UPRIGHT FRAME SHEAR STIFFNESS AND UPRIGHT BIAXIAL BENDING IN THE DESIGN OF COLD-FORMED STEEL STORAGE RACK-SUPPORTED BUILDINGS

NIMA TALEBIAN
B.Eng., M.Sc.

School of Engineering and Built Environment
Griffith University

Submitted in fulfilment of the requirements of the degree of
Doctor of Philosophy

July, 2018
Abstract

Steel storage racks are commonly used worldwide to store goods on pallets and represent freestanding structures to design. Recently, a new type of storage systems has gained popularity in which the rack system supports both the building enclosure and the stored goods. These new rack structures are referred to as “rack-supported buildings” or “clad racks”. Due to combined actions of wind loading and stored pallets, uprights undergo a combination of biaxial bending and compression. Existing design rules may not be adequate for this type of combined loading. Furthermore in clad racks, as the outer rack frames must withstand cross-aisle horizontal actions due to wind loading, accurately determining the transverse shear stiffness of the upright frames is essential. Indeed, this stiffness is needed in calculating the elastic buckling load, performing earthquake design and serviceability checks. This thesis is motivated by the two aforementioned aspects relative to clad racks and investigates first the factor affecting the transverse shear stiffness of steel storage rack upright frames and second the biaxial bending behaviour of the uprights.

International racking design specifications recommend different approaches to evaluate the shear stiffness. The Rack Manufacturers Institute (RMI) specification conservatively uses an analytical solution based on Timoshenko and Gere's theory while the European (EN15512) and Australian (AS4084) specifications recommend experimental testing to be conducted. Discrepancy between Finite Element Analyses (FEA) and experimental test results is likely attributed to the local deformations occurring at the bolted joints. In the first part of this thesis, an advanced FEA model to accurately capture the transverse shear stiffness of upright frames is developed and verified against published experimental test results. Based on the FE model, the factors contributing to the transverse shear deformation of the frames with Cee-bracing members are quantified and discussed for lip-to-lip and back-to-back bracing patterns.
In cold-formed steel structures international specifications, a linear interaction equation is typically used to account for members subject to biaxial bending and may be inaccurate. In the second part of this thesis, the biaxial bending capacity of the uprights is experimentally investigated and the actual interactive relationship between bending of the uprights about the major and minor axes, for local and distortional buckling is determined. Two types of regularly perforated and non-perforated storage rack uprights are investigated. An advanced finite element model to determine the biaxial bending capacity of cold-formed steel storage rack upright sections is validated against the experimental tests and parametric studies are performed to analyse the biaxial response of slender, semi-compact and compact unperforated storage rack upright cross-sections in local and distortional buckling failure modes only. The results from the parametric studies are used to verify the accuracy of different forms of published direct strength method (DSM) equations.
Declaration

This work has not been previously submitted for a degree or diploma in any university. To the best of my knowledge and belief, this thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

Signed:  __  __

Date:  31/07/2018

Nima Talebian

Griffith University
Acknowledgement of Published Papers

Section 9.1 of the Griffith University Code for the Responsible Conduct of Research (“Criteria for Authorship”), in accordance with Section 5 of the Australian Code for the Responsible Conduct of Research, states:

To be named as an author, a researcher must have made a substantial scholarly contribution to the creative or scholarly work that constitutes the research output, and be able to take public responsibility for at least that part of the work they contributed. Attribution of authorship depends to some extent on the discipline and publisher policies, but in all cases, authorship must be based on substantial contributions in a combination of one or more of:

- Conception and design of the research project
- Analysis and interpretation of research data
- Drafting or making significant parts of the creative or scholarly work or critically revising it so as to contribute significantly to the final output.

Section 9.3 of the Griffith University Code (“Responsibilities of Researchers”), in accordance with Section 5 of the Australian Code, states:

Researchers are expected to:

- Offer authorship to all people, including research trainees, who meet the criteria for authorship listed above, but only those people.
- Accept or decline offers of authorship promptly in writing.
- Include in the list of authors only those who have accepted authorship
- Appoint one author to be the executive author to record authorship and manage correspondence about the work with the publisher and other interested parties.
- Acknowledge all those who have contributed to the research, facilities or materials but who do not qualify as authors, such as research assistants, technical staff, and advisors on cultural or community knowledge. Obtain written consent to name individuals.
Included in this thesis are papers in Chapters 3, 4, and 5 which are co-authored with other researchers. My contribution to each co-authored paper is outlined at the front of the relevant chapter. The bibliographic details and status for these papers are:

Chapter 3:

**Talebian N**, Gilbert BP, Baldassino N, Karampour H, Factors contributing to the transverse shear stiffness of bolted cold-formed steel storage rack upright frames with channel bracing members, *Thin-walled Structures*, (Submitted 1st July 2018, under review).

Chapter 4:


Chapter 5:


PhD Candidate:

Date: 31/07/2018___

Principal

Date: 31/07/2018___

Principal Supervisor: Dr Hassan Karampour

v
List of Publications

Journal papers:

Talebian N, Gilbert BP, Baldassino N, Karampour H, Factors contributing to the transverse shear stiffness of bolted cold-formed steel storage rack upright frames with channel bracing members, Thin-walled Structures, (Submitted 1st July 2018, under review).

Talebian N, Gilbert BP, Pham CH, Chariere R, Karampour H, Local and distortional biaxial bending capacities of cold-formed steel storage rack uprights, Journal of Structural Engineering, 144(6), 2018.

Talebian N, Gilbert BP, Pham CH, Chariere R, Karampour H, Parametric studies and design rules for local and distortional biaxial bending of cold-formed steel storage rack uprights, Journal of Structural Engineering (Submitted 31th July 2018, under review).

Conference papers:


Talebian N, Gilbert BP, Pham CH, Chariere R, Local and distortional biaxial bending capacity of perforated cold-formed steel storage rack uprights, 7th International Conference on Coupled Instabilities in Metal structures, Baltimore, Maryland, 2016.

Talebian N, Gilbert BP, Karampour H, Transverse shear stiffness of bolted cold-formed steel storage rack upright frames with channel bracing members, accepted for International Specialty Conference on Cold-Formed Steel Structures, November 7-8, 2018.
Talebian N, Gilbert BP, Pham CH, Chariere R, Karampour H, Biaxial bending of cold-formed steel storage rack uprights - Part I: FEA and parametric studies, accepted for International Specialty Conference on Cold-Formed Steel Structures, November 7-8, 2018.

Talebian N, Gilbert BP, Pham CH, Chariere R, Karampour H, Biaxial bending of cold-formed steel storage rack uprights - Part II: Direct Strength Method, accepted for International Specialty Conference on Cold-Formed Steel Structures, November 7-8, 2018.
Acknowledgements

First and foremost, I would like to express my sincere gratitude to my principal supervisor, Associate Professor Benoit Gilbert for the continuous support of my PhD study and related research, for his patience, motivation, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis. I am very grateful to have had such an excellent supervisor.

I would also like to thank Dr Hassan Karampour my associate supervisor for his encouragement, guidance, support, technical guidance and advice from the initial to final stages of this project.

My sincere thanks also go to Dr Cao Hung Pham from the University of Sydney who have given me invaluable assistance and advice during my study.

I am greatly thankful to Dr Nadia Baldassino from University of Trento for her kind contribution to this research program.

I would like to thank the technical staff; Geoffrey Turner, Chuen Lo, David Bellchambers, Grant Pickering and Ian Underhill for helping me to conduct laboratory experiments.

I thank my fellow postgraduate students; Zia Javanbakht and Hamed Aghaei for all their supports and the fun we have had in the last four years.

Last but not least, I would like to thank my parents for their everlasting moral support and for their financial support during my study, my sister and my brothers. I am grateful to them for supporting me spiritually throughout my study and my life in general, and who provided much love and continuous encouragement to motivate me to its completion. I am proud to have such a great and wonderful family.

This research was made possible through the financial support provided by Griffith University International Postgraduate Research Scholarship (GUIPRS) and Griffith University Postgraduate Research Scholarship (UGPRS).
# Table of Content

Abstract .......................................................................................................................... ii

Declaration .................................................................................................................. iii

Acknowledgement of Published Papers ................................................................. iv

List of Publications .................................................................................................. vi

Acknowledgements .................................................................................................. viii

Table of Content ..................................................................................................... ix

List of Figures ............................................................................................................ xi

List of Tables .............................................................................................................. xviii

List of Symbols ......................................................................................................... xx

CHAPTER 1 ................................................................................................................ 1

Introduction ............................................................................................................... 1
  1.1. Background ........................................................................................................ 1
  1.2 Rack-supported buildings .................................................................................... 2
  1.3 Problems arising in rack supported buildings .................................................... 3
  1.4. Research objectives ........................................................................................ 5
  1.5. Research methodology ................................................................................... 6
  1.6. Thesis outline .................................................................................................. 7

CHAPTER 2 ................................................................................................................ 8

Literature review ....................................................................................................... 8
  2.1 General steel storage racks literature review ................................................... 8
  2.2. International rack design specifications .......................................................... 11
  2.3. Shear stiffness of steel storage racks upright frames ....................................... 17
  2.4. Buckling modes of cold-formed profiles sections ......................................... 26
  2.5. Design methods for thin-walled members ...................................................... 26
  2.6. Biaxial bending of cold-formed members ...................................................... 28
  2.7. Shear stiffness of steel storage upright frames and biaxial bending of the uprights ................................................................. 31

CHAPTER 3 ................................................................................................................ 33

Factors contributing to the transverse shear stiffness of bolted cold-formed steel storage upright frames with channel bracing members ............33
  3.1. Introduction ...................................................................................................... 35
  3.2. Literature review ........................................................................................... 37
3.3. Experimental tests used to verify FE models ........................................41
3.4. Finite element modelling ........................................................................46
3.5. Validation of the FE model and comparison with experimental results ....49
3.6. Contribution of factors affecting the transverse shear stiffness ..............60
3.7. Summary ..................................................................................................67

CHAPTER 4 .....................................................................................................69
Local and distortional biaxial bending capacities of cold-formed steel
storage rack uprights .......................................................................................69
4.1. Introduction ..............................................................................................71
4.2. Test program (methodology) ...................................................................74
4.3. Test Results ..............................................................................................82
4.4. Direct strength method equations to predict bending capacity ..........92
4.5. Comparison of Direct Strength Method design with test results ..........97
4.6. Summary ..................................................................................................104

CHAPTER 5 .....................................................................................................105
Parametric studies and design rules for local and distortional biaxial
bending of cold-formed steel storage rack uprights ........................................105
5.1. Introduction ..............................................................................................107
5.2. Published experimental tests (background) .............................................108
5.3. Finite element model ..............................................................................110
5.4. Validation of the FE model .....................................................................114
5.5. Parametric studies ..................................................................................118
5.6. Biaxial bending response of the uprights and interactive behaviour ......123
5.7. Direct Strength Method equations to predict biaxial bending capacity …128
5.8. Comparison of direct strength method design approaches with parametric
results .............................................................................................................131
5.9. Summary ................................................................................................136

CHAPTER 6 .....................................................................................................138
Conclusions .....................................................................................................138
6.1. Thesis findings .........................................................................................138
6.2. Recommendations for future research .....................................................140

References .....................................................................................................142

Appendix-A Biaxial bending test setup ..........................................................151
Appendix-B Biaxial bending – Parametric studies .........................................152
List of Figures

Figure 1.1 A selective rack structure (Sajja et al., 2008).................................2
Figure 1.2 Elements of a typical rack structure (Gilbert et al., 2012)..............3
Figure 1.3 A rack supported building or Clad rack (Courtesy Modulblok)........4
Figure 2.1 Selective rack (Freitas et al., 2005)............................................9
Figure 2.2 Drive-in rack (Freitas et al., 2005).............................................9
Figure 2.3 Push back rack system (Freitas et al., 2005)...............................10
Figure 2.4 Flow rack (Freitas et al., 2005)................................................11
Figure 2.5 Example of beam end connector (Gilbert, 2010)..........................12
Figure 2.6 Cantilever test set-up from AS4084 (2012).................................13
Figure 2.7 Cantilever test set up from the EN15512 (2009)..........................14
Figure 2.8 Base plate assembly (Gilbert 2010)..........................................15
Figure 2.9 Upright frame shear stiffness test set-up from EN15512 (2009).....18
Figure 2.10 Alternative test set up from AS4084 (2012).............................19
Figure 2.11 Upright frame deformation (a) total deformation, (b) pure bending deformation and (c) pure shear deformation (Gilbert et al., 2012) ..............19
Figure 2.12 Upright frame test setup and LVDT locations (Sajja et al., 2008)...21
Figure 2.13 Upright frame test setup (Sajja et al., 2008).............................22
Figure 2.14 Test configuration of X pattern bracing with CHS members (Gilbert et al., 2012)..........................................................22
Figure 2.15 Test configuration of Z pattern bracing with Cee members (Gilbert et al., 2012)..................................................................................................................23

Figure 2.16 Test set-up for the EN 15512 (2009) test setup with CHS bracing members (Gilbert et al., 2012)..................................................................................................................23

Figure 2.17 Test set-ups for the alternative method in AS4084 (2012) with Cee bracing members (Gilbert et al., 2012)..................................................................................................................24

Figure 2.18 Test set-ups for the test setup in EN15512 (2009) with Cee bracing members (Gilbert et al., 2012)..................................................................................................................24

Figure 2.19 Cold-formed steel profiles buckling modes, (a) local buckling (b) distortional buckling (c) global buckling.................................................................26

Figure 2.20 Fundamental steps in the strength determination of a C-section by (a) Effective Width Method and (b) Direct Strength Method (Schafer, 2008)....27

Figure 2.21 Pure bending test setup (Pham and Hancock, 2013).................29

Figure 2.22 Loading system to apply biaxial bending (Put et al., 1999).........29

Figure 2.23 Biaxial bending strength of Z-sections (Put et al., 1999)...........30

Figure 2.24 Experiment setup for perforated long beam-column uprights (Kumar and Jayachandran, 2016).........................................................................................31

Figure 3.1 Elements of a typical rack structure (Gilbert et al., 2012)..........36

Figure 3.2 Typical upright frame...............................................................36

Figure 3.3 Upright frame shear stiffness test set-up EN15512 (2009)........39

Figure 3.4 Alternative test set up AS4084 (2012)..................................39

Figure 3.5 Z-pattern bracing configuration with channel bracing members.....42

Figure 3.6 Main dimensions of the uprights (dimensions in mm).............43
Figure 3.7 Test set-up performed for all types of the upright frames following the alternative method in AS4084 (2012) and Type A upright frame following the EN15512 (2009) (shown for Type B upright frame in the AS4084) (a) overall view of the upright frame (b) vertical restraint during test (c) schematic view of the vertical restraints. .................................................................44

Figure 3.8 Test set-up performed following the method in EN15512 (2009) for Type B and C upright frames (shown for Type B upright frame) (a) overall view of the upright frame (b) schematic view of the vertical restraints along the loaded upright (c) schematic view of the vertical restraints at the free end of the pinned upright.................................................................45

Figure 3.9 Vertical restraint used in FE model for the upright frames tested following AS4084 (2012) and Type A upright frame tested following EN15512 (2009).................................................................................................................47

Figure 3.10 Vertical restraint in FE model for Types B and C frames tested following EN15512 (2009) (a) Vertical restraints at four locations along the loaded upright (b) Vertical restraint at free end of the pinned upright........47

Figure 3.11 Interactions between elements in bolted connection (master surface shown in red and slave surface shown in purple).................................................50

Figure 3.12 True stress-strain curves for all uprights and bracing members....51

Figure 3.13 FEA and experimental load-deflection curves for AS4084 (2012) test set-up, Type A upright frame.................................................................52

Figure 3.14 FEA and experimental load-deflection curves for AS4084 (2012) test set-up, Type B upright frame.................................................................53

Figure 3.15 FEA and experimental load-deflection curves for AS4084 (2012) test set-up, Type C upright frame.................................................................53
Figure 3.16 FEA and experimental load-deflection curves for all LVDTs and AS4084 (2012) test set up - Type A upright frame

Figure 3.17 Deformed shapes following the AS4084 (2012) test set-up, Type A upright frame, (a) FEA (with deformed scale factor of 1.0 and at a displacement of 70 mm) and (b) experimental observations

Figure 3.18 Deformed shapes following the AS4084 (2012) test set-up, Type B upright frame, (a) FEA (with deformed scale factor of 1.0 and at a displacement of 70 mm) and (b) experimental observations

Figure 3.19 Deformed shapes following the AS4084 (2012) test set-up, Type C upright frame, (a) FEA overall deformation of the upright frame (with deformed scale factor of 1.0 and at a displacement of 70 mm) (b) Overall deformation of the upright frame – experimental observations (c) local deformation of the upright at bolted connection – FEA (d) local deformation of the upright at bolted connection – experimental

Figure 3.20 FEA and experimental load-deflection curves for EN15512 (2009) test set-up, Type A upright frame

Figure 3.21 FEA and experimental load-deflection curves for EN15512 (2009) test set-up, Type B upright frame

Figure 3.22 FEA and experimental load-deflection curves for EN15512 (2009) test set-up, Type C upright frame

Figure 3.23 Deformed shapes following the EN15512 (test set-up, Type A upright frame (a) FEA (with deformed scale factor of 1.0 and at a displacement of 14 mm) and (b) experimental observation

Figure 3.24 EN15512 (2009) test set-up deformed shapes, Type B upright frame (a) FEA (with deformed scale factor of 1.0 and at a displacement of 14 mm) and (b) experimental observation
Figure 3.25 EN15512 (2009) test set-up deformed shapes, Type C upright frame (a) FEA (with deformed scale factor of 1.0 and at a displacement of 14 mm) and (b) experimental tests. 60

Figure 4.1 (a) Typical selective storage rack structure (Gilbert, et al., 2012) and (b) clad rack in construction (Courtesy Modulblok). 74

Figure 4.2 Cross-sections (a) Upright Type A (b) Upright Type B (Dimensions are in mm). 76

Figure 4.3 Experimental four-point bending test setup – Schematic view, (a) side view and (b) top view. 79

Figure 4.4 Experimental test setup – photo. 79

Figure 4.5 Solid pinned connection. 80

Figure 4.6 Cee-shaped plates bolted with the uprights to the pinned connections to prevent uplift in the tension zone. 80

Figure 4.7 Transducer frame to measure cross-sectional deformations at mid-length (a) Type A (b) Type B uprights. 82

Figure 4.8 Short upright deformations (a) Web deformation (LVDT 2) tested in Configuration 2 ($M_x = 2M_y$) (b) Cross-sectional opening (LVDTs 1+3) tested in Configuration 5 ($M_x = -2M_y$) (see Table 4.2 for configurations). 84

Figure 4.9 Local buckling failure modes (a) Non-perforated Type A upright - Configuration 2 ($M_x = 2M_y$) (b) Type B upright - Configuration 2 ($M_x = 2M_y$) (c) Perforated Type A upright - Configuration 5 ($M_x = -2M_y$) (d) Type B upright - Configuration 5 ($M_x = -2M_y$) (see Table 4.2 for configurations). 85

Figure 4.10 Cross-section deformation (LVDT 1 + LVDT 3) for all long uprights (a) Configuration 1($M_y = 0$) (b) Configuration 6 ($M_y = -2M_x$) (see Table 4.2 for configurations). 85
Figure 4.11 (a) Local-distortional buckling interactive failure mode for perforated Type A upright - Configuration 1 ($M_y = 0$) (b) Distortional failure mode for Type B upright - Configuration 1 ($M_y = 0$) (c) Local-distortional buckling interactive failure mode for non-perforated Type A upright - Configuration 6 ($M_y = -2M_x$) (d) Distortional failure mode for Type B upright - Configuration 6 ($M_y = -2M_x$) (see Table 4.2 for configurations).........................................................87

Figure 4.12 Biaxial bending interaction points for local buckling – All uprights..91

Figure 4.13 Biaxial bending interaction points for distortional buckling - All uprights.................................................................................................91

Figure 4.14 Comparison of the DSM predictor curves with test data, (a) Local buckling tests (b) Distortional buckling tests......................................................102

Figure 4.15 Comparison of the DSM predictor curves with test data- (a) NLD (b) NDL approaches..............................................................................................103

Figure 5.1 Location of LVDT lines for imperfection measurements of Type A upright........................................................................................................110

Figure 5.2 Imperfection measurements for one long non-perforated Type A upright........................................................................................................111

Figure 5.3 FE model and boundary conditions (shown for 1100 mm perforated Type A upright)...........................................................................................112

Figure 5.4 Multi-linear stress–strain curve for numerical simulation...........113

Figure 5.5 Deformed shapes of the 400 mm long upright tested in Conf 4 for non-perforated Type A upright a) FE failure mode and b) experimental failure mode.................................................................115

Figure 5.6 Deformed shapes of the 400 mm long upright tested in Conf 3 for perforated Type A upright a) FE failure mode and b) experimental failure mode..............................................................................115
Figure 5.7 Deformed shapes of the 400 mm long upright tested in Conf 2 for perforated Type B upright a) FE failure mode and b) experimental failure mode..................................................................................................................115

Figure 5.8 Deformed shapes of the 1,100 mm long upright tested in Conf 1 for non-perforated Type A upright a) FE failure mode and b) experimental failure mode..................................................................................................................117

Figure 5.9 Deformed shapes of the 1,100 mm long upright tested in Conf 4 for perforated Type A upright a) FE failure mode and b) experimental failure mode..................................................................................................................117

Figure 5.10 Deformed shapes of the 900 mm long upright tested in Conf 1 for perforated Type B upright a) FE failure mode and b) experimental failure mode..................................................................................................................118

Figure 5.11 FEA and experimental web deformation and cross-sectional deformation (a) 400 mm long non-perforated Type A upright in Conf 4 (b) 400 mm long perforated Type A upright in Conf 3 (c) 400 mm long perforated Type B upright in Conf 4 (d) 1,100 mm long non-perforated Type A upright in Conf 1 (e) 1,100 mm long perforated Type A upright in Conf 6 (f) 900 mm long perforated Type B upright in Conf 1..................................................................................................................119

Figure 5.12 Upright cross-sections considered..............................................................121

Figure 5.13 Biaxial bending interaction points for local buckling – All uprights..................................................................................................................124

Figure 5.14 Biaxial bending interaction points for distortional buckling – All uprights..................................................................................................................127

Figure 5.15 Comparison of the DSM curve to parametric studies data for local buckling (Approaches (i), (iv) and (v))..................................................................................................................133

Figure 5.16 Comparison of the DSM curve to parametric studies data for distortional buckling (Approaches (i), (iv) and (v))..................................................................................................................136
List of Tables

Table 3.1 Cross-sectional properties of the uprights and bracing members

Table 3.2 Average measured material properties of uprights and bracing members

Table 3.3 Transverse shear stiffness for FEA and experimental tests

Table 3.4 Contribution of factors on shear stiffness for each structural component- lip-to-lip upright frames

Table 3.5 Contribution of factors on shear stiffness for each structural component – back-to-back upright frames

Table 4.1 Nominal and measured cross-sectional properties of tested uprights

Table 4.2 Tested biaxial bending configurations

Table 4.3 Comparison of experimental results with DSM for local buckling tests (L=400 mm for all uprights)

Table 4.4 Comparison of experimental results with DSM for distortional buckling tests (L=1100 mm for Type A and L=900 mm for Type B uprights)

Table 4.5 Measured material properties of tested uprights

Table 5.1 Comparison of test results with FEA results

Table 5.2 Nominal cross-sectional dimensions and properties of investigated uprights

Table 5.3 Investigated biaxial bending configurations in parametric...
Table 5.4 Biaxial bending capacity in distortional buckling with inward and outward imperfection .................................................................123

Table 5.5 Different amplitude of imperfections for local and distortional buckling......................................................................................123

Table 5.6 Comparison of parametric studies results to linear equation for local buckling........................................................................126

Table 5.7 Comparison of parametric studies results with linear equation for distortional buckling................................................................127

Table 5.8 Comparison of parametric results with DSM for local buckling uprights.................................................................................134

Table 5.9 Comparison of parametric results with DSM for distortional buckling uprights.........................................................................135
NOTATION

The following symbols are used in this thesis:

\( F \) total applied load
\( L_1 \) distance between the supports and the points of application of the loads
\( M \) total moment applied to the upright
\( M_{bd} \) DSM nominal member capacity for distortional buckling
\( M_{bdn} \) DSM nominal distortional member capacity with extended range
\( M_{bdl} \) DSM nominal interactive local-distortional moment capacity
\( M_{bl} \) DSM nominal member capacity for local buckling
\( M_{bd} \) DSM interactive distortional-local moment capacity
\( M_{bln} \) DSM nominal local member capacity with extended range
\( M_{bx} \) and \( M_{by} \) moment capacities about the x- and y-axes
\( M_{crd} \) elastic distortional buckling moment including effect of holes
\( M_{nyd} \) inelastic distortional strength
\( M_{nyl} \) inelastic local strength
\( M_{od} \) elastic buckling moment for distortional buckling
\( M_{ol} \) Elastic buckling moment for local buckling
\( M_p \) plastic moment
\( M_{rig} \) moment applied by the weight of the steel RHS and solid pinned connections to the upright.
\( M_x \) and \( M_y \) applied moments about the two principal cross-sectional x- and y-axes
\( M_y \) yield moment
\( M_{yc} \) moment at which yielding initiates in compression
\( M_{yet} \) yield moment of net cross-section
\( \alpha \) rotational angle of the cross-section about its centroidal axis
\( \lambda_d \) elastic distortional non-dimensional slenderness ratio
\( \lambda_{dl} \) elastic interactive local-distortional non-dimensional slenderness
ratio
\( \lambda_{dn} \) inelastic distortional non-dimensional slenderness ratio
\( \lambda_d \) elastic local non-dimensional slenderness ratio
\( \lambda_{ld} \) elastic interactive distortional-local non-dimensional slenderness ratio
\( \lambda_{ln} \) inelastic local non-dimensional slenderness ratio
\( S_{ti} \) transverse shear stiffness
\( K_{ti} \) slope of the experimental load-displacement curve
d distance between the centroidal axes of the two uprights
h height of the upright frame
\( S_{cti} \) transverse shear stiffness
\( K_{cti} \) slope of the experimental load-displacement curve
\( \sigma_n \) nominal stress
\( \varepsilon_n \) nominal strain
\( \sigma_t \) true stress
\( \varepsilon_t \) true strain
E Young’s modulus
\( \Omega \) influence of analysed factor
\( \Delta F_y \) increase in yield stress
\( F_u \) ultimate strength
\( F_y \) yield stress
r inside bending radius of the corner
t wall thickness
\( S_{ol} \) local buckling imperfection amplitude
\( F_{ol} \) elastic local buckling stress
\( S_{od} \) distortional buckling imperfection amplitude
\( F_{od} \) elastic distortional buckling stress
CHAPTER 1
Introduction

1.1. Background
Steel storage rack systems, commonly made of cold-formed steel profiles, are extensively used in the logistic industry to store goods. Goods are generally placed on pallets which are positioned on the rack using forklift trucks. Steel storage racks are load bearing systems and act as freestanding structures and are designed as lightly as possible (Gilbert et al., 2010). Their benefit is based on their ability of increasing storage capacity by minimising floor space and providing a number of different storage configurations that can be used for specific requirements (Freitas et al., 2005). In industry, selective racks are the most conventionally used pallet system in which pallets are accessible from the structure’s aisle and pallet beams behave as the support for the pallets. There is a wide range of steel storage racks from small light household racks to 45 meter high storage racks. A selective rack structure is shown in Figure 1.1. A typical rack configuration is constructed from components including uprights, beams, bracing and connections as shown in Figure 1.2. The uprights (columns) are vertical members made of open mono-symmetric cross-sections. The beams support the pallets and are generally formed by welding together two thin walled lipped channels to form a closed section (Sajja et al., 2008).

For the down-aisle direction, the stability of these structures is normally ensured by base plate to floor and pallet beams to upright semi-rigid connections. In the cross-aisle direction, stability is ensured by the upright frames. Although, rack structures are comprised of simple beams and uprights, their analysis and design are complicated. The performance of rack elements is influenced by typical problems such as local deformations, local buckling and upright distortion (Sajja et al., 2008).
Moreover, the uprights are generally characterized by the presence of regular perforations to erect quickly the structures and/or to adapt the clear height of the load levels to the pallet sizes. The upright behaviour is further complicated by the presence of perforations along its length. The bolted connection between beams and uprights, which exhibits semi-rigid behaviour, also influences the structural performance of the system (Baldassino and Zandonini, 2003; Prabha et al., 2010).

![Figure 1.1 A selective rack structure (Sajja et al., 2008)](image)

1.2 Rack-supported buildings

As mentioned before, storage racks are typically freestanding structures built inside an enclosed building, with seldom connections between the two structures. In recent years, a new type of building has entered the market place in which the storage rack is integrated into the building and supports both the stored goods and building enclosure. These types of racks are referred to as “rack-supported building” or “Clad racks” (Sajja et al., 2008). The main advantage of this structure is the elimination of building columns, with the wind load now resisted by the rack. A typical rack-supported building is shown in Figure 1.3.
1.3 Problems arising in rack supported buildings

1.3.1 Transverse shear stiffness of rack structures

Stability and stiffness of the upright frames in the cross-aisle direction have a great impact on the behaviour of the complete structure. Since these structures are sensitive to second order effects, determining the shear stiffness of upright frames is of a great importance, in order to ensure the stability of the structure. This becomes more crucial in clad racks as the outer rack frames must withstand cross-aisle horizontal actions due to wind loading. Consequently, accurately determining the transverse shear stiffness of upright frames is essential in the design for serviceability checks, as for determining the elastic buckling load and calculating earthquake forces. Currently, there are three main international specifications that are used to determine the shear stiffness of cold-formed steel storage rack upright frames. These standards adopt different approaches. Timoshenko and Gere’s theory is used in the Rack Manufactures Institute (RMI, 2012). In the European code (EN15512, 2009), experimental testing in the longitudinal direction is proposed to determine the shear stiffness of the upright frame. The newly revised Australian Standard (AS4084, 2012) propose alternative testing approach in the transverse direction to evaluate the combined shear and bending stiffness of cold-formed steel upright frames. It has been shown that calculating the shear stiffness of an upright frame using different design standards leads to different values (Sajja et al., 2008).
In addition to, numerical analyses performed to reproduce the shear stiffness value calculated from experimental results were not successful to capture the experimental shear stiffness and obtained the stiffness value significantly greater than experimental stiffness (Godley and Beale 2008; Sajja et al., 2008; Gilbert et al., 2012). The difference between numerical simulations and test results is attributed to the local deformations occurring at the bolted joints of the upright frames. Consequently, connection between upright and braces is believed to be the key element to understand the shear behaviour of cold-formed steel bolted upright frames (Gilbert et al., 2012) and more importantly in clad racks.

