



Mobile Technologies: How Might Using Mobile Technologies Reshape the Learning and Teaching of Mathematics?

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As our attention moves to the opportunities and constraints that mobile technologies (MT) might afford, app developers, teachers and researchers have become more adept at identifying and enacting opportunities for enhancing mathematical thinking. These opportunities emerge through the various environments, both hardware (i.e., tablets) and software (i.e., applications), and the mathematical activity that these facilitate. The features of MT, for instance the ability to use in-built video and audio tools, allows users to capture authentic data in their everyday world and use the data for modelling, or statistical inference. Processing this data in situ changes the nature of the learning experience. Likewise, the potential for visual, interactive engagement with some learning experiences, coupled with the haptic and oral/aural affordances of the technology, change the nature of the mathematical activity. By inference, this changes the nature of the mathematical thinking. Engaging with number sequences by creating sets of objects that represent numbers as you touch the screen, using an oral count, using concrete materials, or learning a sequence by rote, are all different representations of number that might evoke a variety of understandings and ways of thinking mathematically. Being able to elegantly connect these various representations, and move between them, appears to offer opportunities for deeper conceptual understanding of mathematics concepts.

With MT being a relatively recent addition to the scope of digital technologies that might facilitate learning in mathematics, there is a need for research on the use

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27 of MT, and for this research to be presented in a coherent, multi-faceted manner.
28 Such research is of particular importance as schools are investing heavily in mobile
29 devices, often without a concomitant investment in developing practices regarding
30 how such devices may be used to develop conceptual rather than procedural or
31 declarative knowledge (e.g., Calder, 2011), nor of how such devices connect with
32 other resources in the classroom. The effectiveness of their engagement in shifting
33 conceptual understanding is also contingent on associated professional learning for
34 teachers (O'Malley et al., 2013). Theoretical frameworks for teaching and learning
35 with MT might also be influential in our understanding of the ways that using MT
36 in the learning of mathematics might be examined.

37 Some researchers have noted a lack of theoretical rigour regarding the use of MT
38 (e.g., Larkin, 2015), identifying issues in relation to the lack of mathematical
39 quality of many mathematics apps. He also reported the lack of time and expertise
40 for teachers to accurately evaluate them or their use. Nevertheless, MT offer vast
41 potential to enhance mathematical learning. This book builds on international
42 research (e.g., Attard, 2015; Calder & Campbell, 2016; Moyer-Packenham et al.,
43 2015; Sinclair & Heyd-Metzzyanim, 2014) into the use of MT in mathematics
44 education. It includes an examination of the ways MT might influence student
45 engagement, cognition, collaboration and attitudes, through reshaping the learning
46 experiences across a diverse range of year levels and contexts.

47 Central to learning mathematics through using MT is the nature of the tools and
48 the apps utilised, the learning intentions of the teacher, and the type of activity that
49 the students are engaged with. While there is frequently a focus in schools, and in
50 the media, on *consumable* apps that is, those where students follow a set task at a
51 specified level; more recently there has been a focus on apps that: enable students to
52 create screencasts of their mathematical thinking; can be used for coding, including
53 the programming of small robotic devices; enable students to create visual, dynamic
54 representations of mathematical situations.

55 In this relatively new field of engaging with mathematics learning through using
56 MT, this book reflects the growing understanding of how the learning experience
57 might be reshaped to harness the opportunities that MT afford. It also incorporates
58 an examination of using MT for developing mathematical thinking, enhancing
59 teacher pedagogy, and understanding the embodied cognition inherent when using
60 mobile, touch-screen devices. In addition, the broader assemblages incorporating
61 underlying discourses and political elements are hugely influential in using MT
62 effectively. The book proposes emerging frameworks, or new uses for existing
63 frameworks that encourage educators to better interrogate student engagement and
64 learning aspects as well as to evaluate the vast range of apps that continuously
65 appear. As the field is always in flux, with researchers and teachers often scrambling
66 to keep up with recent innovative developments, we are reminded to look
67 beyond the specific to the general (Mason, 2005). Thus we look to find common
68 themes and trends that enable us to gain insights into and evaluate MT, and the
69 associated learning/teaching practices, from vantage points not dependent on
70 understanding specific examples or apps. Notwithstanding the need for a global
71 approach, the examination of individual apps and experiences are crucial as they



72 mark new initiatives and point toward potential innovation. And what of the
73 methodologies that we use to theorise the terrain? There is an advantage in looking
74 through a range of lenses, as what one lens doesn't highlight, another may.
75 However, with the continual developments in technology, we need to also consider
76 how MT might open up innovative approaches to methodology and research de-
77 sign. As a community, we need to continue to explore the edges, while incorpor-
78 ating the generic ways these innovations inform practice, reshape the learning
79 experience and might enhance students' mathematical thinking.

