The Correct Equation for the Current through Voltage-Dependent Capacitors

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ABSTRACT Two different equations for the current through voltage-dependent capacitances are used in the literature. One equation is obtained from the time derivative of charge that is considered as capacitance–voltage product: \( \frac{dQ}{dt} = \frac{d}{dt} [C(V)\frac{dV}{dt}] = C(V)\frac{dV}{dt} + V\frac{dC(V)}{dt} \). In the second equation, the term \( V\frac{dC(V)}{dt} \) does not exist: \( \frac{dQ}{dt} = C(V)\frac{dV}{dt} \). This paper clears the ongoing confusion caused by the difference between these two equations. We use the voltage-dependent parasitic capacitance of a commercial Schottky diode in reverse bias mode to test experimentally both equations. The result is that it is incorrect to add the term \( V\frac{dC(V)}{dt} \) in the first equation with the measured capacitance. We also perform a theoretical analysis, which shows that the differential capacitance, \( C(V) = \frac{dQ}{dV} \), in the correct current equation corresponds to the physical parameters of the diode capacitance.

INDEX TERMS Capacitors, current, local capacitance, mathematical equation, parasitic capacitance, total capacitance, voltage dependent capacitors.

I. INTRODUCTION

The capacitors used in most applications are usually voltage independent. But semiconductor devices, such as diodes and MOSFETs, contain parasitic capacitances that are highly voltage-dependent [1]. The impact of these voltage-dependent parasitic capacitances is pronounced in power electronic circuits, such as power converters. At high frequencies and high voltages, desirable for power converter operation, the circuit performance depends on these parasitic capacitances [1], [2]. Hence, designing efficient power converters, or other power electronic circuits, requires modelling of these variable capacitances, and so it is necessary to use the proper model.

The current flowing through a capacitor is given by

\[ i_c = \frac{dQ}{dt} \]

(1)

where \( Q \) is the charge on the capacitor plates. For a voltage-dependent capacitance, \( C(V) \), and voltage \( V \) across the capacitor, putting the charge as \( Q(V) = C(V)V \) in (1) leads to the following equation:

\[ \frac{dQ(V)}{dt} = \frac{d}{dt} \left[ C(V)\frac{dV}{dt} \right] = C(V)\frac{dV}{dt} + V\frac{dC(V)}{dt} \]

(2)

The differentiation in (2) can be applied to both terms, leading to one of the two possible equations for the current through a voltage-dependent capacitor:

\[ i_{c1} = C(V)\frac{dV}{dt} + V\frac{dC(V)}{dt} \]

(3)

\[ i_{c1} = C(V)\frac{dV}{dt} + V\frac{dC(V)}{dt} \]

(4)

Another way to determine the current–voltage relationship for voltage-dependent capacitors is by rewriting (1) in the following way:

\[ \frac{dQ(V)}{dt} = \frac{d}{dt} \left[ C(V)\frac{dV}{dt} \right] = C(V)\frac{dV}{dt} + \frac{dC(V)}{dt} \frac{dV}{dt} \]

(5)

Defining the voltage-dependent capacitance as \( C(V) = \frac{dQ}{dV} \), (5) can be written as

\[ i_{c2} = \frac{dC(V)}{dt} \frac{dV}{dt} = \frac{d}{dt} \left[ C(V)\frac{dV}{dt} \right] \]

(6)
Apart from the use of voltage-dependent capacitance, \( C(V) = dQ(V)/dV \), (6) has the same form as the equation for voltage-independent capacitance. Comparing \( i_{C1} \) and \( i_{C2} \) given by (4) and (6), respectively, we can see that there is an extra term in (4). This difference leads to the question which equation should be used for circuit modeling.

There is a view that (4) represents voltage-dependent capacitors incorrectly [1], [3]–[5]. However, at first glance, (4) seems more tempting in the sense that it clearly accounts for the change in capacitance due to the voltage change. Consistent with this view is the conclusion that (4) is the more correct expression than (6) for the voltage-dependent capacitance [6]–[8]. Another view is that (6) is only appropriate for small signals, whereas (4) should be used for large-signal analyses [9]. It was also shown that (4) and (6) can be equated by using different definitions for \( C(V) \), which are the definition of total capacitance \( C(V) = C_i = Q/V \) in (4) and the definition of differential capacitance \( C(V) = C_{di} = dQ/dV \) in (6) [10]–[12]. However, this shifts the question about the correct current equation to the question which of these two different capacitance definitions corresponds to the physical parameters of real voltage-dependent capacitances. The lack of clarity remains, which is manifested very clearly by the fact that MathWorks is providing both (4) and (6) as options in Simulink [13].

