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ANALYTICAL SOLUTION AND FIELD TEST OF CRITICAL BEARING CAPACITY AND SETTLEMENT OF PILE TIP

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ABSTRACT

In order to explore the relationship between the critical bearing capacity and settlement of closed pile tip pierced into the soil, based on the Boussinesq solution and the Kelvin solution, the analytical solution between the critical bearing capacity and the critical settlement of the closed pile tip is derived by combining the stress distribution function. The analytical solution of critical bearing capacity and settlement of pile tip is verified by field test of static pressure pile penetrating into layered soil with a full-section pressure sensor installed at pile tip. The results show that during the penetration process, the bearing capacity increase stage of the pile tip is divided into linear steepening section and nonlinear slow increasing section. The soil in the linear steep increase section behaves as an elastic state. The bearing capacity of the pile tip before the punctured soil layer is linear with the settlement, and the final value of the linear steep increase section is the elastic limit value and the critical bearing capacity of the piercing pile tip. When the residual pile tip force is not considered, the critical settlement of the pile tip is between 0.095-0.119d; when considering the residual pile tip force, the critical settlement is between 0.091-0.105d. In particular, when the Poisson's ratio is 0.5, the analytical solution of the semi-infinite space is equivalent to the analytical solution of the infinite space.

KEYWORDS

Pile tip force, Piercing, Settlement, Critical bearing capacity, Field test

Introduction

The relationship between bearing capacity and settlement of pile has always been the focus of research in the field of pile foundation engineering. Especially for settlement -reducing pile, the bearing capacity within the reasonable settlement range is the basis of piles design [1]-[3]. Many scholars have carried out a lot of research on the relationship between bearing capacity and settlement of pile[4]-[6], but only a few experiments are concerned with the relationship between ultimate bearing capacity and settlement of pile tip in the critical state [7]-[10], the relationship between the critical bearing capacity and settlement of pile is not established. The analytical solution is only for numerical simulation analysis of sand [11]-[12]. The relationship between the critical bearing capacity and settlement of a pile tip is of great significance for different working conditions. For example, for the end-bearing pile, the pile tip bearing capacity within the reasonable settlement can be calculated. For the rigid composite foundation, the thickness of the cushion layer can be reasonably designed according to the critical bearing capacity of the pile top when piercing the cushion layer to better exert the bearing capacity of the composite foundation.

The existing bearing capacity theory of pile tip mainly includes load transfer method [13], elastic theory method [14], ultimate bearing capacity theory [15]-[18], etc., but the problem of penetration is not well explained. The process of dynamic penetration of the pile foundation into the soil layer has a process of acceleration and deceleration. The difficulty of measuring the acceleration will result in difficulty in balancing the pile foundation during the force analysis, and the huge soil compaction effect caused by the dynamic penetration will lead to the change of soil properties. However, the pile is a quasi-static process before the soil layer is pierced. The Boussinesq problem and the Kelvin problem can well solve the ultimate bearing capacity and settlement relationship between shallow and deep soil layers, but the force is still the point load. This requires a point load to be converted into a surface load to find a reasonable stress distribution function. The author deducts the assumed stress distribution function by the inverse solution method to meet the settlement of pile tip, solves the transformation of the pile tip force from the point load to the surface load, and combines the closed pile with the full section pressure sensor installed on the pile tip. Through the layered soil test [19], the analytical solution of the bearing capacity and settlement of the pile tip was verified.

Analytical solution of bearing capacity and settlement of pile tip in shallow soil

The Boussinesq solution assumes that the concentrated load acts on the surface of the semi-infinite space homogeneous elastomer. When the pile tip is in initial contact with the soil, it can be considered that the surface load acts on the surface of the semi-infinite space homogeneous elastomer. At this time, the soil is an undisturbed soil and satisfies the assumptions of uniformity, continuousness, small deformation, and elasticity. As the penetration force increases, the pile tip is more closely in contact with the surface of the soil, and the soil is compressed in the elastic range. At this time, the relevant assumptions of the Boussinesq solution can still be satisfied; As the penetration force continues to increase, the soil at the pile tip will not be able to satisfy the small deformation hypothesis. At this time, the force at the pile tip is the critical bearing capacity of the pile tip, and the Boussinesq solution fails at the next moment.

Boussinesq solution of earing capacity and settlement of pile tip

For the initial penetrating soil at the pile tip, the solution of the vertical concentrated force and settlement in the semi-infinite space can be solved, that is, the spatial vertical settlement of the Boussinesq problem is solved.