1.3.2 Biaxial bending of the uprights in clad rack

Steel storage rack structures have become increasingly complex in the recent years with a large number of bays and beam levels to support heavy storage loads (Sajja et al., 2008). In particular, clad racks are subject to loading patterns significantly different from typical racking structures, and main international racking Specifications (RMI 2012; EN15512, 2009; AS4084, 2012) do not deal
with rack-supported buildings. Due to combined actions of wind and stored goods, uprights undergo biaxial bending in clad racks.

Currently, international cold-formed steel Specifications including the North American Specification AISI-S100 (2016), the Australian and New Zealand Standard AS/NZS 4600:2018 (AS/NZS, 2018) and the Eurocode 3 (EN 1993-1-3, 2006) use a linear relationship between combined bending actions and it is likely that this relationship is non-linear. In order to have safe design, accurate relation between both major and minor axes of bending should be addressed. The aforementioned specifications propose design approaches such as Direct Strength Method (DSM) (Schafer, 2008) to account for properly local, distortional and overall buckling modes. Furthermore, presence of regular perforations on upright can significantly affect the upright performance in bending. Hence, effect of perforations on biaxial bending of the uprights should also be studied to investigate their influence on biaxial bending capacity.

1.4. Research objectives

The research goals of this PhD project are mainly (i) to study the transverse shear stiffness of steel storage rack upright frames and (ii) biaxial bending of steel storage rack uprights. The objectives of this study are summarised as below:

The first phase of the thesis aims:

- To accurately capture transverse shear deformation of the upright frames. Available numerical simulations have not been able to fully capture the shear stiffness.
- To investigate and quantify all the factors affecting the shear deformation of cold-formed steel bolted upright frames to arrive at appropriate design recommendations. Parameters contributing the most to the overall shear deformation of the upright frames are determined.

The second phase of this project investigates:
• Local and distortional biaxial bending behavior of the uprights to better understand the true behavior and relation between bending about major and minor axis.

• Effect of perforation on biaxial bending capacity of the uprights in local and distortional buckling.

• The accuracy of different forms of the Direct Strength Method (DSM) (Schafer, 2008) in predicting the biaxial bending capacity of cold-formed steel storage rack uprights.

1.5. Research methodology

The objectives of this research are achieved as follows:

• Based on experimental tests performed on upright frames at Griffith University and Trento University, a detailed Finite Element model of the upright frames was built using the commercial software Abaqus (2015) to accurately capture the transverse shear stiffness, according to both the EN15512 (2009) and the alternative AS4084 (2012) test set-up procedures.

• The calibrated Finite Element model was then used to investigate and quantify contribution of different factors affecting the shear stiffness of the bolted upright frames with lip-to-lip and back-to-back channel bracing members following the two test set-up. Contribution of (a) bolt bending deformation, (b) local deformation at the bracing member bolt holes, (c) local deformation at the upright bolt holes (d) cross-sectional deformation of the end of the bracing members, (e) cross-sectional deformation of the uprights in the vicinity of the bolted connection, (f) axial elongation of bracing members and (g) upright overall bending and shear deformation were determined by performing appropriate changes to the Finite Element model.

• In order to gain full understanding of biaxial bending behaviour of the uprights, two commercially available upright sections were experimentally tested in local and distortional biaxial bending to obtain biaxial interactive relation. To investigate the effect of perforations, one of the uprights were tested with and without perforations.
Chapter 1

• A detailed Finite Element model of the tested uprights was developed and calibrated against experimental upright biaxial bending tests to accurately capture the local and distortional biaxial bending capacities of cold-formed steel storage rack upright sections. Using Finite Element Analysis, parametric studies are performed to quantify the local and distortional biaxial bending capacities of slender, semi-compact and compact storage rack upright cross-sections.

• Finally, different current forms of the Direct Strength Method (DSM) (Schafer, 2008) in AISI-S100 (2016) and AS/NZS 4600:2018 (AS/NZS, 2018) are used to predict the biaxial bending capacity of cold-formed steel storage rack uprights and investigate their accuracy based on experimental and parametric studies results.

1.6. Thesis outline

This thesis is structured as follows:

• Chapter 1 presents an introduction to the thesis and briefs the research objectives and methodologies.

• Chapter 2 reviews the literature relevant to this thesis. Characteristics of racks in general are detailed as are design particularities of storage racks. Available literature on transverse shear stiffness of upright frames and upright biaxial bending is presented.

• Chapter 3 is based on the submitted journal paper, which details factors contributing to the transverse shear stiffness of bolted cold-formed steel storage rack upright frames with channel bracing members.

• Chapter 4 is based on the published journal paper, which experimentally investigates local and distortional biaxial bending capacities of cold-formed steel storage rack uprights.

• Chapter 5 is based on the submitted journal paper, which presents parametric studies and design rules for local and distortional biaxial bending of cold-formed steel storage rack uprights.

• Chapter 6 summarises the thesis findings and provides the recommendations for future research.
CHAPTER 2

Literature review

2.1 General steel storage racks literature review

Steel storage racks are used throughout the manufacturing and storage industries based on their efficiency and economical value. Based on the high demand for steel storage structures, storage structures suppliers provide various kinds of storage systems. There are different types of steel storage rack buildings. As mentioned in Chapter 1, selective racks (Figure 2.1) are the most conventionally used pallet system. In this type of storage structure, pallets are accessible from the structure’s aisle. In fact, pallet beams behave as the support for the pallets. Although selective racks might be different in types, they are mostly associated with narrow aisle racking, standard and deep reach systems. Narrow aisle racking is used to create optimum space. Although, the system allows for large storage capacity, it requires a specialized narrow lift truck (Freitas et al., 2005).

The second type of storage racks, are drive-in racks (and drive through racks) which allow space for a forklift to drive into the structure’s bay (Figure 2.2). Although, drive-in and drive through racks are mostly similar, drive-in rack structures have one entry/exit way while drive-through racks have entry access on both sides of the bay. This difference typically affects the way materials are loaded in these storages. For instance, items are stored in drive-in racks by last-in, first-out process (also known as LIFO) and storage is not readily accessible. Consequently, drive-in systems are feasible for non-perishable items and products with a low turnover. The drive through system, however, requires the FIFO (first-in first-out) process (Freitas and Souza, 2005).
Push back racks (Figure 2.3) are ideal for bulk storage, as they are capable of storing products that occupy or run several pallets deep. These structures typically measure several levels high. Inclined rails and sliding carts are used in
these structures and loads are stored at the higher end and removed at the lower end point, using the first-in first-out loading system. Basically, when an item (pallet) is stored or loaded on the rack, it pushes the previous item (pallet) back on the rails. When unloading, they are pushed to the front of the structure (Freitas and Souza, 2005).

Flow racks (also known as gravity flow racks) are mostly used for high-density storing purposes. Pallets are loaded at the higher end and unloaded at the lower end point, using the FIFO loading system. These systems involve gravity rollers which generate movement based on the rack load, as pallets are moved on a inclined plane. Furthermore, this system takes the advantage of using brakes, or speed controllers to control the movement of the products. The rails are usually powered by gravity, so no electric equipment is required to operate the system (Freitas and Souza, 2005).
2.2. International rack design specifications

In North-America, Rack Manufacturer Institute specification, RMI (2012) is the American code for design of steel storage racks. In Europe, European Standard, EN15512 (2009) is used for design of steel storage racks, whereas in Australia racking systems are designed according to Standard AS4084 (2012), which is primarily based on EN15512 (2009).

According to all of the above design codes, design of rack structures depend primarily on experimental tests performed on the main structural components of these systems such as beams, uprights, upright frames and different types of connections.

2.2.1. Joints in rack structures

Understanding behaviours of different types of connections, such as the portal beam to column connectoin and the base plate connection to the floor, is of a great importance in rack systems to provide stability in different directions and also achieve economic designs. This is typically performed based on experimental-theoretical approaches in three main rack design specifications (Baldassino and Zandonini, 2003).
2.2.1.1. Pallet Beam end Connector

Typically, pallet beams are connected to the upright frames by tabs which are inserted into perforations in the upright frame and secured by using locking pins. Figure 2.5 shows a typical case of a beam end connector.

![Image of beam end connector](image)

Figure 2.5 Example of beam end connector (Gilbert, 2010)

Determining the stiffness and strength of a particular connector is not analytically achievable since there are variety of available beam end connector’s configurations (Bernuzzi and Castiglioni, 2001). Consequently, testing approach is proposed by main international racking design standards. In the AS4084 (2012), RMI (2012) two alternative test setups are proposed whereas in the EN15512 (2009) code, only one test set up is proposed.

In the “cantilever test” setup proposed in AS4084 (2012), a short length of a beam is connected to a short length of an upright frame as depicted in Figure 2.6. In this testing approach, according to RMI (2012) and AS4084 (2012) the beam end connector stiffness is calculated using the following equation:

\[
k_{RMI,AS} = \frac{R.F.}{\delta_{0.85}} \left( \frac{L_c}{16EI_c} + \frac{L_B}{3EI_B} \right)
\]
where $P_{0.85}$ is 0.85 times the ultimate load, $\delta_{0.85}$ is the deflection at the free end of the cantilever beam at load $P_{0.85}$, $E$ is the Young’s modulus, $L_b$ is the length of pallet beam and $L_c$ is the length of upright frame, $I_c$ and $I_b$ are the second order moments of area of the upright and portal beam, R.F. is a reduction factor taken as 1 when determining the bending moment distributions and $2/3$ when designing pallet beams.

Cantilever test set-up according to the EN15512 (2009) is shown in Figure 2.7. Based on the EN15512 (2009) the bending moment of the connector, $M_{EN}$, is calculated using the following equation:

$$M_{EN} = aF$$

(2.2)

where $a$ is the lever arm of the applied load $F$. As it can be seen in Figure 2.7, two transducers on each side of the connector directly record the displacement. The rotation of the connector $\theta_{EN}$ is calculated and the $M_{EN} - \theta_{EN}$ curve can be used to determine the connector stiffness $K_{EN}$. 

Figure 2.6 Cantilever test set-up from AS4084 (2012)
Steel storage racks are connected to the floor by using a base plate assembly which is basically bolted to the ground. International specifications use different approaches to determine the connection properties. A typical base plate set up is shown in Figure 2.8.

The RMI (2012) determines the stiffness of the base plate connection as follow:

$$M = \frac{1}{12} b d^2 E_c \theta$$

(2.3)

where $b$ and $d$ are the dimensions of upright section and $E_c$ is the Young’s modulus of the floor which is normally concrete. This equation, does not consider the base plate deformation (Salmon et al., 1955). EN15512 (2009) proposes testing approach to determine the base plate stiffness. In section A.2.7 of the EN15512, the semi-rigid stiffness of storage rack base plate is calculated from a test of two uprights which are connected to a concrete block in a dual-actuator set-up.
Godley et al. (1998) introduced the dual-actuator test set up and proved that increase of the axial load of the upright would improve the moment-rotation capacity of the base plate. Furthermore, their study showed that the stiffness of the floor material (such as concrete and wood) directly influences the base plate rotational stiffness.

The influence of the base plate stiffness on the ultimate load capacity of the rack was investigated using the program “Pallet” (Pallet 2000; Godley and Beale, 2002). This study showed that the ultimate load capacity of the rack is not affected by increasing the base plate stiffness above a certain level. This limiting value $K_h$ is calculated by the following equation:

$$M = k_h \theta = \frac{EI}{h} \theta \Rightarrow k_h = \frac{EI}{h} \quad (2.4)$$

where $E$ is Young’s modulus, $I$ is the moment of inertia of the steel upright section and $h$ is the distance from the floor to the first beam (Godley 2007). Gilbert and Rasmussen (2011) proposed an alternative test method to that given in the EN15512 (2009) to determine the base plate stiffness and identified the contributions of deformations of the supporting floor, the base plate assembly itself and the upright.

Figure 2.8 Base plate assembly (Gilbert 2010)
2.2.2. Rack design

2.2.2.1. Upright design

Flanges and webs of steel storage rack uprights are perforated for easy connection of bracing and also pallet beams to uprights. It is clear that these perforations in webs and flanges have influence on the axial and bending capacities of the uprights (Pekoz 1988). Compression test on a short length of an upright is proposed by the RMI (2008), AS4084 (2012) and EN15512 (2009) in order to calculate the section capacity.

Stub column test is used to determine the Q-factor. The Q-factor is a measurement which is used in cold-formed steel design as a reduction coefficient for the effect of local buckling and post-buckling of perforated members. Q-factor can be calculated using the following equation:

\[ Q = \frac{P_u}{f_y A_{\text{net min}}} \leq 1 \]  \hspace{1cm} (2.5)

\( P_u \) is the experimental stub column strength, \( F_y \) is yield stress and \( A_{\text{net min}} \) is the minimum net area. \( A_{\text{net min}} \) can be calculated by the area of a plane cutting through the cross section of the upright, considering perforations.

2.2.2.2. Single upright model

Equivalent single upright models have been proposed to avoid the use of complicated computer models. In 1992, an equivalent single internal free sway upright was proposed by Davies (Davies 1992) for the racks which are subject to horizontal and vertical loads. In this model, base plate and pallet beams are considered as semi-rigid but the flexibility of the upright is merely considered from second beam elevation. Lewis (1997) suggested a single column upright model for unbraced rack structures in which linear and non-linear semi-rigid restraints are taken into consideration for pallet beam end connector, while the base plate connection was assumed as pinned. Although equal vertical loads and equal spacing between pallet beams and small upright bending distortions are assumed in this model, the model was not successful to present the flexibility of pallet storage racks. In 2000, Davies’s (1992) model was improved by Godley et al (2000) and a single steel upright model was proposed. In this
model, they covered the versatility of the upright over its full length and compared analytical displacements and rotations, considering the second order P-Δ effects, to finite element analysis. The results showed a comprehensive agreement. As a result, the single column can be used for design check.

2.2.2.3. Critical load

In order to calculate the ultimate load capacity of an upright frame, Scholz (1990) suggested an approximate method, considering semi-rigid connections. Teh et al (2004) demonstrated that determining the flexural-torsional buckling loads of high-rise storage racks using 2D Finite Element analysis overestimate the results and determining the actual buckling modes requires 3D analysis.

Parametric studies on the buckling load of single upright frames and one bay selective racks were performed by Sangle and Bajoria (2008).

2.3. Shear stiffness of steel storage racks upright frames

To accurately determine the cross-aisle displacement of steel storage rack and cross-aisle elastic buckling load of rack upright frame, the transverse shear stiffness of the upright frame must be calculated. The elastic buckling load, $P_{cr}$, is given by the following equation in the RMI (2012) which is based on Timoshenko and Gere’s (1961) shear formula:

$$P_{cr} = \left(\frac{\pi EI}{k^2 L^2}\right) \left(\frac{1}{1 + \frac{\pi^2 I}{k^2 L^2 A_d \sin \phi (\cos \phi)^2}}\right)$$

(2.6)

where $A_d$ is equal to the cross sectional area of a diagonal bracing, $E$ is the Young’s modulus and $I$ is the minimum net moment of inertia of the upright about an axis perpendicular to the plane of the upright frame. $L$ is equal to the height of the upright frame and $\phi$ is the angle between horizontal and diagonal braces. The $k$ factor is a coefficient depending on the position of the loads. In Timoshenko and Gere’s (1961) theory, the upright frame shear deformation is assumed to be merely from the axial deformation of the bracing members.
In EN15512 (2009), testing approach is used to calculate the transverse shear stiffness per unit length of steel upright frame. Based on this testing approach, the upright frame longitudinal shear stiffness is calculated, and then Timoshenko and Gere’s (1961) shear formula is used to determine the transverse shear stiffness. Figure 2.9 shows the shear stiffness test set up in EN15512 (2009). This test set-up includes a steel upright frame with at least two panels, restrained longitudinally in transverse direction. One of the uprights is pinned at one end and a force F is applied at the end of the other upright through its centroid. Chapter 3 of this thesis explains the testing approach and calculation of shear stiffness based on EN15512 (2009) test setup.

![Figure 2.9 Upright frame shear stiffness test set-up from EN15512 (2009)](image)

Australian Standard AS4084 (2012) uses an alternative testing approach in which the shear stiffness of the upright frame in the cross-aisle direction is determined by testing the frame in combined bending and shear. Figure 2.10 shows the alternative test set up.

As it can be seen, minimum of two bracing panels are connected to a rigid testing frame. Both ends of uprights are restrained using pin connection and the top of the upright is restrained from out of plane displacements. At the top horizontal bracing member level, a load F is applied to the frame. Two displacement transducers are set, one at the top horizontal bracing member elevation, where the load F is applied, and the other one at bottom horizontal bracing member. According to Gilbert et al. (2012), recordings from displacement transducer at the top horizontal bracing member level, is the sum
of a pure bending deformation of the upright frame and a pure shear deformation of the frame, as shown in Figure 2.11. Chapter 3 of this thesis details the testing approach and determination of shear stiffness using the alternative test setup in AS4084 (2012).

Rao et al (2004) numerically and experimentally evaluated the shear stiffness of upright frame. In this study, a simple frame analysis was carried out using the
finite element analysis (FEA) to investigate the factors affecting the shear stiffness of upright frames, such as flexibility of upright, the aspect ratio of frame pales (Length/Depth), eccentric loading on bracing member and bolt bending. In total, 21 tests were performed by changing upright size, number of panels in the frame, aspect ratio of the panel and lacing pattern. They found that the Timoshenko and Gere's (1961) theory overestimates the shear stiffness by a factor up to 20 and recommended a further study to propose a better procedure for determination of the shear stiffness of rack frame.

At Oxford Brookes University, Sajja et al (2008) performed 80 experimental tests in longitudinal direction on upright frames with back-to-back and lip-to-lip bracing members characterised by various numbers of panels, bracing configuration, upright sizes and aspect ratios frame panel (Length/Depth). Figure 2.12 presents the typical upright frame test configuration at Oxford Brookes University. Figure 2.13 shows photo of a tested upright frame. Finite Element Analysis was conducted using beam elements to model the upright and frames, considering the eccentricities between bracing members, uprights and bolts. Upright distortion was not included in the model and developed finite element models were not able to successfully reproduce the experimental results and obtained shear stiffness values 2 to 5 times greater than the tests results. In order to accurately evaluate the upright frame shear stiffness, Sajja et al (2008) and Sajja (2010) concluded that more sophisticated three-dimensional numerical models are required to consider contact behaviour between various elements at the joints.

Gilbert et al (2012) performed 36 experiments following test setups in EN15512 (2009) and AS4084 (2012) and compared the two approaches. The upright frames were built up from three upright sections and two different bracing configurations, Circular Hollow Sections (CHS) set up in X pattern and lip-to-lip Cee sections set up in Z pattern, as shown in Figure 2.14 and 2.15, respectively. These tests have been performed at two different universities. Tests according to the EN15512 (2009) were carried out at the University of Trento and remaining tests were conducted at Griffith University. All the test frames were restrained in three locations from lateral out-of-plane displacements at Griffith University and in four locations at Trento University.
Figure 2.16 and 2.17 show the test set up at Griffith University for the test setup following EN15512 (2009) and AS4084 (2012), respectively. Figure 2.18 shows test setup for EN15512 (2009) setup in Trento university. It was shown that the frame stiffness with Z pattern bracing configuration is lower than the frame stiffness with X pattern bracing configuration by factor of 4 to 11 times, due to torsion of the uprights in Z pattern bracing and the lower axial stiffness of the bracing members. The observed failure modes for upright frames with X pattern bracing configuration was lateral buckling of the diagonal bracing member in compression, shearing of the bracing members at the bolt hole, shearing of the bolts and buckling of the flat part at the ends of the CHS bracing members. For upright frames with Z pattern bracing configuration, lateral-torsional buckling of the diagonal bracing member in compression and shearing of the bracing members at the bolt hole was reported.

Figure 2.12 Upright frame test setup and LVDT locations (Sajja et al., 2008)
Figure 2.13 Upright frame test setup (Sajja et al., 2008)

Figure 2.14 Test configuration of X pattern bracing with CHS members (Gilbert et al., 2012)
Figure 2.15 Test configuration of Z pattern bracing with Cee members
(Gilbert et al., 2012)

Figure 2.16 Test set-up for the EN 15512 (2009) test setup with CHS bracing members (Gilbert et al., 2012)
Gilbert et al. (2012) performed Finite Element Analysis using beam elements, considering geometrical eccentricities of the bracing members. The uprights and bracing members were modelled at their centroid axis. Bolts were not...
modelled explicitly in the Finite Element Analysis. Results showed that Finite Element models overestimates the shear stiffness of upright frame by factor of 9 to 25 times for the EN15512 (2009) test setup and by factor of 3 to 16 times for the AS4084 (2012) test setup. Results illustrated that the two methods do not produce equal shear stiffness. It can be said that due to the fact that alternative method in the AS4084 (2012) measures the deformation in the transverse direction (the direction that deformation would occur during the service), the alternative method is more appropriate than the method in EN 15512 (2009). They concluded local deformation of the connections between bracing members and uprights are the key elements to understand the actual behaviour of cold-formed bolted upright frames.

Sangle et al. (2012) conducted elastic stability analysis of upright frames through 3D finite element analysis and emphasized the need for 3D modelling to study the behaviour of racking systems.

Far et al (2017) experimentally and numerically investigated shear stiffness of upright frames and effect of connection flexibility on the frame response. Developed FE models of the upright frames using solid elements overestimated the shear stiffness by 30%. A simplified modelling approach was also proposed to account for the flexibility in bolted connections. The proposed method results in better prediction of the shear stiffness. They also studied the effects of bracing members configuration on shear stiffness and showed that back-to-back bracing pattern leads to higher shear stiffness compared to lip to lip bracing members.

Roure et al. (2016) experimentally and analytically studied the behaviour of upright frames. Tests were performed on the joints between uprights and braces as well as on the upright frames. A simple practical design approach was proposed based on an adjusted cross-sectional area at both ends of the bracing members to consider that the axial stiffness of the bracing members is affected by the local distortions at the joints.
2.4. Buckling modes of cold-formed profiles sections

Since cold-formed sections are slender and mono-symmetric, they are affected by a range of complicated buckling modes. Hancock (1985) discussed a range of buckling modes including local, distortional and flexural-torsional modes in detail. The main buckling modes of cold-formed thin-walled sections are local, distortional and global buckling as shown in Figure 2.19. In distortional buckling rotation of the flange about the flange-web junction leads to deformation of the edge-stiffened flange elements of the section (Hancock et al., 1994).

![Figure 2.19 Cold-formed steel profiles buckling modes](image)

(a) (b) (c)

Figure 2.19 Cold-formed steel profiles buckling modes, (a) local buckling (b) distortional buckling (c) global buckling

2.5. Design methods for thin-walled members

Basically, two design methods are available for cold-formed steel members in design standards ((AISI-S100 (2016) and AS/NZS 4600:2018 (AS/NZS, 2018)), Effective Width Method and the Direct Strength Method (DSM).

2.5.1. Effective Width Method

The basis of the Effective Width Method is on the reductions in the effectiveness of a cross-section that happens when local buckling takes place as shown in Figure 2.20 (a). In fact, actual nonlinear stress distribution that develops due to buckling is approximated by an effective plate under simplified stress distribution. In the effective cross section the locations in which material
of the member is not able to carry loading, are removed. It is obvious neutral axis is shifted when local buckling occurs that is taken into account by effective cross-section (Schafer, 2008).

2.5.2 Direct Strength Method

Direct Strength Method relies on precise accurate member elastic stability, as shown in Figure 2.20 (b). All the elastic instabilities of the gross section including local (Mcr), distortional (Mcrd) and global buckling (Mcre) are determined and yield moment (My) of the section is calculated. The strength is then determined, i.e. \( Mn = f(Mcr, Mcrd, Mcre, My) \) (Schafer, 2008). Chapter 4 presents current available DSM equations for design of cold-formed members in bending for local and distortional buckling.

![Figure 2.20](image)

(a) An effective C-section determined as a composition of effective plates, with the effective width of the flange plate shown along with the actual flange plate under nonuniform longitudinal stress.

(b) Semi-analytical finite strip solution of a C-section in bending showing local, distortional and lateral-torsional buckling as well as the moment to cause first yield.

Figure 2.20 Fundamental steps in the strength determination of a C-section by (a) Effective Width Method and (b) Direct Strength Method (Schafer, 2008)
2.6. Biaxial bending of cold-formed members

The main current cold-formed steel structures design specifications, including the North American Specification AISI-S100 (2016), the Australian and New Zealand Standard AS/NZS 4600:2018 (AS/NZS, 2018) and the Eurocode 3 (EN 1993-1-3, 2006), consider a linear interaction equation to design members undergoing bi-axial bending. This equation is discussed in detail in chapter 4 and 5.

The capacity of cold-formed steel beams, bent about one of their principal axes, has been widely studied by numerous investigators such as Nguyen (2006), Wang and Zhang (2009), Shifferaw and Schafer (2012). Pham and Hancock (2013) experimentally investigated plain C- and SupaCee sections in pure bending about major axis using a four-point bending test set-up as shown in Figure 2.21. They evaluated the DSM local and distortional bending strength equations and proposed an extended range of the cross-sectional slenderness about the symmetric axis of bending for which the inelastic strength can be applied, for both local and distortional buckling. However, fewer studies have investigated the biaxial bending behaviour of cold-formed steel sections. Shanmugan et al (1989) studied the ultimate capacity of thin-walled steel box columns under biaxial loading and presented interaction diagrams for the studied beam-columns. Duan and Chen (1990) investigated the yield strength of doubly symmetric steel sections including thin-walled hollow sections under axial force and biaxial bending. Put et al (1999) experimentally investigated the biaxial bending of cold-formed steel Z-sections. Interaction design equations were proposed through an extended series of analytical models depending on the angle between the applied load and web inclination. The loading system used to bi-axially bend z-section is shown in Figure 2.22. The vertical load Q was inclined at (α-θ) to the web. The interaction design equations can be used in design of eccentrically loaded cold-formed Z-beams. Figure 2.23 shows nonlinear biaxial interaction curve for z-section. Bedair (2011) analytically investigated the structural behaviour of channel members in combined biaxial bending and compression and presented a method to obtain economical design under this loading pattern. Torabian et al. (2014, 2015, 2016) experimentally investigated the behaviour of cold-formed steel lipped Cee and Zee-sections.
under biaxial bending and compression (beam-column). Test results showed that a non-linear relationship governs the section capacity. The authors further developed new Direct Strength Method (DSM) equations for beam-column design through a comprehensive Finite Element (FE) parametric study. The new method was found to be conservative and predicts, on average, a 20% increase in strength when compared to current design methods.

Figure 2.21 Pure bending test setup (Pham and Hancock, 2013)

Figure 2.22 Loading system to apply biaxial bending (Put et al., 1999)
For steel storage racks, Kumar and Jayachandran (2016) experimentally tested 16 upright specimens and investigated the beam-column behaviour of long perforated storage rack uprights subjected to compression and biaxial bending. The authors evaluated the strength using the current linear interaction equation, as well as the recently proposed DSM beam-column design equations in Torabian et al (2014). It was found that the nonlinear interaction is unconservative in some specimens for linearly varying moment. Figure 2.24 shows their experimental setup. Recently, Bonada et al (2016) experimentally and numerically studied the behaviour of steel storage rack uprights under compression and bending about their major axes. They concluded that the influence of the bending moment about the major axis on the load carrying capacity of the upright due to an eccentrically applied axial force is significant. Pastor et al. (2017) experimentally and numerically investigated the behaviour of the uprights tests performed on uprights under combined compression and bending and compared results to European Standard, EN 15512 (2009). They concluded that EN15512 (2009) generally gives conservative predictions.

Figure 2.23 Biaxial bending strength of Z-sections (Put et al., 1999)
2.7. Shear stiffness of steel storage upright frames and biaxial bending of the uprights

Transverse shear stiffness of storage upright frames is recognised as an important step in design of steel storage rack structures, and yet no scientific studies have been undertaken to quantify the factors affecting shear stiffness of upright frames. It is believed that the main influence on shear stiffness is induced by the deformation of the bolted connections between the bracing members and the upright. Consequently, numerical models that are able to accurately predict the shear stiffness of upright frames and capture the deformation in bolted connections are required. Chapter 3 investigates factors affecting shear stiffness of the upright frames through comprehensive numerical studies.

In the literature, very few investigations have been reported on biaxial bending on cold-formed and rack upright section. To the authors’ best knowledge, actual relationship between bending about major and minor axes for uprights is likely nonlinear and accurately determining interactive equations is required to
achieve economic design. Therefore, Chapter 4 and 5 experimentally and numerically investigate the biaxial bending behaviour of cold-formed steel storage rack upright sections.
CHAPTER 3
Factors contributing to the transverse shear stiffness of bolted cold-formed steel storage rack upright frames with channel bracing members

statement of contribution to co-authored published paper

This chapter includes a co-authored paper. The bibliographic details of the co-authored paper, including all authors, are:

Talebian N, Gilbert BP, Baldassino N, Karampour H, Factors contributing to the transverse shear stiffness of bolted cold-formed steel storage rack upright frames with channel bracing members, Thin-walled Structures (Submitted 1st July 2018, under review).

My contribution to this paper involved: literature review, numerical modelling, run analyses, discussion of the results, writing, editing.