In this paper, we present experimental results to demonstrate that (6) is the correct equation. We also analyze and discuss the two capacitance definitions to show that \( C_{di} \) is real and to explain why \( C_i \) is the result of a confusing and unnecessary mathematical transformation.

II. EXPERIMENT

A commercial Schottky diode (STPS10L25) from STMicroelectronics was selected for the experiment. A reverse-biased Schottky diode is analogous to a parallel plate capacitor with the depletion layer acting as the dielectric between the plates [14]. However, since the depletion-layer width changes with the applied bias, the capacitance is voltage dependent.

Capacitance–voltage (\( C–V \)) measurements for the selected diode were performed with an Agilent Power Device Analyzer (B1505A) using four-point probe measurement. The \( C–V \) measurements, which are shown in Fig. 1, were performed up to the reverse-bias voltage of 25 V with a step size of 50 mV.

The selected Schottky diode in reverse-biased mode was used as a voltage-dependent capacitor in the simple R-C circuit shown in Fig. 2. To avoid unnecessary signal distortion, a sinusoidal voltage, \( v_{in} \), with the frequency of \( f = 1 \) MHz was used as the source in the testing circuit [15]. The sinusoidal voltage is generated from AFG1022 function generator. In order to keep the diode in reverse bias, a DC offset, \( V_{IN} \), was added in series with the sinusoidal voltage from the signal generator. The DC bias and the amplitude of the sinusoidal voltage were selected to cover the maximum change in the capacitance—as such, both the DC bias and the amplitude were set at 5 V. The external resistance, \( R = 56 \Omega \), was used to enable minimum noise in the measured current.

![FIGURE 1. Measured C–V characteristics of STPS10L25 using Agilent Power Device Analyzer.](Image)

![FIGURE 2. Reversed biased Schottky diode (STPS10L25) used as a voltage-dependent capacitor in a R–C circuit.](Image)

Probes and oscilloscopes of adequate bandwidth must be used for sufficient accuracy [16]. For measuring the voltage waveform across the diode capacitance, \( V_{C_{meas}} \), a Tektronix passive voltage probe (P6139B) of 500 MHz bandwidth was used. To measure the current flowing through the capacitance of the reverse-biased diode, \( i_{C_{meas}} \), a Tektronix current probe (TCP0030) of 120 MHz bandwidth was employed. Both of the probes were used in conjunction with a Tektronix DPO7104 oscilloscope of 1 GHz bandwidth.

III. EXPERIMENTAL RESULTS

The measured voltage across and the current through the capacitance of the reverse-biased diode are shown in Fig. 3. Using the measured voltage values and the measured diode capacitance (Fig. 1), (4) and (6) were used to numerically calculate the current—these are also shown in Fig. 3. It is quite evident from Fig. 3 that \( i_{C2} \), calculated by (6), matches the measured current. It is also evident that \( i_{C1} \), calculated by (4), is quite erroneous. Therefore, this straightforward experiment demonstrates that (6) is the correct equation for the current flowing through voltage-dependent capacitances.
As mentioned in the introduction, (4) and (6) can be equated by using two different definitions for a voltage-dependent capacitance [10]-[12]. These two definitions, commonly used in the literature and illustrated in Fig. 4, are the total capacitance $C_t(V) = Q(V)/V$ and the local or differential capacitance $C_d(V) = dQ(V)/dV$ [12], [13]. The capacitances $C_t(V)$ and $C_d(V)$ are also referred to as large-signal capacitance and small-signal capacitance, respectively [11], [17], [18]. Youhana et al. [19] define $C_s(V)$ as static capacitance and $C_d(V)$ as dynamic capacitance. Despite these different interpretations of $C_t(V)$ and $C_d(V)$, the mathematical equations for these two capacitances remain the same. In the following two sub-sections, we address the questions about the meaning and applicability of these two definitions.

### A. THE REALITY OF DIFFERENTIAL CAPACITANCE, $C_d = dQ/dV$

We begin the analysis of the difference between the two capacitance definitions by answering the following specific question: Which capacitance, $C_d$ or $C_t$, corresponds to the actual physical parameters in real devices? To answer this question, we will analyze the capacitance due to semiconductor depletion layers, since the depletion-layer widths determine the value of the capacitance in power devices or in diodes used as varactors.

