Modeling undesirable outputs in multiple objective data envelopment analysis

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Abstract

Recent empirical and conceptual work in data envelopment analysis (DEA) has emphasized its potential importance in highlighting the environmental performance of economic entities. Initial work in this emerging research area has focused on the separation of output factors into desirable and undesirable ones. In this paper, we describe recent developments in the modeling undesirable outputs. In particular, the modeling of undesirable outputs in the range adjusted measure (RAM) is investigated. We discuss some of the difficulties of RAM in assessing the environmental efficiency of decision making units (DMUs) and develop a multiple objective DEA model to overcome these difficulties. The proposed multiple objective model is solved as a linear programming and its applicability as a mechanism for assessing environmental efficiency is demonstrated by evaluating the technical, ecological, and process environmental quality efficiency scores of China's provinces.

Keywords: Data envelopment analysis; Multiple objective; Undesirable outputs; Ecological efficiency; Process environmental quality efficiency.

1. Introduction

Evaluating the relative efficiency of decision making units (DMUs) with data envelopment analysis (DEA) has been studied in a wide range of settings. Many models, with different properties and different purposes, have been developed. Prior to determining which DEA model is appropriate for a particular efficiency evaluation problem, an assessment of various forms of inputs and outputs is required. Generally, outputs with larger values and inputs with smaller values show better performance. However, outputs that are deemed negative (for example, pollution) should be valued differently, with higher values representing a relatively worse performance. In efficiency evaluation of production units, therefore, by-product undesirable outputs need to be incorporated differently into DEA models

DEA measures "technical efficiency" of each DMU in terms of its consumed inputs and produced desirable outputs. Technical efficiency is the ratio of desirable outputs to inputs. Alternatively, if the efficiency score of each DMU is calculated in terms of its desirable and undesirable outputs, the efficiency score is known as "ecological efficiency". Ecological efficiency is defined as the ratio of added values of products or services to added environmental impacts (Zhang et al., 2008; Kuosmanen

and Kortelainen, 2005). Ecological efficiency, in short, assesses the creation of more value with less environmental impact. The ratio of inputs to undesirable outputs can also be measured and is called the "process environmental quality efficiency" (Mahdiloo et al., 2014). This ratio can help decision makers to define potential improvements in the quality of the capital equipment and related processes which create high volumes of pollution.

Figure 1 shows three different trade-offs among inputs, desirable, and undesirable outputs. These different trade-offs can be modeled as three separate ratios and can be used to determine whether a DMU can: (i) efficiently use its resources to produce more desirable outputs (technical efficiency), (ii) produce more desirable outputs with less undesirable outputs (ecological efficiency), and (iii) produce less undesirable outputs with more inputs (process environmental quality efficiency). A similar representation of measuring overall efficiency of activities made of two separate stages was given in Chiou et al. (2010). We adopt the same conceptual model to address the measurement of the overall efficiency of DMUs consisting of technical, ecological and process environmental quality efficiencies.

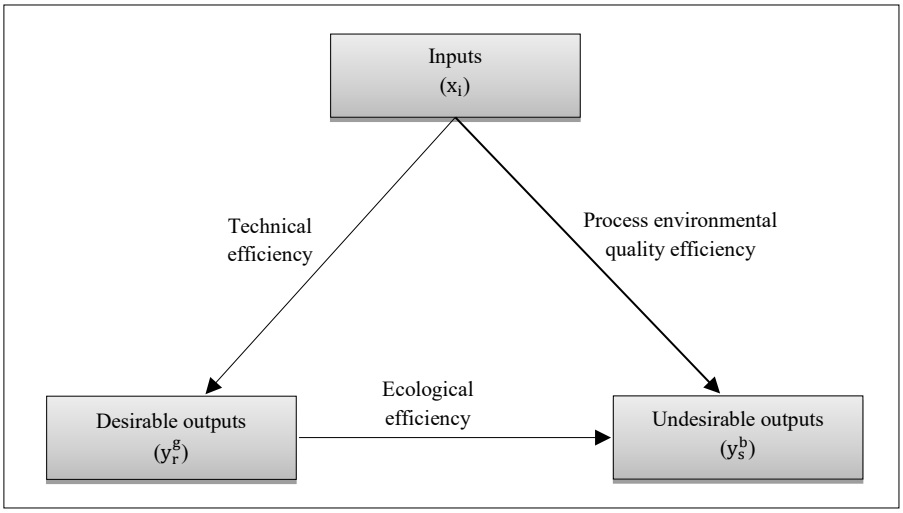

Figure 1. Possible trade-offs between inputs and outputs

Measures of technical efficiency can be obtained either by radial models like CCR (Charnes, Cooper, and Rhodes, 1978) and BCC (Banker, Charnes, and Cooper, 1984) or by the models that directly deal with shortage slack variables of desirable outputs and excess slack variables of inputs such as slacks-based measure (SBM) (Tone, 2001),

additive (Charnes et al., 1985), and range adjusted measure (RAM) (Cooper et al., 1999). Ecological efficiency can be measured by assessing the ratio of desirable outputs to undesirable outputs or similarly assessing the shortage slack variables of desirable outputs and excess slack variables of undesirable outputs. Process environmental quality efficiency can be measured in a similar fashion.

Having three possible efficiency ratios will lead to three linear programs. One possible way to measure overall efficiency is to run three separate and independent linear programming models. However, this way of modeling, aside from the creation of a requirement for intensive computations, cannot address potential conflicts among the ratios. For example, consider a DMU which is inefficient with regards to process environmental quality efficiency ratio. This inefficient DMU may increase its inputs to be projected on the efficiency frontier. However, this leads to an increase in the denominator of the technical efficiency ratio and consequently may lead to a decrease in technical efficiency. A discussion and challenge similar to this can be found in two-stage DEA literature where there is the same sort of conflict between the stages of a two-stage activity. Liang et al. (2008) discussed this potential conflict and developed a game theoretic approach to deal with it. Mahdiloo et al. (2014) applied Liang et al's approach to measure technical, ecological, and process environmental quality efficiencies.

The purpose of our paper is to develop a multiple objective linear programming model to measure the technical, ecological, and process environmental quality efficiencies. The developed model is a linear model and considers the conflicts among different ratios.

The rest of this paper is structured as follows. Section 2, reviews the literature. In Section 3, mathematical models from the literature are introduced. In Section 4, the proposed model is developed. In Sections 5 and 6, the proposed model is tested by a numerical example and a real data set related to the provinces in China, respectively. Section 7 concludes the paper.

2. Literature review

There are different ways in the DEA literature to simultaneously take into account inputs, desirable and undesirable outputs. A benefit of these unified approaches is that they can be used to measure overall efficiency score by combining different efficiency

ratios. Song et al. (2012) provided a comprehensive review of papers dealing with undesirable outputs in DEA.

Färe et al. (1989) were among the first to model undesirable outputs. They introduced the weak disposability of undesirable outputs versus strong disposability of desirable outputs and showed how this assumption can be modeled in DEA. As it was explained later in Färe et al. (1996), strong disposability refers to the fact that lower level of desirable outputs can be achieved with the current level of inputs. However, this is not the case with undesirable outputs, and weak disposability assumes that decreasing undesirable outputs requires either increasing inputs or decreasing desirable outputs.

In an envelopment model, the weak disposability of undesirable outputs was modelled using an equality constraint rather than an inequality constraint. However, undesirable outputs weak disposability has created a lot of confusion in the literature and has been a topic of controversy in a series of studies such as Färe et al. (1996), Hailu and Veeman (2001), Färe and Grosskopf (2003), Hailu (2003), Kuosmanen (2005), Färe and Grosskopf (2009), Kuosmanen and Podinovski (2009).

Modelling of undesirable outputs as inputs is one of the very common approaches in DEA. See for example Hailu and Veeman (2001), Korhonen and Luptacik (2004), Gomes and Lins (2008), Sarkis and Cordeiro (2009), Yang and Pollitt (2009), Wu et al. (2013a), Wu et al. (2017). The logic behind modelling undesirable outputs as inputs is that the lower values of undesirable outputs is a sign of better performance (Sarkis and Cordeiro, 2009).

An alternative approach, presented by Seiford and Zhu (2002), transformed the value of undesirable outputs via multiplying them by -1 and then adding a positive scalar to make transformed values greater than zero. With this new value, undesirable outputs are considered as desirable outputs. Färe and Grosskopf (2004) used the directional distance function to model undesirable outputs. Unlike the model proposed by Seiford and Zhu (2002), the weak disposability property of undesirable outputs was assumed in directional distance function.

Sueyoshi and Goto (2010; 2011) used non-radial RAM model to measure technical efficiency. Then, they measured the environmental efficiency score by modifying RAM. The modified RAM model measures environmental efficiency in terms of inputs and undesirable outputs. Finally, to measure overall efficiency of

DMUs, they used inputs, desirable outputs, and undesirable outputs[1]. The sign of inputs slacks in their environmental efficiency model indicates that inefficient DMUs can reach their reference benchmark DMUs sometimes by increasing their inputs. The models proposed by Sueyoshi and Goto (2010; 2011), however, have some limitations. Sueyoshi and Goto (2010; 2011) calculated technical, environmental, and overall efficiencies by solving three different models. This is technically difficult to implement as calculation of three separate models can be cumbersome. Moreover, overall efficiency models developed by Sueyoshi and Goto (2010; 2011) employed either nonlinear or mixed integer programming to avoid an unbounded solution of their linear model or did not provide a single set of intensity variables for desirable and undesirable outputs. Regarding their nonlinear model, it is not easy to develop its dual form and, therefore, determine the type of returns to scale (RTS). Apart from these difficulties which were also pointed out by Sueyoshi and Goto (2011), we will show that RAM, without convexity constraint ($\sum_{j=1}^{n} \lambda_j = 1,$), may lead to negative efficiency scores.

Our paper is aimed at developing a multiple objective DEA model to measure technical, ecological, and process environmental quality efficiency scores. We believe that the developed model and our paper contributes to the theory and application of DEA in the following ways: (i) the developed model is linear and it guarantees that the solutions found by the model are global optimal solutions. Also, the linearity of the model makes it possible to develop its dual form, (ii) based on our multiple objective model, technical, ecological, and process environmental quality efficiencies are found by running only one linear programming. Therefore, computational complexity is decreased and (iii) we show how these different efficiency ratios can be integrated into a CRS model.

