Thermal radiation as a probe of one-dimensional electron liquids

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(Dated: April 4, 2018)

Motivated by recent developments in the field of plasmonics, we develop the theory of radiation from one-dimensional electron liquids, showing that the spectrum of thermal radiation emitted from the system exhibits signatures of non-Fermi liquid behavior. We derive a multipole expansion for the radiation based on the Tomonaga-Luttinger liquid model. While the dipole radiation pattern is determined by the conductivity of the system, we demonstrate that the quadrupole radiation can reveal important features of the quantum liquid, such as the Luttinger parameter. Radiation offers a probe of the interactions of the system, including Mott physics as well as non-linear Luttinger liquid behavior. We show that these effects can be probed in current experiments on effectively one-dimensional electron liquids, such as carbon nanotubes.

arXiv:1804.00675v1 [cond-mat.str-el] 2 Apr 2018

Plasmons are fundamental excitations of electron liquids that emerge from the coupling between the collective motion of charge and the electromagnetic field. In conventional metals in three-dimensions, plasmons are gapped to high frequencies, typically above the ultraviolet. However, plasmon excitations at the surface of metals [1] (surface plasmons) or in low-dimensional electron systems such as semiconductor heterostructures, graphene or carbon nanotubes become gapless [2–5]. As a result, such plasmons can be studied in more experimentally accessible frequency ranges — from the microwave to the optical domain [6–14]. The exploitation of these low-dimensional plasmons for technological applications including biosensing, optical communication, and information processing is a burgeoning field of research [15–17].

Plasmons play a central role in the emergent physics of low-dimensional electron systems. In particular, one-dimesional (1D) electron liquids, where electron-electron interactions play a crucial role, represent an important exception to Landau's Fermi liquid theory [18–20]. A conventional formalism to treat such 1D systems is the Tomonaga-Luttinger liquid (TLL) framework in which the excitations are described by free bosons with an acoustic-like spectrum. For repulsive interactions, the velocity of these excitations v is renormalized from the Fermi velocity v_F, with $v > v_F$. However, this picture is not exact, and short distance behavior and nonlinearities can give rise to physics beyond the TLL paradigm [21–25]. Experimentally, the most common manner in which 1D systems are probed is by electron transport. However, transport measurements are often dominated by the properties of Fermi-liquid leads and DC measurements do not directly reveal interaction effects [26]. More generally, direct signatures of non-Fermi liquid behavior remain challenging to observe experimentally [27].

In this work, we demonstrate that thermal radiation in the optical range can serve as a novel probe of non-Fermi liquid behavior in 1D systems. Within the TTL framework, we develop the theory of radiation from a generic 1D electron liquid, in terms of a multipole expansion in the small parameter v/c, valid for long systems (i.e., much longer than the wavelength of typical radiation). We show that the dipole radiation offers an alternative probe of the AC conductivity, $\sigma(\omega)$ [28]. The basic features of the radiation are predicted using a baseline theory in which all the bosonic modes of the TLL model are assumed to have a finite lifetime τ. One of the key predictions of the TLL model is the renormalization of v due to interactions. We demonstrate how v can be gleaned from the thermal radiation, by considering both dipole and quadrupole emission. Our work also shows how radiation offers a probe of more subtle effects. We calculate the effect of a Mott gap (as occurs as the doping is tuned to half-filling) on the radiative properties of a 1D liquid. Additionally, we show that the quadrupole radiation field reveals subtle signatures of non-linear TLL effects. Finally, our theory allows us to make detailed predictions of these effects for carbon nanotubes.

Carbon nanotubes are well-suited to studies of thermal radiation. Their large ($\sim$ eV) bandwidths allow the Luttinger liquid regime to be probed at room temperature. Although the radiation from carbon nanotubes has been studied in the context of single electron physics [29] and quantized plasmons [27, 30], the implications of non-Fermi liquid behavior on the radiation has, to our knowledge, not been explored.

