Exploring a Primary Mathematics Initiative in an Indigenous Community School

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Submitted in fulfilment of the requirements of the degree of

Doctor of Philosophy

June 2020
ABSTRACT

An important focus in Indigenous education research focuses on equitable educational experiences to support student outcomes. This study explored the implementation of a mathematics education initiative in an Australian Indigenous community school aimed at raising students’ mathematics proficiency over a 7-month period. The initiative was informed by current scholarship focused on effective practices in mathematics education, and explored how teachers implemented a mathematics education initiative, and factors that influenced the development of students’ mathematical proficiency.

To achieve the aims of the study, an innovative conceptual framework was developed integrating perspectives from sociology and psychology. A mixed methods research design was used, and data sources included standardised mathematics tests, adapted classroom diagnostic tests, problem-solving interviews, and classroom observations. The participants in the study included 50 primary school students (Years two to six) and four primary school teachers.

Findings from the study indicated that students’ mathematical proficiency was below national means at the beginning of the initiative. At the end of the study, positive changes in students’ proficiency were found, with evidence of closing gaps in achievement noted on some measures (i.e., on standardised tests). Other findings indicated there were complex factors associated with students, teachers, the school, cultural-factors, and community factors that acted to influence the development of students’ mathematical proficiency.

The findings inform curriculum recommendations including the need for early years mathematics programs to focus on key number concepts, as well as the inclusion of problem-solving heuristics into problem-solving programs. The importance of supporting Indigenous students’ productive dispositions towards mathematics, developing skills for success at school in the early years, and considering interpersonal interactions when assessing students was also highlighted in the study findings. Teaching practice recommendations include the use of balanced explicit instruction, a diagnostic teaching cycle with effective and immediate feedback, a mastery teaching approach, consistent lesson structures, and an expectation of high academic and
behavioural standards with increased time on task. The findings support the value of mentoring models of professional development to support teachers’ capacity with pedagogical content knowledge, in addition to the need to focus on developing Indigenous students’ familiarity with the school discourse to enable them to engage successfully with this discourse in the early primary years.

Keywords: mathematics education, Indigenous education, primary education, educational initiatives
STATEMENT OF ORIGINALITY

This work has not previously been submitted for a degree or diploma in any university. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

Signed:__________________________________________________________

Bronwyn Reid O’Connor
ACKNOWLEDGEMENTS

I would like to acknowledge my supervisors, Associate Professor Christine McDonald and Dr Stephen Norton, who significantly advanced my thinking throughout my candidature. Your wisdom and guidance are forever appreciated.

I would like to acknowledge my fellow postgraduate peers, who provided invaluable support throughout my candidature.

I would also like to acknowledge the students, teachers, and community who I had the privilege of working with as part of this study. I thank you for your patience, and openness in allowing me into your learning spaces.

Finally, I would like to thank my children and husband, without whom this endeavour would have never begun.
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LIST OF ABBREVIATIONS AND ACRONYMS

AAMT – Australian Association of Mathematics Teachers
ACARA – Australian Curriculum, Assessment, and Reporting Authority
ACER - Australian Council for Educational Research
ACT – Australian Capital Territory
AIATSIS - Australian Institute of Aboriginal and Torres Strait Islander Studies
CRA – Concrete, Representational, Abstract
COAG – Council of Australian Governments
DEEWR – Department of Education, Employment and Workplace Relations
DEST – Department of Education, Science and Training
DETE – Department of Education, Training & Employment
GGSA – Good to Great School Australia
IEA - International Association for the Evaluation of Educational Achievement
ILLANS - Longitudinal Literacy and Numeracy Study for Indigenous Students
NAPLAN – The National Assessment Program – Literacy and Numeracy
NCTM – National Council of Teachers of Mathematics
NHMRC - National Health and Medical Research Council
NSW – New South Wales
NT – Northern Territory
OECD - Organisation for Economic Co-operation and Development
QS - QuickSmart
SA – South Australia
SSLC – Stronger Smarter Learning Communities
TIMSS - Trends in International Mathematics and Science Study
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1 INTRODUCTION

1.1 Overview

Gaps in achievement between Indigenous and non-Indigenous students persist despite several reform programs and government initiatives attempting to address these inequalities. This study explores a mathematics education initiative in an Indigenous community school focused on improving students’ mathematical proficiency. The implemented initiative was informed by a review of related literature, and findings from the study provide educators and researchers with evidence of the impact of specific mathematics education practices on the teaching and learning of mathematics for Indigenous students.

In this chapter a brief overview of terminology is outlined in Section 1.2, followed by background information on Indigenous education and mathematics education in Section 1.3. The context of the study is outlined briefly in Section 1.4, followed by the statement of purpose and research questions (Section 1.5). To conclude the chapter, the significance of the study is outlined (Section 1.6), and the thesis structure is described (Section 1.7).

1.2 Key terminology and constructs

For consistency and clarity, important terminology will be discussed in the following section. Terms and concepts such as Indigenous, numeracy, mathematics, algorithms, computations, procedural fluency, and proficiency will be defined and discussed.

**Indigenous:** The Indigenous people of Australia consist of two diverse cultural groups – Torres Strait Islander people, the original inhabitants of the Torres Strait Islands, and Aboriginal people, the original inhabitants of mainland Australia (Foley, 2013; Purdie, Reid, Frigo, Stone, & Kleinhenz, 2011; Rigney, 2011). The term Indigenous is used as shorthand throughout this study with reference to people of Aboriginal and/or Torres Strait Islander descent, consistent with terminology used by a range of authors (e.g., Foley, 2013; Jorgensen, Grootenboer, Niesche & Lerman, 2010; Jorgensen, Grootenboer, & Sullivan, 2010; Klenowski, 2009; Ockenden, 2014; Purdie, Milgate & Bell, 2011; Purdie, Reid, Frigo, Stone...
The diverse and heterogeneous culture of Indigenous people in Australia must also be acknowledged, particularly with regard to socio-geographic and language distinctions (Martin 2007). Acknowledgement of the socio-geographic and language context (local context) is important to disaggregate data and compare outcomes between communities that are geographically remote, or where the common daily vernacular is not English, a factor considered important by some authors (e.g. Clancy & Simpson, 2002; Mellor & Corrigan, 2004). Beyond these distinctions, no further geographic or cultural classifications of Indigenous students, staff, and community members involved in this study will be undertaken to account for cultural sensitivity, and ethical considerations.

**Numeracy and mathematics:** The terms numeracy and mathematics are often interchanged, however they have different meanings. Numeracy is often used to refer to the broader scope of mathematics application, meaning that a numerate person is one who can work with and understand numbers in a wide range of situations (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2018a, p. 1). Therefore, numeracy is a capability referring to the application of mathematics in other subject domains and real-world applications (Organisation for Economic Co-operation and Development [OECD], 2001). Mathematics is often used to refer to the learning of specific mathematics skills and concepts, such as addition or factorisation (Cohen, 2001). While the distinction between mathematics and numeracy is an artificial construct, it is still useful particularly with reference to the relationship between the two; being numerate (i.e., having competence with numeracy) is founded on mathematical knowledge, fluency, and understanding (Australian Association of Mathematics Teachers [AAMT], 1998; ACARA, 2018a). The emphasis on the application of mathematics has seen the emergence of “numeracy” commonly used in context-relevant documents such as the Australian Curriculum F-10: Mathematics (ACARA, 2018b).