Take the example of a uniformly doped N-type region in a Schottky diode or in a one-sided abrupt P–N junction diode. From the solution of Poisson equation, we know that the depletion-layer width is [14]:

$$W = \sqrt{\frac{2\varepsilon_s\phi}{qN_D}}$$  \hspace{1cm} (7)

where $\varepsilon_s$ is the semiconductor permittivity, $N_D$ is the donor concentration, $q$ is the charge of an electron, $\phi = V + V_b$ is the electric potential at one of the depletion-layer edges with...
respect to the other edge, \( V \) is the applied reverse-bias voltage, and \( V_{bi} \) is the built-in voltage. The charge, \( Q \), in the depletion layer is:

\[
Q = qN_D W A = A \sqrt{2 \varepsilon qN_D} \phi
\]

where \( A \) is the diode area. The differential capacitance, or also referred to as local capacitance, \( (C_d) \) is:

\[
C_d = \frac{dQ}{d\phi} = A \sqrt{\varepsilon qN_D} \phi^{-1/2} = \alpha \phi^{-1/2}
\]

(9)

where, \( \alpha = A \sqrt{\varepsilon qN_D} \).

The total capacitance \( (C_t) \) is:

\[
C_t = \frac{Q}{\phi} = A \sqrt{\varepsilon qN_D} \phi^{-1/2} = 2C_d
\]

(10)

The question now is whether \( C_d \) or \( C_t \) relates to the capacitance defined by the physical parameters of the depletion layer, \( (A \varepsilon_s / W) \):

\[
A \frac{\varepsilon_s}{W} = A \sqrt{\varepsilon qN_D} \phi^{-1/2} = C_d
\]

(11)

As a second example, let us take the case for a linear P–N junction diode, which is the other extreme from the case of the abrupt junction. Expressing the linear change of doping concentration by \( N_D = ax \), and noting that the depletion layer in the N-type region is \( W/2 \), we have [14]:

\[
W = \left( \frac{12 \varepsilon_s}{a} \right)^{2/3} \phi
\]

(12)

The charge stored in the N-type region of this capacitor is:

\[
Q = N_D \frac{W}{2} A = \frac{a}{2} (ax) \frac{W}{2} A
\]

\[
\Rightarrow Q = \frac{1}{2} \left( \frac{a W}{2} \right) \frac{W}{2} A = A \frac{a}{8} W^2
\]

(14)

Differentiating (14), we obtain

\[
dQ = A \frac{a}{4} W dW
\]

(15)

Differentiating (12), we get

\[
dW = \frac{1}{3} \left( \frac{12 \varepsilon_s}{a} \right)^{1/3} \phi^{-2/3} d\phi
\]

(16)

The differential/local capacitance \( (C_d) \) of the depletion layer for a linear P–N junction can now be calculated using (15), and (16):

\[
C_d = \frac{dQ}{d\phi} = A \left( \frac{a}{12} \right)^{1/3} \varepsilon_s^{2/3} \phi^{-1/3} = \beta \phi^{-1/3}
\]

(17)

where, \( \beta = A \left( \frac{a}{12} \right)^{1/3} \varepsilon_s^{2/3} \).

The total capacitance \( (C_t) \) of the depletion layer for a linear P–N junction is:

\[
C_t = \frac{Q}{\phi} = \frac{3}{2} A \left( \frac{a}{12} \right)^{1/3} \varepsilon_s^{2/3} \phi^{-1/3} = \frac{3}{2} C_d
\]

(18)

The capacitance that defines the physical parameters of the depletion layer for a linear P–N junction, \( A (\varepsilon_s / W) \) is:

\[
A \frac{\varepsilon_s}{W} = A \left( \frac{a}{12} \right)^{1/3} \varepsilon_s^{2/3} \phi^{-1/3} = C_d
\]

(19)

Therefore, the capacitances for the two extreme cases—(11) for the abrupt and (19) for the linear P–N junctions—show that \( C_d \) relates to the physical device parameters rather than \( C_t \).

Using \( dQ = C_d d\phi \), and noting again that the electric potential across the depletion region is a sum of the applied reverse bias voltage and built-in voltage, \( \phi = V + V_{bi} \), the current through the voltage-dependent capacitor is

\[
i = \frac{dQ}{dt} = \frac{dQ}{d\phi} \frac{d\phi}{dt} = \frac{dQ}{dV} \frac{dV}{dt}
\]

\[
\Rightarrow i = C_d \frac{dV}{dt}
\]

(20)

(21)

which matches (6) rather than (4).

We should also stress that the capacitance measurements give the differential capacitance. Advanced instruments, such as the Agilent Power Device Analyzer, perform quasi-static capacitance measurements by utilizing a linearly-ramped voltage and measuring the current. The capacitance is then obtained as [20]:

\[
C_{meas}(V) = \frac{i}{dV / dt}
\]

(22)

where, \( C_{meas}(V) \) is the measured diode capacitance. Comparing (22) to (21) shows that this measurement gives the differential capacitance. The other method of measuring the capacitance is to use a high-frequency small-signal voltage superimposed onto a DC bias. The capacitance can then be extracted by analyzing the amplitude and phase of the current signal. Clearly, this also gives the differential capacitance by definition.

