr>
<td>10</td>
<td>$2, $2, $2, $1, $1</td>
<td>2+2+2+2+1+1</td>
<td>$10 - $2</td>
<td>8 + ? = 10</td>
</tr>
</tbody>
</table>

Results

The first set of items involved the perceptual recognition of amounts in coins. The number of students who correctly recognised the total value of the coins, when the coins were shown for 2 seconds, is shown in Table 2 below. Note that once a student experienced difficulty with a sequence of items, we jumped to the start of the next sequence.

<table>
<thead>
<tr>
<th>Amount in coins</th>
<th>Number of students correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>20c, 20c</td>
<td>31</td>
</tr>
</tbody>
</table>

Table 2: **Number of students who correctly recognised amounts of coins (n = 42)**
Around three quarters of the students could do the easier items, and most of them could also do the items of medium difficulty as well. Noting that the coins were only shown briefly, this suggested that many of the students were able to perceptually count the money. In other words, many were not forming a mental image and then adding the value of the coins, but seemed to be using some other process. There were students, including some of the more successful students, who did form such mental images nevertheless.

Overall, the range of results was broad with some young students getting none of the items correct, and a few students (from grades 4 to 6) getting the majority of them correct. There was about one quarter of the students who could not say the money amounts even with the easier items. The diversity in the responses is discussed below.

To explore the nature of the responses and to allow comparison across the sets of items, the responses were compared by grade level for the equivalent items for the item involving 50c, 20c, 10c (see Table 3 below). This was the fourth item in each set and is illustrative of the other results since the profile of responses is similar. Note that the item was not posed to students who experienced difficulty with earlier items.

Table 3:
Number of students, by grade, responding correctly to the items involving 50c, 20c, 10c

<table>
<thead>
<tr>
<th>Grade</th>
<th>n</th>
<th>Money addition</th>
<th>Number addition</th>
<th>Money subtraction</th>
<th>Number subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>18</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>6</td>
<td>7</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td>10</td>
<td>9</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>42</td>
<td>25</td>
<td>26</td>
<td>22</td>
<td>14</td>
</tr>
</tbody>
</table>

It appears that the overall facilities of the money addition, number addition, and money subtraction items were similar. Given that they were only shown the money addition task briefly it confirms that many students have an interesting fluency with this perceptual recognition of the money amounts. The majority of the grade 4, 5 and 6 students were able to do the coin recognition for this combination, but only a minority of the grade 3s. This suggests that the questions were pitched at the appropriate level to probe these students’ capacity at the items.

Most of the students who were not able to recognise the amount of these coins were in grade 3. This suggests that the collective experience of the students influences their capacity. It seems that whatever experiences are needed for this perceptual recognition of the coin amounts, most students had had those experiences by the time they were in grade 4.
It had been anticipated that the money items would be easier for the students than the number ones. Indeed our hypothesis was that the students would be better with the money amounts, and that we would use this facility in designing instructional programs. This was not the case for equivalent pair of items involving addition.

To illustrate the extent to which the responses to one are dependent on the response to the related item, the numbers in each of the four possible combinations of responses to the equivalent addition items are presented in Table 4 (note that some students were not asked all these items as outlined previously).

Table 4:
Comparison of student responses of the pairs of addition items connected to 50c, 20c, 10c

<table>
<thead>
<tr>
<th>Coin recognition correct</th>
<th>Coin recognition incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number addition correct</td>
<td>24</td>
</tr>
<tr>
<td>Number addition incorrect</td>
<td>2</td>
</tr>
</tbody>
</table>

This table shows that 29 out of the 32 students that attempted both were either correct or incorrect on both, and only 3 (less than 10%) were correct on only one. It seems that the students could either add the 50, 20 and 10 or they could not. For the equivalent items in this case, it seems that the capacity to add 50, 20 and 10, is connected to the ability to perceptually recognise this amount in coins. In fact, the cross tabulation for seven sets of equivalent items (items 1, 2, 3, 4, 5, 8 and 9) is very similar to this. The implications of this are discussed later. However, there were three groups of related items (items 6, 7 and 10) that had a different distribution, and one of these is shown below in Table 5 (item 6)