Formula:

$$u_z = \frac{1+\upsilon}{2\pi E}\left[\frac{2(1-\upsilon)}{r} + \frac{z^2}{r^3}\right]F_z \tag{1}$$

Among them,u_z is the vertical settlement, $r = \sqrt{x^2 + y^2 + z^2}$, *E* is the compressive modulus of the soil, υ is the Poisson's ratio of the soil, and F_z is the penetration force of the pile tip.

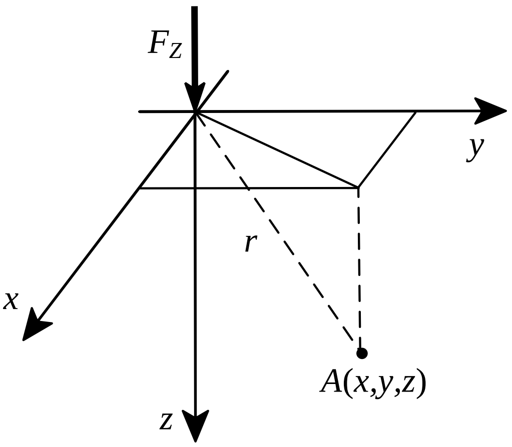

Fig.1 - Boussinesq problem schematic

Take the center of the coordinate axis as the center of the pile tip, and z=0 at the pile tip in the half-space infinite body. The coordinate system is shown in Figure1. The vertical settlement is simplified as:

$$u_z = \frac{(1-\upsilon^2)}{\pi E}\frac{F_z}{r} \tag{2}$$

Where $r = \sqrt{x^2 + y^2}$.

Analytical solution of bearing capacity and settlement of pile tip in semi-infinite space

The force of the Boussinesq problem is a vertical concentration force, and the vertical settlement caused by multiple vertical concentrated forces can be obtained by superposition. In order to study the pile tip penetration force, a finite number of vertical concentrated force superposition methods cannot be used. The stress distribution function is required, and the stress distribution function is assumed according to the dimension:

$$p = p_0(1-\frac{r^2}{a^2})^{-\frac{1}{2}} \tag{3}$$

$$\iint p dxdy = F_z \tag{4}$$

Where p_0 is the compressive stress amplitude, *a* is the pile tip radius, *r* is the arbitrary point to the axial distance; the distribution curve of the stress function in the radius is shown in Figure 2.

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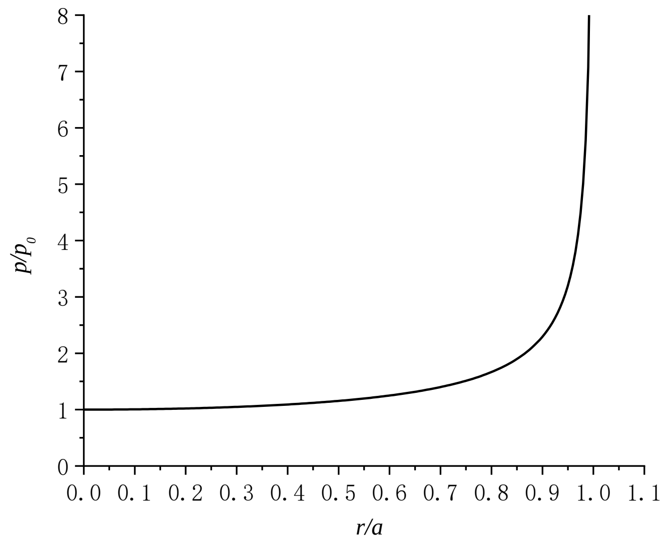

Fig.2 - Stress function curve

Combined with the stress distribution function formula (2) becomes:

$$u_z = \frac{1-\upsilon^2}{\pi E} \iint \frac{p(x,y)}{r} dxdy \tag{5}$$

Due to the rotational symmetry of the pile tip, the vertical settlement of a point is only related to its distance *r* from the center point, for which only the settlement of the point on the x-axis needs to be calculated. Take the point A coordinate (x,y) on the x-axis and any point B coordinate (x',y')on the pile tip. According to the positional relationship shown in Figure3, $t^2 = r^2 + s^2 + 2rs\cos\varphi$.

