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Abstract

This paper employs a stochastic and dynamic intermediate storage model to estimate the optimal stockpile levels at both a mine and port for a coal supply chain. The optimisation model demonstrates that the principle costs incurred from high inventories of coal include working capital, storage costs and double handling costs, whilst costs incurred from low inventories are dominated by train cancellations, spot price purchases of coal to make up shortfalls and demurrage. The optimisation model allows for the dynamic interaction of cost functions across the supply chain and results in optimal inventories that are typically lower than intuitively assumed by logistics managers.

Key words: Optimisation, supply chain.

Introduction

The volume of coal available for export is expected to rise in most major coal producing centres due to an increased global demand for energy and steel. The continued growth in coal production within a physically constrained supply chain provides motivation for the coal industry to enhance methods to improve the cost profile of mining and transporting coal while maximising throughput and meeting stringent environmental regulations. The optimisation of the coal supply chain, from pit to port, is critical for reducing mining, transportation and storage costs.

In addition to the contracted quality specifications of exported coal, the most critical factor driving optimisation of the coal supply chain is ensuring security of coal supply. The effect of coal stock-outs on thermal power producers and steel manufacturers is financially catastrophic. Ensuring supply throughput is therefore a key element of coal production and supply chain management. Historically, to cater for possible disruptions affecting security of supply, logistics managers over-compensate and stock large levels of coal at both the mine and if possible at the port. This paper examines stockpile management practices and focuses on modelling efficient stockpile levels within a dynamic cost environment to obtain optimal levels of coal stockpiles. The main objective is to define the optimal stockpile level at the mine and the port that minimises overall costs to the producer while minimising the risk of supply disruptions.

There are a number of implications for operating coal stocks at sub-optimal levels, both high and low. It is clearly prudent that mines maintain a minimum stock level to avoid train colliery cancellations and vessel loading delays resulting in demurrage costs, as well as helping smooth out variations in coal quality. Minimum product stock levels for each brand of coal are also essential since take or pay port entitlement contracts must be honoured. Coal is not a homogenous product due to the discretion of producers to uncover a range of coal seams at their disposal and the capacity to process coal through wash-plants enhances ash, nitrogen and sulphur levels. Stock levels at the load ports must be sufficient to ensure that no distressed purchases of coal are necessary and demurrage costs are minimised, especially if there is a history of disruptions in the logistics chain between mine and port.

A producer needs to keep sufficient stocks to support existing contractual sales, as well as to cater for a certain amount of spot sales. This also assists in building brand image with key customers that recognise the producer as a reliable supplier of choice. In some locations a

buffer stock is contractually required by the port to allow for optimisation of railing and port operations.

In the coal industry there are a number of excellent supply chain management programs such as QMASTOR which address stockpile management issues as p art of full supply chain optimisation however they generally neglect to optimise stockpile levels across the supply chain efficiently. We estimate the optimal coal inventory level for both a mine and port, in the absence of structural requirements, using a st ochastic and dynamic intermediate storage model to minimise the costs of stockpiling. The results from this study complement rather than contradict the solutions provided by supply chain software systems since the supply chain is not explicitly constrained by stockpile levels. This model merely seeks to minimise the total stockpile costs incurred from either producing too much or too little coal, which relates directly to mining practices rather than supply chain dynamics.

Section 2 discusses the major costs incurred by producers resulting from either excessive or insufficient stocks at either the mine or the port. Section 3 introduces the model and discusses the methodology. Section 4 discusses the results and tests the sensitivity of cost inputs which highlights the two major cost drivers. Section 5 offers some concluding remarks.

Cost implications for stocks above and below an optimal level

Stockpiles at a mine are used as a buffer for mine reconfigurations, production disruptions and to minimise train cancellations. Stockpiles at a port are generally used to reduce demurrage penalties and to properly manage coal qualities. Mine stockpiles are typically much larger and have spare capacity. Mine and port stockpiles are usually connected by rail but barge transport between locations is also common. Additional capacity along the rail is allocated to third party coal purchasers allowing for a degree of flexibility in coal delivery. An efficient supply chain aims to achieve sufficient logistic flexibility to allocate trains for mines to transport quantities of coal with specific qualities that are intended for export as well as coal for domestic consumption.

