REWORKED ILLUSTRATIVE/DESIGN EXAMPLES

IN

REINFORCED AND PRESTRESSED CONCRETE

THIRD EDITION

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PREAMBLE

The Australian Standard AS 3600-2018 Concrete Structures was released while the third edition of Reinforced and Prestressed Concrete was already in press. The illustrative/design examples which are affected by the changes in AS 3600-2018 are reworked and presented here.
3.4.6 Illustrative example for ultimate strength of a singly reinforced rectangular section

Problem
For a singly reinforced rectangular section with \( b = 250 \text{ mm}, d = 500 \text{ mm}, f' = 50 \text{ MPa}, \) and Class N reinforcement only \((f_{sy} = 500 \text{ MPa})\), determine the reliable moment capacity for the following reinforcement cases:

- \((\text{a})\) \( A_{st} = 1500 \text{ mm}^2 \)
- \((\text{b})\) \( A_{st} = 9000 \text{ mm}^2 \)
- \((\text{c})\) a ‘balanced’ design
- \((\text{d})\) with the maximum allowable reinforcement ratio \((p_{\text{all}})\)
- \((\text{e})\) \( A_{st} = 4500 \text{ mm}^2 \).

Then plot \( M' \) against \( p_t \).

Solution

Equation 3.3(2)a: \( \alpha_s = 0.85 - 0.0015 \times 50 = 0.775 \)
Equation 3.3(2)b: \( \gamma = 0.97 - 0.0025 \times 50 = 0.845 \)
Equation 3.4(4): \( k_{\text{st}} = \frac{600}{600 + f_{sy}} = 0.545 \)
Equation 3.4(5): \( p_{\text{hl}} = \frac{0.775 \times 50 \times 0.845 \times 0.545}{500} = 0.0357 \)

\((\text{a})\) \( A_{st} = 1500 \text{ mm}^2 \)

\[ p_t = \frac{1500}{250 \times 500} = 0.012 < p_{\text{hl}} = 0.0357 \text{, therefore the section is under-reinforced.} \]

Equation 3.4(10): \( M_{u} = 1500 \times 500 \times 500 \times \left(1 - \frac{1}{2 \times 0.775} \times \frac{1500}{250 \times 500} \times \frac{500}{50}\right) \times 10^{-6} \)
\( = 346.0 \text{ kNm} \)
Equation 3.4(8): \( k_{u} = \frac{0.012 \times 500}{0.775 \times 0.845 \times 50} = 0.183 \)

By assuming the steel reinforcement to be in one layer, we have \( d = d_0 \) and \( k_{u0} = k_u = 0.183 \).

Equation 3.4(20)a: \( \phi = 1.24 - 13 \times 0.01832/12 = 1.04 \)
For Class N reinforcement, Equation 3.4(20)b: \( \phi = 0.85 \)

Accordingly, Equation 3.4(19): \( M' = \phi M_u = 0.85 \times 346.0 = 294.1 \text{ kNm} \)

\( \text{(b)} \)  \( A_{st} = 9000 \text{ mm}^2 \)

\[ p_r = \frac{9000}{250 \times 500} = 0.072 > p_u = 0.0357, \text{ therefore the section is over-reinforced.} \]

Equation 3.4(16): \( \mu = \frac{600 \times 0.072 \times 500}{0.775 \times 50} = 557.42 \)

Equation 3.4(17): \( a = \frac{\sqrt{557.42^2 + 4 \times 557.42 \times 0.845 \times 500} - 557.42}{2} = 280.92 \text{ mm} \)

Equation 3.4(18): \( M_u = 0.775 \times 50 \times 280.92 \times 250 \times \left( 500 - \frac{280.92}{2} \right) \times 10^6 \)

\[ = 978.5 \text{ kNm} \]

But \( a = \gamma k_u d \) from which

\[ k_u = \frac{280.92}{0.845 \times 500} = 0.665. \]

By assuming one layer of steel, we have \( d = d_0 \) and \( k_{u0} = k_u = 0.665. \)

Then, Equation 3.4(20)a: \( \phi = 0.52 \) but Equation 3.3(20)b stipulates that \( \phi \geq 0.65. \)

Accordingly, Equation 3.4(19): \( M' = \phi M_u = 0.65 \times 978.5 = 636.0 \text{ kNm} \)

This example is for illustrative purposes only. In practice, one layer is not enough to accommodate 9000 mm\(^2\) of bars in the given section.

\( \text{(c)} \) A 'balanced' design (i.e. \( p_{bl} = 0.0357 \))

\[ A_{st} = 0.0357 \times 250 \times 500 = 4462.5 \text{ mm}^2 \]

Equation 3.4(10): \( M_u = 4462.5 \times 500 \times 500 \)

\[ \times \left( 1 - \frac{1}{2 \times 0.775} \times \frac{4462.5}{250 \times 500} \times \frac{500}{50} \right) \times 10^6 = 858.7 \text{ kNm} \]

By assuming one layer of steel, we have \( d = d_0 \) and \( k_{u0} = k_u = k_{uB} = 0.545. \) Then,

Equation 3.4(20)a with b: \( \phi = 0.65 \)

Accordingly, Equation 3.4(19): \( M' = \phi M_u = 0.65 \times 858.7 = 558.2 \text{ kNm} \)

\( \text{(d)} \) With the maximum allowable reinforcement ratio (\( p_{all} \))

Equation 3.4(6): \( p_{all} = 0.4 \times 0.775 \times 0.845 \times \frac{50}{500} = 0.0262 \)

\[ A_{st} = 0.0262 \times 250 \times 500 = 3275 \text{ mm}^2 \]

Equation 3.4(10): \( M_u = 3275 \times 500 \times 500 \)

\[ \times \left( 1 - \frac{1}{2 \times 0.775} \times \frac{3275}{250 \times 500} \times \frac{500}{50} \right) \times 10^6 = 680.4 \text{ kNm} \]

By assuming one layer of steel, we have \( d = d_0 \) and \( k_{u0} = k_u = 0.4. \) Then
Accordingly, Equation 3.4(19): $M' = \phi M_u = 0.807 \times 680.4 = 549.1 \text{ kNm}$

\[ A_{st} = 4500 \text{ mm}^2 \]

$\rho_t = \frac{4500}{250 \times 500} = 0.036 > \rho_{lt} = 0.0357$, therefore the section is over-reinforced.

Equation 3.4(16): $\mu = \frac{600 \times 0.036 \times 500}{0.775 \times 50} = 278.71$

Equation 3.4(17): $a = \frac{\sqrt{278.71^2 + 4 \times 278.71 \times 0.845 \times 500 - 278.71}}{2} = 231.0$

Equation 3.4(18): $M_u = 0.775 \times 50 \times 231.0 \times 250 \times \left(500 - \frac{231.0}{2}\right) \times 10^{-6}$

\[ = 860.4 \text{ kNm} \]

But $a = \gamma k_u d$ from which

$\frac{231.0}{0.845 \times 500} = 0.547$

By assuming one layer of steel, we have $d = d_0$ and $k_{u0} = k_u = 0.547$. Then,

Equation 3.4(20)a with b: $\phi = 0.65$

Accordingly, Equation 3.4(19): $M' = \phi M_u = 0.65 \times 860.4 = 559.3 \text{ kNm}$

The $M'$ versus $\rho_t$ plot is given in Figure 3.4(4). In the region where $\rho_t > \rho_{lt}$ the use of additional $A_{st}$ is no longer as effective. The reason is obvious, since failure is initiated by the rupture of concrete in compression and not by yielding of the steel in tension. Thus, in over-reinforced situations the use of doubly reinforced sections is warranted. This is done by introducing reinforcement in the compressive zone as elaborated in Section 3.6.

![Figure 3.4(4)](M' versus \(\rho_t\) for a singly reinforced section)
3.4.7 Spread of reinforcement

For computing $M_0$, Equations 3.4(10) and 3.4(18) are valid only if the reinforcement is reasonably concentrated and can be represented by $A_{st}$ located at the centroid of the bar group. If the spread of reinforcement is extensive over the depth of the beam, some of the bars nearer to the neutral axis may not yield at failure. This leads to inaccuracies. A detailed analysis is necessary to determine the actual $M_0$. The example below illustrates the general procedure.

Example: Computing $M_0$ from a rigorous analysis

Problem

Compute $M_0$ for the section in Figure 3.4(5), assuming $f'_c = 32$ MPa and $f_{sy} = 500$ MPa.

![Figure 3.4(5) Cross-sectional details of the example problem](image)

Solution

For $f'_c = 32$ MPa, Equations 3.3(2)a and b, respectively, give $\alpha_2 = 0.802$ and $\gamma = 0.89$.

And the reinforcement ratios

$$p_i = \frac{3768}{300 \times 600} = 0.0209$$

$$p_b = \frac{0.802 \times 32 \times 0.89}{500} \times \frac{600}{600 + 500} = 0.025 > p_i = 0.0209,$$

and therefore the section is under-reinforced.

1. Assume all steel yields (i.e. $[$ $] = $ [ $]$).

$$T = A_{st}f_{sy} = 3768 \times 500 \times 10^{-3} = 1884 \text{ kN}$$ and
\[ C = a \times 300 \times 0.802 \times 32 \times 10^{-3} = T = 1884 \text{ kN} \]

from which \( a = 244.7 \text{ mm} \)

Thus,

\[ k_d = \frac{a}{\gamma} = \frac{244.7}{\gamma} = 274.9 \text{ mm} \]

\[ \varepsilon_{sy} = \frac{500}{200000} = 0.0025 \]

**Figure 3.4(6)** Strain distribution on the assumption of all steel yielding

From Figure 3.4(6),

\[ \varepsilon_{s1} = \frac{175.1}{274.9} \times 0.003 = 0.0019 < \varepsilon_{sy} \quad \text{Equation 3.4(21)} \]

\[ \varepsilon_{s2} > \varepsilon_{sy} \]

and

\[ \varepsilon_{s3} > \varepsilon_{sy} \]

Therefore, the assumption is invalid.

2. Assume only the second and third layers yield while the first layer remains elastic (Figure 3.4(7)).

**Figure 3.4(7)** Strain distribution on the assumption of first steel layer not yielding
From Figure 3.4(7),

$$\frac{\epsilon_{s1}}{450-k_u d} = 0.003 \quad \text{Equation 3.4(22)}$$

Therefore

$$f_{s1} = 600 \times \frac{(450-k_u d)}{k_u d}$$

Since $\Sigma F_x = 0$, we have $C = T$, that is

$$0.802 \times 32 \times 0.89k_u d \times 300 = 1256 \times 600 \times \frac{(450-k_u d)}{k_u d} + 2512 \times 500$$

or

$$6.85(k_u d)^2 - 502.4k_u d - 339120 = 0$$

from which $k_u d = 262.2 \text{ mm}$

From Equation 3.4(22), we have $\epsilon_{s1} = 0.00215 < \epsilon_{sy}$.

Since $\epsilon_{s3} > \epsilon_{s2} > \epsilon_{sy}$ (see Equation 3.4(21)), thus Assumption 2 is valid, that is, only the first steel layer is not yielding.

![Figure 3.4(8)](image)

**Figure 3.4(8)** Lever arms between resultant concrete compressive force and tensile forces at different steel layers

Hence, from Figure 3.4(8),

$$a = 0.89 \times 262.2 = 233.4 \text{ mm}$$

$$l_1 = 450 - 233.4 / 2 = 333.3 \text{ mm}$$

$$l_2 = 483.3 \text{ mm}$$

and

$$l_3 = 633.3 \text{ mm}$$

Therefore,

$$M_x = (1256 \times 500 \times (483.3 + 633.3) + 1256 \times 200000 \times 0.00215 \times 333.3) \times 10^{-6}$$

$$= 881.2 \text{ kNm}$$
3.5.3 Design example

Problem
Using the relevant clauses of AS 3600-2018, design a simply supported beam of 6 m span to carry a live load of 3 kN/m and a superimposed dead load of 2 kN/m plus self-weight. Given that $f' = 32$ MPa, $f_y = 500$ MPa for 500N bars, the maximum aggregate size $a = 20$ mm, the stirrups are made up of R10 bars, and exposure classification A2 applies.

Solution
Live-load moment

$$M_q = \frac{wL^2}{8} = \frac{3 \times 6^2}{8} = 13.5 \text{ kNm}$$

Superimposed dead-load moment

$$M_{SG} = \frac{2 \times 6^2}{8} = 9 \text{ kNm}$$

Take $b \times D = 150 \times 300$ mm and assume $\rho_s = 1.4\%$ (by volume). Then

Equation 2.4(1): $\rho_s = 24 + 0.6 \times 1.4 = 24.84 \text{ kN/m}^3$

Thus, self-weight = $0.15 \times 0.30 \times 24.84 = 1.118 \text{ kN/m}$

The moment due to self-weight is

$$M_{sw} = \frac{1.118 \times 6^2}{8} = 5.031 \text{ kNm}$$

and $M_g = M_{SG} + M_{sw} = 9 + 5.031 = 14.031 \text{ kNm}$.

Then

Equation 1.3(2): $M^* = 1.2M_g + 1.5M_q$

or $M^* = 1.2 \times 14.031 + 1.5 \times 13.5 = 37.09 \text{ kNm}$

Equation 3.3(2)a: $\alpha_2 = 0.85 - 0.0015 \times 32 = 0.802$
Equation 3.3(2)b: $\gamma = 0.97 - 0.0025 \times 32 = 0.89$
Equation 2.2(2): $f_{ct}^* = 0.6 \times \sqrt{32} = 3.394 \text{ MPa}$

Adopting N20 bars as the main reinforcement in one layer with 25 mm cover gives

$$d = D - \text{cover} - \text{diameter of stirrup} - \frac{d_b}{2} = 300 - 25 - 10 - 20/2 = 255 \text{ mm}$$
Equation 3.5(5): $p_{\text{min}} = 0.20 \left( \frac{300}{255} \right)^2 \left( \frac{3.394}{500} \right) = 0.00191$

Equation 3.4(6): $p_{\text{all}} = 0.4 \times 0.802 \times 0.89 \times \frac{32}{500} = 0.01827$

Use, say,

$$p_t = \frac{2}{3} p_{\text{all}} = 0.01218 > p_{\text{min}}$$

this is acceptable.

Then

Equation 3.5(3): $bd^2 = \frac{M^*}{\phi p_r f_y \left( 1 - \frac{1}{2\alpha_2} \times p_r \times f_y \right)}$

Equation 3.4(8): $k_u = \frac{0.01218 \times 500}{0.802 \times 0.89 \times 32} = 0.267$

For a single layer of bars, we have $d = d_b$. Thus, $k_{u0} = k_u = 0.267$. Then from Equation 3.4(20)a with $b$, $\phi = 0.85$. Thus

Equation 3.5(3): $150d^2 = \frac{37.09 \times 10^6}{0.85 \times 0.01218 \times 500 \times \left( 1 - \frac{1}{2 \times 0.802} \times 0.01218 \times \frac{500}{32} \right)}$

from which $d = 232.8$ mm

Finally, $A_{st} = p_r bd = 0.01218 \times 150 \times 232.8 = 425.3$ mm$^2$

From Table 2.3(1), there are three options:

1. two N20: $A_{st} = 628$ mm$^2$
2. three N16: $A_{st} = 603$ mm$^2$
3. four N12: $A_{st} = 452$ mm$^2$

Figure 3.5(1) Checking accommodation for 2 N20 bars

Taking Option 1, we have two N20 bars (see Figure 3.5(1)) and Table 1.4(2)
gives a cover $c = 25\text{ mm}$ to stirrups at top and bottom. Hence the cover to the main bars $= c + $ diameter of stirrup $= 25 + 10 = 35\text{ mm}$, and $d = D - $ cover to main bars $- d_b/2 = 300 - 35 - 20/2 = 255\text{ mm} > 232.8\text{ mm}$; therefore this is acceptable (see Figure 3.5(1)).

Table 1.4(4) specifies a minimum spacing $s_{\text{min}}$ of $[25, d_b, 1.5d]_{\text{max}}$. Thus $s_{\text{min}} = [25, 20, 30]_{\text{max}} = 30\text{ mm}$.

The available spacing $= 150 - 2 \times 35 - 2 \times 20 = 40\text{ mm} > s_{\text{min}} = 30\text{ mm}$; therefore, this is acceptable (see Figure 3.5(1)).

Note also that, since $k_u = 0.267 < 0.36$, the design is acceptable without providing any compression reinforcement (see Section 3.4.1).

Taking Option 2, we have three N16 bars as shown in Figure 3.5(2).

The available spacing $= (150 - 2 \times 35 - 3 \times 16)/2 = 16\text{ mm} < s_{\text{min}} = 30\text{ mm}$.

To provide a spacing of 30 mm would require $b > 150\text{ mm}$ (Figure 3.5(2)); therefore, Option 2 is not acceptable.

![Figure 3.5(2) Checking accommodation for 3 N16 bars](image)

Option 3 is similarly unacceptable. Thus, Option 1 should be adopted, but noting the following qualifications:

(a) Option 1 is slightly over-designed (i.e. $d = 255\text{ mm}$ is about 9.5% higher than required and $A_{\text{st}} = 628\text{ mm}^2$ is 47.7% higher than necessary).

(b) The percentage of steel by volume for the section is $[628/(150 \times 300)] \times 100 = 1.396\% = 1.4\%$ as assumed in self-weight calculation; hence this is acceptable.

(c) If the beam is to be used repeatedly or frequently, a closer and more economical design could be obtained by having a second or third trial, assuming different $b \times D$.

(d) If design for fire resistance is specified, ensure that concrete cover of 25 mm is adequate by checking Section 5 of AS 3600-2018.
3.6.3 Illustrative examples

Example 1

Problem

Given a doubly reinforced section as shown in Figure 3.6(3) with $f'_c = 32$ MPa and $f_{sy} = 500$ MPa. Compute $\phi M_o$.

![Cross-sectional details of Example 1](image)

Solution

The reinforcement ratios

$$p_t = \frac{2712}{350 \times 620} = 0.0125$$

and

$$p_c = \frac{339}{350 \times 620} = 0.00156$$

From Section 3.5.3 and for $f'_c = 32$ MPa

$$\alpha_2 = 0.802 \text{ and } \gamma = 0.89$$

Equation 3.6(3): $(p_t - p_c)_{\text{limit}} = \frac{600 \times 0.802 \times 0.89 \times 32 \times 40}{(600 - 500) \times 500} = 0.01768$

But $(p_t - p_c) = 0.01094 < (p_t - p_c)_{\text{limit}} = 0.01768$

Hence $A_{sc}$ does not yield at failure. Then

Equation 3.6(13): $\eta = \frac{0.0125 \times 500 - 600 \times 0.00156}{2 \times 0.802 \times 0.89 \times 32} = 0.1163$
Equation 3.6(14): \( v = \frac{600 \times 0.00156}{0.802 \times 0.89 \times 32} = 0.041 \)

Equation 3.6(12): \( k_u = 0.1163 + \sqrt{0.1163^2 + 0.041 \times \frac{40}{620}} = 0.243 \)

Since \( a = \gamma k_u d = 0.89 \times 0.243 \times 620 = 134.1 \text{ mm} \),

Equation 3.6(15)b: \( M_u = 2712 \times 500 \times \left( 620 - \frac{134.1}{2} \right) + 600 \times 339 \times \left[ 1 - \frac{40}{0.243 \times 620} \left( \frac{134.1}{2} - 40 \right) \right] \times 10^{-6} \)

That is, \( M_u = 753.8 \text{ kNm} \)

With the bars in one layer, we have \( d = d_o, k_{uo} = k_u = 0.243 \), and Equation 3.4(20)a with b gives, for Class N reinforcement \( \phi = 0.85 \)

Finally \( \phi M_u = 0.85 \times 753.8 = 640.7 \text{ kNm} \)

Example 2

Problem

Same as Example 1 (a doubly reinforced section with \( f'_c = 32 \text{ MPa} \) and \( f_y = 500 \text{ MPa} \), but \( d_c = 34 \text{ mm} \) and \( A_{st} \) consists of 6 N28 bars. Compute \( \phi M_u \).

Solution

The reinforcement ratios

\[ p_i = \frac{3696}{350 \times 620} = 0.01703 \]

and

\[ p_c = \frac{339}{350 \times 620} = 0.00156 \]

Equation 3.6(3): \( (p_i - p_c)_{\text{limit}} = \frac{600 \times 0.802 \times 0.89 \times 32 \times 34}{(620 - 500) \times 500} = 0.01503 \)

but \( (p_i - p_c) = 0.01547 > (p_i - p_c)_{\text{limit}} = 0.01503 \)

Hence, \( A_{sc} \) yields at failure. Then

Equation 3.6(6): \( a = \frac{(3696 - 339) \times 500}{0.802 \times 32 \times 350} = 186.9 \text{ mm} \)
Equation 3.6(7): \[ M_u = [339 \times 500 \times (620 - 34) + 0.802 \times 32 \times 186.9 \times 350 \times \left(620 - \frac{186.9}{2}\right)] \times 10^{-6} \], that is \( M_u = 983.3 \text{ kNm} \)

Since \( a = \gamma k_u d \) from which \( k_u = 0.339 \) and for \( d = d_o \) we have \( k_{uo} = k_o = 0.339 \).

Thus, \( \phi = 0.85 \) according to Equations 3.4(20)a and b. Finally, \( \phi M_u = 0.85 \times 983.3 = 835.8 \text{ kNm} \).

### 3.7.2 Illustrative example

**Problem**

If \( b = 230 \text{ mm}, D = 400 \text{ mm}, M^* = 250 \text{ kNm}, f' = 25 \text{ MPa} \) and \( f_{sy} = 500 \text{ MPa} \), and where exposure classification A1 applies, determine \( A_{st} \) and, as necessary, \( A_{sc} \) using only N28 bars. Use R10 ties.

**Solution**

Assume two layers, say, of N28 bars for \( A_{st} \) and one layer for \( A_{sc} \) as shown in Figure 3.7(3).

![Figure 3.7(3) Section layout for illustrative example](image)

Note: all dimensions are in mm.

Thus

\[ d = D - \text{cover up to tie} - \text{tie diameter} - 1.5 \times \text{bar diameter} \]

That is

\[ d = 400 - 28 - 10 - 1.5 \times 28 = 320 \text{ mm}, d_c = 28 + 10 + 28/2 = 52 \text{ mm} \]

Then
Equation 3.3(2)a: $\alpha_1 = 0.85 - 0.0015 \times 25 = 0.8125$ and
Equation 3.3(2)b: $\gamma = 0.97 - 0.0025 \times 25 = 0.9075$

Thus

Equation 3.7(2): $A_{sl} = 0.4 \times 0.8125 \times 0.9075 \times \frac{25}{500} \times 230 \times 320 = 1085.4 \text{ mm}^2$

Equation 3.4(10): $M_{sl} = 1085.4 \times 500 \times 320 \times \left(1 - \frac{1}{2 \times 0.8125} \times \frac{1085.4}{230 \times 320} \times \frac{500}{25}\right) \times 10^{-6}$

That is

$M_{sl} = 142.1 \text{ kNm}$

and

Equation 3.7(3): $M^* = M^* - \phi M_{sl}$

In this case, $k_u = 0.4$

And with $d_o = 400 - 38 - 28/2 = 348$, $k_u = \frac{k_v d}{d_o} = \frac{0.4 \times 320}{348} = 0.368$

The stipulation of Clause 8.1.5 in the Standard that $k_{uo} \leq 0.36$ may not apply to doubly reinforced sections where $k_u \leq 0.4$, provided that the $A_{sc}$ is not less than the specified minimum, which is true in most cases. If in doubt, double check and revise as necessary.

With $k_{uo} = 0.368$, Equation 3.4(20)a with $b$ gives $\phi = 0.841$, with which $M^* = 250 - 0.841 \times 142.1 = 130.5 \text{ kNm}$

And

Equation 3.7(5): $A_s = \frac{130.5 \times 10^6}{0.841 \times 500 \times (320 - 52)} = 1158.0 \text{ mm}^2$

Thus

$A_s = A_{sl} + A_{s2} = 1085.4 + 1158.0 = 2243.4 \text{ mm}^2$

Table 2.3(1) shows that with four N28 bars, $A_{st} = 2464 \text{ mm}^2$ is acceptable.

Also,

Equation 3.7(7): $A_{sl,limit} = \frac{600 \times 0.8125 \times 25 \times 0.9075 \times 52 \times 230}{(600 - 500) \times 500} = 2645.6 \text{ mm}^2$

Since $A_{st1} < A_{sl,limit}$, $A_{sc}$ does not yield, and with $k_u = 0.4$

$\varepsilon_{sc} = 0.003 \times \left(1 - \frac{d}{0.4d}\right) = 0.003 \times \left(1 - \frac{52}{0.4 \times 320}\right) = 0.00178$

Thus, the compression steel stress

$f_{sc} = \varepsilon_{sc} \times E_s \leq f_{sy}$

Or

Or
\[ f_{w} = 0.00178 \times 200000 = 356 \text{ MPa} < f_{y} = 500 \text{ MPa} \]

Hence, \( f_{w} = 356 \text{ MPa} \) requires

\[ A_{w} = \frac{A_{f} f_{y}}{f_{w}} = \frac{1158.0 \times 500}{356} = 1626.4 \text{ mm}^2 \]

With three N28 bars, \( A_{w} = 1848 \text{ mm}^2 \), which is acceptable.

