Numerical Investigation of the Wave/Current–Induced Responses of Transient Soil around a Square Mono-Pile Foundation

Lunliang Duan†, Dong-Sheng Jeng,‡*†, Shaohua Wang†, and Bing Zhu†

†Department of Bridge Engineering
Southwest Jiaotong University
Chengdu, China

‡School of Engineering and Built Environment
Griffith University Gold Coast Campus
Queensland, Australia

ABSTRACT


A mono-pile foundation is a commonly used supporting base for marine infrastructure in shallow water, such as offshore wind turbine foundations and cross-sea bridges and platforms. Sea floor instability around a pile foundation has attracted much attention among offshore geotechnical engineers. In this paper, a three-dimensional numerical model for wave–seabed–structure interactions is proposed using open field operation and manipulation (OpenFOAM). Unlike previous studies, currents are included in the proposed model and applied to a square cross-section mono-pile foundation. In the model, the flow motion is described by the Reynolds-averaged Navier–Stokes (RANS) equation and k–ε turbulence model, and the water free surface is tracked using the volume of fluid (VOF) method. Meanwhile, the soil response is determined by the Biot’s poro-elastic equation (u–p approximation), in which the accelerations of the soil displacements are included. After the model is validated with the previous laboratory experiments, it is further used to investigate the transient soil response around the square mono-pile foundation. Numerical results demonstrate that the cross-section shape has significant influence on the distribution of wave/current–induced transient pore pressure but little influence on the maximum soil liquefaction depth. On the other hand, both the current velocity and the seabed permeability can greatly affect the maximum soil liquefaction depth and slightly affect the distribution of pore pressure. Furthermore, the cross-section dimension significantly affects both the maximum soil liquefaction depth and the distribution of the pore pressure around a mono-pile foundation.

ADDITIONAL INDEX WORDS: Wave-seabed-pile interactions-3D, wave-current interactions, square cross section, OpenFOAM, RANS, liquefaction.

INTRODUCTION

Piled foundations play an important role in transporting load from offshore structures to a porous seabed. However, seabed instability usually occurs in the vicinity of the pile foundation under various hydrodynamic actions, which have been recognized as a major engineering issue that needs to be taken into consideration in the design of offshore foundations. Under cyclic wave loadings, the pore pressure and the effective stresses within the seabed can be induced. Once the effective stresses between the soil particles vanish, the soil will be liquefied, and the water–sediment mixture will behave like a heavy fluid.

Generally speaking, two mechanisms for wave/current–induced soil dynamic response have been reported in the literature (Jeng, 2013; Sumer, 2014; Zen and Yamazaki, 1990). One is the transient soil response, which is caused by the pressure gradient within the seabed in the vertical direction. The other is the residual soil response, which is attributed to the build-up of the pore pressure. The transient soil dynamic response normally appears at the beginning of the cyclic loading. In this study, only transient soil response is considered. For the residual mechanism, readers can refer to Jeng (2013) and Sumer (2014).

Based on the Biot’s poro-elastic model (Biot, 1941, 1956), numerical and theoretical studies for the wave-induced soil response have been carried out since 1970s. Three different approaches have been adopted in the past. They are as follows.

Quasi-Static Model

This approach is based on the consolidation theory (Biot, 1941), in which the acceleration of both soil and fluid displacement are ignored. This approach was first used by Yamamoto et al. (1978), in which an analytical solution is proposed for progressive wave-induced transient soil response in an infinite seabed. Similar work was done by Madsen (1978) for a hydraulically anisotropic seabed. This framework was further extended to a three-dimensional short-crested wave system by Hsu and Jeng (1994). These studies directly solve the boundary value problem and obtain closed-form solutions. Another approximation, boundary-layer approximation, was proposed by Mei and Foda (1981), in which two domains were divided through scale analysis. In the region near the seabed surface (i.e. inner boundary layer), the full solution needs to be considered. For the region outside the boundary layer, the simplified solution is used. The advantage of this approach is simple and identical to previous closed-form analytical solu-
ions for fine sand, but significant differences were found for coarse sand (Hsu and Jeng, 1994). Based on the quasi-static (QS) model, some numerical models are established to study the soil response around the pipelines (Duan et al., 2017b) and breakwaters (Liao, Tong, and Chen, 2018; Liao et al., 2018).

**u–p Approximation**

In this model, the accelerations due to soil displacements are included. It is based on Biot (1956), and simplified by Zienkiewicz, Chang, and Bettess (1980) for earthquake loading. The model was adopted for wave loading by Jeng and Rahman (2000) for an infinite seabed. Recently, this model was used to study the progressive and nonlinear standing wave–induced liquefaction in a loosely deposited seabed (Yang and Ye, 2017, 2018). Based on u–p approximation, numerical models have also been developed and applied to cases of pipelines (Duan et al., 2017a; Dunn et al., 2006) and breakwaters (Jeng et al., 2013; Zhang et al., 2012).