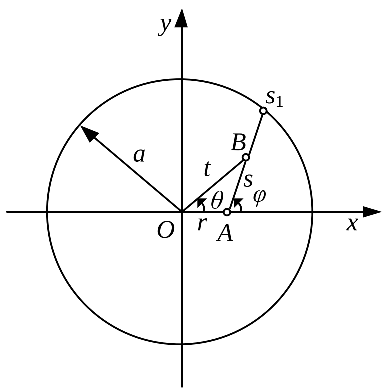

Fig.3 - Position relations of various points in the pile tip area

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Stress at point B:

$$p(s,\varphi) = p_0(1 - \frac{r^2+s^2+2rs\cos\varphi}{a^2})^{-\frac{1}{2}} = p_0 a(a^2 - r^2 - s^2 - 2rs\cos\varphi) \quad (6)$$

The settlement at point A is equal to the superposition of any point at pile tip, and is obtained according to formula (7):

$$u_z = \frac{1-\upsilon^2}{\pi E} p_0 a \int_0^{2\pi} (\int_0^{s_1} (a^2 - r^2 - 2rs\cos\varphi - s^2)^{-\frac{1}{2}} ds) d\varphi \quad (7)$$

Where s_1 is the positive root of the equation $a^2 - r^2 - 2rs\cos\varphi - s^2 = 0$,

$$\int_0^{s_1} (a^2 - r^2 - 2rs\cos\varphi - s^2)^{-\frac{1}{2}} ds = \frac{\pi}{2} - \arctan\left[\frac{r\cos\varphi}{(a^2 - r^2)^{\frac{1}{2}}}\right]$$

Equation (7) is calculated:

$$u_z = \frac{1-\upsilon^2}{\pi E} p_0 a \int_0^{2\pi} \frac{\pi}{2} d\varphi = \frac{\pi(1-\upsilon^2) p_0 a}{E} = \text{constant} \quad (8)$$

The vertical settlement of each point in the contact area of a closed pile tip is the same under the condition of displacement coordination.

Calculated by (4) conversion element integration:

$$F_z = \int_0^a p_0 \left(1 - \frac{r^2}{a^2}\right)^{-\frac{1}{2}} 2\pi dr = 2\pi p_0 a^2 \quad (9)$$

The simultaneous equations (8) and (9) get:

$$u_z = \frac{(1-\upsilon^2)}{2} \frac{F_z}{Ea} \quad (10)$$

Analytical solution of bearing capacity and settlement of pile tip in deep soil

Kelvin's solution is based on the assumption that the concentrated load acts on the surface of the infinite space homogeneous elastomer. The initial contact between the pile tip and the deep soil can also be considered as the surface load acting on the surface of the infinite space homogeneous elastomer. At this time, the deep soil is the original. The soil satisfies the assumptions of uniformity, continuousness, small deformation, and elasticity.

As the penetration force increases, the pile tip is more closely in contact with the surface of the soil, and the soil is compressed in the elastic range, and the relevant assumption of the Kelvin solution can still be satisfied at this time; As the penetration force continues to increase, the soil at the pile tip will produce a critical moment of compaction, and the soil will not be able to satisfy the small deformation hypothesis. At this time, the pile tip force is the critical bearing capacity of the pile tip. At the next moment, Kelvin's solution failed.

Kelvin solution of bearing capacity and settlement of pile tip

For the ultimate bearing capacity of the pile tip in the deep soil layer, the Kelvin problem in the infinite space can be solved, and the vertical settlement solution according to the Kelvin problem:

$$u_z = \frac{1}{2(1-\upsilon)}\frac{(1+\upsilon)}{4\pi E}\left[\frac{2(1-2\upsilon)}{r} + \frac{1}{r} + \frac{z^3}{r^3}\right]F_z \tag{11}$$

Taking the center of the coordinate axis as the center of the pile tip and taking z=0 in the infinite body, the vertical settlement is simplified as:

$$u_z = \frac{(1+\upsilon)}{2(1-\upsilon)}\frac{(3-2\upsilon)}{4\pi E}\frac{F_z}{r} \tag{12}$$

Analytical solution of bearing capacity and settlement of pile tip in infinite space

The stress distribution function formula (12) combined with (3) becomes:

$$u_z = \frac{(1+\upsilon)}{2(1-\upsilon)}\frac{(3-2\upsilon)}{4\pi E}\iint \frac{p(x,y)}{r}dxdy \tag{13}$$

Substituting (6) into (13) gives:

$$u_z = \frac{(1+\upsilon)}{2(1-\upsilon)}\frac{(3-2\upsilon)}{4\pi E}p_0 a\int_0^{2\pi}\left(\int_0^{s_1}(a^2 - r^2 - 2rs\cos\varphi - s^2)^{\frac{1}{2}}ds\right)d\varphi \tag{14}$$

The same reason:

$$\int_0^{s_1}(a^2 - r^2 - 2rs\cos\varphi - s^2)^{-\frac{1}{2}}ds = \frac{\pi}{2} - \arctan\left[\frac{r\cos\varphi}{(a^2 - r^2)^{\frac{1}{2}}}\right]$$

(14) Simplification is obtained:

$$u_z = \frac{(1+\upsilon)}{(1-\upsilon)}\frac{(3-2\upsilon)}{8E}p_0 a\pi = \text{constant} \tag{15}$$

The settlement of each point at the pile tip is constant, and the vertical settlement of each point in the contact area of the closed pile tip is consistent, which is consistent with the displacement coordination condition of the pile tip.

