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A Simple Equation for the Energy Stored by Voltage-Dependent Capacitances

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***Abstract*- The parasitic capacitances of semiconductor power devices that contribute to the switching losses are voltage-dependent, which can make calculations of their stored energy difficult. Typically, manufacturers will provide effective capacitance values to aid in circuit design and component selection. However, stored energy calculations using these effective capacitor values are erroneous. In this paper, we derive a new equation for the stored energy in the voltage-dependent capacitance associated with a semiconductor depletion region, such as in diodes and transistors. In particular, we show that the ½ term in ½CV^2 should be replaced by a new term γ, which depends on the device structure. By applying our proposed method to several commercial diodes and transistors, we show that it matches the measured data much better than using the effective capacitances. The proposed equation will enable better power circuit design by improving the accuracy of stored energy calculations.[1]**

***Index Terms*—C_{OSS} losses, effective capacitance, energy stored, E_{OSS}, switching losses, simple equation, voltage dependent parasitic capacitances.**

I. Introduction

INCREASE in switching frequencies have driven modern power converters towards greater power densities and smaller converter sizes. However, with the increase in the switching frequencies, the switching characteristics—and crucially the switching losses—of the power semiconductor devices become highly impactful and may compromise the overall power efficiency. The switching characteristics of the power semiconductor devices are highly dependent on the voltage-dependent parasitic capacitances. At high frequencies, the effect of these voltage dependent capacitances is profound, but their impact is largely overlooked by the researchers [1]-[4].

The output capacitance (C_{OSS}) of a power semiconductor device is the primary voltage-dependent capacitance of interest as it actively participates in the switching transitions. For a hard switched converter, C_{OSS} is charged during turn-Off interval, and then discharges the same stored energy through the channel of the power semiconductor device during turn-On interval [5]-[7]. Hence, it plays a crucial role in the distribution of switching losses. In soft switching circuits, C_{OSS} plays a significant role in determining the resonant frequency for creating zero-voltage switching (ZVS) conditions [3]. Therefore, C_{OSS} cannot be ignored for power circuits. A greater insight into C_{OSS}, and in particular the impact of its voltage dependencies, will be useful for designing more efficient power converters.

Generally, C_{OSS} versus applied drain-to-source voltage curves are provided by the manufactures in their datasheets, which is useful for circuit analysis. Modern datasheets also provide energy stored in the C_{OSS} with increasing drain-to-source voltage (v_{DS}). The C_{OSS} stored energy can also be computed by integrating $C_{OSS}(v_{DS})\cdot v_{DS}$ product from zero to a chosen V_{DS} value [8]. Additionally, the manufacturers also provide effective capacitances in their datasheets. The primary reason for this is to translate the voltage-dependent capacitances into voltage-independent values of the effective capacitances, so that traditional linear methods can be applied to analyze the power circuits [7], [9].

The effective capacitance that is widely used for calculating the energy stored in the $C_{OSS}(v_{DS})$ is defined as the fixed output capacitance that will store the same amount of energy as $C_{OSS}(v_{DS})$ does when v_{DS} rises from 0 V to 80% of the maximum drain-to-source voltage (V_{DS}=0.8$V_{DS\text{-}MAX}$) [7], [10]. According to this definition, the value of the effective capacitance, which we will label by C_{eff}, is calculated by the following equation [11]:

$$\frac{1}{2}V_{DS}{}^{2}C_{eff} = \int_{0}^{V_{DS}} C_{OSS}(v_{DS}) \times v_{DS}\,dv_{DS} \tag{1}$$

Approximating the voltage-dependent capacitance of power semiconductor devices with the voltage-independent effective capacitance (C_{eff}) simplifies circuit analysis and calculations of the dissipated energy. However, it can also result in significant errors [7]. The concept of C_{eff} is correct for the specific V_{DS} point selected as the upper integral boundary in (1), meaning that the energy calculated using ½$C_{eff}V_{DS}^2$ is correct only for a single specified V_{DS} point and gives wrong results for all other v_{DS} voltages.