**Full Dynamic Model**

In this model, in addition to u–p approximation, the acceleration due to the relative displacement of pore fluid to solid is considered. This is a full dynamic model. Jeng and Cha (2003) may have been the first to adopt a full dynamic model to the wave-induced soil response. This framework has been further applied to cases with caissons (Ulker, Rahman, and Guddati, 2012).

The applicable range of the above three approaches (QS, u–p, and full dynamic) was first identified by Jeng and Cha (2003), and a relationship between wave and seabed characteristics was proposed. Ulker and Rahman (2009) further clarified such a relationship with different soil. These studies can provide theoretical workers a guide of which model should be used.

In natural environments, waves and currents commonly coexist. However, most previous studies for seabed response only considered wave loading. Recently, numerous studies investigated the wave–current interaction (WCI) mechanisms and the wave characteristics caused by wave–current interactions (Umeyama, 2011; Zhang et al., 2014). Meanwhile, the seabed responses under combined wave and current loading are also examined. Among these, Ye and Jeng (2012) may have been the first study in the literature to consider the effects of combined wave and current loading on the seabed response. In their study, the u–p approximation was used together with the third-order analytical solution for wave–current interactions (Hsu et al., 2009). Effects of currents on the seabed response including momentary liquefaction were demonstrated in Ye and Jeng (2012). Similar works with analytical solutions and with various models for combined wave and current loading have been reported in the literature (Liao, Jeng, and Zhang, 2015; Liu, Jeng, and Zhang, 2014; Zhang et al., 2013). However, those investigations mainly focused on the wave/current–seabed interaction without a structure.

When the pile foundation is included in the Wave-Seabed-Structure Interactions system, the problem will become extremely complicated due to the turbulence flow around the structure (Kim, 1993; Li and Lin, 2001; Lin and Li, 2003). A large number of investigations have been performed to classify the mechanism of wave–seabed–pile foundation interaction (WSPI). For example, Li et al. (2011) studied the pore pressure distribution under linear wave loading using the Comsol model, where both transient pore pressure and residual pore pressure were included. Later, Chang and Jeng (2014) investigated the wave–seabed–pile foundation interaction by a 3D integrated numerical model (WSPI-3D), in which the inclined pile was considered. Sui et al. (2016) established an integrated model to explore the small steepness wave-induced transient soil liquefaction around a mono-pile foundation, where the FUNWAVE (Fully nonlinear Boussinesq wave model) code was used to govern the wave motions. Recently, Lin et al. (2017) proposed a 3D numerical model within the framework of open field operation and manipulation (OpenFOAM) to study transient soil dynamic response around a mono-pile foundation due to nonlinear wave action, where the quasi-static Biot’s equation (QS) was applied to govern the transient soil dynamic response. Zhang et al. (2017) examined the transient soil liquefaction around the group-pile foundation due to wave loading. The aforementioned studies are helpful in clarifying the mechanism of transient soil dynamic response around a pile foundation due to wave loading. Nevertheless, all of these studies only considered the wave loading, although the wave and current usually exist simultaneously in the real oceanic environment.

To the author’s best knowledge, the phenomenon of wave/current–induced transient soil response around a piled foundation with a square section has not been fully understood, although square piles have been used as marine structure foundations (Li and Li, 2016). This paper aims to broaden the understanding of wave/current–induced transient soil dynamic response around a square cross-section piled foundation. In particular, a 3D integrated model for wave/current–seabed-structure interaction is developed within the framework of the OpenFOAM, in which the flow model and the Biot’s poro-elastic (u–p approximation) model are incorporated. With this model, the effects of the pile cross-section shape on wave/current–induced transient soil dynamic response will be clarified first. Then the effects of current velocity, wave/current obliquity, and the pile cross-section dimension on wave/current–induced transient soil liquefaction will be examined. The present study can provide some references to predict the stability of the square mono-pile foundation in the marine environment.

**METHODS**

In this study, the 3D numerical model can be divided into two submodels: the flow model and the seabed model. The dynamic wave pressure at the surface of the seabed can be obtained from the flow model first, then it is treated as the seabed surface boundary condition in the seabed model to further determine flow-induced transient soil response around a pile.

As depicted in Figure 1, the phenomenon of wave/current–seabed–pile interactions is considered in this study, in which the pile cross-section shape is square. The parameter \( z \) denotes the angle of the wave/current obliquity; \( h_b \) denotes the penetration depth; \( d \) denotes the water depth; \( h_s \) denotes the seabed depth; \( D \) denotes the cross-section dimension; \( W \) denotes the seabed width; \( L \) denotes the length of computing domain, which is set to four times wave length; the distances from points \( A, B, C, E, F \) to pile center are equal to the cross-section dimension.
Flow Model

In this study, the flow is established using IHFOAM developed by the Institute of Hydraulics, university of Cantabria (Higuera, Lara, and Losada, 2013a,b), in which the Reynolds-Averaged Navier–Stokes (RANS) equation is used to govern the flow motions, and the modified volume of fluid (VOF) method is used to track the water free surface. In this three-dimensional flow model, the governing equations can be expressed as:

$$\frac{\partial (\rho u_i)}{\partial t} = 0$$

(1)