80 This book draws from a diverse range of international studies, where MT have
81 influenced the ways that learning might occur across a range of educational con-
82 texts. The book is divided into four sections: *Looking across the terrain*; *Traversing*
83 *the learning and teaching landscape*; *Navigating content: focussing on particular*
84 *concepts*; and *Exploring new forms of communication to make mathematical*
85 *learning visible*. While the purpose of these groupings is to draw the reader's
86 attention to particular themes within the chapters, there is nevertheless considerable
87 overlap between the sections, and most chapters could have easily been situated in
88 more than one section.

89 The first section, *Looking across the terrain* considers some generic aspects that
90 straddle the diversity in the field. Larkin and Milford use cluster analysis to group
91 apps based on particular processes, features and concepts. Undertaking this process
92 enables the apps to be grouped independently from developers' marketing and
93 highlights particular aspects that mathematics educators consider as important.
94 Through this educators are supported in their selection of apps for the user's
95 intended purpose, and hence enhance app use in mathematics classrooms. Calder
96 and Murphy analyse aspects of a 2-year study involving primary children learning
97 mathematics through the use of mobile devices and apps. With the teachers as
98 co-researchers, they examined teacher practice and the inter-connectivity between
99 teacher pedagogy and the affordances of the apps. The teachers used a diverse range
100 of apps, including ones for screencasting and coding, with the various learning
101 experiences and opportunities for influencing the learning outlined and considered.
102 An interesting theme to emerge across the use of various MT was socio-material
103 assemblages. The final chapter in this section also investigates the interplay
104 between a range of contexts and the types of apps used in these contexts. Attard
105 considers the notion of student engagement when using apps for mathematical
106 learning. Her framework for interrogating aspects of engagement incorporates
107 cognitive elements. She draws together common threads about engagement, while
108 synthesising insights into a collection of different studies related to engagement in
109 various contexts.

110 The second section, *Traversing the teaching and learning landscape*, includes
111 chapters that link teaching and learning related to various learning processes that
112 utilise particular processes or affordances of MT. Kyriakides and
113 Meletiou-Mavrotheris discuss a multifaceted programme designed to provide a
114 group of in-service teachers with the knowledge, skills, confidence, and practical
115 experience required to effectively use tablet devices for enhancing mathematics
116 teaching and learning. The teachers integrated the app *A.L.E.X* into their lesson



117 plans and thus reshaped the students' learning experience. Sollervall, de la Iglesia,
118 and Zbick explore how mathematics classroom teachers can implement an innova-
119 tive mobile learning activity. They report on an ongoing, 5-year study into using
120 a GPS app for geometry, with the focus of the activity involving GPS and spatial
121 orientation tasks that are executed in outdoor settings. Sedaghatjou and Rodney's
122 chapter considers how a particular multitouch app called *TouchCounts*, along with
123 children's collaborative engagements, can enhance mathematical learning of
124 number. They utilise *StudioCode* software to better understand children's collab-
125 orative, gestural practices within the *TouchCounts* environment. In the final chapter
126 of this section, Bokhove, Clark-Wilson, and Pittalis consider two cases of how MT
127 provided opportunities for "mathematics outside the classroom". The examples
128 describe how using mobile phones with augmented reality allowed students to
129 bridge between formal and informal mathematics learning. Their examples, a
130 dynamic Ferris wheel and a static cathedral are used to demonstrate how educators
131 can use *geo-location* and *augmented reality* to enhance the learning of mathematics
132 through MT.

133 In the third section of the book, *Navigating content: focussing on particular*
134 *concepts*, the authors primarily attend to specific mathematical concepts or pro-
135 cesses. Pelton, Milford, and Francis Pelton use an app to develop children's
136 understanding of time. Their chapter details the integration of a researcher-designed
137 iPad app into a series of collaboratively created lessons to facilitate the learning of
138 clock-reading and time concepts. The authors used a lesson study approach to
139 design and refine the intervention that included teacher-led activities and structured
140 use of the iPad app. Lommatsch, Tucker, Moyer-Packenham, and Symanzik
141 examine what patterns were revealed when heatmaps were used with hierarchical
142 clustering to examine pre-schoolers' performance with two touchscreen mathe-
143 matics apps in two different learning sequences: counting and seriation. Their
144 analysis highlighted changes in children's performance, speed, and developmental
145 progressions after using the two apps. The use of hierarchical clustering analysis
146 facilitated the analysis of individual and whole group data leading to the identifi-
147 cation of young children's developmental progressions. Rosen, Palatnik, and
148 Abrahamson explore an embodied-design for engaging particular mathematical
149 concepts with an action level where the virtual objects were either generic (e.g., a
150 circle), or situated, (e.g., a hot-air balloon). They evaluate an instructional
151 methodology whereby students first learn to physically move objects on the screen
152 before eventually generalising these movements as formal mathematical rules.
153 Chorney and Sinclair describe a research project with first-grade children using a
154 multi-touch, dynamic geometry app called *WebSketchpad* to study how the concept
155 of symmetry arises. They analyse the data through the lens of inclusive materialism,
156 considering the intra-actions involved in the child-device-geometry assemblages,
157 and how new mathematical ideas might emerge from these assemblages. Their
158 particular focus is how the multi-touch environment can provide the basis for
159 emerging geometrical ideas. Ferrara and Savioli discuss a classroom-based