System.— We consider the radiation from a long system, such that the length L of the system greatly exceeds the characteristic wavelength $1/q$ of the excitations of the 1D liquid, as shown in Fig. 1 a. Additionally, we take $L \gg \hbar v/k_B T$ so that finite size quantization is not reflected in the emission spectrum [30]. Electromagnetic

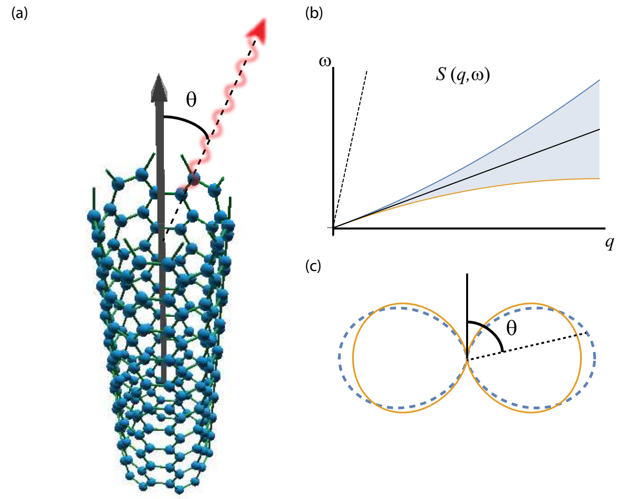

FIG. 1. (a) Thermal radiation from an armchair carbon nanotube. (b) Shaded area indicates regions in the (q, ω)-plane for which the dynamic structure factor $S(q, \omega)$ is non-zero for (i) the Tomonaga-Luttinger model at zero temperature (black line) and (ii) an electron gas at zero temperature (shaded region). The dotted line shows gives the relation $\omega = cq/\cos\theta$, which is the condition for momentum to be conserved. (c) The angular dependence of the radiated power for dipole (solid) and quadrupole (dashed) emission channels (the scale of the quadrupole emission has been exaggerated).

fields couple to matter via

$$H_{\rm int} = -e \int d\mathbf{r}\, \mathbf{j}(\mathbf{r}, t) \cdot \mathbf{A}(\mathbf{r}, t), \tag{1}$$

where $\mathbf{A}$ is the vector potential and $\mathbf{j}$ is the 3D number current. Specializing to the case of a 1D electron liquid oriented along the z-axis, we introduce

$$\mathbf{j}_{\mathbf{k}} = \int dz\; e^{-i|\mathbf{k}|z\cos\theta} j(z, t)\,\hat{z}, \tag{2}$$

which is the Fourier transform of the 1D electron current operator $\mathbf{j}(\mathbf{r})$. The spontaneous emission rate from $|n\rangle$ to $|m\rangle$ with energies $\hbar\omega_{n,m}$ is given by Fermi's golden rule,

$$\frac{d\dot{N}}{d\omega\, d\Omega} = \frac{\pi e^2}{\hbar\varepsilon_0} \frac{\omega^2}{(2\pi c)^3} |\langle m|j_{\mathbf{k}}|n\rangle|^2 |\hat{z}\cdot\hat{\varepsilon}^*|^2, \\ \times \delta\left(\omega_n - \omega_m - \omega\right), \tag{3}$$

where $\mathbf{k}$ is the wave number of the emitted photon and $\hat{\varepsilon}$ is its polarization. The right-hand side of Eq. (3) can be expressed in terms of the (current-current) structure factor $S(q, \omega)$,

$$\frac{1}{L}\frac{d\dot{N}}{d\omega\, d\Omega} = 2\pi\alpha c \frac{\omega^2}{(2\pi c)^3} \frac{S(k_\omega, -\omega)}{\omega} \sin^2\theta, \tag{4}$$

where $k_\omega = \frac{\omega}{c}\cos\theta$, as required by momentum conservation. This expression is the product of the fine structure constant $\alpha \approx 1/137$, a photon phase space factor $\propto \omega^2$, and $S(k, -\omega)$, which for $\omega > 0$ is related to the energy lost by the system [31]. For a system in thermal equilibrium,

$$S(q, -\omega) = \frac{2\chi''(q, -\omega)}{e^{\beta\hbar\omega} - 1}, \tag{5}$$

where $\chi''(q, \omega)$ is the imaginary part of the current-current correlator and $\beta = 1/k_B T$. Equation (5) can be derived from detailed balance, i.e. $S(q, \omega) = e^{\beta\hbar\omega} S(q, -\omega)$.