3. Modeling undesirable outputs

Assume there is a problem of evaluating the relative efficiency of n DMUs which use m different types of inputs and produces s types of desirable and p types of undesirable outputs. Table 1 shows the summary of the notations used in this paper.

[1] Note that the terms "technical efficiency" and "overall efficiency" are referred to as "operational efficiency" and "unified efficiency" measures in Sueyoshi and Goto (2010; 2011), respectively. However, in order to maintain semantic consistency, we refer to the efficiency score calculated in terms of inputs and desirable outputs as technical efficiency score. Also, the term "overall efficiency" is used instead of "unified efficiency".

Table 1. The notations

DMU_o:	The decision making unit under evaluation
$j=1,\ldots,n$	Collection of DMUs
$r=1,\ldots,s$	The set of desirable outputs
$i=1,\ldots,m$	The set of inputs
$k=1,\ldots,p$	The set of undesirable outputs
y_{ro}^{g}:	The rth desirable output of DMU_o
$y_{r_j}^{g}$:	The rth desirable output of DMU_j
u_r:	The weight of rth desirable output
x_{io}:	The ith input of DMU_o
x_{ij}:	The ith input of DMU_j
v_i:	The weight of ith pollution-related input
g_f:	The weight of fth pollution-unrelated input
y_{ko}^{b}:	The kth undesirable output of DMU_o
$y_{k_j}^{b}$:	The kth undesirable output of DMU_j
μ_k:	The weight of kth undesirable output
d_r^g:	Shortages in rth desirable output
d_i^x:	Excesses in ith input
d_k^b:	Excesses in kth undesirable output
λ_j:	Benchmarks associated with DMU_j
θ:	Radial efficiency measure for DMU_o

Models (1) and (2) can evaluate the technical and ecological efficiencies of DMU_o, respectively (Korhonen and Luptacik, 2004).

$$\begin{aligned} \max \quad & \frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{i=1}^{m} v_i x_{io}} \\ \text{s.t.} \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}^{g}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \quad j = 1,2,\ldots,n, \\ & v_i \geq 0 \qquad i = 1,2,\ldots,m, \\ & u_r \geq 0 \qquad r = 1,2,\ldots,s. \end{aligned} \tag{1}$$

$$\begin{aligned} \max \quad & \frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{k=1}^{p} \mu_k y_{ko}^{b}} \\ \text{s.t.} \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}^{g}}{\sum_{k=1}^{p} \mu_k y_{kj}^{b}} \leq 1 \quad j = 1,2,\ldots,n, \\ & u_r \geq 0 \qquad r = 1,2,\ldots,s, \\ & \mu_k \geq 0 \qquad k = 1,2,\ldots,p. \end{aligned} \tag{2}$$

3.1. Radial models

In this section, we review some of the models from the literature which simultaneously consider inputs, desirable and undesirable outputs. These models can be regarded as the combination of the Models (1) and (2). Table 2 summarizes the models proposed by

Färe et al. (1989), Hailu and Veeman (2001), Seiford and Zhu (2002), and Korhonen and Luptacik (2004). The last column of the table shows some characteristics of the models.

Table 2. The models dealing with undesirable outputs

Authors	DEA model	Characteristics
Färe et al. (1989)	$\max \theta$ s.t. $\theta y_{ro}^{g} \leq \sum_{j=1}^{n} \lambda_j y_{rj}^{g} \quad r=1,2,\ldots,s,$ $x_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij} \quad i=1,2,\ldots,m,$ (3) $\theta^{-1} y_{ko}^{b} = \sum_{j=1}^{n} \lambda_j y_{kj}^{b} \quad k=1,2,\ldots,p,$ $\lambda_j \geq 0 \quad j=1,2,\ldots,n.$	• Original values of the undesirable outputs are used, i.e., there is no need to transform undesirable outputs. • Equality sign in the constraint related to the undesirable outputs shows weak disposability assumption.
Hailu and Veeman (2001)	$\min \theta$ s.t. $y_{ro}^{g} \leq \sum_{j=1}^{n} \lambda_j y_{rj}^{g} \quad r=1,2,\ldots,s,$ $\theta x_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij} \quad i=1,2,\ldots,m,$ (4) $y_{ko}^{b} \geq \sum_{j=1}^{n} \lambda_j y_{kj}^{b} \quad k=1,2,\ldots,p,$ $\sum_{j=1}^{n} \lambda_j = 1 \quad j=1,2,\ldots,n,$ $\lambda_j \geq 0 \quad j=1,2,\ldots,n.$	• Original values of the undesirable output are used. • The undesirable outputs are modeled as inputs. • Since θ is only multiplied by the input factors, undesirable outputs can be seen beyond the control of management. In other words, undesirable outputs are considered as nondiscretionary factors[2].
Seiford and Zhu (2002)	$\max \theta$ s.t. $\sum_{j=1}^{n} \lambda_j y_{rj} \geq \theta y_{ro}^{g} \quad r=1,2,\ldots,s,$ $\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} \quad i=1,2,\ldots,m,$ (5) $\sum_{j=1}^{n} \lambda_j \bar{y}_{kj}^{b} \geq \theta \bar{y}_{ko}^{b} \quad k=1,2,\ldots,p,$ $\sum_{j=1}^{n} \lambda_j = 1 \quad j=1,2,\ldots,n,$ $\lambda_j \geq 0 \quad j=1,2,\ldots,n.$	• Undesirable outputs are transformed to desirable outputs by multiplying their values by -1 and adding a positive scalar that is large enough to make transformed values greater than zero. The transformed undesirable outputs are shown by $\bar{y}_{kj}^{b}$. • When this type of transformation is used, translation invariance property should be considered. If the model does not have translation invariance property, by changing the arbitrary positive scalar, inefficiency score of DMUs may change.

[2] Banker and Morey (1986) distinguished between discretionary and non-discretionary inputs via multiplying θ by just those inputs which are completely under control of management.

Authors	DEA model	Characteristics
Korhonen and Luptacik (2004)	$\max \sum_{r=1}^{s} u_r y_{ro}^g$ s.t. $\sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{p} \mu_k y_{ko}^b = 1,$ $\sum_{r=1}^{s} u_r y_{rj}^g - (\sum_{i=1}^{m} v_i x_{ij} + \sum_{k=1}^{p} \mu_k y_{kj}^b) \leq 0 \quad j=1, \ldots, n,$ $u_r \geq 0 \quad r=1, 2, \ldots, s, \quad (6)$ $v_i \geq 0 \quad i=1, 2, \ldots, m,$ $\mu_k \geq 0 \quad k=1, 2, \ldots, p.$	• Original values of undesirable outputs are used. • Undesirable outputs are treated as inputs.
Korhonen and Luptacik (2004)	$\max \sum_{r=1}^{s} u_r y_{ro}^g - \sum_{i=1}^{m} v_i x_{io}$ s.t. $\sum_{k=1}^{p} \mu_k y_{ko}^b = 1,$ $\sum_{r=1}^{s} u_r y_{rj}^g - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y_{kj}^b \leq 0 \quad j=1, \ldots, n,$ $u_r \geq 0 \quad r=1, 2, \ldots, s, \quad (7)$ $v_i \geq 0 \quad i=1, 2, \ldots, m,$ $\mu_k \geq 0 \quad k=1, 2, \ldots, p.$	• Original values of undesirable outputs are used. • Undesirable outputs are treated as inputs. • Inputs are treated as nondiscretionary factors.

3.2. *Non-radial RAM model*

There are also papers which have modeled undesirable outputs in the RAM model. Sueyoshi and Goto (2010; 2011) used RAM to measure technical and environmental efficiencies of DMUs. They used the original RAM model to evaluate the technical efficiency of DMUs, following the model originally developed by Cooper et al. (1999). They then measured the environmental efficiency of DMUs by modifying the original RAM. Technical and environmental efficiency models are then unified in two different ways and overall efficiency is measured. Model (8) is the RAM model to measure the technical efficiency.

$$
\begin{array}{lll}
\max & 1 - \left(\sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_r^g d_r^g\right) & \\
\text{s.t.} & \sum_{j=1}^{n} x_{ij}\lambda_j + d_i^x = x_{io} & i = 1, 2, \ldots, m, \\
& \sum_{j=1}^{n} y_{rj}^g \lambda_j - d_r^g = y_{ro}^g & r = 1, 2, \ldots, s, \\
& \sum_{j=1}^{n} \lambda_j = 1 & \\
& \lambda_j \geq 0 & j = 1, 2, \ldots, n, \\
& d_i^x \geq 0 & i = 1, 2, \ldots, m, \\
& d_r^g \geq 0 & r = 1, 2, \ldots, s.
\end{array}
\tag{8}
$$

where d_i^x and d_r^g are slack variables related to inputs and desirable outputs, respectively. R_i^x and R_r^g are ranges associated with inputs and outputs, respectively and are determined as: $R_i^x = 1/(m+s)(\overline{x_i} - \underline{x_i})$ and $R_r^g = 1/(m+s)(\overline{y_r^g} - \underline{y_r^g})$. The $\overline{x_i}$,

$\underline{x_i}$, $\overline{y_r^g}$, and $\underline{y_r^g}$ are determined by $\overline{x_i}$= max $\{x_{ij}\}$, $\underline{x_i}$ = min$\{x_{ij}\}$, $\overline{y_r^g}$= max $\{y_{rj}^g\}$, and $\underline{y_r^g}$ = min$\{y_{rj}^g\}$. On the other hand, Models (9) and (10) can be used to measure environmental efficiency.