**Algorithms and computations:** In this study, a computation refers to a mathematical calculation employed to answer a mathematical question (Booker, Bond, Sparrow, & Swan, 2014). Computations are often completed or solved using an algorithm, a generalised sequenced procedure (Booker et al., 2014; Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid & Schaar, 2005; Ernest, 2003). For example, to solve a problem-solving question requiring
addition, a computation is required, and the computation may be completed using an addition algorithm.

**Procedural fluency:** In primary mathematics education, a key goal is the development of number sense in relation to place value, and the ability to efficiently carry out computations (ACARA, 2018b; National Council of Teachers of Mathematics [NCTM], 2014; Randolph & Sherman, 2001; Thomson, Hillman, & Wernert, 2012). In order to complete computations efficiently, both procedural fluency and conceptual understanding regarding algorithms and place value are required (Geary, 2004). Procedural fluency is defined as the ability to accurately, efficiently, flexibly, and appropriately apply mathematical procedures such as algorithms to complete computations (NCTM, 2014; National Research Council, 2001). The ability to carry out computations is important as proficiency with mathematical foundations associated with number directly relates to the ability to succeed in further mathematics study due to the hierarchical nature of mathematics (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Calhoon, Wall, Flores & Houchins, 2007; Cawley & Miller, 1989; Cawley, Fitzmaurice, Shaw, Khan, & Bates, 1979; Fleischner, Garnett, & Shepherd, 1982; Gersten, Jordan, & Flojo, 2005; National Mathematics Advisory Panel, 2008; Warner, Schumaker, Alley, & Deshler, 1980).

**Proficiency:** Mathematics proficiency is often referred to as the qualities that students should develop as a result of studying mathematics at school, and commonly comprises five interdependent strands: procedural fluency, conceptual understanding, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick, Swafford & Findell, 2001). Proficiency, as utilised in this study, has been defined by these strands as is common practice in mathematics education research and international standards. These five strands of proficiency also inform the Australian Curriculum: Mathematics (ACARA, 2018b), which is relevant to the national context of this study.

To be proficient in mathematics, students must be able to apply mathematical procedures (i.e., have *procedural fluency*) efficiently, appropriately and accurately (NCTM, 2014), however, they must also possess a *conceptual understanding* of the related mathematical concepts and procedures (Baroody, 2003; Baroody & Ginsburg, 1986; Byrnes & Wasik, 1991; Hiebert & Lefèvre, 1986; Kilpatrick et al., 2001). *Adaptive reasoning* refers to a student’s ability to justify and reflect on their understanding of mathematical concepts, and *strategic competence* is the ability to formulate and represent mathematical problems appropriately.
(Kilpatrick et al., 2001). Essentially, adaptive reasoning is related to a students’ ability to articulate their conceptual understanding, and strategic competence essentially is related to conceptual understanding of the procedures that are employed to appropriately and accurately complete a mathematics task. Therefore, procedural fluency and conceptual understanding are the main overarching categories of mathematics proficiency, whereas strategic competence and adaptive reasoning could be considered as a part of each of these strands.

In addition to the four cognitively focused strands of mathematics proficiency, a productive disposition (i.e., attitude; Moyer, Robison, & Cai, 2018) is a critical affective strand. The development of conceptual understanding, procedural understanding, strategic competence, and adaptive reasoning is dependent on a students’ productive disposition, which is a student’s ability to perceive mathematics as useful and worthwhile, and to envisage themselves as an effective and capable learner of mathematics (Kilpatrick et al., 2001; Jacobson & Kilpatrick, 2015; Philip & Siegfried, 2015; Woodward, Beswick, & Oates, 2017). Consequences of failing to develop a productive disposition towards mathematics can manifest as avoidance (Kilpatrick et al., 2001).

1.3 Background

Relevant background information concerning Indigenous education and mathematics education will be discussed in the following section. Approaching research in Indigenous education will also be discussed to situate the approach to this study.

1.3.1 Indigenous education

Research in Indigenous education has been of importance to Australia for many decades, primarily driven by the presence of equity issues (Hunter & Schwab, 2003). Reports of disparities between Indigenous and non-Indigenous students’ academic achievement levels have been extensively documented (e.g., ACARA, 2013; Australian Council for Educational Research [ACER], 2007; Brown, 2008; Dreise & Thomson, 2014; Eckermann, Dowd, Chong, Nixon, Gray, & Johnson, 2006; Matthews, Howard, & Perry, 2003; Owens, 2015; Purdie et al., 2011; Queensland Studies Authority [QSA], 2006; Schwab & Sutherland, 2001; Thomson, Hillman, Wernert, Schmid, Buckley, & Munene, 2012b; Thomson, McKelvie, & Murnane, 2006; Thomson, Wernert, O'Grady, & Rodrigues, 2016a, 2016b; Tripcony, 2002). Reports
have cited that Indigenous 15-year old students are, on average, over two years behind non-Indigenous students in literacy and numeracy (Department of the Prime Minister & Cabinet, 2017). Further, gaps in achievement on international tests such as the Trends in International Mathematics and Science Study (TIMSS) have remained unchanged since 1995 (Thomson et al., 2016b, Thomson, Wernert, O’Grady, & Rodrigues, 2017).

Addressing disparities in schooling achievement is important considering that, when comparing Indigenous and non-Indigenous outcomes from 1986 to 2001, a significant difference in post-secondary qualifications and secondary school retention rates persists (Hunter & Schwab, 2003). When Indigenous students, or any students, are precluded from achievement in mathematics, educational and employment opportunities become limited (Brown, 2008; Hunter, 1997; Purdie et al., 2000). The first reason for this is because education, particularly the level of education attained, acts as a mechanism by which employment systems evaluate a person’s productive worth (Biddle & Cameron, 2012). Secondly, mathematics education is a key in developing students’ numeracy, which is important due to its role in understanding social, economic, and environmental issues that shape modern society (OECD, 2001). However, research also suggests that the issue goes beyond the simplified equivalence between the development of mathematical knowledge, and economic or job-related advantages (e.g., Atweh & Brady, 2009).

Mathematical knowledge is essential in developing students’ capacity to read and write the world (i.e., understand and change it) (Gutstein, 2006). Beyond conforming ideals of creating capable citizens, mathematical knowledge enables students to understand how the world works, and those who possess mathematical knowledge potentially have the capacity to also create the world in a new way (Atweh & Brady, 2009). As such, research exploring effective education practices in Indigenous education is important as “future Indigenous education policy decisions must be based upon real research findings, and where these findings necessitate policy action, those actions must be taken” (Mellor & Corrigan, 2004, p. iv). These sentiments were re-iterated in the report on Indigenous primary school achievement published by the Productivity Commission (2016).