The spatial dependence of the emitted radiation profile can be studied by expanding $\chi''(q, \omega)$ in the small parameter v/c, which effectively corresponds to a small q expansion

$$\chi''(q, -\omega) = \chi''(0, -\omega) + \frac{1}{2!}\frac{\partial^2\chi''(0, -\omega)}{\partial k^2} q^2 + \ldots \tag{6}$$

Only even powers of q are allowed for a system with inversion symmetry and time-reversal symmetry. Due to the factor $|\hat{z}\cdot\varepsilon|^2 = \sin^2\theta$ in Eq. (4), $\chi''(0, -\omega)$ controls the dipole radiation. The higher order terms in the expansion are related to higher order multipoles of the radiation field [32], however, the two expansions are not equivalent because, as we see from Eq. (4), the terms in Eq. (6) determine the spatial profile of the radiated power rather than the field.

The TLL model describes the low energy properties of a strongly correlated degenerate electron system. For a non-interacting electron gas, the low-energy excitations are particle-hole excitations which move at the Fermi velocity. For short-ranged interactions, the charge excitations of the system can be described by the TLL Hamiltonian

$$H_0 = \frac{\hbar v}{2} \int dx \left[K\left(\partial_x\theta\right)^2 + \frac{1}{K}\left(\partial_x\phi\right)^2\right], \tag{7}$$

where the charge density is given by $\rho = n + \partial_x\phi/\sqrt{\pi}$ while the current is $j = v\partial_x\theta/\sqrt{\pi}$ [19]. This Hamiltonian can be obtained by bosonizing the electron operators. The Hamiltonian (7) assumes a linear fermionic spectrum.

The propagation speed of the plasmons differs from the Fermi velocity, v_F. The quantity $K = v_F/v$ is a measure of the strength of the interactions. The non-interacting limit corresponds to $K = 1$. Repulsive interactions stiffen the plasmons and lead to $v > v_F$ and thus $K < 1$. In a phenomenological model where the Coulomb interactions are treated using elementary electrostatics, $K = \left[1 + (2Ne^2/\pi\hbar v_F)\ln(R_s/R)\right]^{-1/2}$, where R_s is the screening length of the Coulomb interaction and R is the transverse size of the 1D system [3]. The parameter N is the number of channels, e.g., in carbon nanotubes $N = 4$, arising from two spin and valley degrees of freedom, and typically $K \approx 0.2 - 0.3$.

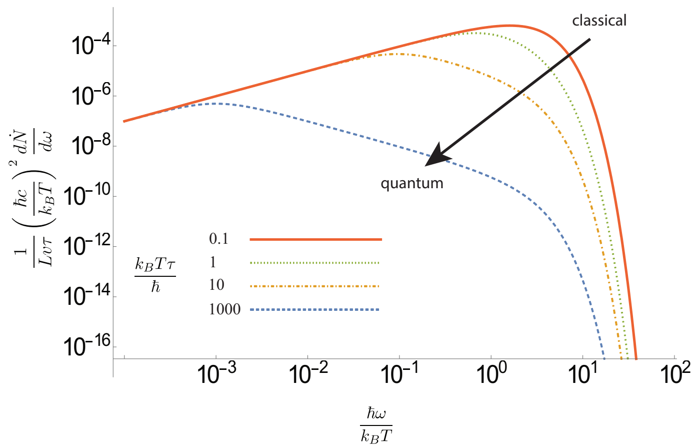

FIG. 2. Log-log plot of the spectrum of photon emission in the dipole approximation. This is obtained by integrating Eq. (9) over all solid angles with $\sigma(\omega)$ given by the Drude conductivity. The plotted curves are for various values of the parameter $\eta = k_BT\tau/\hbar$ which controls the shape of the spectrum.

Near $q = 0$, neither interactions nor finite temperatures give rise to plasmon damping in the TLL model [19]. The retarded charge current-current correlator for charge excitations near $q = 0$ is given by

$$\chi(q,\omega) = \frac{v}{2\pi}\left(\frac{\omega}{\omega - vq + i/\tau} + \frac{\omega}{\omega + vq + i/\tau}\right), \quad (8)$$

where $\tau \to \infty$ [19, 20]. The two terms in Eq. (8) represent right- and left-moving charged excitations moving with a speed v.