$$
\begin{array}{lll}
\max & 1-\left(\sum_{i=1}^{m} R_i^x d_i^x + \sum_{k=1}^{p} R_k^b d_k^b\right) & \\
\text{s.t.} & \sum_{j=1}^{n} x_{ij}\lambda_j - d_i^x = x_{io} & i = 1,2,\dots,m, \\
 & \sum_{j=1}^{n} y_{kj}^b \lambda_j + d_k^b = y_{ko}^b & k = 1,2,\dots,p, \\
 & \sum_{j=1}^{n} \lambda_j = 1 & \\
 & \lambda_j \geq 0 & j = 1,2,\dots,n, \\
 & d_i^x \geq 0 & i = 1,2,\dots,m, \\
 & d_k^b \geq 0 & k = 1,2,\dots,p.
\end{array}
\tag{9}
$$

$$
\begin{array}{lll}
\max & 1-\left(\sum_{i=1}^{m} R_i^x (d_i^{x+} + d_i^{x-}) + \sum_{k=1}^{p} R_k^b d_k^b\right) & \\
\text{s.t.} & \sum_{j=1}^{n} x_{ij}\lambda_j - d_i^{x+} + d_i^{x-} = x_{io} & i = 1,2,\dots,m, \\
 & \sum_{j=1}^{n} y_{kj}^b \lambda_j + d_k^b = y_{ko}^b & k = 1,2,\dots,p, \\
 & \sum_{j=1}^{n} \lambda_j = 1 & \\
 & d_i^{x+} d_i^{x-} = 0 & i = 1,2,\dots,m, \\
 & \lambda_j \geq 0 & j = 1,2,\dots,n, \\
 & d_i^{x+} \geq 0 & i = 1,2,\dots,m, \\
 & d_i^{x-} \geq 0 & i = 1,2,\dots,m, \\
 & d_k^b \geq 0 & k = 1,2,\dots,p.
\end{array}
\tag{10}
$$

The d_k^b is the slack variable related to the undesirable outputs. Ranges are calculated as $R_i^x = 1/(m+p)(\overline{x_i} - \underline{x_i})$ and $R_k^b = 1/(m+p)(\overline{y_k^b} - \underline{y_k^b})$. $\overline{y_k^b}$ and $\underline{y_k^b}$ are determined by $\overline{y_k^b}$= max $\{y_{kj}^b\}$ and $\underline{y_k^b}$ = min$\{y_{kj}^b\}$.

Assuming two-dimensional space in which inputs are on the horizontal axis and desirable and undesirable outputs are on the vertical axis, projections of technically inefficient DMUs in Model (8) are toward the North West. The projection toward North West is determined by the positive sign of input slacks (d_i^x) and the negative sign of desirable output slacks (d_r^g). Therefore, a DMU can be technically efficient by decreasing inputs or/and increasing desirable outputs. The projection suggested by Model (9) requires that inefficient DMUs move toward the South East, indicating a decrease in undesirable outputs and/or an increase in inputs. Therefore, it is recommended that inefficient DMUs increase their inputs in order to decrease their undesirable outputs. This method of projecting inefficient DMUs on the efficiency

frontier requires corporate and managerial efforts. These projections are explained in more details by Sueyoshi and Goto (2011).

Model (10) is slightly different from Model (9). The *i*th input slack variable is separated into d_i^{x+} and d_i^{x-} and nonlinear constraint $d_i^{x+}d_i^{x-} = 0$ shows that the two slacks cannot simultaneously be strictly positive. Model (10) is a nonlinear programming problem. This model can also be run as a linear or a mixed-integer programming problem with some minor modifications (for details see Sueyoshi and Goto, 2011). Given the $d_i^{x+}d_i^{x-} = 0$, the inefficient DMUs are projected by moving toward either South West or South East. Sueyoshi and Goto (2010 and 2011) based their models on RAM since it is a slack based efficiency model and they believe that it can be easily adapted to combine different models into a single and unified form.

Models (11) and (12) measure overall efficiency of DMUs. Model (11) is the combination of the Models (8) and (9) and Model (12) is the combination of the Models (8) and (10).

$$
\begin{array}{lll}
\max & 1 - \left(\sum_{i=1}^{m} R_i^x d_i^{xg} + \sum_{r=1}^{s} R_r^g d_r^g + \sum_{i=1}^{m} R_i^x d_i^{xb} + \sum_{k=1}^{p} R_k^b d_k^b\right) & \\
\text{s.t.} & \sum_{j=1}^{n} x_{ij}\lambda_j^g + d_i^{xg} = x_{io} & i = 1, 2, \dots, m, \\
& \sum_{j=1}^{n} y_{rj}^g \lambda_j^g - d_r^g = y_{ro}^g & r = 1, 2, \dots, s, \\
& \sum_{j=1}^{n} \lambda_j^g = 1 & \\
& \sum_{j=1}^{n} x_{ij}\lambda_j^b - d_i^{xb} = x_{io} & i = 1, 2, \dots, m, \\
& \sum_{j=1}^{n} y_{kj}^b \lambda_j^b + d_k^b = y_{ko}^b & k = 1, 2, \dots, p, \\
& \sum_{j=1}^{n} \lambda_j^b = 1 & \\
& \lambda_j^g \geq 0 & j = 1, 2, \dots, n, \\
& \lambda_j^b \geq 0 & j = 1, 2, \dots, n, \\
& d_i^{xg} \geq 0 & i = 1, 2, \dots, m, \\
& d_i^{xb} \geq 0 & i = 1, 2, \dots, m, \\
& d_r^g \geq 0 & r = 1, 2, \dots, s, \\
& d_k^b \geq 0 & k = 1, 2, \dots, p.
\end{array}
\tag{11}
$$

$$
\begin{array}{lll}
\max & 1-\left(\sum_{i=1}^{m} R_i^x (d_i^{x+}+d_i^{x-})+\sum_{r=1}^{s} R_r^g d_r^g+\sum_{k=1}^{p} R_k^g d_k^b\right) & \\
\text{s.t.} & \sum_{j=1}^{n} x_{ij}\lambda_j - d_i^{x+} + d_i^{x-} = x_{io} & i=1,2,\dots,m, \\
 & \sum_{j=1}^{n} y_{rj}^g \lambda_j - d_r^g = y_{ro}^g & r=1,2,\dots,s, \\
 & \sum_{j=1}^{n} y_{kj}^b \lambda_j + d_k^b = y_{ko}^b & k=1,2,\dots,p, \\
 & \sum_{j=1}^{n} \lambda_j = 1 & \\
 & d_i^{x+} d_i^{x-} = 0 & i=1,2,\dots,m, \\
 & \lambda_j \geq 0 & j=1,2,\dots,n, \\
 & d_i^{x+} \geq 0 & i=1,2,\dots,m, \\
 & d_i^{x-} \geq 0 & i=1,2,\dots,m, \\
 & d_r^g \geq 0 & r=1,2,\dots,s, \\
 & d_k^b \geq 0 & k=1,2,\dots,p.
\end{array}
\tag{12}
$$

The ranges are determined as $R_i^x = 1/(m+s+p)(\overline{x_i} - \underline{x_i})$, $R_r^g = 1/(m+s+p)(\overline{y_r^g} - \underline{y_r^g})$, and $R_k^b = 1/(m+s+p)(\overline{y_k^b} - \underline{y_k^b})$. The unified Model (11) uses two different intensity variables (λ_j^g and λ_j^b).

Models (11) and (12) measure overall efficiency of DMUs. Model (12) uses a single intensity variable (λ_j) to combine technical and environmental efficiency models. This is an advantage for Model (12) in comparison with Model (11) which has two intensity variables (λ_j^g and λ_j^b). Model (11) is a linear programming problem while Model (12) is a nonlinear or mixed integer programming problem.

4. Proposed model

After introducing the above two groups of models to measure the efficiency with undesirable outputs, this section deals with developing a new model. Developing this new model is important since the radial models introduced above can be regarded as overall efficiency models and do not decompose efficiency to technical and ecological efficiencies. Regarding the non-radial RAM models, we will show that they cannot be used with CRS assumption. This, therefore, requires a different way of modelling undesirable outputs.

The model developed in this paper is an extension to Korhonen and Luptacik (2004) and Sarkis and Cordeiro (2009) who have used two efficiency ratios of technical and ecological efficiencies in their analysis of overall efficiency. Therefore, we will use the same assumptions used in Korhonen and Luptacik (2004) and Sarkis and Cordeiro (2009). The purpose of our paper is to show how two efficiency ratios of technical and ecological efficiencies used in Korhonen and Luptacik (2004) and Sarkis and Cordeiro

(2009) in addition to the process environmental efficiency ratio used in Mahdiloo et al. (2014) can be integrated into a single DEA model.

A linear multiple objective model is developed which is a combination of the three separate models of the technical, ecological, and process environmental quality efficiency models. At first, the process environmental quality efficiency model which was developed by Mahdiloo et al. (2014) is introduced. We slightly modify their model and explain the application and importance of the model herewith.

4.1 Measuring process environmental quality efficiency

Model (13) was developed by Mahdiloo et al. (2014) to measure the quality of the production process from an environmental perspective. The ratio $\frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{k=1}^{p} \mu_k y_{ko}^b}$ is called the process environmental quality efficiency measure.

$$\begin{aligned} \max \quad & \frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{k=1}^{p} \mu_k y_{ko}^b} \\ \text{s.t.} \quad & \frac{\sum_{i=1}^{m} v_i x_{ij}}{\sum_{k=1}^{p} \mu_k y_{kj}^b} \leq 1 \quad j = 1, 2, \dots, n, \\ & v_i \geq 0 \qquad i = 1, 2, \dots, m, \\ & \mu_k \geq 0 \qquad k = 1, 2, \dots, p. \end{aligned} \tag{13}$$

We slightly change Model (13) by separating input factors as pollution-related and pollution-unrelated inputs. The different types of fuel consumed in the production process, and also different environmental investment on infrastructure and environmental R&D, are examples of pollution-related inputs. These inputs have a direct impact on the amount of the pollution in the production process. Model (14) is the modified process environmental quality efficiency model and the superscript *P-R* in the inputs stands for pollution-related.

$$\begin{aligned} \max \quad & \frac{\sum_{f=1}^{z} g_f x_{fo}^{P-R}}{\sum_{k=1}^{p} \mu_k y_{ko}^b} \\ \text{s.t.} \quad & \frac{\sum_{f=1}^{z} g_i x_{fj}^{P-R}}{\sum_{k=1}^{p} \mu_k y_{kj}^b} \leq 1 \quad j = 1, 2, \dots, n, \\ & g_f \geq 0 \qquad f = 1, 2, \dots, z, \\ & \mu_k \geq 0 \qquad k = 1, 2, \dots, p. \end{aligned} \tag{14}$$

The purpose of Model (14) is to find the best possible weights for inputs and undesirable outputs that can maximize the ratio of the weighted sum of inputs to the weighted sum of undesirable outputs. Since inputs are separated as the pollution-related and pollution-unrelated inputs, we can measure the efficiency of the DMUs in terms of their undesirable outputs relative to their consumed pollution-related inputs. The development of this new measure will help decision makers to evaluate their level of undesirable outputs production not only compared to their desirable outputs production (Model 2) but also compared to the consumed inputs. This measure can identify some potential improvements in the quality of the processes leading to a high volume of pollution.