Signed: ___________________________ Date: 31/07/2018 ______

PhD Candidate: Nima Talebian

Countersigned: _________________________________ Date: 31/07/2018 ___

Principal Supervisor: Associate Professor Benoit P. Gilbert

Countersigned: _________________________________ Date: 3107/2018____

Principal Supervisor: Dr Hassan Karampour
Factors contributing to the transverse shear stiffness of bolted cold-formed steel storage rack upright frames with channel bracing members

Abstract

Accurately determining the transverse shear stiffness of steel storage rack upright frames is essential in calculating the elastic buckling load, performing earthquake design and serviceability checks. International racking design specifications recommend different approaches to evaluate this stiffness. The Rack Manufacturers Institute (RMI) specification conservatively uses an analytical solution based on Timoshenko and Gere's theory while the European (EN15512) and Australian (AS4084) specifications recommend experimental testing to be conducted. Previous studies have shown that Finite Element Analyses (FEA), solely using beam elements, fail to reproduce experimental test results and may overestimate the transverse shear stiffness by a factor up to 25. This discrepancy is likely attributed to the local deformations occurring at the bolted joints. In this chapter, a model to capture the transverse shear stiffness of upright frames is developed using shell elements and advanced FEA. Its accuracy is verified against published experimental test results performed on three commercially used upright frame configurations with lip-to-lip bracing pattern. The model accurately reproduces the experimental stiffness, with differences ranging from 2% to 17%. Based on the FE model, the factors contributing to the transverse shear deformation of the frames are quantified and discussed for both lip-to-lip and back-to-back bracing patterns. For lip-to-lip upright frames, results show that the local deformations at the end of bracing members contributes the most to the shear deformation of the frames. For back-to-back upright frames, bolt bending and axial deformation of braces contribute the most to the shear deformation of the frames. Results from this chapter would assist the racking industry in improving their products by focusing on the factors influencing the most the behavior of the frames.
Keywords

Steel storage racks, Cold-formed steel, Upright frames, Shear stiffness, Finite Element Analysis

3.1. Introduction

Steel storage rack systems, commonly assembled from cold-formed steel profiles, are extensively used in industry to store goods. Goods are placed on pallets which are positioned on the racks using forklift trucks. They act as freestanding structures and are designed as lightly as possible, while still capable of carrying heavy loads (Gilbert et al., 2012). Their popularity lies in their ability to increase storage capacity by both minimising the floor space and providing a number of different storage configurations (Freitas et al., 2005). The most common type of rack is referred to as “selective” storage rack and typically consists of uprights, pallet beams, bracing members and connectors, as illustrated in Figure 3.1. In the down-aisle direction, the stability of unbraced racks is solely ensured by the base plate-to-floor and pallet beam-to-upright semi-rigid connections (Bajoria et al., 2010; Davies, 1980; Godley and Beale, 2008; Gilbert et al., 2012). In the cross-aisle direction, the stability is ensured by the upright frames, each consisting of two uprights connected by bracing members, as shown in Figure 3.1 and Figure 3.2. The bracing members are commonly cold-formed lipped channel-sections bolted to the upright flanges. Welded connections are also encountered. Other forms of cold formed profiles, such as circular hollow sections, are also used in practice.

Although, the configuration of steel storage rack structures is simple, as they are assembled from beams, uprights and bracings, their analysis and design are complicated. Due to the nature of the cold-formed steel elements, their performance is influenced, among others, by local deformations at the uprights and bracing members at the connections (Sajja et al., 2008). The base plate-to-floor and beam-to-upright semi-rigid connections also influence the structural behaviour of the system (Baldassino and Bernuzzi, 2000; Prabha et al., 2010).
Chapter 3

The transverse shear stiffness of the upright frames has a significant impact on the behaviour of the overall structure in the cross-aisle direction. As rack structures are sensitive to second order effects, precisely determining the shear stiffness is of a great importance for serviceability checks, and to calculate the elastic buckling loads and earthquake design forces. This is especially relevant for high-bay racks that can reach heights greater than 20 metres and racks supporting the building enclosure which are subjected to cross-aisle horizontal forces due to wind loading. The shear behaviour of the upright frames is currently poorly understood and investigations are still needed to advance the knowledge.

Figure 3.1 Elements of a typical rack structure (Gilbert et al., 2012)

Figure 3.2 Typical upright frame
To understand all factors affecting the transverse shear behaviour of bolted cold-formed steel storage rack upright frames, numerical models able to capture the deformations at the bolted connections are required (Gilbert et al., 2012; Sajja et al., 2008; Sajja 2010). This chapter develops and details an advanced Finite Element (FE) model to accurately predict the transverse shear stiffness of storage rack upright frames with channel bracing members. The accuracy of the model is verified against experimental tests performed on three frame configurations in (Gilbert et al., 2012) and tested following the two different set-ups. The various factors influencing the shear stiffness of the rack upright frames with lip-to-lip and back-to-back channel bracing members are then numerically identified, quantified and discussed. These results would provide rack manufacturers the possibility to improve their design by targeting the factors influencing the most the shear stiffness of the frames.

3.2. Literature review
3.2.1. Design Specifications
Currently, different approaches are adopted by the three main international storage rack design specifications AS4084 (2012), EN15512 (2009) and RMI (2012) to determine the shear stiffness of cold-formed steel storage rack upright frames. Timoshenko and Gere’s (1961) theory is used in the Rack Manufacturers Institute specification (2012) in which the upright frame shear deformation is assumed to purely arise from the axial deformation of the bracing members. In the European design specification (2009), experimental testing of the frame in the longitudinal direction is proposed to determine the stiffness per unit length. The test set-up consists of an upright frame restrained from rigid body rotation and with at least two bracing panels. One of the uprights is pinned at one end and a longitudinal force $F$ is applied at the opposite end of the other upright through its centroidal axis, as depicted in Figure 3.3. The longitudinal displacement of the upright, on which the force is applied, is recorded and the slope $k_{ti}$ of the linear part of the experimental load-displacement curve is calculated. The stiffness $k_{ti}$ is then used in conjunction with Timoshenko and
Gere’s (1961) theory to estimate the upright frame transverse shear stiffness $S_{ti}$ as:

$$S_{ti} = \frac{k_{ti}d^2}{h}$$

(3.1)

where $d$ is the distance between the centroidal axes of the two uprights and $h$ is the height of the frame.

The Australian Standard AS4084 (2012) proposes the use of the testing method suggested by the European design specification EN15512 (2009) and an alternative approach in which the frame is loaded in the transverse direction to evaluate the combined shear and bending stiffness, as shown in Figure 3.4. In the alternative approach, the frame is composed of a minimum number of two bracing panels. The bottom ends of the uprights are pinned to a rigid frame and the top ends of the uprights are prevented from out-of-plane displacements. A load $F$ is applied to the frame at the elevation of the top horizontal bracing member. Two displacement transducers are positioned at the elevations of the top (i.e. where the load $F$ is applied) and bottom horizontal bracing members. The combined bending and shear transverse stiffness $S_{cti}$ is then calculated as:

$$S_{cti} = k_{cti}h$$

(3.2)

where $k_{cti}$ is the slope of the experimental load-displacement curve, with the displacement being calculated as the difference between the two transducers, and $h$ is height of the frame.
Figure 3.3 Upright frame shear stiffness test set-up EN15512 (2009)

Figure 3.4 Alternative test set up AS4084 (2012)

3.2.2. Frame behaviour

Very limited studies have investigated the transverse shear stiffness of cold-formed steel storage rack bolted upright frames. Rao et al. (2004), Sajja et al. (2008) and Sajja (2010) experimentally and numerically investigated the shear stiffness of various upright frames. Developed models with beam elements
which included upright bending stiffness, the eccentricity of bracing members and the bending of the bolt connecting the bracings to the upright, were not able to successfully reproduce the experimental test results. The models overestimated the shear stiffness by a factor of 2 to 5. The discrepancy was attributed to the “torsional distortion” of the uprights, not being considered in the model. It was recommended that it is essential to consider the contact behaviour between various elements at the connections to accurately determine the shear stiffness.

Gilbert et al. (2012) performed 36 tests of bolted upright frames following both the EN15512 (2009) and alternative AS4084 (2012) test set-ups. The two methods were compared and the practical use of the alternative method proposed by the AS4084 was demonstrated. The conclusions were based on three upright section types and two different bracing cross-sections (CHS and channels), totalling six different upright frame types. Finite Element models were also developed in Gilbert et al. (2012) using beam elements. The analyses overestimated the upright frame shear stiffness by a factor of 9 to 25 for the test set-up in the EN15512 (2009) and 3 to 16 for the alternative test set-up in the AS4084 (2012). This difference was mainly attributed to the local deformation of the connections between bracing members and uprights, not being considered in the model.

Sangle et al. (2012) conducted elastic stability analysis of upright frames through 3D finite element analysis and emphasized the need for 3D modelling to study the behaviour of racking systems. Far et al. (2017) numerically and experimentally investigated the shear behaviour of the upright frames. Developed FE models of the upright frames using solid elements overestimated the shear stiffness by 30%. A simplified modelling approach was also proposed to account for the flexibility in bolted connections. Roure et al. (2016) experimentally and analytically studied the behaviour of upright frames. Tests were performed on the joints between uprights and braces as well as on the upright frames. A simple practical design approach was proposed based on an adjusted cross-sectional area at both ends of the bracing members to consider that the axial stiffness of the bracing members is affected by the local distortions.
at the joints. Finite element modelling of storage rack frames using shell elements was performed and validated against test results by Cardoso and Rasmussen (2016). The semi-rigid behaviour of connections was implemented in the model by moment-rotation curves obtained from available component tests.

3.3. Experimental tests used to verify FE models

As mentioned in the literature review, Gilbert et al. (2012) reported tests performed on three different upright frames with channel bracing members to determine both the shear stiffness $S_{\text{hi}}$ (Eq (3.1)) and the combined transverse bending and shear stiffness, $S_{\text{ti}}$ (Eq (3.2)). In total 17 tests were performed both in Australia and Italy with channel bracing members. These tests are used in this chapter to validate the FE model. The upright frames were assembled from three upright types (referred to as “Type A”, “Type B” and “Type C”) and with lip-to-lip channel-sections in a Z-pattern, as shown in Figure 3.5. Each configuration was tested following both the EN15512 (2009) and the alternative AS4084 (2012) test set-up procedures. Three repeat tests have been performed for each upright frame configuration. When tested Type A upright frame following the EN15512 (2009), only two tests were performed. The main dimensions of the uprights used are shown in Figure 3.6 and the cross-sectional properties of the uprights and bracing members are presented in Table 3.1. Type A upright frames were tested with bracing type C35x20x1.2, while Types B and C upright frames were tested with bracing type C35x35x1.5. End plates were welded to the ends of the uprights to ensure easy connection and restrain warping. For the frames tested following AS4084 (2012) displacement was recorded at each bracing elevation, from LVDT 4 at the bottom bracing elevation to LVDT 1 at the top bracing elevation, as shown in Figure 3.5.

The tests were performed with two different boundary conditions, depending in which country the tests took place. The test set-up for all the upright frames tested following the alternative method in the AS4084 (2012) and Type A upright frame tested following the method in the EN15512 (2009) is shown in Figure 3.7 (a). In these tests, the frames were restrained from lateral out-of-plane displacements at six locations along the height of the frame, as depicted
in Figure 3.7 (a). Figure 3.7 (b) and (c) shows the vertical restraint and their schematic view, respectively. Bottom Nylon pads sprayed with silicone were placed underneath the uprights so as to allow the horizontal displacement of the frame. The top Nylon pads were pinned above the centroid axis of the uprights. Specifically, the latter pads were loosely connected to steel square hollow sections (SHS) using steel balls to both avoid out-of-plane movement and allow the uprights to rotate. The supported steel SHS in Figure 3.7 were bolted to the strong floor through threaded bars.

On the other hand, Figure 3.8 (a) shows the experimental test set-up for Types B and C upright frames following the method in the EN15512 (2009) in which the frames were restrained at five locations to prevent out-of-plane displacement. Figure 3.8 (b) shows a schematic view of the vertical restraint at four locations along the loaded upright. Steel plates were placed underneath the upright and a non-frictional material was inserted between the steel plate and the upright so as to allow the longitudinal displacement of the upright. The gap between the upright and top steel plate was to allow the upright to twist without excessive out-of-plane displacement. Figure 3.8 (c) shows a schematic view of the vertical restraint at free end of the pinned upright. This restraint prevented out-of-plane displacement, horizontal displacement, and rotation about the upright longitudinal axis.

![Figure 3.5 Z-pattern bracing configuration with channel bracing members](image-url)
Figure 3.6 Main dimensions of the uprights (dimensions in mm)

Table 3.1 Cross-sectional properties of the uprights and bracing members

<table>
<thead>
<tr>
<th>Member</th>
<th>Gross area (mm²)</th>
<th>I_{major axis} (mm⁴)</th>
<th>I_{minor axis} (mm⁴)</th>
<th>J (mm⁴)</th>
<th>I_{warping} (mm⁶)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upright Type A</td>
<td>484.0</td>
<td>4.34 × 10⁵</td>
<td>1.91 × 10⁵</td>
<td>491.9</td>
<td>2.21 × 10⁸</td>
</tr>
<tr>
<td>Upright Type B</td>
<td>674.0</td>
<td>1.34 × 10⁶</td>
<td>2.60 × 10⁵</td>
<td>802.2</td>
<td>6.00 × 10⁸</td>
</tr>
<tr>
<td>Upright Type C</td>
<td>784.6</td>
<td>2.01 × 10⁶</td>
<td>7.73 × 10⁵</td>
<td>991.6</td>
<td>5.80 × 10⁹</td>
</tr>
<tr>
<td>C35 × 20 × 1.2</td>
<td>100.0</td>
<td>1.83 × 10⁴</td>
<td>4.60 × 10³</td>
<td>47.5</td>
<td>1.79 × 10⁶</td>
</tr>
<tr>
<td>C35 × 35 × 1.5</td>
<td>168.6</td>
<td>3.49 × 10⁴</td>
<td>2.43 × 10⁴</td>
<td>125.4</td>
<td>8.45 × 10⁶</td>
</tr>
</tbody>
</table>
Figure 3.7 Test set-up performed for all types of the upright frames following the alternative method in AS4084 (2012) and Type A upright frame following the EN15512 (2009) (shown for Type B upright frame in the AS4084) (a) overall view of the upright frame (b) vertical restraint during test (c) schematic view of the vertical restraints
Figure 3.8 Test set-up performed following the method in EN15512 (2009) for Type B and C upright frames (shown for Type B upright frame) (a) overall view of the upright frame (b) schematic view of the vertical restraints along the loaded upright (c) schematic view of the vertical restraints at the free end of the pinned upright
3.4. Finite element modelling

A FE model built using the Finite Element software ABAQUS (2015) is detailed hereafter to capture both the global and local deformations of the upright frames.

3.4.1. Boundary conditions

Due to the loose connection between the top Nylon pads and the uprights (Figure 3.7), the influence of the top pads on FE response can be physically ignored for all the upright frames tested following the alternative method in the AS4084 (2012) and Type A upright frame tested following the method in the EN15512 (2009). Only bottom Nylon pads were consequently modelled in the FEA simulations and fixed to the ground. The interaction between the uprights and bottom pads was modelled using a frictionless surface-to-surface contact. During tests, upright frames rotate and displace towards the bottom Nylon pads and remain in contact with them. Therefore, only modelling the bottom pads is enough to prevent out-of-plane displacements and this was found to accurately represent the effect of vertical restraint on the frame response, as detailed in Section 3.5. Figure 3.9 shows the details of the vertical restraints used in the FE models.

As shown earlier in Figure 3.8, the vertical displacement of the frames was restrained at five locations for Types B and C upright frames tested following the methodology in the EN15512 (2009). Figure 3.10 (a) and (b) details the vertical restraint used in the FE model. For the four restraints located on the loaded upright, steel plates were fixed to the ground and a frictionless surface-to-surface contact was used to model the interaction between the upright and the steel plates, as shown in Figure 3.10 (a). To prevent twisting, transvers and vertical displacement, the free end of the pinned upright was restrained at specified points, as shown in Figure 3.10 (b).
3.4.2. Geometry, element types and mesh

Reduced integration four node shell elements, S4R, were used to model the uprights, upright end plates and bracing members at their wall center-line. Only largest perforations of the uprights were considered in the FE model. Bolts and pinned supports were modelled using C3D10 solid elements. Based on convergence studies performed on a single upright and bracing member, the
size of the S4R elements was found to be about 10 mm × 15 mm for the uprights and 10 mm × 10 mm for the bracing members to well capture the frame deformation. In the vicinity of the bolted connections, 3 mm × 3 mm mesh size was found to be fine enough to accurately capture the local deformations of the connections. The mesh size was further refined locally around the bolt holes to account for the presence of stress concentrations (Kim and Kuwamura, 2007). Five integration points through the thickness of the shell elements were considered.

3.4.3. Upright and bracing member bolted connections

To replicate the actual behaviour at the bolted connections, the interaction between elements was modelled using contacts as shown in Figure 3.11. Especially, the contacts between (i) the bolts and the bolt holes, and (ii) upright flange edges and bracing webs were modelled using the node-to-surface discretization method, with small sliding and zero initial clearance. Consequently, the looseness in the connections was ignored. The contacts between (i) the bracing members and upright flanges, and (ii) the bolt head/nut and uprights were simulated using the surface-to-surface discretization method with finite sliding. Hard contact with friction coefficient equal to 0.3 was considered as interaction behaviour for all contacts (Barth et al., 2002). The bracing members were assumed to fit perfectly between the upright flanges, i.e. no gap was considered between these elements.

3.4.4. Material properties

The material non-linearity was modelled using the von Mises yield criteria and isotropic hardening. The stress-strain curves were inputted into ABAQUS as multi-linear curves and derived from tensile tests on coupons cut from the uprights and bracing members. To account for the change of cross-sectional dimensions of the coupons during testing, true engineering stress and strain were employed in the numerical model. The nominal stress ($\sigma_n$) and strain ($\varepsilon_n$) are converted to true stress ($\sigma_t$) and strain ($\varepsilon_t$) using the following equations (Lim and Nethercot, 2004; Chung and Ip, 2000):

\begin{align*}
\sigma_t &= \sigma_n + \frac{1}{2} \frac{E}{1 + \nu} \sigma_n^2 \\
\varepsilon_t &= \varepsilon_n + \frac{1}{2} \frac{E}{1 + \nu} \sigma_n \varepsilon_n
\end{align*}

Where $E$ is the Young's modulus and $\nu$ is the Poisson's ratio.
\[
\sigma_i = \sigma_n (1 + \varepsilon_i) \tag{3.3}
\]
\[
\varepsilon_i = \ln(1 + \varepsilon_n) \tag{3.3}
\]

Figure 3.12 shows the \( \sigma_t - \varepsilon_t \) curves used for all uprights and bracing members. Bolts were modelled using an elastic perfectly plastic material with yield stress equal to 640 MPa (AS/NZS1252, 1996). All other components including upright end plates and pinned supports (test rig) were modelled as elastic materials. Table 3.2 summarises the average measured material properties for all upright types and bracing members. Three tensile tests were performed per upright type and bracing member. The Poisson’s ratio was assumed equal to 0.3.

Table 3.2 Average measured material properties of uprights and bracing members

<table>
<thead>
<tr>
<th>Member</th>
<th>E (Mpa)</th>
<th>( F_y ) (Mpa)</th>
<th>( F_u ) (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upright-Type A</td>
<td>202252</td>
<td>375</td>
<td>435</td>
</tr>
<tr>
<td>Upright-Type B</td>
<td>217254</td>
<td>452</td>
<td>499</td>
</tr>
<tr>
<td>Upright-Type C</td>
<td>197116</td>
<td>360</td>
<td>434</td>
</tr>
<tr>
<td>C35 × 20 × 1.2</td>
<td>212656</td>
<td>356</td>
<td>393</td>
</tr>
<tr>
<td>C35 × 35 × 1.5</td>
<td>200218</td>
<td>441</td>
<td>475</td>
</tr>
</tbody>
</table>

3.5. Validation of the FE model and comparison with experimental results

To consider all deformations occurring in the frames, nonlinear geometry and material analyses were ran. Yielding at the bolt holes were found to occur at an early stage of loading (particularly for the frames tested following the alternative AS4084 test set-up) and this is captured by the FEA. As the chapter focuses on the initial shear stiffness of the frames, analyses were stopped when the applied load reached about 65\% (AS4084 test set-up) and 60\% (EN15512 test set-up) of the experimental failure load. Note that in terms of failure, the models predict that, similar to the experimental results, Type A upright frame fails in flexural-torsional buckling of the bracing members while Type B and C upright frames fail by shearing of the bracing members at the bolt hole. However, due to large local deformations at the bolt holes for Type B and C upright frames, the FEA stopped converging before reaching the peak load. Further investigations would be needed outside the scope of this research to improve the convergence of the models and consider the high level of nonlinearity involved.
Chapter 3

Surface-to-surface contact between the lip of the bracing members

Surface-to-surface contact between the diagonal bracing member and upright flange

Node-to-surface contact between bolt and upright holes

Node-to-surface contact between the horizontal bracing member and upright flange edge

Node-to-surface contact between the diagonal bracing member and upright flange edge

Figure 3.11 Interactions between elements in bolted connection (master surface shown in red and slave surface shown in purple)
**3.5.1. Tests following the AS4084 test set-up**

Figure 3.13 to 3.15 compare the load-deflection curves (deformation taken as the difference between the displacements at the top and bottom bracing elevations – Figure 3.4) between FEA and experimental test results for the three upright frames configurations. The initial nonlinear response observed in the FEA load-deflection curve in Figure 3.13 is attributed to nonlinear frictional contacts in the model. Table 3.3 gives the stiffness $k_{sti}$ (Eq. (3.2)) calculated from both the FEA and experimental results. The stiffness is calculated by performing a linear regression on the load-deflection curves, i.e. between 1 kN and 3 kN for Type A and C upright frames, and between 1 kN and 2.5 kN for frames with Type B upright frames. Figure 3.13 to 3.15 and Table 3.3 show a reasonable agreement between the FEA and experimental tests. The average difference in stiffness is 7% and reaches up to 16.9% for Type B upright frame. Figure 3.16 compares the displacements recorded by all LVDTs (Figure 3.5) (i.e. at all bracing elevations) with FEA results for the 1st test performed on Type A upright frame. The FEA is able to well predict the overall frame displacement.

Figure 3.17 compares the FE deformation of the frame to available experimental photos of tested Type A upright frame. The developed FE model...
predicts well the overall deformed shape. Similarly, Figure 3.18 compares the FE and experimental overall deformation of Type B upright frame. Figure 3.19 (a) and (b) compare the FE and experimental overall deformation of Type C upright frame and Figure 3.19 (c) and (d) compare the FE and experimental local deformations at the bolted connections for the latter upright. Similar to Figure 3.17 to 3.19 show that FEA is able to well capture the global and local behaviour of the upright frames. Note, to magnify the deformation of the frame, Figure 3.17 to 3.19 were shown at a FE displacement greater than the one shown in Figure 3.13 to 3.15.

Figure 3.13 FEA and experimental load-deflection curves for AS4084 (2012) test set-up, Type A upright frame
Figure 3.14 FEA and experimental load-deflection curves for AS4084 (2012) test set-up, Type B upright frame

Figure 3.15 FEA and experimental load-deflection curves for AS4084 (2012) test set-up, Type C upright frame
Figure 3.16 FEA and experimental load-deflection curves for all LVDTs and AS4084 (2012) test set up - Type A upright frame

Table 3.3 Transverse shear stiffness for FEA and experimental tests

<table>
<thead>
<tr>
<th>Upright type</th>
<th>FEA (N/mm)</th>
<th>Experiment(^{(1)}) (N/mm)</th>
<th>FEA/Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AS4084 (2012) (k_{C_{ii}})</td>
<td>EN215512 (2009) (k_{ii})</td>
<td></td>
</tr>
<tr>
<td>Type A upright frame</td>
<td>84</td>
<td>89</td>
<td>0.94</td>
</tr>
<tr>
<td>Type B upright frame</td>
<td>59.4</td>
<td>50.8</td>
<td>1.17</td>
</tr>
<tr>
<td>Type C upright frame</td>
<td>84.4</td>
<td>75.7</td>
<td>1.11</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>AS4084 (2012) (k_{C_{ii}})</td>
<td>EN215512 (2009) (k_{ii})</td>
<td></td>
</tr>
<tr>
<td>Type A upright frame</td>
<td>1.22</td>
<td>1.14</td>
<td>1.07</td>
</tr>
<tr>
<td>Type B upright frame</td>
<td>0.55</td>
<td>0.56</td>
<td>0.98</td>
</tr>
<tr>
<td>Type C upright frame</td>
<td>0.75</td>
<td>0.78</td>
<td>0.96</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

\(^{(1)}\) Average of three tests
Figure 3.17 Deformed shapes following the AS4084 (2012) test set-up, Type A upright frame, (a) FEA (with deformed scale factor of 1.0 and at a displacement of 70 mm) and (b) experimental observations

Figure 3.18 Deformed shapes following the AS4084 (2012) test set-up, Type B upright frame, (a) FEA (with deformed scale factor of 1.0 and at a displacement of 70 mm) and (b) experimental observations
Figure 3.19 Deformed shapes following the AS4084 (2012) test set-up, Type C upright frame, (a) FEA overall deformation of the upright frame (with deformed scale factor of 1.0 and at a displacement of 70 mm) (b) Overall deformation of the upright frame – experimental observations (c) local deformation of the upright at bolted connection – FEA (d) local deformation of the upright at bolted connection – experimental

3.5.2. Tests following the EN15512 test set-up

Figure 3.20 to 3.22 compare the load-deflection curves, at the load application point of the EN15512 test set-up, between the FEA and experimental test results for the three types upright frames configurations. The initial nonlinear responses observed in the FEA load-deflection curves are attributed to nonlinear frictional contacts in the model, but it manifests differently to Figure 3.13 due to the different boundary conditions. Table 3.3 gives the stiffness $k_i$ (Eq. (3.1)) calculated for both the FEA and experimental results. In Table 3.3,
the stiffness were determined by performing a linear regression on the load-deflection curves, specifically between 5 kN and 12 kN for Type A upright frame, between 2 kN and 8 kN for Type B upright frame, and between 4 kN and 14 kN for Type C upright frame. Figure 3.20 to 3.22 and Table 3.3 show a good agreement between the FEA and experimental tests. The average predicted to experiment ratio is 1.00 and the maximum ratio is 1.07 for Type A upright frame. Figure 3.23 presents the deformed shape of the FEA and experimentally tested Type A upright frame and shows that the developed FE model predicts well the overall deformed shape. Figure 3.24 shows the upright rotation and the deformation of the bracing members at the bolted connections for Type B upright frame. Similarly, Figure 3.25 shows the local deformation at the bolted upright-bracing connection for Type C upright frame. Figure 3.24 to 3.25 also show that FE models are able well capture the global and local deformation of the upright frames when tested following the EN15512 test set-up. Note, to magnify the deformation of the frame, Figure 3.23 to 3.25 were shown at a FE displacement greater than the one shown in Figure 3.20 to 3.22.

![Image](image_url)

Figure 3.20 FEA and experimental load-deflection curves for EN15512 (2009) test set-up, Type A upright frame
Figure 3.21 FEA and experimental load-deflection curves for EN15512 (2009) test set-up, Type B upright frame

Figure 3.22 FEA and experimental load-deflection curves for EN15512 (2009) test set-up, Type C upright frame
Figure 3.23 Deformed shapes following the EN15512 test set-up, Type A upright frame (a) FEA (with deformed scale factor of 1.0 and at a displacement of 14 mm) and (b) experimental observation.

Figure 3.24 EN15512 (2009) test set-up deformed shapes, Type B upright frame (a) FEA (with deformed scale factor of 1.0 and at a displacement of 14 mm) and (b) experimental observation.
3.6. Contribution of factors affecting the transverse shear stiffness

3.6.1. General

The verified FE model is used herein to determine the contribution of factors affecting the shear stiffness of the frames. The following factors are investigated:

(a) bolt bending deformation, (b) local deformation at the bracing member bolt holes, (c) local deformation at the upright bolt holes (d) cross-sectional deformation of the end of the bracing members, (e) cross-sectional deformation of the uprights in the vicinity of the bolted connection, (f) axial elongation of bracing members and (g) upright overall bending and shear deformation.

Specifically, the contribution of the previous factors were determined by performing the following changes to the FE model:

(a) Bolt bending deformation: the bending stiffness of the bolts was increased by multiplying their Young’s modulus $E$ by 100 (Yu and Schafer, 2007) effectively creating rigid bodies.

(b) Local deformation at bolt hole of bracing members: a 2 mm wide circular strip around the upright holes was modelled with a high Young’s modulus (multiplied by 100). This strip width was found to be efficient in preventing ovalization of the holes.
(c) **Local deformation at bolt hole of uprights:** Similar to (b), a 2 mm wide circular strip around the holes was modelled with a high Young’s modulus (multiplied by 100).

(d) **Cross-sectional deformation of bracing members:** to prevent the cross-sectional deformation of the bracing members at the bolted connections (as in Figures 3.19 (d) and 3.24 (b)), the ends of the bracing members were made rigid, by increasing their Young’s modulus by a factor of 100, on a length of 70 mm.

(e) **Bracing axial deformation:** the Young’s modulus of the entire bracing members was increased by 100 to prevent their axial shortening and elongation.

(f) **Cross-sectional deformation of uprights:** to prevent the cross-sectional deformation of the uprights at the connections, the Young’s modulus of the uprights on a length of +/- 75 mm about the bolted connection, was multiplied by 100.

(g) **Upright shear and bending deformation:** similar to (e) the Young’s modulus of the entire upright was multiplied by 100, so the uprights act as rigid bodies.

The contribution of the deformations of bracing members and uprights to the overall shear stiffness of the upright frames is treated separately in this chapter. The combinations of factors analysed are presented in Table 3.4. Note that while the influence of the upright and bracing member deformations are treated separately, preventing the bracing members to deform would have an influence on the local deformation of the uprights. The same applies to the local behaviour of the bracing members when preventing the uprights to deform.