When coal is extracted in either an open-cut or underground mine, it is typically crushed, screened and sometimes further processed and then stockpiled at the mine. Mine managers rely on experience and intuition to set a stockpile level that caters for train or barge scheduling, forecast changes in mine geology which will affect product volumes, anticipate and react to weather impacts that halt mining operations and seek to minimise the non-trivial costs associated with storage and stockpile management. Logistics managers also rely heavily

on experience and intuition to set stockpile levels at the port that cater for train delays, vessel loading delays and storage costs.

The costs incurred for maintaining larger stockpiles than necessary is perceived as a form of insurance by site managers and logistics teams, to secure supply for thermal power generators and steel producers. There has been a growing trend towards extremely high stockpile levels in some mines where stock volumes can be as high as 25 per cent of the total coal produced annually. An inventory of hundreds of millions of dollars on a mine's balance sheet distorts the true value of the operation, implies that the nature of the storage cost profile is not understood and hints at inefficiencies in the off-take process.

Stockpile management at an operational level seeks to maintain the coal in good storage conditions and control for coal heating and dusty conditions. Operational issues associated with stockpiling and reclaiming of coal include the accurate measurement of tonnage going onto stockpile and of reclaimed tonnage, changes in moisture content as a result of rain or drying out of material (representing either a weight gain or loss), stockpile losses due to wind, water erosion and carpeting, coal transferral and product contamination avoidance, variations in bulk density (the bulk density of a stockpile can vary between $0.8t/m^3$-$1.2t/m^3$ or higher for compacted coal), spontaneous combustion if stockpiled for long periods and the potential for some trace elements in coal to leach out into groundwater or run-off into nearby streams/dams/rivers.

It is usually regarded to the benefit of all coal yards to try and implement a 'first in-first out' basis for stock rotation in order to reduce the amount of time the coal is kept in stock. One method to achieve this is the 'walking stockpile' where stacking is conducted on one side while reclamation is conducted from the other. This method also allows for the maximisation of the use of the stocking area however the main drawback is that it requires additional storage area which isn't always available. Stockpiles of different grades of material are typically separated into discrete areas with a 10-meter gap between to minimise contamination. This is not always possible and is an explicit constraint in the supply chain.

For very large tonnages stockpiling can be achieved by putting coal down in extended beds, restricted only by the amount of space available. Where there is a stacker-reclaimer available, coal is laid down by means of a traversing conveyor and reclaimed in a series of horizontal strata, or by full face reclamation. This method of stacking and reclaiming fulfils the requirements of a blending system where the variations in quality are reduced. The preferred methods for achieving suitable blends can include but are not limited to gravity reclamation,

portal scrapers, slewing bucket wheels, bridge scraper, silos, bridge bucket wheel and barrel. If a mechanical stacker-reclaimer is not available, blending of coal is normally achieved on the conveyor belt by reclaiming different grades of material simultaneously. The other alternative for blending is to stack different grades of material on the same pile.

Notwithstanding the operational issues discussed above, stockpiling represents a real cost to a coal producer. The logistics situation differs markedly across coal exporting regions since the logistics chain is typically stockpile constrained at either the mine or the port. This is generally due to the temporal misalignment of infrastructure upgrades to expand capacity in both rail and port. This constraint forces stockpile management to be optimised where possible.

Mine site stockpiling

Low inventories at mines usually result in train cancellations Train cancellations lose significant value for coal producers as they typically face take-or-pay costs along with the lost revenue from the cargo, and it also jeopardises the mine's perceived reliability for train scheduling capacity. This also results in port inventories not being in line with the logistics plan thereby triggering demurrage payments and potential quality adjustment penalties.

High inventories translate into high working capital costs since a large stockpile of coal is housed on the mine's balance sheet implying that the inventory turnover duration is much higher than is necessary. This generally indicates an inefficient supply chain with an array of hidden costs associated with double handling stocks, preparation of additional storage areas, increased costs for dust control and greater working capital. Excess stock at the mine can also hamper effective blending operations so stockpile management is often governed by intuition and physical constraints rather than by what is cost efficient.

Port stockpiling

Insufficient inventories render it difficult if not impossible to 'sweeten' low quality cargoes, in which case value-leakage occurs. For most thermal coal contracts, cargoes may be rejected by counterparties if net calorific value drops below 5,850kcal/kg NAR while double penalties apply below 5,900 kcal/kg NAR. Similar contract penalty structures exist for metallurgical coal also. Cargo rejection results in the full value of the coal being temporarily surrendered with additional costs being incurred to either renegotiate contract terms or to source another buyer for the now distressed parcel of coal.