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial }{\partial x_j} \left( \mu \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial \rho}{\partial x_j} \left( -\rho \left( \mu u_i \mu' j \right) \right) + \rho g_l$$

(2)

where, $u_i$ and $u_j$ denote the ensemble averaged velocity; $x_i$ and $x_j$ denote the Cartesian coordinates; $\rho$ denotes the water density; $t$ denotes the time; $p$ denotes the water pressure; $\mu$ denotes the kinematic viscosity; $g_l$ denotes the gravity acceleration; $-\rho \left( \mu u_i \mu' j \right)$ denotes the Reynolds stress term, which can be estimated by:

$$-\rho \left( \mu u_i \mu' j \right) = \mu_0 \left( \frac{\partial (u_i)}{\partial x_j} + \frac{\partial (u_j)}{\partial x_i} \right) - \frac{2}{3} \rho \delta_{ij} k$$

(3)

in which, $\mu_0$ denotes the turbulent viscosity; $\delta_{ij}$ denotes the Kronecker delta, and $k$ denotes the turbulence kinetic energy (TKE). Based on Equation (3), Equation (2) can be written as:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = \frac{\partial }{\partial x_j} \left( \mu_0 \frac{\partial u_i}{\partial x_j} \right) + \rho g_l$$

(4)

where, $\mu_0 = \mu + \mu_t$ denotes the total effective viscosity.

The standard $k$–$\varepsilon$ turbulence model used to make the governing equations enclosed in this study can be written as (Lauder and Spalding, 1974):

$$\frac{\partial \rho k}{\partial t} + \frac{\partial (\rho u_i k)}{\partial x_j} = \frac{\partial }{\partial x_j} \left( \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right) + \rho P_k - \rho \varepsilon$$

(5)

$$\frac{\partial \rho \varepsilon}{\partial t} + \frac{\partial (\rho u_i \varepsilon)}{\partial x_j} = \frac{\partial }{\partial x_j} \left( \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right) + \frac{\varepsilon}{k} (C_{1\varepsilon} P_k - C_{2\varepsilon} \rho \varepsilon)$$

(6)

where, $k$ denotes the TKE; $\varepsilon$ denotes the dissipation ratio of the TKE; and other parameters are constant values: $\sigma_k = 1.00$, $C_{1\varepsilon} = 0.09$, $\sigma_\varepsilon = 1.30$, $C_{1\varepsilon} = 1.44$, $C_{2\varepsilon} = 1.92$.

The modified VOF method used to accurately track the water free surface can be expressed as (Hirt and Nichols, 1981):

$$\frac{\partial \alpha}{\partial t} + \frac{\partial \alpha u_i}{\partial x_i} = \frac{\partial (1 - \alpha) u_i}{\partial x_i} = 0$$

(8)

where, $|u_i| = \min[c_s |u_i|, \text{max}(|u_i|)]$, and $\alpha$ is used to define the quantity of water per unit volume in each cell.

In the flow model, the static boundary wave generation method is used to generate waves. Therefore, the wave generation involves setting the values of velocity and the VOF function at the inlet side. To avoid the influence of wave reflection, the “full 3D absorption” boundaries in IHFOAM are applied at the outlet side, which can provide a reasonable fair wave absorption, as suggested by Higuera, Lara, and Losada (2013a).

Seabed Model

In the seabed model, Biot’s poro-elastic equation (Biot, 1956; Zienkiewicz, Chang, and Bettess, 1980) is adopted to govern the transient soil dynamic response, and the porous seabed is assumed to be unsaturated, elastic, and hydraulically permeable media. The governing equation used in this study can be expressed as:

$$\nabla^2 P_w - \frac{r_n n_s}{K_w} \frac{\partial^2}{\partial x^2} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right) = \frac{r_n}{K_w} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right)$$

(9)

in which, $\nabla^2$ denotes the Laplace’s operator; $P_w$ denotes the flow-induced transient pore pressure; $r_n$ denotes the unit weight of water; $n_s$ denotes the soil porosity; $k$ denotes the soil permeability; and $u_x, u_y, u_z$ denote the soil displacements along the $x, y, z$ directions respectively. $\beta$ denotes the pore-water compressibility, which can be expressed as:

$$\beta = \frac{1}{K_w} \left[ 1 - S_{sat} \right]$$

(10)

where, $K_w$ denotes the true modulus; $P_{abs}$ denotes the absolute water pressure; and $S_{sat}$ denotes the degree of seabed saturation.

The equations of force equilibrium can be expressed as the following:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \frac{\partial p_s}{\partial x} + \rho \frac{\partial^2 u_x}{\partial t^2}$$

(11)

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = \frac{\partial p_s}{\partial y} + \rho \frac{\partial^2 u_y}{\partial t^2}$$

(12)
\[
\begin{align*}
\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \sigma'_z}{\partial z} &= \frac{\partial p_s}{\partial z} + \rho \frac{\partial^2 u_s}{\partial t^2} \\
\frac{\partial \tau_{xy}}{\partial y} &= \frac{\partial \sigma'_x}{\partial x} = 0
\end{align*}
\]  

(13)

where, \( \tau_{xx}, \tau_{xy}, \) and \( \tau_{yy} \) denote the shear stresses and \( \sigma'_x, \sigma'_y, \) and \( \sigma'_z \) denote the effective stresses along the \( x, \) the \( y, \) and the \( z \) directions, respectively.