Substituting (9) into (15) gives:

$$u_z = \frac{(1+\upsilon)}{(1-\upsilon)}\frac{(3-2\upsilon)}{16}\frac{F_z}{Ea} \tag{16}$$

Boussinesq solution is equivalent to Kelvin solution

When the semi-infinite space load-settlement analytical solution is equal to the infinite space load-settlement analytical solution, when equations (10) and (16) are equivalent:

$$u_z = \frac{(1-\upsilon^2)}{2}\frac{F_z}{Ea} = \frac{(1+\upsilon)}{(1-\upsilon)}\frac{(3-2\upsilon)}{16}\frac{F_z}{Ea}$$

Solving $\upsilon = 0.5$ or $\upsilon = 1.25$, when Poisson's ratio is equal to 0.5 means that formula (10) is equivalent to formula (16) under the incompressible material or under the combined effects of shear dilatation and reduction;

The Poisson's ratio can be greater than 0.5 due to the shear dilatation of the soil, but the case of $\upsilon = 1.25$ has not yet appeared in the soil. Therefore, the analytical solution of the semi-infinite space load-settlement is equivalent to the infinite space load-settlement analytical solution with Poisson's ratio $\upsilon = 0.5$.

Verification and analysis of bearing capacity and settlement of pile tips in shallow and deep soil

Field test overview

In order to verify the relationship between the critical bearing capacity and the settlement of the pile tip, an on-site static pressure pile test was carried out. The test site is located in Dongying, Shandong Province, about 20 kilometers away from the Yellow River. It belongs to the Yellow River impact plain and has clear soil layers. It is mainly composed of silt and silty clay. The soil layer is shown in Table 1. The pile diameter is 400mm and the pile length is 12m. A 100t full-section pressure sensor is placed at the end of the closed pile. The test position and sensor[19] are shown in Figure 4.

Tab. 1 - Stratigraphic parameters

Depth/m	Soil layer	Compression modulus E_{s1-2} / MPa	Void ratio *e*	Water content w / %	Cone penetration test Q_c / MPa	$\bar{N}$
0～3.08	①Plain fill(q4ml)	4.19	0.867	30.4	1.130	2.9
3.08～4.58	②Silt(q4al)	8.55	0.803	27.7	2.796	5.4
4.58～5.08	③Silty clay(q4al)	4.90	0.876	30.5	0.928	3.4
5.08～6.98	④Silt(q4al)	9.11	0.794	28.0	4.930	9.4
6.98～10.38	⑤Silty clay(q4al)	4.67	0.895	31.5	0.799	2.7
10.38～13.88	⑥Silt(q4al)	10.54	0.793	28.0	7.379	16.2