To check bar accommodations for \( b = 230 \text{ mm} \): \( b > (5 \times 28 + 2 \times 10) = 160 \text{ mm} \) is acceptable (use two layers of two bars) or \( b > (7 \times 28 + 2 \times 10) = 216 \text{ is acceptable} \) (use three bars in the bottom layer plus one bar above). Details of the two possible reinforcement layouts are shown in Figure 3.7(4).

**Figure 3.7(4)** Section details for illustrative example

Note: all dimensions are in mm.
4.2.7 Illustrative examples

Two numerical examples are provided here. Examples 1 and 2 consider the analysis and design of singly reinforced T-sections, respectively.

Example 1: Analysis of singly reinforced T-sections

Problem

Given a T-beam as shown in Figure 4.2(8), reinforced with one layer only of Class-N bars. Take $f' = 25$ MPa, $f_{sy} = 500$ MPa and compute $M'$. 

Solution

For $f' = 25$ MPa, $\alpha = 0.8125$ and $\gamma = 0.9075$ and

Equation 4.2(9): 

$$t = \frac{8000 \times 500}{0.8125 \times 25 \times 1100} = 179.02 \text{ mm} > t = 120 \text{ mm}$$

Thus, the NA at failure is located within the web.

For the flange-beam

Equation 4.2(11): 

$$M_{fl} = 0.8125 \times 25 \times 120(1100 - 400) \times \left(650 - \frac{120}{2}\right) \times 10^{-6}$$

$$= 1006.7 \text{ kNm}$$

and

Equation 4.2(12): 

$$A_{fl} = \frac{0.8125 \times 25 \times 120 \times (1100 - 400)}{500} = 3412.5 \text{ mm}^2$$

Therefore $A_{fl} = 8000 - 3412.5 = 4587.5 \text{ mm}^2$

To check the condition of $A_{fl}$ at failure
Equation 4.2(18): \[ 0.8125 \times 0.9075 \times \frac{25}{500} \times 400 \times 650 \times \frac{600}{600 + 500} = 5228.4 \text{ mm}^2 > A_{s1} \]

Therefore

\( A_{s1} \) will yield at failure.

For the web-beam

Equation 4.2(14): \[ M_{u1} = 4587.5 \times 500 \times 650 \times \left( 1 - \frac{1}{2 \times 0.8125} \times \frac{4587.5 \times 500}{400 \times 650 \times 25} \right) \times 10^{-6} \]

\[ = 1167.2 \text{ kNm} \]

and

Equation 3.4(7): \[ k_u = \frac{4587.5 \times 500}{0.8125 \times 0.9075 \times 25 \times 400 \times 650} = 0.479 \]

Assuming that the bars are located in a single layer, we have \( d = d_o \) and \( k_{uo} = k_u = 0.479 \). Thus Equations 3.4(20)a with \( \phi = 0.721 \) and \( M' = 0.721(1167.2 + 1006.7) \]

\[ = 1567.3 \text{ kNm}. \]

Since \( k_{uo} > 0.36 \), appropriate compression reinforcement must be provided.

With reference to Figure 4.2(8), the required \( A_{sc} = 0.01 \times [1100 \times 120 + [(0.479 \times 650) - 120] \times 400] = 2085.4 \text{ mm}^2 \), which may all be placed in the flange at mid-depth.

**Example 2: Design of singly reinforced T-sections**

**Problem**

Given a T-beam with the dimensions shown in Figure 4.2(9), \( f_c = 32 \text{ MPa; } f_{sy} = 500 \text{ MPa} \) and \( M^* = 600 \text{ kNm} \). Design the reinforcement for the section.

![Diagram of T-beam](image.png)

**Figure 4.2(9)** Cross-sectional details of the design T-beam

Note: all dimensions are in mm.
Solution

For $f' = 32$ MPa, $\alpha_2 = 0.802$ and $\gamma = 0.89$.

To use an alternative method to Section 4.2.3 for criterion checking, assume a rectangular section of $b \times d = 750 \times 475$ and that $a = t$. Then, the effective moment

$$M' = \phi M_u = \phi \alpha_2 f'bt \left( d - \frac{t}{2} \right) = 533.8 \phi < M^*$$

Therefore, the neutral axis at the ultimate state lies in the web. Or $a > t$.

For the flange-beam

Equation 4.2(11): $M_{\alpha} = 0.802 \times 32 \times 62.5 \times (750 - 250) \times \left( 475 - \frac{62.5}{2} \right) \times 10^6$

$$= 355.9 \text{ kNm}$$

and

Equation 4.2(12): $A_{\alpha} = \frac{355.9 \times 10^6}{500 \left( 475 - \frac{62.5}{2} \right)} = 1604.1 \text{ mm}^2$

Since $a = \gamma k_ui d = t = 62.5 \text{ mm}$, we have $k_i = 0.148$. Assuming that the bars are located in a single layer, we have $d = d_e$ and $k_{e0} = k_i = 0.148$. Thus $\phi = 0.85$ as per Equations 3.4(20)a and b.

For the web-beam

Equation 4.2(22): $M_{w} = 600 - 0.85 \times 355.9 = 297.5 \text{ kNm}$

and

Equation 3.5(7): $\bar{\xi} = \frac{0.802 \times 32}{500} = 0.0513$

with which

Equation 3.5(6): $p_{i1} = 0.0513 - \sqrt{0.0513^2 - \frac{2 \times 0.0513 \times 297.5 \times 10^6}{0.85 \times 250 \times 475^2 \times 500}} = 0.0144$

Thus, the web reinforcement

$A_{i1} = 0.0144 \times 250 \times 475 = 1710 \text{ mm}^2$

As per Equation 4.2(24)

$$p_{i1} = 0.4 \times 0.802 \times 0.89 \times \frac{32}{500} = 0.01827 > p_{i1}, \text{ which is acceptable.}$$

The right-hand side of Equation 4.2(18) gives

$$0.802 \times 0.89 \times \frac{32}{500} \times 250 \times 475 \times \frac{600}{600 + 500} = 2958.9 \text{ mm}^2 > A_{i1}$$

Therefore, $A_{i1}$ yields at the ultimate state.
Finally, \( A_{ct} = 1710 + 1604.1 = 3314.1 \text{ mm}^2 \)

Further considerations to complete the design are to:

- select a bar group and check accommodation
- double-check the capacity \( M_b \) as necessary.

### 4.3.2 Illustrative examples

#### Example 1

**Problem**

For the doubly reinforced section with an irregular shape as shown in Figure 4.3(3), compute the ultimate moment \( (M_u) \). Take \( f' = 25 \text{ MPa} \).

![Figure 4.3(3) Cross-sectional details of the irregular shape example problem](image)

Note: all dimensions in mm.

**Solution**

For \( f' = 25 \text{ MPa} \), \( \alpha_2 = 0.8125 \) and \( \gamma = 0.9075 \) as per Equations 3.3(2)a and b, respectively.

But, as per AS 3600-2018, for sections where width reduces from the neutral axis towards the compression face, \( \alpha_2 \) shall be reduced by 10%.

Thus, for this example, \( \alpha_2 = 0.9 \times 0.8125 = 0.7313 \)

Based on the strain diagram given in Figure 4.3(4), we obtain

\[
\varepsilon_{bc} = \frac{0.003(d_{NA} - 50)}{d_{NA}}
\]

\[ \text{Equation (i)} \]

and
\[
\varepsilon_s = \frac{0.003(580 - d_{NA})}{d_{NA}}
\]

**Equation (ii)**

**Figure 4.3(4)** Stress and strain distribution across the example section

For the concrete stress over the top area (see the stress diagram in Figure 4.3(4))

\[
C_1 = 150 \times 100 \times 0.7313 \times 25 = 274237.5 \text{ N}
\]

**Equation (iii)**

and for the remaining area

\[
C_2 = 400(\gamma d_{NA} - 100) \times 0.7313 \times 25 = 7313(\gamma d_{NA} - 100)
\]

**Equation (iv)**

The trial and error process below will lead to the required \(M_u\).

**Trial 1:** Assume \(d_{NA} = 200\) mm.

Equation (i): \(\varepsilon_{sc} = 0.00225 < \varepsilon_{sy} = \frac{500}{200000} = 0.0025\); that is, \(A_{sc}\) would not yield.

Therefore, \(f_c = 0.00225 \times 200000 = 450\) MPa.

Through Equation (ii), we observe that \(\varepsilon_s > \varepsilon_{sy}\). Or \(f_s = 500\) MPa.

The total horizontal force in compression is given as

\[
C = C_1 + C_2 + C_s = C_1 + C_2 + A_s f_{sc}
\]

\[
= [274237.5 + 7313(0.9075 \times 200 - 100) + 1232 \times 450] \times 10^{-3}
\]

\[
= 1424.7 \text{ kN}
\]

The total tensile force \(T = A_{st} f_s = 4080 \times 500 \times 10^{-3} = 2040\) kN

Since \(T > C\), assume a larger \(d_{NA}\) in the next trial.

**Trial 2:** Assume \(d_{NA} = 250\) mm, and we have

\(\varepsilon_{sc} = 0.0024 < \varepsilon_{sy}\) and \(\varepsilon_s > \varepsilon_{sy}\)
or

\[ f_{\text{sec}} = 0.0024 \times 200000 = 480 \text{ MPa} \text{ and } f_c = 500 \text{ MPa}. \]

Then \( C = C_1 + C_2 + C_3 = 1793.4 \text{ kN} < T. \)

Try a still larger \( d_{\text{NA}}. \)

**Trial 3:** Assuming \( d_{\text{NA}} = 285 \text{ mm} \) and in a similar process, we obtain

\[ C = 2043.9 \text{ kN} \approx T. \]

Accept \( d_{\text{NA}} = 285 \text{ mm} \), and by taking moments about the level of \( T \), we have

\[ M_u = C_1 \left( \frac{580 - 100}{2} \right) + C_2 \left[ 580 - 100 - \frac{(\gamma d_{\text{NA}} - 100)}{2} \right] + C_3 \times 530 \]

\[ = \left( 274 \times 237.5 \times 530 + 1160 \times 116.0 \times \left[ 480 - \frac{(0.9075 \times 285 - 100)}{2} \right] + 609 \times 515.8 \times 530 \right) \times 10^{-6} \]

\[ = 933.2 \text{ kNm} \]

Note that for beam sections made up of rectangles and other simple shapes, the exact value of \( d_{\text{NA}} \) may be determined by equating the total tensile and compressive forces.

In our case

\[ C = [274 \times 237.5 + 7313(\gamma d_{\text{NA}} - 100) + 1232 \times 500] \times 10^{-3} \]

and

\[ T = 2040 \text{ kN} \]

But

\[ C = T \]

from which

\[ d_{\text{NA}} = 283.4 \text{ mm} \]

However, in all cases before accepting such an 'exact' \( d_{\text{NA}} \), ensure that the resulting stress conditions in \( A_{\text{st}} \) and \( A_{\text{sc}} \) (i.e. yielding or otherwise) are as assumed in the first place.

**Example 2**

**Problem**

Figure 4.3(5) illustrates the section of a beam in a structure containing prefabricated elements. The total width and total depth are limited to 450 mm and 525 mm,
respectively. Tension reinforcement used is 4 N32 bars. Using $f' = 32$ MPa, determine the moment capacity $M_o$ of the section.

**Solution**

For $f' = 32$ MPa, $\alpha = 0.802$ and $\gamma = 0.89$ as per Equations 3.3(2)a and b, respectively.

But, as per AS 3600-2018, for sections where width reduces from the neutral axis towards the compression face, $\alpha$ shall be reduced by 10%.

Thus, for this example, $\alpha = 0.9 \times 0.802 = 0.7218$

Assuming that tension reinforcement $A_t$ yields at failure, and based on the stress diagram where $d_{NA} = k_d d$ as given in Figure 4.3(6), we obtain

$$C = [0.7218 \times 32 \times (\gamma d_{NA} - 75) \times 450 + 2 \times 0.7218 \times 32 \times 125 \times 75] \times 10^{-3}$$

and

$$T = 3216 \times 500 \times 10^{-3} = 1608 \text{ kN}$$
But
\[ C = T \]
from which
\[ d_{sA} = k_a d = 211.3 \text{ mm} \]

Check stress condition for \( A_{sA} \):
Now,
\[ k_a = \frac{k_d d}{d_o} = \frac{211.3}{(525 - 60)} = 0.454 < k_{\text{th}} = 0.545 \]
(from Equation 3.4(4))

This confirms the assumption of tension failure or \( A_{sA} \) yielding at failure.

Finally, taking moment about the level of \( T \) gives

\[ M_u = 0.7218 \times 32 \times (0.89 \times 211.3 - 75) \times 450 \times \left( 525 - 60 - 75 - \frac{(0.89 \times 211.3 - 75)}{2} \right) \times 10^{-6} \]
\[ + 2 \times 0.7218 \times 32 \times 75 \times 125 \times \left( 525 - 60 - 75/2 \right) \times 10^{-6} \]
\[ = 577.0 \text{ kNm} \]
5.3.4 Illustrative example

Problem
Given a simply supported beam (with \( L_d = 10 \text{ m}, b = 350 \text{ mm}, d = 580 \text{ mm}, D = 650 \text{ mm} \) and \( \rho_t = 0.01 \)), compute the midspan deflection under a combination of dead load including self-weight (\( g = 8 \text{ kN/m} \)) and live load (\( q = 8 \text{ kN/m} \)). Take \( E_c = 26000 \text{ MPa}, E_s = 200000 \text{ MPa} \) and \( f' = 32 \text{ MPa} \); assume that the beam forms part of a domestic floor system; ignore the shrinkage effects.

Solution
The gross moment of inertia (see Figure 5.3(1))

\[
I_g = 350 \times \frac{650^3}{12} = 8010 \times 10^4 \text{ mm}^4
\]

Equation 5.3(5): \( M_{cr} = 0.6 \sqrt{32 \left( \frac{I_g}{325} \right)} 10^{-6} = 83.65 \text{ kNm} \)

With \( n = \frac{200000}{26000} = 7.69 \) and \( \rho_t = 0.01 \), Equation A(5) (from Appendix A) yields

\( k = 0.3227. \) In turn, Equation 5.3(3) gives \( I_{cr} = 3174 \times 10^6 \text{ mm}^4 \).

Figure 5.3(1). Cross-sectional details of the example simply supported beam
Note: all dimensions are in mm.

For domestic floor systems, \( \psi_s \) and \( \psi_l \) are given in Table 1.3(1) as 0.7 and 0.4, respectively. For short-term deflection, the third formula in Equation 1.3(8) governs and we have combined load \( = g + 0.7q = 8 + 0.7 \times 8 = 13.6 \text{ kN/m} \).
The moment at midspan is

\[ M_s = 13.6 \times \frac{10^2}{8} = 170 \text{ kNm} \]

Equation 5.3(2) thus gives

\[ I_{ef} = \frac{3174 \times 10^6}{1 - \left(1 - \frac{3174 \times 10^6}{8010 \times 10^6}\right) \left(\frac{83.65}{170}\right)^2} \]

\[ = 3717.4 \times 10^6 \text{ mm}^4 < I_e, \text{ which is acceptable.} \]

Finally, Equation 5.2(1) in conjunction with Table 5.2(1) gives

\[ \Delta = \frac{5}{384} \times \frac{13.6 \times 10000}{26000 \times 3717.4 \times 10^6} = 18.3 \text{ mm} \]

It may be apparent in Table 5.2(4) that the immediate deflection under dead and live loads is not a criterion for serviceability design. Its calculation, however, is essential in the analysis of long-term deflection as discussed in Section 5.4 and the total deflections under normal or repeated loading (Section 5.6).  

### 5.4.3 Illustrative example

**Problem**

For the beam analysed in Section 5.3.4, compute the total deflection \( \Delta_T \) using the multiplier method assuming that the dead load only is the sustained load and

\[ \frac{A_s}{bd} = 0.0025. \]

**Solution**

From Section 5.3.4 and with reference to the first formula in Equation 1.3(8), the sustained load moment

\[ M_s = 8 \times \frac{10^2}{8} = 100 \text{ kNm} \]

Equation 5.3(2) then gives

\[ I_{ef} = \frac{3174 \times 10^6}{1 - \left(1 - \frac{3174 \times 10^6}{8010 \times 10^6}\right) \left(\frac{83.65}{100}\right)^2} \]

\[ = 5495.7 \times 10^6 \text{ mm}^4 < I_e, \text{ which is acceptable.} \]

and, applying Equation 5.2(1),
Based on Equation 5.4(3), the multiplier
\[ k_a = 2 - 1.2 \times \frac{0.0025}{0.01} = 1.7 \]
Finally, Equation 5.4(2) yields
\[ \Delta_k = 18.3 \times 1.7 \times 7.3 = 30.7 \text{ mm} \]
The limit for the total deflection of a beam, as given in Table 5.2(4), is not to be greater than
\[ \frac{L_{ul}}{250} = \frac{10000}{250} = 40 \text{ mm} > \Delta_k = 30.7 \text{ mm}, \text{ which is acceptable} \]
Thus, the beam in question is satisfactory as far as total deflection is concerned.

### 5.6.2 Illustrative example

**Problem**
Re-analyse the beam in Section 5.4.3, taking into consideration the effects of 31200 repetitions of full live load (i.e. \( q = 8 \text{ kN/m} \)).

**Solution**

1. **\( I_{ef} \)**
   
   For \( q = 8 \text{ kN/m} \)
   
   \[ M_q = 8 \times 10^2 / 8 = 100 \text{ kNm}; \ M_s = 200 \text{ kNm} \]
   
   Note that using \( I_{ef,q} \) instead of \( I_{ef,g} \) will lead to a conservative \( I_{rep} \).
   
   Thus, Equation 5.3(2) yields
   
   \[ I_{ef} = \frac{3174 \times 10^6}{1 - \left(1 - \frac{3174 \times 10^6}{8010 \times 10^6}\right)^2 \left(\frac{83.65}{200}\right)} = 3548.8 \times 10^6 \text{ mm}^4 \]

2. **Yield moment \((M_y)\)**
   
   The yield moment is the upper limit of the working stress bending moment.
   
   For under-reinforced rectangular sections, we have
   
   \[ M_y \approx A_s f_y d \left(1 - \frac{k}{3}\right) \]
   
   where \( k \) is obtained using Equation A(5) in Appendix A.
   
   From the example given in Section 5.3.4, \( k = 0.3227 \). Thus
3. Intensive creep factor and $\Delta A_g$

Equation 5.6(5): $k_i = 1.18 + \frac{0.029}{0.01} \frac{200 - 100}{525.4 - 83.65} = 1.84$

Equation 5.6(6): $R = \frac{0.0015}{0.01} \frac{200 - 100}{525.4 - 83.65} = 0.03396$

Equation 5.6(4): $k_g = 1.84 + 0.03396 \times \log_{10}(31200) = 1.993$

From Section 5.4.3, $\Delta I_g = 7.3$ mm. Thus

$\Delta A_g = 1.993 \times 7.3 = 14.55$ mm

4. $I_{rep}$ and $\Delta g$

Equation 5.6(8): $M_z = \frac{(200 - 83.65)^2}{(525.4 - 83.65)} + 83.65 = 114.3$ kNm

Equation 5.6(7): $I_{rep} = \frac{(83.65)}{114.3}^2 \times 8010 \times 10^6 + \left[1 - \left(\frac{83.65}{114.3}\right)^2\right] \times 3548.8 \times 10^6$

$= 5938.2 \times 10^6$ mm$^4$

Comparing Equations 5.6(1) and 5.6(2), we have

$\Delta_i = \frac{5}{48} \times \frac{100 \times 10^6 \times 10000^2}{26000 \times 5938.2 \times 10^6} = 6.75$ mm

5. Grand total deflection

From Section 5.4.3 and using Equation 5.6(10),

$\Delta_t = 1.7 \times 7.3 = 12.41$ mm

Finally, Equation 5.6(9) gives

$\Delta_t = 14.55 + 6.75 + 12.41 = 33.71$ mm.
6.3.8 Design example

Problem

A T-beam with a simply supported span of 6 m is subjected to a concentrated live load $P = 700$ kN, as shown in Figure 6.3(4)a; the cross-sectional details are given in Figure 6.3(4)b. Design the beam for shear, assuming $f' = 20$ MPa.

![Figure 6.3(4)](image)

**Note:** Cross-sectional dimensions are in mm unless specified otherwise.

Solution

**Design shear**

From Table 2.3(1) we obtain $A_{st} = 4928$ mm$^2$ and the gross sectional area $A_g = 1200 \times 100 + 770 \times 300 = 351000$ mm$^2$, based on which the steel percentage by volume

$v = \frac{4928}{351000} \times 100 = 1.4\%$

Then Equation 2.4(1): $\rho_w = 24 + 0.6 \times 1.4 = 24.84$ kN/m$^3$ and the self-weight

$g = (0.1 \times 1.2 + 0.77 \times 0.3) \times 24.84 = 8.72$ kN/m

The maximum shear can be determined via Equation 1.3(2) as

$V' = 1.2V_q + 1.5V_q$

$= 1.2 \times 8.72 \times 3 + (1.5 \times 700) / 2$

$= 31.4 + 525$

$= 556.4$ kN
The design shear may be taken to be the shear at a distance \( d_0 \) (= 828 mm) from the support. Or from Figure 6.3(4)a

\[
V' = 525 + \frac{3-d_0}{3} \times 31.4 = 525 + \frac{3-0.828}{3} \times 31.4 = 547.7 \text{kN}
\]

**Section adequacy**

Check the maximum section capacity in shear using Equation 6.3(1) and we have

\[
V_{u,\text{max}} = 0.55 \left[ f', b_v d_v \left( \cot \theta_{c} \right) \right]
\]

For the given section,

\[
d_v = [0.72D, 0.9d]_{\text{max}} = [(0.72 \times 870), (0.9 \times 800)]_{\text{max}} = [626.4, 720] = 720 \text{ mm}
\]

and using the simplified method, \( \theta_{c} = 36^\circ \)

Finally, \( V_{u,\text{max}} = 0.55 \left[ 20 \times 300 \times 720 \times \left( \cot 36^\circ \right) \right] \times 10^{-3} = 1129.9 \text{ kN} \) from which

\[
\phi V_{u,\text{max}} = 0.75 \times 1129.9 = 847.4 \text{ kN} > V' = 547.7 \text{ kN}
\]

Thus \( D = 870 \text{ mm} \) is acceptable.

**Concrete shear capacity**

Next, compute \( V_{uc} \)

\[
\sqrt{f'_c} = \sqrt{20} = 4.47 \text{ MPa} < 8 \text{ MPa}, \text{ which is acceptable.}
\]

Using simplified method, for \( \frac{A_w}{s} \geq \frac{A_{v,\text{min}}}{s}, \quad k_v = 0.15 \)

Then Equation 6.3(4):

\[
V_{uc} = k_v b_v d_v \sqrt{f'_c} = 0.15 \times 300 \times 720 \times 4.47 \times 10^1 = 144.8 \text{ kN}
\]

and

\[
\phi V_{uc} = 0.75 V_{uc} = 108.6 \text{ kN} < V' = 547.7 \text{ kN}
\]

Thus, shear reinforcement is required.

**Shear reinforcement**

If, say, vertical ties made of N12 bars are used, we have

\[
A_w = 2 \times 113 = 226 \text{ mm}^2
\]

Equation 6.3(17): \( V_w = \frac{(547.7 - 108.6)}{0.75} = 585.5 \text{ kN} \)

That is,
Reinforcement arrangement

The layout is as shown in Figure 6.3(5).

For the full length of the beam, we require a total of \((16 + 16) = 32\) ties, with the spacing between the two ties on either side of the load at mid-span increased to 300 mm for obvious reasons. As the shear force is due mainly to the 700 kN of concentrated load and the distribution is almost uniform, it is acceptable that the chosen tie spacing \((s)\) is used throughout. In cases where the shear distribution varies greatly, \(s\) may vary accordingly along the span to suit, but the detailing requirements specified in Clause 8.3.2.2 of the Standard (AS 3600-2018) must be met (see Section 6.3.7).

6.4.5 Design example

Problem

For the T-beam in the preceding design example (Section 6.3.8), check and design for longitudinal shear, if applicable. Take \(f' = 20\) MPa and use N12 or N16 bars for ties as necessary. Assume monolithic construction.

Solution

From Section 6.3.8, self-weight = 8.72 kN/m and \(V^* = 547.7\) kN.
The critical shear plane, although not specified, may be taken as at the rectangular stress block depth level located (in this case) in the web – thus $d = 300$ mm.