Based on the generalized Hooke’s law, the relationships between effective stresses and soil displacements could be written by:

\[
\begin{align*}
\sigma'_x &= 2G \left[ \frac{\partial u_x}{\partial x} + \frac{\mu}{1-2\mu} \left( \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right) \right] \\
\sigma'_y &= 2G \left[ \frac{\partial u_y}{\partial y} + \frac{\mu}{1-2\mu} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) \right] \\
\sigma'_z &= 2G \left[ \frac{\partial u_z}{\partial z} + \frac{\mu}{1-2\mu} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) \right] \\
\tau_{xx} &= G \left[ \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right] = \tau_{xx} \\
\tau_{xy} &= G \left[ \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right] = \tau_{xy} \\
\tau_{yy} &= G \left[ \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right] = \tau_{yy}
\end{align*}
\]  

(14)–(19)

in which, \( G \) denotes the shear modulus. Substituting Equations (14)–(19) into Equations (11)–(13), the force equilibrium equations become:

\[
\begin{align*}
GV^2 u_s + G \frac{\partial}{\partial x} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right) &= \frac{\partial p_s}{\partial x} + \rho \frac{\partial^2 u_s}{\partial t^2} \\
GV^2 v_s + G \frac{\partial}{\partial y} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right) &= \frac{\partial p_s}{\partial y} + \rho \frac{\partial^2 v_s}{\partial t^2} \\
GV^2 w_s + G \frac{\partial}{\partial z} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right) &= \frac{\partial p_s}{\partial z} + \rho \frac{\partial^2 w_s}{\partial t^2}
\end{align*}
\]  

(20)–(22)

where, the second term on the right side of the above three equations denotes the inertial acceleration of the soil particles, which only appear in the \( u-p \) model, not the QS model.

Boundary Conditions

The flow-induced soil dynamic response can be derived by solving the seabed governing equations under certain boundary conditions. Thus, boundary conditions for flow model and boundary conditions for seabed model should be specified.

As mentioned before, the HIFoAM solver is applied in the flow model in this study, which has been widely used to study the problem of flow–structure interactions (Higuera, Lara, and Losada, 2014a,b). The details of the flow model boundary condition can be found in Higuera, Lara, and Losada (2013a), and this paper will only specify the seabed boundary conditions.

The wave pressure at the seabed surface derived from the flow model will be taken as the seabed surface boundary condition in the seabed model to further determine the soil dynamic response. In addition, the seabed bottom boundary conditions and pile–seabed interface boundary conditions should also be specified.

Seabed Surface Boundary Conditions

In this model, the shear stresses as well as the vertical effective normal stresses disappear at the seabed surface, in other words:

\[
\tau_{xx} = \tau_{xy} = \sigma'_z = 0
\]  

(23)

Moreover, the pore pressure at the seabed surface should be equal to the wave pressure.

Seabed Bottom Boundary Conditions

Assuming that the seabed bottom is impermeable and rigid, both the vertical flow and the soil displacements vanish at the seabed bottom, in other words:

\[
p_s = u_s = v_s = w_s = 0
\]  

(24)

Seabed–Pile Foundation Interface Boundary Conditions

In this study, the pile foundation is considered to be a rigid object, and the soil–structure interface is considered to be impermeable. Therefore, the pore pressure gradient should be zero at the seabed–pile foundation interface, in other words:

\[
\frac{\partial p_s}{\partial n} = 0
\]  

(25)

Integration Process of Flow Model and Seabed Model

As mentioned previously, the present model is composed of two submodels: the flow model and the seabed model. The flow model is responsible for the flow simulations and flow–structure interactions, while the seabed model is responsible for the flow-induced transient soil dynamic response. After the wave parameters and current velocity are specified in the flow model, the wave pressure at the seabed surface can be obtained from the flow model. Then it is taken as the boundary condition of the seabed model to further determine wave/current–induced transient pore pressure and soil displacements by solving the \( u-p \) equations. Therefore, a data exchange should be required at the interface of the flow model and the seabed model, and the details of the integrated process are illustrated in Figure 2.

RESULTS

In this section, the model validation will be performed first by comparing the numerical results with previous experimental data available in the literature. After validation, the numerical model will be further applied to investigate the differences between the transient soil response around the circular monopile and that around the square mono-pile.

Model Validation

As mentioned previously, it is necessary to conduct a model validation before it is applied to investigate a wave/current–induced transient soil dynamic response. In this part, the newly developed 3D integrated model will be validated by previous laboratory experiments, and the validations in this paper include three cases: (1) Comparison of the presented
numerical model with the laboratory experiments for the wave–current interactions without a structure. (2) Comparison of the presented numerical model with the laboratory experiments for the wave-induced transient pore pressure in a sandy seabed without a cylinder. (3) Comparison of the presented numerical model with the laboratory experiments for the wave/current–induced transient pore pressure around a cylinder. Since there are no experimental data for square cross-section piles available in the literature, the newly developed model is only compared with a circular cylinder.