Demurrage is incurred when customers do not receive their cargo at the intended time due to insufficient stocks at the port and it inherently erodes a producer's image as a reliable supplier of coal. Delays may also result in re-negotiated delivery dates and/or quality specifications. For instance, if a shipment is rolled to the following month a change in the index price can result in significant deterioration in value. As an example the API4 index for free-on-board coal at Richard's Bay in South Africa fell by US$38 per tonne between September and October 2009, which means a capesize vessel of coal that suffered a delay of a few days may have lost as much as US$6m – a quarter of the cargo value.

Data and Research Method

The model aims to minimise the cost profile of stockpiles at both a mine and its respective port via stochastic simulation given satisfactory delivery performance per coal grade. The model is subject to the following input constraints:

- Uncertainty in production, quality, railing, laycans and shipping tolerance;
- Additional costs associated with moving coal between mine stockpiles, rail cancellations, working capital, demurrage, low quality & distressed purchases;
- Excess stock managed by selling coal under distressed conditions or moving the product to an alternative stockpile at extra cost; and
- Insufficient stocks resulting in demurrage penalties and potentially high costs of buying third party coal at a premium to the spot or contract price.

The model includes deterministic estimates of demurrage rates, volume options, rail performance, working capital costs (interest rates and coal prices), using stochastic simulation of expected production and production variance and the expected lay-can intra-month split. The model excludes explicit modelling of expected coal quality, apart from an assumption of coal degradation over time for extended stockpiling of coal and explicit modelling of the variance in quality along with coal blending optionality.

The optimal stockpile level calculation is based on minimising the expected aggregate liabilities of having excess or insufficient stock at a mine and its associated port.

Main assumptions

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The main cost assumptions used in the model relate to the costs associated with excess or insufficient stocks and the level at which these costs escalate. We choose a mine with average annual production of 10Mt which exports up to ten different coal quality products. No coal is destined for domestic production. Domestically consumed coal is typically unprocessed and deployed via conveyors so the logistic implications of run of mine coal for domestic consumption are minimal.

We assume that coal stocks at the mine in excess of 700kt coal must be stockpiled at a secondary location which will incur double handling costs. To minimise impacts on stockpile capacity and handling problems, we assume coal is sold at a discount for levels above 700kt with an associated cost. Too low stockpiles however carry the danger of paying demurrage and buying in supplementary coal. For capital costs we will assume that the stockpile is depreciated by current interest rates adjusted for credit quality and the inflation rate. We adjust the value of the stockpiled coal by 0.3% per day to cater for the calorific value degradation of stockpiled coal (assuming the coal is not compacted and covered for long term storage). All costs are in US dollars.

To simplify the analysis and to incorporate production constraints, we assume one single stockpile and do not distinguish between different qualities. Coal is always first stacked on the so-called 'live' pile and thereafter any excess material is stacked on the so-called 'dead' pile to minimise or eliminate secondary stockpiling costs. Typical daily transport rates are 3 trains of 8kt of coal each. This implies that the cargo for a capesize vessel can be gathered over a 6-day period.

The dynamic feature of the model incorporates sources of uncertainty relating to variability of production, variability in contracted volume off-take and variability in the timing of the off-take in a delivery period. Production is modelled as a stochastic process using a lognormal distribution calibrated to three years of production history. Average production is 200kt per week and variability in production is 100kt per week. Variability in off-take volumes includes the ±10% option (as per a typical coal off-take contract to allow for vessel loading constraints) which is also modelled as a stochastic process. Variability in timing for the vessel's laycan within the delivery period is modelled deterministically as one of two scenarios that help dictate the dynamics of stockpile management; firstly 75 per cent of laycans occur in the first half of the contracted delivery period and secondly 75 per cent of laycans occur in the second half of the contracted delivery period. Unquantifiable costs to reputation incurred from delivery delays or logistic inefficiencies are not explicitly modelled.

The model simulates the expected volume beyond the high and low stock thresholds. The expected volume outside thresholds is multiplied with the cost and summed over a four-week period. The stockpile that gives the lowest expected total cost is then estimated to be the optimal stockpile level. The four-week period is used to represent the total duration of the stock recovery period as historical observation suggests it takes about a month to normalise stocks from extremely high or low levels.