Table 5:
Comparison of responses to the pairs of items connected to 50c, 20c, 10c, 10c, 5c, 5c

<table>
<thead>
<tr>
<th>Coin recognition correct</th>
<th>Coin recognition incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number addition correct</td>
<td>7</td>
</tr>
<tr>
<td>Number addition incorrect</td>
<td>4</td>
</tr>
</tbody>
</table>

Note that the coin recognition item has six coins, involving four different amounts, and that this would perhaps be an unfamiliar combination. Indeed, it is noteworthy that 11 students could recognise the total value of the coins in the two seconds. There were 14 students who could perform the addition task correctly. The task was written on a card and spoken as well, but it still requires the addition of six amounts. There were seven students who could add the numbers but not recognise the coins. The normal assumption is that the number fluency would precede the application to money, so this type of response could have been expected. On the other hand, there were four who could state the coin total but not add the numbers. This is worthy of further investigation since it seems counter intuitive. There was a similar distribution of responses on the pairs of items 7 and 10. These three items are the more difficult ones in that there were more coins, increased total value, and a larger range of coin values. One of the key issue for this exploration is the extent to which there is widespread facility with the coin recognition items so that the skill could be used as the basis of instruction on other number ideas. Again it seems that fluency with the coin prompt was related to capacity to answer the straight number question.

Across the ten items, the facility on the items involving the difference between coin amounts is very close to the coin recognition items, and therefore, to the number addition
items. This suggests that whatever experience is needed to answer any one of the types of items is needed for each. It appears that fewer students could complete the number subtraction items, although it is noted that this might be due to the unfamiliar formulation of the item. To explore this further, a comparison of the coin difference and the number subtraction items for the equivalent amounts to those presented Table 4, is presented in Table 6.

Table 6: Comparison of responses to the pairs of items connected to $80c - ? = 60c$

<table>
<thead>
<tr>
<th></th>
<th>Coin item correct</th>
<th>Coin item incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number subtraction correct</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>Number subtraction incorrect</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

While the pattern is not quite as stark with the subtraction items as it was with the addition items (shown in Table 4), it still appears that generally students could either do both or neither. Only 8 out of these 25 students could do just one or the other, with 6 being able to do the coin item but not the number one.

Conclusion

In terms of the research questions, it seems that many of these primary students from remote communities are able to state the amount of collections of coins very quickly and readily. This was compatible with our observations in the classroom.

It also seems that nearly all students who could do this were able to respond to straight number questions using comparable numbers. While it makes sense that this should happen, it seems that the students were more fluent with the number calculation tasks than seemed evident from observations of their classroom responses. It is possible that the interview environment is more conducive to concentration.

An inference overall is that the facility with the straight number tasks is connected directly to the ability to perceptually recognise the money amounts, although it is not clear which skill informs the other.

The students were able to perform similar levels of tasks involving hidden coins, simulating subtraction, but the straight number subtraction items were more difficult. One possible inference is that the form of the task was perhaps confusing, so students need experience with various forms of posing such problems.

This perceptual counting of coins seems an extraordinary strength of the students, and teachers could explore the extent to which this strength could be used to introduce students to some number concepts. It seems though that students can do the coin tasks as the same times as they can do the strength number tasks. Therefore, while there is limited potential for one to be used to introduce the other, the facility with coin recognition is clearly a strength of many of these students so it can be used for developing fluency with addition tasks, for promoting confidence in solving contextual problems, for challenging those students who are ready, and for allowing students to recognise the advantages of developing number fluency. In other words, the context with which these students are familiar can be used to enrich their learning of other mathematical ideas.

One additional comment is that, while most students seemed fluent with the money recognition task, there were others who were not, and indeed had trouble even identifying the value of the coins. It seems that the Aboriginal Education Workers could assist by providing small group intensive instruction using money that could assist such students.
References