160 intervention with a group of first-grade children using the multi-touch app
161 *TouchCounts* to develop children’s number sense. They investigate how under-
162 standing might emerge out of the relational entanglement of numbers, iPads, and
163 learners, engendering new kinds of mathematical experiences with number and
164 providing the basis for emerging relational meanings of number. In the final chapter
165 of this section, Soldano and Arzarello examine an approach to geometry in a
166 secondary-school context. Their chapter illustrates a way of using MT to support
167 the transition from an empirical to a theoretical approach to geometry. Drawing on
168 Zbiek’s et al., (2007) notions of pedagogical, mathematical and cognitive fidelities,
169 they implement group game-activities whereby students investigate the geometric
170 property upon which the game is designed.

171 In the final section of the book, *Exploring new forms of communication to make*
172 *mathematical learning visible*, the notion of screencasting is the focus. The use of
173 screencasting opens up opportunities for mathematical thinking of learners to
174 become more transparent as students and teachers might create individual expla-
175 nations of their thinking using a blend of both digital tools and their associated
176 social elements. Galligan and Hobohm examine a case study of the use of mobile
177 devices and screencasting in university mathematics education teaching. They
178 incorporate this with an evaluative tool for teachers and students to evaluate their
179 own and others’ screencasts, with the intention of developing pre-service teachers’
180 understanding of mathematics and ways to teach it. The chapter concludes with
181 recommendations for using screencasting to assist with developing mathematical
182 understanding and pedagogical content knowledge. Prescott and Maher explore the
183 ways primary-school students worked collaboratively to solve a problem,
184 explaining their mathematical thinking. The students used screencasting apps such
185 as *Explain Everything* and *Educreations* to produce create-alouds, which helped
186 them to collaboratively understand and explain mathematical concepts. The apps
187 also assisted teachers in providing formative assessment and feedback to the stu-
188 dents. In the final chapter of this section, Ingram, Pratt, and Williamson-Leadley
189 discuss how a Show and Tell app can make the students’ thinking more observable
190 in problem solving. They consider how using Show and Tell apps for problem
191 solving can lead to improvements in the level and quality of student engagement.
192 Students were encouraged to socially negotiate their understandings, making stu-
193 dent thinking more visible during this process. The apps can also scaffold students
194 in reflecting upon the processes they used for problem solving.

195 The learning opportunities provided, and the evolution of the ways of promoting
196 engaging mathematics learning and thinking through MT, exist in a fast moving,
197 dynamic space; one where the comparative costs for mobile devices and connec-
198 tivity are dropping markedly. There are also emergent MT that might quickly come
199 to dominate the field: virtual reality is already developing rapidly, as is artificial
200 intelligence and robotics. Some trusts and educational systems are distributing, to
201 all schools in their community, 3-D printers that can print using materials as diverse
202 as wood and titanium. Due to the reduction in the costs of MT, many of the



203 previous equity and accessibility issues are alleviated. The potential to envisage
204 space, location, shape, number, movement and rates of change has already been
205 transformed. Likewise, ways of analysing data to model real-life situations in situ
206 have changed. Yet, despite the rapid change we have witnessed in recent times, we
207 do not really know where the technology, or its potential as a digital pedagogical
208 medium, is headed. What might the landscape look like in 5 years, let alone in 20
209 years?

210 Underpinning each of the chapters in this book is the understanding that
211 mathematical thinking must be given primacy—in the end it is our guiding premise
212 and intention. We need to ask ourselves whether what we do with MT enhances
213 mathematical thinking and understanding and to reflect on how the MT might be
214 changing what counts as mathematical activity. The chapters in the book contribute
215 to mathematics teaching and learning by providing readers with opportunities to
216 reflect on their practice. It is not possible, nor wise, to ignore the role of MT in
217 enhancing teaching and learning in mathematics. Therefore, all educators need to be
218 alert to the potential that MT provide to enhance student learning.

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