The first validation is to compare the numerical results with the laboratory experiments conducted by Umeyama (2011), in which both the porous seabed and the structure are not involved. Umeyama (2011) studied the free water waves propagating with a current through a series of laboratory experiments in a wave flume, where a piston-type wave maker is used to generate waves and a pipe under the tank is used to generate currents.

Figure 3 illustrates the comparison results of the experimental and simulated water free surface profiles under the same wave and current conditions. Figure 3 shows that the numerical model can provide a satisfactory prediction for wave–current interaction.

The second validation is to compare the simulated wave-induced pore pressure in a porous seabed with the laboratory experiments by Liu et al. (2015), in which 10 pore pressure points are fixed along the seabed depth of 1.8 m. More details of the laboratory experiments can be found in Liu et al. (2015).

Figure 4 displays the comparison results of the simulated and experimental wave-induced pore pressure in the porous seabed. As shown in the figure, the simulated results and the experimental data (Liu et al., 2015) have the same trend.

In the third validation, the laboratory experiments by Qi and Gao (2014a, personal communication) are selected to validate the presented model for wave/current–induced soil response, in...
which the wave flume has a length of 52 m and a cross section of 1 m wide by 1.5 m deep. In this validation, the measured point next to the pile foundation (2 mm away from the pile) is selected to validate the wave/current–induced pore pressure, in which the effects of structure on soil response can be included.

Figure 5 illustrates the comparison results of the simulated and experimental wave/current–induced pore pressure within the sandy bed. As shown in the figure, the trend of the simulated results overall agrees with experimental results (Qi and Gao, 2014a, personal communication).

Based on the above three validations, the presented numerical model within the framework of OpenFOAM can accurately predict the wave/current–induced soil dynamic response around a circular mono-pile foundation.

Comparison of Transient Soil Response around the Circular and Square Cross-Section Piles

As reported in previous studies, the existence of the marine structure can greatly affect the wave propagation profiles around the structure due to the blockage effects, which may further influence the wave/current–induced soil dynamic response within the porous seabed. Therefore, the distribution of the water free surface as well as the wave/current–induced transient pore pressure around a mono-pile foundation will be examined first in this section.

To compare the water free surface around the circular and square cross-section pile foundation, Figure 6 presents the variations of the water free surface around the circular mono-pile foundation with time, and Figure 7 illustrates the variations of the water free surface around the square mono-pile foundation with time in a complete wave period. The pile cross-section areas in Figures 6 and 7 are equivalent, to make sure the axial bearing capacities are the same. By comparing Figure 6 with Figure 7, the cross-section shape can greatly affect the water free surface near the mono-pile foundation, and the phenomenon of waves running up in front of the square pile foundation is more obvious than in front of the circular pile.

In order to clarify the difference of the wave/current–induced transient soil dynamic response around the circular and square cross-section pile foundation, Figure 8 shows the distribution of wave/current–induced pore pressure, and Figure 9 illustrates the distribution of the soil liquefaction depth around the mono-pile foundation. As shown in Figure 8, the pore pressure in front of the mono-pile foundation is larger than that in other zones regardless of the cross-section shape. The distribution of wave/current–induced transient pore pressure and the soil liquefaction around the mono-pile foundation can be greatly

Figure 5. Validation of the variations of wave/current–induced pore pressure ($p_s$) at the gauge 2 with time ($t$) against the experimental data (Qi and Gao, 2014a; Qi and Gao, personal communication). Input data: $H = 0.05$ m, $d = 0.5$ m, $T = 1.0$ s, $U = 0.05$ m/s, $G = 1.0 \times 10^3$ N/m$^2$, $S_r = 1.0$, $\mu = 0.3$, $n = 0.771$, $k_s = 1.8 \times 10^3$ m/s.

Figure 6. Variations of the water free surface around a circular mono-pile foundation with time in a complete wave period.

Figure 7. Variations of the water free surface around a square mono-pile foundation with time in a complete wave period.

Figure 8. Distribution of wave/current–induced transient pore pressure in the porous seabed around a mono-pile foundation: (a) circular mono-pile foundation, (b) square mono-pile foundation.
affected by the cross-section shape. This is because the cross-section shape can influence the boundary conditions around the structure in the flow model and the seabed model. Moreover, the cross section can also influence the distribution of the initial effective stress within the porous seabed, which is a key factor to determine the soil liquefaction. Another obvious conclusion from Figures 8 and 9 is that the effects of the cross-section shape on wave/current--induced soil liquefaction depth are relatively small.

Comparing the circular cross-section pile foundation and the square cross-section pile foundation shows that the transient soil dynamic response can be significantly affected by the cross-section shape. Since the wave-induced soil response around the circular cross-section pile foundation has been extensively studied previously, the following investigations mainly focus on the square cross-section mono-pile foundation.