It is also important to consider why achievement gaps exist considering that the Council of Australian Governments (COAG) goal of halving the gap in reading, writing, and numeracy achievement for Indigenous students by 2018 was unrealised (Department of the Prime Minister & Cabinet, 2018). Numerous reform programs and initiatives such as Yu Mi Deadly,
Stronger Smarter Learning Communities, Connected Communities, Make it Count, and Good to Great Schools have attempted to address reported issues in the field (AAMT, 2011; Good to Great Schools Australia [GGSA], 2017; NSW Department of Education and Communities, 2012; Stronger Smarter, 2014; YuMi Deadly Centre, 2020), however it appears that the focus of current policies and programs requires evaluation as the gaps in achievement still exist and appear to be an intractable, ongoing challenge (Brown, 2008; De Bortoli & Thomson, 2009; Department of Education, Training and Employment [DETE], 1999; Dreise & Thomson, 2014; Mellor & Corrigan, 2004; Productivity Commission, 2016; Thomson et al., 2006, 2012a, 2012b, 2016a, 2016b, 2017; Tripcony, 2002).

Many important factors relevant to educational success for Indigenous students related to achievement disparities have been identified in the literature. Commonly cited factors that impact on learning and achievement for Indigenous students include absenteeism, health, and other social factors (Bourke, Rigby & Burdon, 2000; Hancock, Shepherd, Lawrence, & Zubrick, 2013; Mellor & Corrigan, 2004; Sarra, 2003). However, although these factors are relevant to the education of Indigenous students, it must be acknowledged that considering these factors in isolation places the blame for poor academic performance solely on Indigenous students and their communities (Sarra, 2003, 2011; Sarra, Matthews, Ewing, & Cooper, 2011). It is important to consider how specific factors within the educational system, such as the role of the teacher, and classroom factors, can help to explain variance in student achievement levels (Hattie, 2008; Hayes, Mills, Christie, & Lingard, 2006; Hill & Rowe, 1996; Lingard & Ladwig, 2001; Rowe, 2003).

Overall, despite a long history of attention in policy development and implementation, no consistent improvements in mathematics achievement have been realised for Indigenous students (Productivity Commission, 2016). Therefore, it is important to continue investigating the impact of educational practices to further understand the underlying reasons for these gaps and to guide the development of future successful practices.

1.3.2 Mathematics education

Primary mathematics education research has identified several factors that are relevant to all students’ success in school mathematics and beyond. Critical themes in primary
mathematics education discussed in this section include number fact fluency, computational fluency, and problem-solving.

Developing basic mathematics skills is considered to be an important factor for successful mathematics learning in the primary school years (Poncy, McCallum, & Schmitt, 2010). Specifically, efficient and accurate recall of number facts is important for success in primary mathematics as it facilitates computation and estimation, and is fundamental to further studies in mathematics for all students (Cumming & Elkins, 1999; Isaacs & Carroll, 1999; Poncy et al., 2010; Woodward, 2006). In primary mathematics, number facts refers to the fluent recall of addition, subtraction, multiplication, and division facts. Fluency with number facts is essential to meet classroom demands as it works to reduce other educational concerns such as anxiety, frustration, and avoidance of mathematical tasks (Poncy et al., 2010). The ability to recall number facts fluently also reduces the load on working memory, which allows for more effective problem solving since students can focus on problem structures rather than being preoccupied with attempting to recall basic facts and processes (Baxter, Olson, & Woodward, 2001; Woodward, 2006). The influence of number fact proficiency on future mathematics study illustrates the hierarchical nature of mathematics knowledge (Muller & Taylor, 1995). Specifically, lack of multiplication facts not only impedes carrying out multiplication and division computations, but also hinders fraction computations (Woodward, 2006). In turn, further algebra study, such as factorising algebraic expressions for example (Kotsopoulos, 2007), can be significantly impacted.

In addition to number fact fluency, analysis of findings from the 2011 TIMSS demonstrates that the majority of students on an international scale struggle with simple computation questions, such as four-digit subtraction, or two-digit multiplication sums (International Association for the Evaluation of Educational Achievement [IEA], 2013). This fact is important to consider as computational skills and number concepts associated with place value are vital components of mathematics curricula, and play a fundamental role in mathematics education (NCTM, 2016; Randolph & Sherman, 2001). In terms of the hierarchical nature of mathematics, just as number facts facilitate computation, computational fluency facilities problem-solving (Geary, 2004). The critical role of computations is supported by research which indicates that students who do not gain foundational understanding related to computations will continue to struggle in mathematics as concepts increase in complexity (Bryant et al., 2008; Calhoon et al., 2007; Cawley et al., 1979; Cawley & Miller, 1989;
Alternatively, if students possess procedural fluency and a conceptual understanding of algorithms to carry out computations, the load on working memory is reduced, as short-term memory is not employed for procedural purposes.

Another difficulty identified in primary mathematics research relates to students’ challenges with problem-solving (IEA, 2013; Tambychik & Meerah, 2010) despite its continued importance in curricula (e.g., ACARA, 2018b). Analysis of the TIMSS (2011) indicated that most Year 4 students, both nationally and internationally, had difficulties identifying which operation was appropriate for a problem-solving question involving division (IEA, 2013). A simple addition question in a problem-solving setting also proved difficult for approximately 30% of Year 4 students in Australia (IEA, 2013). Problem-solving is important as proficient application of mathematics is the crux of developing numerate students. Therefore, giving further attention to these reported difficulties is essential.

This section outlined some examples of the difficulties primary students exhibit, both nationally and internationally, with computation and application as well as transfer to problem-solving settings. Due to the importance of critical number concepts in developing numerate students, there is a need to further evaluate practices in mathematics education to effectively address these concerns.

1.3.3 Primary mathematics achievement data for Indigenous students

In the previous section, several factors were identified as relevant to all students’ success in school mathematics and beyond. In the context of Indigenous education, achievement data indicates that these factors are particularly important for Indigenous students due to reported achievement gaps (ACARA, 2013; ACER, 2007; Brown, 2008; Eckermann et al., 2006; Matthews et al., 2003; Owens, 2015; QSA, 2006; Schwab & Sutherland, 2001; Thomson et al., 2012a, 2012b, 2016a, 2016b; Thomson et al., 2006; Tripcony, 2002).

Analysis of the 2011 and 2015 TIMSS data carried out by ACER internationally for Year 4 and Year 8 students demonstrates disparities between Indigenous and non-Indigenous students’ achievement in foundational mathematics concepts. Year 4 Indigenous students are, on average, achieving lower than the intermediate international benchmark whereas achievement for non-Indigenous students is nearing the high international benchmark in Year
4 (Thomson et al., 2012, 2016b, 2017). Approximately 61% of Indigenous students performed at the low or below low international benchmark in TIMSS 2015 (Thomson et al., 2017), and the low Year 4 benchmark describes students being able to add and subtract whole numbers (Provasnik, Kastberg, Ferraro, Lemanski, Roey, & Jenkins, 2012), which is considerably below Year 4 standards both nationally and internationally (ACARA, 2018b, NCTM, 2017). In comparison, students achieving the high benchmark can apply all operations to whole numbers, and can apply place value knowledge (Provasnik et al., 2012).