To indicate the importance and the meaning of this measure, we refer to an example from Mahdiloo et al. (2014). Assume that there are two DMUs which consume one type of pollution-related input (coal consumption) and produce one type of undesirable output (CO_2). DMU 1 uses 1 unit of coal and produces 100 units of CO_2 while DMU 2 uses 100 units of coal and produces 1 unit of CO_2. Both DMUs produce 1 unit of desirable output. Based on the process environmental quality efficiency ratio ($\frac{\sum_{f=1}^{z} g_f x_{fo}^{P-R}}{\sum_{k=1}^{p} \mu_k y_{ko}^{b}}$), DMU 2 is a better DMU than DMU 1. The figures indicate that there might be a problem with the quality of fuel, technology, labor or process used by DMU 1, leading to the relatively high emission of CO_2. DMU 1, with 1 unit of input and 100 units of pollution, is environmentally inefficient since there is another DMU (DMU 2) which uses more units of input but is still able to produce less undesirable output. Here, Model (14) should be applied to measure the process environmental quality inefficiency of DMU 1.

Given different possible projections of the DMUs, the projection of inefficient DMUs by the process environmental quality efficiency model is explained. Sueyoshi and Goto (2011) addressed two projections for environmentally inefficient DMUs toward "South West" and "South East" to achieve efficiency frontier. Again, consider the DMUs 1 and 2 being plotted in a two-dimensional space with input on the horizontal axis and undesirable output on the vertical axis. In moving towards the South West, the inefficient DMU 1 is projected on efficiency frontier by decreasing both input and undesirable output. On the other hand, inefficient DMU 1 is projected toward the South East by decreasing the amount of its undesirable output and/or increasing its input. The CO_2 is emitted from combustion of the coal. If a manager decides to decrease coal

consumption, the amount of CO_2 emission will also decrease and the DMU will be projected toward South West. Sueyoshi and Goto (2011) called this a ‘‘natural reduction’’ and claimed that any DMU can achieve this CO_2 emission without any technological or process related changes. On the other hand, projection towards the South East refers to “a managerial or engineering effort” to decrease undesirable outputs.

4.2 A linear multiple objective model to simultaneously measure technical, ecological, and process environmental quality efficiency scores

Models (1), (2), and (14) can be separately run to measure technical, ecological, and process environmental quality efficiency scores, respectively. However, this sort of modeling is computationally intensive and, more importantly, it does not express the potential conflicts between the ratios. One alternative is Model (15) which is an integration of three separate models. Note that in technical efficiency, the inputs are separated as pollution-related (x_{fo}^{P-R}) and pollution-unrelated inputs (x_{io}^{P-UR}).

$$
\begin{aligned}
\max \quad & \frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{i=1}^{m} v_i x_{io}^{P-UR} + \sum_{f=1}^{z} g_f x_{fo}^{P-R}} + \frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{k=1}^{p} \mu_k y_{ko}^{b}} + \frac{\sum_{f=1}^{z} g_f x_{fo}^{P-R}}{\sum_{k=1}^{p} \mu_k y_{ko}^{b}} \\
\text{s.t.} \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}^{g}}{\sum_{i=1}^{m} v_i x_{ij}^{P-UR} + \sum_{f=1}^{z} g_f x_{fj}^{P-R}} \leq 1 && j = 1, 2, \dots, n, \\
& \frac{\sum_{r=1}^{s} u_r y_{rj}^{g}}{\sum_{k=1}^{p} \mu_k y_{kj}^{b}} \leq 1 && j = 1, 2, \dots, n, \\
& \frac{\sum_{f=1}^{z} g_f x_{fj}^{P-R}}{\sum_{k=1}^{p} \mu_k y_{kj}^{b}} \leq 1 && j = 1, 2, \dots, n, \\
& u_r \geq 0 && r = 1, 2, \dots, s, \\
& v_i \geq 0 && i = 1, 2, \dots, m, \\
& g_f \geq 0 && f = 1, 2, \dots, z, \\
& \mu_k \geq 0 && k = 1, 2, \dots, p.
\end{aligned}
\tag{15}
$$

However, this model is a nonlinear model and it is a challenging task to find the global optimal solutions for the model. This limitation is a motivation for the following section. The purpose of this section is to develop a linear model which simultaneously maximizes technical, ecological and process environmental efficiency ratios. We wish to maximize three different efficiency ratios $\frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{i=1}^{m} v_i x_{io}^{P-UR} + \sum_{f=1}^{z} g_f x_{fo}^{P-R}}$, $\frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{i=1}^{m} \mu_k y_{ko}^{b}}$, and $\frac{\sum_{f=1}^{z} g_f x_{fo}^{P-R}}{\sum_{k=1}^{p} \mu_k y_{ko}^{b}}$. In order to maximize the first ratio ($\frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{i=1}^{m} v_i x_{io}^{P-UR} + \sum_{f=1}^{z} g_f x_{fo}^{P-R}}$), it is enough

to consider numerator equal to 1 and then minimize the difference between denominator and numerator $\left(\sum_{i=1}^{m} v_i\, x_{io}^{P-UR} + \sum_{f=1}^{z} g_f\, x_{fo}^{P-R} - \sum_{r=1}^{s} u_r\, y_{ro}^{g}\right)$. By fixing $\sum_{r=1}^{s} u_r\, y_{ro}^{g}$ in the first ratio, we can maximize the second ratio by minimizing $\sum_{k=1}^{p} \mu_k\, y_{ko}^{b} - \sum_{r=1}^{s} u_r\, y_{ro}^{g}$ (in Appendix we show how minimizing the difference between denominator and numerator of a ratio leads to maximization of the ratio). By fixing $\sum_{r=1}^{s} u_r\, y_{ro}^{g}$ to 1, the first fraction is minimizing the expression $\sum_{i=1}^{m} v_i\, x_{io}^{P-UR} + \sum_{f=1}^{z} g_f\, x_{fo}^{P-R}$and the second fraction is minimizing the expression $\sum_{k=1}^{p} \mu_k\, y_{ko}^{b}$. Therefore, we can define a linear model which minimizes Expression (16) as the objective function while $\sum_{r=1}^{s} u_r\, y_{ro}^{g} = 1$, $\sum_{r=1}^{s} u_r\, y_{rj}^{g} - \sum_{i=1}^{m} v_i\, x_{io}^{P-UR} - \sum_{f=1}^{z} g_f\, x_{fo}^{P-R} \leq 0$, and $\sum_{r=1}^{s} u_r\, y_{rj}^{g} - \sum_{k=1}^{p} \mu_k\, y_{kj}^{b} \leq 0$ are used as the constraints of the model.

$$\left(\left(\sum_{i=1}^{m} v_i\, x_{io}^{P-UR} + \sum_{f=1}^{z} g_f\, x_{fo}^{P-R} - \sum_{r=1}^{s} u_r\, y_{ro}^{g}\right) + \left(\sum_{k=1}^{p} \mu_k\, y_{ko}^{b} - \sum_{r=1}^{s} u_r\, y_{ro}^{g}\right)\right) \qquad (16)$$

Using optimal weights of this linear programming problem we can measure the technical and ecological efficiency of the DMU under observation. However, we are also interested to observe DMUs performance in the $\frac{\sum_{f=1}^{z} g_f x_{fo}^{P-R}}{\sum_{k=1}^{p} \mu_k y_{ko}^{b}}$ ratio. If we simply add the difference between the denominator and the numerator of this ratio to the objective function and also $\sum_{f=1}^{z} g_f\, x_{fo}^{P-R} - \sum_{k=1}^{p} \mu_k\, y_{kj}^{b} \leq 0$ as a constraint, DMUs which are able to produce less pollution by using more pollution-related inputs will be identified. However, $\sum_{f=1}^{z} g_f\, x_{fo}^{P-R}$ has appeared in both the denominator of the first ratio and the numerator of the third ratio. This means that projection of an inefficient DMU on the process environmental quality efficiency frontier may decrease the technical efficiency of the DMU.

We wish to measure the process environmental quality efficiency while keeping the technical efficiency unchanged. We can show and solve this multiple objective problem as a linear goal programming problem. Linear goal programming, introduced by Charnes et al. (1955) and Charnes and Cooper (1961), deals with multiple objectives. In goal programming we set specific goals (here in our problem, the maximum efficiency score of 1) for each of the objectives and then seek a solution to minimize sum of deviations of the objective functions from their related goals (full efficiency

score of 1). Therfore, $\frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{i=1}^{m} v_i x_{io}^{P-UR} + \sum_{f=1}^{z} g_f x_{fo}^{P-R}} = 1$, $\frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{k=1}^{p} \mu_k y_{ko}^{b}} = 1$, and $\frac{\sum_{f=1}^{z} g_f x_{fo}^{P-R}}{\sum_{k=1}^{p} \mu_k y_{ko}^{b}} = 1$, are the goals. In other words, if the DMU under evaluation is technically, ecologically, and process environmentally efficient, the following three conditions are satisfied:

$$\begin{cases} \sum_{r=1}^{s} u_r y_{ro}^{g} - \sum_{i=1}^{m} v_i \, x_{io}^{P-UR} - \sum_{f=1}^{z} g_f \, x_{fo}^{P-R} = 0 \\ \sum_{r=1}^{s} u_r y_{ro}^{g} - \sum_{k=1}^{p} \mu_k y_{ko}^{b} = 0 \\ \sum_{f=1}^{z} g_f \, x_{fo}^{P-R} - \sum_{k=1}^{p} \mu_k y_{ko}^{b} = 0 \end{cases} \tag{17}$$