**DISCUSSION**

In this section, four issues will be discussed in detail with the new integrated model: effects of current on wave/current--seabed interactions, effects of wave obliquity on seabed response, effects of seabed permeability on soil liquefaction, and effects of pile configuration on wave/current--induced pore pressure/liquefaction. The parameters used in the following numerical examples can be found in Table 1 unless otherwise specified, and the wave/current--induced transient soil liquefaction can be determined with the following criterion (Zen and Yamazaki, 1990):

$$\sigma'_0 - (p_s - p_b) \leq 0$$  \hspace{1cm} (26)

in which, $\sigma'_0$ denotes mean initial effective stress after preconsolidation, $p_s$ denotes wave/current--induced transient pore-water pressure, and $p_b$ denotes water pressure at the seabed surface.

**Effects of the Current Velocity on Transient Soil Response around a Square Pile**

As mentioned previously, the wave and current generally exist simultaneously in the real oceanic environment. The wave characteristics, such as wave height and wave length, may be greatly affected after the strongly nonlinear interaction between the ocean wave and marine current. Therefore, it is of great interest to investigate the effects of the current velocity on the wave/current--induced soil dynamic response around the square mono-pile foundation.

To examine the effects of current velocity on wave/current--induced transient soil dynamic response, the current velocity ($U$) in this section varies from −1 m/s to 1 m/s with an increment of 0.5 m/s, in which the positive current velocity means that the current travels following the wave propagation direction, and the negative current velocity means that the current travels against the wave propagation direction. The wave/current--induced transient pore pressure is a key factor to determine the soil stability in the vicinity of the marine structure, and Figure 10 displays the distribution of maximum pore pressure along the vertical line through points A, B, C, E, F, respectively. As shown in Figure 10, the maximum pore pressure at the above five points increases incrementally with the current velocity ($U$). Moreover, the pore pressure at point A is larger than that

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Scale for Numerical Computations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave height $H$ (m)</td>
<td>2 or various</td>
</tr>
<tr>
<td>Water depth $d$ (m)</td>
<td>6</td>
</tr>
<tr>
<td>Wave period $T$ (s)</td>
<td>5</td>
</tr>
<tr>
<td>Wave length $L$ (m)</td>
<td>32.173</td>
</tr>
<tr>
<td>Current velocity $U$ (m/s)</td>
<td>0.5</td>
</tr>
<tr>
<td>Sealed thickness $h$ (m)</td>
<td>20</td>
</tr>
<tr>
<td>Shear modulus $G$ (N/m²)</td>
<td>10⁶</td>
</tr>
<tr>
<td>Poisson’s ratio $\mu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Soil permeability $k_s$ (m/s)</td>
<td>10⁻⁵</td>
</tr>
<tr>
<td>Degree of saturation $S_r$</td>
<td>0.98</td>
</tr>
<tr>
<td>Pile cross-section dimension $D$ (m)</td>
<td>4</td>
</tr>
<tr>
<td>Burial depth of pile $h_b$ (m)</td>
<td>10</td>
</tr>
</tbody>
</table>
at other points regardless of current velocity ($U$). To provide a better understanding of the effects of the current velocity on the transient soil liquefaction, Figure 11 displays the distribution of the soil liquefaction depth around a square cross-section mono-pile foundation for different current velocities ($U$). Figure 11 shows that the liquefaction depth for the case where the current travels following the wave propagation direction is larger than that for the case where the current travels against the wave propagation direction due to the differences of the vortex structures around the mono-pile (Qi and Gao, 2014b). In other words, the horseshoe vortex can emerge at the upstream pile side, and wake vortices may be intensified on the downstream pile side when the current travels following the wave propagation direction. The horseshoe vortex will produce extra pressure on the bed and increase the amplitude of the bed wave pressure, which can further increase the liquefaction depth. Moreover, the soil liquefaction zone extends incrementally with the current velocity ($U$) when the current travels following the wave propagation direction.

![Figure 11. Variations of the soil liquefaction depth around a square mono-pile foundation with the current velocity ($U$): (a) $U = -1$ m/s; (b) $U = -0.5$ m/s; (c) $U = 0$ m/s; (d) $U = 0.5$ m/s; (e) $U = 1$ m/s.](image)

![Figure 12. Distribution of wave/current–induced transient pore pressure around a square mono-pile foundation at 1 m below the seabed surface under various wave/current obliquities ($\alpha$): (a) $\alpha = 0^\circ$; (b) $\alpha = 15^\circ$; (c) $\alpha = 30^\circ$; (d) $\alpha = 45^\circ$.](image)

![Figure 13. Distribution of wave/current–induced maximum pore pressure along the vertical line through points A, B, C, E, F, where the wave obliquity ($\alpha$) varies from 0° to 45° with an interval of 15°: (a) Point A; (b) Point B; (c) Point C; (d) Point E; (e) Point F.](image)
Effects of the Wave Obliquity on Transient Soil Response around a Square Pile

In addition to the current velocity ($U$), the wave obliquity ($\alpha$) should be another important factor to affect the wave/current–induced transient soil dynamic response. Therefore, the effects of the wave obliquity ($\alpha$) on the wave/current–induced transient soil dynamic response around a square cross-section pile foundation will be investigated in this section.