The achievement gaps identified in Year 4 persist into Year 8 with 68% of Indigenous students achieving below the intermediate benchmark in 2015 (Thomson et al., 2017). The low proportion of Indigenous students achieving international benchmarks by secondary school is concerning considering that the low benchmark is described as “...an elementary understanding of whole numbers and decimals and ... basic computations” (Thomson et al., 2012a, p. 12). In other words, over half of Indigenous students potentially are entering high school without the ability to add, subtract, multiply, or divide. Similar trends can be seen in Australia’s national assessment with approximately 13% fewer Indigenous students meeting national minimum standards on the 2018 National Assessment Program – Literacy and Numeracy [NAPLAN] test in comparison to non-Indigenous students (ACARA, 2018c). Such difficulties are likely to have profound impacts on Indigenous student’s engagement and success in further mathematics study.

The question of why Indigenous students are not experiencing success with performing basic computations in middle primary school is not well understood. Overall, evidence suggests that, in general, Indigenous students are struggling to understand basic mathematics concepts such as basic computation, and current practices are not supporting Indigenous students’ mathematical proficiency effectively. The need to focus on number (i.e., counting, place value, and operations) has been noted in other studies in remote Indigenous contexts with a focus on competency, confidence, and fluency (Jorgensen, 2018). Therefore, critical foundational concepts such as number facts, computational skills, and problem solving need to be considered and addressed specifically in future studies that aim to improve Indigenous student’s mathematical achievement.
1.3.4 Approaching research in Indigenous mathematics education

Research in the field of Indigenous education must be approached in an ethical and respectful manner, and consider the core values of reciprocity, respect, equality, responsibility, survival and protection, and spirit and integrity (National Health and Medical Research Council [NHMRC], 2015). As noted earlier, there are several examples of national and international standardised tests that demonstrate gaps in mathematics achievement for Indigenous students. However, it has been suggested that care must be taken in considering results from standardised tests (both nationally and internationally) as examples of Indigenous peoples’ learning capabilities due to perceived economic and cultural biases in test items (Carstairs, Myers, Shores, & Fogarty, 2006; Eckermann et al., 2006; Hambleton, & Rodgers, 1995; Klenowski, 2014; Mellor & Corrigan, 2004; Sarra, 2009). This consideration is necessary as it is important that educational approaches are not derived from deficit-based beliefs, which are conceptualised as beliefs that Indigenous people are unable to, or lack the necessary intellectual or cultural capital to, learn in Western systems (Ascher & Ascher, 1986; Beresford, 2003; Harrison, 2007; Phillips, 2011).

This study also acknowledges the diversity of Indigenous students, as it is potentially limiting to view all Indigenous students as a homogenous group with clearly defined and pre-determined learning preferences. Ethical research must recognise the diversity and uniqueness of individuals (Australian Institute of Aboriginal and Torres Strait Islander Studies [AIATSIS], 2012; Harrison, 2007; Jamieson et al., 2012; Mellor & Corrigan, 2004; Nicholls, Crowley, & Watt, 1998; Perry, Lowrie, Logan, MacDonals & Greenless, 2012). Some researchers have expressed concern that “there has been a focus on the uniqueness of the Indigenous experience of education” (Mellor & Corrigan, 2004, p. v). The current study has been designed to avoid this conceptualisation by acknowledging that students will have varied learning preferences and capabilities irrespective of their cultural or ethnic background.

Another key consideration related to Indigenous education, as proposed by Meaney, McMurchy-Pilkington, and Trinick (2012) relates to Indigenous knowledge:

One way that this institutional racism manifests itself is through the assumption that school mathematics is the only valuable mathematics that Indigenous students should know and their performance in tests of this is what defines them in regards to their ability to contribute to society. (p. 69)
The current study is underpinned by a recognition that although school mathematics is only one aspect of developing numerate students, it is an important form of knowledge that has equity connotations. Thornton surmises that to ignore the value of this mathematical knowledge “runs the risk of further excluding the already disenfranchised from knowledge that can liberate and empower” (2020, p. 11). As such, the primary focus of the study is mathematics as defined and prescribed in current schooling systems and their associated syllabuses. Accordingly, fluency and conceptual understanding of critical number concepts and skills are a primary focus due to their important role in developing numerate students as recommended by both Australian and International curriculum organisations (AAMT, 1998; ACARA, 2018a; NCTM, 2014).

1.4 Context

The students in this study are of Aboriginal and Torres Strait Islander heritage, which encompasses a diverse range of people. The local context of students was an urban community in southeast Queensland. The context of the study school is a community school for Indigenous students operating across the primary and secondary years (P-12). Being community run, the school is culturally inclusive, and is considered to provide a culturally safe learning environment.

The researcher had an established rapport and professional relationship with staff and students through her previous role as a teacher at the school. In the study, the researcher played an interactive role in the classroom with teachers and students both during and outside of regular mathematics lessons. This interaction mainly took place during the collection of data such as interviews, observations, and discussions with both teacher and student participants. The role of the researcher’s values and potential biases were addressed through the documentation of observations in the form of field notes during the study, as well as the use of multiple modes of data to support any assertions. This method supported triangulation of findings (Creswell, 2012). In addition, consultation was maintained with a reference group at the school consisting of teachers, school board members, and auxiliary staff including an educational psychologist. The reference group acted as an important source of professional and cultural knowledge in relation to maintaining cultural and site sensitivity both during the implementation of the study, and in the analysis and reporting of data.
1.5 Purpose and research questions

The purpose of the study was to implement and evaluate the effectiveness of a mathematics initiative designed to improve students’ mathematical proficiency in an Indigenous community school. The implemented initiative was informed by an analysis of current recommendations in the literature regarding effective teaching practices in mathematics education and Indigenous education. Current reform programs in mathematics education and Indigenous mathematics education were critiqued and linked to literature regarding best practice recommendations. The following research questions were explored:

Research Question 1: How do teachers implement a mathematics initiative in an Indigenous community school?

Research Question 2a: What is primary students’ mathematics proficiency prior to a mathematics initiative in an Indigenous community school?

Research Question 2b: What is primary students’ mathematics proficiency after a mathematics initiative in an Indigenous community school?

Research question 3: What factors related to the implemented initiative influenced primary students’ mathematics proficiency in an Indigenous community school?

A mixed methods research design was used to provide evidence to answer the research questions. To determine how teachers implemented the initiative, qualitative evidence was collected regarding classroom practices. The sources of data included observations of classroom lessons, and informal interviews with teachers. To assess the effectiveness of the initiative, the current state of students’ mathematics proficiency prior to commencing the initiative was evaluated through testing, followed by subsequent re-testing at the conclusion of the initiative. The sources of data comprised written tests including adapted classroom diagnostic tests (Booker, 2011), a standard mathematics achievement test (Progressive Achievement Test, Mathematics [PAT-M]), and problem-solving interviews.

Addressing the third research question involved identifying aspects of the initiative that influenced students’ mathematical proficiency. This was achieved through the use of qualitative data to explain and triangulate the findings reported from the quantitative testing. The sources of data used to answer this question included informal interviews with teachers,
problem solving interviews (Newman interviews), and classroom observations in the form of field notes.