Coal off-take at the port is assumed to equal coal production at the mine aggregated over a four-week period. Stock levels at the mine have a direct influence on stock levels at the port. The throughput of coal in the supply chain (via rail) remains at a steady state equal to the weekly rate of coal production. The probability of low or even zero stocks at the mine directly translates into low or zero stocks at the port with third party coal purchases making up the difference until coal stocks recover from own production.

Cost model

The cost function for both high and low volume stockpiles with reference to respective costs at the given thresholds, as well as the working capital cost of inventories is derived as

$$C(i,j,t) = \sum_{t=1}^{4} \sigma_{p,t} c_n^j \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{(x_t^j)^2}{2}} - x_t^j . \frac{1}{\sqrt{2\pi}} e^{-\frac{(1-x_t^j)^2}{2}} + x_t^j \right) + S_t P_t (e^{r_t} - 1), \qquad (1)$$

where

$$x_t^j = \frac{K_t^j - S_t}{\sigma_{p,t}}. \qquad (2)$$

In (1) $\sigma_{p,t}$ is the standard deviation of production at week t, c_n^j is the fixed cost in USD per tonne for excess or insufficient volumes n at volume level j, S_t is the stockpile level at the mine or port at time t, P_t is the short-term coal off-take price at time t and r_t is the working capital cost rate at time t represented as a fixed 200 basis point spread over the bank bill swap rate minus inflation. In (2) K_t^j is the threshold volume level j: $j=1,2,...,5$ at time t. The threshold volume levels j: $j=1,2,...,5$ represent the threshold for the live stockpile ($j=1$), the live plus dead stockpile ($j=2$) and the live plus dead stockpile plus a residual stock area ($j=3$) for high stock levels. For low stockpiles the threshold volume level $j=4$ represents the threshold impacting train cancellations and quality penalties and $j=5$ is the threshold for demurrage costs and the distressed purchasing of coal.

The cost function in (1) and (2) is essentially an option contract priced using the stockpile at each week-end valued against the costs of excess or insufficient stocks measured against a threshold (strike) and summed over a four week period. The expected volume above threshold levels is calculated by using the partial expectation $P(x \le a) = \sigma(L(z_a) + z_a)$ and $P(x > a) = \sigma L(z_a)$ where z_a is the normalised z-score, and the normal distribution loss function is $L(z_a) = \psi(z_a) - (1 - \Phi(z_a))z_a$ where ψ is the normal density and Φ is the normal cumulative function.

Due to the mine stocks directly influencing the port stocks, there are multiple uncertainties impacting the expected stockpile level at each location. Instead of calculating the bivariate or multivariate partial expectation, we add the variances of the random variables $\sigma_{i,j} = \sqrt{(\sigma_i^2 + \sigma_i\sigma_j\rho_{ij})}$ for all i and $j > i$ with minimal loss of accuracy.

The cost function (1) is minimised across all low and high stock scenarios over a four-week forecast period subject to the constraints around laycan and vessel loading times, train throughput and input costs addressed above. The model conducts an iterative simulation of forward production, production variability and laycan probabilities to estimate the total cost of a range of stock levels at the mine and port. Cost inputs and associated threshold levels remain static throughout the simulation run. The simulation is run for 5000 paths.

Results

The model was calibrated to efficient estimates of forward contracts beyond the longest-maturity futures contract. Using a key assumption that coal is sold off when the stockpile level reaches 700kt, we analyse how this assumption influences the results.

Stockpile Optimiser - Mine FY10

Models inputs

MAIN INPUTS

scenario	All	
initial stockpile level	200	'000 t

STATISTICAL MODEL PRODUCTION

average rate	'000 t/week	200
total variability	000 t/weeks	100

DEMURRAGE COST (NOT USED DIRECTLY IN MODEL)

demurrage cost	$/day	76,000
coal days shortage		3 days
demurrage cost		2 $/mt
laycan volume % week on week		25.0%

CAPITAL COST

capital cost rate		8.5%
reference price	US$/t	120.00
time for Cap. Cost calc.		4.00 weeks