Similarly to the model presented in section 3.4, nonlinear geometry and material analyses are performed with the Young’s moduli and yield stresses reported in Table 3.2 used as input. The influence of each analysed factor on the overall shear stiffness is quantified as:

\[
\Omega = \frac{S_{\text{modified}}}{S_{\text{initial}}} \tag{3.5}
\]
where $S_{\text{initial}}$ and $S_{\text{modified}}$ are the initial numerical (reported in Table 3.3) and modified (i.e. with increased $E$ for selected parts of the frame) shear stiffness of the upright frames, respectively, calculated from Eqs (3.2) and (3.3) for the EN15512 and AS4084 test set-ups, respectively.

The same three upright frame configurations tested in Gilbert et al. (2012) with lip-to-lip channel bracing members, and reported in section 3.3 are used as a case study in this research. In addition, to further analyse the behaviour of the frames, the previous analyses were re-run with back-to-back channel bracing members. Detailed description of the back-to-back upright frames is similar to one described in Section 3.4 for the lip-to-lip upright frames. The contribution of the aforementioned factors is analysed for these three frames for both the AS4084 and EN15512 test set-ups.

3.6.2. Lip-to-lip bracing configuration

3.6.2.1. Contribution of factors according to AS4084 test set-up

Table 3.4 shows the contribution of the analysed factors on the shear stiffness of the upright frames following the AS4084 test set-up and lip-to-lip bracing configuration. The values in brackets show the increase in shear stiffness between two consecutive factors. For all upright frames, the effect of the bolt bending on the frame shear stiffness is less than 5%. This results from the load mainly being transferred in shear in the bolt from the web of the bracing members to the uprights (Sajja, 2008; Far et al., 2017)

From Table 3.4, the effect of the local deformation at the bracing holes contributes the most to the shear stiffness for Type A and C upright frames (about 14%). This is explained by higher contact stress at bracing holes when compared to Type B upright frames due to (i) thinner bracing members used for Type A upright frames and (ii) a slender cross-section for Type C uprights which is prone to cross-sectional deformation, therefore leading to significant deformations of the uprights at the connections and high contact forces between the bolts and other elements. This is observed in the FEA by plastification occurring at the bracing holes earlier for Types A and C than for Type B upright frames. The cross-sectional deformation at the ends of the
bracing members is found to contribute the most to the shear stiffness of the frames, especially for Type B upright frames, with the stiffness being increased by more than 85%. This is attributed to Type B upright being a compact cross-section, therefore not prone to cross-sectional deformation, and forcing the bracing members to deform with the upright when it is twisting Figure 3.18). Consequently, local deformation is mainly concentrated to the flanges of the bracing members. The axial stiffness of the bracing members was found to contribute more to the overall stiffness of the frame for Type A upright frames (12%) than for Types B and C.

Regarding the deformation of the uprights, the local deformation at the holes and the cross-sectional deformation at the connections contribute more to the shear stiffness of the frame for Type C upright frames than for the other two types. This is due to a slender upright cross-section and consequently significant cross-sectional deformation of the uprights at the connections. Having rigid uprights increases the shear stiffness by 57% and 63% for Types A and B upright frames, respectively. For Type C upright frames, the upright bending stiffness contributes to the overall stiffness, about 14%.

### 3.6.2.2. Contribution of factors according to EN15512 test set-up

In general, when tested using the EN15512 test set-up, the analysed factors contribute to the overall shear stiffness of the frames following a similar trend than when tested following the AS4084 test set-up. For Type C upright frames, the axial deformation of bracing members contributes more to the frame shear stiffness, about 11%, when compared to the AS4084 test set-up. For Type A upright frames, the local deformation of the upright affects more the overall shear stiffness when tested following the EN15512 test set-up (about 29%) than when tested following the AS4084 test set-ups (about 6%). The contribution of the upright bending stiffness is also different between the two test set-up for Types B and C upright frames. The differences above are attributed to the different loading directions between the two test set-ups, resulting in different deformations of the frames.
<table>
<thead>
<tr>
<th>Structural component</th>
<th>Factors</th>
<th>Contribution (Ω) - AS4084 test set-up(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Type A upright frame</td>
</tr>
<tr>
<td>Bolts</td>
<td>(a) Bolt bending</td>
<td>1.02</td>
</tr>
<tr>
<td>Bracing members</td>
<td>(b) Local deformation at the bolt holes</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>(b) + (d) Local deformation at the end of braces</td>
<td>1.64 (+ 0.50)</td>
</tr>
<tr>
<td></td>
<td>(b) + (d) + (e) Axial deformation of braces</td>
<td>1.76 (+ 0.12)</td>
</tr>
<tr>
<td>Uprights</td>
<td>(c) Local deformation at the bolt holes</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>(c) + (f) Local deformation at the connections</td>
<td>1.07 (+ 0.06)</td>
</tr>
<tr>
<td></td>
<td>(c) + (f) + (g) Upright bending stiffness</td>
<td>1.64 (+ 0.57)</td>
</tr>
<tr>
<td></td>
<td>Contribution (Ω) – EN15512 test set-up</td>
<td></td>
</tr>
<tr>
<td>Bolts</td>
<td>(a) Bolt bending</td>
<td>1.02</td>
</tr>
<tr>
<td>Bracing members</td>
<td>(b) Local deformation at the bolt holes</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>(b) + (d) Local deformation at the end of braces</td>
<td>1.75 (+ 0.52)</td>
</tr>
<tr>
<td></td>
<td>(b) + (d) + (e) Axial deformation of braces</td>
<td>1.91 (+ 0.16)</td>
</tr>
<tr>
<td>Uprights</td>
<td>(c) Local deformation at the bolt holes</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>(c) + (f) Local deformation at the connections</td>
<td>1.31 (+ 0.29)</td>
</tr>
<tr>
<td></td>
<td>(c) + (f) + (g) Upright bending stiffness</td>
<td>1.85 (+ 0.54)</td>
</tr>
</tbody>
</table>

(1) The values in brackets show the increase in shear stiffness between two consecutive factors.
3.6.3. Contribution of factors affecting the transverse shear stiffness of back-to-back upright frames

3.6.3.1. Contribution of factors according to AS4084 test set-up

Back-to-back bracing configurations typically show higher shear stiffness than lip-to-lip bracing configurations (Sajja et al., 2008; Rao et al., 2004; Far et al., 2017). Table 3.5 shows the contribution of the analysed factors on the shear stiffness of the back-to-back upright frames following the AS4084 test set-up. The effect of bolt bending on the shear stiffness is significant due to the back-to-back bracing pattern, now, resulting in high bending moments in the bolts. This effect is less pronounced for Type A upright frames due to shorter bolts being used when compared to Type B and C upright frames.

Similar to the lip-to-lip configurations, the effect of the local deformation at the bracing holes contributes to the frame shear stiffness more for Types A and C upright frames than for Type B. Cross-sectional deformation at the ends of the bracing members contributes to the shear stiffness more for Type B upright frames (about 40%) than for the other two types. Unlike the lip-to-lip upright frames, the axial stiffness of the bracing members is found to significantly contribute to the overall stiffness of the frame, especially for Type A (141%) upright frames.

Regarding the deformation of the uprights and similar to lip-to-lip upright frames, effect of the local deformation at the holes and cross-sectional deformation at the connections contribute more to the shear stiffness, about 7% and 19% respectively, for Type C upright frames than for the other two types. Upright bending stiffness contributes the most to the shear stiffness for Type A upright frames (about 55%) and the least for Type C upright frames (about 25%). In overall, the upright bending stiffness contributes the most to the shear stiffness of upright frames for all upright types. Compared to lip-to-lip upright frames, back-to-back frames show less local deformations of the uprights at the bolted connections.
3.6.3.2. Contribution of factors according to EN15512 test set-up

Similar to lip-to-lip upright frames, when tested using the EN15512 test set-up, the contribution of the analysed factors to the overall shear stiffness of the frames presents a similar trend as when tested following the AS4084 test set-up. When compared to the AS4084 test set-up, the local deformation at the bolt holes of the bracing members is insignificant and about 3% for all upright frames. Unlike AS4084 test set-up, the local deformation at the ends of bracing members contributes more to the shear stiffness for Type A upright frames than Types B and C. The differences above are attributed to different deformations of the frames due to different loading directions between the two test set-ups.
Chapter 3

Table 3.5 Contribution of factors on shear stiffness for each structural component – back-to-back upright frames

<table>
<thead>
<tr>
<th>Structural component</th>
<th>Factors</th>
<th>Contribution (Ω) – AS4084 test set-up(1)</th>
<th>Type A upright frame</th>
<th>Type B upright frame</th>
<th>Type C upright frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolts</td>
<td>(a) Bolt bending</td>
<td>1.14</td>
<td>1.78</td>
<td>2.02</td>
<td></td>
</tr>
<tr>
<td>Bracing members</td>
<td>(b) Local deformation at the bolt holes</td>
<td>1.12</td>
<td>1.02</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) + (d) Local deformation at the end of braces</td>
<td>1.42 (+0.30)</td>
<td>1.42 (+0.40)</td>
<td>1.30 (+0.24)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) + (d) + (e) Axial deformation of braces</td>
<td>2.81 (+1.41)</td>
<td>2.08 (+0.66)</td>
<td>1.83 (+0.53)</td>
<td></td>
</tr>
<tr>
<td>Uprights</td>
<td>(c) Local deformation at the bolt holes</td>
<td>1.01</td>
<td>1.04</td>
<td>1.07</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) + (f) Local deformation at the connections</td>
<td>1.15 (+0.14)</td>
<td>1.18 (+0.14)</td>
<td>1.26 (+0.19)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) + (f) + (g) Upright bending stiffness</td>
<td>1.70 (+0.55)</td>
<td>1.54 (+0.36)</td>
<td>1.51 (+0.25)</td>
<td></td>
</tr>
</tbody>
</table>

Contribution (Ω) – EN15512 test set-up

| Bolts                | (a) Bolt bending | 1.19 | 1.89 | 2.03 |
| Bracing members      | (b) Local deformation at the bolt holes | 1.03 | 1.03 | 1.03 |
|                      | (b) + (d) Local deformation at the end of braces | 1.53 (+0.50) | 1.45 (+0.42) | 1.36 (+0.33) |
|                      | (b) + (d) + (e) Axial deformation of braces | 3.80 (+2.27) | 2.12 (+0.67) | 1.70 (+0.34) |
| Uprights             | (c) Local deformation at the bolt holes | 1.02 | 1.06 | 1.14 |
|                      | (c) + (f) Local deformation at the connections | 1.20 (+0.18) | 1.32 (+0.26) | 1.53 (+0.39) |
|                      | (c) + (f) + (g) Upright bending stiffness | 1.64 (+0.44) | 1.60 (+0.28) | 1.76 (+0.23) |

(1) The values in brackets show the increase in shear stiffness between two consecutive factors.

3.7. Summary

In this chapter, an advanced shell Finite Element model of bolted cold-formed steel storage rack upright frames, with channel-bracing members, was developed for both lip-to-lip and back-to-back bracing configurations. The nonlinear interaction (contact) behaviour between components at the bolted connections was modelled to capture the local deformation at these locations. Results show that the FE model is able to accurately capture the shear stiffness of the frames when compared to published experimental tests, with differences...
ranging from 2% to 17%. The model was then used to quantify the contribution of factors influencing the transverse shear stiffness of six configurations of upright frames, including the deformation of the bolts, bracing members and uprights. Analyses were ran by deforming the frame following the testing methods in the AS4084 and EN15512 specifications. Results showed that plastification at the bolt holes starts at an early stage of loading and particularly for the frames tested following the alternative AS4084 test set-up. For lip-to-lip upright frames, (i) effect of bolt bending on the shear stiffness is insignificant and is less than 5%. (ii) local deformation at the end of the bracing members contributes the most to the overall shear stiffness of the frames, about 62% on average for both the AS4084 and EN15512 test set-ups, and (iii) effect of the upright bending stiffness on the shear stiffness is significant, about 44% on average for both the AS4084 and EN15512 test set-ups. For back-to-back upright frames, (i) bolt bending significantly influences the shear stiffness, 68% on average for the two test set-ups, (ii) unlike the lip-to-lip frames, axial deformation of bracings significantly influence the frame shear stiffness, about 81% on average for both the AS4084 and EN15512 test set-ups, and (iii) effect of upright bending stiffness on shear stiffness is significant, about 35% on average for both the AS4084 and EN15512 test set-ups.
CHAPTER 4

Local and distortional biaxial bending capacities of cold-formed steel storage rack uprights

Statement of contribution to co-authored published paper

This chapter includes a co-authored paper. The bibliographic details of the co-authored paper, including all authors, are:

Talebian N, Gilbert P., Pham CH, Chariere R, Karampour H, Local and distortional biaxial bending capacities of cold-formed steel storage rack uprights, Journal of Structural Engineering, 144(6), 2018.

My contribution to this paper involved: literature review, Test setup, conduct experiments, analyses of experimental results, discussion of the results, writing, editing and response to reviewers.

Signed: __________________________ Date: 31/07/2018____

PhD Candidate: Nima Talebian

Countersigned: __________________________ Date: 31/07/2018____

Principal Supervisor: Associate Professor Benoit P. Gilbert

Countersigned: __________________________ Date: 31/07/2018____

Principal Supervisor: Dr Hassan Karampour
Local and distortional biaxial bending capacities of cold-formed steel storage rack uprights

Abstract

Cold-formed steel storage rack-supported buildings, also referred to as “clad racks”, support both the building enclosure and the stored goods. Due to combined actions of wind loading and stored pallets, uprights undergo a combination of biaxial bending and compression. The focus of attention of this chapter is only on pure biaxial bending capacity of the uprights. In cold-formed steel structures international specifications, a linear interaction equation is typically used to account for members subject to biaxial bending and may be inaccurate. In order to produce safe and economical design guidelines, this chapter experimentally investigates the actual interactive relationship between bending of the uprights about the major and minor axes, for local and distortional buckling. Two types of regularly perforated and non-perforated storage rack uprights are investigated. Results show that a nonlinear interactive relationship governs the biaxial bending of the studied uprights and the linear interaction equation in design specifications underestimates the biaxial bending capacity by up to 44% and 68% for local and distortional buckling, respectively. Also, the accuracy of the Direct Strength Method (DSM) to directly predict the local and distortional buckling strengths of the uprights under biaxial bending is investigated. Results show that DSM equations provide better predictions but still underestimate the biaxial bending capacity by up to 27% and 36% for local and distortional buckling, respectively.

Keywords

Steel storage racks; Biaxial bending; Local buckling; Distortional buckling
4.1. Introduction

Cold-formed steel storage racks are common structural systems extensively used in industry. Their applications range from light shelving racks for domestic, archive or shopping to heavy load bearing industrial systems that can reach up to 30 metres high. They typically consist of upright frames connected by pallet beams. While the vertical uprights are usually mono-symmetric and open sections, the horizontal pallet beams are often doubly-symmetric and closed sections (Baldassino and Bernuzzi, 2000; Freitas, et al., 2005). To ensure fast erection through simple connections between the uprights and the pallet beams, tab connectors are typically welded to the ends of the pallet beams and inserted into the perforated uprights. This results in the upright structural behaviour being further complicated by presence of the regular perforations (Casafont, et al., 2012; Moen and Schafer, 2010; Moen and Schafer, 2008). Figure 4.1 (a) shows a typical selective storage rack.

While steel storage racks are usually free-standing structures built inside an independent building with seldom connections between the two structures, rack-supported buildings, also referred to as “clad racks”, are currently gaining popularity in the logistic industry. In this type of buildings, shown in Figure 4.1 (b), the storage racks support both the stored goods and the building enclosure, resulting in more economical buildings and more complex structural systems subjected to a combination of combined biaxial bending and compression. The vertical load of the stored goods creates a bending moment in the upright about its axis of symmetry while the wind loading, acting on the building walls, creates a bending moment about the axis perpendicular to the upright axis of symmetry. With this combined actions, the uprights undergo biaxial bending. The main current cold-formed steel structures design specifications, including the North American Specification AISI-S100 (2016), the Australian and New Zealand Standard AS/NZS 4600:2018 (AS/NZS, 2018) and the Eurocode 3 (EN 1993-1-3, 2006), consider a linear interaction equation to design members undergoing bi-axial bending in the form of:

\[ \frac{M_x}{M_{tx}} + \frac{M_y}{M_{ty}} \leq 1.0 \]  

(4.1)
where \( M_x \) and \( M_y \) are the applied moments about the two principal cross-sectional \( x \)- and \( y \)-axes, respectively, and \( M_{bx} \) and \( M_{by} \) are the moment capacities about the \( x \)- and \( y \)-axes, respectively. However, it is likely that a representative interaction equation is not linear (Put, et al., 1999; Torabian, et al., 2016; Torabian, et al., 2014; Torabian, et al., 2015). In order to produce safe and economical design guidelines, the actual relationship for bending the uprights about their two principal cross-sectional axes requires further investigation. Moreover, the presence of regular perforations along the upright length may influence the relationship and also needs to be studied.

The capacity of cold-formed steel beams, bent about one of their principal axes, has been widely studied by numerous investigators such as Nguyen (2006), Wang and Zhang (2009), Shifferaw and Schafer (2012), and Pham and Hancock (2013). However, fewer studies have investigated the biaxial bending behaviour of cold-formed steel sections. Shanmugan et al (1989) studied the ultimate capacity of thin-walled steel box columns under biaxial loading and presented interaction diagrams for the studied beam-columns. Duan and Chen (1990) investigated the yield strength of doubly symmetric steel sections including thin-walled hollow sections under axial force and biaxial bending. Put et al (1999) experimentally investigated the biaxial bending of cold-formed steel Z-sections. Interaction design equations were proposed through an extended series of analytical models depending on the angle between the applied load and web inclination. Bedair (2011) analytically investigated the structural behaviour of channel members in combined biaxial bending and compression and presented a method to obtain economical design under this loading pattern. Torabian et al. (2016, 2014, 2015) experimentally investigated the behaviour of cold-formed steel lipped Cee and Zee-sections under biaxial bending and compression (beam-column). Test results showed that a non-linear relationship governs the section capacity. The authors further developed new Direct Strength Method (DSM) equations for beam-column design through a comprehensive Finite Element (FE) parametric study. The new method was found to be conservative and predicts, on average, a 20% increase in strength when compared to current design methods.
For steel storage racks, Kumar and Jayachandran (2016) experimentally investigated the beam-column behaviour of long storage rack uprights subjected to compression and biaxial bending. The authors evaluated the strength using the current linear interaction equation, as well as the recently proposed DSM beam-column design equations in Torabian et al (2014). It was found that the nonlinear interaction is unconservative in some specimens for linearly varying moment. Recently, Bonada et al (2016) experimentally and numerically studied the behaviour of steel storage rack uprights under compression and bending about their major axes. They concluded that the influence of the bending moment about the major axis on the load carrying capacity of the upright due to an eccentrically applied axial force is significant.

This chapter presents and discusses the experimental results of seventy-eight biaxial bending tests performed on two different types of storage rack uprights. In order to investigate both local and distortional buckling failure modes, two different upright lengths per upright type were considered. Uprights with and without regular perforations were also investigated to study the effect of the perforations on the biaxial bending behaviour of the upright. The test results are also compared to the linear interaction equation currently considered in international design specifications and the newly proposed DSM for beam-column equations (Torabian, et al., 2014).

This study aims to investigate the buckling behaviour of steel storage racks under pure biaxial bending only. The interaction between axial compression and biaxial bending is not considered within the scope of this study and requires further investigation, when the pure biaxial bending behaviour is understood.
Figure 4.1 (a) Typical selective storage rack structure (Gilbert, et al., 2012) and (b) clad rack in construction (Courtesy Modulblok)

4.2. Test program (methodology)

4.2.1. Tested uprights

Two different types of rack uprights with different cross-sections, referred herein as “Type A” and “Type B” and shown in Figure 4.2, are investigated. These uprights vary in cross-sectional areas and shapes. Type A upright has a nominal wall thickness of 1.5 mm and a width-to-depth ratio of 0.71, while Type B upright has a nominal wall thickness of 2.0 mm and a width-to-depth ratio of 0.5. The main cross-sectional dimensions and properties of the two upright types are given in Figure 4.2 and Table 4.1, respectively. Both uprights are commercially available but Type A has been specifically rolled-formed for the purpose of this research to a thinner thickness than that of the commercialised uprights. This is to ensure that Type A upright has local and distortional nominal capacities ($M_{bl}$ and $M_{bd}$) according to the Direct Strength Method (DSM) (Schafer, 2008) in Clause 7.2.2 of the AS/NZS 4600: 2018 (AS/NZS, 2018) which are lower than the yield moment $M_y$. On the contrary, Type B has a compact section with the nominal moment capacities $M_{bl}$ and $M_{bd}$ equal to the yield moment $M_y$.

To investigate the effect of perforations on the member capacity and biaxial bending interaction, Type A uprights were tested with and without regular perforations along their length whereas all tested Type B uprights were
Chapter 4

perforated. Both ends of the uprights were welded to 220 mm × 220 mm square steel plates of 10 mm thickness. Uprights of the same type were rolled-formed from the same steel coil.

To enforce the uprights failing either in local or distortional buckling, the length of the tested uprights were determined based on elastic buckling analyses performed using the Finite Element (FE) package Abaqus (2015). S4R shell elements with a fine mesh density (3 mm x 3 mm) (Schafer, et al., 2010) were used. The nominal thickness and cross-sectional dimensions were used in the models. Similar boundary conditions as in the experimental tests were used at the upright walls (see Section “Biaxial Bending Test Set-up”). Warping was restrained by using end plates rigidly connected to the ends of the upright (Bonada, et al., 2016; Crisan, et al., 2012; Roure, et al., 2011). The upright was then simply supported by pinning the end plates at the location of the upright centroidal axis.

The lengths of the tested uprights (per upright type and buckling mode) were solely determined from buckling analyses of the non-perforated uprights bent about the x-axis of symmetry, i.e. by applying a concentrated bending moment at the pinned joints to replicate the test set up described in later sections. These lengths were then used for all biaxial bending configurations in Section “Tested Configurations”. The lengths of the uprights based on the FE elastic buckling analyses were chosen as 400 mm for local buckling to occur, and 1100 mm and 900 mm for distortional buckling to occur for both Types A and B, respectively. These lengths are short enough to ensure that global buckling does not occur and therefore the study only focusses on local and distortional buckling.
Figure 4.2 Cross-sections (a) Upright Type A (b) Upright Type B (Dimensions are in mm)

Table 4.1 Nominal and measured cross-sectional properties of tested uprights

<table>
<thead>
<tr>
<th>Upright</th>
<th>Thickness (mm)</th>
<th>Depth (mm)</th>
<th>Width (mm)</th>
<th>Gross area / Net area</th>
<th>Second moment of area $I_x / I_y$</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>1.5</td>
<td>140</td>
<td>100</td>
<td>1.108</td>
<td>2.52</td>
<td>1.54</td>
</tr>
<tr>
<td>Type B</td>
<td>2.0</td>
<td>120</td>
<td>60</td>
<td>1.124</td>
<td>5.05</td>
<td>1.95</td>
</tr>
</tbody>
</table>
4.2.2. Biaxial bending test set-up

A four-point bending test set-up shown in Figure 4.3 (a) was used to test the uprights under pure biaxial bending. Both ends of the uprights were bolted to solid pinned connections which were connected to two short segments of steel Rectangular Hollow Sections (RHS) to form a beam, as shown in Figure 4.3 (b) (schematic) and Figure 4.4 (overall view of test rig). The load was applied using a 500 kN universal MTS testing machine, through a spreader beam, to the steel RHS, as shown in Figure 4.3. In order to bend the upright about its two principal cross-sectional axes, the upright was rotated about its centroidal axis, as shown in Figure 4.3 (a). The pinned connections (see Figure 4.5) were manufactured using heavy duty roller bearings in order to resist the applied bending moment, while allowing the ends of the uprights to freely rotate about the axis perpendicular to the applied moment. This testing arrangement creates a statically determinate system. The total moment M applied to the upright is calculated as follows:

\[ M = \frac{FL_1}{2} + M_{\text{rig}} \tag{4.2} \]

where \( F \) is the total applied load, \( L_1 = 455 \text{ mm} \) is the distance between the supports and the points of application of the loads, as shown in Figure 4.3, and \( M_{\text{rig}} = 0.864 \text{ kN.m} \) is the measured moment applied by the weight of the steel RHS and solid pinned connections to the upright. The moments \( M_x \) and \( M_y \) developed about the \( x \)-axis (axis of symmetry) and \( y \)-axis (perpendicular to the \( x \)-axis), respectively, are calculated as follows:

\[ M_x = M \cos(\alpha) \quad \text{and} \quad M_y = M \sin(\alpha) \tag{4.3} \]

where \( \alpha \) is the rotational angle of the cross-section about its centroidal axis shown in Figure 4.3.

To prevent the uprights from uplifting in the tension zone through deformation of the end plates, thick Cee-shaped steel plates were bolted to the uprights and to the pinned connections shown in Figure 4.6.
4.2.3. Tested configurations

In order to obtain a sufficient number of points to apprehend the biaxial bending interaction curve, seven different bending configurations per upright type were tested as presented in Table 4.2. Tests were repeated twice for each configuration and upright type. However, due to the compactness of the cross-section, Type B uprights, which were tested for local buckling, tended to fail in the welded zone between the upright and the end plates. Several unsuccessful trials were initially made to reinforce the weld, resulting in lack of samples to test all configurations twice. However, all configurations were tested once but Configuration 6 in which was tested twice. In total, 78 valid tests were performed in displacement control at a stroke rate of 1.25 mm/min for local buckling and 1.5 mm/min for distortional buckling, so as to reach failure in 10 to 15 mins.
Figure 4.3 Experimental four-point bending test setup – Schematic view, (a) side view and (b) top view

Figure 4.4 Experimental test setup – photo
Figure 4.5 Solid pinned connection

Figure 4.6 Cee-shaped plates bolted with the uprights to the pinned connections to prevent uplift in the tension zone
Table 4.2 Tested biaxial bending configurations

<table>
<thead>
<tr>
<th>Configuration 1</th>
<th>Configuration 2</th>
<th>Configuration 3</th>
<th>Configuration 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_x &gt; 0$ and $M_y = 0$</td>
<td>$M_x &gt; 0$, $M_y &gt; 0$ and $M_x = 2M_y$</td>
<td>$M_x &gt; 0$, $M_y &gt; 0$ and $M_y = 2M_x$</td>
<td>$M_x = 0$ and $M_y &gt; 0$</td>
</tr>
</tbody>
</table>

$M_x > 0$ generates compression in the top flange (as drawn), $M_y > 0$ generates compression in the web and $M_y < 0$ generates compression in the lip stiffeners.

4.2.4. LVDT frame to capture deformations

Three Linear Variable Displacement Transducers (LVDT) were used to measure the cross-sectional deformation at mid-length of each specimen. For each upright type, by using the similar method reported in Trouncer and Rasmussen (2014), a frame was laser cut out of 4.5 mm thick acrylic sheets and designed to fit around the upright section. The LVDTs were attached to the frame and recorded the displacement at the free end of each flange (LVDTs 1 and 3) and at the middle of the web (LVDT 2) as shown in Figure 4.7. As the frame moved with the upright, only the actual cross-sectional deformation due to local and distortional buckling was recorded.

4.2.5. Material properties

Three tensile coupon samples per upright type were waterjet cut from the flat steel coil used to manufacture the uprights in the rolling direction, i.e. in the longitudinal direction of the uprights. The coupons were tested according to the Australian standard AS1391 (2007) in a 30 kN LLOYD universal testing
machine. The strain rates were chosen to best match with the ones experienced by the uprights tested in Configuration 1 (see Section “Tested configurations”). However, due to a manipulation error, the strain rate of Type A coupons were 2.3 times higher than the ones experienced by the uprights during the biaxial bending tests. At the slow stroke rates adopted in the bending tests (1.25 mm/min for local buckling and 1.5 mm/min for distortional buckling), such a difference in strain rate marginally affects the measured yield stress of the steel, by no more than 4% (Soroushian and Choi, 1987). Two coupons out of three were fitted with strain gauges, glued on each face of the coupons to accurately measure the Young’s modulus using the average strain deformation of the two gauges.

![Figure 4.7 Transducer frame to measure cross-sectional deformations at mid-length (a) Type A (b) Type B uprights](image)

**4.3. Test Results**

**4.3.1. Biaxial bending response of the short uprights**

Elastic and inelastic local buckling failures were observed for all 400 mm long specimens. Typically, local buckling initially occurred in the compression flange for all uprights tested in Configurations 1, 5, 6 and 7, and in the web tested in Configurations 2, 3 and 4.

Figure 4.8 (a) plots the deformation of the web (LVDT 2) versus the normalised applied moment \((M/M_{\text{max}})\), where \(M\) and \(M_{\text{max}}\) are the applied and maximum recorded moments of the individual test, respectively, for all tested short uprights and Configuration 2 (i.e. \(M_x = 2M_y\) and web in compression). Despite
being a compact section, Type B uprights have buckling starting at about $M/M_{\text{max}} = 0.7$ and showed a more distinct post buckling behaviour than that of Type A uprights, where buckling occurred at about $M/M_{\text{max}} = 0.8$. Figure 4.8 (b) plots the cross-sectional opening at mid-length, as the sum of LVDTs 1 and 3, versus the normalised failure moment ($M/M_{\text{max}}$) for all tested short uprights in Configuration 5 (i.e. $M_x = -2M_y$ and lip stiffeners in compression). In this configuration, Type A uprights showed a larger post buckling behaviour, with buckling starting at about $M/M_{\text{max}} = 0.7$, than that of Type B uprights, where buckling occurred at about $M/M_{\text{max}} = 0.8$. In general, when local buckling occurred in the web, Type B uprights showed more distinct post buckling behaviour than that of Type A, and the opposite was observed when local buckling occurring in the flange. The observed failure modes for Type A and B uprights, tested in Configurations 2 and 5, are shown in Figure 4.9. The normalised ultimate moment capacities ($M_x/M_{\text{bx}}$ or $M_y/M_{\text{by}}$) for all upright types and associated failure modes are summarised in Table 4.3. The moment capacities $M_{\text{bx}}$ and $M_{\text{by}}$ are calculated as the average between two tests, when two tests were performed.