Taking the imaginary part of Eq. (8) yields $\chi''(q,-\omega) \propto \sum_\pm \delta(\omega \pm vq)$ (with $\tau \to \infty$). The on-shell condition $\omega = \pm qv$ is incompatible with momentum conservation, $\omega = cq/\cos\theta$ (see Fig. 1 a). We are apparently led to the conclusion that the TLL model does not admit radiation. The vanishing of the off-shell spectral weight for the TLL model can ultimately can be traced to the integrability of the model (see e.g. [33]). Integrable models have the property that the excitations of the system are infinitely long-lived. In a real quantum liquid, the quasiparticle lifetimes are finite and give rise to off-shell spectral weight. The origin of the finite lifetime arise from processes internal to the 1D system such as charge-phonon coupling and numerous irrelevant perturbations as well as from external coupling to the environment.

As a theoretical baseline, we consider a model in which every bosonic mode decays at the energy-independent rate $1/\tau$. This approximation is valid when the plasmon-scattering rate is approximately independent of energy. Exceptions will occur, for instance, near Van Hove singularities of the phonon density of states [34].

Dipole contribution.—The dipole contribution to the radiation is determined by the conductance since, according to the Kubo formula, the first term of the expansion Eq. (6) can be written $\lim_{q\to 0}\chi''(q,-\omega) = \frac{\hbar\omega}{e^2}\mathrm{Re}\,\sigma(\omega)$. The dipole spectrum can, thus, be written as

$$\frac{1}{L}\frac{d^2\dot{N}_{\rm dip}}{d\omega\, d\Omega} = \frac{1}{8\pi^3\varepsilon_0 c^3}\frac{\omega^2\,\mathrm{Re}\,\sigma(\omega)}{e^{\beta\hbar\omega}-1}\sin^2\theta, \quad (9)$$

where $\dot{N}_{\rm dip}$ is the rate at which photons are emitted in the dipole channel. The spontaneous emission described by Eq. (9) arises from currents generated by vacuum fluctuations of the electric field parallel to the system. This result demonstrates that the spectrum of emitted radiation offers a powerful probe of $\sigma(\omega)$. Figure 2 shows the spectrum of the dipole emission for the baseline model in which τ is taken to be a constant. The emission spectrum (integrated over all solid angles) is given by

$$\frac{1}{L}\frac{d\dot{N}_{\rm dip}}{d\omega} = \frac{4\alpha}{3\pi^2}\left(\frac{v}{c}\right)\frac{1}{c\tau}\left[\frac{(\omega\tau)^2}{1+(\omega\tau)^2}\right]\frac{1}{e^{\beta\hbar\omega}-1.} \quad (10)$$

The baseline approximation for the dipole emission is equivalent to taking the Drude conductivity form for $\sigma(\omega)$.

As shown in Fig. 2, the salient features of the baseline spectrum depend on the ratio $\eta = k_BT\tau/\hbar$. In the regime $\eta < 1$, the system acts like a classical black-body and is characterized by a lack of quantum coherence as the radiating dipoles are disrupted faster than their oscillation period. For $\eta \ll 1$,

$$\frac{\dot{N}_{\rm dip}}{L} = \frac{8\zeta(3)\alpha}{3\pi^2}\frac{v\tau\,(k_BT)^3}{c^2\hbar^3} \quad (11)$$

and the emission spectrum has the black-body form for a hypothetical cylinder of radius $\alpha v\tau$ with an emissivity $\varepsilon = 0.04$. In the opposite regime $\eta \gg 1$, the TLL bosons are phase coherent and one naively expects the strongest signatures of non-Fermi liquid behavior in the radiation. In this case, the total rate at which photons are radiated goes as $\dot{N}_{\rm dip} \propto T$, and the spectrum $d\dot{N}_{\rm dip}/d\omega$ achieves its maximum near $\omega = \tau^{-1}$. If the only damping mechanism was from the back action of the radiation itself, the radiation would be in the regime $\eta \gg 1$.

Drude behavior in $\sigma(\omega)$ itself should not be taken as evidence of TLL behavior. Although Eq. (8) formally depends on v, one finds that if τ is independent of energy, then $\sigma(\omega)$ becomes insensitive to interactions and the renormalization effects of v. This follows from the f-sum rule $\int_0^\infty \mathrm{Re}\,\sigma(\omega)\,d\omega = \pi e^2 n/2m$. Consequently, the Luttinger parameter cannot be extracted from the dipole radiation in the baseline model. As we will show below, in the regime $\eta \gg 1$, v can be extracted if the quadrupole emission is also known. The observation of a plasmon velocity v with $v > v_F$ would be a signature of TLL physics.