1.6 Significance

Considering the importance of numeracy and mathematics in developing informed and capable citizens (Barr et al., 2008; Brown, 2008), investigating the persistent achievement gap between Indigenous and non-Indigenous students in mathematics is essential due to equity concerns (Hunter & Schwab, 2003). Therefore, this study is significant for its role in continuing to explore the implementation of high quality and effective educational practices in mathematics education for Indigenous students.

This study contributes to developing a deeper understanding of mathematics teaching and learning in Indigenous settings through the implementation and evaluation of an evidenced-based mathematics initiative. Specifically, this study will build on knowledge surrounding the effectiveness of pedagogical practices for Indigenous students. This exploration will build on knowledge pertaining to the complex interplay between student, teacher, school, and cultural factors that influence Indigenous students’ mathematical proficiency.

In addition to exploring these factors, the study also contributes to research surrounding the implementation of an evidence-based mathematics initiative with teachers in an Indigenous setting. This provides insight into how teachers implement (or do not implement) recommended practices that are supported by research, as well as the challenges surrounding pedagogical choices in this setting. Broadly, the study also contributes to understanding the role of community schools in influencing and supporting Indigenous students’ achievement in school.

1.7 Structure of the thesis

This chapter provided a rationale for exploring the influence of practices in primary mathematics education for Indigenous students, specified the context for the study, and outlined the three research questions that will be answered. Chapter 2 will provide a comprehensive description of the conceptual framework guiding the study. The purpose of this is to give insight into the guiding approaches for the study as well as to introduce the conceptual
framework which informed the analysis and interpretation of findings. An overview of research relevant to this study will be outlined in a literature review in Chapter 3. This review critically evaluates pedagogical approaches in mathematics education, and educational initiatives in mathematics and Indigenous education. The importance of number fact, computation, and problem-solving proficiency was outlined in this chapter, and therefore the literature review will explore research surrounding approaches for these mathematical skills and concepts. Effective professional development will also be explored within the literature review.

Chapter 4 will provide a detailed summary of the research design developed to address the research questions for this study. The purpose of this chapter is to provide a justification for the implemented research design. Analysis of the findings will be presented in Chapter 5, including a comprehensive analysis of how teachers implemented the initiative, and how students’ mathematical proficiency changed as a result of the initiative. Findings from this analysis will provide evidence to address the research questions. A summation of how the initiative influenced students’ mathematical proficiency, followed by discussion of the specific factors that influenced students’ proficiency are outlined in Chapter 6. In Chapter 7, the findings from the study will be summarised, and implications and recommendations arising from the critical analysis of findings will be outlined. Limitations of the study, and direction for future research will also be presented.
2 CONCEPTUAL FRAMEWORK

2.1 Introduction

The guiding conceptual framework for this study utilises and integrates perspectives from sociology and psychology to consider how students learn, and what is important in influencing the learning process. Pollard (1990) discussed the potential validity and value of integrating theoretical perspectives by surmising:

The demarcations between disciplines must be seen as socio-historical products, maintained by people and institutions … reliance on detailed intra-disciplinary development could result in the establishment of theoretical perspectives and empirical procedures which in fact fail to engage with the more complex and enduring realities of social processes and phenomena. Indeed, I would argue that, as our understanding of the social world becomes more sophisticated, it is becoming increasingly apparent that the validity of the study of many issues cannot be maximised unless each of the relevant disciplines is drawn on in sustained study (p. 26).

The critical need for integration of perspectives stems from the need to further consider the cognition of the individual learner, which is not distinctly or deeply considered in some sociological frameworks. The integration of perspectives gives credit to the socio-cultural influences that implicitly and explicitly impact cognition (Daniels, 2012). To address this specifically, cognitive psychology will be utilised in this study to explain the process of learning for the individual. Cognitive psychology also provides a valuable lens through which pedagogic practices can be evaluated against what is understood about how individuals learn. The work of sociologist Michael Apple (1986) also acknowledges that in schools we are not teaching an abstract ‘learner’, but rather people who are shaped by their class, race, and gender as well as the economic status, political views, and ideologies of the community and family in which they socialise. In essence, it is not enough to stop at the individual level of understanding learning as individuals encounter and generate concepts in an interpersonal, group context (Haste, 1987; Youniss, 1981). Therefore, there is a need to go beyond considering an individual’s cognition in influencing learning, as the individual also exists and learns in a
unique environment (Bruner & Haste, 1987). To reconcile cognitive psychology and the way in which the social context influences learning, Haste’s (1987) framework provides a valuable conceptualisation of three factors affecting learning: the intra-individual domain; the interpersonal domain; and the socio-historical domain. The intra-individual domain is underpinned by understandings stemming from the field of cognitive psychology as it concerns the way an individual constructs understanding (i.e., ‘learns’). The interpersonal domain focuses on small-group interactions and relates to the fact that individuals learn and experience learning and teaching in a social environment influenced by cultural and social conventions (Pollard, 1990). The socio-historical domain, informed by culturally and historically relevant values and norms, broadly influences both interpersonal interactions, and intra-individual values and understandings (Haste, 1987).

However, in addition to the historically-shaped social environment that individual students learn in, consideration of the epistemology of mathematics is also called for as this informs the way in which mathematical knowledge is communicated via pedagogy. That is, it is important to consider both the learner and what is being learnt, as both are relevant when considering the process of learning (Vygostky, 1962). The epistemology of mathematics informs how the structuring of knowledge in mathematics is viewed, thus influencing how mathematics, as a domain, might be interpreted and transformed into classroom pedagogy. Therefore, the two important questions that will be considered in the following sections are: What are the rules structuring mathematical knowledge?, and How is mathematical knowledge disseminated to learners?

In this study, an innovative conceptual framework (Figure 1) has been created for the purposes of understanding factors associated with teaching and learning mathematics. In the conceptual framework for this study, the domains influencing learning have been conceptualised as concentric circles as the domains cannot be understood in isolation (Pollard, 1990). The individual learner is placed as the common centre. The links between each of the domains, as exemplified in the framework outlined in Figure 1, describe how the way in which an individual learns cannot be conceptualised without understanding both the interpersonal experiences facilitating and regulating learning, as well as the socio-historic circumstances that shape how an individual operates in and understands the guiding rules of a particular discourse.
The socio-historical, inter-personal, and intra-individual model will also be further expanded to include the three distinct fields in which the pedagogic device operates by proposing that the fields of the classroom, and the school operate at the porous boundaries of each of the domains. At the outer boundary of the socio-historical domain of learning, the field of production exists and operates. The field of production has been purposefully placed at this boundary as the development of new knowledge and content of knowledge categories is regulated and driven by the cultural and historical relevance of the domain, and social norms (Bernstein, 2000; Singh, 2002). The field of production sits at the very outer boundary of this framework as it is also shaped by the nature of mathematics as a knowledge form (Maton & Muller, 2007). At the boundary separating the socio-historical and inter-personal domain is the field of recontextualization, which is the school. The recontextualisation of mathematical knowledge is placed at this boundary, as it is dependent on the socio-historical valuing of the knowledge domain, as well as the inter-personal negotiation of meaning that occurs in the
classroom. The boundary between the inter-personal and intra-individual domain contains the field of reproduction, relating to the actual teaching and learning that happens in the classroom. What occurs in the field of reproduction, that is, the classroom, is influenced by the inter-personal negotiation of meaning creating cognitive disequilibrium driving intra-individual cognition.