4.3.2. Biaxial bending response of the long uprights

Local, distortional and local-distortional interaction buckling modes were observed in biaxial bending tests of the 900 mm and 1100 mm long uprights. Typically, Type A uprights failed in local-distortional buckling interaction, while Type B uprights failed in distortional buckling at an applied moment close to the first yield moment $M_y$. However, when tested with the web in compression and the lip stiffeners in tension (Configuration 4), all uprights failed in inelastic local buckling. Table 4.3 summarises all observed failure modes.

Figure 4.10 (a) plots the cross-sectional opening, as the sum of LVDTs 1 and 3, versus the normalised failure moment ($M/M_{\text{max}}$) for all tested long uprights and Configuration 1 (i.e. $M_y = 0$). As shown in Figure 4.10 (a), both Type A and Type B uprights experienced large post-buckling deformation, with buckling starting at about $M/M_{\text{max}} = 0.6 - 0.7$. For Type A uprights, the cross-sectional deformation at mid-length is primarily attributed to the tip of the flange in compression. Local buckling was initially observed and the deformation continued to increase until
distortional buckling occurred at a deformation equal to about 20-25 mm, which triggered the failure for perforated and non-perforated uprights, as shown in Figure 4.10 (a). In Figure 4.10 (b) normalised moments are plotted against deformations for Configuration 6 tests (i.e. $M_y = -2M_x$ and lip stiffeners in compression). Contrary to Type B uprights that showed little to no post-buckling behaviour, Figure 4.10 (b) indicates that Type A uprights experienced large post-buckling deformations, with buckling starting at about $M/M_{\text{max}} = 0.5$. This behaviour is somewhat different to the short tested lengths of upright where post-buckling was observed for Type B uprights. In general, for long uprights both Type A and Type B showed large post buckling deformations in Configurations 1, 2 and 3, whereas only Type A showed large post buckling deformation in Configurations 4, 5, 6 and 7.

Figure 4.11 (a) and (b) present the local-distortional buckling interactive failure mode for Type A uprights with perforations and distortional buckling failure mode for Type B uprights, respectively, tested in Configuration 1. Similarly, Figure 4.11 (c) and (d) show the local-distortional buckling interactive failure mode for Type A uprights with no perforations and distortional failure mode for Type B uprights, respectively, tested in Configuration 6. The average normalised ultimate moment capacities ($M_x/M_{bx}$ or $M_y/M_{by}$) for all uprights are tabulated in Table 4.4.
Figure 4.9 Local buckling failure modes (a) Non-perforated Type A upright - Configuration 2 ($M_x = 2M_y$) (b) Type B upright - Configuration 2 ($M_x = 2M_y$) (c) Perforated Type A upright - Configuration 5 ($M_x = -2M_y$) (d) Type B upright - Configuration 5 ($M_x = -2M_y$) (see Table 4.2 for configurations)

Figure 4.10 Cross-section deformation (LVDT 1 + LVDT 3) for all long uprights (a) Configuration 1 ($M_y = 0$) (b) Configuration 6 ($M_y = -2M_x$) (see Table 4.2 for configurations)
4.3.3. Interactive behaviour

As discussed earlier, the North American Specification (AISI-S100 (2016), the Australian Standard and New Zealand Standards AS/NZS 4600:2018 (AS/NZS, 2018) and the Eurocode 3 (EN 1993-1-3, 2006) design specifications use a linear interaction equation (Eq. (4.1)) to account for biaxial bending. Figure 4.12 shows Eq. (4.1) versus the normalised biaxial bending experimental results obtained from the seven different investigated configurations and for the short uprights. Figure 4.12 shows that the governing interaction relationship is not linear and that Eq. (4.1) is conservative for all tested uprights. When tested in Configurations 3 and 6, i.e. \( M_y = +/-2M_x \), the uprights were usually able to sustain a moment \( M_y \) similar to or greater than the moment capacity \( M_{by} \), when solely bent about the y-axis (Configurations 4 and 7). For Configurations 2 and 3 (\( M_y > 0 \), web in compression), at failure Eq. (4.1) gives an interaction ratio significantly higher than unity, ranging from 1.16 (Type B upright tested in Configuration 3) to 1.46 (perforated Type A upright tested in Configuration 2). For \( M_y < 0 \) (Configurations 5 and 6, lip stiffeners in compression), Eq. (4.1) gives a ratio less than that obtained from the \( M_y > 0 \) configurations, but still higher than unity and ranging from 1.10 to 1.26, both ratios for non-perforated Type A upright type.
Figure 4.11 (a) Local-distortional buckling interactive failure mode for perforated Type A upright - Configuration 1 ($M_y = 0$) (b) Distortional failure mode for Type B upright - Configuration 1 ($M_y = 0$) (c) Local-distortional buckling interactive failure mode for non-perforated Type A upright - Configuration 6 ($M_y = -2M_x$) (d) Distortional failure mode for Type B upright - Configuration 6 ($M_y = -2M_x$) (see Table 4.2 for configurations)

The actual interactive biaxial moment capacity and the one obtained from Eq. (4.1) for the short uprights are summarised in Table 4.3.

Similar to Figure 4.12, Figure 4.13 shows the linear interaction equation (Eq. (4.1)) versus all normalised biaxial experimental results obtained for the long uprights (distortional buckling). Similar to the short uprights, Eq. (4.1) is conservative and gives an interaction ratio at failure ranging from 1.04 (Type B upright tested in Configuration 5) to 1.69 (perforated Type A upright tested in Configuration 2) for all biaxial bending configurations. Biaxial bending responses of Type B uprights tend to be closer (about 14%) to the linear interaction curve than Type A uprights. This is more highlighted in Configurations 2 and 5 ($M_x = +/- 2M_y$). While no main biaxial behavioural
differences between perforated and non-perforated uprights can be observed, Type B uprights tend to be closer (about 10%) to the linear interaction curve than Type A uprights, especially for long uprights.

Uprights Type A tested in Configuration 2 (\(M_x = 2M_y\) and web in compression) exhibited a moment capacity \(M_x\) greater (about 15%) than the moment capacity \(M_{bx}\) observed in Configuration 1, i.e. when only bent about the x-axis. This can be explained by the different failure modes that are developed in the cross-section between the two configurations. Configuration 2 principally failed in local buckling, at a moment similar to the one observed in tests performed on short uprights. Configuration 1 failed in local-distortional buckling interaction, and therefore at a lower moment than for pure local buckling (Dinis et al., 2014). In general, the failure moments for Configurations 2-4 are similar (within 4 %) to the moments obtained from the local buckling tests. The right hand part of the interaction curve in Figure 4.13 represents a transition between a local-distortional buckling interactive mode in Configuration 1 to a pure local buckling mode in Configuration 4.

Similar to the short uprights, in the long uprights when lip stiffeners are in compression (Configuration 5 and 6), the biaxial bending capacities are closer (about 12%) to the linear interactive curve.

4.3.4. Material Properties

Average results of material properties including measured Young’s modulus, yield stress calculated as 0.2% tensile proof stress and ultimate tensile strength obtained from the tensile coupon tests are summarized in Table 4.5.
Table 4.3 Comparison of experimental results with DSM for local buckling tests (L=400 mm for all uprights)

<table>
<thead>
<tr>
<th>Upright Type</th>
<th>Number of tests performed</th>
<th>Configuration</th>
<th>Failure Mode</th>
<th>$\lambda_1$</th>
<th>$M_y/M_{bx}$</th>
<th>$M_y/M_{by}$</th>
<th>$M_{Exp}/M_{DSM}$ (No reserve)</th>
<th>$M_{Exp}/M_{DSM}$ (With reserve)</th>
<th>$M_{Exp}/M_{DSM}$ (Pham and Hancock)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A- non Perforated</td>
<td>2</td>
<td>1</td>
<td>L</td>
<td>0.69</td>
<td>1.00</td>
<td>0</td>
<td>1.00</td>
<td>1.12</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>L</td>
<td>0.58</td>
<td>0.80</td>
<td>0.62</td>
<td>1.42</td>
<td>1.24</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>L</td>
<td>0.52</td>
<td>0.29</td>
<td>0.96</td>
<td>1.25</td>
<td>1.58</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>L</td>
<td>0.52</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.63</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>L</td>
<td>1.02</td>
<td>0.63</td>
<td>0.62</td>
<td>1.25</td>
<td>1.16</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>L</td>
<td>0.97</td>
<td>0.21</td>
<td>0.90</td>
<td>1.11</td>
<td>1.33</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>L</td>
<td>0.91</td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.41</td>
<td>1.41</td>
</tr>
<tr>
<td>Type A- perforated</td>
<td>2</td>
<td>1</td>
<td>L</td>
<td>0.80</td>
<td>1.00</td>
<td>0</td>
<td>1.00</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>L</td>
<td>0.77</td>
<td>0.83</td>
<td>0.61</td>
<td>1.44</td>
<td>1.13</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>L</td>
<td>0.56</td>
<td>0.31</td>
<td>0.97</td>
<td>1.28</td>
<td>1.44</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>L</td>
<td>0.57</td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.41</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>L</td>
<td>1.06</td>
<td>0.65</td>
<td>0.56</td>
<td>1.21</td>
<td>1.08</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>L</td>
<td>1.00</td>
<td>0.25</td>
<td>0.92</td>
<td>1.17</td>
<td>1.37</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>L</td>
<td>0.95</td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.37</td>
<td>1.14</td>
</tr>
<tr>
<td>Type B- Perforated</td>
<td>1</td>
<td>1</td>
<td>L</td>
<td>0.37</td>
<td>1.00</td>
<td>0</td>
<td>1.00</td>
<td>1.20</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>L</td>
<td>0.42</td>
<td>0.57</td>
<td>0.72</td>
<td>1.29</td>
<td>1.03</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>L</td>
<td>0.40</td>
<td>0.18</td>
<td>0.98</td>
<td>1.16</td>
<td>1.32</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>L</td>
<td>0.41</td>
<td>0.00</td>
<td>1</td>
<td>1.00</td>
<td>1.29</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
<td>L</td>
<td>0.48</td>
<td>0.55</td>
<td>0.61</td>
<td>1.16</td>
<td>0.99</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>L</td>
<td>0.41</td>
<td>0.21</td>
<td>0.99</td>
<td>1.20</td>
<td>1.52</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>7</td>
<td>L</td>
<td>0.39</td>
<td>0</td>
<td>1</td>
<td>1.00</td>
<td>1.47</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Average</td>
<td>1.29</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>CoV</td>
<td>0.14</td>
<td>0.14</td>
</tr>
</tbody>
</table>

(1) L = Local buckling; (2) No inelastic reserve capacity; (3) Inelastic reserve capacity as in AISI-S100 (2016); (4) Extended reserve strength in Pham and Hancock (2013)
Table 4.4 Comparison of experimental results with DSM for distortional buckling tests (L=1100 mm for Type A and L=900 mm for Type B uprights)

<table>
<thead>
<tr>
<th>Upright Type</th>
<th>Number of tests performed</th>
<th>Configuration</th>
<th>Failure(1) Mode</th>
<th>λ(1)</th>
<th>λ(2)</th>
<th>Mx/Mbx</th>
<th>My/Myby</th>
<th>Eq. (4.1) (Linear)</th>
<th>MExp/MDSM (No reserve)(2)</th>
<th>MExp/MDSM (With reserve)(3)</th>
<th>MExp/MDSM (Pham and Hancock)(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type A- non Perforated</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>L+D</td>
<td>0.72</td>
<td>0.80</td>
<td>1.00</td>
<td>0</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>L+D</td>
<td>0.58</td>
<td>0.58</td>
<td>0.99</td>
<td>0.58</td>
<td>1.57</td>
<td>1.26</td>
<td>1.19</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>L+D</td>
<td>0.52</td>
<td>0.58</td>
<td>0.37</td>
<td>0.93</td>
<td>1.30</td>
<td>1.65</td>
<td>1.49</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>L</td>
<td>0.52</td>
<td>1.01</td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.77</td>
<td>1.41</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>L+D</td>
<td>1.06</td>
<td>0.65</td>
<td>0.61</td>
<td>1.33</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>L+D</td>
<td>1.00</td>
<td>0.90</td>
<td>0.22</td>
<td>0.98</td>
<td>1.20</td>
<td>1.29</td>
<td>1.29</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>L+D</td>
<td>0.94</td>
<td>0.85</td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.23</td>
<td>1.23</td>
<td>1.09</td>
</tr>
<tr>
<td><strong>Type A- perforated</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>L+D</td>
<td>0.80</td>
<td>0.83</td>
<td>1.00</td>
<td>0</td>
<td>1.00</td>
<td>0.97</td>
<td>0.97</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>L+D</td>
<td>0.77</td>
<td>0.64</td>
<td>1.07</td>
<td>0.61</td>
<td>1.68</td>
<td>1.19</td>
<td>1.19</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>L+D</td>
<td>0.57</td>
<td>0.64</td>
<td>0.40</td>
<td>0.96</td>
<td>1.36</td>
<td>1.53</td>
<td>1.46</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>L</td>
<td>0.58</td>
<td></td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.52</td>
<td>1.26</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>L+D</td>
<td>1.10</td>
<td>1.03</td>
<td>0.66</td>
<td>0.57</td>
<td>1.23</td>
<td>1.12</td>
<td>1.12</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>L+D</td>
<td>1.03</td>
<td>0.91</td>
<td>0.24</td>
<td>0.87</td>
<td>1.11</td>
<td>1.24</td>
<td>1.24</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>L+D</td>
<td>0.99</td>
<td>0.86</td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.28</td>
<td>1.28</td>
<td>1.15</td>
</tr>
<tr>
<td><strong>Type B- Perforated</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>D</td>
<td>-</td>
<td>0.60</td>
<td>1.00</td>
<td>0</td>
<td>1.00</td>
<td>1.02</td>
<td>0.99</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>D</td>
<td>-</td>
<td>0.49</td>
<td>0.67</td>
<td>0.62</td>
<td>1.29</td>
<td>1.00</td>
<td>0.88</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>D</td>
<td>-</td>
<td>0.47</td>
<td>0.24</td>
<td>0.95</td>
<td>1.19</td>
<td>1.47</td>
<td>1.23</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>L</td>
<td>0.48</td>
<td></td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.49</td>
<td>1.24</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>D</td>
<td>-</td>
<td>0.71</td>
<td>0.55</td>
<td>0.51</td>
<td>1.06</td>
<td>0.85</td>
<td>0.85</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>D</td>
<td>-</td>
<td>0.58</td>
<td>0.23</td>
<td>0.91</td>
<td>1.14</td>
<td>1.39</td>
<td>1.28</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>L</td>
<td>0.40</td>
<td></td>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.48</td>
<td>1.17</td>
<td>1.10</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.16</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.17</td>
<td>0.19</td>
</tr>
</tbody>
</table>

(1) L = Local buckling, D = Distortional, L+D = Local-Distortional interaction; (2) No inelastic reserve capacity; (3) Inelastic reserve capacity as in AISI-S100 (2016); (4) Extended reserve strength in Pham and Hancock (2013)
Table 4.5 Measured material properties of tested uprights

<table>
<thead>
<tr>
<th>Upright Type</th>
<th>E (MPa)</th>
<th>Fy (MPa)</th>
<th>Fu (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>194645</td>
<td>436.3</td>
<td>509.3</td>
</tr>
<tr>
<td>Type B</td>
<td>199497</td>
<td>424.7</td>
<td>478.3</td>
</tr>
</tbody>
</table>

Figure 4.12 Biaxial bending interaction points for local buckling – All uprights

Figure 4.13 Biaxial bending interaction points for distortional buckling - All uprights
4.4. Direct strength method equations to predict bending capacity

The newly developed DSM equations for beam-columns (Torabian, et al., 2014), with an axial force equal to zero, give the classical DSM equations for pure bending ((AS/NZS, 2018) (AISI-S100:2016). The biaxial bending elastic and yield moments (i.e. calculated about their overall axis of bending) are used as input in the equations. This approach in the DSM is used in the following sections.

Three different approaches are investigated in this study to compare the test results with DSM results, namely; (i) By using the DSM equations given in the AS/NZS 4600:2018 (AS/NZS, 2018), with the nominal member capacity equal to the yield moment for compact cross-sections, (ii) Through exploiting the inelastic reserve capacity for compact cross-sections, as permitted in the new AISI-S100 (2016) and (iii) By adopting an extended inelastic reserve strength proposed by Pham and Hancock (2013). These methods are described in the following Sections.

4.4.1. DSM equations

4.4.1.1. Local buckling

The DSM nominal member capacity \( M_{bl} \) for local buckling, ignoring inelastic reserve capacity, is defined as (AISI-S100, 2016, AS/NZS, 2018, Schafer, 2008):

\[
M_{bl} = M_y \quad \text{if} \quad \lambda_t \leq 0.776 \quad (4.4)
\]

\[
M_{bl} = \left[ 1 - 0.15 \left( \frac{M_{ol}}{M_y} \right)^{0.4} \right] \left( \frac{M_{ol}}{M_y} \right)^{0.4} M_y \quad \text{if} \quad \lambda_t > 0.776 \quad (4.5)
\]

where \( M_{bl} \) and \( M_y \) are the elastic local buckling moment and yield moment respectively, and \( \lambda_t \) is a non-dimensional slenderness ratio defined as:

\[
\lambda_t = \sqrt{\frac{M_y}{M_{ol}}} \quad (4.6)
\]
Furthermore, the recent AISI-S100 (2016) permits the use of the local inelastic reserve capacity, i.e. allowing the nominal member capacity to range between $M_y$ and the plastic moment $M_p$ for compact cross-sections, i.e. if $\lambda_l \leq 0.776$. When the first yield is in compression:

$$M_{bl} = M_y + (1 - 1/C_{yl}^2)(M_p - M_y) \quad (4.7)$$

where

$$C_{yl} = \sqrt{0.776/\lambda_l} \leq 3 \quad (4.8)$$

and when the first yield is in tension:

$$M_{bl} = M_{yc} + (1 - 1/C_{yl}^2)(M_p - M_y) \leq M_{yr3} \quad (4.9)$$

where

$$M_{yr3} = M_y + 8/9(M_p - M_y) \quad (4.10)$$

and $M_{yc}$ is the moment at which yielding initiates in compression (after yielding in tension). $M_{yc}$ has been conservatively taken as $M_y$ herein (AISI-S100, 2016, Torabian, et al., 2014).

Pham and Hancock (2013) proposed an extended range of the cross-sectional slenderness about the symmetric axis of bending for which the inelastic strength can be applied. For local buckling, the inelastic strength can be applied when $\lambda_l \leq 1.55$ and $C_{yl}$ in Eq. (4.8) becomes:

$$C_{yl} = \sqrt{1.55/\lambda_l} \leq 3 \quad (4.11)$$

and the inelastic local strength is calculated as:

$$M_{yl} = M_y + (1 - 1/C_{yl}^2)(M_p - M_y) \quad (4.12)$$

$M_{yl}$ is then used in the classical DSM Eq. (4.4-4.5) instead of $M_y$, and $\lambda_{ln}$ defined as:

$$\lambda_{ln} = \sqrt{\frac{M_{yl}}{M_{ol}}} \quad (4.13)$$
is used instead of \( \lambda \) to obtain the new nominal member capacity with extended range \( M_{bln} \). It should be noted that member yield moment of net cross-section, \( M_{ynet} \) (net section modulus reference to the extreme fiber at yield multiplied by yield stress), has been used for perforated sections AS/NZS (2018).

4.4.1.2 Distortional Buckling

Similarly, the DSM nominal member capacity \( M_{bd} \) for distortional buckling, ignoring inelastic reserve capacity, is as follows (AISI-S100, 2016, AS/NZS, 2018, Schafer, 2008):

\[
M_{bd} = M_y \quad \text{if} \quad \lambda_d \leq 0.673
\]  \hspace{1cm} (4.14)

\[
M_{bd} = \left[ 1 - 0.22 \left( \frac{M_{od}}{M_y} \right)^{0.5} \right] \left( \frac{M_{od}}{M_y} \right)^{0.5} M_y \quad \text{if} \quad \lambda_d > 0.673
\]  \hspace{1cm} (4.15)

where \( M_{od} \) is the elastic buckling moment for distortional buckling, and \( \lambda_d \) is a non-dimensional slenderness ratio defined as:

\[
\lambda_d = \sqrt{\frac{M_y}{M_{od}}}
\]  \hspace{1cm} (4.16)

According to AISI-S100 (2016), distortional inelastic reserve capacity is permitted to be taken into account if \( \lambda_d \leq 0.673 \). The same equations as for local buckling Eqs (4.7-4.10) are used with \( C_{yd} \) in Eqs (4.7, 4.9) replaced by:

\[
C_{yd} = \sqrt{0.673 / \lambda_d} \leq 3
\]  \hspace{1cm} (4.17)

For distortional buckling, the inelastic strength with extended range proposed by Pham and Hancock (2013) can be applied when \( \lambda_d \leq 1.45 \) and \( C_{yd} \) in Eq. (4.17) becomes:

\[
C_{ydn} = \sqrt{1.45 / \lambda_d} \leq 3
\]  \hspace{1cm} (4.18)

and the inelastic distortional strength is calculated as:

\[
M_{nyd} = M_y + \left(1 - 1/C_{ydn}^2\right) (M_p - M_y)
\]  \hspace{1cm} (4.19)
The $M_{nyd}$ is then used in the classical DSM Eq. (4.14-4.15) instead of $M_y$, and $\lambda_{dn}$ defined as:

$$\lambda_{dn} = \sqrt{\frac{M_{rod}}{M_{od}}}$$

is used instead of $\lambda_d$ to obtain the new nominal member capacity with extended range $M_{bdn}$.

Moreover, the recent AISI-S100 (2016) considers the following equations for members with holes failing in distortional buckling:

$$M_{bd} = M_{ynet}$$

if

$$\lambda_d \leq \lambda_{d1}$$

(4.21)

$$M_{bd} = M_{ynet} - \left( \frac{M_{ynet} - M_{d2}}{\lambda_{d2} - \lambda_{d1}} \right) (\lambda_d - \lambda_{d1}) \leq$$

$$\sqrt{1 - 0.22 \left( \frac{M_{od}}{M_y} \right)^{0.5}} \left( \frac{M_{od}}{M_y} \right)^{0.5} M_y$$

if

$$\lambda_{d1} \leq \lambda_d \leq \lambda_{d2}$$

(4.22)

where

$$\lambda_d = \sqrt{\frac{M_y}{M_{crd}}}$$

(4.23)

$$\lambda_{d1} = 0.673 (M_{ynet} / M_y)^3$$

(4.24)

$$\lambda_{d2} = 0.673 \left[ 1.7 (M_y / M_{ynet})^{2.7} - 0.7 \right]$$

(4.25)

$$M_{d2} = \left[ 1 - 0.22 (1/\lambda_{d2}) \right] (1/\lambda_{d2}) M_y$$

(4.26)

$M_{crd}$ is elastic distortional buckling moment including effect of holes, and $M_y$ and $M_{ynet}$ are yield moment and yield moment of net cross-section, respectively.

### 4.4.1.3. Local-distortional buckling interaction

In the experimental tests, local and distortional interactive failure modes were observed. Eqs (4.4-4.6, 4.14-4.16) cannot fully capture this failure phenomenon (Dinis and Camotim, 2010; Silvestre, et al., 2012). Consequently, the NLD (Nominal Local-Distortional) and NDL (Nominal Distortional-Local) approaches
which were proposed to consider this interaction into the DSM by Schafer (2002) are used herein. The method replaces $M_y$ by either (i) $M_{bl}$ obtained from Eqs (4.4-4.6) in Eqs (4.14–4.16) (NDL approach) or (ii) $M_{bd}$ obtained from Eqs (4.14-4.16) in Eqs (4.4-4.6) (NLD approach).

Therefore, the NLD approach predicts the interactive distortional-local nominal moment capacity $M_{bld}$, when the inelastic reserve capacity is not considered, as:

$$M_{bld} = M_{bd} \quad \text{if} \quad \lambda_{bl} \leq 0.776 \quad (4.27)$$

$$M_{bld} = \left[1 - 0.15 \left(\frac{M_{od}}{M_{bd}}\right)^{0.4}\right] \left(\frac{M_{od}}{M_{bd}}\right)^{0.4} M_{bd} \quad \text{if} \quad \lambda_{bl} > 0.776 \quad (4.28)$$

$$\lambda_{bl} = \frac{M_{bd}}{M_{od}} \quad (4.29)$$

and the NDL approach predicts the interactive local-distortional nominal moment capacity $M_{bdl}$ as:

$$M_{bdl} = M_{bl} \quad \text{if} \quad \lambda_{dl} \leq 0.673 \quad (4.30)$$

$$M_{bdl} = \left[1 - 0.22 \left(\frac{M_{od}}{M_{bl}}\right)^{0.5}\right] \left(\frac{M_{od}}{M_{bl}}\right)^{0.5} M_{bl} \quad \text{if} \quad \lambda_{dl} > 0.673 \quad (4.31)$$

$$\lambda_{dl} = \frac{M_{bl}}{M_{od}} \quad (4.32)$$

Regarding the inelastic reserve capacity for configurations failing in local-distortional interaction, the NLD and NDL approaches were adapted to meet the inelastic reserve capacity criterion in AISI-S100 (2016). For the NDL approach, the nominal member capacity for local buckling $M_{bl}$ is calculated following the inelastic reserve capacity equations in the AISI (Eqs. (4.7-4.10) and then used instead of $M_y$ in the classical DSM equations for distortional buckling (Eqs. (4.14-4.16). The opposite is done for the NLD approach where the nominal member capacity for distortional buckling $M_{bd}$, calculated following the inelastic
reserve capacity equations in the AISI (Eqs. (4.7, 4.9-4.10, 4.17)), is used instead of M_y in the classical DSM equations for local buckling (Eqs. (4.4-4.6)). The NLD and NDL approaches were also adapted herein to the extended range in Pham and Hancock (2013) using the same approach as the one described above. For the NDL approach, M_y in the classical DSM equations for distortional buckling is replaced by the nominal member capacity for local buckling M_{bln} calculated from the method described in Pham and Hancock (Eqs. (4.11-4.13)). The opposite applies for the NLD approach.

4.4.1.4. Elastic buckling, yield and plastic Moments

Elastic buckling moments (M_{ol} and M_{od}) for each tested configuration were calculated and input in the DSM expressions running linear buckling analyses (LBA) in Abaqus (2015). A similar model to the one described in Section “Tested uprights” was used but the measured thickness of the upright was used instead of the nominal one. Concentrated bending moments about major and minor axes were applied at pinned boundary conditions to replicate the test set up. To consider the effect of perforations of the perforated uprights in calculating M_{ol} and M_{od}, the perforations were modelled in Abaqus before running the elastic buckling analyses.

For each of tested configurations, M_y (yield moment) and M_p (plastic moment) were calculated about the axis about which the combined bending moment was applied using the yield stress reported in Table 4.5. For perforated uprights, net cross-sectional areas were used to calculate M_y and M_p.

4.5. Comparison of Direct Strength Method design with test results

The comparison of the test results to the previous DSM equations is performed in this section. Note that as addressed in the appendix A “Testing procedure” of Eurocode 3 (EN 1993-1-3, 2006), in order to compare experimental and numerical design strengths, experimental results have to be reduced to the characteristic values (taken as the 5th percentile value of the test results, estimated with 75% confidence). The characteristic value is then used as the design value to which the partial resistance factor is applied. The AISI-S100 (2016) specification uses the average value of all tests as the design value.
When a design equation is used to predict the test results, the equation is directly compared to the test results. The DSM curves in the AS/NZS 4600:2018 (AS/NZS, 2018) and AISI-S100 (2016) standards have been directly calibrated to the test results (Schafer, 2008), not characteristic values as calculated in the Eurocode 3 (EN 1993-1-3, 2006). Therefore, the approach taken in this chapter of comparing the average test results to the DSM design equation is in accordance with the AISI-S100 (2016) specification and the derivation of the DSM design curves.

4.5.1. Local buckling

Table 4.3 provides the elastic local slenderness and the average ratio of the experimental biaxial failure moment \(M_{\text{exp}}\) to the DSM predicted moment \(M_{\text{DSM}}\), with and without the inelastic reserve capacity, for all experiments performed on short uprights. Figure 4.14 (a) also graphically compares the DSM local buckling curve to local buckling tests results (normalized to \(M_y\) and \(M_{nyl}\)). As shown in Table 4.3, the DSM without the inelastic reserve capacity typically conservatively estimates the bending capacity of non-perforated Type A uprights with the experimental to predicted capacity ratios ranging between 1.12 (Configuration 1) and 1.63 (Configuration 4). For perforated Type A uprights, it also results in conservative predictions with experimental to predicted capacity ratios ranging between 1.04 (Configuration 1) and 1.44 (Configuration 3). For perforated Type B uprights, the method gives ratio ranging from 0.99 (Configuration 5) to 1.52 (Configuration 6). On average, the DSM without the inelastic reserve capacity conservatively estimates the bending capacity by about 29% with Coefficient of Variation (CoV) of 15% for all tested uprights and is generally more accurate in predicting the moment capacity when bending solely occurs about the major axis than about any other axis, with the worse prediction occurring for bending about the minor axis.

For Type A uprights, the observed experimental ultimate bending moment was found to be between \(M_y\) and \(M_p\) for 10 configurations out of 14. However, for all configurations but six, the slenderness ratio \(\lambda\) is greater than 0.776 and therefore the use of the DSM, with and without considering the AISI-S100 (2016) inelastic reserve capacity, gives the same results for these
configurations (see Table 4.3). Overall, considering the inelastic reserve capacity for Type A uprights only results in a marginal improvement of the DSM predictions. For all configurations, considering the AISI-S100 (2016) inelastic reserve capacity underestimates the bending capacity by 22% on average with CoV of 10%. For Type B uprights, the AISI-S100 (2016) inelastic reserve capacity can be used for all 7 configurations resulting in only underestimating the bending capacity by 2% on average with CoV of 14%. This is a significant improvement, as when the inelastic reserve capacity is ignored the DSM overestimates the bending capacity by 26% on average with CoV of 16%.