With reference to Figure 6.3(4), imposing $\sum F_x = 0$ gives

$$4928 \times 500 = 0.82 \times 20 \times [1200 \times 100 + 300(\gamma_{kud} - 100)]$$

from which

$$\gamma_{kud} = 200.8 \text{ mm}$$

Then, in Figure 6.4(4), the distance between the centre of gravity of the compressive resultant and the extreme compressive fibre

$$z = \frac{1200 \times 100 \times 100}{2 + (100.8 \times 300) \times (100 + 100.8/2)} = 70.2 \text{ mm}$$

Equation 6.4(3): $z = d - \bar{z} = 800 - 70.2 = 729.8 \text{ mm}$

Equation 6.4(2): $r^* = \frac{1 \times 547.7 \times 10^3}{729.8 \times 300} = 2.50 \text{ MPa}$

The quantities required for Equation 6.4(5) are:

- $A_{sf}/s$ for the existing transverse reinforcement or N12 ties @ 190 mm = $226/190 = 1.19 \text{ mm}^2/\text{mm}$
- $g_\Phi = (100 \times 1200 + 100.8 \times 300) \times 10^{-6} \times 24.84 \times 10^3 = 3732.0 \text{ N/m} = 3.73 \text{ N/mm}$
- From Equation 2.2(3), $f_{c}' = 0.36\sqrt{20} = 1.61 \text{ MPa}$
- From Table 6.4(1), $\mu = 0.9$; $k_{co} = 0.5$

Thus Equation 6.4(5):

$$r_n = 0.9 \left( \frac{1.19 \times 500}{300} + \frac{3.73}{300} \right) + 0.5 \times 1.61 = 2.6 \text{ MPa}$$
Check Equation 6.4(1):
\[
\phi r_u = 0.75 \times 2.6 = 1.95 < \tau_b = 2.50 \text{ MPa}, \text{ which is inadequate.}
\]

Thus, additional shear reinforcement is required.

Equation 6.4(6): \[
\frac{A_s}{s} = \frac{300}{500} \times \left[ \left( \frac{2.50}{0.75} - 0.5 \times 1.61 \right) / 0.9 - (3.73 / 300) \right] \]
\[
= 1.68 \text{ mm}^2 / \text{mm}
\]

Try N12 ties: \[
s = \frac{2 \times 113}{1.68} = 134.5 \text{ mm}, \text{ which is rather close}
\]

Thus, try N16 ties and

\[
s = \frac{2 \times 201}{1.68} = 239.3 \text{ mm}
\]

Say, use N16 ties @ 230 mm, which is less than \(3.5 \bar{s} = 3.5 \times 100 = 350 \text{ mm}\) as required in Equation 6.4(7).

In summary, the final reinforcement arrangement for both transverse and longitudinal shear is N16 closed ties @ 230 mm, and the total number required over the 6-m span: \((26 + 1) = 27\).
7.4.4 Design example

Problem

A cantilever bent beam subjected to torsion, shear and bending is detailed in Figure 7.4(2). Transverse reinforcement (i.e. N10 ties at 300 mm) is provided to resist transverse and longitudinal shear only.

(a) Is the beam section adequate to resist the torsion?

(b) If not, what should the $A_{sw}/s_w$ be and the corresponding longitudinal steel?

Take $f'_c = 20$ MPa.

**Solution**

*For (a)*

1. Compute design shear, torsion and moment.

From Table 2.3(1), we obtain the total reinforcement area for the section

$$A_{st} + A_{sc} = 6160 + 3080 = 9240 \text{ mm}^2$$

Thus, the percentage of steel by volume

$$v = \frac{9240}{300 \times 700} \times 100 = 4.4\%$$

According to Equation 2.4(1), $\rho_w = 24.0 + (0.6 \times 4.4) = 26.64 \text{ kN/m}^3$

Then the self-weight = $0.3 \times 0.7 \times 26.64 = 5.6 \text{ kN/m}$. 

Figure 7.4(2) Details of the example beam for torsion design
At support A:

\[ I^* = 1.5 \times 50 + 1.2 \times 5.6 \times 10.5 = 145.56 \text{ kN} \] (which is a conservative value)

\[ T^* = 1.5 \times 50 \times 0.5 + 1.2 \times 5.6 \times 0.5 \times \frac{0.5}{2} = 38.34 \text{ kNm} \]

\[ M^* = 1.5 \times 50 \times 10 + 1.2 (5.6 \times 0.5 \times 10 + 5.6 \times \frac{10^2}{2}) = 750 + 369.6 \]

\[ = 1119.6 \text{ kNm} \]

2. Check acceptability of the concrete section.

Check the maximum section capacity in shear using Equation 6.3(1) and we have

\[ V_{\text{max}} = 0.55 \left( f_{c} \cdot b_{w} \cdot d \left( \frac{\cot \theta_{c}}{1 + \cot^2 \theta_{c}} \right) \right) \]

For the given section,

\[ d_{w} = [0.72D, 0.9d]_{\text{max}} = [(0.72 \times 700), (0.9 \times 630)]_{\text{max}} = [504, 567]_{\text{max}} \]

\[ = 567 \text{ mm} \]

and \[ \theta_{c} = (29 + 7000\varepsilon_{c}) \] in which

\[ \varepsilon_{c} = \frac{M^*}{d_{w}} + \sqrt{\left(\frac{N^*}{2} \right)^2 + \left[ \frac{0.9T^*u_{b}}{2A_{o}} \right]^2} + 0.5N^* \]

\[ \leq 3 \times 10^{-3} \]

where \[ u_{b} = \text{perimeter of the centre-line of the closed transverse torsion reinforcement} = 2(254 + 654) = 1816 \text{ mm} \]

\[ A_{o} = \text{area enclosed by shear flow path, including any area of holes therein} = 300 \times 700 = 210000 \text{ mm}^2 \]

\[ N^* = 0 \text{ and } d_{w} \sqrt{\left(\frac{N^*}{2} \right)^2 + \left[ \frac{0.9T^*u_{b}}{2A_{o}} \right]^2} = \]

\[ \left[ 567 \times \sqrt{\left(145.56 \times 10^3 \right)^2 + \left[ \frac{0.9 \times 38.34 \times 10^6 \times 1816}{2 \times 210000} \right]^2} \right] \times 10^{-6} = 118.2 \text{ kNm} < \]

\[ M = 1119.6 \text{ kNm, which is acceptable.} \]

Thus,

\[ \varepsilon_{c} = \frac{1119.6 \times 10^6}{567} + \sqrt{\left(145.56 \times 10^3 \right)^2 + \left[ \frac{0.9 \times 38.34 \times 10^6 \times 1816}{2 \times 210000} \right]^2} \]

\[ \times 2 \times 200000 \times 6160 \]

\[ = 0.000886 < 3 \times 10^{-3} \text{ which is acceptable.} \]
Hence, \( \theta = (29 + 700 \times 0.000086) = 35.2' \)

Finally, \( V_{u,\text{max}} = 0.55 \left[ 20 \times 300 \times 567 \times \left( \frac{\cot 35.2^\circ}{1 + \cot^2 35.2^\circ} \right) \right] \times 10^{-3} = 881.3 \text{ kN} \) from which

\[
\phi V_{u,\text{max}} = 0.75 \times 881.3 = 661.0 \text{ kN}
\]

Now, Equation 7.2(1):

\[
\left( \frac{V^2}{b_w d_v} \right)^2 + \left[ \frac{T u_b}{1.7 A_{ob}^2} \right]^2 = \text{in which}
\]

\( A_{ob} = \) area enclosed by centre-line of exterior closed transverse torsion reinforcement, including area of holes, if any = 254 \times 654 = 166116 \text{ mm}^2

Thus, Equation 7.2(1):

\[
\left( \frac{145.56 \times 10^4}{300 \times 567} \right)^2 + \left( \frac{(38.34 \times 10^6) \times 1816}{1.7 \times 166116^2} \right)^2 = 1.71 \text{ MPa} < \phi V_{u,\text{max}} / b_w d_v
\]

\( = 661.0 \times 10^3 / (300 \times 567) = 3.89 \text{ MPa} \)

Therefore, the section is reinforceable.

3. Check torsional reinforcement requirement.

Equation 7.3(2): \( T_{cr} = \left( 0.33 \sqrt{f_c} \right) \frac{A_{cp}^2}{u_c} \) where

\( A_{cp} = \) total area enclosed by outside perimeter of concrete section

\( = 300 \times 700 = 210000 \text{ mm}^2 \)

\( u_c = \) the length of the outside perimeter of concrete cross-section

\( = 2 \times (300 + 700) = 2000 \text{ mm} \)

Thus, Equation 7.3(2):

\[
T_{cr} = \left[ 0.33 \sqrt{20} \right] \left( \frac{210000^2}{2000} \right) \times 10^{-6} = 32.5 \text{ kNm}
\]

Equation 7.3(1): Right-hand side = 0.25 \times 0.75 \times 32.5 = 6.09 \text{ kNm} < T^*

Thus, the beam section is inadequate and torsional reinforcement is required.

For (b)

1. Compute \( A_{sw} / s_w. \)

Equation 7.4(3): \( A_{sw} / s_w = \frac{T^*}{\phi f_{y,d} 1.7 A_{th} \cot \theta} = \frac{38.34 \times 10^6}{0.75 \times 500 \times 1.7 \times 166116 \times \cot 35.2^\circ} \)

That is, \( A_{sw} / s_w = 0.2554 \text{ mm}^2 / \text{mm}. \)
2. Minimum reinforcement requirement checks

\[ \frac{A_{sw}}{s_w} = \frac{0.2 \cdot y_i}{f_{y,ef}} = \frac{0.2 \times 664}{500} = 0.2656 \text{ mm}^2/\text{mm} \]

in the above equation \( y_i \) = the larger overall dimension of the closed fitment = 664 mm.

But \( \frac{A_{sw}}{s_w} = 0.2656 > 0.2554 \), which is not acceptable.

(Check 1)

Equation 7.4(5) which gives, \( \frac{A_{sw}}{s_w} = \frac{T_{\alpha}}{1.7 f_{y,ef} A_{\alpha} \cot \theta} = \frac{32.5 \times 10^6}{1.7 \times 500 \times 166116 \times \cot 35.2^\circ} = 0.1624 \)

< 0.2554, which is acceptable.

(Check 2)

Thus, the required transverse torsional reinforcement

\[ \frac{A_{sw}}{s_w} = 0.2554 \text{ mm}^2/\text{mm}. \]

Note that the stipulation in Clause 8.2.4.5 of the Standard regarding the torsional effects on shear strength (as mentioned in the last paragraph of Section 6.3.4) is naturally satisfied because the beam is reinforced against shear (as well as torsion).

3. Longitudinal reinforcement

\[ \cot \theta \left( \sqrt{\left[ \frac{V_{eq}^*}{0.5 \phi V_{us}} \right]^2 + \left[ \frac{0.457 T_{u,ef}}{2 A_{\alpha}} \right]^2} \right) \geq 0 \]

Equation 7.4(6): \( \Delta A_{eq} = \Delta A_{uc} = \phi f_{y,ef} \]

where

\[ V_{eq}^* = \sqrt{\left( V_{eq}^* \right)^2 + \left( \frac{0.9 T_{u,ef}}{2 A_{\alpha}} \right)^2} = \sqrt{\left( 145.56 \times 10^3 \right)^2 + \left( \frac{0.9 \times (38.34 \times 10^6) \times 1816}{2 \times 210000} \right)^2} \times 10^{-3} \]

\[ = 208.4 \text{ kN} \]

and Equation 6.3(17): \( V_{us} = \left( \frac{V_{eq}^* - \phi V_{uc}}{\phi} \right) \) in which \( V_{uc} = k_b d \sqrt{f_c} \)
and \( k_x = \left[ \frac{0.4}{1 + 1500 \varepsilon_x} \right] = \left[ \frac{0.4}{1 + 1500 \times 0.000886} \right] = 0.17 \) from which

\[
V_{uc} = \left( 0.17 \times 300 \times 567 \times \sqrt{20} \right) \times 10^{-3} = 129.3 \text{ kN}
\]

Thus, Equation 6.3(17): \( V_{us} = \left( \frac{208.4 - 0.75 \times 129.3}{0.75} \right) = 148.6 \text{ kN} \)

Hence, Equation 7.4(6):

\[
\Delta A_{st} = \Delta A_{sc} = \frac{\cot 35.2^\circ \sqrt{(208.4 \times 10^3 - 0.5 \times 0.75 \times 148.6 \times 10^3)^2 + \left[ \frac{0.45 \times 38.34 \times 10^6 \times 1816}{2 \times 210000} \right]^2}{0.75 \times 500}
\]

\( = 642.4 \text{ mm}^2 > 0 \), which is acceptable.

4. Check adequacy of bending reinforcement.

So that AS 3600-2018, Clauses 8.2.7 and 8.2.8 are complied with, the given cross-section less \( \Delta A_{st} \) and \( \Delta A_{sc} \) should be able to resist \( M_t = 1119.6 \text{ kNm} \).

\( A_{st} = 6160 - 642.4 = 5517.6; p_t = 0.0292 \)
\( A_{sc} = 3080 - 642.4 = 2437.6; p_c = 0.0130 \)

\( p_t - p_c = 0.0162 \)

Equation 3.3(2)a: \( \alpha_2 = 0.85 - 0.0015 \times 20 = 0.82 \)
Equation 3.3(2)b: \( \gamma = 0.97 - 0.0025 \times 20 = 0.92 \)

Equation 3.6(3): \( (p_t - p_c)_{\text{lim}} = \frac{0.92 \times 600 \times 0.82 \times 20 \times 42}{630} = \frac{0.0121}{(600 - 500) \times 500} < p_t - p_c \),

and \( A_{sc} \) would yield at ultimate.

Equation 3.6(6): \( a = \frac{(5517.6 - 2437.6) \times 500}{0.82 \times 20 \times 300} = 313.0 \)

Further, \( k_a = \frac{a}{\gamma d} = \frac{313.0}{0.92 \times 630} = 0.54 \) and \( k_{so} = \frac{0.54 \times 630}{658} = 0.52 \)

Thus, Equation 3.4(20)a with b: \( \phi = 0.677 \) and finally

Equation 3.6(8): \( \phi M_{so} = 0.677 \times \left[ 5517.6 \times 500 \times \left( \frac{313.0}{2} \right) + 2437.6 \times 500 \times \left( \frac{313.0}{2} - 42 \right) \right] \times 10^{-6} = 978.8 \text{ kNm} \)
That is, \( \phi M_{uo} = 978.8 \text{kNm} < M_{A^*} = 1119.6 \text{kNm} \), which is not acceptable.

Since the moment capacity of the section (see Figure 7.4(2)) less \( \Delta A_{st} \) and \( \Delta A_{sc} \) is not close enough to the design moment \( (M_{A^*}) \), the existing section needs an increase in \( A_{st} \) and \( A_{sc} \) by 642.4 mm\(^2\) each, to resist torsion.

5. Transverse and longitudinal reinforcement details and final design

From (b) Step 2, above, we have \( A_{sw}/s_{sw} = 0.2554 \text{ mm}^2/\text{mm} \). Use N10 closed ties and the required spacing \( s_{sw} = 78/0.2554 = 305.4 \text{ mm} \).

For the 10-m span, the number of N10 ties = \( 10000/305.4 = 32.7 \), and the existing number of N10 ties as \( A_{sv} = 10000/300 = 33.3 \).

These make a total of 66 N10 ties @ \( s = 10000/(66 - 1) = 153.8 \text{ mm} \). Say, use 67 N10 ties @ 150 mm, covering a total length of \( 66 \times 150 = 9.90 \text{ m} < 10 \text{ m} \), which is acceptable.

Note that N10 ties @ 150 mm complies with the spacing requirements given in Equation 7.4(11).

For additional longitudinal reinforcement, 2 N20 bars (= 628 mm\(^2\) close to required 642.4 mm\(^2\)) are to be provided both at top and bottom of the section.

6. The final design

Details of the final design are as shown in Figure 7.4(3).
8.4 Illustrative examples

8.4.1 Example 1

Problem

Given N28 bars in a simply supported beam as shown in Figure 8.4(1) and \( f' = 25 \text{ MPa} \), determine the position from the support beyond which the yield stress can be developed in the bars, if

(a) the bars are extended straight into the support
(b) standard 180° hooks are used.

Solution

(a) Straight bars

For Equation 8.2(1)

\[
k_1 = 1.0
\]

\[
k_3 = \frac{(132 - 28)}{100} = 1.04
\]

\[
k_4 = 1.0 - 0.15 \frac{(28/2 - 28)}{28} > 1.0 \text{ use 1.0.}
\]

Thus Equation 8.2(1):

\[
L_{yo,b} = \frac{0.5 \times 1.0 \times 1.0 \times 500 \times 28}{1.04 \times \sqrt{25}} = 1346 \text{ mm} > 0.058 \times 500 \times 1.0 \times 28 = 812 \text{ mm},
\]

which is acceptable.

(b) Hooks

\[
L_{yo,1} = 1346 / 2 = 673 \text{ mm}
\]

The points beyond which yield stress can be developed in the two cases are illustrated in Figure 8.4(2).
8.4.2 Example 2

Problem
The stirrups specified for the beam in Figure 8.4(1) are made of R10 bars, each of which is required to develop $k_y$ at mid-depth of the section. Determine and sketch the dimensional details of the stirrup.

Solution
For Equation 8.2(1)

$k_1 = 1.0$
$k_2 = (132 - 10)/100 = 1.22$
$k_3 = 1.0 - 0.15(25 - 10)/10 = 0.78.$

Thus, Equation 8.2(1): $L_{sy,t} = 0.5 \times 1.0 \times 0.78 \times 250 \times 10/(1.22 \times 25) = 160 < 0.058 \times 500 \times 1.0 \times 10 = 290.$ Use 290 mm

Equation 8.2(8): $L_{sy,t} = 1.5 \times 290 = 435 \text{ mm} \geq 300 \text{ mm},$ which is acceptable.

Since each end of the stirrup takes the form of a hook, we have $L_{sy,t}/2 = 435/2 = 218 \text{ mm}.$

However, it can be seen in Figure 8.4(1) that at mid-depth the available development length is only 175 mm. It is therefore necessary to extend the hook by a minimum of $2 \times (218 - 175) = 86 \text{ mm}.$

The details of the stirrup are given in Figure 8.4(3). In Equation 8.2(9), $d_h = 30 \text{ mm}$ and $s_i = 70 \text{ mm}.$

Figure 8.4(3) Dimensional details of the stirrup in Example 2
9.2.4 Design example

Problem

A multistorey reinforced concrete and brick structure as part of an office building may be idealised as shown in Figure 9.2(6). Design the typical floor as a one-way slab. Take $f' = 25$ MPa, $D = 210$ mm, $q = 4$ kPa and the floor finish to be 0.5 kPa. Exposure classification A1 and fire resistance period of 60 minutes are assumed. The effects of wind may be ignored.

Solution

The design is to be carried out on a 1-metre wide strip.

First, the loading must be considered. Assuming the total steel reinforcement is 0.5% by volume in the slab, Equation 2.4(1) gives

$$\rho_u = 24 + 0.6 \times 0.5 = 24.3 \text{ kN/m}^3.$$  

Thus, the dead load

$$g = 1 \times 0.21 \times 24.3 + \text{floor finish} = 5.1 + 0.5 = 5.6 \text{ kN/m}$$

and the live load $q = 4 \text{ kN/m}$

The design load according to Equation 9.2(4) is

$$F_d = 1.2 \times 5.6 + 1.5 \times 4 = 12.72 \text{ kN/m}$$

Following Figure 9.2(2), the 'beam' moment coefficients $a$ for the various sections are shown in Figure 9.2(7) together with the $M^*$ values.
Moment coefficients ($\alpha$) for and moments ($M^*$) at various sections of the slab

**Bending design**

We may now proceed to design the main steel reinforcement using 500 N12 bars.

Table 1.4(2): cover = 20 mm for exposure classification A1

Table 5.5.2(B) of AS 3600-2018: cover ≥ 10 mm for fire resistance period of 60 minutes

Thus, as shown in Figure 9.2(8),

$$d = 210 - 20 - 6 = 184 \text{ mm}$$

![Figure 9.2(8)](image)

**Beam section A**

As shown in Figure 9.2(7),

$$M^* = -13.25 \text{ kN} \cdot \text{m}$$

Equation 3.3(2)a: $\alpha \varepsilon = 0.85 - 0.0015 \times 25 = 0.8125$

Equation 3.5(7): $\xi = \frac{0.8125 \times 25}{500} = 0.0406$

Assuming $\phi = 0.85$,

Equation 3.5(6):

$$p_i = 0.0406 - \sqrt{0.0406^2 - \frac{2 \times 0.0406 \times 13.25 \times 10^6}{0.85 \times 1000 \times 184^2 \times 500}} = 0.00093$$

Now check for $\phi$. 
Equation 3.3(2) b: \( \gamma = 0.97 - 0.0025 \times 25 = 0.9075 \)

Equation 3.4(8): \( k_u = \frac{0.00093 \times 500}{0.8125 \times 0.9075 \times 25} = 0.025 \)

For a single layer of steel, we have \( d = d_0 \) and \( k_{u_0} = k_u = 0.025 \). Then

Equations 3.4(20)a with b: \( \phi = 0.85 \), which is acceptable.

But

Equation 3.5(5): \( p_{t_{\text{min}}} = 0.20 \left( \frac{D}{d} \right) ^2 \frac{f_{\text{st}}}{f_y} = 0.20 \times \left( \frac{210}{184} \right) ^2 \times \frac{0.6 \times \sqrt{25}}{500} = 0.001563 \)

Since \( p_t < p_{t_{\text{min}}} \) use \( p_{t_{\text{min}}} \) or

\[
A_t = p_{t_{\text{min}}} bd = 0.001563 \times 1000 \times 184 = 288 \text{ mm}^2 / \text{m}
\]

From Table 2.3(2) we have for N12 bars at a spacing of \( s = 300 \text{ mm} \)

\[
A_t = 377 \text{ mm}^2 / \text{m} (> 288 \text{ mm}^2 / \text{m})
\]

This is acceptable, as

Equation 9.2(7): \( s \leq \) the lesser of \( 2.0D = 2 \times 210 = 420 \text{ mm} \) and \( 300 \text{ mm} \), and thereby \( s = 300 \text{ mm} \).

Thus, use N12 @ 300 mm (\( A_{st} = 377 \text{ mm}^2 / \text{m} > 288 \text{ mm}^2 / \text{m} \)) for the top bars.

**Beam section B**

As per Figure 9.2(7),

\[
M^* = 28.91 \text{ kNm/m}
\]

Following the same process, we have

\( p_t = 0.00206 > p_{t_{\text{min}}} \), which is acceptable

\[
A_t = 379 \text{ mm}^2 / \text{m}
\]

for N12 at 275 mm, \( A_{st} = 411 \text{ mm}^2 / \text{m} \), which is acceptable.

Thus, use N12 @ 275 mm for the bottom bars.

**Beam section C1**

As shown in Figure 9.2(7),

\[
M^* = -31.80 \text{ kNm/m}
\]

Following the same process, we have

\( p_t = 0.00227 > p_{t_{\text{min}}} \), which is acceptable

\[
A_t = 418 \text{ mm}^2 / \text{m}
\]

for N12 bars at 250 mm, \( A_{st} = 452 \text{ mm}^2 / \text{m} \), which is acceptable. Thus, use N12 @ 250 mm for the top bars.
**Beam section C2**

As shown in Figure 9.2(7),

\[ M^* = -45.79 \text{ kNm/m} \]

In view of the heavier moment, we may use N16 bars. This gives \( d = 182 \text{ mm} \). Similarly, we have

\[ \rho_t = 0.0034 > \rho_{t, \text{min}}; \text{ this is acceptable} \]
\[ A_{st} = 619 \text{ mm}^2/\text{m} \]

for N16 bars at 300 mm, \( A_{st} = 670 \text{ mm}^2/\text{m} \), which is acceptable. This also satisfies Equation 9.2(7).

Thus, use N16 @ 300 mm for the top bars.

**Beam section D**

As shown in Figure 9.2(7),

\[ M^* = 28.62 \text{ kNm/m} \]

Using, say, N12 bars, \( d = 184 \text{ mm} \). Then we have:

\[ \rho_t = 0.00204 > \rho_{t, \text{min}}, \text{ which is acceptable} \]
\[ A_{st} = 375 \text{ mm}^2/\text{m} \]

for N12 bars at 300 mm, \( A_{st} = 377 \text{ mm}^2/\text{m} \).

Thus, use N12 @ 300 mm for the bottom bars.

**Beam section E1**

As shown in Figure 9.2(7),

\[ M^* = -41.63 \text{ kNm/m} \]

In view of the heavier moment, we may use N16 bars. This gives \( d = 182 \text{ mm} \). Similarly, we have

\[ \rho_t = 0.00307 > \rho_{t, \text{min}}, \text{ this is acceptable} \]
\[ A_{st} = 559 \text{ mm}^2/\text{m} \]

for N16 bars at 300 mm, \( A_{st} = 670 \text{ mm}^2/\text{m} \), which is acceptable.

Also,

Equation 9.2(7): \( s \leq \) the lesser of \( 2.0 \times D = 2 \times 210 = 420 \text{ mm} \) and 300 mm, and thereby \( s = 300 \text{ mm} \)

Thus, use N16 @ 300 mm (\( A_{st} = 670 \text{ mm}^2/\text{m} > 596 \text{ mm}^2/\text{m} \)) for the top bars.
**Shear design**

It is apparent in Figure 9.2(3) that the shear force is a maximum at section Cl of continuous strip if all spans are equal. For our case, $L_{n2} > L_{n1}$ and the maximum shear occur in section C2.

Equation 9.2(5) thus gives

$$V^* = 0.5 \times 12.72 \times 6 = 38.16 \text{ kN}$$

Equation 6.3(4): $V_{uc} = k_v b_d d_c \sqrt{f_c'}$

where $\sqrt{f_c'} = \sqrt{25} = 5 \text{ MPa} < 8 \text{ MPa}$, which is acceptable.