This paper addresses the problem in calculating energy stored using C_{eff}. For the case of constant capacitances, E = ½CV^2 is valid, but this simple equation does not apply for the case of voltage-dependent capacitors. In this paper, we show

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that the factor ½ in the constant capacitance equation can be modified and used with the actual measured capacitance of the power semiconductor device to obtain correct values of stored energy (E_{OSS}) at any voltage across the capacitor. Our approach provides a simple equation that can fit the stored energy in the voltage-dependent capacitances of diodes, power MOSFETs, superjunction (SJ) MOSFETs, and HEMTs.

To demonstrate the theoretical foundation of this approach, which is to modify the ½ term rather than the voltage-dependent capacitance, we derive the equation for energy stored by the parasitic capacitance in semiconductor diodes in the next section.

II. Derivation of the Fundamental Equation for Energy Stored in Parasitic Capacitances of Semiconductor Diodes

A. Linear P–N Junction

The depletion-layer width (W) of linear P–N junction is given by [12]:

$$W = \left(\frac{12\varepsilon_s}{a} \varphi \right)^{1/3} \tag{2}$$

where, ε_s is the semiconductor permittivity, φ is the electric potential, and we have expressed the linear change of doping concentration by $N_D = ax$. Note that the depletion layer in the N-type region is equal to $W/2$. The charge stored in the N-type region (the cathode) of this capacitor is given as [12]:

$$Q = A\left(a/8 \right) W^2 \tag{3}$$

where, A is the area of the linear P–N junction. Differentiating (3) and (2), we obtain

$$dQ = A\left(a/4 \right) W dW \tag{4}$$

$$dW = \frac{1}{3}\left(\frac{12\varepsilon_s}{a} \right)^{1/3} \varphi^{-2/3} d\varphi \tag{5}$$

The differential capacitance of the depletion layer for linear P–N junction can now be calculated using (2), (4), and (5):

$$C_d = \frac{dQ}{d\varphi} = A\left(\frac{a}{12} \right)^{1/3} \varepsilon_s^{2/3} \varphi^{-1/3} = \beta \varphi^{-1/3} \tag{6}$$

where $\beta = A\left(a/12 \right)^{1/3} \varepsilon_s^{2/3}$.

In the case of a constant capacitance, C, the well-known equation for the stored energy is obtained in the following way:

$$E = \int_0^V \varphi dQ = \int_0^V \varphi C d\varphi = \frac{1}{2} C V^2 \tag{7}$$

However, in the case of a voltage-dependent capacitor, (7) has to be modified. The voltage-dependent capacitance has to be integrated together with the electric potential, and the boundaries of the integral need to account for the relationship between the applied voltage and the electric potential across the depletion layer of the diode. Using (6) for the case of a linear P–N junction, we have

$$E = \int_{V_{bi}}^{V+V_{bi}} \varphi C_d d\varphi = \frac{3}{5} \beta \left[\left(V + V_{bi} \right)^{5/3} - V_{bi}^{5/3} \right] \tag{8}$$

In terms of $C_d = \beta \left(V + V_{bi} \right)^{-1/3}$, (8) can be rewritten as:

$$E = \frac{3}{5} C_d \left(V + V_{bi} \right)^2 - \frac{3}{5} C_d \left(V + V_{bi} \right)^{1/3} V_{bi}^{5/3} \tag{9}$$

For high reverse bias voltages, $V_{bi} << V$, and V_{bi} can be ignored in (9). Therefore, the energy stored in the voltage-dependent capacitance of a linear P–N junction can be approximated by:

$$E \approx \frac{3}{5} C_d V^2 \tag{10}$$

The comparison between (10) and (7) shows the difference between the stored energy in a voltage-dependent capacitor and in a fixed capacitor.

B. One-Sided Abrupt P–N Junction and Schottky Diodes

The derivation for one-sided abrupt P–N junctions and Schottky diodes, which is the opposite extreme case from the linear P–N junction, is analogous [12]. The following is the result for the energy stored in one-sided abrupt P–N junctions and Schottky diodes:

$$E \approx \frac{2}{3} C_d V^2 \tag{11}$$

Again, the comparison between (11) and (7) shows the difference between the stored energy in a voltage-dependent capacitance and in a fixed capacitor.