Regarding the DSM predictions using the extended range of the inelastic reserve strength, Table 4.3 and Figure 4.14 show that the proposed method in Pham and Hancock (2013) provides better strength predictions as compared to the DSM equations with and without the AISI-S100 (2016) inelastic reserve capacity. On average for all configurations and upright types, this method overestimates the capacity by experimental to predicted ratios equal to 1.00 and CoV of 12%. However, for Type B upright, it underestimates the bending capacity by 9%. As can be seen in Figure 4.14 (a), for short Type B uprights tested in Configurations 2 and 5, the proposed method in Pham and Hancock (2013) is quite unconservative and underestimates the bending capacity by 25%. This likely results from the sections reaching their maximum capacity for these two configurations due to localised failure at the perforations (as shown in Figure 4.9 (b)), and at an applied moment close to or slightly greater than \( M_y \). As Type B upright is a stocky section, the proposed DSM predicts a large post-buckling reserve capacity that does not develop in practice. It is worth mentioning that, for perforated Type A uprights tested in Configurations 2 and 5, similar localised failure at the holes were observed. Yet, the cross-sections are more slender than Type B cross-sections and the proposed DSM predicts a capacity with little inelastic reserve capacity and therefore similar to the test results. More investigations need to be performed on the influence of the holes on the inelastic reserve capacity.
4.5.2. Distortional buckling

Table 4.4 provides the elastic local and distortional slenderness and the ratio $M_{\text{exp}}/M_{\text{DSM}}$, with and without the inelastic reserve capacity, for all experiments performed on long uprights. Figure 4.14 (b) compares the DSM distortional buckling curve to distortional buckling tests results normalized to $M_y$ and $M_{\text{nyd}}$. As shown in Figure 4.15 (a) the DSM interactive distortional-local buckling (NLD) curve is compared to uprights test results normalized to $M_{\text{bd}}$, $M_{\text{bd}}$ with AISI inelastic reserve capacity and $M_{\text{bdn}}$. Figure 4.15 (b) plots local- distortional buckling (NDL) curve compared to uprights test results normalized to $M_{\text{bl}}$, $M_{\text{bl}}$ with AISI inelastic reserve capacity and $M_{\text{bln}}$. For each configuration, the DSM equations corresponding to the observed mode in Table 4.3 and reported in Section “DSM equations” were used. In Table 4.4, the minimum of the two NLD and NDL approaches is reported. According to Table 4.4, the DSM without considering the inelastic reserve capacity typically conservatively estimates the bending capacity of non-perforated Type A uprights by an experimental to predicted ratio up to 1.77 (Configuration 4). For perforated Type A uprights, the method provides better predictions but still results in typically conservative predictions with experimental to predicted ratios ranging between 0.97 (Configuration 1) to 1.53 (Configuration 3). For Type B uprights, the experimental to predicted ratios vary from 0.85 (Configuration 5) to 1.49 (Configuration 4). It should be noted that for the perforated Type B uprights failing in distortional buckling, using Eqs (4.14-4.16) with $M_y$ replaced by $M_{\text{nyd}}$ results in slightly better predictions (3%) than using Eqs (4.21-4.26). Therefore, Eqs (4.14-4.16) have been used herein to predict distortional buckling moments. For all configurations and upright types, the DSM results in an average experimental to predicted ratio of 1.28 with a CoV of 19% when the inelastic reserve capacity is ignored. Similar to the results for local buckling, the DSM without considering the inelastic reserve capacity better predicts the bending capacity for bending about major axis.

For Type A uprights failing in interactive local-distortional buckling, the observed ultimate bending moment for 4 configurations out of 12 was found to be between $M_y$ and $M_p$. However, and similar to local buckling predictions, the slenderness ratios $\lambda_i$ and $\lambda_y$ are greater than 0.776 and 0.673, respectively, for
Chapter 4

7 configuration out of 12, therefore preventing the use of the AISI-S100 (2016) inelastic reserve capacity in the DSM, when using the NLD or NDL approach. This leads to an average underestimation of the bending capacity of 24% with CoV of 12%. For Type B uprights failing in distortional buckling, the AISI-S100 (2016) inelastic reserve capacity can be used and the DSM underestimates the bending capacity by an average of 9%, compared to an average of 24% when the inelastic reserve capacity is ignored.

Similar to local buckling and as also can be seen in Figure 4.14 and Figure 4.15, the DSM predictions using the extended range of the inelastic reserve strength proposed by Pham and Hancock (2013) provides better strength predictions compared to the other two DSM predictions investigated herein. On average, this method overestimates the capacity about 3% with a CoV of 13%. For Type B uprights, the method underestimates the bending capacity by 6% on average. However, as can be seen in Figure 4.14 and similar to short uprights, for long Type B uprights tested in Configurations 2 and 5 failing in distortional buckling the proposed method in Pham and Hancock (2013) is quite unconservative and underestimates the bending capacity by 26%, due to localised failure at the perforations.
Figure 4.14 Comparison of the DSM predictor curves with test data, (a) Local buckling tests (b) Distortional buckling tests
Figure 4.15 Comparison of the DSM predictor curves with test data - (a) NLD (b) NDL approaches
4.6. Summary

This chapter presented biaxial bending experimental results on tests performed on cold-formed steel storage rack uprights. In total, 78 tests were performed on two upright types, with and without regular perforations. Accuracy of the (i) linear biaxial bending design equation in international design specifications (AISI-S100 (2016), AS/NZS 4600:2018 (AS/NZS, 2018) and EN 1993-1-3 (2006)) and (ii) the DSM for biaxial bending were evaluated. Tests were performed for local and distortional failure modes only. The biaxial bending interaction relationship was found to be highly nonlinear and the existing linear design equation to be significantly conservative. Failure was found to occur at ratios given by the linear design equation ranging from 1.00 to 1.68. The DSM calculated using the biaxial elastic buckling moment in the nominal moment capacity equations either with or without considering the inelastic reserve capacity prescribed in the AISI-S100 (2016), was found to underestimate the biaxial bending capacity for the majority of the tested configurations, especially when bending about the minor axis is dominant. On average, the experimental to predicted ratio was 1.17 when considering the inelastic reserve capacity and 1.29 when not taking that into consideration. When using the extended inelastic reserve capacity range proposed by Pham and Hancock (2013), the DSM inelastic reserve strength equations give better predictions with an average experimental to predicted ratio of 1.02. More rack types with different cross-sections are required to be tested in order to extend the outcomes of this study.

Acknowledgments

The authors would like to express their gratitude to Mr Tito Cudini from Modulblok S.p.A., Italy, and Modulblok S.p.A. for providing the upright sections free of charge and rolled-forming Type A uprights for the sole purpose of this research. The authors are grateful to the continuous support from Mr Cudini and Modulblok S.p.A.
CHAPTER 5

Parametric studies and design rules for local and distortional biaxial bending of cold-formed steel storage rack uprights

Statement of contribution to co-authored published paper
This chapter includes a co-authored paper. The bibliographic details of the co-authored paper, including all authors, are:

Talebian N, Gilbert BP, Pham CH, Chariere R, Karampour H, Parametric studies and design rules for local and distortional biaxial bending of cold-formed steel storage rack uprights, Journal of Structural Engineering (Submitted 31 July 2018, under review).

My contribution to this paper involved: literature review, FE validation, parametric study, Analyses of results, discussion of the results, writing, editing.

Signed: _________________________________ Date: 31/07/2018

PhD Candidate: Nima Talebian

Countersigned: _________________________________ Date: 31/07/2018

Principal Supervisor: Associate Professor Benoit P. Gilbert

Countersigned: _________________________________ Date: 31/07/2018

Principal Supervisor: Dr Hassan Karampour
Chapter 5

Parametric studies and design rules for local and distortional biaxial bending of cold-formed steel storage rack uprights

Abstract

This chapter first introduces an advanced finite element model to determine the biaxial bending capacity of cold-formed steel storage rack upright sections. The model is found to accurately predict published experimental results with an average predicted to experimental capacity ratio of 1.02. Second, the validated model is used to run parametric studies and analyse the biaxial response of slender, semi-compact and compact unperforated storage rack upright cross-sections. Analyses are run for local and distortional buckling failure modes only. Nine biaxial bending configurations are considered per cross-section and buckling mode. Results show that a nonlinear interactive relationship typically governs the biaxial bending of the studied uprights. This relationship is discussed in some details and analysed for the different failure modes and cross-sectional slenderness. The results from the parametric studies are used to verify the accuracy of different forms of published direct strength method (DSM) equations. They consist of the classical DSM equations and considering the inelastic reserve capacity into these equations, with and without an extended range of the cross-sectional slenderness. Results show that for all investigated buckling modes, the DSM results in better predictions when the inelastic reserve capacity is considered. The appropriate form of the DSM to predict the biaxial capacity of unperforated cold-formed steel storage rack uprights is discussed.

Keywords

Cold-formed; Biaxial bending; Local and distortional buckling; Parametric study; Finite element modelling; Direct strength method
5.1. Introduction

Rack-supported buildings, also referred to as “clad racks”, are gaining popularity. In this type of buildings, stored goods and building enclosure are both supported by the storage racks, resulting in more economical buildings but also complex structural systems. The uprights, i.e. the vertical members of the storage racks which are usually perforated monosymmetric open sections, undergo biaxial bending due to the combined actions of the wind and the vertical loads of the stored goods (Talebian et al. 2018). Current cold-formed steel structures design specifications (North American Specification AISI-S100 (AISI 2016), the Australian and New Zealand Standard AS/NZS 4600:2018 (AS/NZS 2018) and the Eurocode 3 EN1993-1-3 (CEN 2006)) consider a linear interaction equation to design members in biaxial bending. Nevertheless, previous studies have shown that a nonlinear relationship governs the biaxial bending behaviour of cold-formed steel members and the linear equation is conservative (Put et al. 1999; Torabian et al. 2016, 2014, 2015; Talebian et al. 2018).

An experimental investigation on the local and distortional biaxial bending behaviour of cold-formed steel storage rack uprights (Talebian et al. 2018) has recently been performed at Griffith University, Australia. The investigation included tests on two different types of storage rack uprights. One of the upright sections was tested with and without regular perforations while the other one was perforated. Results showed that the linear interaction equation is conservative and underestimates the biaxial bending capacity by up to 68%.

The aim of this chapter is to fully capture the local and distortional biaxial bending behaviour of cold-formed steel storage uprights through numerical parametric studies. To do so, this chapter presents first an advanced Finite Element (FE) model to accurately capture the local and distortional biaxial bending capacities of cold-formed steel storage rack upright sections. The software package ABAQUS (2015) is used for this purpose and the experimental results in Talebian et al. (2018) (Chapter 4) are compared to the numerical ones to verify the accuracy of the FE model. Second, the model is subsequently used to run parametric studies and quantify the local and distortional biaxial bending capacities of slender, semi-compact and compact
unperforated storage rack upright cross-sections. Ten and six different cross-sections are considered for local and distortional buckling, respectively. Analyses are run for nine biaxial bending configurations per cross-section type and buckling mode. Biaxial bending responses of all studied uprights are discussed and presented in the chapter.

Finally, the accuracy of different forms of the Direct Strength Method (DSM) (Schafer, 2008) in predicting the biaxial bending capacity of cold-formed steel storage rack uprights is also investigated. The results from the parametric studies are used for this purpose. Three different DSM approaches are investigated in this study, namely (i) by using the classical DSM equations given in the AS/NZS 4600:2018 (AS/NZS, 2018), with the nominal member moment capacity equal to the yield moment for compact cross-sections, (ii) through exploiting the inelastic reserve capacity for compact cross-sections, as permitted in the new AISI-S100 (2016) and AS/NZS (2018), and (iii) by adopting an extended range of the cross-sectional slenderness for the inelastic reserve capacity, as proposed by Pham and Hancock (2013).

5.2. Published experimental tests (background)

5.2.1. Experimental set-up

In Chapter 4, two different types of storage rack upright cross-sections, referred to as “Type A” and “Type B”, were tested. Type A upright had a nominal wall thickness of 1.5 mm, a width-to-depth ratio of 0.71 and a semi-compact cross-sectional shape. Type B upright had a nominal wall thickness of 2.0 mm, a width-to-depth ratio of 0.5 and was compact. To investigate the effect of perforations on the member capacity and biaxial bending interaction, Type A uprights were tested with and without regular perforations along their length, whereas all tested Type B uprights were perforated. To ensure local and distortional buckling failure modes, the lengths of the uprights were equal to 400 mm in local buckling, and 900 mm (Type B) and 1,100 mm (Type A) in distortional buckling. 10 mm thick, 220 mm × 220 mm steel plates were welded to both ends of the uprights to connect to the test rig and restrain warping. The full test rig is detailed in Chapter 4.
To obtain a sufficient number of points and apprehend the biaxial bending interaction curve, seven different biaxial bending configurations per upright type were tested in Chapter 4. These included bending about x-axis of symmetry (Conf 1), \( M_x = 2M_y \) (Conf 2), \( M_y = 2M_x \) (Conf 3), bending about minor y-axis when web is in compression (Conf 4), \( M_x = -2M_y \) (Conf 5), \( M_y = -2M_x \) (Conf 6) and bending about minor y-axis when flanges are in compression (Conf 7), where \( M_x \) and \( M_y \) are the moments applied about x- and y-axes, respectively. Tests were typically repeated twice for each configuration and upright type. In total, 78 tests were performed.

Three Linear Variable Displacement Transducers (LVDT) were used to measure the cross-sectional deformation at mid-length of each specimen by using a frame designed to fit around the upright sections (Chapter 4). The LVDTs were attached to the frame and recorded the displacement at the free end of each flange (cross-sectional deformation) and at the middle of the web (web deformation).

### 5.2.2. Geometric imperfection measurements

The structural performance of cold-formed members is highly sensitive to initial geometric imperfections (Dubina, et al., 2000; Schafer and Peköz, 1998). Accurately measuring these imperfections is essential to reproduce the observed buckling behaviour in FEA (Dubina and Ungureanu, 2002). Therefore, geometric imperfections of semi-compact Type A upright, with and without perforations, were measured prior to testing for all local and distortional specimens. As Type B upright had a compact cross-section, imperfections were not recorded.

An imperfection measurement set-up, similar to the one used by Schafer and Pekoz (1998), was built to capture imperfections along the upright length using Linear Variable Displacement Transducers (LVDT). Locations of LVDT were chosen to account for local and distortional buckling modes. In total, imperfections were measured along fifteen lines, as shown in Figure 5.1. The measured imperfections of LVDT lines 2, 5, 8, 11 and 14, for one long non-perforated Type A upright, are illustrated in Figure 5.2.
Figure 5.1 Location of LVDT lines for imperfection measurements of Type A upright

5.3. Finite element model

5.3.1. Element type, mesh size and boundary conditions

The uprights and their end plates were modelled using S4R shell elements (ABAQUS, 2015). Convergence studies showed that an element size of approximately 3 mm × 3 mm was adequate for all cases. Similar boundary conditions as in the experimental tests were used: (i) warping was restrained by using end plates rigidly connected to the ends of the uprights, and (ii) the uprights were simply supported by pinning the end plates at the location of the uprights centroidal axis. A concentrated biaxial bending moment was then applied at the pinned joints to replicate the test set-up. Figure 5.3 shows the FE model and boundary conditions for a 1,100 mm Type A upright.
Figure 5.2 Imperfection measurements for one long non-perforated Type A upright against AS4084 imperfection values

5.3.2. Material modelling

Material non-linearity in the specimens was considered using with the von Mises yield criteria and isotropic hardening. The average coupon test results reported in Chapter 4 were used for the material properties of the flat parts of the cross-sections. The stress-strain relationships (derived from the coupon tests) were described by multi-linear curves, as showed in Figure 5.4 for all upright types. As the coupon material tests also measured the effect of residual stresses in the material, the membrane residual stresses were ignored in this model.

The enhanced yield $\Delta F_y$ stress in the corner zones of the upright sections was determined by the following equations (Karren, 1967):

$$\Delta F_y = 0.6 \left[ \frac{B_0}{(r/t)^n} - 1.0 \right] F_y$$  \hspace{1cm} (5.1)

$$B_0 = 3.69 \left( \frac{F_y}{F_{tu}} \right) - 0.819 \left( \frac{F_y}{F_{tu}} \right)^2 - 1.79$$  \hspace{1cm} (5.2)
where $F_u$ is the ultimate strength, $F_y$ the yield stress, $r$ the inside bending radius of the corner and $t$ the wall thickness. The corner zone consists of the curved areas and two equivalent flat areas on both sides of each curved area of length equal to $1/2\pi r$. The measured thickness of the uprights was used to calculate the enhanced corner strength. The inside bending radius of the corners was 3 mm and 2 mm, for Type A and B uprights, respectively. Elastic–perfectly plastic behaviour was assumed for the corners with enhanced yield strength as per the stress-strain curves in Karren (1967).

$\begin{align*}
m = 0.192 \left( \frac{F_u}{F_y} \right) - 0.068
\end{align*}$

Figure 5.3 FE model and boundary conditions (shown for 1100 mm perforated Type A upright)
Chapter 5

113

Figure 5.4 Multi-linear stress–strain curve adopted in the numerical simulations

To account for the change of cross-sectional dimensions of the coupons during testing, true engineering stress and strain were employed in the numerical model. The nominal stress ($\sigma_n$) and strain ($\varepsilon_n$) were converted to true stress ($\sigma_t$) and strain ($\varepsilon_t$) using the following equations (Chung and Ip, 2000):

$$\sigma_t = \sigma_n (1 + \varepsilon_t)$$  \hspace{1cm} (5.4)

$$\varepsilon_t = \ln(1 + \varepsilon_n)$$  \hspace{1cm} (5.5)

5.3.3. Geometric imperfections

As mentioned earlier, imperfections were measured for semi-compact Type A upright. The readings collected by the data logger were smoothed using a Fourier Transform to filter the noise. The geometric imperfections at each measured line were then added to the “perfect” model assuming an undeformed cross-section at both ends of the uprights. Linear interpolations were assumed between each measured lines in the “imperfect” model.

For Type B upright, geometric imperfections were introduced in the model using axial compressive buckling modes. An initial linear buckling analysis (LBA) was carried out on a “perfect” model to generate the deformed shape of the local or distortional buckling mode. The geometric imperfections were then introduced to the “perfect” mesh by means of linearly superimposing the first local (for the 400 mm long specimens) or the first distortional (for the 900 mm long specimens) elastic buckling mode onto the mesh. The elastic buckling deformed...
shapes were scaled using the recommendations in the Australian standard AS4084 (2012). For the first local buckling mode, the following amplitude $S_{ol}$ was used:

$$S_{ol} = 0.3t \sqrt{\frac{F_y}{F_{ol}}}$$  \hspace{1cm} (5.6)

and for the first distortional buckling mode, the amplitude $S_{od}$:

$$S_{od} = 0.3t \sqrt{\frac{F_y}{F_{od}}}$$  \hspace{1cm} (5.7)

where $t$ is the thickness of the upright, $F_y$ is the yield stress, $F_{ol}$ the elastic local buckling stress and $F_{od}$ the elastic distortional buckling stress.” Comparison between the measured imperfections and the AS4084 values given by Equations 5.6 and 5.7 for a non-perforated Type A upright is illustrated in Figure 5.2. As it can be seen, the AS4084 predicts similar imperfections.

5.3.4. Analysis

The arc-length method (Riks) was selected to perform geometric and material nonlinear analyses in ABAQUS.

5.4. Validation of the FE model

Table 5.1 shows the ultimate test to predicted bending moment ratios ($M_{test}/M_{FEA}$) for the local and distortional buckling investigations and for all tested configurations. The table shows that the FE model is able to accurately predict the ultimate experimental moment capacities with a maximum difference between the predicted and experimental ultimate bending moment of 10%. The mean values of the test-to-predicted bending strength ratios are 0.98 and 1.03 for all local and distortional buckling tests, respectively, and the corresponding coefficients of variation (COV) are 5% and 5.8%, respectively.

Figures 5.5 to 5.7 show the FEA and experimental failure modes of the 400 mm long uprights. Similarly, Figures 5.8 to 5.10 show the FEA and experimental failure modes of the 900 mm and 1,100 mm long uprights. The FEA model is
also able to well capture the different experimentally observed biaxial bending failure modes of the uprights.

Figure 5.5 Deformed shapes of the 400 mm long upright tested in Conf 4 for non-perforated Type A upright a) FE failure mode and b) experimental failure mode

Figure 5.6 Deformed shapes of the 400 mm long upright tested in Conf 3 for perforated Type A upright a) FE failure mode and b) experimental failure mode

Figure 5.7 Deformed shapes of the 400 mm long upright tested in Conf 2 for perforated Type B upright a) FE failure mode and b) experimental failure mode
<table>
<thead>
<tr>
<th>Upright type</th>
<th>Local</th>
<th></th>
<th></th>
<th>Distortional</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of tests</td>
<td>Conf.</td>
<td>((M_{\text{test}}/M_{\text{FEA}}))</td>
<td>Number of tests</td>
<td>Conf.</td>
<td>((M_{\text{test}}/M_{\text{FEA}}))</td>
</tr>
<tr>
<td>Non-perforated Type A</td>
<td>2</td>
<td>1</td>
<td>0.93</td>
<td>2</td>
<td>1</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>1.02</td>
<td>2</td>
<td>2</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>1.04</td>
<td>2</td>
<td>3</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>1.00</td>
<td>2</td>
<td>4</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>0.98</td>
<td>2</td>
<td>5</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>0.94</td>
<td>2</td>
<td>6</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>0.99</td>
<td>2</td>
<td>7</td>
<td>1.08</td>
</tr>
<tr>
<td>Perforated Type A</td>
<td>2</td>
<td>1</td>
<td>0.90</td>
<td>2</td>
<td>1</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>0.97</td>
<td>2</td>
<td>2</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>1.01</td>
<td>2</td>
<td>3</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>1.00</td>
<td>2</td>
<td>4</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>0.93</td>
<td>2</td>
<td>5</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>0.94</td>
<td>2</td>
<td>6</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>1.00</td>
<td>2</td>
<td>7</td>
<td>1.08</td>
</tr>
<tr>
<td>Perforated Type B</td>
<td>1</td>
<td>1</td>
<td>1.03</td>
<td>2</td>
<td>1</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>0.96</td>
<td>2</td>
<td>2</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>0.92</td>
<td>2</td>
<td>2</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>1.03</td>
<td>2</td>
<td>3</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
<td>1.01</td>
<td>2</td>
<td>3</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>1.10</td>
<td>2</td>
<td>4</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>7</td>
<td>1.08</td>
<td>2</td>
<td>5</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>7</td>
<td>1.08</td>
<td>2</td>
<td>6</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>7</td>
<td>1.08</td>
<td>2</td>
<td>7</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>7</td>
<td>1.08</td>
<td>2</td>
<td>7</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>0.98</td>
<td>Average</td>
<td>1.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>COV (%)</td>
<td>5.00</td>
<td>COV (%)</td>
<td>5.80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Chapter 5
Figure 5.8 Deformed shapes of the 1,100 mm long upright tested in Conf 1 for non-perforated Type A upright a) FE failure mode and b) experimental failure mode

Figure 5.9 Deformed shapes of the 1,100 mm long upright tested in Conf 4 for perforated Type A upright a) FE failure mode and b) experimental failure mode
Figure 5.10 Deformed shapes of the 900 mm long upright tested in Conf 1 for perforated Type B upright a) FE failure mode and b) experimental failure mode

Figure 5.11 plots the FEA and experimental web deformation and cross-sectional deformation versus the normalised applied moment \((M/M_{\text{max}})\), where \(M\) and \(M_{\text{max}}\) are the applied moment and applied maximum moment, respectively, for short and long uprights. The FEA model is also able to well capture the web deformations and cross-sectional deformations observed in the biaxial bending experimental tests of the uprights.

5.5. Parametric studies

Parametric studies are performed in this chapter over a wider range of section slenderness values than the ones encountered in Chapter 4 to fully capture the biaxial bending behaviour of cold-formed steel storage rack uprights. Slender, semi-compact and compact unperforated upright cross-sections are considered for both local and distortional buckling failure modes. Note that unperforated uprights are considered for simplicity as experimental results in Chapter 4 tend to show that the biaxial bending behaviour of the uprights is not influenced by the regular perforations along their length.
Figure 5.11 FEA and experimental web deformation and cross-sectional deformation (a) 400 mm long non-perforated Type A upright in Conf 4 (b) 400 mm long perforated Type A upright in Conf 3 (c) 400 mm long perforated Type B upright in Conf 4 (d) 1,100 mm long non-perforated Type A upright in Conf 1 (e) 1,100 mm long perforated Type A upright in Conf 6 (f) 900 mm long perforated Type B upright in Conf 1
5.5.1. Biaxial configurations and upright lengths

Ten and six upright cross-sectional shapes were investigated for local and distortional buckling failure modes, respectively. These upright cross-sectional shapes are shown in Figure 5.12 and are either commercially available or taken from the literature (Lau and Hancock, 1987; Bernuzzi and Simoncelli, 2015). In total, thirteen different cross-sectional shapes are considered with Types C, D and F used for both local and distortional analyses. The thickness of Types J, K and L has been intentionally reduced to increase their slenderness ratio. The main cross-sectional dimensions and properties of all upright types are given in Table 5.2. Note that depending on the values of the biaxial moments, it is possible to have different ranges of slenderness ratio per upright type. In general, a section is considered to be slender when its slenderness ratio is greater than 1.25 (Martins et al. 2016).

Nine biaxial bending configurations, shown in Table 5.3, were considered per buckling mode and upright type. The numerical analyses were run using similar models to the ones presented in Section “Finite Element model”. Characteristics specific to the parametric studies and used in the present models are given later in Section “Modelling characteristics”.

The length of the tested uprights was determined based on elastic buckling analyses performed in CUFSM (2006), with simply supported and free-to-warp beams. For local buckling, the upright length of each upright type was taken as four times the longest local buckling half-wave length of the nine biaxial bending configurations. This criterion ensured that the uprights were short enough so distortional buckling did not occurred. For distortional buckling, the upright length of each upright type was taken equal to 1 to 2 times the longest distortional buckling half-wave length of the nine investigated configurations, effectively preventing global buckling. To avoid local-distortional buckling interaction to occur, LBA were carried out in ABAQUS on the warping restrained beams for all nine biaxial bending configurations. Any configuration for which the ratio of the elastic local bending moment ($M_{ol}$) to the elastic distortional bending moment ($M_{od}$) was less than 1.3 (Martins et al., 2016) was excluded from the analyses. The lengths of all uprights are given in Table 5.2.
Figure 5.12 Upright cross-sections considered

<table>
<thead>
<tr>
<th></th>
<th>Thick. (mm)</th>
<th>Depth (mm)</th>
<th>Width (mm)</th>
<th>( \frac{I_{\text{Major}}}{I_{\text{Minor}}} )</th>
<th>Local buckling upright length (mm)</th>
<th>Distortional buckling upright length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type C</td>
<td>2.0</td>
<td>140</td>
<td>100</td>
<td>2.53</td>
<td>200</td>
<td>1200</td>
</tr>
<tr>
<td>Type D</td>
<td>1.2</td>
<td>90</td>
<td>72</td>
<td>1.58</td>
<td>120</td>
<td>860</td>
</tr>
<tr>
<td>Type E</td>
<td>1.2</td>
<td>90</td>
<td>72</td>
<td>2.06</td>
<td>120</td>
<td>--</td>
</tr>
<tr>
<td>Type F</td>
<td>1.5</td>
<td>125</td>
<td>100</td>
<td>1.79</td>
<td>200</td>
<td>1240</td>
</tr>
<tr>
<td>Type G</td>
<td>1.5</td>
<td>100</td>
<td>110</td>
<td>0.94</td>
<td>220</td>
<td>--</td>
</tr>
<tr>
<td>Type H</td>
<td>1.5</td>
<td>100</td>
<td>90</td>
<td>1.41</td>
<td>350</td>
<td>--</td>
</tr>
<tr>
<td>Type I</td>
<td>1.5</td>
<td>100</td>
<td>80</td>
<td>2.13</td>
<td>240</td>
<td>--</td>
</tr>
<tr>
<td>Type J</td>
<td>0.6</td>
<td>140</td>
<td>100</td>
<td>2.53</td>
<td>300</td>
<td>--</td>
</tr>
<tr>
<td>Type K</td>
<td>0.8</td>
<td>90</td>
<td>72</td>
<td>1.57</td>
<td>200</td>
<td>--</td>
</tr>
<tr>
<td>Type L</td>
<td>0.8</td>
<td>90</td>
<td>72</td>
<td>2.03</td>
<td>260</td>
<td>--</td>
</tr>
<tr>
<td>Type M</td>
<td>1.8</td>
<td>80</td>
<td>60</td>
<td>2.17</td>
<td>--</td>
<td>800</td>
</tr>
<tr>
<td>Type N</td>
<td>1.5</td>
<td>80</td>
<td>90</td>
<td>1.17</td>
<td>--</td>
<td>600</td>
</tr>
<tr>
<td>Type O</td>
<td>2.0</td>
<td>120</td>
<td>60</td>
<td>5.05</td>
<td>--</td>
<td>900</td>
</tr>
</tbody>
</table>
5.5.2. Modelling characteristics

In the parametric studies, the stress-strain curve of the flat parts of the upright sections used in all analyses is similar to the one for Type A presented in Figure 5.4, but with a Young’s modulus of 200 GPa and a yield stress of 450 MPa. An elastic-perfectly plastic material is also used for the corner zones with the yield stress calculated from Eqs. 5.3-5.5.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Configuration 1</th>
<th>Configuration 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_x &gt; 0$ and $M_y = 0$</td>
<td>$M_x &gt; 0$, $M_y &gt; 0$ and $M_x = 2.5M_y$</td>
<td>$M_x &gt; 0$, $M_y &gt; 0$ and $M_x = M_y$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Configuration 3</th>
<th>Configuration 4</th>
<th>Configuration 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_x &gt; 0$, $M_y &gt; 0$ and $M_y = 2.5M_x$</td>
<td>$M_x = 0$ and $M_y &gt; 0$</td>
<td>$M_x &gt; 0$, $M_y &lt; 0$ and $M_x = -2.5M_y$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Configuration 6</th>
<th>Configuration 7</th>
<th>Configuration 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_x &gt; 0$, $M_y &lt; 0$ and $M_x = -M_y$</td>
<td>$M_x &gt; 0$, $M_y &lt; 0$ and $M_y = -2.5M_x$</td>
<td>$M_x = 0$ and $M_y &lt; 0$</td>
</tr>
</tbody>
</table>

$M_x > 0$ generates compression in the bottom flange, $M_y > 0$ generates compression in the lip stiffeners and $M_y < 0$ generates compression in the web.