2.2 The knowledge structure of mathematics

It is important to consider the nature of mathematical knowledge as this influences the organising and structuring principles of the domain. Bernstein’s work distinguishes between vertical and esoteric, or horizontal and mundane knowledge forms (1999, 2000). Horizontal knowledge forms refer to everyday, common-sense knowledge that is accessible to all. This type of knowledge is segmentally organised, and context specific; that is, a particular skill is not dependent on knowledge of another (Bernstein, 2000; Maton & Muller, 2007). Contrastingly, vertical knowledge forms are hierarchical in nature, and require a “coherent, explicit, and systematically principled structure” (Bernstein, 1999, p. 159). Bernstein envisages hierarchical knowledge structures as a triangle (1999). These knowledge forms consist of specialised symbolic structures of explicit knowledge, and procedures are linked to other procedures hierarchically. Privileged vertical knowledge is transmitted through official institutions such as schools. The privileged nature of this knowledge means that the literal inability to access such knowledge forms can be associated with the perpetuation of social inequality. Vertical knowledge forms are particularly relevant to abstract disciplines such as mathematics.

Mathematics knowledge is a form of esoteric knowledge. It has long been recognised that mathematics is hierarchically structured where some understanding cannot be developed before another has been established (Freudenthal, 1973; Sfard & Linchevski, 1994). In Chapter 1, the hierarchical nature of mathematics has been referred to in the explanation of why fluency with number facts is essential to computational fluency, and why computational proficiency is essential for problem-solving and further mathematics study. As a result, the acquisition of symbolic mastery in the form of computations or problem-solving in mathematics acts as a gateway to further conceptualisation and application of increasingly advanced mathematical concepts and procedures. Continuing to focus on esoteric mathematical knowledge is important in school curriculums (Thornton, 2020).
Building on Treffers’ (1978) work, Freudenthal (1991) further conceptualised these knowledge distinctions specific to mathematics as vertical and horizontal mathematisation. What such a distinction appears to suggest is that mathematics can originate from mundane forms, as well as esoteric. When considering the examples and definitions, horizontal (‘mundane’) mathematisation correlates with conceptualisations and definitions of numeracy, that is ‘mathematics’ that is applied to and originates from real-world applications. The implication of this is that the distinction between vertical and horizontal mathematisation is, in many ways, similar to the distinctions between numeracy and mathematics. Such a view concurs with Freudenthal’s definition of horizontal mathematisation as having originated from the world of life, which then develops to the world of symbols. That is, in horizontal mathematisation, the knowledge is gained from real-world experiences (similar to the everyday, common-sense definition of mundane knowledge forms from Bernstein). This horizontal view of mathematics originating from reality also aligns with an Indigenous conceptualisation of the epistemology of mathematics by Matthews (2008). Matthews summarises that mathematics stems from a particular observation from perceived reality, which is then mathematised (to use Freudenthal’s language) through the creation of an abstract representation of the real-life scenario using symbols, forming what is known as mathematics.

An example of horizontal mathematisation is solving five times eight via a graphical model using the rectangular scheme of 5 rows of 8 each (Freudenthal, 1991). Vertical mathematisation was conversely conceptualised as the world of symbols. Relating to the example of solving five times eight above, a vertical solution to the problem would be to utilise the sequence 8, 16, 24, 32, 40 (Freudenthal, 1991). Unfortunately, overemphasis on horizontal mathematisation strategies can mean that procedures are not generalisable as the mathematics is too tightly tied to a particular context or application. For example, conceptualising five times eight using a graphical model, whilst useful for this context, becomes quickly redundant if we asked a student to answer 24 times 38. In this instance, structured procedures such as algorithms are most efficient and effective, and these would be an example of vertical mathematisation.

As suggested by Freudenthal’s imagining of vertical and horizontal mathematisation, the boundaries between vertical and horizontal mathematical knowledge forms, or origins of mathematical knowledge, are not distinct or rigid. The implication of this is that concepts can belong to both worlds simultaneously depending on the conceptualisation of the task, and the selected solving mechanisms (i.e., procedures). Such a sentiment builds on Bernstein’s
conceptualisation of vertical knowledge structures as a triangle. Therefore, it is proposed that mathematics knowledge is potentially more accurately conceptualised as a continuum, as opposed to distinct and clearly defined dichotomies of ‘mundane’ and ‘esoteric’ forms (Figure 2). This idea also begins to echo mathematics educators’, researchers’, and educational policy makers’ continual difficulties with a consistent, clear-cut definition or distinction between ‘mathematics’ and ‘numeracy’. Hence, visualising mathematical knowledge as a continuum relates directly to the somewhat artificial and superfluous distinction between numeracy (as relating to real-world application) and mathematics (knowledge of, and proficiency with specific mathematical structures and concepts), in that numeracy occurs at the lower end of the imagined triangle of hierarchically-structured mathematical knowledge where less abstraction and domain specific knowledge is required as outlined in Figure 2.

![Diagram: The hierarchical nature of mathematical knowledge](image)

**Figure 2**: The hierarchical nature of mathematical knowledge

An example of tasks that require knowledge operating at the less abstract end of the hierarchical knowledge structure can be seen when conceptualising the answer to the multiple choice question in Figure 3 from a standardised mathematics test.
In this question, students are asked to find the missing piece. This question is, in essence, testing numeracy as answering the question is not contingent on possessing any specific mathematical knowledge or understanding developed from school instruction. That is, a person with spatial awareness and experience with two dimensional shapes is likely to be able to answer the question. Using Freudenthal’s definitions of horizontal mathematisation, it could be argued that answering such a question is learnt from the world of life, and therefore requires basic reasoning to answer. In contrast, an example of a question operating at the increasingly abstract end of the hierarchical knowledge structure is outlined in Figure 4. Answering such a question requires understanding of, and proficiency with, highly specific mathematical structures (i.e., multiplication algorithms). It would be unlikely that such a question could be answered through horizontal mathematisation techniques, such as representing diagrams of sets or arrays.

To answer this question, the student needs to comprehend that they need to find the total amount of watered rows (i.e., $34 + 52 = 86$), followed by multiplying the total rows by 68.
to find the total amount of plants (i.e., $86 \times 68$). To carry out this process correctly, a student needs to be able to comprehend the task and recognise that the rows must be added, which requires proficiency with number facts and place value. Carrying out the final multiplication requires students to have proficiency with multiplication facts, the multiplication algorithm, and associated place value knowledge (particularly associated with renaming). The process of solving this question illustrates the need for a high level of both conceptual understanding and procedural fluency.