$d_c = [0.72 D, 0.9 d]_{\text{max}} = [0.72 \times 210, 0.9 \times 182]_{\text{max}} = [151.2, 163.8]_{\text{max}}$

and using the simplified method of Clause 8.2.4.3, AS 3600-2018, for

$$\frac{A_{sv}}{s} < \frac{A_{sv, \text{min}}}{s}, \quad k_v = \frac{200}{(1000 + 1.3 d_c)} = \frac{200}{(1000 + 1.3 \times 163.8)} = 0.165 > 0.10,$$

which is not acceptable, for which, take $k_v = 0.10$

Finally, Equation 6.3(4): $V_{uc} = 0.10 \times 1000 \times 163.8 \times \sqrt{25} \times 10^{-3} = 81.9 \text{ kN}$

and

$$V_{uc} = 0.75 \times 81.9 = 61.4 \text{ kN} \quad \Rightarrow \quad V_{uc} > V^* = 38.16 \text{ kN}$$

This is acceptable. Note that if $\phi V_{uc} < V$, a thicker slab should be used.

**Shrinkage and temperature steel (in y-direction)**

For unrestrained edge conditions, Equation 9.4(14) gives:

$$A_{sv, \text{min}} = 1.75 \times 1000 \times 210 \times 10^{-3} = 368 \text{ mm}^2/m$$

for N12 @ 300 mm, $A_s = 377 \text{ mm}^2/m (> 368 \text{ mm}^2/m)$.

Thus, use N12 @ 300 mm for the bottom bars.

**Serviceability check**

Figure 9.2(7) shows that, for the exterior span, Equation 5.2(4) requires that

$$L_{ef} = \text{the lesser of } L \text{ (5250 mm) and } L_s + D \text{ (5210 mm)}$$

Thus,

$$L_{ef} = 5210 \text{ mm}$$

For total deflection:

Table 5.2(4): $(\Delta/L_{ef}) = 1/250$

Equation 5.4(3):

$$k_{cs} = 2 - 1.2 \frac{A_{sv}}{A_s} = 2.0$$
Table 1.3(1): for office building $\psi_k = 0.7$ and $\psi_l = 0.4$.

Then Equation 5.5(8) gives

$$ F_{d,e} = (1 + 2) \times 5.6 + (0.7 + 2 \times 0.4) \times 4 = 22.8 \text{kN/m}^2 = 22.8 \times 10^{-3} \text{ MPa} $$

Equations 9.2(9) and 9.2(13) give $k_3 = 1.0$ for one-way slab and $k_4 = 1.75$ for an endspan, respectively.

From Table 2.2(1), for $f_c' = 25 \text{ MPa}$, $E_c = 26 \text{ 700 MPa}$

Finally, Equation 9.2(8) gives

$$ d_{min} = \frac{5210}{1 \times 1.75 \times \sqrt{\frac{(1/250) \times 26700}{22.8 \times 10^{-3}}}} = 177.9 \text{ mm} < d = 184 \text{ mm} $$

This is acceptable.

For the interior span, $L_{di} = 6210 \text{ mm}$ and $k_4 = 2.1$. These lead to

$$ d_{min} = 176.7 \text{ mm} < d = 182 \text{ mm} $$

This is also acceptable.

**Drawings**

The design results are presented in Figure 9.2(9).

---

**Figure 9.2(9)** Reinforcement layout for the design slab

a. Or half the amount provided near the top and bottom faces may also use steel fabrics

b. $\leq 50\%$ may be curtailed
9.3.6 Design example

Problem
The corner slab shown in Figure 9.3(8) is part of a hotel complex. Sides AB and BE are supported on brick walls, and sides AC and CE on reinforced concrete spandrel beams. The beams are supported at C by a column cast monolithically with the slab.

![Diagram of corner slab in a hotel complex]

Figure 9.3(8) Corner slab in a hotel complex

Design the slab in accordance with the Standard, given $f'_{c} = 25$ MPa, exposure classification A1 and fire-resistance period = 90 minutes. Use N bars only.

Solution

Loading
Table 3.1 of AS/NZS 1170.1-2002 gives $q = 4$ kPa. Assuming $D = 300$ mm and the total amount of steel in the slab is 0.5% by volume, Equation 2.4(1) gives

$$\rho_{w} = (24 + 0.6 \times 0.5) = 24.3 \text{kN/m}^3$$

Then

$$g = 0.3 \times 24.3 = 7.29 \text{kPa}$$

Also, the design load is

$$F_{d} = 1.2 \times g + 1.5q = 14.75 \text{kPa}$$

Design moments
Now, $L_x = 10$ m and $L_y = 12$ m yield $L_y / L_x = 1.2$. With the two adjacent edges being continuous, Table 9.3(1) gives

$$\beta_x = 0.046 \quad \text{and} \quad \beta_y = 0.035$$
Positive moments

Equation 9.3(1): \( M^+ = 0.046 \times 14.75 \times 10^2 = 67.85 \text{kNm/m} \)

Equation 9.3(2): \( M^+ = 0.035 \times 14.75 \times 10^2 = 51.63 \text{kNm/m} \)

Negative moments

About edge AB

Equation 9.3(7): \( M^- = -1.33 \times 51.63 = -68.67 \text{kNm/m} \)

About edge CE

Equation 9.3(5): \( M^- = -0.5 \times 51.63 = -25.82 \text{kNm/m} \)

About edge BE

Equation 9.3(6): \( M^- = -1.33 \times 67.85 = -90.24 \text{kNm/m} \)

About edge AC

Equation 9.3(4): \( M^- = -0.5 \times 67.85 = -33.93 \text{kNm/m} \)

Minimum effective depth

The shorter effective span is \( L_{ef} = 10000 \text{mm} \). For total deflection,

Table 5.2(4): \( (\Delta/L_{ef}) = 1/250 \)

Equation 5.4(3): \( k_{sc} = 2 - 1.2 \cdot \frac{A_k}{A_s} = 2.0 \)

Table 1.3(1): \( \psi_s = 0.7 \) and \( \psi_i = 0.4 \)

Then,

Equation 5.5(8): \( F_{def} = (1.0 + 2) \times 7.29 + (0.7 + 2 \times 0.4) \times 4 \)

\( = 27.87 \text{kN/m}^2 = 27.87 \times 10^{-3} \text{MPa} \)

From Section 9.3.4, we have \( k_3 = 1.0 \) and by interpolation from Table 9.3(3)

\( k_i = 2.50 + [(0.05/0.25) \times (2.95 - 2.5)] = 2.59 \)

Equation 2.2(5): \( E_s = 25280 \text{MPa} \)

Finally, Equation 9.2(8) gives

\[ d_{\text{min}} = \frac{10000}{1 \times 2.59 \times \sqrt{\frac{1/250 \times 25 \times 280}{27.87 \times 10^{-3}}}} = 251.3 \text{mm} \]

Since \( D = 300 \text{mm} \), use N16 bars. From Table 1.4(2), the concrete cover is 20 mm for exposure classification A1 and from Table 5.5.2(B) of the Standard, only 15 mm is needed for fire resistance. Thus

\[ d_s = 300 - 20 - 8 = 272 \text{ mm} > d_{\text{min}} = 251.3 \text{ mm} \]
This is acceptable. Now
\[ d_y = 272 - 16 = 256 \text{ mm} > d_{\text{min}} = 251.3 \text{ mm} \]
This is also acceptable. Finally, the average
\[ d_{\text{average}} = 272 - 8 = 264 \text{ mm} > d_{\text{min}} = 251.3 \text{ mm} \]
This, too, is acceptable.

**Bending design**

Equation 3.3(2)a: \( \alpha_s = 0.85 - 0.0015 \times 25 = 0.8125 \)

and

Equation 3.5(7): \( \xi = \frac{0.8125 \times 25}{500} = 0.0406 \)

Assuming \( \phi = 0.85 \), Equation 3.5(6) yields the \( p_t \) values for all the design sections. The results are summarised in Table 9.3(4).

### Table 9.3(4)

<table>
<thead>
<tr>
<th>Location</th>
<th>( M_x^* )</th>
<th>( M_y^* )</th>
<th>( p_t )</th>
<th>( A_{\text{st}} ) (actual)</th>
<th>Top or bottom</th>
<th>( x ) or ( y ) direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central region</td>
<td>67.85</td>
<td>–</td>
<td>0.00222</td>
<td>603.8</td>
<td>B</td>
<td>( x )</td>
</tr>
<tr>
<td>Central region</td>
<td>–</td>
<td>51.63</td>
<td>0.002</td>
<td>512</td>
<td>B</td>
<td>( y )</td>
</tr>
<tr>
<td>Outside the central region</td>
<td>–</td>
<td>0.002</td>
<td>528 ( (d_{\text{average}}) )</td>
<td>300 670 B</td>
<td>( x ) and ( y )</td>
<td></td>
</tr>
<tr>
<td>About AB</td>
<td>–</td>
<td>–68.67</td>
<td>0.0026</td>
<td>665.6</td>
<td>T</td>
<td>( y )</td>
</tr>
<tr>
<td>About CE</td>
<td>–</td>
<td>–25.82</td>
<td>0.00156(a)</td>
<td>399.4</td>
<td>T</td>
<td>( y )</td>
</tr>
<tr>
<td>About BE</td>
<td>–90.24</td>
<td>–</td>
<td>0.003</td>
<td>816</td>
<td>T</td>
<td>( x )</td>
</tr>
<tr>
<td>About AC</td>
<td>–33.93</td>
<td>–</td>
<td>0.00139(a)</td>
<td>378.1</td>
<td>T</td>
<td>( x )</td>
</tr>
</tbody>
</table>

\( a \): \( p_{\text{min}} \) values, since the minimum reinforcement requirements govern in these cases. See Equation 9.3(3).

Note: check for \( \phi \). Take the largest \( p_t \) value of 0.003, Equation 3.4(8) gives \( k_s = 0.0814 \) and for \( d = d_0, k_{uo} = k_s = 0.0814 \) and Equations 3.4(20) \( a \) with \( b \) gives \( \phi = 0.85 \). Hence, for all cases, the assumption of \( \phi = 0.85 \) is acceptable.
**Corner reinforcement**

**Corner C**
At corner C, the uplift is prevented. Thus

Equation 9.3(17): $A'_{st} = 0.75 \times 670 = 502.5 \text{ mm}^2/\text{m}$

The existing top and bottom reinforcement in the $x$ and $y$ directions exceed 502.5 mm$^2$/m. Therefore, no additional reinforcement is required over corner C.

**Corners A and E**
Equation 9.3(18): $A'_{st} = 0.5 \times 670 = 335 \text{ mm}^2/\text{m}$

All existing top and bottom reinforcement in the $x$ and $y$ directions exceed 335 mm$^2$/m. Therefore, no additional reinforcement is required over corners A and E.

**Reinforcement layout**
A sketch of the reinforcement arrangement is given in Figure 9.3(9).

---

<table>
<thead>
<tr>
<th>Main steel</th>
<th>Corner steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) M16@300 mm8</td>
<td>No additional reinforcement required over corners A, C and E.</td>
</tr>
<tr>
<td>(2) M16@300 mm (T)</td>
<td></td>
</tr>
<tr>
<td>(3) M16@225 mm (T)</td>
<td></td>
</tr>
<tr>
<td>(4) M16@300 mm (T)</td>
<td></td>
</tr>
<tr>
<td>(5) M16@300 mm (T)</td>
<td></td>
</tr>
<tr>
<td>(6) M16@300 mm (T)</td>
<td></td>
</tr>
<tr>
<td>(7) M16@300 mm (T)</td>
<td></td>
</tr>
</tbody>
</table>

Note: all dimensions are in mm.
9.6.10 Illustrative example

Problem

An interior idealised frame is illustrated in Figure 9.6(4)a with the details of connection A shown in Figure 9.6(4)b. Given $M_v^* = 120$ kNm, $N_z^* = 1500$ kN, $D_s = 350$ mm and $d = 300$ mm. Is the connection adequate in resisting the punching shear? Design a torsion strip as necessary. Assume $f' = 25$ MPa and that a shear head is provided. Use N12 bars for $A_{sw}$.

Figure 9.6(4) An interior idealised frame with the details of its connection at A
Note: all dimensions are in mm.

Solution

**Punching shear strengths**

Equation 9.6(4): $u = 2 \times (350 + 150) + (400 + 150 + 150) = 1700$ mm

Equation 9.6(8): $V_{wu} = 0.5 \times 1700 \times 300 \times \sqrt{25 \times 10^3} = 1275$ kN

Equation 9.6(9): $V_u = \frac{1275}{1700 \times 120 \times 10^6 \times 1 + \frac{8 \times 1500 \times 10^3 \times 500 \times 300}{1700 \times 120 \times 10^6}} = 1145$ kN

But

$\phi V_u = 0.75 \times 1145 = 858.8$ kN < $N_z^* = 1500$ kN

Thus, the slab–column connection has insufficient punching shear strength.
**Torsion strip**

Equation 9.6(23): \( y_1 = 500 - 25 - 6 = 469 \text{ mm} \)

Equation 9.6(22): \[
\left( \frac{A_{sw}}{s} \right)_{\text{min}} \approx \frac{0.2 \times 469}{500} = 0.1876 \text{ mm}^2 / \text{mm}
\]

Equation 9.6(24): \( V_{\text{min}} = \frac{1.2 \times 1275}{1 + \frac{1700 \times 120 \times 10^3}{2 \times 1500 \times 10^3 \times 500^2}} = 1202.83 \text{ kN} \)

But
\[
\phi V_{\text{min}} = 0.75 \times 1202.83 = 902.1 \text{ kN} < N_{y^*} = 1500 \text{ kN}
\]

Thus, \((A_{sw}/s)_{\text{min}}\) is inadequate and the required amount of torsional steel, according to Equation 9.6(25), is
\[
\left( \frac{A_{sw}}{s} \right) = \frac{0.2 \times 469 \times (1500/902.1)}{500} = 0.519
\]

Equation 9.6(26): the right-hand side = \( 3 \times 902.1 \times \sqrt{350/500} = 2264.3 \text{ kN} > N_{y^*} = 1500 \text{ kN} \)

This is acceptable.

**Detailing**

Using N12 closed ties, Table 2.3(1) gives \( A_{sw} = 113 \text{ mm}^2 \). Thus, the spacing is
\[
s = \frac{113}{0.519} = 217.7 \text{ mm}
\]

![Figure 9.6(5) Reinforcement details of the torsion strip](image)

a Over a distance not less than a quarter of the respective centre to centre span of the next column

b The cover of 35 mm is inadequate for B1 exposure classification.

Note: all dimensions are in mm.
We can now take \( s = 215 \text{ mm} \), and from Equation 9.6(32) we see that \( s \) is less than the greater of \( 300 \text{ mm} \) and \( D_s = 350 \text{ mm} \). This is acceptable. The layout of the torsion strip is given in Figure 9.6(5).

### 9.7.7 Design of column and middle strips

The overall thickness of the slab has been given as \( 400 \text{ mm} \). Each of the column and middle strips can be designed as a wide beam using the restricted design process discussed in Section 3.5.2. Two equations need to be used repeatedly. They are:

\[
\xi = \frac{\alpha'_s f'_s}{f_{sy}}
\]

and

\[
p_c = \xi - \frac{2 \xi M^*}{\phi b d^2 f_{sy}}
\]

As per Equation 9.4(13)

\[
p_{c,\min} \geq \frac{0.24(D/d)^3 f_{y,\min}}{f_{sy}}
\]

For exposure classification Bl with \( f'_s = 25 \text{ MPa} \), Table 1.4(2) indicates that a clear cover of \( 60 \text{ mm} \) is required. This is greater than the \( 25 \text{ mm} \) cover required for the 90-minute fire-resistance period as given in Table 5.5.2(A) of the Standard (AS 3600-2018). Thus, using N28 bars for both the positive and negative steel, or the bottom and top reinforcement,

\[
d = 400 - 60 - 28/2 = 326 \text{ mm}
\]

Note that with \( d = 326 \text{ mm} \) for bending in the \( x-z \) plane, the corresponding \( d \) for the \( y-z \) plane has to be \( d = 326 - 28 = 298 \text{ mm} \). These are illustrated in Figure 9.7(5).

![Figure 9.7(5) Definitions of \( d \) for the top and bottom reinforcement layers in the slab](image)

Note: all dimensions are in mm.
In practice, it is economical to adopt the larger \( d \) for the plane having a larger moment in a majority of the sections. Alternatively, it is acceptable to use the average \( d \) value (312 mm in this case). In the present example, let us assume that the bending in the \( x\)-\( z \) plane is more severe. Thus we adopt

\[ d = 326 \text{ mm} \]

Note that this value is acceptable for fire-resistance considerations since Table 5.5.1 of the Standard indicates that, for a period of 90 minutes, the minimum effective or actual thickness \( (D) \) is 100 mm for insulation. For structural adequacy on the other hand, the minimum \( D \) required is 200 mm as per Table 5.5.2(A) of the Standard. Both values are smaller than the available \( D = 400 \text{ mm} \).

With \( d \) now calculated and using Equations 3.5(6) and 3.5(7), the bending reinforcement can readily be computed.

**The column strip**

\[ (b = 5500 \text{ mm}, \ d = 326 \text{ mm}) \]

**Section L**

\[ M^* = 2442.4 \text{ kNm} \]

Consider a 1-metre width of the column strip.

\[ M^* = 2442.4/5.5 = 444.1 \text{ kNm/m} \]

For \( f'_c = 25\text{MPa} \), \( \alpha_2 = 0.8125 \) and \( \xi = 0.8125 \times \frac{25}{500} = 0.0406 \).

Assuming \( \phi = 0.85 \):

\[ p_t = 0.0406 - \sqrt{0.0406^2 - \frac{2 \times 0.0406 \times 444.1 \times 10^6}{0.85 \times 1000 \times 326^2 \times 500}} = 0.01145 \]

Equation 9.7(1): \( p_{t\text{min}} = 0.0022 \)

Equation 3.4(6): \( p_{st} = 0.01475 \)

As \( p_{t\text{min}} \) \( \lessgtr \) \( p_t \) and \( p_t < p_{st} \), which is acceptable. Therefore,

\[ p_t = 0.01145 \]

and

\[ A_{st} = p_t bd = 0.01145 \times 1000 \times 326 = 3732.7 \text{ mm}^2/\text{m} \]

Provide N28 @ 150 mm top bars \( (A_{st} = 4107 \text{ mm}^2/\text{m}) \).

**Section C**

\[ M^* = 1049.4 \text{ kNm} \]

Consider a 1-metre width of the column strip.

\[ M^* = 1049.4/5.5 = 190.8 \text{ kNm/m} \]

\[ p_t = 0.0045 > p_{t\text{min}} = 0.0022 \]

\[ p_t = 0.0045 < p_{st} = 0.01475 \]
This is acceptable. Therefore,

\[ p_t = 0.0045 \]

and

\[ A_{st} = 0.0045 \times 1000 \times 326 = 1467 \text{ mm}^2/\text{m} \]

Provide N28 @ 300 mm bottom bars \((A_{st} = 2053 \text{ mm}^2/\text{m})\). Also,

Equation 9.2(7): \( s = \) the lesser of \( 2.0D \) and \( 300 \text{ mm} = 300 \text{ mm} \)

Thus, provide N28 @ 300 mm bottom bars \((A_{st} = 2053 \text{ mm}^2/\text{m})\).

**Section R**

The reinforcement is the same as Section L for reason of symmetry.

Therefore, provide N28 @ 150 mm top bars \((A_{st} = 4107 \text{ mm}^2/\text{m})\).

**Half middle strip 1 (HMS1)**

\((b = 3000 \text{ mm}, d = 326 \text{ mm})\)

**Sections L and R**

\[ M' = 305.3 \text{ kNm} \]

Consider a 1-metre width.

\[ M' = \frac{305.3}{3} = 101.77 \text{ kNm/m} \]

\[ p_t = 0.00232 > p_{\text{min}} = 0.0022 \]

\[ p_t = 0.00232 < p_{\text{all}} = 0.01475 \]

This is acceptable. Therefore,

\[ p_t = 0.00232 \]

and

\[ A_{st} = 0.00232 \times 1000 \times 326 = 756.3 \text{ mm}^2/\text{m} \]

Provide N28 @ 300 mm top bars \((A_{st} = 2053 \text{ mm}^2/\text{m})\).

**Section C**

\[ M' = 349.8 \text{ kNm} \]

Consider a 1-metre width.

\[ M' = \frac{349.8}{3} = 116.6 \text{ kNm/m} \]

\[ p_t = 0.00267 > p_{\text{min}} = 0.0022 \]

\[ p_t = 0.00267 < p_{\text{all}} = 0.01475 \]

This is acceptable. Therefore,

\[ p_t = 0.00267 \]

and

\[ A_{st} = 0.00267 \times 1000 \times 326 = 870.4 \text{ mm}^2/\text{m} \]

Provide N28 @ 300 mm bottom bars \((A_{st} = 2053 \text{ mm}^2/\text{m})\).
Half middle strip 2 (HMS2)  
\((b = 2500 \text{ mm}, d = 326 \text{ mm})\)

**Sections L and R**  
\(M^* = 305.3 \text{ kNm}\)

Consider a 1-metre width.

\[
M^* = \frac{305.3}{2.5} = 122.12 \text{ kNm/m} \\
p_t = 0.0028 > p_{\text{min}} = 0.0022 \\
p_t = 0.0028 < p_{\text{all}} = 0.01475
\]

This is acceptable. Therefore,

\(p_t = 0.0028\)

and

\[
A_e = 0.0028 \times 1000 \times 326 = 912.8 \text{ mm}^2/\text{m}
\]

Provide N28 @ 300 mm top bars \((A_{st} = 2053 \text{ mm}^2/\text{m})\).

**Section C**  
\(M^* = 349.8 \text{ kNm}\)

Consider a 1-metre width.

\[
M^* = \frac{349.8}{2.5} = 139.92 \text{ kNm/m} \\
p_t = 0.00323 > p_{\text{min}} = 0.0022 \\
p_t = 0.00323 < p_{\text{all}} = 0.01475
\]

This is acceptable. Therefore,

\(p_t = 0.00323\)

and

\[
A_e = 0.00323 \times 1000 \times 326 = 1052.98 \text{ mm}^2/\text{m}
\]

Provide N28 @ 300 mm bottom bars \((A_{st} = 2053 \text{ mm}^2/\text{m})\).

The design of the bending reinforcement is now completed. However, note that, in practice, it is acceptable to combine the total moments of the two adjacent half middle strips from the two adjoining idealised frames. Based on this combined total moment, the \(A_{st}\) for the middle strip can be computed accordingly.

It should also be pointed out that in all the bending design calculations, \(\phi = 0.85\) is assumed. This, as in Equation 3.4 (20)\(a\), requires \(k_{\text{so}} \leq 0.36\). It may be shown that this criterion is satisfied throughout.
10.3.3 Interaction diagram

For a given column section subjected to an axial load \((N_0)\) at an eccentricity \((e')\) giving an ultimate moment of \(M_0 = e'N_0\), the failure mode and strength depend upon the combined effect of \(N_0\) and \(M_0\). The interaction diagram of a column prescribes all the combinations of \(N_0\) and \(M_0\) that can cause failure to the column. The procedure in detail for constructing an interaction diagram is illustrated using the following example.

Illustrative example

Problem

A column section subjected to bending in the \(x-y\) plane is detailed in Figure 10.3(4). Construct the interaction diagram.

Take \(f' = 25\) MPa, \(f_y = 500\) MPa, \(\alpha_1 = 0.8125\), \(\gamma = 0.9075\) and \(A_{sc} = A_{st} = 2\) N32 bars @ 804 mm\(^2\) = 1608 mm\(^2\).

Solution

The following are given: \(b = 300\) mm; \(d = 450\) mm; \(d_k = 50\) mm.

Since \(A_{sc} = A_{st} = 1608\) mm\(^2\), we have \(p_t = p_e = 0.01191\).

For \(f' = 25\) MPa, \(\alpha_1 = 0.8125\) as per Equation 3.3(2)a.
Under different combinations of $N_0$ and $M_0$, the column could fail in one of three modes.

For compression failure, Equation 10.3(6) for the squash load capacity yields:

$$N_u = 0.8125 \times 0.9075 k_u \times 25 \times 300 \times 450 + 1608 \times (500 - 0.8125 \times 25) - 1608 f'_u$$

where $f'_u$ is given in Equation 3.4(13), or

$$f'_u = 600(1 - k_u) / k_u$$

We then have

$$N_u = [2.489 k_u + 0.771 - 0.965 (1 - k_u) / k_u] \times 10^3 \text{ (kN)}$$  \hspace{1cm} \text{Equation 10.3(17)}$$

The moment equation (Equation 10.3(9)) becomes

$$eN_u = [0.8125 \times 0.9075 k_u \times 25 \times 300 \times 450^2 \times (1 - 0.9075 k_u / 2) + 1608 \times (500 - 0.8125 \times 25) \times (450 - 50)] \times 10^8$$

$$= [11.20 k_u (1 - 0.454 k_u) + 3.085] \times 10^2 \text{ (kNm)}$$  \hspace{1cm} \text{Equation 10.3(18)}$$

For tension failure, Equation 10.3(18) remains valid, but for $N_0$ Equation 10.3(17) gives

$$N_u = [2.489 k_u - 0.033] \times 10^3 \text{ (kN)}$$  \hspace{1cm} \text{Equation 10.3(19)}$$

Note that either the tension or compression failure strength equations may be used for the balanced failure analysis.