TIMING

volatility time		1.5 weeks
average time		0.5 weeks

LIABILITIES

|		
high threshold 1 (e.g. max live stock)	400	'000 t
high threshold 2 (e.g. live+dead stock)	700	'000 t
high threshold 3 (e.g. live+dead+2nd stock)	1,000	'000 t
low threshold 1 (e.g. train cancel / quality pena	200	'000 t
low threshold 2 (e.g. demurrage / stressed 3rd	75	'000 t
high threshold 1 cost	3.00	$/t
high threshold 2 cost	4.00	$/t
high threshold 3 cost	12.00	$/t
low threshold 1 cost	1.00	$/t
low threshold 2 cost	12.00	$/t
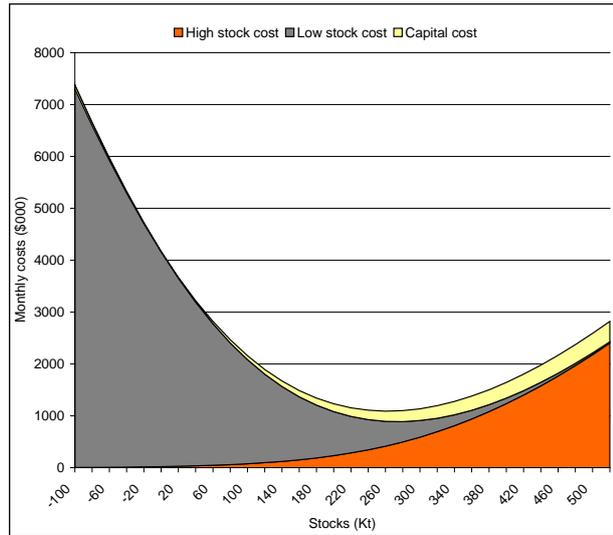
Figure 1: Stockpile optimal level at the mine using the given market parameters, thresholds and costs. The optimal level is given at around 280kt.

The model indicates that low stocks generally have the potential for incurring greater liabilities than high stocks at the mine. Conversely the model shows that the opposite is the case at the port since stockpile capacity can only be sourced at a significant premium. Figure 1 shows a sample cost profile for a range of stock levels given a set of static inputs. The optimum stock level can be visually estimated at the minimum total cost level. Input parameters can obviously be adjusted to cater for the different market inputs and cost thresholds.

Stockpile Optimiser - Port FY10

Models inputs

MAIN INPUTS

scenario	All	
initial stockpile level	450	'000 t

STATISTICAL MODEL PRODUCTION

average rate	'000 t/week	200
total variability	000 t/weeks	100

DEMURRAGE COST (NOT USED DIRECTLY IN MODEL)

demurrage cost	$/day	76,000
coal days shortage		3 days
demurrage cost		2 $/mt
laycan volume % week on week		25.0%

CAPITAL COST

capital cost rate		8.5%
reference price	US$/t	96.00
time for Cap. Cost calc.		4.00 weeks