Table 5.4 shows sensitivity of the model to the shape (inward or outward in distortional buckling) for slender Type N upright and Table 5.5 shows influence of amplitude of the imperfections for Type I in local buckling and Type N in distortional buckling.

Geometric imperfections are introduced in the analyses following the methodology described in Section “Geometric imperfections” for Type B upright. In this method, the first local or distortional buckling mode deformed shape in pure compression is used and scaled by the factors obtained from Eqs. 5.6-5.7.
Table 5.4 Biaxial bending capacity in distortional buckling with inward and outward imperfection

<table>
<thead>
<tr>
<th>Upright Type</th>
<th>Conf.</th>
<th>M_{FEA} with Inward Imperfection (kN.m)</th>
<th>M_{FEA} with Outward Imperfection (kN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type N</td>
<td>1</td>
<td>3.15</td>
<td>3.15</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.73</td>
<td>2.77</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.23</td>
<td>2.32</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2.04</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.65</td>
<td>1.65</td>
</tr>
</tbody>
</table>

Table 5.5 Different amplitude of imperfections for local and distortional buckling

<table>
<thead>
<tr>
<th>Upright Type</th>
<th>Conf.</th>
<th>M_{FEA} (Imperfection Amplitude=0.6 mm) (kN.m)</th>
<th>M_{FEA} (Imperfection Amplitude=1 mm) (kN.m)</th>
<th>M_{FEA} (Imperfection Amplitude=1.5 mm) (kN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local buckling: Type I</td>
<td>0</td>
<td>6.98</td>
<td>6.97</td>
<td>6.97</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6.21</td>
<td>6.21</td>
<td>6.08</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.85</td>
<td>4.83</td>
<td>4.81</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.95</td>
<td>4.94</td>
<td>4.94</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.56</td>
<td>4.55</td>
<td>4.54</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5.70</td>
<td>5.70</td>
<td>5.70</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>4.40</td>
<td>4.40</td>
<td>4.40</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>4.09</td>
<td>4.09</td>
<td>4.08</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>4.04</td>
<td>4.04</td>
<td>4.03</td>
</tr>
<tr>
<td>Distortional buckling: Type N</td>
<td>0</td>
<td>3.15</td>
<td>3.14</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2.73</td>
<td>2.71</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.23</td>
<td>2.22</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.04</td>
<td>1.95</td>
<td>1.94</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.65</td>
<td>1.65</td>
<td>1.65</td>
</tr>
</tbody>
</table>

5.6. Biaxial bending response of the uprights and interactive behaviour

5.6.1. Local buckling

Elastic and inelastic local buckling failure modes were observed for all specimens investigated for local buckling. Denoting, the bending moment capacities about the x- and y-axes, M_{bx} and M_{by}, respectively, the normalised biaxial ultimate moment capacities (M_{x}/M_{bx} and M_{y}/M_{by}) for all upright types are summarised in Table 5.6 with local slenderness ratio \( \lambda_l \) (determined from FE model running LBA) and the associated interactive biaxial moment capacity obtained from the linear equation (AISI-S100, 2016; AS/NZS 4600, 2018; EN 1993-1-3, 2006). Figure 5.13 illustrates the linear equation versus the normalised biaxial bending results obtained from the nine different investigated configurations and local buckling. Similar observations to the ones presented in
Chapter 4, but on a larger range of cross-sections can be established. Table 5.6 and Figure 5.13 show that the governing interaction relationship is not linear and that the linear equation is conservative for all investigated uprights. For Configurations 1 to 3 ($M_y > 0$, lip stiffeners in compression), the linear equation gives interaction ratios ranging from 1.12 (Type H and Configuration 1) to 1.35 (Type L and Configuration 1). For Configurations 5 to 7 ($M_y < 0$, web in compression), the linear equation gives ratios ranging from 1.03 (Type L and Configuration 7) to 1.39 (Type E and Configurations 5 and 6). When the web is in compression, the biaxial bending responses of the uprights tend to be closer to the linear interaction curve. This is more highlighted for Type K and L uprights.

![Figure 5.13 Biaxial bending interaction points for local buckling – All uprights](image)

5.6.2. Distortional buckling

For distortional buckling analyses, Type D, E and M uprights tested with the web in compression (Configurations 5 to 8) and Type C uprights (Configurations 2 to 6) did not meet the $M_{ol}/M_{od}$ ratio of less than 1.3 (Martins et al., 2016) and would fail in local buckling or local-distortional buckling interaction. Therefore, these configurations were excluded from the analyses. The normalised ultimate moment capacities ($M_x/M_{bx}$ and $M_y/M_{by}$) for the upright types considered for
distortional buckling are summarised in Table 5.7 with distortional slenderness ratio $\lambda_d$ (also determined from LBA in Abaqus) and the interactive biaxial moment linear equation. Similar to Figure 5.13, Figure 5.14 illustrates the linear interaction equation versus all normalised biaxial bending numerical results obtained for pure distortional buckling failure modes.

The linear equation is also found to be conservative for distortional buckling and gives interaction ratios ranging from 1.00 (Type F and Configuration 1) to 1.46 (Type N and Configuration 2) for all biaxial bending configurations. Biaxial bending responses of Type F upright tend to be closer to the linear interaction curve than other uprights.
Table 5.6 Comparison of parametric studies results to linear equation for local buckling

| Type | Up- | Conf. | \( \lambda_i \) | \( M_{u}/M_{\max} \) | \( M_{l}/M_{\max} \) | Linear equation | Up- | Conf. | \( \lambda_i \) | \( M_{u}/M_{\max} \) | \( M_{l}/M_{\max} \) | Linear equation |
|------|-----|-------|----------------|-----------------|----------------|----------------|-----|-------|----------------|-----------------|----------------|----------------|-----|-------|----------------|-----------------|----------------|----------------|
| C    | 0   | 0.57  | 1.00 | 0.00 | 0.00 | 1.00 | 0 | 0.57  | 1.00 | 0.00 | 0.00 | 1.00 |
|      | 1   | 0.78  | 0.70 | 0.60 | 1.30 | 1.17 | 0.76 | 0.36 | 1.12 |
|      | 2   | 0.83  | 0.42 | 0.89 | 1.30 | 1.08 | 0.54 | 0.64 | 1.18 |
|      | 3   | 0.74  | 0.19 | 1.03 | 1.22 | 1.03 | 0.29 | 0.85 | 1.14 |
|      | 4   | 0.71  | 0.00 | 1.00 | 1.00 | 1.06 | 0.00 | 1.00 | 1.00 |
|      | 5   | 0.52  | 0.80 | 0.52 | 1.32 | 1.06 | 0.85 | 0.36 | 1.22 |
|      | 6   | 0.43  | 0.50 | 0.80 | 1.30 | 1.01 | 0.60 | 0.63 | 1.23 |
|      | 7   | 0.40  | 0.24 | 0.95 | 1.19 | 1.09 | 0.32 | 0.86 | 1.18 |
|      | 8   | 0.41  | 0.00 | 1.00 | 1.00 | 1.23 | 0.00 | 1.00 | 1.00 |
| D    | 0   | 0.58  | 1.00 | 0.00 | 1.00 | 0 | 0.83  | 1.00 | 0.00 | 1.00 |
|      | 1   | 0.61  | 0.74 | 0.41 | 1.15 | 1 | 0.72  | 0.83 | 0.51 | 1.33 |
|      | 2   | 0.62  | 0.51 | 0.70 | 1.21 | 2 | 0.72  | 0.49 | 0.75 | 1.24 |
|      | 3   | 0.60  | 0.26 | 0.89 | 1.15 | 3 | 0.64  | 0.26 | 1.01 | 1.27 |
|      | 4   | 0.61  | 0.00 | 1.00 | 1.00 | 4 | 0.62  | 0.00 | 1.00 | 1.00 |
|      | 5   | 0.47  | 0.83 | 0.42 | 1.25 | 5 | 0.90  | 0.76 | 0.52 | 1.28 |
|      | 6   | 0.46  | 0.59 | 0.75 | 1.34 | 6 | 1.02  | 0.45 | 0.77 | 1.22 |
|      | 7   | 0.44  | 0.29 | 0.94 | 1.24 | 7 | 0.98  | 0.22 | 0.94 | 1.16 |
|      | 8   | 0.45  | 0.00 | 1.00 | 1.00 | 8 | 1.01  | 0.00 | 1.00 | 1.00 |
| E    | 0   | 0.90  | 1.00 | 0.00 | 1.00 | 0 | 1.88  | 1.00 | 0.00 | 1.00 |
|      | 1   | 1.19  | 0.71 | 0.58 | 1.30 | 1 | 1.65  | 0.67 | 0.61 | 1.29 |
|      | 2   | 1.30  | 0.43 | 0.87 | 1.30 | 2 | 1.74  | 0.38 | 0.87 | 1.25 |
|      | 3   | 1.08  | 0.19 | 0.99 | 1.18 | 3 | 1.54  | 0.17 | 0.99 | 1.16 |
|      | 4   | 1.05  | 0.00 | 1.00 | 1.00 | 4 | 1.48  | 0.00 | 1.00 | 1.00 |
|      | 5   | 0.52  | 0.91 | 0.48 | 1.39 | 5 | 1.71  | 0.88 | 0.41 | 1.29 |
|      | 6   | 0.48  | 0.59 | 0.79 | 1.39 | 6 | 1.39  | 0.58 | 0.68 | 1.26 |
|      | 7   | 0.42  | 0.29 | 0.96 | 1.24 | 7 | 1.29  | 0.31 | 0.90 | 1.21 |
|      | 8   | 0.39  | 0.00 | 1.00 | 1.00 | 8 | 1.30  | 0.00 | 1.00 | 1.00 |
| F    | 0   | 0.64  | 1.00 | 0.00 | 1.00 | 0 | 0.88  | 1.00 | 0.00 | 1.00 |
|      | 1   | 0.71  | 0.69 | 0.47 | 1.15 | 1 | 0.91  | 0.75 | 0.44 | 1.19 |
|      | 2   | 0.69  | 0.47 | 0.80 | 1.26 | 2 | 0.90  | 0.51 | 0.75 | 1.26 |
|      | 3   | 0.66  | 0.22 | 0.94 | 1.17 | 3 | 0.87  | 0.26 | 0.97 | 1.23 |
|      | 4   | 0.68  | 0.00 | 1.00 | 1.00 | 4 | 0.89  | 0.00 | 1.00 | 1.00 |
|      | 5   | 0.58  | 0.78 | 0.43 | 1.21 | 5 | 1.18  | 0.63 | 0.46 | 1.09 |
|      | 6   | 0.52  | 0.53 | 0.74 | 1.28 | 6 | 1.36  | 0.40 | 0.73 | 1.13 |
|      | 7   | 0.47  | 0.26 | 0.89 | 1.15 | 7 | 1.47  | 0.20 | 0.91 | 1.11 |
|      | 8   | 0.47  | 0.00 | 1.00 | 1.00 | 8 | 1.65  | 0.00 | 1.00 | 1.00 |
| G    | 0   | 1.03  | 1.00 | 0.00 | 1.00 | 0 | 1.43  | 1.00 | 0.00 | 1.00 |
|      | 1   | 0.87  | 0.98 | 0.29 | 1.27 | 1 | 1.89  | 0.74 | 0.61 | 1.35 |
|      | 2   | 0.72  | 0.76 | 0.57 | 1.32 | 2 | 1.90  | 0.43 | 0.89 | 1.31 |
|      | 3   | 0.50  | 0.45 | 0.84 | 1.29 | 3 | 1.72  | 0.19 | 0.98 | 1.17 |
|      | 4   | 0.42  | 0.00 | 1.00 | 1.00 | 4 | 1.68  | 0.00 | 1.00 | 1.00 |
|      | 5   | 0.92  | 0.90 | 0.29 | 1.18 | 5 | 1.31  | 0.71 | 0.44 | 1.14 |
|      | 6   | 0.84  | 0.72 | 0.57 | 1.29 | 6 | 1.45  | 0.43 | 0.67 | 1.10 |
|      | 7   | 0.67  | 0.45 | 0.90 | 1.35 | 7 | 1.41  | 0.21 | 0.82 | 1.03 |
|      | 8   | 0.52  | 0.00 | 1.00 | 1.00 | 8 | 1.46  | 0.00 | 1.00 | 1.00 |
Figure 5.14 Biaxial bending interaction points for distortional buckling – All uprights

Table 5.7 Comparison of parametric studies results with linear equation for distortional buckling

<table>
<thead>
<tr>
<th>Type of Upright</th>
<th>Conf.</th>
<th>( \lambda_d )</th>
<th>( M_x/M_{bx} )</th>
<th>( M_y/M_{by} )</th>
<th>Linear equation</th>
<th>Type of Upright</th>
<th>Conf.</th>
<th>( \lambda_d )</th>
<th>( M_x/M_{bx} )</th>
<th>( M_y/M_{by} )</th>
<th>Linear equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0</td>
<td>0.76</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>D</td>
<td>0</td>
<td>0.74</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.55</td>
<td>0.61</td>
<td>0.74</td>
<td>1.35</td>
<td></td>
<td>1</td>
<td>0.76</td>
<td>0.71</td>
<td>0.39</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.50</td>
<td>0.29</td>
<td>0.89</td>
<td>1.18</td>
<td></td>
<td>2</td>
<td>0.76</td>
<td>0.50</td>
<td>0.69</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.51</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td>3</td>
<td>0.72</td>
<td>0.25</td>
<td>0.87</td>
<td>1.12</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0.78</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td></td>
<td>4</td>
<td>0.74</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.83</td>
<td>0.64</td>
<td>0.36</td>
<td>1.00</td>
<td></td>
<td>0</td>
<td>0.63</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.80</td>
<td>0.45</td>
<td>0.63</td>
<td>1.08</td>
<td></td>
<td>1</td>
<td>0.66</td>
<td>0.74</td>
<td>0.40</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.75</td>
<td>0.23</td>
<td>0.81</td>
<td>1.03</td>
<td></td>
<td>2</td>
<td>0.63</td>
<td>0.46</td>
<td>0.62</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.77</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td>3</td>
<td>0.57</td>
<td>0.24</td>
<td>0.81</td>
<td>1.05</td>
</tr>
<tr>
<td>N</td>
<td>0</td>
<td>1.29</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td></td>
<td>4</td>
<td>0.59</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1.39</td>
<td>0.80</td>
<td>0.61</td>
<td>1.42</td>
<td></td>
<td>5</td>
<td>0.36</td>
<td>0.85</td>
<td>0.43</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.32</td>
<td>0.50</td>
<td>0.96</td>
<td>1.46</td>
<td></td>
<td>6</td>
<td>0.31</td>
<td>0.59</td>
<td>0.75</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.26</td>
<td>0.24</td>
<td>1.15</td>
<td>1.39</td>
<td></td>
<td>7</td>
<td>0.30</td>
<td>0.30</td>
<td>0.94</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.25</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td>8</td>
<td>0.33</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>O</td>
<td>0</td>
<td>0.60</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td></td>
<td>1</td>
<td>0.69</td>
<td>0.57</td>
<td>0.53</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.69</td>
<td>0.34</td>
<td>0.80</td>
<td>1.14</td>
<td></td>
<td>2</td>
<td>0.69</td>
<td>0.34</td>
<td>0.80</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.56</td>
<td>0.17</td>
<td>0.99</td>
<td>1.16</td>
<td></td>
<td>3</td>
<td>0.56</td>
<td>0.17</td>
<td>0.99</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.54</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td>4</td>
<td>0.45</td>
<td>0.65</td>
<td>0.58</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.50</td>
<td>0.39</td>
<td>0.86</td>
<td>1.25</td>
<td></td>
<td>5</td>
<td>0.45</td>
<td>0.39</td>
<td>0.86</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.45</td>
<td>0.17</td>
<td>0.96</td>
<td>1.14</td>
<td></td>
<td>6</td>
<td>0.45</td>
<td>0.17</td>
<td>0.96</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.46</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td>8</td>
<td>0.46</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
5.7. Direct Strength Method equations to predict biaxial bending capacity

The accuracy of five different DSM approaches in predicting the biaxial bending capacity of steel storage rack uprights are investigated in this study. These approaches are detailed in the following sections and are: (i) using the classical DSM equations given in the AISI-S100 (AISI 2016) and AS/NZS 4600:2018 (AS/NZS, 2018), (ii) through exploiting the conservative inelastic reserve capacity for compact cross-sections in the AISI-S100 (AISI 2016) and AS/NZS 4600:2018 (AS/NZS, 2018) with the moment at which yielding initiates in compression taken as the yield moment, (iii) using the inelastic reserve capacity by considering the actual moment at which yielding initiates in compression (after yielding in tension), (iv) by adopting the extended inelastic reserve strength proposed by Pham and Hancock (2013), and (v) using the extended inelastic reserve strength in (iv) but with the actual moment at which yielding initiates in compression.

5.7.1. Local buckling

The classical DSM (Approach (i)) nominal member moment capacity $M_{bl}$ for local buckling, ignoring inelastic reserve capacity, is defined as (AISI-S100, 2016; AS/NZS, 2018; Schafer, 2008):

$$M_{bl} = M_y$$

if $\lambda_l \leq 0.776$ (5.8)

$$M_{bl} = \left[1 - 0.15 \left(\frac{M_{ol}}{M_y}\right)^{0.4}\right]^{0.4} M_y$$

if $\lambda_l > 0.776$ (5.9)

where $M_{bl}$ and $M_y$ are the elastic local buckling moment and yield moment respectively, and $\lambda_l$ is a non-dimensional slenderness ratio defined as:

$$\lambda_l = \frac{\sqrt{M_y}}{M_{ol}}$$

(5.10)

The recent AISI-S100 (2016) and AS/NZS (2018) now allow the nominal member moment capacity to range between $M_y$ and the plastic moment $M_p$ for compact cross-sections, i.e. if $\lambda_l \leq 0.776$ (local inelastic reserve capacity). When the first yield is in compression:
\[ M_{by} = M_y + (1 - 1/C_{yl}^2)(M_p - M_y) \]  
where
\[ C_{yl} = \sqrt{0.776/\lambda_y} \leq 3 \]  
and when the first yield is in tension:
\[ M_{bl} = M_{yc} + (1 - 1/C_{yl}^2)(M_p - M_y) \leq M_{yr3} \]
where
\[ M_{yr3} = M_y + 8/9(M_p - M_y) \]
and \( M_{yc} \) is the moment at which yielding initiates in compression (after yielding in tension). \( M_{yc} \) can be conservatively taken as \( M_y \) (AISI-S100, 2016, Torabian, et al., 2014) and the two options (Approaches (ii) and (iii)) are investigated when considering the inelastic reserve capacity.

Pham and Hancock (2013) proposed an extended range of the cross-sectional slenderness for which the inelastic strength can be applied (Approach (iv)). For local buckling, the inelastic reserve capacity can be applied when \( \lambda_l \leq 1.55 \) and \( C_{yl} \) in Eq. (5.12) becomes:
\[ C_{ylh} = \sqrt{1.55/\lambda_y} \leq 3 \]
and the inelastic local strength is calculated as:
\[ M_{nyl} = M_y + (1 - 1/C_{ylh}^2)(M_p - M_y) \]  
\( M_{nyl} \) is then used in the classical DSM (Eqs. (5.8-5.9)) instead of \( M_y \), and \( \lambda_{ln} \) defined as:
\[ \lambda_{ln} = \frac{M_{nyl}}{M_{ol}} \]  
is used instead of \( \lambda_l \) to obtain the new nominal member capacity with extended range \( M_{bln} \). Approach (v) is the same as Approach (iv) above but for configurations at which yielding initiates in compression (after yielding in tension), \( M_{yc} \) is used in Eq. (5.16) instead of \( M_y \) to calculate \( M_{nyl} \), and \( M_{nyl} \) must be less than \( M_{yr3} \) (as in Eq. (5.13)).
5.7.2. Distortional buckling

Similarly, the classical DSM (Approach (i)) nominal member moment capacity \( M_{bd} \) for distortional buckling, ignoring inelastic reserve capacity, is as follows (AISI-S100, 2016; AS/NZS, 2018; Schafer, 2008):

\[
\begin{align*}
M_{bd} &= M_y & \text{if } \lambda_d \leq 0.673 \\
M_{bd} &= \left[1 - 0.22 \left(\frac{M_{od}}{M_y}\right)^{0.5} \left(\frac{M_{od}}{M_y}\right)^{0.5} M_y \right] & \text{if } \lambda_d > 0.673
\end{align*}
\] (5.18-5.19)

where \( M_{od} \) is the elastic buckling moment for distortional buckling and \( \lambda_d \) is a non-dimensional slenderness ratio defined as:

\[
\lambda_d = \frac{M_y}{M_{od}}
\] (5.20)

According to AISI-S100 (2016) and AS/NZS (2018), distortional inelastic reserve capacity (Approaches (ii) and (iii)) is permitted to be taken into account if \( \lambda_d \leq 0.673 \). The same equations as for local buckling (Eqs (5.11-5.14)) are used with \( C_{yi} \) in Eqs (5.11-5.13) replaced by:

\[
C_{yd} = \sqrt{0.673/\lambda_d} \leq 3
\] (5.21)

For distortional buckling, the inelastic strength with extended range proposed by Pham and Hancock (2013) can be applied when \( \lambda_d \leq 1.45 \) (Approach (iv)) and \( C_{ydn} \) in Eq. (5.21) becomes:

\[
C_{ydn} = \sqrt{1.45/\lambda_d} \leq 3
\] (5.22)

and the inelastic distortional strength is calculated as:

\[
M_{myd} = M_y + (1 - 1/C_{ydn}^2)(M_p - M_y)
\] (5.23)

\( M_{myd} \) is then used in the classical DSM (Eqs. (5.18-5.19)) instead of \( M_y \) and \( \lambda_{dn} \) defined as:
is used instead of $\lambda_d$ to obtain the new nominal member capacity with extended range $M_{b_{dn}}$.

In Approach (v), $M_{yc}$ is used in Eq. (5.23) instead of $M_y$ to calculate $M_{nyd}$ (with $M_{nyd} \leq M_{yt3}$) when yielding initiates in compression after yielding occurred first in tension.

5.7.3. Elastic buckling, yield and plastic Moments

Elastic buckling moments ($M_{ol}$ and $M_{od}$) for each tested configuration were calculated and input in the DSM expressions running linear buckling analyses (LBA) in Abaqus (2015). A similar model to the one described in Section “Finite element model” was used.

For each of tested configurations, the yield moment $M_y$ and plastic moment $M_p$ were calculated about the axis about which the biaxial bending moment was applied using a yield stress equal to 450 MPa, as used in parametric studies.

5.8. Comparison of direct strength method design approaches with parametric results

5.8.1. Local buckling

Table 5.8 provides the elastic local slenderness ratio $\lambda_l$ (Eq. (5.10)) and the FEA biaxial failure moment ($M_{FEA}$) to the DSM predicted moment ($M_{DSM}$) ratio for the five different DSM approaches and local buckling.

Figure 5.15 also graphically compares the DSM local buckling curve to the normalised FEA predicted capacities for the classical DSM (Approach (i)), and the proposed method in Pham and Hancock (2013) with and without the use of $M_{yc}$ (Approaches (iv) and (v)). As shown in Table 5.8, the classical DSM (without the inelastic reserve capacity) typically conservatively estimates the bending capacity of the studied uprights, with the FEA to DSM capacity ratios ranging between 0.99 and 2.05, both values for Type J upright in Configurations 1 and 8, respectively. On average, the DSM without the inelastic reserve capacity conservatively estimates the bending capacity by 45% with a
Coefficient of Variation (COV) for all tested uprights and configuration of 17%. The classical DSM is generally more accurate in predicting the moment capacity when bending solely occurs about the major axis than about any other axis.

The use of the DSM with inelastic reserve capacity and conservatively taken \( M_{yc} = M_y \) (Approach (ii)), as in the AISI-S100 (2016) and AS/NZS (2018), results in an 11% improvement of the predictions, when compared to the classical DSM. Considering \( M_{yc} \) for configurations at which yielding initiates in compression (after yielding in tension – Approach (iii)) marginally improves the use of the DSM with inelastic reserve capacity.

Regarding the DSM predictions using the extended range of the inelastic reserve capacity, Table 5.8 and Figure 5.15 show that the proposed method in Pham and Hancock (2013) (Approach iv)) provides better strength predictions when compared to the previous three DSM approaches. On average, for all configurations and upright types, this method overestimates the FEA capacity by 20%, with a COV of 16%. As can be seen in Figure 5.15, the proposed method in Pham and Hancock (2013) is mainly conservative for slenderness ratio greater than about 1.15. Considering \( M_{yc} \) for configurations at which yielding initiates in compression (after yielding in tension – Approach (v)) in the method proposed by Pham and Hancock (2013) results in a 5% improvement of the predictions (mainly for slender sections) with a COV of 13%.

### 5.8.2. Distortional buckling

Table 5.9 provides the elastic distortional slenderness \( \lambda_d \) and the \( M_{FEA}/M_{DSM} \) ratios for the five different DSM approaches and all analyses failing in distortional buckling. Figure 5.16 compares the DSM distortional buckling curve to normalised FEA predicted capacities for the classical DSM (Approach (i)), and the proposed method in Pham and Hancock (2013) with and without \( M_{yc} \) (Approaches (iv) and (v)).

Similar conclusions to the local buckling can be drawn for distortional buckling. Table 5.9 shows that the DSM without considering the inelastic reserve capacity usually conservatively estimates the bending capacity of the investigated uprights, with a FEA to DSM biaxial moment capacity ratio up to 1.91 (Type M
and Configuration 7). For all configurations and upright types, the classical DSM overestimates on average the FEA capacity by 29%, with a COV of 28%.

The use of the DSM with inelastic reserve capacity with $M_{yc} = M_y$ (Approach (ii)), as in the AISI-S100 (2016) and AS/NZS (2018), leads to an average underestimation of the bending capacity of 18%, with COV of 17%. However, considering $M_{yc}$ (Approach (iii)) in the inelastic reserve capacity improves the prediction by 6% with a COV of 15%.

The DSM predictions using the extended range of the inelastic reserve capacity proposed by Pham and Hancock (2013) (Approach (iv)) provide better strength predictions when compared to the previous three DSM approaches investigated herein. On average, this method overestimates the capacity by about 1% with a COV of 14%. Considering $M_{yc}$ in Pham and Hancock (2013) (Approach (v)) for configurations at which yielding initiates in compression (after yielding in tension) results in a 2% underestimation of the predictions, with a COV of 13%.