With Equations 10.3(17), 10.3(18) and 10.3(19) in hand, the interaction curve can be obtained by appropriately varying the value of $k_u$. However, for compression failure $k_u > k_{uB}$; for tension failure $k_u < k_{uB}$; according to Equation 10.3(12) $k_{uB} = 0.5454$.

- $N_{u0}$ (i.e. $e' = 0$)
  Equation 10.2(1): $N_{u0} = 4727.2 \text{ kN}$

- $M_{u0}$ (i.e. $N_u = 0$)
  Equation 3.6(15)a or b: $M_{u0} = 328.3 \text{ kNm}$

- Balanced failure (i.e. $k_{uB} = 0.5454$).
  Equation 10.3(17): $N_{uB} = 1324.2 \text{ kN}$
  Equation 10.3(18): $eN_{uB} = 768.1 \text{ kNm}$

But from Figure 10.3(4),

$$M_{uB} = e'_u N_{uB} = N_{uB} (e - 0.2) = eN_{uB} - 0.2N_{uB} = 768.1 - 0.2 \times 1324.2 = 503.3 \text{ kNm}$$

- Compression failure (i.e. $k_u > k_{uB}$).
The variable $k_u$ may be given some appropriate values and $N_u$ and $M_u$ can be computed using Equations 10.3(17) and 10.3(18), as for the balanced failure case given above.

For $k_u = 1$

Equation 10.3(17): $N_u = 3260.0$ kN
Equation 10.3(18): $eN_u = 920.0$ kNm
$M_u = eN_u - 0.2N_u = 268.0$ kNm

For $k_u = 0.9$

Equation 10.3(17): $N_u = 2903.9$ kN
Equation 10.3(18): $eN_u = 904.6$ kNm
$M_u = 323.8$ kNm

For $k_u = 0.8$

Equation 10.3(17): $N_u = 2521.0$ kN
Equation 10.3(18): $eN_u = 879.1$ kNm
$M_u = 374.9$ kNm

For $k_u = 0.7$

Equation 10.3(17): $N_u = 2099.7$ kN
Equation 10.3(18): $eN_u = 843.3$ kNm
And $M_u = 423.4$ kNm

- Tension failure (i.e. $k_u < k_uB$ and $f = f_y$)

For tension failure, Equations 10.3(19) and 10.3(18), respectively, should be used for computing $N_u$ and $eN_u$. Note that in Equations 10.3(6) and 10.3(9), yielding of $A_{sc}$ is assumed. For this to be valid, $k_u$ must be greater than a certain lower limit.

Considering $k_u = 0.4$ as the limiting value,

Equation 10.3(19): $N_u = 962.6$ kN
Equation 10.3(18): $eN_u = 675.1$ kNm
$M_u = 482.6$ kNm

For $k_u = 0.5$ ($< k_uB = 0.5454$),

Equation 10.3(19): $N_u = 1211.5$ kN
Equation 10.3(18): $eN_u = 741.4$ kNm
$M_u = 499.1$ kNm

- Decompression mode

For the threshold beyond which no tensile stress exists in the section, Equation 10.3(16) gives

$$f_s = 600 \left( \frac{500}{450} - 1 \right) = 66.67 \text{ MPa}.$$
Then from Equation 10.3(15),
\[ N_{u,dc} = [0.8125 \times 0.9075 \times 25 \times 300 \times 500 + 1608(500 - 0.8125 \times 25) + 1608 \times 66.67] \times 10^{-3} \]
\[ = 3643.6 \text{kN} \]

Further,
Equation 10.3(8): \[ j_d = (450 - 0.9075 \times 500 / 2) = 223.1 \text{mm} \]
Equation 10.3(9): \[ eN_{u,dc} = [0.8125 \times 0.9075 \times 25 \times 300 \times 500 \times 223.1 + 1608(500 - 0.8125 \times 25) \times (450 - 50)] \times 10^{-4} = 925.4 \text{kNm} \]

and
\[ M_{u,dc} = eN_{u,dc} - 0.2N_{u,dc} = 925.4 - 0.2 \times 3643.6 = 196.7 \text{kNm} \]

Finally, with all the above coordinates of \( N_u \) and \( M_u \), the interaction diagram can be drawn. This is shown in Figure 10.3(5).

![Interaction diagram for the example column section](image)

**Figure 10.3(5)** Interaction diagram for the example column section

10.3.4 Approximate analysis of columns failing in compression

Illustrative example

**Problem**

For the column shown in Figure 10.3(4), calculate the approximate values of \( N_u \) and \( M_u \) for the case with \( e' = 0.0822 \text{m} \) (i.e. for \( k_u = 1 \)).

**Solution**

From the example in Section 10.3.3, we have
\[ N_{ul} = 1324.2 \text{ kN} \]
\[ M_{ul} = 503.3 \text{ kNm} \]
\[ \epsilon'_u = 0.380 \text{ m}, \]

and

\[ N_u = 4727.2 \text{ kN}. \text{ Thus} \]

Equation 10.3(22): \[ N_u = \frac{4727.2}{1 + \left( \frac{4727.2}{1324.2} \right) \frac{0.0822}{0.380}} = 3038.2 \text{ kN} \]

Equation 10.3(21): \[ M_u = \frac{4727.2 - 3038.2}{4727.2 - 1324.2} \times 503.3 = 249.8 \text{ kNm} \]

Note that the approximate formulas have underestimated the values of \( N_u \) and \( M_u \) by 6.80% and 6.79%, respectively.

### 10.4.2 Illustrative example of iterative approach

**Problem**

An irregular-shaped column section is shown in Figure 10.4(3). Compute \( N_u \) using the iterative procedure. Take \( f'c = 32 \text{ MPa} \) and \( f_{sy} = 500 \text{ MPa} \).

![Figure 10.4(3)](image)

**Solution**

Equation 3.3(2a): \[ \alpha_2 = 0.85 - 0.0015 \times 32 = 0.802, \] but as per AS 3600-2018, for sections where the width reduces from the neutral axis towards the compression face, \( \alpha_2 \) shall be reduced by 10%.

Hence, in this case \[ \alpha_2 = 0.9 \times 0.802 = 0.7218 \]

Equation 3.3(2b): \[ \gamma = 0.97 - 0.0025 \times 32 = 0.89 \]

And there are two layers of steel or \( m = 2 \).
**Trial 1**

**Step 1**
Assume $d_{NA} = 240$ mm.

**Step 2**
For the top steel layer 2 (or $A_{sm}$ with $m = 2$ in this case)

$$\varepsilon_{s2} = 0.003 \times (240 - 60)/240 = 0.00225 < \varepsilon_y = 0.0025$$

that is, $f_{s2} = 200000\varepsilon_{s2} = 450$ MPa

$$T_2 = 450 \times 1232 \times 10^{-3} = 554.4 \text{ kN}$$

For the bottom steel layer 1 (or $A_{s1}$),

$$\varepsilon_{s1} = 0.003 \times (240 - 300)/240 = -0.00075 \text{ (tension)}$$

and

$$f_{s1} = 200000\varepsilon_{s1} = -150 \text{ MPa}$$

$$T_1 = -150 \times 3164 \times 10^{-3} = -474.6 \text{ kN (tension)}$$

**Step 3**
The height of the concrete area in compression as shown in Figure 10.4(4) is given as

$$\gamma d_{NA} = 0.89 \times 240 = 213.6 \text{ mm}$$

and

$$c = (213.6/240) \times 120 = 106.8 \text{ mm}$$

![Figure 10.4(4)](image)

**Figure 10.4(4)** Determination of neutral axis (NA) for the irregular-shaped column section

Note: all dimensions are in mm.

Thus, the concrete area

$$A' = 120 \times 213.6 + 2 \times 0.5 \times 213.6 \times 106.8 = 25,632 + 22,812.5 = 48,444.5 \text{ mm}^2$$

Using Equation 10.4(4),

$$d_c = (25,632 \times 213.6/2 + 22,812.5 \times 213.6 \times 2/3)/48,444.5 = 123.6 \text{ mm}$$

Equation 10.4(2): $C_c = 0.7218 \times 32 \times 48,444.5 \times 10^{-3} = 1119.0 \text{ kN}$
Step 4

Equation 10.4(5): \( N_u = 1119.0 + 554.4 - 474.6 = 1198.8 \text{ kN} \)
Equation 10.4(6): \( e_{\text{computed}} = \left[ 1119.0 \times (300 - 123.6) + 554.4 \times (300 - 60) \right] / 1198.8 \)
\[ = 275.6 \text{ mm} \]

Thus, the eccentricity
\( e'_{\text{computed}} = 275.6 - (300 - 218) = 193.6 \text{ mm} \)

Step 5
\( e'_{\text{computed}} < e'_{\text{given}} \ (= 380 \text{ mm}) \)

Thus, reduce \( d_{NA} \) and repeat the process.

Trial 2
Step 1
Assume \( d_{NA} = 210 \text{ mm} \).

Step 2
\( e_{\text{max}} = 0.00214 < e_{\text{max}} \), therefore
\( T_2 = 527.3 \text{ kN} \)
\( e_{\text{min}} = -0.001286 \), therefore
\( T_1 = -813.8 \text{ kN} \)

Step 3
\( \gamma d_{NA} = 186.9 \text{ mm} \)

Accordingly,
\( A' = 39893.8 \text{ mm}^2 \)
and \( d_e = 107.1 \text{ mm} \)
Thus, \( C_e = 921.5 \text{ kN} \)

Step 4
\( N_u = 921.5 + 527.3 - 813.8 = 635.0 \text{ kN} \), and
\( e'_{\text{computed}} = 479.2 \text{ mm} \), therefore
\( e'_{\text{computed}} = 397.2 \text{ mm} \)

Step 5
\( e'_{\text{computed}} > e'_{\text{given}} \)

but the value is getting larger. Thus, increase \( d_{NA} \) to 211.5 mm and try again.

Trial 3
Step 1
\( d_{NA} = 211.5 \text{ mm} \)
Step 2
\[ T_2 = 529.8 \text{ kN} \]
\[ T_1 = -797.3 \text{ kN} \]

Step 3
\[ C_c = 930.7 \text{ kN} \]

Step 4
\[ N_u = 663.2 \text{ kN}, \text{ and} \]
\[ e' \text{ computed} = 461.3 \text{ mm} \]
Or, \[ e'' \text{ computed} = 379.3 \text{ mm} \]

Step 5
\[ e' \text{ computed} \approx e'' \text{ given} \]
Therefore, we take \( N_u = 663.2 \text{ kN} \).

10.4.4 Illustrative example of semi-graphical method

Problem
A rectangular column section, reinforced with four layers of steel (a total of twelve 20-mm diameter bars) is subjected to uniaxial bending in the \( x \)-\( y \) plane. Details of the section are shown in Figure 10.4(6). Take \( f'_c = 32 \text{ MPa}, \ f_y = 500 \text{ MPa}, \ \alpha_2 = 0.802 \) and \( \gamma = 0.89 \). Determine \( N_u \) using the semi-graphical method.

![Figure 10.4(6)](image)
Note: all dimensions are in mm.
Solution

There are four layers of steel (or \( m = 4 \)) and for the symmetrical section, the position of the plastic centre \( d_{pc} = 225 \) mm.

The scaled drawing of the cross-section including the reinforcement positions is given in Figure 10.4(7), together with the steel stress diagram. Note that for the rectangular section \( b_h = 360 \) mm, which is constant, there is no need to draw the entire cross-section. The drawing of the full section is mandatory for an irregular section.

Figure 10.4(7) Scaled drawing on graph paper for the semi-graphical method example
The working is given in detail in Table 10.4(1). For Trial 1, assume $d_{iA} = 200$ mm. Then the two sums in Equation 10.4(9) are given, respectively, in columns 9 and 6 of Table 10.4(1). We have $N_0 = 1596.9 - 210.3 = 1386.6$ kN.

### Table 10.4(1)

Calculations for semi-graphical method example

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<tr>
<th>1</th>
<th>2</th>
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<td>Element</td>
<td>$A_i$</td>
<td>$f_{iA}$</td>
<td>$(d_{pc} - d_i)$</td>
<td>$A_i f_{iA}$</td>
<td>$\Delta A_{ij}$</td>
<td>$(d_{pc} - x_j)$</td>
<td>$\alpha_j f_{iA} \Delta A_{ij}$</td>
<td>$\Delta (e^* N_0)$</td>
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<td>225 - 390 = -165</td>
<td>-628 kN</td>
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<td>103.6 kNm</td>
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<td>420.1</td>
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</table>

Equation 10.4(10) is represented by column 10 of Table 10.4(1) and the sum $e'N_0 = 417.7$ kNm.

Thus

\[ e'_{\text{computed}} = \frac{417.7}{1386.6} = 0.30 \text{ m}, \] which is greater than $e'_{\text{given}} (= 0.25 \text{ m}).$

For Trial 2, $d_{\text{HA}} = 225 \text{ mm.}$ This gives $N_0 = 1800.2 - 3.0 = 1797.2 \text{ kN},$ and

\[ e'_{\text{computed}} = \frac{417.8}{1797.2} = 0.23 \text{ m}, \] which is less than $e'_{\text{given}}.$

For Trial 3, $d_{\text{HA}} = 215 \text{ mm.}$ This gives $N_0 = 1717.0 - 103.4 = 1613.6 \text{ kN},$ and

\[ e'_{\text{computed}} = \frac{420.1}{1613.6} = 0.26 \text{ m}, \] which is also greater than $e'_{\text{given}} = 0.25 \text{ m}.$

Thus, take $N_0 = \text{greater than } 1613.6 \text{ kN, say } 1620 \text{ kN}.$
10.6.2 Illustrative example

Problem
Given \( M_g = 34 \text{kNm} \), \( M_q = 38.3 \text{kNm} \); \( N_g = 340 \text{kN} \), \( N_q = 220 \text{kN} \) where subscripts \( g \) and \( q \), respectively indicate dead- and live-load effects. Assume \( f' = 32 \text{MPa} \) and \( f_y = 500 \text{MPa} \), and proportion a symmetrically reinforced square section.

Solution
By invoking Equation 1.3(2), we have
\[
M^* = 1.2 \times 34 + 1.5 \times 38.3 = 98.25 \text{kNm}
\]
and
\[
N^* = 1.2 \times 340 + 1.5 \times 220 = 738 \text{kN}
\]
1. Assuming \( p = p_c = 0.008 \), Equation 10.6(1) gives
\[
\frac{bD}{(738 \times 10^3) / [0.65 \times (0.802 \times 32 + 500 \times 0.016) = 33.727.0 \text{mm}^2}
\]
from which \( b = D = 184 \text{mm} \).
2. Use \( b = D = 225 \text{mm}, \text{say} \).
3. Equation 10.6(3) leads to
\[
\frac{b^2}{d} = (98.25 \times 10^3) / [0.85 \times (0.9 \times 500 \times 0.008)] = 34.11 \times 10^4 \text{mm}^4
\]
Take \( b = 1.1 \), \( d = 315 \). This means
\[
b = D = 0.9 \times 1.1 \times 315 = 312 \text{mm} \approx 320 \text{mm, say}.
\]
4. Thus, the details of the preliminary column section are
\[
b = D = 320 \text{mm} \text{ and}
\]
\[
A_{st} = A_{sc} = 0.008 \times 0.9 \times 320^2 = 738 \text{mm}^2
\]
or use 4 N24 bars, one each at the four corners, giving a total of 1808 mm², which is greater than \( 2 \times 738 = 1476 \text{mm}^2 \).
11.6 Illustrative examples

11.6.1 Example 1 – load-bearing wall

Problem
A rectangular wall is 300 mm thick with an unsupported height of 6 m and a length of 5 m. The vertical axial loading from the floor beams above acts with a 10-mm eccentricity. Assuming that the wall is restrained against rotation at the top and bottom ends by the floors, and that $f' = 32$ MPa and $f_{sy} = 500$ MPa, compute the design axial strength values for the following side-restraint conditions:

(a) without side restraints using the Standard procedure
(b) without side restraints using the ACI 318-2014 formula
(c) with restraints at both sides of the wall.

Solution

(a) Without side restraint – Standard procedure
Equation 11.3(1) is applicable in this case, for which $H_{we} = kH_w = 0.75 \times 6 = 4.5$ m, which is less than the wall length of 5 m.
Hence, adopt $H_{we} = 4.5$ m.
Since the eccentricity $e = 10$ mm $< 0.05tw = 0.05 \times 300 = 15$ mm, use $e = 15$ mm.
The slenderness ratio $H_{we}/tw = 4500/300 = 15 < 30$, which is acceptable.
Equation 11.3(2): $e_3 = (H_{we})^2/2500tw = (4500)^2/(2500 \times 300) = 27$ mm.
With these values:
Equation 11.3(1): $\phi N_0 = \phi (tw - 1.2e - 2e_a)0.6f'$
$= 0.65 [300 - 1.2(15) - 2(27)] \times 0.6 \times 32 \times 5000 \times 10^{-3}$
$= 14 227$ kN

(b) Without side restraint – ACI 318-2014 procedure
Apply Equation 11.3(6) for which
$L/tw = 5000/300 = 16.7 < 25$, which is acceptable.
$t_w > 100$, which is acceptable.

$k = 0.8$ as the top and bottom ends are restrained against rotation.

Thus, Equation 11.3(6):

$$\phi N_w = \phi 0.55 f'c' A_k [1 - (kH/32t_w)^2]$$

$$= 0.7 \times 0.55 \times 32 \times (300 \times 5000) \times [1 - \{(0.8 \times 6000)/(32 \times 300)\}^2] \times 10^{-3}$$

$$= 13\,860 \text{kN}$$

Note that Equation 11.3(6) gives a 2.6% less capacity than Equation 11.3(1).

This is because the ACI 318-2014 equation does not consider the effect of eccentricity, or the additional eccentricity due to the secondary $P-\Delta$ effects.

(c) With both sides restrained laterally

The Standard's simplified method in the form of Equation 11.3(1) is also applicable when both sides are restrained laterally. The solution is obtained by using the appropriate input parameters for the wall in two-way action.

Since $H_w > L_1,$

Equation 11.3(5):

$$k = \frac{L_1}{2H_w} = \frac{5000}{2 \times 6000} = 0.4167$$

That is, $H_{we} = kH_w = 0.4167 \times 6 = 2.5 \text{ m}$

Similarly, the eccentricity $e = 10 \text{ mm} < 0.05 t_w = 0.05 \times 300 = 15 \text{ mm}$.

Therefore, adopt $e = 15 \text{ mm}$.

Equation 11.3(2):

$$e_w = (H_{we})^2/2500 t_w = (2500)^2/(2500 \times 300) = 8.3333 \text{ mm}$$

Then

Equation 11.3(1):

$$\phi N_w = \phi (t_w - 1.2e - 2e_w) 0.6 f'c'$$

$$= 0.65 [300 - 1.2(15) - 2(8.3333)] \times 0.6 \times 32 \times 5000 \times 10^{-3}$$

$$= 16\,557 \text{kN}$$

Comparing the results in cases (a) and (c) indicates that, with both sides of the wall restrained laterally, the design axial strength increases by $(16\,557 - 14\,227)/14\,227 \times 100\% = 16.4\%$.

11.6.2 Example 2 – tilt-up panel

Problem

A two-storey office building is to be constructed as an assembly of tilt-up panels. The (continuous) external panels are each 7.5 m high by 5 m long. They are designed to support the loading from the first floor as well as the roof. The first floor is 3.6 m above
the base slab; the roof is a further 3.6 m above the first floor. The dead and live loads from the roof are 45 kN/m and 40.5 kN/m, respectively, along the wall. The dead and live loads from the first floor are 76.5 kN/m and 66 kN/m, respectively. Assume $f'_c = 32$ MPa and that the floor slab bearing into the panels is 25 mm.

Is a 150-mm thick concrete wall with minimum vertical and horizontal reinforcement adequate for the design?

**Solution**

(a) Design at 1.8 m (mid-height of wall between base and first floor)

Assuming $\rho_w = 24$ kN/m$^3$, wall dead load = $24 \times (7.5 - 1.8) \times 0.15 = 20.52$ kN/m

Total dead load $g =$ self-weight + roof dead load + floor dead load

$= 20.52 + 45 + 76.5 = 142.02$ kN/m

Total live load $q =$ roof live load + floor live load = $40.5 + 66 = 106.5$ kN/m

Design load = $(1.2 \times g) + (1.5 \times q) = 330.17$ kN/m

(b) Eccentricity of load at 1.8 m height

Vertical load of first floor = $(1.2 \times 76.5) + (1.5 \times 66) = 190.8$ kN/m

As in Clause 11.5.4 of the Standard, vertical load is assumed to act at one-third the depth of the bearing area from the span face of the wall. Thus, the eccentricity of the load above the first floor

$e = (150/2) - (25/3) = 66.667$ mm $> e_{\text{min}} = 0.05t = 7.5$ mm, which is acceptable.

At 1.8 m, the eccentricity of load, $e = 66.667 \times 190.8/330.17 = 38.525$ mm.

Also,

Equation 11.3(2): $e_i = (0.75 \times 3600)/(2500 \times 150) = 19.44$ mm

Equation 11.3(1): $N_u = 0.65[150 – 1.2(38.525) – 2(19.44)] \times 0.6 \times 32$

$= 809.83$ kN/m $> 330.17$ kN/m, which is satisfactory.

### 11.6.3 Example 3 – the new strength formula

**Problem**

Repeat the example in Section 11.6.1 using the new design formula for a wall

(a) without side restraint
Solution

(a) Design strength without side restraint.

Since $H/t_w < 27$

Equation 11.3(10): $\beta = 1$. Hence,
Equation 11.3(9): $H_{we} = \beta H = 6$ m and
Equation 11.3(8): $e_a = (6000)^2/(2500 \times 300) = 48$ mm.

Thus

Equation 11.3(7): $\phi N_a = 0.6 \times 2 \times 32^0.7[300 - 1.2(10) - 2(48)] \times 10^{-3}$

$= 13033$ kN

(b) Design strength with all sides restrained.

For $H/t_w < 27$

Equation 11.3(14): $\alpha = \frac{1 - \frac{e}{t_w}}{1 - \frac{10}{300}} = 1.0345$

Hence, for $H = 6$ m > $L = 5$ m

Equation 11.3(13): $\beta = \alpha \frac{L}{2H} = 1.0345 \frac{5000}{2 \times 6000} = 0.4310$

Thus $H_{we} = 0.4310 \times 6000 = 2586$ mm

and

Equation 11.3(2): $e_a = (2586)^2/(2500 \times 300) = 8.917$ mm

Finally,

Equation 11.3(7): $\phi N_a = 0.6 \times 2 \times 32^0.7[300 - 1.2(10) - 2(8.917)] \times 10^{-3}$

$= 18340$ kN

It should be noted that the new formula yields higher load-carrying capacities. In particular, when side restraints are considered, the capacity is about 29% greater than that allowed for a one-way-action wall. This shows that Equation 11.3(1) is unduly conservative when applied to walls with lateral restraints at all the four sides.

11.6.4 Example 4 – design shear strength

Problem

A reinforced concrete wall is required to resist vertical and horizontal in-plane forces in a multistorey building. The overall first-floor height and length of the wall are both 4.5
The wall is 250 mm thick with two 500 mm × 500 mm boundary columns. A structural analysis of the building gives \( N^* = 6925 \text{ kN} \), \( V^* = 3750 \text{ kN} \) and the in-plane moment \( M^* = 19300 \text{ kNm} \).

Compute the shear strength of the wall and provide suitable reinforcement. Assume \( f'_c = 50 \text{ MPa} \) and \( f_{yy} = 500 \text{ MPa} \). Compare the strength result with the ACI 318-2014 provisions.

**Solution**

The boundary columns serve to resist the in-plane bending moment in the wall, as well as the axial compression. The shear force due to horizontal load on the other hand is carried by the wall.

(a) Check the maximum shear capacity.

Equation 11.4(2): \( V_{u,max} = 0.2 \times 50 (0.8 \times 4500 \times 250) \times 10^{-3} = 9000 \text{ kN} \)

And \( \phi V_{u,max} = 0.75 \times 9000 = 6750 \text{ kN} > V^* \), which is acceptable.

(b) Calculate the shear strength of the wall.

**Trial 1**

Use N16 bars @ 200 mm in each face of the wall in both the vertical and horizontal directions.

**Equation 11.4(3):**

\[
V_{uc} = [0.66 \sqrt{50} - 0.21(4.5/4.5) \sqrt{50}] 0.8 \times 4500 \times 250 \times 10^{-3} = 2863.8 \text{ kN} > 0.17 \sqrt{f'_c} (0.8 L_c t_w), \text{ which is acceptable.}
\]

The steel ratio, \( \rho_w = 201/(200 \times 250) = 0.004 \)

**Equation 11.4(6):** \( V_{us} = 0.004 \times 500 \times (0.8 \times 4500 \times 250) \times 10^{-3} = 1800 \text{ kN} \)

Then

**Equation 11.4(1):**

\( \phi V_u = 0.75 (2863.8 + 1800) = 3497.9 \text{ kN} < 3750 \text{ kN} \), which is not acceptable.

**Trial 2**

Use N20 bars @ 220 mm.