Quadrupole contribution.—The quadrupole contribution to the radiation can be gleaned from the q^2 term in the expansion of $\chi''(q,\omega)$ [see Eq. (6)]. For the baseline theory in which τ is a constant, the correction to the distribution of emitted radiation arising from the quadrupole channel is given by

$$\frac{1}{L}\frac{d\dot{N}_{\rm quad}}{d\omega} = \frac{4\alpha}{15\pi^2}\left(\frac{v}{c}\right)^3 \frac{1}{c\tau}\left[\frac{3\,(\omega\tau)^6-(\omega\tau)^4}{\left(1+(\omega\tau)^2\right)^3}\right]\frac{1}{e^{\beta\hbar\omega}-1}. \tag{12}$$

For $\eta \gg 1$, the correction to the radiated power goes as $\dot{N}_{\rm quad} \propto T$. Relative to Eq. (9), Eq. (12) is suppressed by a factor $(v/c)^2$. This relation allows for a determination of v and thus gives K provided v_F is known.

So far we have focused on the gapless phase of the electron liquid, however, 1D systems are sensitive to perturbations which can open up a gap which will also affect the radiation. For example, exactly at half-filling, carbon nanotubes can exhibit a Mott gap [35] which is characterized by $\sigma(\omega) = 0$ for $0 < \omega < 2\Delta$, where Δ is the size of the gap. In the Luttinger liquid framework, a careful microscopic description of the Coulomb interaction at this filling gives rise to a variety of sine-Gordon like terms in the bosonization treatment [36–38]. A Luttinger liquid calculation for carbon nanotubes then shows that for $\Delta \ll \omega \ll k_BT$, we have $\sigma(\omega) \sim \Delta^2 T^{1-2K}$. Based on Eq. (11), the integrated flux $\dot{N}_{\rm dip}$ would receive a correction scaling as $\sim T^{4-2K}$ [36]. This would contribute a small correction to the spectrum and the total radiated flux due to this effect would be suppressed by a factor $(\Delta/D)^2$ from the leading contribution (11). Values of the Mott gap range 1 to 10 meV, depending on the size of the nanotube, and thus this scaling gives a correction to the baseline theory at the level of one part in $\sim 10^5$ or 10^6.

Moreover, the measurement of the total radiated power as a function of the filling allows one to determine the Mott gap that appears at half-filling. Since the integral $\int_0^\infty d\omega\,\sigma(\omega)$ is independent of the doping due to the f-sum rule, we can estimate the change in radiative output due to the opening of a Mott gap by supposing that $\sigma(\omega)$ is simply shifted in frequency by an amount $2\Delta/\hbar$, i.e., $\sigma(\omega) \to \sigma(\omega - 2\Delta/\hbar)$. Given the photon phase space factor $\propto \omega^2$ in Eq. (9), this leads to an increase in $\dot{N}$. In the case that most of $\sigma(\omega)$'s spectral weight is below $k_BT/\hbar$, the f-sum rule can be employed to give

$$\frac{\Delta\dot{N}_{\rm dip}}{L} \sim \alpha\left(\frac{k_BT}{m_ec^2}\right)\frac{n\Delta}{\hbar}, \tag{13}$$

where $\Delta\dot{N}_{\rm dip}$ is the increase in the photon count rate resulting in the gap opening.

Quadrupole radiation also arises due to non-linearities which are not accounted for in the Luttinger theory. For instance, density dependence of the Luttinger parameters v and K give rise to scattering among the bosonic quasiparticles [39]. The interaction which is relevant to radiation is that between right- and left-moving bosons, as described by

$$H_\gamma = \gamma\int dx\left[(\partial_x\varphi_L)^2\,\partial_x\varphi_R - (\partial_x\varphi_R)^2\,\partial_x\varphi_L\right], \tag{14}$$

where the chiral fields $\varphi_{R/L} = \frac{1}{2}(\theta \mp \phi)$. The parameter

$$\gamma \propto \hbar\left[\partial_n(vK) - \partial_n(v/K)\right], \tag{15}$$

and is thus controlled by the density dependence of the Luttinger parameters v and K; as such, γ vanishes for the standard Luttinger liquid case in which v and K are taken to be independent of electron density n.