The objective is to find the weights u_r, v_i, g_f, and μ_k to meet three goals defined by (17). Therefore, a similar objective function defined by (16) can be shown as follows:

$$\min \begin{pmatrix} \text{The deviation from full technical efficiency} + \text{ the deviation from full ecological} \\ \text{efficiency} + \text{the deviation from full environmental quality efficiency} \end{pmatrix} \tag{18}$$

To show (18) algebraically, some auxiliary variables, which are called deviation variables, are defined as follows:

$$\begin{cases} d_o = \sum_{i=1}^{m} v_i \, x_{io}^{P-UR} + \sum_{f=1}^{z} g_f \, x_{fo}^{P-R} - \sum_{r=1}^{s} u_r y_{ro}^{g} \\ d_o' = \sum_{k=1}^{p} \mu_k y_{ko}^{b} - \sum_{r=1}^{s} u_r y_{ro}^{g} \\ d_o'' = \sum_{k=1}^{p} \mu_k y_{ko}^{b} - \sum_{f=1}^{z} g_f \, x_{fo}^{P-R} \end{cases} \tag{19}$$

Therefore, the purpose is to minimize deviations under the target. In a similar effort to simultaneously optimize two ratios of the technical and ecological efficiencies, Mahdiloo et al. (2015) developed a linear model. They did not, however, separate inputs into pollution-related and pollution-unrelated categories, and also the process environmental quality efficiency ratio was not considered in their model. The same approach was later used by Mahdiloo et al. (2016) to jointly measure and optimize efficiencies of activities which are made of two stages. The approach used by Mahdiloo et al. (2015) and Mahdiloo et al. (2016) to maximize two efficiency ratios by minimizing two deviation variables was originally suggested by Li and Reeves (1999) but for maximizing only one efficiency ratio.

By considering all the goals as roughly of equal importance, non-preemptive goal programming can be modeled and solved. In non-preemptive goal programming,

penalty weights of missing all these goals are equal to each other. However, if we wish to measure process environmental quality efficiency without deteriorating technical efficiency, preemptive goal programming should be used for this purpose. Using preemptive goal programming, we will allocate primary importance to technical and ecological efficiencies, ensuring their first priority of attention. Therefore, the initial focus is on achieving, as closely as possible, to technical and ecological efficiency scores of 1, after which the second priority goal (process environmental quality efficiency) will be considered.

The two ways of dealing with multiple goals in a preemptive goal programming approach include sequential and streamlined procedures. In the sequential approach, the model is run only by the first priority goal(s). If the obtained solution is unique, it will be accepted without considering other goals. Otherwise, if there are multiple optimal solutions, we can break the ties by considering other goals. On the other hand, the streamlined procedure allows finding solutions by one run rather than running a sequence of linear programming models (Hillier and Lieberman, 2005). We can find optimal solution(s) based solely on technical and ecological goals and break the ties among multiple optimal solutions (if there is any) by considering the process environmental quality efficiency goal. Using the basic idea of streamlined preemptive goal programming, we need to multiply the first goals (technical and ecological efficiencies) by an extremely large and positive penalty weight like M. Given penalty weights in the objective function and also previously defined constraints, the multiple objective model is developed as follows:

$$
\begin{aligned}
\min \quad & M d_o + M' d_o' + d_o'' \\
\text{s.t.} \quad & \textstyle\sum_{r=1}^{s} u_r\, y_{ro}^{g} = 1 \\
& \textstyle\sum_{i=1}^{m} v_i\, x_{ij}^{P-UR} + \sum_{f=1}^{z} g_f\, x_{fj}^{P-R} - \sum_{r=1}^{s} u_r\, y_{rj}^{g} \geq 0 && j = 1, 2, \dots, n, \\
& \textstyle\sum_{i=1}^{m} v_i\, x_{io}^{P-UR} + \sum_{f=1}^{z} g_f\, x_{fo}^{P-R} - \sum_{r=1}^{s} u_r\, y_{ro}^{g} - d_o = 0 \\
& \textstyle\sum_{k=1}^{p} \mu_k\, y_{kj}^{b} - \sum_{r=1}^{s} u_r\, y_{rj}^{g} \geq 0 && j = 1, 2, \dots, n, \\
& \textstyle\sum_{k=1}^{p} \mu_k\, y_{ko}^{b} - \sum_{r=1}^{s} u_r\, y_{ro}^{g} - d_o' = 0 \\
& \textstyle\sum_{k=1}^{p} \mu_k\, y_{kj}^{b} - \sum_{f=1}^{z} g_f\, x_{fj}^{P-R} \geq 0 && j = 1, 2, \dots, n, \\
& \textstyle\sum_{k=1}^{p} \mu_k\, y_{ko}^{b} - \sum_{f=1}^{z} g_f\, x_{fo}^{P-R} - d_o'' = 0 \\
& u_r \geq 0 && r = 1, 2, \dots, s, \\
& v_i \geq 0 && i = 1, 2, \dots, m, \\
& g_f \geq 0 && f = 1, 2, \dots, z, \\
& \mu_k \geq 0 && k = 1, 2, \dots, p, \\
& d_o \geq 0 \\
& d_o' \geq 0 \\
& d_o'' \geq 0.
\end{aligned}
\tag{20}
$$

Basically, the efficiency score is calculated using the optimal weights of the factors. To maximize the efficiency ratio, optimal weights are determined via a linear programming model. For example, consider the ecological efficiency ratio which is $\frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{k=1}^{p} \mu_k y_{ko}^{b}}$. The linear programming model seeks the best possible weights for u_r and μ_k to maximize the ecological efficiency ratio. Optimal weights of u_r should make the $\sum_{r=1}^{s} u_r\, y_{ro}^{g}$ equal to one and optimal weights of μ_k should make the $\sum_{k=1}^{p} \mu_k\, y_{ko}^{b} \geq 1$. This idea is slightly different in its application for the process environmental quality efficiency ratio ($\frac{\sum_{f=1}^{z} g_f x_{fo}^{P-R}}{\sum_{k=1}^{p} \mu_k y_{ko}^{b}}$), since this ratio has a common expression ($\sum_{f=1}^{z} g_f\, x_{fo}^{P-R}$) with the technical efficiency ratio. The deviation of each DMU from the full technical efficiency of 1 (d_o) is multiplied by an extremely large penalty weight (M) and the optimal amount of the $\sum_{f=1}^{z} g_f\, x_{fo}^{P-R}$ is determined to maximize the technical efficiency score of each DMU. Then, if there are multiple optimal solutions for μ_k, the tie is broken by maximizing the process environmental quality efficiency score[i].

The Model (20) is a multiple objective DEA model which finds solutions for u_r, v_i, g_f, and μ_k to maximize ratios $\frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{i=1}^{m} v_i x_{io}^{P-UR} + \sum_{f=1}^{z} g_f x_{fo}^{P-R}}$, $\frac{\sum_{r=1}^{s} u_r y_{ro}^{g}}{\sum_{i=1}^{m} \mu_k y_{ko}^{b}}$, and $\frac{\sum_{f=1}^{z} g_f x_{fo}^{P-R}}{\sum_{k=1}^{p} \mu_k y_{ko}^{b}}$ with the condition that all three ratios are less than or equal to 1. This model is linear and measures the technical, ecological, and process environmental quality efficiencies

in one step. This model suggests decreasing pollution-related inputs by minimizing d_o. In other words, minimizing d_o will lead to a natural reduction of undesirable outputs. Also, minimizing d_o'' suggests an increase in pollution-related inputs in the process environmental quality efficiency ratio. Minimizing d_o'' will lead to a managerial and corporate effort to decrease undesirable outputs.

After running the model and finding optimal weights, technical, ecological, process environmental quality and overall efficiency scores can be calculated by (21) – (24).

$$\text{Technical efficiency } = \frac{\sum_{r=1}^{s} u_r^* y_{ro}^g}{\sum_{i=1}^{m} v_i^* x_{io}^{P-UR} + \sum_{f=1}^{z} g_f^* x_{fo}^{P-R}} \tag{21}$$

$$\text{Ecological efficiency } = \frac{\sum_{r=1}^{s} u_r^* y_{ro}^g}{\sum_{k=1}^{p} \mu_k^* y_{so}^b} \tag{22}$$

$$\text{Process environmental quality efficiency } = \frac{\sum_{f=1}^{z} g_f^* x_{fo}^{P-R}}{\sum_{k=1}^{p} \mu_k^* y_{so}^b} \tag{23}$$

$$\text{Overall efficiency } = \frac{1}{3} \times \left(\frac{\sum_{r=1}^{s} u_r^* y_{ro}^g}{\sum_{i=1}^{m} v_i^* x_{io}^{P-UR} + \sum_{f=1}^{z} g_f^* x_{fo}^{P-R}} + \frac{\sum_{r=1}^{s} u_r^* y_{ro}^g}{\sum_{k=1}^{p} \mu_k^* y_{so}^b} + \frac{\sum_{f=1}^{z} g_f^* x_{fo}^{P-R}}{\sum_{k=1}^{p} \mu_k^* y_{so}^b} \right) \tag{24}$$

To find the overall efficiency score, we use the arithmetic mean of the technical, ecological, and process environmental quality efficiency scores. Since we may not have at least one DMU with the overall efficiency score equal to 1, averaged overall efficiency scores are normalized. Overall efficiency scores are normalized by dividing them by the maximum score of all DMUs.