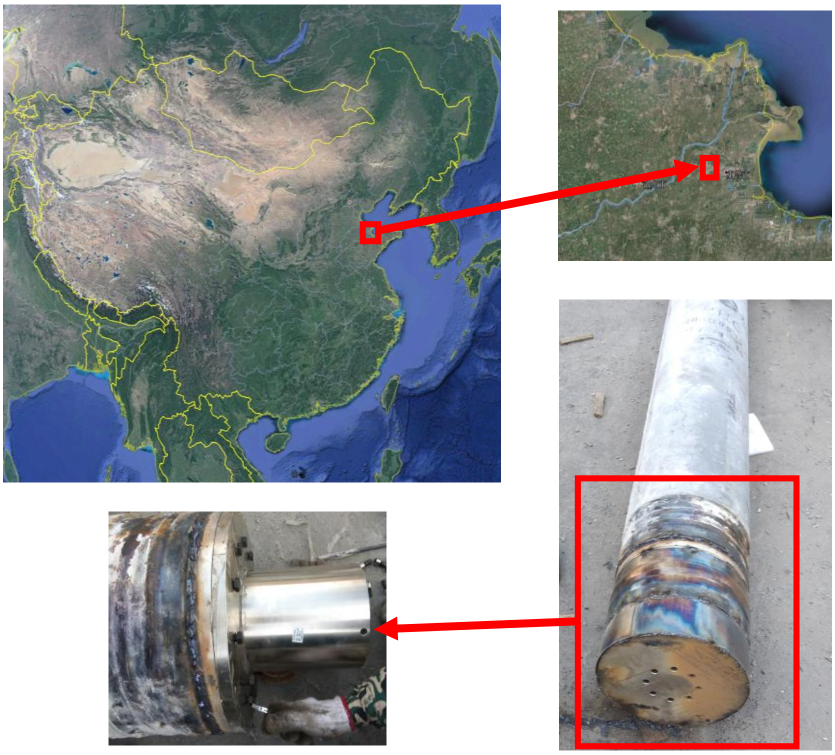

Fig.4 - Test position and sensor

Test analysis and verification

The static pressure pile penetration test was carried out in 9 strokes, and the penetration depth of each stroke was about 1.4 meters, the penetration rate of each stroke is 1.2 m / min, the penetration time of each stroke is about 70 seconds. The relationship between the pile tip resistance and the penetration depth is shown in Figure 5. After the static pressure penetration of each stroke is started, there is a stage of accumulating pressure for each stroke before the pile tip punctures the soil layer. During this period, the deformation of the soil is mainly compressed. When the pile tip force breaks through the penetration threshold, the pile will penetrate into the soil layer quickly.

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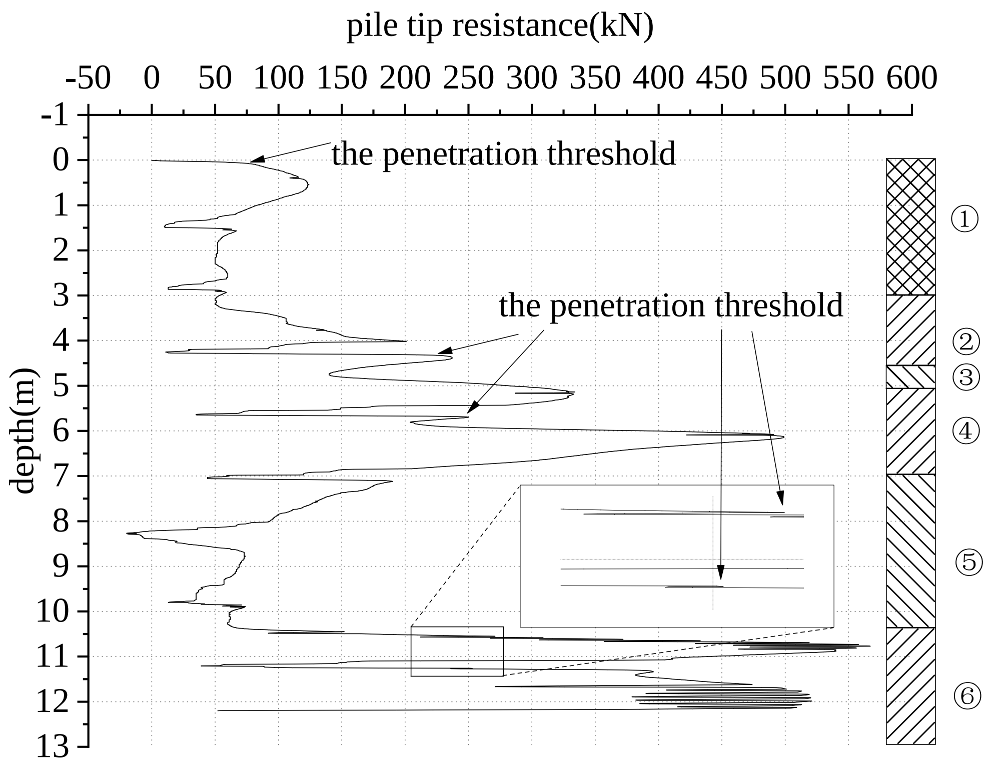

Fig.5 - Relationship between resistance and depth of pile tip

The process of pile penetration will produce severe soil squeezing effect. When the pile tip penetrates the hard layer, it has a large soil squeezing effect on the soft soil layer under the hard layer. The nature of the soft layer soil has changed and cannot be obtained by geological survey parameters[20]-[21]. The process of the pile tip penetrating from the soft layer to the hard layer. The soil squeezing effect from the soft layer to the hard layer under the pile tip is small, and the hard layer soil is less affected and can be obtained by geological survey parameters

In order to study the penetration threshold of the pile tip and eliminate extrusion effect on the soil layer, the stroke 1, the stroke 4, the stroke 5, the stroke 8 and the stroke 9 are selected for analysis. The relationship between the pile tip force and depth of each stroke is shown in Figures 6-8. It can be seen from Figures 6-8 that the pile tip force increases linearly before the pile penetrates, and the final value of linear growth is the critical bearing capacity of the piercing (or the penetration threshold). When the critical bearing capacity is reached, the vertical settlement of the pile is small, and the soil deformation is mainly compressed; after the pile penetrates into the soil layer, the bearing capacity reaches the peak with the development of soil squeezing, but the peak of bearing capacity requires large vertical settlement. The maximum bearing capacity of piles cannot be used in the design of structures, and the vertical settlement of the critical bearing capacity is small for design reference.