TIMING

volatility time		1.5 weeks
average time		0.5 weeks

LIABILITIES

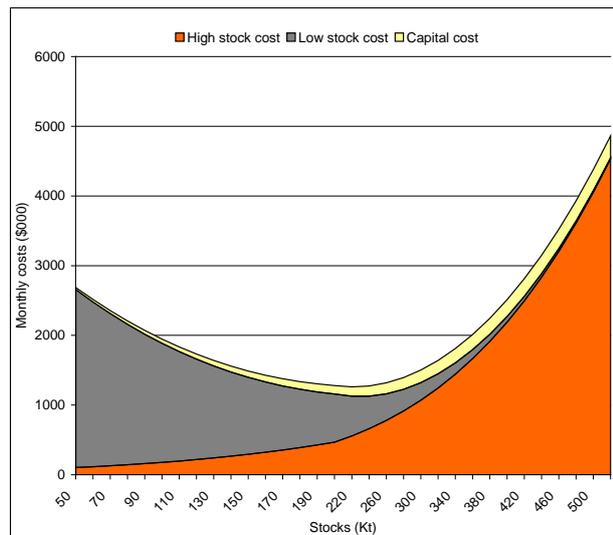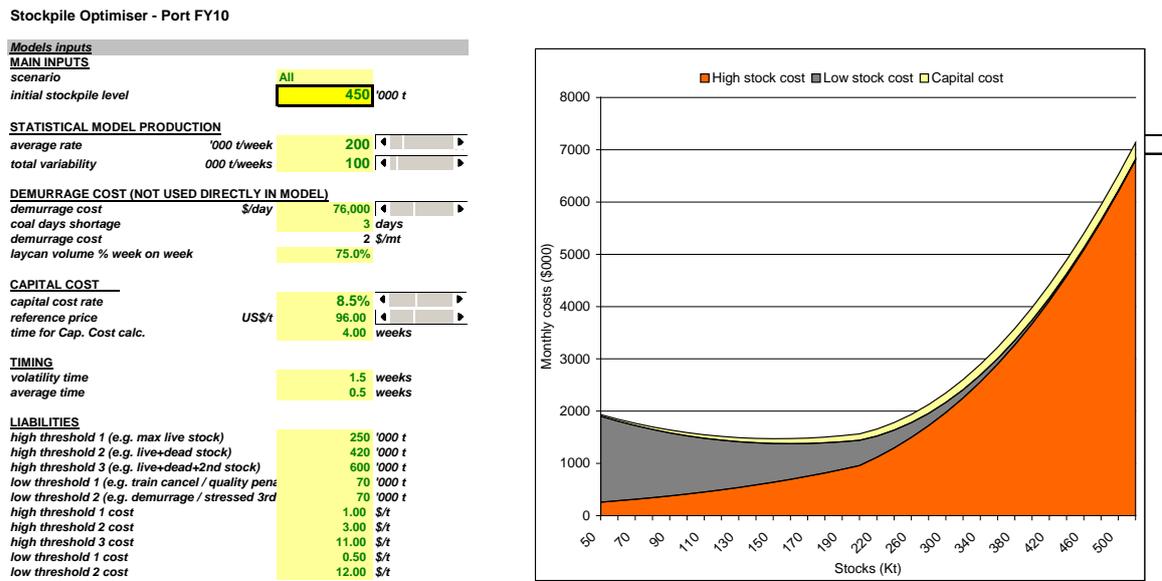|		
high threshold 1 (e.g. max live stock)	250	'000 t
high threshold 2 (e.g. live+dead stock)	420	'000 t
high threshold 3 (e.g. live+dead+2nd stock)	600	'000 t
low threshold 1 (e.g. train cancel / quality pena	70	'000 t
low threshold 2 (e.g. demurrage / stressed 3rd	70	'000 t
high threshold 1 cost	1.00	$/t
high threshold 2 cost	3.00	$/t
high threshold 3 cost	11.00	$/t
low threshold 1 cost	0.50	$/t
low threshold 2 cost	12.00	$/t

Figure 2: Stockpile optimal level at the port using the given market parameters, thresholds and costs. The optimal level is given at around 240kt.

Figure 2 demonstrates the optimal level of stocks at the port given a set of cost inputs and coal throughput as well as the stock level at the mine. The port has greater restrictions for storing higher volumes of coal and thus the costs generally escalate beyond the optimum point. Furthermore if more laycans fall in the second half of the delivery period the optimal level reduces quite significantly as the costs of having excess stocks continuing to build over time become extreme. This is shown in Figure 3.

Stockpile Optimiser - Port FY10

| Models inputs		
MAIN INPUTS		
scenario	All	
initial stockpile level	450	'000 t
STATISTICAL MODEL PRODUCTION		
average rate	'000 t/week	200
total variability	000 t/weeks	100
DEMURRAGE COST (NOT USED DIRECTLY IN MODEL)		
demurrage cost	$/day	76,000
coal days shortage	3 days	
demurrage cost	2 $/mt	
laycan volume % week on week	75.0%	
CAPITAL COST		
capital cost rate	8.5%	
reference price	US$/t	96.00
time for Cap. Cost calc.	4.00	weeks
TIMING		
volatility time	1.5	weeks
average time	0.5	weeks
LIABILITIES		
high threshold 1 (e.g. max live stock)	250	'000 t
high threshold 2 (e.g. live+dead stock)	420	'000 t
high threshold 3 (e.g. live+dead+2nd stock)	600	'000 t
low threshold 1 (e.g. train cancel / quality pena	70	'000 t
low threshold 2 (e.g. demurrage / stressed 3rd	70	'000 t
high threshold 1 cost	1.00	$/t
high threshold 2 cost	3.00	$/t
high threshold 3 cost	11.00	$/t
low threshold 1 cost	0.50	$/t
low threshold 2 cost	12.00	$/t

Figure 3: Stockpile optimal level at the port using the given market parameters, thresholds and costs with more vessels programmed to laycan in the second half of the delivery period. The optimal level is given at around 160kt.

The model can also be run using deterministic distribution functions for production and production variability which greatly simplifies the calculation and provides optimal stockpile estimates rapidly. However the ability for coal stocks at the mine to directly influence the downstream stock level at the port using this method is somewhat limited.