As an alternative to (21) - (24), we could also use the values of d_o, d_o', and d_o'' to measure technical, ecological, and process environmental quality efficiency scores as follows:

Since technical efficiency $= \frac{\sum_{r=1}^{s} u_r^* y_{ro}^g}{\sum_{i=1}^{m} v_i^* x_{io}^{P-UR} + \sum_{f=1}^{z} g_f^* x_{fo}^{P-R}}$, $\sum_{r=1}^{s} u_r^* y_{ro}^g = 1$, and $\sum_{i=1}^{m} v_i^* x_{io}^{P-UR} + \sum_{f=1}^{z} g_f^* x_{fo}^{P-R} = 1 + d_o$, the technical efficiency can be calculated by:

$$\text{Technical efficiency } = \frac{1}{1+d_o} \tag{25}$$

Since ecological efficiency $= \frac{\sum_{r=1}^{s} u_r^* y_{ro}^g}{\sum_{k=1}^{p} \mu_k^* y_{so}^b}$, $\sum_{r=1}^{s} u_r^* \, y_{ro}^g = 1$, and $\sum_{k=1}^{p} \mu_k^* \, y_{so}^b = 1 + d_o'$, the ecological efficiency can be measured by:

$$\text{Ecological efficiency} = \frac{1}{1+d_o'} \tag{26}$$

Since process environmental quality $= \frac{\sum_{f=1}^{z} g_f^* x_{fo}^{P-R}}{\sum_{k=1}^{p} \mu_k^* y_{so}^b}$, $\sum_{k=1}^{p} \mu_k^* \, y_{so}^b = 1 + d_o'$, $\sum_{f=1}^{z} g_f^* \, x_{fo}^{P-R} = \sum_{k=1}^{p} \mu_k^* \, y_{so}^b - d_o''$, and $\sum_{f=1}^{z} g_f^* \, x_{fo}^{P-R} = 1 + d_o' - d_o''$, the process environmental quality efficiency can be measured by:

$$\text{Process environmental quality efficiency} = \frac{1+d_o'-d_o''}{1+d_o'} \tag{27}$$

Finally, overall efficiency is calculated as follows:

$$\text{Overall Efficiency} = \frac{1}{3} \times \left(\frac{1}{1+d_o} + \frac{1}{1+d_o'} + \frac{1+d_o'-d_o''}{1+d_o'} \right) \tag{28}$$

5. Numerical example and validation of the new model

Here, a simple numerical example is presented to validate the developed model. Assume that there are 10 power plants with one input (energy consumption), one desirable output (electricity generation), and one undesirable output (CO_2). Energy and CO_2 are measured in terms of tons and electricity is measured in terms of KWh. Table 3 shows the data set.

Table 3. Data set

DMU	x_1	y_1^b	y_1^g
1	1	100	1
2	20	10	1
3	30	10	1
4	300	100	1
5	100	1	1
6	180	60	2
7	100	50	2
8	70	70	2
9	90	40	2
10	20	80	2

First, we run five different models developed by Sueyoshi and Goto (2010; 2011) and show the results in Table 4.

Table 4. Efficiency scores obtained from the models developed by Sueyoshi and Goto (2010; 2011)

DMU	Model (8)	Model (9)	Model (10)	Model (11)	Model (12)	Model (8) with CRS assumption
1	1.0000	0.3344	0.3344	0.5563	0.3654	1.0000
2	0.5000	0.8208	0.8208	0.5472	0.8464	-8.5000
3	0.4833	0.8375	0.8375	0.5472	0.8575	-13.5000
4	0.0318	1.0000	0.1656	0.3545	0.4437	-148.5000
5	0.3662	1.0000	1.0000	0.5775	1.0000	-48.5000
6	0.7325	0.9020	0.5682	0.7563	0.8323	-88.0000
7	0.8662	0.7525	0.7525	0.7458	0.9552	-48.0000
8	0.9164	0.6013	0.6133	0.6785	0.8767	-33.0000
9	0.8830	0.7863	0.7314	0.7795	1.0000	-43.0000
10	1.0000	0.4672	0.6464	0.6448	0.7873	-8.0000

The last column of Table 4 shows the technical efficiency score of DMUs when convexity constraint $\sum_{j=1}^{n} \lambda_j = 1$ is removed from the RAM model. In other words, the model is solved assuming CRS. As is shown, the CRS efficiency score is positive only for DMU 1. In the next stage, we run our developed Model (20) with M and M' equal to 10,000 and show the results in Table 5.

Table 5. Efficiency scores obtained from the developed model

DMU	Technical efficiency	Ecological efficiency	Process environmental quality efficiency	Overall efficiency
1	1.0000	0.0001	0.0001	0.9806
2	0.0500	0.0010	0.0200	0.0696
3	0.0333	0.0010	0.0300	0.0630
4	0.0033	0.0001	0.0300	0.0327
5	0.0100	0.0100	1.0000	1.0000
6	0.0111	0.0003	0.0300	0.0406
7	0.0200	0.0004	0.0200	0.0396
8	0.0286	0.0003	0.0100	0.0381
9	0.0222	0.0005	0.0225	0.0443
10	0.1000	0.0003	0.0025	0.1008

To validate the efficiency scores obtained from the developed model, some comparisons between the results of the model with the models developed by Sueyoshi and Goto (2010; 2011) are required.

- *Technical efficiency scores obtained from Model (8) versus the developed Model (20):*

Both models deal with inputs and desirable outputs. Both models identified DMU 1 as technically efficient. Based on Model (8), DMU 10 is also technically efficient. The

efficiency frontier in Model (8) is made by DMUs 1 and 10 while in the developed model is made by DMU 1. The efficiency scores of the other DMUs are calculated by their distance from the DMUs on the frontier. The average technical efficiency obtained by Model (8) is 0.68 and by the developed model is 0.13. The developed model has better discriminatory power than Model (8). To validate the technical efficiency scores obtained from the developed model, we can measure technical efficiency of each DMU relative to technically efficient DMU 1. The following ratio gives the same scores obtained by Model (20).

$$\text{Technical efficieny} = \frac{\dfrac{weighted\ sum\ of\ desirable\ outputs\ of\ DMU_o}{weighted\ sum\ of\ inputs\ of\ DMU_o}}{\dfrac{weighted\ sum\ of\ desirable\ outputs\ of\ DMU_1}{weighted\ sum\ of\ inputs\ \ of\ DMU_1}}$$

In this ratio, the relative efficiency of each DMU is calculated relative to the efficient DMU 1. Rankings of DMUs by the developed model and Model (8) are different. For example, using Model (8), DMUs 2 and 7 are ranked seventh and fifth, respectively. Using our developed model, rankings of the DMUs 2 and 7 are third and seventh, respectively. In other words, using the developed model, DMU 2 is better than DMU 7. Comparing inputs and outputs of these two DMUs with the technically efficient DMU 1 indicates that DMU 2 is relatively better than DMU 7. Using Model (8), efficiency scores of DMUs 2 and 7 should be calculated relative to both DMUs 1 and 10, or combination of them. Since the intensity variable (λ_j) corresponding to technically efficient DMU 10 for technically inefficient DMUs 2 and 7 is equal to one, their efficiency scores should only be calculated relative to DMU 10. The efficiency scores found in this manner contradict the results of the Model (8), with DMU 7 achieving a better rank than DMU 2.

- *Results of Models (9) and (10) versus the developed Model (20):*

Sueyoshi and Goto (2010; 2011) developed both Models (9) and (10) to measure the environmental efficiency of DMUs. This measure is termed 'process environmental quality efficiency' in our model. The first very important point to make is that the efficiency scores and ranking results obtained from Models (9) and (10), which were developed for the same purpose, are not stable. Spearman correlation coefficient between ranking results of these two models is 0.22. Therefore, there is no significant correlation between ranking results of these two models. For example, by shifting from Model (9) to Model (10), the environmental efficiency of DMU 4 is changed from 1 to

0.166. This shows that the results are sensitive to the used model. Models (9) and (10), and also our developed Model (20), identified DMU 5 as efficient. The ranking results of our developed model are significantly correlated with the Model (9) which is also a linear model. As another example, consider DMUs 3 and 4. DMU 3 consumes 30 units of inputs and produces 10 units of undesirable outputs. DMU 4 consumes 10 times more inputs than DMU 3 (300 units of inputs) and produces 10 times more undesirable outputs (100 units of the undesirable output). Based on the developed model, these two DMUs have the same efficiency scores. This is consistent with the CRS assumption of the developed model.

Based on the developed model there is only one process environmental quality efficient DMU (DMU 5). The efficiencies of the other DMUs are calculated by their distance from this DMU. The following ratio gives the same scores found by the developed model.

$$\text{Process environmental quality efficiency} = \frac{\dfrac{weighted\ sum\ of\ inputs\ of\ DMU_o}{weighted\ sum\ of\ undesirable\ outputs\ of\ DMU_o}}{\dfrac{weighted\ sum\ of\ inputs\ of\ DMU_5}{weighted\ sum\ of\ undesirable\ outputs\ of\ DMU_5}}$$

- *Results of Models (11) and (12) vs. the overall efficiency score obtained by the developed Model (20):*

Both Models (11) and (12) are developed by Sueyoshi and Goto (2010; 2011) to measure overall efficiency of DMUs. The Spearman correlation coefficient between ranking results of DMUs obtained by these two models is 0.51. The efficiency scores and rankings of DMUs are not stable when the evaluation model is changed from Model (11) to Model (12). DMU 5 is identified as an overall efficient DMU by our developed model and also Model (12).

6. Testing the developed model on a real data set of China's provinces

In this section, we test the developed model using a data set related to China's provinces. Efficiency evaluation of Chinese provinces in terms of their used resources and generated desirable and undesirable outputs has been studied by various authors previously. Li et al. (2013), for example, measured China's regional environmental efficiency by Super-SBM model. They considered undesirable outputs in the Super-SBM model, and also used Tobit regression model to identify most influential factors in environmental efficiency of the regions. Wu et al. (2013b) used DEA to measure

congestion in China's regions. They combined the models developed by Seiford and Zhu (2002) and Wei and Yan (2004) and measured congestion in the presence of desirable and undesirable outputs. Song et al. (2013) used Super-SBM and measured energy efficiency of BRICS' economies (i.e. Brazil, Russia, India, China, and South Africa). They discussed that in their small sample, DEA led to biased estimates. Therefore, they used bootstrap-DEA to modify efficiencies. Bootstrap-DEA was used for the statistical test by providing confidence intervals of efficiency measures. Zhang et al. (2008), Bian and Yang (2010), and Wang et al. (2012) are also other researchers who worked in this area. These studies contributed to research stream in efficiency evaluation of China's provinces and provided us with an adequate understanding of the problem.