The steel ratio, \( \rho_w = 314/(220 \times 250) = 0.00571 \)

**Equation 11.4(6):**
\[ V_{ut} = 0.00571 \times 500 \times 0.8 \times 4500 \times 250 \times 10^{-3} = 2569.5 \text{kN} \]

**Equation 11.4(1):**
\[ \phi V_u = 0.75(2863.8 + 2569.5) = 4075.0 \text{kN} > 3750 \text{kN}, \text{which is acceptable.} \]

(c) **Calculate the shear strength using the ACI 318-2014 recommendations.**

Equation 11.4(8): \[ V_{us} = 0.275 \sqrt{50(0.8 \times 4500 \times 250)\times 10^{-3} + 0.2 \times 6925} \]
\[ = 3135.1 \text{kN} \]
Equation 11.4(9): \[ V_{us} = 3028 \text{kN} < 3135.1 \text{kN}. \text{Thus, adopt this value.} \]

Then
Equation 11.4(10): \[ V_{us} = 314 \times 500 \times (0.8 \times 4500)\times 10^{-3}/220 \]
\[ = 2569.1 \text{kN} \]

Finally, Equation 11.4(1):
\[ \phi V_u = 0.75(3028 + 2569.1) = 4197.8 \text{kN} > 3750 \text{kN}, \text{which is acceptable.} \]
Note that the ACI method gives a higher capacity load. This is to be expected as it also accounts for the beneficial effects of axial loading.

(d) **Check related design requirements.**

The wall panel should be checked for adequate crack control in both vertical and horizontal directions, each having a reinforcement ratio of 0.00571. As quoted in Section 11.5, this ratio is more than adequate for a moderate degree of control, which requires a minimum \( \rho_w > 0.0035 \) in exposure classification A1 or A2. It is just short of the required 0.006 for a strong degree of crack control.

Finally, the flexural strength of the wall must also be checked. This is done by adopting the rectangular stress block for the compression zone and applying the general column analysis given in Chapter 10. The moment capacity of the section \( \phi M_u \) must be greater than or equal to \( M^* \).
CHAPTER 12

12.2.5 Design example

The example given in this section is for an asymmetrical footing under eccentric loading. It demonstrates the use of the equations and process developed in Section 12.2.2 for symmetrical wall footings under eccentric loading. However, as discussed in Sections 12.2.3 and 12.2.4, concentrically loaded footings and asymmetrical footings under eccentric loading may be treated as special cases of a symmetrical footing under eccentric loading. Thus, the example below also illustrates the design of the other two types of wall footings.

Numerical example

Problem

Given a reinforced concrete wall, 300 mm thick and subjected to a dead load $DL = 200$ kN/m and a live load $LL = 150$ kN/m, each having the same eccentricity of $e = 100$ mm from the centre plane of the wall. Take $f'_c = 25$ MPa, the effective soil bearing capacity $q_f = 250$ kPa; A2 exposure classification applies. Design an asymmetrical (strip) footing in such a way that the subsoil pressure is uniformly distributed. Use N16 or N20 bars in one layer and use no shear reinforcement.

Solution

Details of the footing of a 1-metre run are depicted in Figure 12.2(5). Note that the clear cover of 30 mm is in accordance with the recommendation given in Table 1.4(2). This assumes that the base of the footing is well prepared and compacted following excavation and before casting the concrete. If this cannot be assured, a larger cover ought to be used.

The design action per metre run (i.e. $b = 1000$ mm) is

$N^* = 1.2 \times 200 + 1.5 \times 150 = 465$ kN

$M^* = 0$ at the centre line of the asymmetrical footing; see Figure 12.2(5).

The breadth of the footing as per Equation 12.2(10) and with an assumed $\rho_e = 24$ kN/m³

$$L = \frac{465}{250 - 1.2 \times 24D} = \frac{465}{250 - 28.8D}$$

Equation (a)
With Equation (a) in hand, the footing design may follow the trial and error process detailed in Section 12.2.2.

Figure 12.2(5) Details of the design asymmetrical wall footing

**Trial 1**

Assume \( D = 600 \text{ mm} \), and Equation (a) gives

\[
L = \frac{465}{250 - 28.8 \times 0.6} = 1.998 \text{ m}
\]

Use \( L = 2.0 \text{ m} \).

**Step 1**

For N20 bars with exposure classification A2,

\[
d_o = 600 - 30 - 20/2 = 560 \text{ mm}
\]

**Step 2**

The design shear force per metre run is

\[
V^* = \left( \frac{L}{2} + 0.1 - 0.15 - d_o \right) \times (q_f - 0.9 \times 24D)
= (1.0 + 0.1 - 0.15 - 0.56) \times (250 - 0.9 \times 24 \times 0.6) = 92.45 \text{ kN}
\]

Note that a load factor of 0.9 is applied to the self-weight of the footing to produce a more critical \( V^* \).
Step 3
Equation 6.3(1) gives
\[ V_{u,\text{max}} = 0.55 \left[ f'_c b_u d_c \left( \frac{\cot \theta_v}{1 + \cot^2 \theta_v} \right) \right] \]
where using the simplified method of Clause 8.2.4.3, AS 3600-2018, \( \theta_v = 36^\circ \) and, as for N20 bars in one layer, \( d = d_0 = 560 \text{ mm} \)
\[ d_c = [0.72D, 0.9d]_{\text{max}} = [(0.72 \times 600), (0.9 \times 560)]_{\text{max}} = [432, 504]_{\text{max}} = 504 \text{ mm} \]
Thus,
\[ V_{u,\text{max}} = 0.55 \left[ 25 \times 1000 \times 504 \times \left( \frac{\cot 36^\circ}{1 + \cot^2 36^\circ} \right) \right] \times 10^{-3} = 3295.4 \text{ kN} \]
and
\[ \phi V_{u,\text{max}} = 0.75 \times 3295.4 = 2471.6 \text{ kN} > V^* \]
The assumed section with \( D = 600 \text{ mm} \) is more than adequate (i.e. it is reinforceable). Thus, this step will be omitted in subsequent trials.

Step 4
Equation 6.3(4): \( V_u = k_c b_u d_c \sqrt{f_c} \) where \( \sqrt{f_c} = \sqrt{25} = 5 \text{ MPa} < 8 \text{ MPa} \) which is acceptable
and \[ k_c = \left[ \frac{200}{1000 + 1.3d_c} \right] = \left[ \frac{200}{1000 + 1.3 \times 504} \right] = 0.12 > 0.10 \text{ ; hence, take } k_c = 0.10 \]
Finally, Equation 6.3(4) gives
\[ V_u = 0.10 \times 1000 \times 504 \times \sqrt{25} \times 10^{-3} = 252 \text{ kN} \]

Step 5
The specification was that shear reinforcement is not to be used. Thus, check that
\[ 0.75 \times 252 = 189 \text{ kN} > V^* = 92.45 \text{ kN} \]
Thus, \( D = 600 \text{ mm} \) is adequate but too conservative.

Trial 2
Assume \( D = 550 \text{ mm} \), and Equation (a) gives
\[ L = \frac{465}{250 - 28.8 \times 0.55} = 1.986 \text{ m} \]
Use \( L = 2.0 \text{ m} \).

Step 1
\[ d_n = 550 - 30 - 20/2 = 510 \text{ mm} \]
**Step 2**
\[ V^* = (1.0 + 0.1 - 0.15 - 0.51) \times (250 - 0.9 \times 24 \times 0.55) = 104.77 \text{ kN} \]

**Step 3**
As explained in step 3 in trial 1, this step is skipped here.

**Step 4**
For Equation 6.3(4), \( d_i = 459 \text{ mm} \) and \( k_v = 0.10 \). Thus Equation 6.3(4):
\[ V_{uc} = 0.10 \times 1000 \times 459 \times \sqrt{25} \times 10^{-3} = 229.5 \text{ kN} \]

**Step 5**
As \( \phi V_{uc} = 0.75 \times 229.5 = 172.1 \text{ kN} > V^* = 104.77 \text{ kN} \),
\( D = 550 \text{ mm} \) is acceptable. Proceed to the bending steel design in step 9, skipping steps 6–8 which are no longer required for this example.

**Step 9**
Design for \( A_{st} \).

For a single layer of N20 bars, \( d = d_0 = 510 \text{ mm} \) and

Equation 3.3(2)a: \( \alpha_c = 0.8125 \)
Equation 3.3(2)b: \( \gamma = 0.9075 \)

According to Figure 9.4(2) and for the wall-footing layout as shown in Figure 12.2(6), the location of the critical bending section is at \( L_b \) from the right edge where
\[ L_b = \frac{L}{2} + e - 0.7a_i \]

where \( e \) is the eccentricity of the loading and \( a_i \) is half the wall thickness.

Thus,
\[ L_b = 1 + 0.1 - 0.7 \times 0.15 = 0.995 \text{ mm} \]

With \( \rho_w = 24 \text{ kN/m}^3 \) and
\[ M^* = (250 - 24 \times 0.55^2) \times 0.995^2 / 2 = 117.2 \text{ kNm} \]

Equation 3.5(7): \( \xi = \frac{0.8125 \times 25}{500} = 0.0406 \)

Assuming \( \phi = 0.85 \), Equation 3.5(6) gives
\[ \rho_i = 0.0406 - \frac{0.0406^2 - 2 \times 0.0406 \times 117.2 \times 10^5}{0.85 \times 1000 \times 510^2 \times 500} = 0.001075 \]

Check that the assumed \( \phi = 0.85 \) is correct.
\[ k_i = k_{W0} = \frac{0.001075 \times 500}{0.8125 \times 0.9075 \times 25} = 0.029 \text{ from Equation 3.4(8)} \]
and then Equations 3.4(20)a with $b \phi = 0.85$. Therefore, the assumption is correct.

Figure 12.2(6) Defining critical bending section for the trial footing

Now, similar to one-way slabs, as wall footings may be designed as beams of a unit width, Equation 3.5(5):

$$p_{t, \min} = 0.20 \times (550 / 510)^2 \times (0.6 \times \sqrt{25}) / 500 = 0.001396 > p_t$$

Therefore, $p_t = 0.001396$.

**Step 10**

For a 1-metre run of the footing

$$A_{st} = 0.001396 \times 1000 \times 510 = 712.0 \text{ mm}^2 / \text{m}$$

Using Table 2.3(2), N20 bars @ 300 mm yield $A_{st} = 1047 \text{ mm}^2 / \text{m} > 712.0 \text{ mm}^2 / \text{m}$. This is acceptable.

Also, for crack control purposes in this case, the maximum spacing is $s = 300 \text{ mm}$ as shown in Table 1.4(5). Therefore, use N16 bars and from Table 2.3(2), N16 @ 275 mm yields $A_{st} = 731 \text{ mm}^2 / \text{m} > 712.0 \text{ mm}^2 / \text{m}$. This is acceptable.

With the reduced bar size, $d$ is increased and the stress development length is shortened accordingly. In consequence, no extra check is necessary for this bar size variation.
**Step 11**
Check the bond.

For Equation 8.2(1)

\[
\begin{align*}
    k_1 &= 1.3; \\
    k_2 &= (132 - 16)/100 = 1.16; \\
    k_3 &= 1 - 0.15(32 - 16)/16 = 0.85
\end{align*}
\]

Equation 8.2(1), thus gives

\[
L_{sy,t} = (0.5 \times 1.3 \times 0.85 \times 500 \times 16)/(1.16 \times \sqrt{25}) = 762 \text{ mm}
\]

Since the available \( L_{sy,t} < (L_b - \text{cover}) = 0.995 - 0.03 = 0.965 \) m on the right of the critical bending section, and \( L_{sy,t} < (L - L_b - \text{cover}) = 0.975 \) m on the left of the critical bending section, straight bars (without hooks or cogs) of length \((2000 - 2 \times 30) = 1940 \) mm @ \(275 \) mm are adequate for bond.

In view of Equation 8.2(9), or the fact that the actual tensile stress in \( A_{st} \) or \( \sigma_{st} \ll \sigma_y \), the bond strength is more than adequate, thereby rendering step 11 superfluous.

**Step 12**
Provide shrinkage and temperature steel.

Equation 9.4(14) gives

\[
A_{s,\text{min}} = 1.75 \times 1000 \times 550 \times 10^{-3} = 962.5 \text{ mm}^2
\]

Use five N16, \( A_s = 1005 \text{ mm}^2 \). This is acceptable. Note that \( s \leq 300 \) mm does not apply to shrinkage and temperature steel.

The final design is detailed in Figure 12.2(7).

![Wall Design](image-url)

**Figure 12.2(7)** Details of the final design of the asymmetrical wall footing
**Comments**

For $p_t = p_{t,\text{min}} = 0.001396$ in step 9 and $p_{\text{min}} = 0.00175$ in step 12, the steel content of the footing is 0.315%. Then, from Equation 2.4(1), we have

$$\rho_w = 24 + 0.6 \times 0.315 = 24.189 \text{ kN/m}^3$$

This review indicates that for the (lightly reinforced) footings, Equation 2.4(1) may be ignored for simplification and $\rho_w$ taken as 24.0 kN/m$^3$.

**12.3.3 Eccentric loading**

For an eccentrically loaded column footing, the design may follow the approach described in Section 12.2.4 in which the footing dimensions ($b_f$ and $D_f$) can be proportioned in such a way to produce a uniform pressure in the subsoil.

**Illustrative example**

**Problem**

Figure 12.3(2) details a rectangular column ($b_c \times D_c$) subjected to a combination of $N^*$ and $M^*$. Proportion a suitable rectangular footing ($b_f \times D_f$) such that the uniform subsoil pressure is equal to the effective soil bearing capacity ($q_f$). Discuss the procedure for the complete design.

![Figure 12.3(2)](Footing for eccentric column loading)
Solution

For the given $N^*$ and $M^*$, the eccentricity of the equivalent $N^*$ is

$$ e = \frac{M^*}{N^*} \tag{Equation 12.3(8)} $$

The bearing area of the footing is

$$ b_tD_t = \frac{N^*}{q_t - 1.2\rho_w D} \tag{Equation 12.3(9)} $$

where $\rho_w$ is the unit weight of the footing.

But

$$ \frac{b_t}{D_t} = \frac{b_c}{D_c} $$

or

$$ b_t = \frac{b_c}{D_c}D_t \tag{Equation 12.3(10)} $$

Substituting Equation 12.3(10) into Equation 12.3(9) leads to

$$ \frac{b_c}{D_c}D_t = \frac{N^*}{q_t - 1.2\rho_w D} \tag{Equation 12.3(11)} $$

from which

$$ D_t = \sqrt{\frac{D_cN^*}{b_c(q_t - 1.2\rho_w D)}} \tag{Equation 12.3(12)} $$

Equation 12.3(10) then yields $b_c$.

With Equations 12.3(8), 12.3(10) and 12.3(12) in hand, the plan dimensions and layout of the footing, the effective shear depth of the footing $d_v$, and the bending reinforcement can be obtained following an iterative process similar to that described in Section 12.2.5 for wall footings. In view of shear being the dominant action and most column footing dimensions resembling those of a rectangular beam, Equation 6.3(4) may also apply

$$ v_{wc} = k_v b_v d_v \sqrt{f_c'} \tag{Equation 12.3(13)} $$

The critical transverse shear section is located in the longer side of the footing and in this case, the right side. When deriving Equation 12.3(12), the self-weight of the
footing, or \(D\) in Figure 12.3(2), is unknown. Therefore, an iterative process is needed to ensure that \(q_f\) is not exceeded. This may be done by ignoring the self-weight in computing \(b_f\) and \(D_f\), but suitably increasing their values before proceeding to obtain \(d_f\), and then checking Equation 12.3(9). Revise as necessary until either \(q_f\) is not exceeded or \(b_fD_f\) is adequate. Also, a final check for punching shear strength is required by treating \(N^*\) as the design load from an edge column. The process given in Sections 9.6.3–9.6.5 may be used.

\[\text{Figure 12.3(3)}\] Asymmetrical footings for eccentrically loaded columns, either (a) T-shaped or (b) trapezoidal
It should be recognised that, on certain construction sites, the length $L$ may be restricted (e.g. where the footings are close to the boundary lines of the property). In such cases, footings with a stepped or trapezoidal plan shape may be mandatory. Such irregular plan shapes, as shown in Figure 12.3(3), can readily be obtained by ensuring that the geometric centre of the plan ($cg$) has the same eccentricity $e$ from the column axis 0, where $e = M^e/N^e$.

### 12.3.7 Design example

**Problem**

Given a rectangular column with $D_c = 500$ mm and $b_c = 300$ mm. The design actions $N^e = 1400$ kN and $M^e = 200$ kNm; the effective soil bearing capacity $q_f = 300$ kPa; $f' = 20$ MPa; and exposure classification A1 applies.

Design an asymmetrical (pad) footing in such a way that the subsoil pressure is uniformly distributed. Use N20 bars for bending in one layer each way and provide shear reinforcement as required.

**Solution**

The design specifications are illustrated in Figure 12.3(6). The eccentricity $e = M^e/N^e = 142.9$ mm as per Equation 12.3(8). Note that, as cautioned in Section 12.2.5, the concrete cover must be increased unless the base of the footing is well levelled and compacted.

Using Equation 12.3(12) and assuming $\rho_w = 24$ kN/m$^3$

\[ D_t = \left( \frac{0.5 \times 1400}{0.3(300 - 1.2 \times 24 \times D)} \right)^{\frac{1}{2}} = \left( \frac{700}{90 - 8.64D} \right)^{\frac{1}{2}} \]  

Equation (a)

and

Equation 12.3(10): $b_t = 0.6D_t$  

Equation (b)

With Equations (a) and (b) in hand, follow the relevant steps given in Section 12.2.2, which would lead to the required design.
**Figure 12.3(6)** Design specifications for the example asymmetrical pad footing

**Trial 1**

**Step 1**
Assume \( D = 1000 \) mm and:

Equation (a): \( D_f = 2.933 \) m (rounded to 3.0 m)

Equation (b): \( b_h = 1.8 \) m

\( d_0 = 1000 - 20 - 20 / 2 = 970 \) mm

**Step 2**
The design shear force at the critical shear section – see Figure 12.3(6) – is

\[
V^* = b_h \left( D_f / 2 + e - D_t / 2 - d_0 \right) \cdot q_t \cdot 0.9 \rho_w D
\]

Equation (c)

Therefore

\[
V^* = 1.8(1.5 + 0.1429 - 0.5 / 2 - 0.97)(300 - 0.9 \times 24 \times 1) = 211.92 \text{ kN}
\]

**Step 3**
As the section is deep enough to be reinforceable, this step is not required to be checked.
**Step 4**

For a single layer of steel, 

\[ d = d_s = 970 \text{ mm} \] and thereby 

\[ d_e = [0.72D, 0.9d]_{\text{max}} = [(0.72 \times 1000), (0.9 \times 970)]_{\text{max}} = [720, 873]_{\text{max}} = 873 \text{ mm} \]

Equation 6.3(4): 

\[ V_{uc} = k_v b_s d_e \sqrt{f'_c} \] where \( \sqrt{f'_c} = \sqrt{20} = 4.47 \text{ MPa} < 8 \text{ MPa} \) which is acceptable 

and 

\[ k_v = \left[ \frac{200}{1000 + 1.3d_e} \right] = \left[ \frac{200}{1000 + 1.3 \times 873} \right] = 0.094 < 0.10, \] which is acceptable.

Finally, Equation 6.3(4) gives 

\[ V_{uc} = 0.094 \times 1800 \times 873 \times \sqrt{20} \times 10^{-3} = 660.6 \text{ kN} \]

**Step 5**

As \( \phi V_{uc} = 0.75 \times 660.6 = 495.5 \text{ kN} > V^* = 211.92 \text{ kN} \), the assumed \( D = 1000 \text{ mm} \) leads to an unduly large safety margin. Revise using \( D = 750 \text{ mm} \).

**Trial 2**

**Step 1**

Assume \( D = 750 \text{ mm} \), then:

Equation (a): \( D_t = 2.895 \text{ m} \) (rounded to 3.0 m)

Equation (b): \( b_r = 1.8 \text{ m} \)

\[ d_s = 750 - 20 - 20/2 = 720 \text{ mm} \]

**Step 2**

Equation (c): \( V^* = 1.8 \times (1.5 + 0.1429 - 0.5/2 - 0.72) \times (300 - 0.9 \times 24 - 0.72) \)

\[ = 344.53 \text{ kN} \]

**Step 3**

Not required as explained for trial 1.

**Step 4**

For the assumed total depth of 750 mm:

\[ d = d_s = 720 \text{ mm} \] and thereby 

\[ d_e = [0.72D, 0.9d]_{\text{max}} = [(0.72 \times 750), (0.9 \times 720)]_{\text{max}} = [540, 648]_{\text{max}} = 648 \text{ mm} \]

Equation 6.3(4): 

\[ V_{uc} = k_v b_s d_e \sqrt{f'_c} \] where \( \sqrt{f'_c} = \sqrt{20} = 4.47 \text{ MPa} < 8 \text{ MPa} \) which is acceptable and 

\[ k_v = \left[ \frac{200}{1000 + 1.3d_e} \right] = \left[ \frac{200}{1000 + 1.3 \times 648} \right] = 0.11 > 0.10, \] which is not acceptable, hence take \( k_v = 0.10 \)
Finally, Equation 6.3(4) gives
\[ V_{uc} = 0.10 \times 1800 \times 648 \times \sqrt{20} \times 10^{-3} = 521.6 \text{ kN} \]

**Step 5**

As \( \phi V_{uc} = 0.75 \times 521.6 = 391.2 \text{ kN} > V^* = 344.53 \text{ kN} \), the assumed \( D = 750 \text{ mm} \) requires no shear reinforcement. This is acceptable.

**Step 6**

Not required for this example.

**Step 7**

Not required for this example.

**Step 8**

Punching shear check

The footing can be treated as an edge column with overhang on the short (left or heel) side of the footing. To be on the conservative side, the overhang may be ignored in computing the critical shear perimeter. Noting that \( d_{om} = 720 - 10 = 710 \text{ mm} \) and:

- Equation 9.6(4): \( u = 2(500 + 710/2) + (300 + 710) = 2720 \text{ mm} \)
- Equation 9.6(6): \( f_y = 0.17 \left( 1 + \frac{2}{500 / 300} \right) \sqrt{f'_{cu}} = 0.374 \sqrt{f'_{cu}} \leq 0.34 \sqrt{f'_{cu}} \)

Thus \( f_y = 0.34 \sqrt{20} = 1.521 \text{ MPa} \) and:

- Equation 9.6(3): \( V_{uc} = 2720 \times 710 \times 1.521 \times 10^{-3} = 2937.4 \text{ kN} \)

and as shown in Figure 12.3(6), \( M_u^* = 200 \text{ kNm} \)

- Equation 9.6(9): \( V_u = \frac{2937.4}{2720 \times 200 \times 10^{-3}} = 2780.2 \text{ kN} \)

- Equation 9.6(10): \( \phi V_u = 0.75 \times 2780.2 = 2085.2 > N^* = 1400 \text{ kN} \)

Therefore, the punching shear strength is more than adequate.

**Steps 9 and 10**

Bending design covers the moments about the major and minor axes. First, \( A_{st} \) design for the moment about the major axis must be determined. For a single layer of N20 bars:

- \( d = d_o = 720 \text{ mm} \)
- Equation 3.3(2a): \( \alpha = 0.82 \) (for \( f_y = 20 \text{ MPa} \))
- Equation 3.3(2b): \( \gamma = 0.92 \).
Following the recommendations illustrated in Figure 9.4(2) and referring to Figure 12.2(6), the critical bending section is located at $L_b$, from the right edge of the footing where

$$L_b = \frac{D}{2} + e - \frac{0.7D}{2}$$

That is, $L_b = \frac{3}{2} + 0.1429 - 0.7 \times \frac{0.5}{2} = 1.4679$ m

Thus

$$M^b = (300 - 0.9 \times 24 \times 0.75) \times 1.8 \times 1.4679^2 / 2 = 550.36 \text{ kNm}$$

and Equation 3.5(7): $\xi = 0.82 \times 20 / 500 = 0.0328$.

Assuming $\phi = 0.85$, then Equation 3.5(6):

$$\rho_i = 0.0328 - \sqrt{0.0328^2 - \frac{2 \times 0.0328 \times 550.36 \times 10^6}{0.85 \times 1800 \times 720^2 \times 500}} = 0.001419$$

Check to ensure that the assumed value of $\phi = 0.85$ is valid. From Equation 3.4(8)

$$k_{u1} = \frac{0.001419 \times 500}{0.82 \times 0.92 \times 20} = 0.047$$

Since $d = d_o$, then $k_{u0} = k_o = 0.047$ and Equations 3.4(20)a with b yield $\phi = 0.85$.

This is acceptable.

Equation 12.3(18): $\rho_{u1, min} = 0.20 \times (750 / 720)^2 \times 0.6 \times \sqrt{20 / 500} = 0.001165 < 0.001419$

Therefore, $A_{st} = 0.001419 \times 1800 \times 720 = 1839.0$ mm$^2$ and from Table 2.3(1), six N20 gives $A_{st} = 1884$ mm$^2$. This is acceptable.