![Figure 5.15](image.png)  
Figure 5.15 Comparison of the DSM curve to parametric studies data for local buckling (Approaches (i), (iv) and (v))
### Table 5.8 Comparison of parametric results with DSM for local buckling uprights

<table>
<thead>
<tr>
<th>Type</th>
<th>Conf</th>
<th>M_{up}/M_{IC}</th>
<th>M_{up}/M_{IC}</th>
<th>M_{up}/M_{IC}</th>
<th>M_{up}/M_{IC}</th>
<th>M_{up}/M_{IC}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(With reserve)</td>
<td>(No reserve)</td>
<td>(With reserve)</td>
<td>(No reserve)</td>
<td>(With reserve)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Pham and Hancock)</td>
<td></td>
<td>(Pham and Hancock)</td>
<td></td>
<td>(Pham and Hancock)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>with M_{IC}</td>
<td></td>
<td>with M_{IC}</td>
<td></td>
<td>with M_{IC}</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1)</td>
<td></td>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(3)</td>
<td></td>
<td>(4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.57</td>
<td>1.31</td>
<td>1.22</td>
<td>1.22</td>
<td>1.12</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.57</td>
<td>1.31</td>
<td>1.22</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>0.41</td>
<td>2.00</td>
<td>1.40</td>
<td>1.11</td>
<td>1.20</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.41</td>
<td>2.00</td>
<td>1.40</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>1.57</td>
<td>1.30</td>
<td>1.09</td>
<td>1.19</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.45</td>
<td>1.57</td>
<td>1.30</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>0.56</td>
<td>1.27</td>
<td>1.19</td>
<td>1.09</td>
<td>1.09</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.56</td>
<td>1.27</td>
<td>1.19</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>1.19</td>
<td>1.37</td>
<td>1.37</td>
<td>1.30</td>
<td>1.21</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.19</td>
<td>1.37</td>
<td>1.37</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>0.69</td>
<td>1.48</td>
<td>1.37</td>
<td>1.30</td>
<td>1.09</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.69</td>
<td>1.48</td>
<td>1.37</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.43</td>
<td>1.25</td>
<td>1.25</td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
<td>0.43</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>1.08</td>
<td>1.64</td>
<td>1.64</td>
<td>1.38</td>
<td>1.38</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.08</td>
<td>1.64</td>
<td>1.64</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>1.05</td>
<td>1.52</td>
<td>1.52</td>
<td>1.29</td>
<td>1.29</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.05</td>
<td>1.52</td>
<td>1.52</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td>0.89</td>
<td>1.91</td>
<td>1.33</td>
<td>1.08</td>
<td>1.16</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.89</td>
<td>1.91</td>
<td>1.33</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>0.64</td>
<td>1.26</td>
<td>1.21</td>
<td>1.21</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.64</td>
<td>1.26</td>
<td>1.21</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>1.71</td>
<td>1.27</td>
<td>1.27</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.71</td>
<td>1.27</td>
<td>1.27</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>0.69</td>
<td>1.48</td>
<td>1.37</td>
<td>1.30</td>
<td>1.09</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.69</td>
<td>1.48</td>
<td>1.37</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>0.66</td>
<td>1.54</td>
<td>1.37</td>
<td>1.37</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.66</td>
<td>1.54</td>
<td>1.37</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td>0.68</td>
<td>1.29</td>
<td>1.20</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.68</td>
<td>1.29</td>
<td>1.20</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>0.58</td>
<td>1.43</td>
<td>1.25</td>
<td>1.05</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.58</td>
<td>1.43</td>
<td>1.25</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>0.52</td>
<td>1.68</td>
<td>1.35</td>
<td>1.12</td>
<td>1.13</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.52</td>
<td>1.68</td>
<td>1.35</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>0.47</td>
<td>1.77</td>
<td>1.33</td>
<td>1.02</td>
<td>1.11</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.47</td>
<td>1.77</td>
<td>1.33</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>0.47</td>
<td>1.56</td>
<td>1.29</td>
<td>1.14</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.47</td>
<td>1.56</td>
<td>1.29</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>0.87</td>
<td>1.38</td>
<td>1.38</td>
<td>1.21</td>
<td>1.24</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.87</td>
<td>1.38</td>
<td>1.38</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>0.72</td>
<td>1.34</td>
<td>1.29</td>
<td>1.08</td>
<td>1.08</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.72</td>
<td>1.34</td>
<td>1.29</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.43</td>
<td>1.25</td>
<td>1.25</td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
<td>0.43</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>1.05</td>
<td>1.62</td>
<td>1.30</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.05</td>
<td>1.62</td>
<td>1.30</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>0.68</td>
<td>1.29</td>
<td>1.20</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.68</td>
<td>1.29</td>
<td>1.20</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>0.89</td>
<td>1.32</td>
<td>1.32</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.89</td>
<td>1.32</td>
<td>1.32</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>0.84</td>
<td>1.33</td>
<td>1.33</td>
<td>1.15</td>
<td>1.14</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.84</td>
<td>1.33</td>
<td>1.33</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>0.67</td>
<td>1.62</td>
<td>1.48</td>
<td>1.23</td>
<td>1.18</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.67</td>
<td>1.62</td>
<td>1.48</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>0.52</td>
<td>1.54</td>
<td>1.29</td>
<td>1.11</td>
<td>1.11</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.52</td>
<td>1.54</td>
<td>1.29</td>
<td>1.11</td>
</tr>
</tbody>
</table>

**Average (all uprights)**: 1.45 1.34 1.30 1.20 1.15

**COV (%)**: 16.76 14.07 16.17 16.14 13.00

---

(1) Classical DSM with no inelastic reserve capacity (Approach (ii)); (2) Conservative inelastic reserve capacity as in AISI-S100 (2016) and AS/NZS (2018) (Approach (ii)); (3) Inelastic reserve capacity with M\text{IC} (Approach (iii)); (4) Extended reserve strength in Pham and Hancock (2013) (Approach (iv)); (5) Extended reserve strength in Pham and Hancock (2013) with M\text{IC} (Approach (v))
| Upright | Conf | $\lambda_d$ | $M_{FEA}$/$M_{DSM}$ (No reserve) | $M_{FEA}$/$M_{DSM}$ (With reserve) | $M_{FEA}$/$M_{DSM}$ (Pham and Hancock) | $M_{FEA}$/$M_{DSM}$ (Pham and Hancock) with $M_{Lc}$ | Upright | Conf | $\lambda_d$ | $M_{FEA}$/$M_{DSM}$ (No reserve) | $M_{FEA}$/$M_{DSM}$ (With reserve) | $M_{FEA}$/$M_{DSM}$ (Pham and Hancock) | $M_{FEA}$/$M_{DSM}$ (Pham and Hancock) with $M_{Lc}$ |
|---------|------|------------|---------------------------------|---------------------------------|---------------------------------|--------------------------------|---------|------|------------|---------------------------------|---------------------------------|--------------------------------|--------------------------------|--------------------------------|
| Type C  | 0    | 0.76       | 1.05                            | 1.05                            | 0.96                            | 0.96                          | Type D  | 0    | 0.74       | 0.92                            | 0.92                            | 0.92                          | 0.92                          | 0.85                            | 0.85                          |
|         | 6    | 0.55       | 1.32                            | 1.14                            | 1.08                            | 0.86                          |         | 1    | 0.76       | 1.01                            | 1.01                            | 1.01                          | 1.01                          | 0.83                            | 0.83                          |
|         | 7    | 0.50       | 1.78                            | 1.45                            | 1.10                            | 1.12                          |         | 2    | 0.76       | 1.13                            | 1.13                            | 1.13                          | 1.13                          | 0.92                            | 0.92                          |
|         | 8    | 0.51       | 1.97                            | 1.61                            | 1.09                            | 1.24                          |         | 3    | 0.72       | 1.17                            | 1.17                            | 1.17                          | 1.17                          | 0.92                            | 0.92                          |
|         |      |            |                                 |                                 |                                 |                               |         | 4    | 0.74       | 1.06                            | 1.06                            | 1.06                          | 1.06                          | 0.91                            | 0.91                          |
| Type F  | 0    | 0.78       | 0.97                            | 0.97                            | 0.97                            | 0.90                          | Type M  | 4    | 0.59       | 1.55                            | 1.46                            | 1.46                          | 1.46                          | 1.20                            | 1.20                          |
|         | 1    | 0.83       | 0.94                            | 0.94                            | 0.94                            | 0.81                          |         | 5    | 0.36       | 1.37                            | 1.08                            | 1.08                          | 1.08                          | 0.96                            | 0.96                          |
|         | 2    | 0.80       | 1.10                            | 1.10                            | 1.10                            | 0.91                          |         | 6    | 0.31       | 1.62                            | 1.20                            | 1.20                          | 1.20                          | 0.97                            | 0.97                          |
|         | 3    | 0.75       | 1.18                            | 1.18                            | 1.18                            | 0.94                          |         | 7    | 0.30       | 1.91                            | 1.35                            | 1.35                          | 1.35                          | 1.07                            | 1.07                          |
|         | 4    | 0.77       | 1.18                            | 1.18                            | 1.18                            | 1.01                          |         | 8    | 0.33       | 1.64                            | 1.32                            | 1.15                          | 1.15                          | 1.20                            | 1.20                          |
| Type N  | 0    | 1.29       | 1.07                            | 1.07                            | 1.07                            | 1.06                          |         | 4    | 0.59       | 1.55                            | 1.46                            | 1.46                          | 1.46                          | 1.20                            | 1.20                          |
|         | 1    | 1.39       | 1.26                            | 1.26                            | 1.26                            | 1.24                          |         | 5    | 0.36       | 1.37                            | 1.08                            | 1.08                          | 1.08                          | 0.96                            | 0.96                          |
|         | 2    | 1.32       | 1.30                            | 1.30                            | 1.30                            | 1.25                          |         | 6    | 0.31       | 1.62                            | 1.20                            | 1.20                          | 1.20                          | 0.97                            | 0.97                          |
|         | 3    | 1.26       | 1.27                            | 1.27                            | 1.27                            | 1.18                          |         | 7    | 0.30       | 1.91                            | 1.35                            | 1.35                          | 1.35                          | 1.07                            | 1.07                          |
|         | 4    | 1.25       | 0.93                            | 0.93                            | 0.93                            | 0.88                          |         | 8    | 0.33       | 1.64                            | 1.32                            | 1.15                          | 1.15                          | 1.20                            | 1.20                          |
| Type O  | 0    | 0.60       | 1.16                            | 1.13                            | 1.13                            | 1.02                          |         | 4    | 0.59       | 1.55                            | 1.46                            | 1.46                          | 1.46                          | 1.20                            | 1.20                          |
|         | 1    | 0.69       | 0.99                            | 0.99                            | 0.99                            | 0.83                          |         | 5    | 0.36       | 1.37                            | 1.08                            | 1.08                          | 1.08                          | 0.96                            | 0.96                          |
|         | 2    | 0.69       | 1.06                            | 1.06                            | 1.06                            | 0.86                          |         | 6    | 0.31       | 1.62                            | 1.20                            | 1.20                          | 1.20                          | 0.97                            | 0.97                          |
|         | 3    | 0.56       | 1.59                            | 1.42                            | 1.42                            | 1.10                          |         | 7    | 0.30       | 1.91                            | 1.35                            | 1.35                          | 1.35                          | 1.07                            | 1.07                          |
|         | 4    | 0.54       | 1.55                            | 1.39                            | 1.39                            | 1.13                          |         | 8    | 0.33       | 1.64                            | 1.32                            | 1.15                          | 1.15                          | 1.20                            | 1.20                          |
|         |      |            |                                 |                                 |                                 |                               |         | Average (all uprights) | 1.29 | 1.18 | 1.12 | 1.01 | 0.98 |
|         |      |            |                                 |                                 |                                 |                               |         | COV (%) | 0.28 | 0.17 | 0.15 | 0.14 | 0.13 |

(1) Classical DSM with no inelastic reserve capacity (Approach (i)); (2) Conservative inelastic reserve capacity as in AISI-S100 (2016) and AS/NZS (2018) (Approach (ii)); (3) Inelastic reserve capacity with $M_{Lc}$ (Approach (iii)); (4) Extended reserve strength in Pham and Hancock (2013) (Approach (iv)); (5) Extended reserve strength in Pham and Hancock (2013) with $M_{Lc}$ (Approach (v))
5.9. Summary

This chapter presented a FE model to capture the biaxial bending response of cold-formed steel storage rack uprights. The model was validated against experimental results and found to be accurate. Parametric studies were then performed to evaluate the accuracy of the linear biaxial bending design equation in international design specifications (AISI-S100 (2016), AS/NZS 4600:2018 (AS/NZS, 2018) and EN 1993-1-3 (2006)). Analyses were performed for local and distortional buckling failure modes only. The biaxial bending interaction relationship was found to be nonlinear and the linear biaxial bending design equation to be conservative with failure occurring at ratios given by the design equation ranging from 1.00 to 1.46. The accuracy of different Direct Strength Method approaches to estimate the biaxial bending capacity of cold-formed steel storage rack uprights failing in local and distortional buckling was also investigated. The DSM, as published in the AISI-S100 (2016), with or without considering the inelastic reserve capacity, was found to underestimate the biaxial bending capacity for the majority of the tested configurations. On
average, the capacity to DSM prediction ratios were equal to 1.45 and 1.29 for local and distortional buckling, respectively, when the inelastic reserve capacity was ignored. When considering it, these ratios changed to 1.34 and 1.18 for local and distortional buckling, respectively. When using the extended inelastic reserve capacity range proposed by Pham and Hancock (2013), the DSM equations better predicted the biaxial capacity, with a capacity to prediction equal to 1.20 and 1.01 for local and distortional buckling, respectively. Considering the actual moment $M_{yc}$ at which yielding initiates in compression (after yielding in tension) results in significant improved of the predictions for slender sections in local buckling.
CHAPTER 6
Conclusions

6.1. Thesis findings

This thesis experimentally and numerically investigates the transverse shear stiffness of steel storage rack upright frames and biaxial bending of the uprights in rack supported buildings. The main contributions and the outcomes in this research are summarised as below:

- Chapter 3 details an advanced shell Finite Element model of bolted cold-formed steel storage rack upright frames, with channel-bracing members. This model is found to accurately capture the shear stiffness of the frames when compared to published experimental tests, with differences ranging from 2% to 17%. The model is then used to quantify the contribution of factors influencing the transverse shear stiffness of six configurations of upright frames following the AS4084 and EN15512 specifications. These factors include the deformation of the bolts, bracing members and uprights. Following the testing methods in the AS4084 and EN15512 specifications, analyses were ran by deforming the frame. Results showed that plastification at the bolt holes starts at an early stage of loading and particularly for the frames tested following the alternative AS4084 test set-up. For lip-to-lip upright frames, (i) effect of bolt bending on the shear stiffness is insignificant and is less than 5%. (ii) local deformation at the end of the bracing members contributes the most to the overall shear stiffness of the frames, about 62% on average for both the AS4084 and EN15512 test set-ups, and (iii) effect of the upright bending stiffness on the shear stiffness is significant, about 44% on
average for both the AS4084 and EN15512 test set-ups. For back-to-back upright frames, (i) bolt bending significantly influences the shear stiffness, 68% on average for the two test set-ups, (ii) unlike the lip-to-lip frames, axial deformation of bracings significantly influence the frame shear stiffness, about 81% on average for both the AS4084 and EN15512 test set-ups, and (iii) effect of upright bending stiffness on shear stiffness is significant, about 35% on average for both the AS4084 and EN15512 test set-ups.

- Chapter 4 presents biaxial bending experimental results performed on two cold-formed steel storage rack upright types, with and without regular perforations. Tests are performed for local and distortional failure modes only. The biaxial bending interaction relationship is found to be highly nonlinear and the existing linear design equation in (AISI-S100 (2016), AS/NZS 4600:2018 (AS/NZS, 2018) and EN 1993-1-3 (2006)) to be significantly conservative. Failure is found to occur at ratios given by the linear design equation ranging from 1.00 to 1.68. For the majority of the tested configurations, the DSM calculated using the biaxial elastic buckling moment in the nominal moment capacity equations either with or without considering the inelastic reserve capacity prescribed in the AISI-S100 (2016) was found to underestimate the biaxial bending capacity especially when bending about the minor axis is dominant. On average, the experimental to predicted ratio was 1.17 when considering the inelastic reserve capacity and 1.29 when not taking that into consideration. The DSM inelastic reserve strength equations using extended inelastic reserve capacity range proposed by Pham and Hancock (2013) give better predictions with an average experimental to predicted ratio of 1.02.

- Chapter 5 presents a validated FE model to capture the experimental biaxial bending response of cold-formed steel storage rack uprights, reported in Chapter 4. Parametric studies were then performed to evaluate the accuracy of the linear biaxial bending design equation in international design specifications (AISI-S100 (2016), AS/NZS 4600:2018 (AS/NZS, 2018) and EN 1993-1-3 (2006)). Analyses were performed for local and distortional buckling failure modes only. The
biaxial bending interaction relationship was found to be nonlinear and the linear biaxial bending design equation to be conservative with failure occurring at ratios given by the design equation ranging from 1.00 to 1.46. The accuracy of different Direct Strength Method approaches to estimate the biaxial bending capacity of cold-formed steel storage rack uprights failing in local and distortional buckling was also investigated. The DSM, as published in the AISI-S100 (2016), with or without considering the inelastic reserve capacity, was found to underestimate the biaxial bending capacity for the majority of the tested configurations. On average, the capacity to DSM prediction ratios are equal to 1.45 and 1.29 for local and distortional buckling, respectively, when the inelastic reserve capacity is ignored. When considering it, these ratios change to 1.34 and 1.18 for local and distortional buckling, respectively. When using the extended inelastic reserve capacity range proposed by Pham and Hancock (2013), the DSM equations better predict the biaxial capacity, with a capacity to prediction equal to 1.20 and 1.01 for local and distortional buckling, respectively. When the actual moment $M_{yc}$ at which yielding initiates in compression (after yielding in tension) is considered, local buckling DSM equations result in significantly improved predictions for slender sections

6.2. Recommendations for future research

The findings of this research have highlighted some research gaps need to be addressed in future studies. These can be summarized by the following topics:

- General applicability of the results obtained in Chapter 3 is not yet fully determined. Different cross-frame configurations, upright types, bracing members and types of connections need to be experimentally and numerically investigated to determine to what extent the results of Z-bracing configuration with lip-to-lip and back-to-back pattern can be generalized for all types of upright frames.
- Contribution of the factors affecting the transverse shear stiffness of steel storage rack upright frames with CHS bracing members.
• Biaxial bending response of steel storage rack uprights in global buckling and evaluation of the current available DSM equations.

• Biaxial bending response of perforated steel storage rack uprights in local, distortional and global buckling for a wide range of commercially available sections.

• Improve the existing DSM equations for biaxial bending of steel storage rack uprights in local and distortional buckling. Further investigations are needed to evaluate all the studied DSM approach to finalize DSM equations that are able to capture biaxial bending capacity for wide range of slenderness ratio.

• Response of steel storage rack uprights under combined compression and biaxial bending in local, distortional and global buckling
References


AISI-S100, North American Specification for the design of cold-formed steel structural members, American Iron and Steel Institute, 2016.


AS4084, Steel storage racking, Standards Australia, Sydney, 2012.

AS/NZS4600, Cold-formed steel structures, Standards Australia, Sydney, 2018.

AS/NZS1252, High-strength steel bolts with associated nuts and washers for structural engineering, Sydney, Australia/New Zealand Standard, 1996.


Davies MJ, Stability of unbraced pallet racks, 5th International Specialty Conference on Cold-Formed Steel Structures, 409-428, 1980.

Davies JM, Down-aisle stability of rack structures, 11th International Specialty Conference on Cold-Formed Steel Structure, pp. 417-435, St Louis, Missouri, U.S.A. 1992.


EN15512, Steel static storage systems – adjustable pallet racking systems – principles for structural design, European Committee for Standardization (CEN), Brussels, Belgium, 2009.


Gilbert PB, Cross-aisle shear stiffness of pallet rack upright frames, University of Sydney, 2010.


Martins AD, Camotim D, Dinis PB, On the direct strength design of cold-formed steel columns failing in local-distortional interactive modes, *in*
References


Pekoz T, Design of cold-formed steel columns, 9th International Specialty Conference on Cold-Formed Steel Structure, pp. 27-43, St Louis, Missouri, USA, 1988.


References


Talebian N, Gilbert BP, Pham CH, Chariere R, Karampour H, Local and distortional biaxial bending capacities of cold-formed steel storage rack uprights, *Journal of Structural Engineering*, 144(6), 2018.


## Appendix-A

### Biaxial bending test setup

Table A.1 NLD and NDL approach for local-distortional interactive buckling tests

(L=1100 mm for Type A uprights)

<table>
<thead>
<tr>
<th>Upright Type</th>
<th>Conf.</th>
<th>Test Number</th>
<th>(\lambda_{ld})</th>
<th>(\lambda_{dl})</th>
<th>(\lambda_{ld_n})</th>
<th>(\lambda_{dl_n})</th>
<th>(M_{nyl}) (kN.m)</th>
<th>(M_{nyd}) (kN.m)</th>
<th>(M_{bdn}) (kN.m)</th>
<th>(M_{bhn}) (kN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A non-perforated</td>
<td>1</td>
<td>1.1</td>
<td>0.68</td>
<td>0.79</td>
<td>0.71</td>
<td>0.86</td>
<td>8.99</td>
<td>8.99</td>
<td>11.57</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.1</td>
<td>0.58</td>
<td>0.58</td>
<td>0.64</td>
<td>0.65</td>
<td>8.09</td>
<td>8.09</td>
<td>10.11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3.1</td>
<td>0.52</td>
<td>0.58</td>
<td>0.62</td>
<td>0.71</td>
<td>4.73</td>
<td>4.73</td>
<td>7.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4.26</td>
<td>-</td>
<td>6.45</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5.1</td>
<td>0.93</td>
<td>0.91</td>
<td>0.96</td>
<td>0.95</td>
<td>6.26</td>
<td>6.63</td>
<td>7.18</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>6.1</td>
<td>0.91</td>
<td>0.82</td>
<td>0.99</td>
<td>0.89</td>
<td>3.98</td>
<td>4.03</td>
<td>4.71</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>7.1</td>
<td>0.88</td>
<td>0.79</td>
<td>0.96</td>
<td>0.86</td>
<td>3.72</td>
<td>3.76</td>
<td>4.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type A perforated</td>
<td>1</td>
<td>1.1</td>
<td>0.75</td>
<td>0.82</td>
<td>0.79</td>
<td>0.86</td>
<td>7.14</td>
<td>7.87</td>
<td>8.77</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.85</td>
<td>8.77</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.1</td>
<td>0.76</td>
<td>0.63</td>
<td>0.83</td>
<td>0.67</td>
<td>6.81</td>
<td>6.81</td>
<td>7.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.07</td>
<td>7.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3.1</td>
<td>0.56</td>
<td>0.63</td>
<td>0.65</td>
<td>0.77</td>
<td>4.09</td>
<td>4.09</td>
<td>6.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.49</td>
<td>6.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3.80</td>
<td>-</td>
<td>5.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5.1</td>
<td>0.95</td>
<td>0.92</td>
<td>0.98</td>
<td>0.95</td>
<td>5.20</td>
<td>5.45</td>
<td>5.84</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.54</td>
<td>5.84</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>6.1</td>
<td>0.94</td>
<td>0.83</td>
<td>1.02</td>
<td>0.89</td>
<td>3.40</td>
<td>3.40</td>
<td>4.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.01</td>
<td>3.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>7.1</td>
<td>0.91</td>
<td>0.79</td>
<td>1.00</td>
<td>0.86</td>
<td>3.28</td>
<td>3.26</td>
<td>3.95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.95</td>
<td>3.86</td>
<td></td>
</tr>
</tbody>
</table>
**Appendix-B**

**Biaxial bending – Parametric studies**

Table B.1 Parametric study – biaxial bending capacity with yield, elastic and plastic moments for local buckling uprights

<table>
<thead>
<tr>
<th>Type</th>
<th>Up-right</th>
<th>Conf</th>
<th>$M_{yA}$ (kN.m)</th>
<th>$M_{yB}$ (kN.m)</th>
<th>$M_{yC}$ (kN.m)</th>
<th>$M_{yD}$ (kN.m)</th>
<th>$M_{yE}$ (kN.m)</th>
<th>$M_{yF}$ (kN.m)</th>
<th>$M_{yG}$ (kN.m)</th>
<th>$M_{yH}$ (kN.m)</th>
<th>$M_{yI}$ (kN.m)</th>
<th>$M_{yJ}$ (kN.m)</th>
<th>$M_{yK}$ (kN.m)</th>
<th>$M_{yL}$ (kN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type G</td>
<td>0</td>
<td>11.59</td>
<td>9.18</td>
<td>9.18</td>
<td>11.64</td>
<td>22.10</td>
<td>0</td>
<td>1.88</td>
<td>2.07</td>
<td>2.07</td>
<td>2.61</td>
<td>1.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>8.59</td>
<td>6.78</td>
<td>6.78</td>
<td>10.65</td>
<td>13.57</td>
<td>1</td>
<td>1.50</td>
<td>1.77</td>
<td>1.97</td>
<td>2.45</td>
<td>0.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7.69</td>
<td>5.21</td>
<td>5.21</td>
<td>9.02</td>
<td>10.80</td>
<td>2</td>
<td>1.14</td>
<td>1.29</td>
<td>1.29</td>
<td>2.08</td>
<td>0.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6.96</td>
<td>4.52</td>
<td>4.52</td>
<td>8.39</td>
<td>10.35</td>
<td>3</td>
<td>0.96</td>
<td>0.95</td>
<td>0.95</td>
<td>1.83</td>
<td>0.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>6.84</td>
<td>5.32</td>
<td>5.32</td>
<td>8.20</td>
<td>11.62</td>
<td>4</td>
<td>0.85</td>
<td>0.95</td>
<td>0.95</td>
<td>1.74</td>
<td>0.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5.70</td>
<td>4.78</td>
<td>4.78</td>
<td>10.65</td>
<td>20.13</td>
<td>5</td>
<td>1.43</td>
<td>1.77</td>
<td>1.77</td>
<td>2.45</td>
<td>0.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>6.74</td>
<td>5.21</td>
<td>5.21</td>
<td>9.02</td>
<td>19.37</td>
<td>6</td>
<td>1.15</td>
<td>1.29</td>
<td>1.52</td>
<td>2.08</td>
<td>0.61</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>7.99</td>
<td>4.52</td>
<td>6.29</td>
<td>8.39</td>
<td>20.20</td>
<td>7</td>
<td>1.07</td>
<td>0.95</td>
<td>1.43</td>
<td>1.83</td>
<td>0.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>8.32</td>
<td>5.32</td>
<td>8.34</td>
<td>8.40</td>
<td>23.64</td>
<td>8</td>
<td>1.21</td>
<td>0.95</td>
<td>1.94</td>
<td>1.74</td>
<td>0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type H</td>
<td>0</td>
<td>7.23</td>
<td>7.30</td>
<td>7.30</td>
<td>8.93</td>
<td>6.86</td>
<td>0</td>
<td>2.83</td>
<td>4.11</td>
<td>4.11</td>
<td>5.19</td>
<td>1.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>7.60</td>
<td>5.90</td>
<td>6.57</td>
<td>8.71</td>
<td>7.85</td>
<td>1</td>
<td>2.05</td>
<td>3.41</td>
<td>3.66</td>
<td>4.86</td>
<td>1.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7.74</td>
<td>5.76</td>
<td>5.76</td>
<td>8.88</td>
<td>11.10</td>
<td>2</td>
<td>1.53</td>
<td>2.63</td>
<td>2.63</td>
<td>4.06</td>
<td>0.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.75</td>
<td>5.41</td>
<td>5.41</td>
<td>9.13</td>
<td>21.89</td>
<td>3</td>
<td>1.32</td>
<td>1.77</td>
<td>1.77</td>
<td>3.35</td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>9.68</td>
<td>5.88</td>
<td>5.88</td>
<td>9.89</td>
<td>32.87</td>
<td>4</td>
<td>1.24</td>
<td>1.73</td>
<td>1.73</td>
<td>3.19</td>
<td>0.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.99</td>
<td>5.90</td>
<td>5.90</td>
<td>8.71</td>
<td>6.92</td>
<td>5</td>
<td>2.68</td>
<td>3.41</td>
<td>3.41</td>
<td>4.86</td>
<td>1.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>7.33</td>
<td>5.76</td>
<td>5.81</td>
<td>8.88</td>
<td>8.22</td>
<td>6</td>
<td>2.33</td>
<td>2.63</td>
<td>2.77</td>
<td>4.06</td>
<td>1.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>8.79</td>
<td>5.41</td>
<td>6.62</td>
<td>9.13</td>
<td>12.18</td>
<td>7</td>
<td>2.37</td>
<td>1.77</td>
<td>2.45</td>
<td>3.35</td>
<td>1.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>9.06</td>
<td>5.88</td>
<td>8.83</td>
<td>9.29</td>
<td>22.17</td>
<td>8</td>
<td>2.44</td>
<td>1.73</td>
<td>3.45</td>
<td>3.19</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B.2 Parametric study – biaxial bending capacity with yield, elastic and plastic moments for distortional buckling uprights

<table>
<thead>
<tr>
<th>Type</th>
<th>M FEA (kN.m)</th>
<th>M y (kN.m)</th>
<th>M yc (kN.m)</th>
<th>M p (kN.m)</th>
<th>M o (kN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>13.40</td>
<td>13.70</td>
<td>13.70</td>
<td>17.31</td>
<td>23.70</td>
</tr>
<tr>
<td></td>
<td>11.48</td>
<td>8.72</td>
<td>9.25</td>
<td>13.55</td>
<td>29.04</td>
</tr>
<tr>
<td></td>
<td>10.47</td>
<td>5.87</td>
<td>8.17</td>
<td>11.15</td>
<td>23.15</td>
</tr>
<tr>
<td></td>
<td>10.57</td>
<td>5.57</td>
<td>11.51</td>
<td>10.62</td>
<td>21.69</td>
</tr>
<tr>
<td>F</td>
<td>8.20</td>
<td>9.18</td>
<td>9.18</td>
<td>11.44</td>
<td>15.21</td>
</tr>
<tr>
<td></td>
<td>5.65</td>
<td>6.78</td>
<td>6.78</td>
<td>10.65</td>
<td>9.95</td>
</tr>
<tr>
<td></td>
<td>5.19</td>
<td>5.21</td>
<td>5.21</td>
<td>9.02</td>
<td>8.17</td>
</tr>
<tr>
<td></td>
<td>5.04</td>
<td>4.52</td>
<td>4.52</td>
<td>8.39</td>
<td>8.01</td>
</tr>
<tr>
<td></td>
<td>5.81</td>
<td>5.32</td>
<td>5.32</td>
<td>8.20</td>
<td>9.01</td>
</tr>
<tr>
<td>N</td>
<td>3.15</td>
<td>4.58</td>
<td>4.58</td>
<td>5.42</td>
<td>2.75</td>
</tr>
<tr>
<td></td>
<td>2.73</td>
<td>3.58</td>
<td>3.58</td>
<td>5.13</td>
<td>1.86</td>
</tr>
<tr>
<td></td>
<td>2.23</td>
<td>2.73</td>
<td>2.73</td>
<td>4.69</td>
<td>1.56</td>
</tr>
<tr>
<td></td>
<td>2.04</td>
<td>2.46</td>
<td>2.46</td>
<td>4.67</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>1.65</td>
<td>2.69</td>
<td>2.69</td>
<td>4.68</td>
<td>1.72</td>
</tr>
<tr>
<td>O</td>
<td>13.14</td>
<td>11.32</td>
<td>11.32</td>
<td>13.88</td>
<td>31.67</td>
</tr>
<tr>
<td></td>
<td>8.11</td>
<td>8.31</td>
<td>8.31</td>
<td>12.95</td>
<td>17.30</td>
</tr>
<tr>
<td></td>
<td>6.40</td>
<td>6.11</td>
<td>6.11</td>
<td>10.12</td>
<td>13.01</td>
</tr>
<tr>
<td></td>
<td>6.05</td>
<td>3.81</td>
<td>3.81</td>
<td>6.65</td>
<td>11.95</td>
</tr>
<tr>
<td></td>
<td>5.68</td>
<td>3.68</td>
<td>3.68</td>
<td>5.82</td>
<td>12.62</td>
</tr>
<tr>
<td></td>
<td>5.26</td>
<td>6.31</td>
<td>9.06</td>
<td>12.95</td>
<td>40.46</td>
</tr>
<tr>
<td></td>
<td>7.18</td>
<td>6.11</td>
<td>7.15</td>
<td>10.12</td>
<td>24.09</td>
</tr>
<tr>
<td></td>
<td>6.11</td>
<td>3.81</td>
<td>4.93</td>
<td>6.65</td>
<td>18.80</td>
</tr>
<tr>
<td></td>
<td>5.88</td>
<td>3.68</td>
<td>5.78</td>
<td>5.92</td>
<td>17.53</td>
</tr>
</tbody>
</table>