III. Proposed Method

The equation for the depletion-layer capacitance of an arbitrary P–N junction, which is widely used by the researchers for modelling the voltage-dependent capacitance of power semiconductor devices, is given by [5], [7], [12]:

$$C_d(V) = C_d(0) \left[1 + \left(V/V_{bi} \right) \right]^{-m} \tag{12}$$

Here, $C_d(0)$ is the zero bias capacitance and m is the grading coefficient. Note that the grading coefficient m is equal to ½ for the case of an abrupt P–N junction, ⅓ for the case of a linear P–N junction, and it can take a value between ½ and ⅓ for the case of an arbitrary doping profile at the P–N junction. For high voltages, $V/V_{bi} >> 1$, and (12) can be rewritten as:

$$C_d(V) = C_d(0) V_{bi}^{m} V^{-m} \tag{13}$$

The energy stored in this voltage dependent capacitance is:

$$E = \int_0^V C_d(V) V dV = \int_0^V C_d(0) V_{bi}^{m} V^{-m} V dV \tag{14}$$

$$E = \frac{C_d(0) V_{bi}^{m} V^{-m}}{2-m} V^2 = \frac{C_d(V) V^2}{2-m} \tag{15}$$

To simplify (15), we introduce the term $\gamma = 1/(2-m)$, which leads to the final form of the proposed equation:

$$E = \gamma C_d(V) V^2 \tag{16}$$

Note that (16) becomes $E = ½\ CV^2$ for $m = 0$, which is the energy stored in fixed capacitance, exactly like (7).

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Furthermore, (16) represents the energy stored in both the abrupt and linear P–N junctions, as derived in Section II, with $\gamma = 2/3$ and $\gamma = 3/5$, respectively (the two extreme cases described in the previous section). For an arbitrary doping profile at the P–N junction, γ values could be between 3/5 and 2/3. Therefore, we propose (16) to be used as a simple equation for calculating the energy stored in voltage-dependent capacitances, where γ is a variable parameter that can be used to fit different doping profiles.

The next section will show the implementation of (16) in commercial diodes.

IV. Implementation in Commercial Diodes

As mentioned in the previous section, the doping profile of real junctions lies between the abrupt and liner extremes. Therefore, the energy stored in the parasitic capacitances of a real diode will be between the extreme values given by (10) and (11), which means that $3/5 < \gamma < 2/3$. The actual value of this parameter for a specific diode can be determined by fitting (16) to the dependence of stored energy on voltage, obtained by $\int C_d(V)VdV$. Note that by proposing (16) we enable device manufacturers to provide the value of γ in their datasheets, instead of either the simple but incorrect C_{eff} or the graphs of stored energy versus voltage that are not suitable for analyses by analytical equations.

To verify the proposed approach, we applied it to three commercial diodes: STPS10L25D, RFNL20TJ6S, and SB5100. Capacitance–voltage (*C*–*V*) measurements for the respective diodes were performed with an Agilent Power Device Analyzer (B1505A) using four-point probe measurement. The proposed method was applied to the aforementioned diodes and the results are shown in Fig. 1. Note that C_{eff} for each of the diodes was calculated using (1) and the respective C_{eff} values are: 761 pF for STPS10L25D, 107.9 pF for RFNL20TJ6S, and 92.4 pF for SB5100.

The two simple equations, the usual $(1/2)C_{eff}V^2$ and the newly proposed $\gamma C_d(V)V^2$, are compared to the correct value of the stored energy, $\int C_d(V)VdV$, which was calculated from the measured C_d–V data. It is clear from Fig. 1 that a significant error is being made when using C_{eff}. On the other hand, the proposed equation (16) fits very closely to the correct values of the stored energy for all diodes and at all values of applied voltage. It can be also noticed that the γ values for all the diodes are in the derived theoretical range for diodes with doping profiles between the abrupt and linear extremes.

V. Applications to Power MOSFETs, Superjunction MOSFETs, and HEMTs

While our proposed method has been derived for the case of diodes, we can also apply it to the output capacitances of power MOSFETs, and HEMTs. This is because these parasitic capacitances are primarily caused by depletion regions in the device structure and the voltage dependences are quite similar to those of diodes.