Computing the corrections to $\chi''(q,\omega)$ which arise from H_γ at 1-loop, we find

$$\chi''(q,\omega) \sim \frac{\gamma^2}{\hbar^2 v}q^2, \tag{16}$$

for $\omega \gg vq$ and $\omega \ll D$. This result may also be obtained from the continuity equation and the analogous result for the density-density correlator [23, 24]. Physically, the spectral weight in Eq. (16) arises from the relaxation of left- and right-moving bosons such that their momenta nearly cancel. The coordination of these processes requires electron-electron interactions (i.e., $K \neq 1$) and that there is band curvature ($\partial_n v \neq 0$). The off-shell contribution to $\chi''(q,\omega)$ is a small correction to the quadrupole moment. In carbon nanotubes, $\partial_n v$ is strongly suppressed due to the underlying particle-hole symmetry of the graphene lattice [40]. The best chance to observe the effects of Eq. (16) are in systems with very small effective electron mass.

Experimental considerations.— There is well-developed experimental technology to observe the effects described here. For instance, bolometers can measure the total power radiated by photons in a particular energy window. The correspondence between the radiated power $\dot{P}$ and the photon count rate is readily estimated and depends on the regime set by η. For $\eta \ll 1$, the dipole spectrum peaks at frequencies $\hbar\omega \approx k_BT$, and thus $\dot{P} \sim k_BT\dot{N}$. For $\eta \gtrsim 1$, the spectrum peaks at $\omega \approx 1/\tau$ and thus $\dot{P} = k_BT\dot{N}/\eta$.

We briefly describe relevant experimental numbers for carbon nanotubes. A typical value of the plasmon velocity is $v = 3.0 \times 10^6$ m/sec, with $K \approx 0.3$. At $T = 800$ K, with $\tau = 1$ psec, integrating Eq. (10) gives $N_{\rm dip} \approx 1.6 \times 10^7$ photons/(sec $\cdot\mu$m). This corresponds to $\eta \approx 100$, and gives a radiated power of $k_BT\dot{N}_{\rm dip}/\eta \approx 10^4$ eV/(sec$\cdot\mu$m). The quadrupole radiation is given by that the rate of photon emission in the quadrupole channel is $\dot{N}_{\rm quad} \approx 0.3\,(v/c)^2\,\dot{N}_{\rm dip} \approx 5 \times 10^2$ photons/(sec$\cdot\mu$m).

Measurement of the quadrupole contribution to the radiation requires first establishing the symmetry axis of the emitter. Then, a comparison the radiation emitted (either the number of photons or the power per unit time) at two distinct angles θ_1 and θ_2 can be used to fix the ratio of radiation in the dipole and quadrupole channels.

As shown above, this ratio gives a direct measure of v and the Luttinger parameter K. It is important to point out that in addition to the 'intrinsic' quadrupole moment predicted here, an additional contribution to the quadrupole radiation would arise for a finite length system even if the radiation were emitted only in the dipole channel. This geometric effect can be distinguished from the quadrupole radiation predicted here by measuring the scaling of the quadrupole radiation with the characteristic system size L. The electric quadrupole has units of an electric dipole times a length and the power radiated due to this geometric effect would scale as the square of the quadrupole moment, giving a contribution to the quadrupole radiation which scales as L^2. In contrast, the quadrupole effects described here scale linearly with the length of the system [see e.g. Eq. (12)].

Conclusions— In this work, we have developed a general theory of radiation from a 1D non-Fermi electron liquid. We find that dipole and quadrupole radiation taken together can be used to pinpoint Luttinger behavior in a 1D system. Other signatures of TLL physics, including characteristic power-law renormalizations as well as a changes in radiative output arising from a Mott gap appear as corrections to our baseline theory. Quadrupole radiation also bears subtle signatures of nonlinear TLL effects by revealing the interactions between bosonic quasiparticles. Our work is readily generalized to experimental setups that measure optical conductivity, and demonstrates that rich physics is encoded in the quadrupole radiation profile such as the Luttinger liquid parameter. For conceptual simplicity, we examined the radiative properties of an isolated electronic system; however, integrating such strongly-correlated electron systems with nanophotonic devices could allow for more novel probes of the radiative output.

The authors would like to thank K. A. Matveev, Gordon Baym, James Williams, and Dennis Drew for insightful discussions. This research was supported by the Sloan Foundation and the Physics Frontier Center at the Joint Quantum Institute.

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