These distinctions between the mundane (or horizontal mathematisation) and esoteric (or vertical mathematisation) in mathematics are important as to focus on the mundane (or \textit{numeracy}) can sacrifice proficiency with esoteric forms (Hogan, 2012). This has equity connotations, and students should have access to both forms of knowledge (Dowling, 1998; Thornton, 2020). The knowledge structure of esoteric mathematics as a hierarchical discourse has ramifications for the intellectual shape of the way in which mathematical knowledge is communicated as pedagogy. A student’s depth of understanding of esoteric mathematics knowledge forms that are generally transferable has applications for their future study of mathematics. The way in which the nature of mathematical knowledge influences the dissemination of mathematical knowledge to learners through the pedagogic device will now be explored.

2.3 The pedagogic device

Bernstein conceptualised the way in which knowledge is organised and converted into pedagogic communication as the pedagogic device, which regulates what is communicated and to whom, when, and how (Dowling, 2014; Singh, 2002). The pedagogic device regulates the production of school curriculum and its dissemination (Apple, 2002) through three sets of interrelated, hierarchical rules: distributive (which social groups are privy to the distribution of particular knowledge); recontextualising (how knowledge is transformed into a pedagogic discourse amenable to pedagogic transmission); and evaluative (how the recontextualised knowledge is realised in specific pedagogic practices) (Maton & Muller, 2007; Singh, 2002). Recontextualising rules are derived from distributive rules whilst evaluative rules are derived from the recontextualising rules (Bernstein, 1990). Where the pedagogic device operates has been defined in the work of Bernstein (1990, 2001) and Bourdieu (1992) as specific \textit{fields}, which refers to the space where agents establish norms over the types of capital that are
effective in it. The three main hierarchical fields of the pedagogic device are the fields of production, recontextualisation, and reproduction (Bernstein, 1990, 2001). Commonly, each of these fields can be associated with specific sites.

The field of production, that is, where new knowledge is produced, takes place mainly in higher education and research institutions. The field of recontextualisation, that is, where the pedagogic discourse is established (i.e., knowledge is de-located from the field of production and relocated as a pedagogic discourse), mainly occurs in institutions specific to education such as, state departments of education and training, curriculum authorities, and teacher education institutions (Singh, 2002). The field of reproduction, that is, where pedagogic transmission and acquisition occurs, is the primary concern of schools (Maton & Muller, 2007). The boundaries between these fields are strong and insulated, which means that each has specialised agents, agencies and discourses. The relationship between the fields of the pedagogic device, potential sites, and potential agents is outlined in Table 1.

<table>
<thead>
<tr>
<th>Field</th>
<th>Sites</th>
<th>Agents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>- Higher education institutions</td>
<td>- Field experts &amp; researchers</td>
</tr>
<tr>
<td></td>
<td>- Research institutions</td>
<td></td>
</tr>
<tr>
<td>Official</td>
<td>- State departments of education</td>
<td>- Experts within and specific to the outlined sites</td>
</tr>
<tr>
<td></td>
<td>- Curriculum authorities</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Teacher education institutions (tertiary institutions)</td>
<td></td>
</tr>
<tr>
<td>Pedagogical</td>
<td>- Schools</td>
<td>- School leaders (e.g., curriculum leaders) and teachers</td>
</tr>
<tr>
<td>Reproduction</td>
<td>- Primary and secondary schools</td>
<td>- Teachers</td>
</tr>
</tbody>
</table>

In some empirical studies, the fields identified in Bernstein’s work, as surmised in Table 1, have been described and utilised as macro, mezzo, and micro levels of analyses (e.g., Singh, 1995). In place of macro, mezzo, and micro classification, the conceptual framework for this study utilises the titles for the three domains of learning initially proposed by Haste (socio-historical, inter-personal, and intra-individual). These titles more clearly frame the
domains that influence learning, and the subsequent description of factors associated with each of the domains.

2.3.1 The field of production

The field of production, the domain of mathematicians typically carried out in tertiary institution sites, is directly influenced by the esoteric structure of mathematics knowledge. However, the field of production is also driven by socio-historical factors surrounding the valuing of esoteric knowledge. In essence, what is within the purview of agents operating in the field of production will be influenced by what is socially and culturally valued at the time. Due to particular social, historical, and cultural factors, how particular groups of people may choose to interact (or not interact) with particular knowledge will also influence this field.

2.3.2 The field of recontextualisation

Recontextualisation is achieved through two means, one being the official recontextualising field where policy makers and administrators are the influencing agents, and the other being the pedagogical recontextualising field where teachers are the agents (Apple, 2002; Dowling, 2014). Within the field of recontextualisation, the school makes decisions about how syllabi are interpreted into curricula. The rules of recontextualisation are important when considering effective teaching and learning as they are informed by the rules of the site (i.e., the school) so that, subsequently, the pedagogic discourse is formed. This then constitutes the school curriculum.

The field of recontextualisation operating at the school site has been linked to influences functioning at the border of the socio-historical and inter-personal domains within the conceptual framework for this study. Similar to the knowledge valuing in the field of production, pedagogic decisions are influenced by factors within the socio-historical domain. That is, what is valued as school learning is driven by societal, and cultural norms of the time. The way in which particular groups choose to interact (or not interact) with the recontextualisation sites (schools) is also driven by social and cultural factors.

Following the relocation of mathematical knowledge as a pedagogic discourse in the form of curricula, specific pedagogical practice decisions are also influenced by the inter-personal domain and the nature of the classroom being a social environment. That is, teachers
are agents who participate and influence the way in which the pedagogic discourse operates within the classroom.

2.3.3 The field of reproduction

The importance of the field of reproduction in relation to teaching and learning is clear as it relates to the highly specific practices that occur within a classroom. The field of reproduction has been linked in the conceptual framework to the boundary of the inter-personal and intra-individual domains of learning. Learning through the negotiation of meaning in a classroom environment is innately social as both the group of students and the individual student will influence learning. This negotiation of meaning creates cognitive disequilibrium, driving an individual student’s cognitive construction of understanding.

2.4 The domains of learning

In the following section, critical factors central to each domain of learning will be unpacked. As the learner is the key focus of this study, discussion of the domains will begin with the intra-individual domain. Following this, the way in which the inter-personal domain, and socio-historical domain influence learners, particularly Indigenous learners, will be discussed.

2.4.1 Intra-individual domain: Cognitive load theories

In this study, the way in which individual students learn, and the effectiveness of teaching practices will be evaluated through the lens of human cognitive architecture. The concept of human cognitive architecture, as defined by the Atkinson and Shiffrin model (1968), describes the link between environmental inputs and memory. Essentially, this theory of cognition describes three components of human cognition related to learning and memory: sensory inputs, short-term store (or working memory), and long-term store. The goal of learning is to add information to long-term stores and this alteration of long-term memory means that learning has occurred. Creating these changes is not a simple transfer as the short-term memory, responsible for processing new and novel information, is limited in its capacity and duration (Miller, 1956; Peterson & Peterson, 1959). When processing information within the working memory, certain information will be forgotten and it is through purposeful
rehearsal that information will be moved to, and stored in long-term memory to form schema for later retrieval (Bjork, 2011). The process of forgetting and the importance of rehearsal has revealed that short practice sessions spaced out over time are most effective in helping students learn (Bjork, 1994). This is because what is forgotten is easier to retrieve than developing what is not known, so the process of forgetting can be utilised to enhance learning.