Once the $A_{st}$ design for moment about the major axis is completed as above, $A_{st}$ design for the moment about the minor axis is to be carried out. For a single layer of N20 bars, $d = d_{major} - 20 = 700$ mm. For the minor direction

$$L_b = h_1 / 2 - 0.7h_1 / 2 = 1.8 / 2 - 0.7 \times 0.3 / 2 = 0.795$$

Thus

$$M^b = (300 - 0.9 \times 24 \times 0.75) \times 3 \times 0.795^2 / 2 = 269.05 \text{ kNm}$$

If we assume $\phi = 0.85$, then Equation 3.5(6):

$$\rho_i = 0.0328 - \sqrt{0.0328^2 - \frac{2 \times 0.0328 \times 269.05 \times 10^6}{0.85 \times 3000 \times 700^2 \times 500}} = 0.000434$$

Check for $\phi$ as follows.

From Equation 3.4(8)

$$k_{u1} = \frac{0.000434 \times 500}{0.82 \times 0.92 \times 20} = 0.0144$$

and since $d_o = 700$ mm
Equations 3.4(20)a with b yield \( \phi = 0.85 \). This is acceptable.

Now, Equation 12.3(18):

\[
p_{\text{min}} = 0.20 \times (750 \div 700)^2 \times 0.6 \times \sqrt{20} / 500 = 0.001232 > p_c
\]

Therefore, \( A_{st} = 0.001232 \times 3000 \times 700 = 2588 \text{ mm}^2 \). From Table 2.3(1), nine N20 gives \( A_{st} = 2826 \text{ mm}^2 \). This is acceptable.

The layout of the reinforcing bars is illustrated in Figure 12.3(7).

![Diagram](image)

**Figure 12.3(7).** Bending reinforcement: (a) plan and (b) section a-a

Note: all dimensions are in mm.

**Step 11**

Check the bond of the major moment direction. For Equation 8.2(1):

\[
\kappa_1 = 1.3
\]

\[
\kappa_2 = (132 - 20) / 100 = 1.12
\]

\[
\kappa_3 = 1 - 0.15(20 - 20) / 20 = 1.
\]

Therefore, Equation 8.2(1):

\[
L_{y,th} = 0.5 \times 1.3 \times 1 \times 500 \times 20 / (1.12 \times \sqrt{20}) = 1298 \text{ mm} (> 300 \text{ mm})
\]

Since the available
\[ L_{sy,1} < (L_h - \text{cover}) = 1468 - 20 = 1448 \text{ mm} \]

and
\[ L_{sy,1} < (D_f - L_h - \text{cover}) = 3000 - 1468 - 20 = 1512 \text{ mm} \]

then straight bars of length 3000 – 2 \times 20 = 2960 mm have adequate development length at the ultimate state. Depending on the quality of excavation, a shorter bar length should be used resulting in more concrete cover.

Check the bond of the minor moment direction. Similar to the major moment direction calculations, Equation 8.2(1):
\[ L_{sy,m} = 1298 \text{ mm} \]

At either end of the N20 bar, the available length for stress development is
\[ \frac{b_c}{2} - \text{cover} - 0.7 \frac{b_c}{2} = 900 - 20 - 0.7 \times 300 / 2 = 775 \text{ mm} < L_{sy,th} = 1298 \text{ mm} \]

which appears inadequate. However, Equation 8.2(7) indicates that the required stress development length is
\[ L_{st} = M_{u,\text{th}} / M_{u,\text{min}} = 269.05 \times L_{sy,\text{th}} / M_{u,\text{min}} \]

where, as per Equation 3.4(10), the minor moment capacity
\[ M_{u,\text{min}} = 2826 \times 500 \times 700 \times \left( 1 - \frac{2826 \times 500}{2 \times 0.82 \times 3000 \times 700 \times 20} \right) \times 10^{-4} = 968.8 \text{ kNm} \]

or
\[ L_{st} = 269.05 \times 1298 / 968.8 = 360.5 \text{ mm} < 775 \text{ mm}. \]

Thus straight bars of 1800 – 2 \times 20 = 1760 mm have adequate bond strength in the minor bending direction. A shorter bar length should be used to provide more concrete cover as necessary.

**Step 12**

Now the shrinkage and temperature steel must be determined. The \( A_{st} \) in the major bending direction is greater than the \( A_{st,\text{min}} \) specified in Equation 12.3(18), whereas in the minor bending direction \( A_{st} = A_{st,\text{min}} \). Hence, no additional shrinkage and temperature steel is required.
12.4.1 Concentric column loading

Illustrative example

Problem
Details of a centrally loaded square column to be supported on four circular concrete piles are shown in Figure 12.4(2). Given $N^* = 2000$ kN and $f'_c = 25$ MPa, obtain the overall depth ($D$) of the pile cap.

![Diagram of centrally loaded square column supported on four circular concrete piles](image)

Figure 12.4(2) A centrally loaded square column supported on four circular concrete piles

Note: all dimensions in mm.

Solution
The depth of the section is to be determined by shear strength consideration. The pile load

$$P_p^* = (N^* + 1.2\rho_n b_D D) / 4$$

Equation (a)
Assuming $D = 700$ mm and using N20 bars for reinforcement,

\[ d_u = 700 - 75 - 75 - 20 - 20/2 = 520 \text{ mm}. \]

For bending shear consideration, the critical section is located at $d_u$ from the face of the support, which in this case is either the column or any pair of the four piles. Figure 12.4(2) shows that the clear distance between the column and the piles is only 275 mm (i.e. the critical shear section would cut through the column or the pair of piles). This means that bending shear is not critical. Therefore, the design is governed by the punching shear.

### Punching shear design

For punching shear design, we have:

- **Equation (a):**
  \[
  P_p = \left(2000 + 1.2 \times 24 \times 1.95 \times 1.95 \times 0.7\right)/4 = 519.16 \text{ kN}
  \]

- **Equation 9.6(6):**
  \[
  f_{ce} = 0.17 \left(1 + \frac{2}{1.95/1.95}\right) \sqrt{f'_c} = 0.51 \sqrt{f'_c} > 0.34 \sqrt{f'_c}
  \]

Adopt $f_{ce} = 0.34 \sqrt{25} = 1.7 \text{ MPa}$ and the mean value of

\[ d_{om} = (700 - 2 \times 75 - 20) \times 10^{-3} = 0.53 \text{ m} \]

Then:

- **Equation 9.6(5):**
  \[
  \mu = 2(0.4 + 0.53 + 0.49 + 0.53) = 3.72 \text{ m}
  \]

- **Equation 9.6(3):**
  \[
  V_u = 3720 \times 530 \times 1.7 \times 10^{-3} = 3351.72 \text{ kN}
  \]

Since the square pile cap is symmetrically loaded, $M^* = 0$. Hence:

- **Equation 9.6(9):**
  \[
  V_u = V_{um} = 3351.72 \text{ kN}
  \]

- **Equation 9.6(10):**
  \[
  \phi V_u = 0.75 V_u = 2513.8 \text{ kN}
  \]

In Figure 12.4(2), $N^* = 2000 + 1.2 (0.4 + 0.53)^2 \times 0.7 \times 24 = 2017.4 \text{ kN}$

Since, $\phi V_u = 2513.8 \text{ kN} > N^* = 2017.4 \text{ kN}$, the assumed $D = 700$ mm is acceptable, albeit conservative. Note, however, that each pile resembles a corner column in a flat plate system around which the punching shear strength should be checked. This may be carried out using semi-empirical formulas (see Loo and Falamaki 1992). Alternatively, a conservative assessment may be made of the punching shear strength of the cap around a pile. This is done in a process similar to that described in Section 12.3.7. In this case, and as illustrated in Figure 12.4(3)

\[
M^* < P^* \left(0.125 + 0.53/2\right) = 202.5 \text{ kNm}
\]

Let $M^{*}_v = 202.5 \text{ kNm}$
Conservatively, and referring to Figure 12.4(3), the circular pile section may be converted into a square with $b_c = D_c = \sqrt{\pi \times 125^2} = 221.6 \text{ mm}$.

**Figure 12.4(3)** Determination of critical shear perimeter for the punching shear check of the pile cap

Note: all dimensions are in mm.

Then, the critical shear perimeter is obtained using Equation 9.6(4) or

$$u = 2a + a_2$$

where again conservatively,

$$a = 221.6 + d_{cm}/2 = 486.6 \text{ mm}$$

and

$$a_2 = 221.6 + d_{cm} = 751.6 \text{ mm}$$

Thus $u = 1724.8 \text{ mm}$.

Note that this $u$ is shorter and hence more critical than another possible shear perimeter – see Figure 12.4(3). Then

Equation 9.6(3): $V_u = 1724.8 \times 530 \times 1.7 \times 10^{-3} = 1554.0 \text{ kN}$

Equation 9.6(9): $V_a = \frac{1554.0}{1 + \frac{1.7248 \times 202.5}{8 \times 519.16 \times 0.4866 \times 0.53}} = 1171.9 \text{ kN}$

Equation 9.6(10): $\phi V_u = 878.9 \text{ kN} > P_p = 519.16 \text{ kN}$

Thus, punching shear strength around the pile appears to be adequate.

To complete the pile-cap design, proceed to compute the bending steel $A_{st}$ in each of the $x$ and $y$ directions – see Figure 12.4(2). The process is similar to that used in Sections 12.2.5 and 12.3.7 for wall and column footings, respectively. Note however that the bending span in either direction is

$$L_b = 400 + 0.3 \times \frac{400}{2} = 460 \text{ mm}$$
\[ M^x = M^y = 2 \times 519.16 \times 0.460 - 24 \times 1.95 \times 0.7 \times 0.460^2 / 2 = 474.2 \text{ kNm} \]
\[ d_s = 700 - 75 - 75 - 20 - 20 / 2 = 520 \text{ mm} \] and \[ d_r = d_s + 20 = 540 \text{ mm}. \]

After computing the bending reinforcement in both directions, ensure that the bars have adequate stress development length. If they don’t, use hooks or cogs.

### 12.5.7 Illustrative example

**Problem**

Figure 12.5(14) shows details of a reinforced concrete cantilever retaining wall. The superimposed live load, \( p = 15 \) kPa, and the wall and base dimensions are the outcomes of a global geotechnical analysis, including sliding stability check, followed by a preliminary design exercise.

Given the unit weights of the cohesionless backfill and the front surcharge \( \rho_c BF = \rho_c FS = 21 \text{ kN/m}^3 \), with a characteristic effective internal friction angle \( \phi = 35^\circ \), the effective subsoil bearing capacity \( q_f = 250 \text{ kPa} \) and \( f' = 25 \text{ MPa} \). Use D500N bars only.

Compute the subsoil pressures at the toe and heel, \( f_{toe} \) and \( f_{heel} \), respectively, and check the overturning stability. Then carry out a full reinforced concrete design of the retaining structure. The passive earth pressure due to the front surcharge may be ignored.
Figure 12.5(14) Details of the example reinforced concrete cantilever retaining wall
Note: all dimensions are in mm.

Solution
The design steps required are enumerated below.

Step 1
Compute $K_a$ and the lateral active earth pressures. For the levelled backfill, $\theta = 0$, and Equation 12.5(2) gives

$$K_a = \frac{1 - \sqrt{1 - \cos^2 35^\circ}}{1 + \sqrt{1 - \cos^2 35^\circ}} = 0.2710$$
At a given depth \( z \), the earth pressures due to the superimposed load and the backfill as per Equation 12.5(1) are

\[
p_{z,SL} = 0.2710 \times 15 \times 1 = 4.065 \text{ kPa}
\]

and

\[
p_{z,BF} = 0.2710 \times 21 \times 1 = 5.691z \text{ kPa}
\]

respectively.

**Step 2**

Now compute various vertical and lateral forces. Assuming \( \rho_{w,concrete} = 25 \text{ kN/m}^3 \) with the given \( \rho_{w,BF} = \rho_{w,FS} = 21 \text{ kN/m}^3 \) and for a 1-metre run of the retaining wall, the quantities required in Equations 12.5(5) and (6) are as follows.

**Forces (kN)**:

(a) \( \beta_{W} W_{W1} = (0.25 \times 6 \times 1) \times 25 \beta_{W} = 37.5 \beta_{W} \)

(b) \( \beta_{W} W_{W2} = \left( \frac{0.2}{2} \times 6 \times 1 \right) \times 25 \beta_{W} = 15.0 \beta_{W} \)

(c) \( \beta_{W} W_{W3} = (0.45 \times 3.5 \times 1) \times 25 \beta_{W} = 39.375 \beta_{W} \)

(d) \( \beta_{SL} W_{SL} = (15 \times 2.1 \times 1) \times \beta_{SL} = 31.5 \beta_{SL} \)

(e) \( \beta_{BF} W_{BF} = (6 \times 2.1 \times 1) \times 21 \beta_{BF} = 264.6 \beta_{BF} \)

(f) \( \beta_{FS} W_{FS} = (0.55 \times 0.95 \times 1) \times 21 \beta_{FS} = 10.973 \beta_{FS} \)

(g) \( F_{SL} = 4.065 \beta_{SL} \times (6 + 0.45) = 26.22 \beta_{SL} \)

(h) \( F_{BF} = 5.691 \times 6.45 \beta_{BF} \times 6.45/2 = 118.38 \beta_{BF} \)

**Lever arms with respect to \( Ob \) (m)**:

(i) \( L_{W1} = \frac{0.25}{2} + 2.1 - 1.75 = 0.475 \)

(ii) \( L_{W2} = \frac{0.2}{3} + 0.25 + 2.1 - 1.75 = 0.70 \)

(iii) \( L_{FS} = \frac{0.95}{2} + 0.45 + 2.1 - 1.75 = 1.275 \)

(iv) \( L_{SL} = L_{BF} = 1.75 - \frac{2.1}{2} = 0.70 \)

(v) \( L_{SL,H} = (6 + 0.45)/2 = 3.225 \)

(vi) \( L_{BF,H} = (6 + 0.45)/3 = 2.15 \)

**Step 3**

Now calculate the load combinations (LC). Considering the aggravating and reversal effects of each of the various vertical and induced lateral forces, three load combination
cases are required to produce the most critical outcomes. The details are presented in Table 12.5(3).

<table>
<thead>
<tr>
<th>Load combination</th>
<th>$\beta_w$</th>
<th>$\beta_{FS}$</th>
<th>$\beta_{SL}$</th>
<th>$\beta_{BF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1</td>
<td>1.2</td>
<td>1.2</td>
<td>1.5</td>
<td>1.2</td>
</tr>
<tr>
<td>LC2</td>
<td>1.2</td>
<td>1.2</td>
<td>0.4</td>
<td>0.9</td>
</tr>
<tr>
<td>LC3</td>
<td>0.9</td>
<td>0.9</td>
<td>1.5</td>
<td>1.2</td>
</tr>
</tbody>
</table>

**Step 4**

Compute $R$ and $M_R$ at $O_B$ for LC1, 2 and 3. With values obtained in step 2, the resultant vertical force as per Equation 12.5(5) is

$$R = (37.5 + 15.0 + 39.375)\beta_w + 31.5\beta_{SL} + 264.6\beta_{BF} + 10.973\beta_{FS}$$

$$= 91.875\beta_w + 31.5\beta_{SL} + 264.6\beta_{BF} + 10.973\beta_{FS}$$

Equation (a)

Substituting the corresponding load factors, which are the subcripted $\beta$ values for LC1 in Table 12.5(3)

$$R = 488.19 \text{kN}$$

where

$$\beta_w = 1.2; \ \beta_{SL} = 1.5; \ \beta_{BF} = 1.2; \ \text{and} \ \beta_{FS} = 1.2.$$  

The moment about $O_B$ (positive anticlockwise) as per Equation 12.5(6) and with reference to Table 12.5(3), is

$$M_R = 26.22\beta_{SL} \times 3.225 + 118.38\beta_{BF} \times 2.15 + 37.5\beta_{WF} \times 0.475 + 15.0\beta_{WF}$$

$$\times 0.70 + 10.973\beta_{FS} \times 1.275 - 31.5\beta_{SL} \times 0.70 - 264.6\beta_{BF} \times 0.70$$

which is similar to

$$M_R = 84.560\beta_{SL} + 254.517\beta_{BF} + 28.313\beta_{WF} + 13.991\beta_{FS} - 22.050\beta_{SL}$$

$$- 185.220\beta_{WF}$$

Equation (b)

from which for LC1, $MR = 227.686 \text{kNm}$

Following the same process, Equation (a) gives

$$R = 374.16 \text{ kN} \ \text{ for } \ \text{LC2}$$

and

$$R = 457.33 \text{ kN} \ \text{ for } \ \text{LC3}$$
Then, Equation (b) leads to
\[ M_R = 138.13 \text{ kNm} \quad \text{for} \quad \text{LC2} \]
and
\[ M_R = 214.99 \text{ kNm} \quad \text{for} \quad \text{LC3} \]

**Step 5**

Compute \( f_{\text{toe}} \) and \( f_{\text{heel}} \) and check overturning stability. The subsoil pressures at the toe and heel are obtained through Equations 12.5(9) and (10), respectively, as
\[ f_{\text{toe}} = \frac{R}{3.5} + \frac{6M_R}{3.5^2} \]
and
\[ f_{\text{heel}} = \frac{R}{3.5} - \frac{6M_R}{3.5^2} \]

The outcomes for \( R \) and \( M_R \) due to the three critical load combinations are summarised below:

- **LC1**: \( R = 488.19 \text{ kN} \); \( M_R = 227.69 \text{ kNm} \)
- **LC2**: \( R = 374.16 \text{ kN} \); \( M_R = 138.13 \text{ kNm} \)
- **LC3**: \( R = 457.33 \text{ kN} \); \( M_R = 214.99 \text{ kNm} \)

Now the subsoil pressures for the three load combination cases are computed using Equations (c) and (d):

- **LC1**: \( f_{\text{toe}} = 251.00 \text{ kPa} \); \( f_{\text{heel}} = 27.96 \text{ kPa} \)
- **LC2**: \( f_{\text{toe}} = 174.56 \text{ kPa} \); \( f_{\text{heel}} = 39.25 \text{ kPa} \)
- **LC3**: \( f_{\text{toe}} = 235.97 \text{ kPa} \); \( f_{\text{heel}} = 25.36 \text{ kPa} \)

These results show that none of the subsoil pressures exceed the given effective bearing capacity \( q_f = 250 \text{ kPa} \). Note, however, that for LC1 it is slightly larger but can be taken as 250 kPa. This means that the retaining wall dimensions are acceptable. Note also that in practice, \( q_f \) may be calculated in accordance with Section 5 of *AS 4678-2002: Earth-Retaining Structures*, where design requirements for serviceability and durability, inter alia, are also specified.

Since all the pressure values are positive (compressive), the retaining wall is stable against overturning.

**Step 6**

Compute the design moments (\( M^\circ \)). The root moment of the wall for LC1 is given by Equation 12.5(12) with \( \beta_{\text{SL}} = 1.5 \) and \( \beta_{\text{BF}} = 1.2 \) as
\[ M_{o1}^* = 1.5 \times 4.065 \times 6 \times l_{s1} + 0.5 \times 1.2 \times 0.271 \times 21 \times 6^2 \times l_{s2} \]
\[ = 36.585 l_{s1} + 122.926 l_{s2} \]

where from Equation 12.5(13)
\[ l_{s1} = 6/2 + 0.15 \times 0.45 = 3.068 \text{ m} \]
and Equation 12.5(14) gives
\[ l_{s2} = 6/3 + 0.15 \times 0.45 = 2.068 \text{ m} \]
or
\[ M_{o1}^* = 366.45 \text{ kNm} \]

For LC2, \( \beta_{SL} = 0.4 \) and \( \beta_{BF} = 0.9 \), which gives
\[ M_{o1}^* = 9.756 l_{s1} + 92.194 l_{s2} \]
from which and with \( l_{s1} = 3.068 \text{ m} \) and \( l_{s2} = 2.068 \text{ m} \)
\[ M_{o1}^* = 220.59 \text{ kNm} \]

For LC3, with \( \beta_{SL} = 1.5 \) and \( \beta_{BF} = 1.2 \), the moment is the same as for LC1, which is
\[ M_{o1}^* = 366.45 \text{ kNm} \]

For the heel moment, Equation 12.5(15) yields
\[ M_{o2}^* = (31.5 \beta_{SL} + 264.6 \beta_{BF})(2.1/2 + 0.15 \times 0.45) + \beta_{w} (0.45 \times 2.1) \times 1 \times 25 (2.1 + 0.15 \times 0.45) / 2 - P_{1,\text{heel}} (2.1 + 0.15 \times 0.45) / 2 \]
\[ - P_{2,\text{heel}} (2.1 + 0.15 \times 0.45) / 3 \]
\[ = 35.201 \beta_{SL} + 295.691 \beta_{BF} + 25.604 \beta_{w} - 1.084 P_{1,\text{heel}} - 0.723 P_{2,\text{heel}} \]

For LC1, \( \beta_{SL} = 1.5 \), \( \beta_{BF} = 1.2 \), \( \beta_{W} = 1.2 \),
\[ P_{1,\text{heel}} = 27.97 \times (2.1 + 0.15 \times 0.45) = 27.97 \times 2.1675 = 60.625 \text{ kN} \]
and
\[ P_{2,\text{heel}} = \left[ (250 - 27.97 \times \frac{2.1675}{3.5}) \times 2.1675 / 2 \right] = 149.02 \text{ kN} \]

based on which Equation (e) gives
\[ M_{o2}^* = 264.90 \text{ kNm} \]

For LC2, \( \beta_{SL} = 0.4 \), \( \beta_{BF} = 0.9 \), \( \beta_{W} = 1.2 \),
\[ P_{1,\text{heel}} = 39.25 \times (2.1 + 0.15 \times 0.45) = 39.25 \times 2.1675 = 85.074 \text{ kN} \]
and
\[ P_{2,\text{heel}} = \left[ (174.56 - 39.25) \times \frac{2.1675}{3.5} \right] \times 2.1675 / 2 = 90.81 \text{ kN} \]

Then from Equation (e)
\[ M_{o2}^* = 153.05 \text{ kNm} \]
For LC3, \( \beta_{cl} = 1.5 \), \( \beta_{bf} = 1.2 \), \( \beta_w = 0.9 \),

\[
P_{1.heel} = 25.36 \times (2.1 + 0.15 \times 0.45) = 25.36 \times 2.1675 = 54.968 \text{ kN}
\]

and

\[
P_{2.heel} = \left(235.97 - 25.36\right) \times \frac{2.1675}{3.5} \times 2.1675/2 = 141.35 \text{ kN}
\]

Then from Equation (e)

\[
M^o_2 = 268.89 \text{ kNm}
\]

Similarly, the toe moment is given by Equation 12.5(16) as

\[
M^o_{toe} = P_{1.toe} + P_{2.toe} = \frac{250 - 25.36}{2} \times 1.0175 - \frac{250 - 185.45}{2} \times 1.0175 = 18.8695 \text{ kN}
\]

Equation (f) thus gives

\[
M^o_{toe} = 104.15 \text{ kNm}
\]

For LC2, \( \beta_{fs} = \beta_w = 1.2 \)

\[
P_{1.toe} = \left[27.97 + (250 - 27.97) \times \frac{3.5 - 1.0175}{3.5}\right] \times 1.0175 = 188.695 \text{ kN}
\]

and

\[
P_{2.toe} = \frac{1}{2} \times (250 - 185.45) \times 1.0175 = 32.84 \text{ kN}
\]

from which Equation (f) gives

\[
M^o_{toe} = 69.45 \text{ kNm}
\]

For LC3, \( \beta_{fs} = \beta_w = 0.9 \)

\[
P_{1.toe} = \left[25.36 + (235.97 - 25.36) \times \frac{3.5 - 1.0175}{3.5}\right] \times 1.0175 = 177.80 \text{ kN}
\]

and

\[
P_{2.toe} = \frac{1}{2} \times (235.97 - 174.743) \times 1.0175 = 31.15 \text{ kN}
\]

Hence, Equation (f) gives

\[
M^o_{toe} = 100.99 \text{ kNm}
\]
Finally, the root, heel and toe moments for the three load combination cases are shown in Table 12.5(4).

Table 12.5(4) Root, heel and toe moments for the three load combination cases

<table>
<thead>
<tr>
<th>Load combination case</th>
<th>Root moment, ( M^*_{01} ) (kNm)</th>
<th>Heel moment, ( M^*_{02} ) (kNm)</th>
<th>Toe moment, ( M^*_{03} ) (kNm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1</td>
<td>366.45</td>
<td>264.90</td>
<td>104.15</td>
</tr>
<tr>
<td>LC2</td>
<td>220.59</td>
<td>153.05</td>
<td>69.45</td>
</tr>
<tr>
<td>LC3</td>
<td>366.45</td>
<td>268.89</td>
<td>100.99</td>
</tr>
</tbody>
</table>

From Table 12.5(4), the design moments are obtained as
- root moment, \( M^*_{01} = 366.45 \) kNm
- heel moment, \( M^*_{02} = 268.89 \) kNm
- toe moment, \( M^*_{03} = 104.15 \) kNm

**Step 7**

Now determine the design bending reinforcement, \( A_{st} \).

For the root section of the wall, \( M^*_{01} = 366.45 \) kNm, \( d = 410 \) mm and Equation 3.5(7) gives
\[
\xi = 0.8125 \times 25/500 = 0.0406
\]
Assuming \( \phi = 0.85 \) and Equation 3.5(6) gives
\[
p_i = 0.0406 - \sqrt{0.0406^2 - \frac{2 \times 0.0406 \times 366.45 \times 10^6}{0.85 \times 1000 \times 410^2 \times 500}} = 0.005502
\]
Equation 3.4(8) gives
\[
k_u = \frac{0.005502 \times 500}{0.8125 \times 0.9075 \times 25} = 0.1492
\]
Since \( k_{uo} = k_u = 0.1492, \phi = 0.85 \) is confirmed as per Equations 3.4(20)a with b.