To investigate the effects of the wave obliquity ($\alpha$) on the wave/current–induced soil dynamic response around a square mono-pile, Figure 12 shows the distribution of wave/current–induced transient pore pressure at 1 m below the seabed surface, in which the wave obliquity ($\alpha$) varies from 0° to 45° with an interval of 15°. Meanwhile, Figure 13 displays the distribution of wave/current–induced maximum pore pressure along the vertical line through points A, B, C, E, F, respectively.

As shown in Figures 12 and 13, the distribution pattern of the wave/current–induced transient pore pressure around the square cross-section mono-pile foundation can be greatly affected by the wave obliquity ($\alpha$), and the maximum pore pressure along the points in the upstream zone is obviously larger than that in the downstream zone. Moreover, the wave/current obliquity ($\alpha$) has little influence on the maximum pore pressure at a certain point.

To provide a clearer understanding of the transient soil dynamic response under various wave obliquities ($\alpha$), Figure 14 displays the distribution of wave/current–induced soil liquefaction depth around a square mono-pile, and Figure 15 shows the variations of the maximum soil liquefaction depth with the wave obliquities ($\alpha$) under various current velocities ($U$). As shown in Figure 14, wave obliquity ($\alpha$) can greatly affect the distribution of the soil liquefaction depth, and the liquefaction zone next to the pile foundation also varies with the wave obliquity ($\alpha$). Figure 15 shows that the influence of wave obliquity ($\alpha$) on maximum soil liquefaction depth is small regardless of the current velocity. Similarly, the variation of the maximum soil liquefaction depth with the current velocity ($U$) almost has nothing to do with the wave obliquity ($\alpha$). In addition, another conclusion from Figure 15 is that as the current velocity ($U$) increases, the increment of wave/current–induced soil liquefaction depth grows slowly.

Effects of Seabed Permeability on the Transient Soil Response around a Square Pile

Seabed permeability, one of the most important soil properties, is normally used to describe the difficulty of pore fluid passing through pore skeletons. In this section, the effects of seabed permeability on the transient soil response to wave and current loadings will be investigated. Considering that the seabed is assumed to be an isotropic porous medium in this study, the seabed permeabilities along the $x$, $y$, and $z$ directions are the same. The values of seabed permeability are $k_x = 1 \times 10^{-6}$ m/s; $k_y = 1 \times 10^{-5}$ m/s; $k_z = 1 \times 10^{-4}$ m/s; and $k_x = 1 \times 10^{-5}$ m/s.

Figure 16 illustrates the distribution of wave/current–induced transient pore pressure around a square mono-pile foundation at 1 m below the seabed surface under various seabed permeabilities ($k_x$): (a) $k_x = 1 \times 10^{-6}$ m/s; (b) $k_x = 1 \times 10^{-5}$ m/s; (c) $k_x = 1 \times 10^{-4}$ m/s; (d) $k_x = 1 \times 10^{-5}$ m/s.

As shown in Figures 12 and 13, the distribution pattern of the wave/current–induced transient pore pressure around the square cross-section mono-pile foundation can be greatly affected by the wave obliquity ($\alpha$), and the maximum pore pressure along the points in the upstream zone is obviously larger than that in the downstream zone. Moreover, the wave/current obliquity ($\alpha$) has little influence on the maximum pore pressure at a certain point.

To provide a clearer understanding of the transient soil dynamic response under various wave obliquities ($\alpha$), Figure 14 displays the distribution of wave/current–induced soil liquefaction depth around a square mono-pile, and Figure 15 shows the variations of the maximum soil liquefaction depth with the wave obliquities ($\alpha$) under various current velocities ($U$). As shown in Figure 14, wave obliquity ($\alpha$) can greatly affect the distribution of the soil liquefaction depth, and the liquefaction zone next to the pile foundation also varies with the wave obliquity ($\alpha$). Figure 15 shows that the influence of wave obliquity ($\alpha$) on maximum soil liquefaction depth is small regardless of the current velocity. Similarly, the variation of the maximum soil liquefaction depth with the current velocity ($U$) almost has nothing to do with the wave obliquity ($\alpha$). In addition, another conclusion from Figure 15 is that as the current velocity ($U$) increases, the increment of wave/current–induced soil liquefaction depth grows slowly.

Effects of Seabed Permeability on the Transient Soil Response around a Square Pile

Seabed permeability, one of the most important soil properties, is normally used to describe the difficulty of pore fluid passing through pore skeletons. In this section, the effects of seabed permeability on the transient soil response to wave and current loadings will be investigated. Considering that the seabed is assumed to be an isotropic porous medium in this study, the seabed permeabilities along the $x$, $y$, and $z$ directions are the same. The values of seabed permeability are $k_x = 1 \times 10^{-6}$ m/s; $k_x = 1 \times 10^{-5}$ m/s; $k_x = 1 \times 10^{-4}$ m/s; and $k_x = 1 \times 10^{-5}$ m/s. Other wave/current and soil parameters can be found in Table 1.