**B. THE MATHEMATICS OF TOTAL CAPACITANCE, \( C_t = Q/V \)**

Referring to (9), we can see that the charge \( Q(V) \) in the definition for total capacitance, \( C(V) = Q(V)/V \), can be obtained by the following integration:

\[
Q(V) = \int_{V_{bi} + V}^{V} C_d(\phi) d\phi
\]

(23)

where \( V \) is the applied reverse-bias voltage, \( V_{bi} \) is the built-in voltage, and \( \phi \) is the electric potential across the depletion-layer width (W). As indicated by (8), the result is

\[
Q(V) = A \sqrt{2 \varepsilon qN_D (V_{bi} + V)}
\]

(24)
This charge is equal to the total charge of the donor atoms in the depletion layer, which is equal to the donor concentration \(N_D\) multiplied by the volume of the depletion layer \(AW\) and by the unit charge \(q\): \(Q(V) = qN_DAW\). Using (7) for the depletion-layer width, where \(\phi = V_{bi} + V\), we obtain (24).

This charge due to donor ions in the depletion layer is positive and it is exactly equal to the negative charge in the metal of a Schottky diode, or to the negative charge of acceptor ions in the P-type depletion layer of a P–N junction diode. The positive charge, \(Q(V)\), is distributed through the depletion layer, which means that it is not separated by the depletion-layer width \(W\) from the balancing negative charge. Given that \(Q(V)\) and \(-Q(V)\) are not separated by \(W\), the total capacitance \(C = Q(V)/V\) is not equal to the capacitance due to the depletion-layer width, \(\varepsilon AW/\).

This shows that to consider the total capacitance,

\[
C_t(V) = \frac{Q(V)}{V} = \frac{1}{V} \int_{0}^{V_{bi}+V} C_d(\phi) d\phi
\]

as no more than a mathematically defined capacitance, which is related to the real capacitance \(C_d\) through the integration in (25). Given that the integral in (25) transforms the real capacitance \(C_d\) into \(C_t\) as a variabe in an abstract mathematical space, we can transform \(C_t\) back to reality by the inverse first-derivative function. Related to the question of current through a voltage-dependent capacitance, this can be achieved in the following way:

\[
i_c = \frac{dQ(V)}{dt} = \frac{d[C_t(V) V]}{dt}
\Rightarrow i_c = C_t(V) \frac{dV}{dt} + V \frac{dC_t(V)}{dV} \frac{dV}{dt}
\]

(27)

Note that (27) is the same as (4) when \(C(V) = C_t(V)\), which is the suggested use of (4) by the authors of [10]-[12]. However, the use of the mathematical \(C_t\) in (27) is unnecessary because the first derivative \(dC_t(V)/dV\) in (27) transforms \(C_t\), defined by (25) back to the real capacitance \(C_d\). Given that \(dV = d\phi\), we have

\[
V \frac{dC_t(V)}{dV} = V \frac{d}{dV} \left( \frac{1}{V} \int_{0}^{V_{bi}+V} C_d(\phi) d\phi \right)
\Rightarrow V \frac{dC_t(V)}{dV} = C_d(V) - C_t(V)
\]

(28)

Inserting the result obtained in (29) into (27) shows that the mathematical \(C_t\) disappears as (27) is transformed back to the current equation with the real capacitance \(C_d\):

\[
i_c = C_d \frac{dV}{dt}
\]

(30)

\[i_c = \frac{dQ(V)}{dt} = \frac{d[C_t(V) V]}{dt} \Rightarrow i_c = C_t(V) \frac{dV}{dt} + V \frac{dC_t(V)}{dV} \frac{dV}{dt}
\]

(27)

V. CONCLUSION

We have demonstrated experimentally that the current through a voltage-dependent capacitor \(C(V)\) should be calculated by (6), where \(C(V) = dQ(V)/dV\). We have also demonstrated by theoretical analysis that the differential capacitance relates to the physical parameters of real capacitances in semiconductor devices. Finally, we have shown that the total capacitance is not a different capacitance in reality but a mathematical transformation of the measurable differential capacitance. Although this transformation can be reversed by the additional term in (4) to yield the correct result for current through voltage-dependent capacitance, these back and forth transformations are unnecessary. This result shows that—to avoid the confusion caused by the two capacitance definitions and by the two equations for current through voltage-dependent capacitances—the total-capacitance definition, \(C = Q(V)/V\), should not be used.

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