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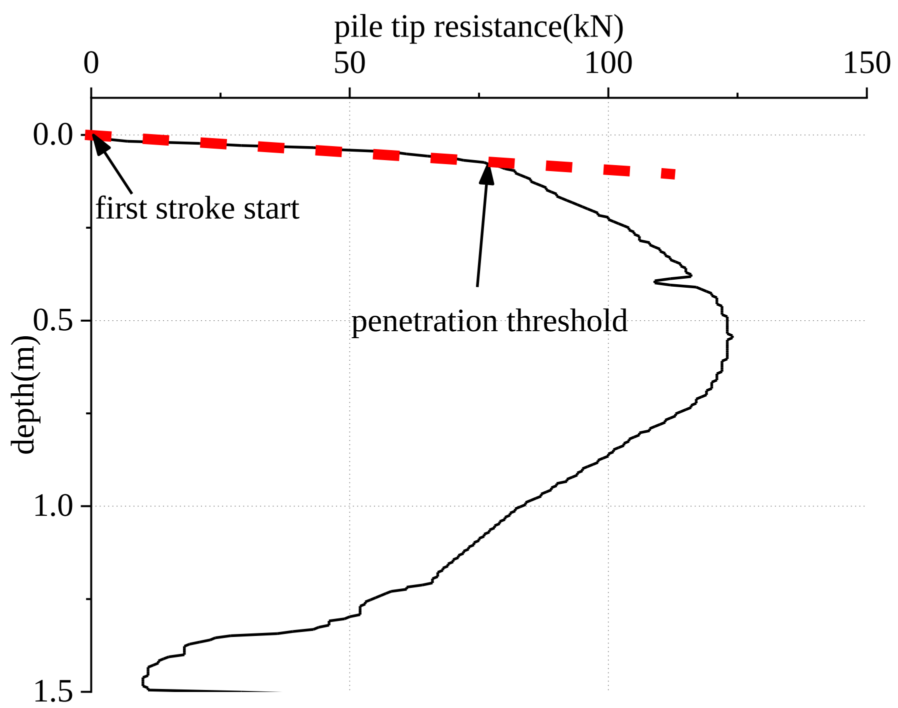

Fig.6 - The relationship between pile tip resistance and depth during the first stroke

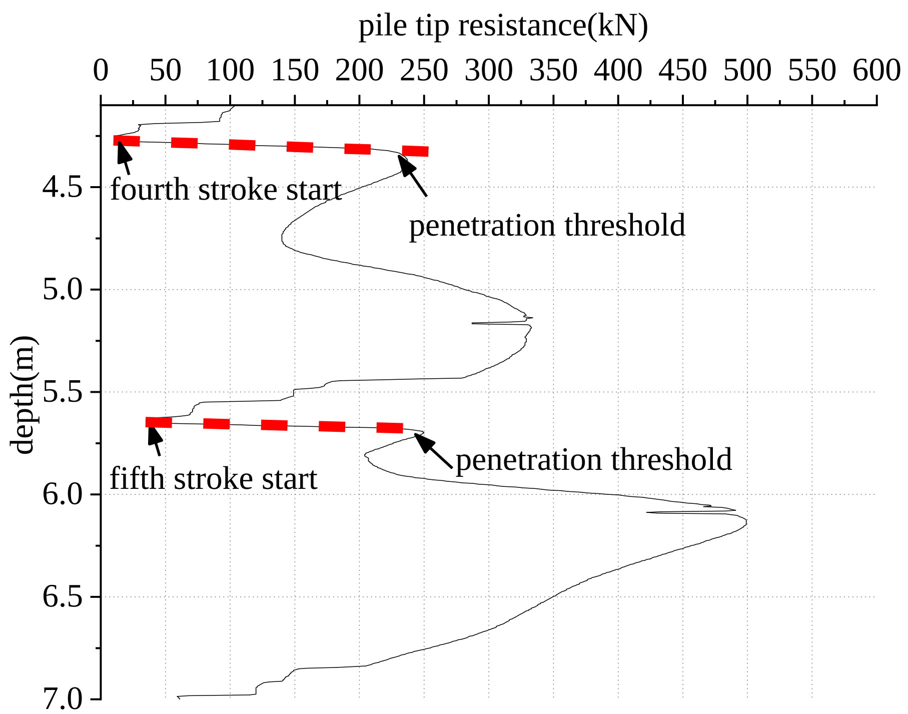

Fig.7 - The relationship between pile tip resistance and depth during the fourth and fifth stroke