The inputs and outputs used in this section are similar to Zhang et al. (2008). The four inputs of this research include water resources (x_1^{P-UR}), fixed assets (x_2^{P-UR}), number of entities[3] (x_1^{P-R}), and energy (x_2^{P-R}). The gross regional product (y_1^g) is the desirable output and chemical oxygen demand (COD) (y_1^b), sulfur dioxide emission (SO_2) (y_2^b), soot (y_3^b), dust (y_4^b), and solid waste (y_5^b) are considered as undesirable outputs. To validate our developed model, the third and the fourth inputs are considered as pollution-related inputs. Table 6 shows the dataset which is collected from China Statistical Yearbook and China Energy Statistical Yearbook for the year 2011. The same data set was also used by Mahdiloo et al. (2014). Tibet is not included in our analysis since some of its data were unavailable.

[3] This factor refers to number of legal entities working in industry. This factor does not include number of international organizations.

Table 6. Data set for provinces in China

Provinces	x_1^{P-UR}	x_2^{P-UR}	x_1^{P-R}	x_2^{P-R}	y_1^g	y_1^b	y_2^b	y_3^b	y_4^b	y_5^b
Beijing	23.08	5402.95	38.45	6954.05	14113.58	9.20	5.68	2.13	1.66	1269
Tianjin	9.20	6278.09	16.85	6818.08	9224.46	13.20	21.76	5.38	0.80	1862
Hebei	138.92	15083.35	34.58	27531.11	20394.26	54.61	99.42	32.26	32.09	31688
Shanxi	91.55	6063.17	20.52	16808.03	9200.86	33.31	114.71	43.23	36.51	18270
Inner Mongolia	388.54	8926.46	13.85	16820.30	11672.00	27.51	119.30	47.59	16.04	16996
Liaoning	606.67	16043.03	36.90	20946.52	18457.27	54.16	85.94	39.79	16.73	17273
Jilin	686.68	7870.38	11.80	8297.31	8667.58	35.22	30.06	20.96	5.30	4642
Heilongjiang	853.48	6812.56	17.58	11233.51	10368.60	44.44	41.71	29.68	5.71	5405
Shanghai	36.81	5108.90	39.95	11201.13	17165.98	21.98	22.15	4.18	0.97	2448
Jiangsu	383.53	23184.28	86.22	25773.71	41425.48	78.80	100.24	29.91	15.11	9064
Zhejiang	1398.55	12376.04	68.55	16865.29	27722.31	48.68	65.39	16.54	13.92	4268
Anhui	922.82	11542.94	24.26	9706.60	12359.33	41.11	48.39	20.74	26.37	9158
Fujian	1652.71	8199.12	30.78	9808.52	14737.12	37.26	39.12	10.00	14.01	7487
Jiangxi	2275.49	8772.27	18.46	6354.88	9451.26	43.11	47.10	13.90	22.36	9407
Shandong	309.12	23280.52	77.36	34807.77	39169.92	62.05	138.29	29.12	18.94	16038
Henan	534.89	16585.86	40.07	21437.76	23092.36	61.97	116.29	47.37	22.70	10714
Hubei	1268.72	10262.70	34.58	15137.59	15967.61	57.23	51.60	14.51	14.65	6813
Hunan	1906.61	9663.58	29.72	14880.06	16037.96	79.81	62.74	23.50	39.44	5773
Guangdong	1998.79	15623.70	80.28	26908.02	46013.06	85.84	98.91	25.33	10.43	5456
Guangxi	1823.57	7057.56	20.50	7918.97	9569.85	93.69	84.80	25.01	31.77	6232
Hainan	479.82	1317.04	3.76	1358.51	2064.50	9.23	2.82	0.65	0.66	212
Chongqing	464.30	6688.91	17.79	7855.52	7925.58	23.45	57.27	10.21	8.36	2837
Sichuan	2575.29	13116.72	35.30	17891.83	17185.48	74.08	93.76	25.97	14.14	11239
Guizhou	956.54	3104.92	11.03	8175.43	4602.16	20.79	63.78	11.32	8.64	8188
Yunnan	1941.45	5528.71	17.09	8674.17	7224.18	26.83	43.96	8.92	9.18	9392
Shaanxi	507.50	7963.67	20.67	8882.11	10123.48	30.77	70.70	11.50	18.55	6892
Gansu	215.25	3158.34	10.84	5923.13	4120.75	16.76	45.25	9.81	9.26	3745
Qinghai	741.11	1016.87	2.83	2568.26	1350.43	8.31	13.31	5.19	9.75	1783
Ningxia	9.32	1444.16	3.60	3681.10	1689.65	12.17	28.04	13.62	6.09	2465
Xinjiang	1113.14	3423.24	9.67	8290.20	5437.47	29.60	51.84	24.84	18.46	3914

*x_1^{P-UR} :100 million cu.m; x_2^{P-UR}: 100 million Yuan; x_1^{P-R}: 10000 units; x_2^{P-R}: 10000 tons of Standard coal equivalent; y_1^g: 100 million Yuan; y_1^b: 10000 tons; y_2^b: 10000 tons; y_3^b: 10000 tons; y_4^b: 10000 tons; y_5^b: 10000 tons.

We first run Models (8) - (12) and show the results in Table 7. All these models measure efficiency scores in a range between 0 and 1, where 1 shows full efficiency score. The Model (8) measures technical efficiency. Twelve provinces are recognized as technically efficient DMUs and average technical efficiency of all provinces is 0.93. The Models (9) and (10) measure the environmental efficiency of provinces. Due to the constraint $d_i^{x+}d_i^{x-} = 0$, Model (10) is a nonlinear model.

The environmental efficiency of provinces and their corresponding rankings are not stable when environmental efficiency model is changed from Model (9) to Model (10). Spearman correlation coefficient between ranking results obtained by these two models is 0.15. Based on Model (9), there are 16 environmentally efficient provinces (DMUs) and average environmental efficiency of provinces is 0.93. Given Model (10), there are only 8 environmentally efficient provinces and average environmental efficiency is also decreased to 0.76. The same analyses are provided for overall efficiency scores obtained by Models (11) and (12). Spearman correlation coefficient is

equal to 0.51. The average overall efficiency of provinces obtained by Models (11) and (12) are 0.90 and 0.81, respectively.

Table 7. Technical, environmental, and overall efficiency scores obtained by models developed by Sueyoshi and Goto (2010; 2011)

Provinces	Model (8)	Model (9)	Model (10)	Model (11)	Model (12)
Beijing	1.00	1.00	0.88	1.00	1.00
Tianjin	1.00	1.00	1.00	1.00	1.00
Hebei	1.00	1.00	0.37	1.00	0.58
Shanxi	0.92	0.66	0.50	0.66	0.59
Inner Mongolia	1.00	1.00	0.58	1.00	0.62
Liaoning	0.85	0.85	0.56	0.79	0.66
Jilin	1.00	0.90	0.80	0.91	0.81
Heilongjiang	0.93	0.87	0.75	0.85	0.76
Shanghai	1.00	1.00	0.83	1.00	1.00
Jiangsu	1.00	1.00	0.38	1.00	0.83
Zhejiang	0.89	1.00	0.57	0.94	0.89
Anhui	0.88	0.88	0.68	0.84	0.71
Fujian	0.86	1.00	0.76	0.93	0.79
Jiangxi	0.81	1.00	1.00	0.90	1.00
Shandong	1.00	1.00	1.00	1.00	1.00
Henan	0.96	0.80	1.00	0.80	1.00
Hubei	0.84	0.94	0.70	0.86	0.75
Hunan	0.83	1.00	0.55	0.92	0.61
Guangdong	1.00	1.00	1.00	1.00	1.00
Guangxi	0.83	0.75	0.55	0.69	0.59
Hainan	1.00	1.00	1.00	1.00	1.00
Chongqing	0.93	0.90	0.84	0.87	0.84
Sichuan	0.72	1.00	0.53	0.86	0.61
Guizhou	0.90	0.89	0.78	0.85	0.78
Yunnan	0.81	1.00	0.76	0.91	0.77
Shaanxi	0.93	0.85	0.76	0.83	0.78
Gansu	0.96	0.87	1.00	0.86	0.88
Qinghai	1.00	1.00	1.00	1.00	1.00
Ningxia	1.00	0.86	0.86	0.87	0.84
Xinjiang	0.91	0.86	0.72	0.83	0.73

In the next step, the technical, ecological, process environmental quality, and overall efficiency scores of provinces are measured by Model (20) and the results are depicted in Table 8. The M and M' are set to 10,000.

Table 8. Technical, ecological, process environmental quality, and overall efficiency scores obtained by the developed Model (20)

Provinces	Technical efficiency	Ecological efficiency	Process environmental quality efficiency	Overall efficiency
Beijing	1.000	1.000	1.000	1.000
Tianjin	1.000	0.801	0.777	0.859
Hebei	0.985	0.243	0.152	0.460
Shanxi	0.842	0.180	0.145	0.389
Inner Mongolia	1.000	0.277	0.143	0.473
Liaoning	0.762	0.222	0.174	0.386
Jilin	0.771	0.180	0.099	0.350
Heilongjiang	0.762	0.192	0.120	0.358
Shanghai	1.000	1.000	0.941	0.980
Jiangsu	1.000	0.411	0.400	0.604
Zhejiang	0.891	0.584	0.656	0.710
Anhui	0.772	0.196	0.254	0.407
Fujian	0.869	0.258	0.297	0.475
Jiangxi	0.875	0.143	0.163	0.394
Shandong	0.951	0.411	0.294	0.552
Henan	0.913	0.243	0.165	0.440
Hubei	0.683	0.211	0.168	0.354
Hunan	0.759	0.250	0.170	0.393
Guangdong	1.000	0.758	0.758	0.839
Guangxi	0.728	0.138	0.190	0.352
Hainan	0.903	0.876	0.970	0.916
Chongqing	0.673	0.251	0.311	0.412
Sichuan	0.640	0.151	0.114	0.302
Guizhou	0.627	0.144	0.127	0.299
Yunnan	0.596	0.176	0.152	0.308
Shaanxi	0.755	0.214	0.238	0.402
Gansu	0.627	0.160	0.149	0.312
Qinghai	0.638	0.106	0.081	0.275
Ningxia	0.819	0.091	0.071	0.327
Xinjiang	0.758	0.125	0.082	0.322

This model is run for each province to find the best weights of factors that maximize technical, process environmental quality, ecological, and overall efficiency scores. To find different types of efficiency scores, either u_r^*, v_i^*, g_f^*, μ_k^* or d_o^*, $d_o^{*\prime}$, and $d_o^{*\prime\prime}$ can be used in (21)-(24) or in (25)-(28), respectively. Only one province, Beijing, is identified as overall efficient. The average technical, ecological, process environmental quality and overall efficiency scores of provinces are equal to 0.82, 0.33, 0.31, and 0.49, respectively.