The model was tested for sensitivities relating to changes in production variability and laycan scheduling in the early or later part of the delivery period. Variability is measured as a proportion of weekly production and the optimum level is also expressed as a proportion of weekly production. As shown on the left side of Figure 4, as production variability increases the optimum mine stock level also increases. The optimum level at the port reaches a maximum when the production variability exceeds 30% of weekly production. In times of severe supply chain constraints caused by train derailments, strikes or weather events, it is optimal to build stock at the mine rather than the port regardless of the actual capacity at each end of the supply chain. This also illustrates that when a mine is operating efficiently and little or no variation in production occurs, very low stockpiles can be maintained at the mine

and port, greatly reducing total costs associated with stockpiling. If mine production variability increases from 20 per cent to 40 per cent of weekly production the stockpiling costs escalate from $0.70 per tonne to $2.80 per tonne. Mine management and production throughput are therefore critical for maintaining low costs associated with stockpile management.
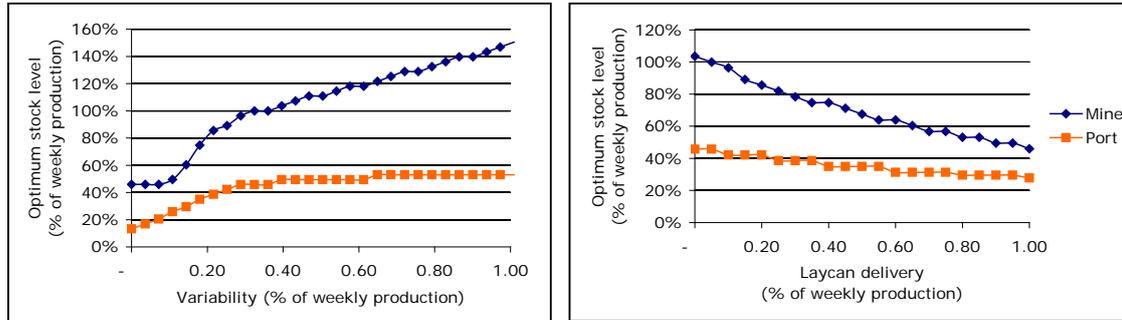
Figure 4: Sensitivities of the optimum stock levels with respect to production variability and laycan delivery scheduling as a proportion of weekly production using a set of market parameters, thresholds and costs at both the mine and the port.

The right hand graph in Figure 4 illustrates sensitivities associated with laycan delivery flow-on effects, measured as a proportion of weekly production. Once again the optimum level is expressed as a proportion of weekly production. When vessels are expected at the port in the front half of the delivery period, target stock levels should be higher than when vessels are expected towards the latter half of the period. But critically the optimal stock level is much higher and the stock profile is more dramatic at the mine than the port as vessels push loading to the latter half of the period. Stocks typically build over the first part of the period as vessels choose to laycan later in the period however the optimal level should decline to an optimal level, which is essentially the target stock level at period end. This seems to be a logical outcome that can be arrived at intuitively however the dynamic cost model demonstrates that the optimal port stock level does not significantly change as vessel are scheduled within a contracted delivery period, with the spare storage capacity that minimises the cost function for such changes located at the mine (where stockpiling costs are less expensive). This figure also illustrates the differential stock levels to be held at mine and port respectively as laycan scheduling changes.

A more general observation of stockpile levels from the model using inputs aligned with current costs and associated thresholds in the Hunter Valley supply chain in Australia suggests that the optimal stock levels at both the mine and the port are significantly lower than the stock levels generally applied by mine and logistics managers. A mine with an annual production of around 10Mt typically stockpiles over 1Mt at the mine and 400kt at the port,

while the model suggests that these levels should be around 280kt at the mine and 240kt at the port. At a coal price of US$100 per tonne this difference translates into an annualised cost saving of US$88m.

Concluding Remarks

A dynamic intermediate storage model is used to estimate the optimal stockpile levels at both a mine and port within a coal supply chain. The optimisation model demonstrates that the principle costs incurred from high inventories of coal are working capital, storage costs and double handling costs whilst costs incurred from low inventories are dominated by train cancellations, spot price purchases of coal to make up shortfalls and demurrage. The optimisation model allows for the dynamic interaction of cost functions across the supply chain and results in optimal inventories that are typically lower than intuitively assumed by logistics managers.

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