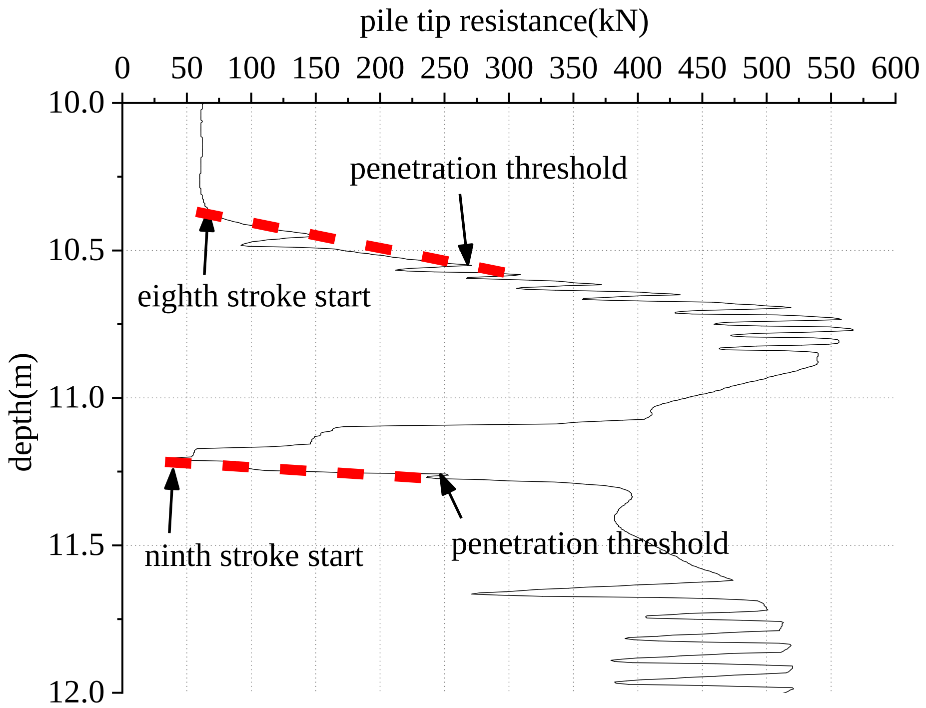

Fig.8 - The relationship between pile tip resistance and depth during the eighth and ninth stroke

According to the reference [22], the Poisson's ratio of the surface layer is stable at 0.42 and the deep layer is stable at 0.49. According to formula (10) and formula (16), the vertical settlement u_z and the pile diameter *d* is normalized, and the starting values of each research stroke and the penetration threshold are listed in Table 2.

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Tab. 2 - The relationship between the critical bearing capacity and settlement of pile tip

Soil layer	Compression modulus E_{s1-2} / MPa	Residual pile tip force / kN	Critical bearing capacity(penetration threshold)/ kN	The value of $\frac{u_z}{d}$		
				Not considering residual force	Considering residual force	Field test value
①Plain fill(q4ml)	4.19	0	77	0.095	0.095	0.200 (Ground Hard Shell)
②Silt (q4al)	8.55	15	209	0.113	0.105	0.095
④Silt (q4al)	9.11	36	217	0.110	0.092	0.097
⑥Silt (q4al)	10.54	62	271	0.119	0.091	0.450 (Junction of soil layer)
⑥Silt (q4al)	10.54	39	253	0.111	0.094	0.136

It can be seen from the data in Table 2 when the residual pile tip force is not considered, the theoretical value of the ultimate settlement and diameter ratio is between 0.095-0.119d, and the critical settlement to diameter ratio is between 0.091-0.105d when considering the residual pile tip force. When the field test value in a single soil layer and the indoor model test data in reference [12], the critical settlement is within 0.1d, which is basically consistent with the elastic phase test data. However, at the junction of the soil layer and the hard shell on the ground, the critical settlement to diameter ratio will have a large difference. This is due to the difference in the elastic modulus of the soft and hard layers. It is proved that formula (10) and formula (16) are suitable for calculating the relationship between the bearing capacity and settlement of the pile tip in the indoor and field test, but they are nearly suitable for pile tips acting on stable soil layers.

CONCLUSION

(1) Based on the Boussinesq solution and the Kelvin solution, the analytical solution between the critical bearing capacity and the critical settlement of the closed pile tip is derived by combining the stress distribution function. When the Poisson's ratio is 0.5, the analytical solution of the semi-infinite space is equivalent to the analytical solution of the infinite space.

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(2) Through the field test of the static pressure pile with the full-section pressure sensor penetrating into the layered soil, it is found that the bearing capacity of the pile tip is linear with the settlement before the piercing, and the final value of linear growth is the critical bearing capacity of the pile tip.