In summary, we suggest using Model (20) as an alternative to Models (8) – (12) and justify its use for the following reasons. First, running Models (8) – (12) with the CRS assumption is not recommended as these models might determine negative efficiency score for some DMUs. Second, Sueyoshi and Goto (2010; 2011) have proposed two different models (Models 9 and 10) to measure the environmental

efficiency of DMUs and two different models (Models 11 and 12) to measure overall efficiency while they may lead to different results.

Figure 2 shows the environmental efficiency of DMUs with Models (9) and (10). It is easy to see that the environmental efficiency scores are significantly different with these two models. Obviously, this creates confusion in using these models. More importantly, Model (12) itself can be run in three different ways (as a linear, nonlinear or mixed-integer programming model) which will add to the complexity and confusion in the results. When Model (12) is run as a linear model (without the constraint $d_i^{x+} d_i^{x-} = 0$), it might generate an unbounded solution. This is explained by Sueyoshi and Goto (2011) and is confirmed in our Chinese provinces example where Model (12) is unbounded for DMU #1. At the same time, when the model is run as a nonlinear or mixed integer programming, the model can produce the local optimal solution.

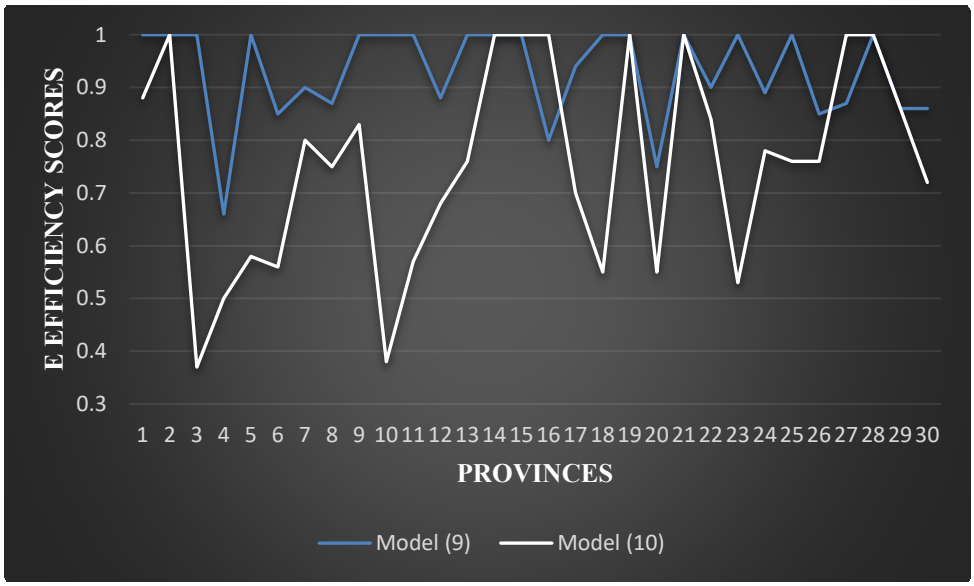

Figure 2. Environmental efficiency scores; Model (9) vs. Model (10)

Third, Sueyoshi and Goto (2010; 2011) do not include the ecological efficiency of the DMUs in their overall efficiency assessment. This is done in the developed model which has included three different efficiency measures in measuring the overall efficiency.

It is apparent that the proposed model, with its strong discriminating power, offers more insightful benchmark exercises for the Chinese provincial data. This subsequently leads to more meaningful decisions and policy development at the provincial level. China is a large country and there are many differences among provinces, including

geography, resources, workforce, and level of economic development, etc. Finding the right benchmark or model of economic development would be highly important for provincial governments. Provinces can rely on the proposed DEA model to identify provinces that suit their development needs and 'borrow' experience from their benchmark counterparts. This, at the same time, helps reduce the possibility of trying to replicate models or policies from unsuitable benchmarks. The benchmarking exercise also provides the opportunity for provinces to check their performance gaps, therefore targeting their efforts accordingly. Actions could include either adjusting the resources for inputs or targeting efficiency improvement to generate more good outputs or less bad outputs. The DEA model can also be applied within each province with a finer granularity, such as at the city level.

7. Concluding remarks

Our paper addresses the problem of modeling undesirable outputs in DEA. This problem was discussed in the literature and was considered in a variety of models. RAM is one of the models which has been used in the literature for this purpose. We showed that RAM measures overall efficiency of DMUs in three steps, where, in the first two steps, technical and environmental efficiencies are measured and in the last step, these two measures are unified in a single model and the overall efficiency is calculated. Our paper has, therefore, argued against the computational complexity of this approach. We also showed that RAM may find negative efficiency scores when it is run with the CRS assumption.

We considered similar concepts in a multiple objective model and argued its advantages compared to RAM. The multiple objective problem discussed in our paper was solved by linear goal programming. We show this to have more discrimination power than RAM and more valid efficiency scores. The proposed model determined the technical, ecological, process environmental quality, and overall efficiency scores in one step. This is computationally less complex than RAM.

Further research can be done based on the results of this paper. The developed model in this paper was an extension to Korhonen and Luptacik (2004) and Sarkis and Cordeiro (2009) and was developed based on the assumptions used in these two papers. We suggest modelling of weak disposability in our integrated model to be explored in the future.

Acknowledgments

The authors would like to thank two anonymous reviewers for their constructive comments which significantly improved an earlier version of this paper.

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Appendix

Here, we show how minimizing the difference between the denominator and the numerator of an efficiency ratio can lead to maximization of the ratio. Model (1) is the basic CCR model to measure the technical efficiency of the DMU under evaluation. This model in its linear form can be shown as Model (29). This model finds the weights of inputs and outputs to maximize the technical efficiency of each DMU.

$$\begin{array}{lll} \max & \sum_{r=1}^{s} u_r y_{ro}^{g} & \\ \text{s.t.} & \sum_{i=1}^{m} v_i\, x_{io} = 1 & \\ & \sum_{r=1}^{s} u_r y_{rj}^{g} - \sum_{i=1}^{m} v_i\, x_{ij} \le 0 & j = 1, 2, \dots, n, \\ & v_i \ge 0 & i = 1, 2, \dots, m, \\ & u_r \ge 0 & r = 1, 2, \dots, s. \end{array} \tag{29}$$

Also, consider Model (30) which was developed by Li and Reeves (1999) and is an equivalent to the Model (29). This model finds the weights of inputs and outputs to minimize inefficiency score (d_o) of each DMU, where $1-d_o$ gives the same efficiency score found by Model (29).

$$
\begin{array}{lll}
\min & d_o & \\
\text{s.t.} & \sum_{i=1}^{m} v_i\, x_{io} = 1 & \\
 & \sum_{r=1}^{s} u_r y_{ro}^{g} - \sum_{i=1}^{m} v_i\, x_{io} + d_o = 0 & \\
 & \sum_{r=1}^{s} u_r y_{rj}^{g} - \sum_{i=1}^{m} v_i\, x_{ij} \leq 0 & j = 1, 2, \dots, n, \\
 & v_i \geq 0 & i = 1, 2, \dots, m, \\
 & u_r \geq 0 & r = 1, 2, \dots, s.
\end{array}
\tag{30}
$$

On the other hand, moving $\sum_{r=1}^{s} u_r y_{ro}^{g}$ and $\sum_{i=1}^{m} v_i\, x_{io}$ in the second constraint of the Model (31) to the other side of the equation gives:

$$
d_o = \sum_{i=1}^{m} v_i\, x_{io} - \sum_{r=1}^{s} u_r\, y_{ro}^{g} \tag{31}
$$

In other words, to find optimal weights which can maximize efficiency scores (or minimize inefficiency score), we can minimize the difference between the denominator and numerator $\left(\sum_{i=1}^{m} v_i\, x_{io} - \sum_{r=1}^{s} u_r\, y_{ro}^{g}\right)$. When $\sum_{i=1}^{m} v_i\, x_{io} = \sum_{r=1}^{s} u_r\, y_{ro}^{g}$, the DMU under evaluation is efficient. That is why we need to minimize the difference between denominator ($\sum_{i=1}^{m} v_i\, x_{io}$) and the numerator ($\sum_{r=1}^{s} u_r\, y_{ro}^{g}$). The minimum value that the difference between $\sum_{i=1}^{m} v_i\, x_{io}$ and $\sum_{r=1}^{s} u_r\, y_{ro}^{g}$ can get is zero. When the difference is equal to zero, it means that the inefficiency score of the DMU is equal to zero or the DMU is efficient. Also, since $\sum_{i=1}^{m} v_i\, x_{io}$ =1, minimizing $\sum_{i=1}^{m} v_i\, x_{io} - \sum_{r=1}^{s} u_r\, y_{ro}^{g}$ is similar to minimizing $-\sum_{r=1}^{s} u_r\, y_{ro}^{g}$ or maximizing $\sum_{r=1}^{s} u_r\, y_{ro}^{g}$.

---

[i] The $\sum_{f=1}^{z} g_i\, x_{fj}^{P-R}$ is a common expression between the technical and process environmental quality efficiency ratios. Basically, the optimal amount of each variable or an expression like $\sum_{f=1}^{z} g_i\, x_{fj}^{P-R}$ should be determined to maximize its own objective function. However, since d_o is multiplied by M, the expression $\sum_{f=1}^{z} g_f^{*}\, x_{fo}^{P-R}$ is determined to maximize the technical efficiency of each DMU first and then the process environmental quality efficiency. This might lead to having the $\sum_{f=1}^{z} g_f^{*}\, x_{fo}^{P-R}$ =0 and consequently the process environmental quality efficiency of the DMU becomes zero. To avoid zero process environmental quality efficiency, adding a constraint like $\sum_{f=1}^{z} g_i \geq \varepsilon$ is suggested.