Furthermore, the proposed equation can be applied to superjunction (SJ) MOSFETs with a slight modification to

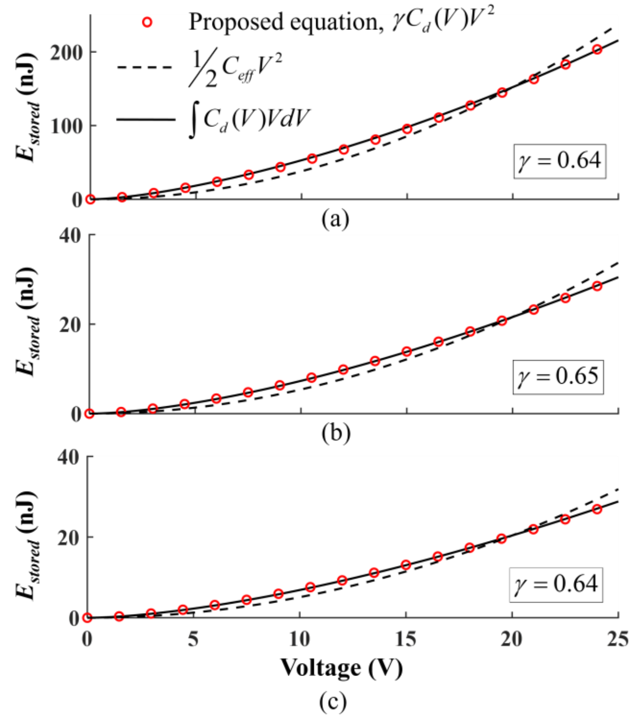

Fig. 1. Energy stored in the parasitic capacitance of (a) STPS10L25 (b) RFNL20TJ6S (c) SB5100 using different methods.

account for the complex structure of this device. The typical capacitance–voltage dependence of these devices consists of two distinct ranges [13]. The first is a low-voltage range, which is mainly determined by the 3-D surface of the blocking P–N junctions. However, the N-type region between the P-type columns is fully depleted at relatively low voltages (typically, less than 10% of the blocking voltage). When this occurs, any further voltage increase beyond this boundary voltage increases the depletion layer in the N-type drift region. This is the second range, where the equation we have derived for the drift region in diodes is applicable. Since this high-voltage range begins at voltages smaller than 10% of the blocking voltage, we can conclude that the specific *E(V)* dependence in the low voltage range is of no interest. However, the energy stored at the boundary voltage when the device enters the high-voltage range has to be added to the *E(V)* equation for the high-voltage range. Therefore, we propose a modification in the derived equation by adding a constant-energy term, E_{CONST}, to account for the energy stored at the boundary voltage: $E = \gamma C_d(V)V^2 + E_{CONST}$.

A 300-V Si power MOSFET (IRFB4137PbF) from Infineon, a 100-V enhancement-mode GaN HEMT (GS61004B) from GaN Systems, a 650-V SJ MOSFET (IPB65R045C7) from Infineon, and a 650-V SJ MOSFET (FCHD125N65S3R0) from ON Semiconductor were selected to test the proposed approach. The manufacturers have provided the E_{OSS} plot for the respective devices, which will be used to verify the proposed equation. We used the g*rabit.m* command in the MATLAB to extract the C_{OSS} and E_{OSS} data from the respective datasheets. For better resolution, data points between those taken from the datasheet were generated via interpolation. C_{eff} values quoted in the datasheet for IRFB4137PbF, GS61004B, IPB65R045C7,