(3) When the residual pile tip force is not considered, the critical settlement of the pile tip is between 0.095-0.119d; when considering the residual pile tip force, the critical settlement is between 0.091-0.105d. The analytical solution between the critical bearing capacity and the settlement of the pile tip can be used as the design basis of the pile foundation.

REFERENCES

[1] LIU R, XU Y, YAN S W. Finite element analysis of action characteristics of anti-settlement pile foundation[J]. Engineering Mechanics, 2006 (02): 144-148.

[2] ZAI J M. Concept of plastically bearing pile and its practical application[J]. Chinese Journal of Geotechnical Engineering, 2001, 23(3): 273-278. (in Chinese)

[3] ZHENG G, LIU R. Engineering examples and finite element analysis of interaction mechanism between reducing pile and soil[J]. Journal of Tianjin University, 2001 (02): 209-213.

[4] CHEN L Z, LIANG G Q, ZHU J Y, et al. An Analytical Method for the Axial Load-Settlement Curve of Piles [J]. Journal of Geotechnical Engineering, 1994 (06): 30-38.

[5] CHI Y J, SONG E X, GAO W X, et al. Experimental Study on Bearing and Deformation Characteristics of Rigid Pile Composite Foundation [J]. Journal of China University of Mining and Technology, 2002 (03): 24-28.

[6] Crispin, J. J., Leahy, C. P., & Mylonakis, G. (2018). Winkler model for axially loaded piles in inhomogeneous soil. Géotechnique Letters, 8(4), 290-297. doi:10.1680/jgele.18.00062

[7] ZHOU Z J, ZHENG H. Working characteristics of cushion during the whole process of pile penetration [J]. Journal of Civil and Environmental Engineering (Chinese and English), 2019 (03): 41-48.

[8] ZHANG D G, ZHANG Z, YAN M L, et al. [J].Architectural Science, 2014, 30 (05): 52-57.

[9] ZHOU J, WANG Q Q, GUO J J. Centrifuge Test Study on pile tip penetration [J]. Industrial Building, 2012, 42 (08): 75-78.

[10] WANG H, ZHOU J, DENG Z H. Mobilization of toe resistance of piles with local displacement in sand[J]. Chinese Journal of Geotechnical Engineering, 2006, 28(5): 587-593.(in Chinese)

[11] ZHOU J, HUANG J, ZHANG J, et al. Numerical simulation of pile tip penetration in stratified media based on three-dimensional discrete-continuous coupling method [J]. Journal of Rock Mechanics and Engineering, 2012, 31 (12): 2564-2571.

[12] ZHOU J, GAO B, GUO J J, et al. Model test and numerical simulation of pile tip force under different penetration depth [J]. Journal of Tongji University (Natural Science Edition), 2012, 40 (03): 379-384.

[13] YANG H, YANG M. A review of load-settlement relationship of single pile by load transfer method [J]. Journal of Underground Space and Engineering, 2006 (01): 155-159+165.

[14] AI Z Y, YANG M. Application of generalized Mindlin solution in single pile analysis of multi-layer foundation [J]. Journal of Civil Engineering, 2001 (02): 89-95.

[15] TERZAGHI K. Theoretical soil mechanics[M]. New York: Wiley & Sons Inc, 1943.

[16] MEYERHOF G G. The ultimate bearing capacity of foundations [J]. Geotechnique, 1951, 2 (4): 301.

[17] BEREZANTZEV V G, KHRISTOFOROV V, GOLUKOV V. Load bearing capacity and deformation of piled foundations [C] // Proceedings of the 5th International Conference on Soil Mechanics and Foundation Engineering. Paris:[s. n.], 1961:1-15.

[18] VESIC A S. Ultimate loads and settlements of deep foundations in sand [C] // Proc of a symposium bearing capacity and settlement of foundations. Durham: Duke University, 1967: 53-68.

[19] YANG S C, ZHANG M Y, WANG Y H, et al. [J].Geotechnical Mechanics, 2018,39 (S2): 91-99.

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[20] PAN H, CHEN G X, SUN T. Experimental study on the Poisson's ratio of undisturbed marine soil [J]. Geotechnical Mechanics, 2011, 32 (S1): 346-350.
[21] He W M, Li D Q, Yang J, et al. Research progress of dynamic shear modulus, damping ratio and Poisson's ratio of soil [J]. Journal of Seismic Engineering, 2016, 38 (02): 309-317.
[22] GAO W P, CHEN Y K, LIU F. Preliminary analysis of Poisson's ratio characteristics of shallow strata in Tianjin [J]. Journal of Seismic Engineering, 2014, 36 (01): 47-53.