Figure 16 illustrates the distribution of wave/current–induced transient pore pressure around a square mono-pile foundation at 1 m below the seabed surface under various...
seabed permeabilities \( k_s \): (a) \( k_s = 1 \times 10^{-6} \text{ m/s} \); (b) \( k_s = 1 \times 10^{-5} \text{ m/s} \); (c) \( k_s = 1 \times 10^{-4} \text{ m/s} \); (d) \( k_s = 1 \times 10^{-3} \text{ m/s} \). Figure 16 shows that the transient pore pressure increases as the seabed permeability increases; this is because the pore fluid in the seabed with a large seabed permeability can flow through a pore skeleton quickly under the periodic wave action, which can further lead to incremental increase in the transient pore pressure. Moreover, the transient pore pressure in front of the square pile is larger than that in other zones. To further examine the effects of the seabed permeability \( k_s \) on the soil liquefaction around the square pile, Figure 17 displays the variation of the maximum liquefaction depth around the square mono-pile with seabed permeability \( k_s \) under various current velocities \( U \). As shown in Figure 17, the maximum liquefaction depth around the square mono-pile increases incrementally with the seabed permeability regardless of the current velocity. In other words, the seabed around the square mono-pile is liquefied more easily when the seabed has a larger permeability.

Effects of Cross-Section Dimension on the Transient Soil Response around a Square Pile

As reported in the previous investigations, the configuration of the structure can greatly affect the wave field and the soil dynamic response in the vicinity of the structure. As for the square mono-pile foundation, the cross-section dimension \( D \) is one of the most important factors to affect the wave characteristics and the soil response. Therefore, the effects of the cross-section dimension \( D \) on wave/current–induced transient soil dynamic response will be studied in this section, in which the cross-section dimension \( D \) increases from 2 m to 8 m with an interval of 2 m.

Figure 18 displays the distribution of wave/current–induced transient pore pressure around the square mono-pile foundation at 1 m below the seabed surface for different cross-section dimensions \( D \). As shown in the figure, the cross-section dimension \( D \) can significantly affect the distribution pattern of wave/current–induced transient pore pressure, and the maximum pore pressure around the piled foundation increases with the increment of the cross-section dimension \( D \).

To provide a more comprehensive understanding of the effects of cross-section dimension \( D \) on the wave/current–induced soil liquefaction, Figures 19 and 20 show the distribution of wave/current–induced soil liquefaction zone around a square pile foundation under four different cross-section dimensions \( D \) and the variation of the maximum liquefaction depth with the cross-section dimension \( D \) for various current velocities \( U \). The figures show that the cross-section dimensions \( D \) not only affect the distribution pattern of the soil liquefaction zone but also affect the maximum liquefaction depth around a square mono-pile. Furthermore, Figure 20 shows that the difference of the maximum liquefaction depth between the adjacent current velocities \( U \) increases with the incremental increase of the cross-section dimension \( D \); this is because the blockage effect of the mono-pile to the incident wave with fixed wavelength increases as the cross-section dimension \( D \) increases, which can further lead
to incremental increase of the wave pressure around the structure.

CONCLUSIONS

In this study, a three-dimensional (3D) numerical model based on the RANS equation and Biot’s poro-elastic theory ($u$–$p$ approximation) was proposed to investigate the transient soil dynamic response around a square mono-pile foundation under combined wave and current loadings. With the presented model, the effects of cross-section shape, the current velocity, the wave obliquity, the seabed permeability, and the pile cross-section dimension on the wave/current–induced transient pore pressure/liquefaction were studied. It was found that the cross-section shape could significantly affect the wave field and the distribution of the soil liquefaction zone in the vicinity of the mono-pile foundation, provided that the cross-section area was the same. Inclusion of current velocity could obviously affect the wave/current–induced soil response, and the current travels following the wave propagation could lead to an increase in the maximum soil liquefaction depth. Unlike the circular cross-section pile foundation, the wave obliquity had a significant effect on the distribution of the wave/current–induced pore pressure, but little effect on the maximum soil liquefaction depth around the square mono-pile foundation. Moreover, the maximum liquefaction depth decreased slightly as the wave obliquity increased. The soil with a larger seabed permeability liquefied more easily regardless of the wave–current interactions, and the maximum pore pressure at a certain depth increased as the seabed permeability increased.

The cross-section dimension could not only affect the distribution pattern of the wave/current–induced pore pressure and soil liquefaction zone but also significantly affect the maximum soil liquefaction depth around the square mono-pile foundation. In addition, the maximum soil liquefaction depth around the square mono-pile foundation increased with the increment of the cross-section dimension regardless of the current velocity.

ACKNOWLEDGMENTS

The authors are grateful for the support from National Natural Science Foundation Council of China (NSFC) Grant 41176073. The authors also thank Professor Fuping Gao at Institute of Mechanics, Chinese Academy of Science for providing the raw data of experiments (Qi and Gao, 2014a, personal communication).

LITERATURE CITED


effective stresses in the porous seabed. *Ocean Engineering*, 30(16), 2065–2089.