Equation 12.3(18) gives
\[
p_{\text{min}} = 0.20 \times \left( \frac{450}{410} \right)^2 \times 0.6 \sqrt{25/500} = 0.001446 < p_i = 0.005502
\]
Therefore,
\[
A_{st} = 0.005502 \times 1000 \times 410 = 2256 \text{ mm}^2/\text{m}
\]
When Table 2.3(2) is consulted, N20 @ 125 mm gives \( A_{st} = 2512 \text{ mm}^2/\text{m} \), which is larger than 2256 mm\(^2\)/m by 11.3%. This is acceptable.
This amount of steel is required only at the root of the cantilever wall. As desired, for a more economical design, some of the bars may be curtailed at levels towards the top of the wall where the moment diminishes rapidly, remembering that the bars must be extended beyond the curtailment level to provide adequate stress development for the curtailed bars.

In a practical design, a deflection check should be performed. However, in the present case of an inwardly tapered wall, even excessive deflection would not be apparent to a layperson.

For the heel section

$M_{e2} = 268.89 \text{ kN/m}, d = 410 \text{ mm}$

and Equation 3.5(6) gives

$$p_t = 0.0406 - \sqrt{0.0406^2 - \frac{2 \times 0.0406 \times 268.89 \times 10^6}{0.85 \times 1000 \times 410^2 \times 500}} = 0.00396 > p_{t,\text{min}}$$

Therefore,

$$A_{st} = 0.00396 \times 1000 \times 410 = 1624 \text{ mm}^2/\text{m}$$

From Table 2.3(2), N20 @ 175 mm gives $A_{st} = 1794 \text{ mm}^2/\text{m} > 1624 \text{ mm}^2/\text{m}$. This is acceptable.

For the toe section,

$M_{e3} = 104.15 \text{ kN/m}, d = 410 \text{ mm}$

and Equation 3.5(6) gives

$$p_t = 0.0406 - \sqrt{0.0406^2 - \frac{2 \times 0.0406 \times 104.15 \times 10^6}{0.85 \times 1000 \times 410^2 \times 500}} = 0.001485 > p_{t,\text{min}}$$

Therefore,

$$A_{st} = 0.001485 \times 1000 \times 410 = 609 \text{ mm}^2/\text{m}$$

From Table 2.3(2), N20 @ 300 mm gives $A_{st} = 1047 \text{ mm}^2/\text{m} > 609 \text{ mm}^2/\text{m}$. This is acceptable.

Also, for crack control purposes, as shown in Table 1.4(5) (slabs), $s < 300 \text{ mm}$. Therefore, use N20 @ 300 mm.

For a more economical solution, use N16 @ 300 mm, which gives $A_{st} = 670 \text{ mm}^2/\text{m}$. This is acceptable.

**Step 8**

Compute design shear $V^*$ for the wall. LC1 governs as per Table 12.5(3). With reference to Figure 12.5(15)
\[ d_0 \approx 410 - \frac{410 \times (450 - 250)}{6000} = 396 \text{ mm} \]

\[ p_{a,SL} = 4.065 \text{ kPa} \]

\[ p_{a,BF} = 5.691 \times 5.604 = 31.892 \text{ kPa} \]

For a 1-metre run

\[ V^*_{\text{root}} = 1.5 \times 4.065 \times 5.604 \times 1 + 1.2 \times 31.892 \times 5.604 / 2 \times 1 = 141.404 \text{ kN} \]

**Figure 12.5(15)** Location and determination of critical shear force for the wall

**Figure 12.5(16)** Location and determination of critical shear force for the heel

Now calculate design shear \( V^* \) for the heel. LC1 governs here.

In Figure 12.5(16)

\[ d_{\text{o,heel}} = 410 \text{ mm} \]

\[ W'_{\text{SL}} = 1.5 \times 15 \times 1.69 \times 1 = 38.025 \text{ kN} \]
Earth pressure resultant is

\[
W'_{\text{heal}} = 27.97 \times 1.69 \times \left(1 + \frac{1.69}{3.5} \times (251 - 27.97) \times \frac{1.69}{2}\right) = 47.269 + 90.999 = 138.268 \text{ kN}
\]

\[
V'^*_{\text{toe}} = 38.025 + 255.528 + 22.815 - 138.268 = 178.10 \text{ kN}
\]

Compute design shear \( V'^* \) for the toe, LC3 governs here.

In Figure 12.5(17),

\[V'_{\text{toe}} = 25.36 \text{ kPa}\]

\[d_{o,\text{toe}} = 410 \text{ mm}\]

Earth pressure resultant is

\[
0.54 \times 235.97 - 0.54 \times (235.97 - 25.36) \times \frac{0.54}{2} = 118.65 \text{ kN}
\]

\[
W'_{FS} = 0.9 \times 0.55 \times 0.54 \times 21 = 5.6133 \text{ kN}
\]

\[
W'_{\text{toe}} = 0.9 \times 0.45 \times 0.54 \times 25 = 5.4675 \text{ kN}
\]

\[
V'^*_{\text{toe}} = 118.650 - 5.613 - 5.468 = 107.569 \text{ kN}
\]

**Step 9**

Check the shear capacity of the wall.

\[d = 410 \text{ mm}\]
and

\[ D = 450 \text{ mm} \]

For the simplified method of Clause 8.2.4.3, AS 3600-2018,

\[ d_v = [0.72D, 0.9d]_{\text{max}} = [(0.72 \times 450), (0.9 \times 410)]_{\text{max}} = [324, 369]_{\text{max}} = 369 \text{ mm} \]

Equation 6.3(4): \( V_{uc} = k_v d_v \sqrt{\bar{f}_c} \) where \( \sqrt{\bar{f}_c} = \sqrt{25} = 5 \text{ MPa} < 8 \text{ MPa} \) which is acceptable

and \( k_v = \left[ \frac{200}{1000 + 1.3d_v} \right] = \left[ \frac{200}{1000 + 1.3 \times 369} \right] = 0.14 > 0.10 \); hence, take \( k_v = 0.10 \)

Finally, Equation 6.3(4) gives

\[ V_{ui} = 0.10 \times 1000 \times 369 \times \sqrt{25} \times 10^{-3} = 184.5 \text{ kN} \]

As, \( \phi V_{uc} = 0.75 \times 184.5 = 138.4 \text{ kN} \approx V_{\text{ext}}^{*} = 141.404 \text{ kN} \), no shear reinforcement is required.

Thus, shear reinforcement is not required for the wall.

Now check the shear capacity of the heel. With \( V_{\text{heel}}^{*} = 178.10 \text{ kN} \) and \( d = d_0 = 410 \text{ mm} \), \( \phi V_{uc} = 0.75 \times 184.5 = 138.4 \text{ kN} < V_{\text{heel}} = 178.10 \text{ kN} \). Thus, shear reinforcement is required. As per the provisions in the simplified method of Clause 8.2.4.3, AS 3600-2018, for \( A_{sv} > A_{sv,\text{min}} \), \( k_v = 0.15 \) and hence, \( V_{ui} = 0.15 \times 1000 \times 369 \times \sqrt{25} \times 10^{-3} = 276.75 \text{ kN} \) and

\[ \phi V_{uc} = 0.75 \times 276.75 = 207.6 \text{ kN} > V_{\text{heel}} = 178.10 \text{ kN} \]

Thus, the minimum shear reinforcement is required. For members not greater than 1.2 m in depth, the spacing (s) for minimum shear reinforcement is the lesser of 0.5D and 300 mm or \( s = 0.5 \times 450 = 225 \text{ mm} \). Equation 6.12(3) gives

\[ A_{sv,\text{min}} = \frac{0.08 \sqrt{f_y} b_w s}{f_{cy}} = \frac{0.08 \times \sqrt{25} \times 1000 \times 225}{500} = 180 \text{ mm}^2 \]

Use N12 ties @ 225 mm (\( A_{sv} = 226 \text{ mm}^2 > 180 \text{ mm}^2 \)).

Finally, check the shear capacity of the toe. With \( V_{\text{toe}}^{*} = 107.569 \text{ kN} \) and \( d = d_0 = 410 \text{ mm} \), relevant computations confirm that shear reinforcement is not required for the toe.

**Step 10**

Provide shrinkage and temperature steel using Equation 9.4(14) which gives

\[ A_{s,\text{min}} = 1.75 \times 1000 \times 410 \times 10^{-3} = 717.5 \text{ mm}^2/\text{m} \]

Use two layers of N12 bars @ 300 mm, which gives \( A_s = 754 \text{ mm}^2/\text{m} \). This is acceptable.
**Step 11**
Check the stress development, that is, the adequacy of the bond strength at the base section of the wall, and the root sections of the toe and heel. The process is similar to that used in Section 12.2.5 – step 11.

**Step 12**
The final design for the reinforced concrete retaining wall structure is shown in Figure 12.5(18).

![Reinforcement layout for the reinforced concrete retaining wall structure](image)

**Figure 12.5(18)** Reinforcement layout for the reinforced concrete retaining wall structure

Note: all dimensions are in mm; clear cover for bending steel 30 mm; for temperature steel not less than 30 mm
13.6.5 Illustrative example

Problem
Using the strut-and-tie modelling (STM) approach of the Standard, design a 600-mm-wide transfer girder spanning 5 m with a column at midspan with ultimate factored load of 5340 kN. The girder is supported by 600 mm square columns. The overall depth of the girder is 2.25 m as shown in Figure 13.6(4). The ultimate shear diagram is shown in Figure 13.6(5). Use $f' = 50$ MPa and N bars only for reinforcement.

![Figure 13.6(4)] STM design example – transfer girder

![Figure 13.6(5)] STM design example – shear diagram
Solution

*Select and establish strut-and-tie model and node locations*

Assume that the loads are carried by a strut-and-tie system consisting of two direct struts running from the top loading column to the supporting columns and a tie connecting the struts horizontally. The geometry of the assumed STM is shown in Figure 13.6(6).

Because of the heavy loads applied on the structure and the minimum allowable height being used, much deeper node locations are required. After multiple iterations, the node at location C at the loading point is determined to be 300 mm from the top of the girder, and the node location at the supports as 350 mm from the bottom of the girder as shown in Figure 13.6(6). Note that after the design, if the final nodal locations show a difference of roughly 50 mm or less, the original locations are deemed acceptable because the forces in the strut may increase only by 1–2%, which should not change the final design.

![Diagram](https://example.com/diagram.png)

*Figure 13.6(6)* STM design example – assumed STM and node locations

Based on the assumed nodal locations as above (see Figure 13.6(6)), the angle between the struts and the tie $= \tan^{-1}\left(\frac{\frac{2250-300-350}{2500}}{600}\right) = 32.6^\circ > 30^\circ$. As per Clause 7.1 of AS 3600-2018, it is acceptable.
Determine forces in struts and ties

From the geometry of the girder with reference to Figure 13.6(6),

length of strut CA = \( \sqrt{(2250 - 300 - 350)^2 + (2500)^2} = 2968.2 \) mm

length of strut CB = \( \sqrt{(2250 - 300 - 350)^2 + (2500)^2} = 2968.2 \) mm

Thus,

force in strut CA = \( 2760 \times \frac{2968.2}{2250 - 300 - 350} = 5120.1 \) kN

force in strut CB = \( 2760 \times \frac{2968.2}{2250 - 300 - 350} = 5120.1 \) kN

force in tie AB = \( 2760 \times \frac{2500}{2250 - 300 - 350} = 4312.5 \) kN

Determine effective concrete strength in nodes and struts

Enough space exists within the girder for bottle-shaped struts to be formed in struts CA and CB. Also, bursting reinforcement will be provided to resist cracking. Thus, Equation 13.6(1):

\[
\beta_s = \frac{1}{1.0 + 0.66 \cot^2 32.6^\circ} = 0.383 > 0.3 \text{ which is acceptable}
\]

Making use of Equation 13.6(2), the effective concrete strength is:

\[
\phi \beta_s 0.9 f'_c = 0.65 \times 0.383 \times 0.9 \times 50 = 11.2 \text{ MPa}
\]

The struts within the columns do not have enough space for a bottle-shaped strut to form. Thus \( \beta_s = 1.0 \) and the effective concrete strength:

\[
\phi \beta_s 0.9 f'_c = 0.65 \times 1 \times 0.9 \times 50 = 29.25 \text{ MPa}
\]

For the nodal region at C, a CCC situation prevails. Thus, \( \beta_n = 1.0 \) and the principal compressive stress on the nodal face is:

\[
\phi \beta_s 0.9 f'_c = 0.65 \times 1 \times 0.9 \times 50 = 29.25 \text{ MPa}
\]

For the nodal region at A and B, a CCT situation prevails. Thus, \( \beta_n = 0.80 \) and the principal compressive stress on the nodal face:

\[
\phi \beta_s 0.9 f'_c = 0.65 \times 0.8 \times 0.9 \times 50 = 23.4 \text{ MPa}
\]

Determine STM geometry

Hydrostatic nodal regions are used herein. Hence, the stresses on each face of the region must be identical and the faces are perpendicular to the axis of the struts. Extended nodal zones may be used, but hydrostatic nodal regions are easy to use for this type of loading and also add some conservatism in the design by requiring a larger nodal zone.
As hydrostatic nodal zones are used, the minimum value for the effective concrete strength (i.e. 11.2 MPa) must be used in calculating the widths of the struts and the height of the tie to ensure a static situation.

Using Equation 13.6(2) with $A_c = d_c t$, where $t$ is the thickness of the strut (which is 600 mm in this example), the strut width

$$d_c = \frac{\phi \beta_{0.9} \sigma}{f_{ct}}$$

Thus,

- width of strut CA, $d_{c,CA} = \frac{5120.1 \times 10^3}{11.2 \times 600} = 761.9$ mm
- width of strut CB, $d_{c,CB} = \frac{5120.1 \times 10^3}{11.2 \times 600} = 761.9$ mm
- width of strut A, $d_{c,A} = \frac{2760 \times 10^3}{11.2 \times 600} = 410.7$ mm
- width of strut B, $d_{c,B} = \frac{2760 \times 10^3}{11.2 \times 600} = 410.7$ mm
- width of strut C1, $d_{c,C1} = \frac{2670 \times 10^3}{11.2 \times 600} = 397.3$ mm
- width of strut C2, $d_{c,C2} = \frac{2670 \times 10^3}{11.2 \times 600} = 397.3$ mm
- height of tie AB, $d_{c,AB} = \frac{4312.5 \times 10^3}{11.2 \times 600} = 641.7$ mm

For the extra compression strut width required ($\geq (397.3 + 397.3) = 794.6$ mm) within the column applying the loads, a $800 \text{ mm} \times 600 \text{ mm}$ column is required. All other dimensions fit within the girder and supporting columns and follow the STM guidelines stipulated in Clause 7.1 of the Standard, as shown in Figure 13.6(7).

The actual node locations are determined using geometry as shown in Figure 13.6(7). The node at C is $650.1/2 = 325.05$ mm from the top of the girder, which is within 50 mm of the initial 300 mm used for the design, and the nodes at A and B are $641.7/2 = 320.85$ mm from the bottom of the girder, which is also within 50 mm of the 350 mm initially selected.

If these nodes were much further apart, new initial node locations would have to be selected and every quantity recalculated until the differences were appropriate.
Determine steel in the tie

For the force in tie AB, $F_{AB} = 4312.5$ kN, the area of steel should be

$$A_{st} \geq \frac{F_{AB}}{\phi f_{yw}} = \frac{4312.5 \times 10^3}{0.85 \times 500} = 10147.1 \text{ mm}^2$$

Use four rows of four N32 bars in each row or $A_{st} = 12864 \text{ mm}^2 > 10147.1 \text{ mm}^2$, which is acceptable.

The tension tie reinforcement of four rows of four N32 bars in each row spaced at 64 mm centre to centre is shown in Figure 13.6(8).

---

**Figure 13.6(7)** STM design example – STM and node geometry

**Figure 13.6(8)** STM design example – tension tie reinforcement
The centroid of the tie reinforcement should line up with the node location; that is, the centroid of the bottom tie reinforcement should start 254 mm above the bottom of the girder. Thus, the distance of the centroid of tie reinforcement from the bottom of the girder = 350 mm and as a result \( d = 2250 - 350 = 1900 \) mm.

The total effective height of the reinforcement = 350 + 2 rows of bars @ 32 mm + 1.5 rows of spacing @ 32 mm = 462 mm.

Check against the height of the tie which is 350 \( \times \) 2 = 700 mm > 462 mm. It is acceptable.

**Determine bursting reinforcement in bottle-shaped struts**

The angle between stirrups and struts, \( \alpha = 90^\circ - 32.6^\circ = 57.4^\circ \) for which \( \tan \alpha = 1.76 > 1/2 \) for serviceability and 1/5 for strength. These are acceptable.

Because of symmetry, the left or right shear span (see Figure 13.6(7))

\[ a = 2500 \text{ mm} \]

The component normal to the shear span

\[ z = 2250 - 650.1/2 - 641.7/2 = 1604.1 \text{ mm} \]

Thus, with \( d_{CA} = d_{CB} = 761.9 \text{ mm} \), Equation 13.6(4) gives

\[ l_z = \sqrt{1604.1^2 + 2500^2} - 761.9 = 2208.5 \text{ mm} \]

For strength, with \( \tan \alpha = 1.76 \), the bursting force

\[ T^* = f_{str} \times \tan \alpha = 5120.1 \times 1.76 = 9011.4 \text{ kN} \]

and the area of steel

\[ A_s \geq \frac{T^*}{(\phi f_y)} = \frac{9011.4 \times 10^3}{(0.85 \times 500)} = 21203 \text{ mm}^2 \]

At the loading point where the bursting crack forms, the force carried by the concrete and to be transferred to the bursting reinforcement is given by Equation 13.6(3) as

\[ T_{s_*} = 0.7b_i f'_{ct} = 0.7 \times 600 \times 2208.5 \times (0.36 \times \sqrt{50}) \times 10^{-3} = 2361 \text{ kN} \]

As per Clause 7.2.4 of the Standard, since \( T_{s_*} > T_{b,cr} \), only the minimum web reinforcement is required or

\[ A_{s,min} = \frac{T_{b,cr}}{\phi f_y} = \frac{2361 \times 10^3}{0.85 \times 500} = 5555.3 \text{ mm}^2 \]

Comparing the above \( A_s \) and \( A_{s,min} \), the minimum bursting reinforcement governs. Thus, the area of bursting reinforcement normal to strut CA or CB is:

\[ A_{st} = 5555.3 \text{ mm}^2 \]
Thus the reinforcement ratio $\rho = A_s / h_t = 5555.3 / (2208.5 \times 600) = 0.00419$.

The force across the bursting plane is maintained if orthogonal reinforcement is placed parallel and normal to the axis of the member such that

$$p_{vh} = p_v \sin \theta = 0.00419 \times \sin 32.6^\circ = 0.00205$$

$$p_{vh} = p_v \cos \theta = 0.00419 \times \cos 32.6^\circ = 0.00365$$

The vertical web reinforcement in each shear span, considering bursting reinforcement, is $p_{vh} = 0.00365$. Adopting two layers (one layer at each face) of 16-mm-diameter bars, with a total bar area across the 600-mm-thick section of 402 mm$^2$, gives a bar spacing requirement of

$$s = 402 / (0.00365 \times 600) = 183.6 \text{ mm}$$

Use N16 stirrups at 175 mm spacing for the vertical reinforcement for the total span.

For the horizontal reinforcement, $p_{vh} = 0.00205$. Using two layers of N16 bars ($A_s = 402 \text{ mm}^2$) gives a bar spacing requirement of

$$s = 402 / (0.00205 \times 600) = 326.8 \text{ mm}$$

Use two layers of N16 bars at 300 mm spacing as the longitudinal reinforcement for the total span.

**Final design**

The completed design of the girder is shown in Figure 13.6(9), with dimensions and the reinforcement.

![Figure 13.6(9) STM design example – final design section](image-url)
CHAPTER 17

17.5.2 Illustrative example

Problem

For the section with unbonded tendons shown in Figure 17.5(1), compute the ultimate moment $M_u$ using Equation 17.3(8).

![Figure 17.5(1)](image)

**Figure 17.5(1)** Details of a partially prestressed beam with unbonded tendons

Note: all dimensions are in mm.

- $f'_c = 40 \text{ MPa}$ ($\alpha_2 = 0.79$; $\gamma = 0.87$) $A_{pt} = 405 \text{ mm}^2$ unbonded
- $f_{ps} = 1710 \text{ MPa}$; $f_p = 1850 \text{ MPa}$ $\sigma_{pt} = \eta f_{ps} = 1100 \text{ MPa}$
- $L/D < 35$

Solution

Using Equation 17.3(10), determine that

$$\sigma_{pu} = 1100 + 70 + 40 \times 200 \times 625 / (100 \times 405)$$

$$= 1293.5 \text{ MPa} < 1100 + 400 = 1500 \text{ MPa}$$

This is acceptable.

Now using Equation 17.3(1)

$$k_d = (1293.5 \times 405 + 500 \times 1356) / (0.79 \times 40 \times 0.87 \times 405) = 218.6 \text{ mm}$$

and Equation 17.3(2) gives

$$d = (1293.5 \times 405 \times 625 + 500 \times 1356 \times 700) / (1293.5 \times 405 + 500 \times 1356)$$

$$= 667.3 \text{ mm}$$

However, $d_0 = 700 \text{ mm}$, and
\[ k_{\omega} = \frac{218.6}{700} = 0.312 < 0.36 \]

which is acceptable.

The section is considered under-reinforced and complies with Equation 17.4(1).

Finally, Equation 17.3(8) gives

\[ M_u = \left[ 1293.5 \times 405 \times 625 + 500 \times 1356 \times 700 - 0.79 \times 40 \times 200 \right] \times (0.87 \times 218.6)^2 / 2 \times 10^{-6} = 687.7 \text{ kNm} \]

17.6.2 Illustrative example

Problem

For the standard bridge-beam section as adopted in AS 5100.5-2017 and shown in Figure 17.6(3), compute the ultimate moment based on relevant recommendations from the Standard. All given values are identical to those prescribed for the rectangular beam in Section 17.5.2, except that \( A_{pt} = 910 \text{ mm}^2 \) and the cable is bonded.

![Figure 17.6(3) Standard bridge-beam section as per AS 5100.5–2017](image)

Note: all dimensions are in mm.

Solution

From the following equations we have:

Equation 17.3(5): \( k_1 = 0.28 \)

Equation 17.3(6): \( k_2 = (1850 \times 910 + 500 \times 1356) / (200 \times 625 \times 40) \)

\[ = 0.4723 \]

Equation 17.3(4): \( \sigma_{pc} = 1850 \times (1 - 0.28 \times 0.4723 / 0.87) = 1568.8 \text{ MPa} \)

Equation 17.3(2): \( d = (1568.8 \times 910 \times 625 + 500 \times 1356 \times 700) / (1568.8 \times 910 + 500 \times 1356) = 649.1 \text{ mm} \)

Equation 17.6(3): \( A' = (1568.8 \times 910 + 500 \times 1356) / (0.79 \times 40) \)

\[ = 66,633 \text{ mm}^2. \]
Equating the above $A'$ with the dimensions of the concrete area in compression, as shown in Figure 17.6(4), we have

$$66\,633 = 200 \times 100 + (120 + 200) \times 40/2 + (\gamma k_d - 140) \times 120$$

from which $\gamma k_d = 475.3$ mm. Thus

$$k_u = 475.3 / (0.87 \times 649.5) = 0.84 \text{ and } k_w = \frac{0.84 \times 649.5}{700} = 0.78 > 0.36$$

Imposing $k_{uw}= 0.36$ as required by Equation 17.4(1) leads to

$$\gamma k_d d = 0.87 \times 0.36 \times 700 = 219.2 \text{ mm}$$

![Figure 17.6(4) Concrete area in compression](image)

**Figure 17.6(4) Concrete area in compression**

Note: all dimensions are in mm.

With reference to Figure 17.6(4) we can obtain

$$A''_c = (219.2 - 140) \times 120 + (120 + 200) \times 40/2 + 200 \times 100 = 35,904 \text{ mm}^2$$

By taking moment about the top edge of the section, the location of the compressive force $C$ (see Figure 17.6(4)), can be computed as

$$d''_c = [200 \times 100 \times 50 + 120 \times 40 \times 120 + (2 \times 40 \times 40/2) \times (100 + 40/3) + 79.2 \times 120 \times (140 + 79.2 / 2)]/A''_c = 96.5 \text{ mm}$$

Hence

Equation 17.6(4): $A_{pt,ef} = (0.79 \times 40 \times 35,904 - 500 \times 1356)/1568.8 = 291.0 \text{ mm}^2$

Equation 17.6(5): $M_{ud} = (1568.8 \times 291.0 \times 625 + 500 \times 1356 \times 700 - 0.79 \times 40 \times 35,904 \times 96.5) \times 10^{-6} = 650.4 \text{ kNm}$

Finally, to comply with the ductility requirement, the required compression steel is given by Equation 17.4(4) as

$$A_{sc} = 0.01 A''_c = 359 \text{ mm}^2$$

Therefore, we can use two N16 or 402 mm².