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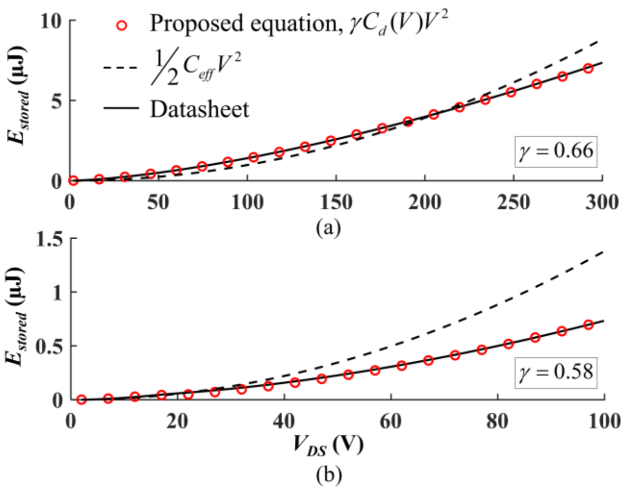

Fig. 2. Energy stored in C_{OSS} of (a) Power MOSFET IRFB4137PbF and (b) GaN-HEMT GS61004B.

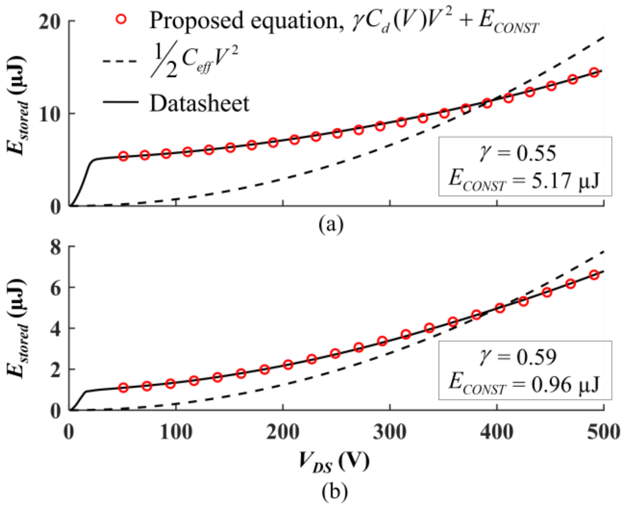

Fig. 3. Energy stored in C_{OSS} of (a) SJ MOSFET IPB65R045C7 and (b) SJ MOSFET FCHD125N65S3R0.

and FCHD125N65S3R0 are equal to 196 pF, 276 pF, 146 pF, and 62 pF, respectively.

Unlike the diodes in the previous section, we allowed γ for these selected devices to be unbounded. The proposed approach was applied to these selected devices and the corresponding energy stored in the C_{OSS} is shown in Fig. 2 and Fig. 3. The values of γ for IRFB4137PbF, GS61004B, IPB65R045C7, and FCHD125N65S3R0 obtained by fitting (16) to the energy values from the datasheets are 0.66, 0.58, 0.55, and 0.59, respectively. The values of E_{CONST} for IPB65R045C7 and for FCHD125N65S3R0 are 5.17 µJ and 0.96 µJ, respectively. It can be clearly seen that the proposed simple equation (16), with the modification for SJ MOSFETs, can match the datasheet values of E_{OSS}. As distinct from this, the energy calculation using C_{eff} only matches the energy stored at V_{DS} = 240 V for IRFB4137PbF, at V_{DS} = 400 V for the two SJ MOSFETs, and at smaller voltages for GS61004B. Importantly, the approach based on C_{eff} cannot represent the entire voltage range. This clearly shows the error in energy calculations by the simple equation with constant capacitance C_{eff}. Therefore the energy calculation from our proposed equation, as compared to using C_{eff}, is much more accurate.

VI. Conclusion

In this paper, we have proposed a replacement of the equation $E = ½C_{eff}V^2$ for the energy stored in the parasitic capacitances of diodes and power switches. We have demonstrated both theoretically and experimentally that the equation $E = \gamma C_d(V)V^2$, where γ is a variable parameter and $C_d(V)$ is the actual measured capacitance, provides an excellent fit to the actual dependencies of stored energy on applied voltages for the case of commercial diodes, power MOSFETs, and HEMTs. For the case of superjunction MOSFETs, we have added a constant term, E_{CONST}, to enable its application to the high-voltage range of interest for circuit design and analysis. The proposed equation will be useful for power circuit design by enabling fast, simple, and accurate calculations of the switching losses.

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