It has been proposed that we can hold between five and nine items of information at one time in our working memory, however this does vary between individuals (Hattie & Clarke, 2019). The limitations of working memory bring about cognitive load theories that suggest that it is important to consider the demands placed on the limited working memory by particular tasks and activities. That is, if the cognitive load of a task places demands on working memory beyond its limited storage capacity, cognitive overload occurs which impacts negatively on performance and the learning process is inhibited (Paas, Renkl & Sweller, 2004). In this way, cognitive load theory attempts to explain the processes of learning based on memory and processing power (Sweller, 2011, 2016). It has been proposed in educational research that effective and efficient instruction is unlikely to occur if it does not take these cognitive structures into consideration (Kirschner, Sweller, & Clark, 2006; Paas & van Merrienboer, 1994), particularly in highly structured subjects such as mathematics where conceptual understanding is developed over several years as new knowledge is generally built on the foundation of prerequisite knowledge (Baxter et al., 2001; Sweller 2016).

When considering specific pedagogical approaches, the question of how much guidance should be provided to facilitate students’ learning is central in mathematics education (Chen, Kalyuga & Sweller, 2016a). A useful consideration when comparing the effectiveness of particular levels of guidance is the worked example effect and its counterpart - the generation effect. The worked example effect utilises cognitive load theory, and suggests that providing worked examples (i.e., scaffolded problem solving guidance) is more effective for student outcomes than alternative pedagogical approaches (Chen et al., 2016b; Kirschner et al., 2006; Retnowati, Ayres, & Sweller, 2010; Schwonke, Renkl, Krieg, Wittwer, Aleven, & Salden, 2009). These conclusions are supported by a meta-analysis that found that worked examples increased student achievement above what would typically be expected (Crissman, 2006). The success of worked examples is potentially because the procedures and processes learnt from worked examples contribute to schema development, and can be transferred to long-term memory and later retrieved for application in other settings (Chen et al., 2016b; van
Schema development involves the integration of new knowledge into existing memory structures that gradually become more sophisticated and less likely to be forgotten due to the multiple connections between aspects of the schema.

In contrast to this, the generation effect suggests that more effective learning outcomes are obtained when students are allowed to generate their own answers to problems (i.e., problem solving is completed with minimal guidance). However, recent findings suggest that the complexity (or element interactivity) of problems dictates whether worked examples or minimal guidance are more effective for learners (Chen et al., 2016b). If problems are of high element interactivity, either due to the learners being novices or the problem itself being complex, worked examples are more beneficial. However, as learners increased in expertise and/or the problems reduce in element interactivity, allowing students to generate their own answers is more effective. Previously, this relationship has been found in the expertise reversal effect which suggests that, as learners increase in their expertise, the advantages of scaffolding and guidance first reduces, disappears, and then reverses thus becoming detrimental to learning (Kalyuga, Ayres, Chandler, & Sweller, 2003). These findings suggest that initially providing high levels of guidance and scaffolding and then removing scaffolding as students’ progress in their learning is highly effective (e.g., Salden, Aleven, Schwonke, & Renkl, 2010). Researchers have termed this the fading effect (Atkinson, Renkl, & Merrill 2003; Renkl, Atkinson, & Maier, 2000). This understanding of brain functioning is important when planning to teach mathematics to students with different knowledge bases and different learning needs. In particular, when teaching mathematical processes involving computation where there is a reliance on short term memory, and in the interactions between short and long-term memory to carry out computations, understanding of brain functioning is imperative. Individual learning is also influenced by affective factors which influence cognitive engagement. The role of engagement and motivation will be explored in later discussion.

2.4.1.1 Metacognitive skills

In conjunction with considerations of individual cognition, students’ metacognitive skills also play an important role in learning, particularly in relation to problem-solving in mathematics (Desoete, Roeyers & Buysse, 2001). In meta-analyses, metacognition has been identified as a key variable in influencing students’ learning (Wang, Haertel, & Walberg, 1993). Metacognition relates to a students’ knowledge of their cognition, as well as their
regulation and control of their cognitive actions (Garofalo & Lester, 1985; Flavell, 1976). The role that students’ metacognition plays as a factor in determining students’ success with mathematics tasks has been discussed in literature from the work of Skemp’s (1979) consideration of *reflective intelligence*, or Piaget’s *reflexive abstraction* (1976) to more current work (e.g., Schneider & Artelt, 2010). Focusing on mathematical problem-solving, currently it is understood that, to be successful in problem-solving, cognitive problem-solving heuristics to support metacognition (e.g., Polya, 1988) as well as the effective use of strategic competence is required (Ozsoy & Ataman, 2009). Using metacognitive strategies has been shown in research to support students’ academic performance (Weinstein & Mayer, 1983).

2.4.2 Intra-individual domain: Affective factors

Beyond cognitive and metacognitive factors, mathematical proficiency is also influenced by other affective factors related to students’ self-efficacy, that is, their belief in and perception of their ability to succeed and learn successfully (Bandura, 1997). The specific strand of mathematical proficiency related to affective factors is productive dispositions (Woodward, Beswick, & Oates, 2017). Affective factors are important to consider as there is a relationship between academic achievement and students’ attitudes towards mathematics (i.e., productive dispositions) (Ma, 1997; Ma & Xu, 2004). This is because students’ productive dispositions influence students’ motivation, specifically their willingness to persevere with tasks until completion (Eccles & Wigfield, 2002). For these reasons, it is important to consider the impacts of students’ productive dispositions on learning in mathematics.

As productive dispositions support academic achievement, it is important to be aware of attributes of students who do not possess productive dispositions towards mathematics. The 2016 report by the Organisation for Economic Cooperation and Development (OECD) highlighted that student characteristics of low perseverance, motivation, and self-confidence were exemplified in low performers (Guy, Cornick, & Beckford, 2015). Lacking a productive disposition has been linked to behaviours such as avoidance of mathematical tasks (Woodward, Beswick, & Oates, 2017). Avoidance of tasks, and low perseverance means that the development of the cognitive strands of mathematical proficiency, such as, conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning (Kilpatrick et al., 2001) is contingent on the presence of a productive disposition. However, the work of Ma
and Xu (2004) outlines that the relationship between achievement and students’ attitudes towards mathematics is cyclical but not reciprocal, with achievement having causal dominance over productive dispositions. That is, student achievement develops their productive dispositions towards mathematics with success building productive dispositions. In addition to these considerations, learners do not generally operate in isolation and the process of learning is dependent upon social or inter-personal factors.

2.4.3 Inter-personal domain: Access to mathematics knowledge forms

Philosophies of social constructivism centre on the understanding that the social domain influences the development of an individual’s understanding in a meaningful way, and individuals construct their own understanding in response to social experiences (Ernest, 1994, 1999; Kilpatrick, 1987; Lerman, 1989). Factors that influence the social aspects of learning have been noted by authors to include linguistic factors, cultural factors, interpersonal interactions, as well as teaching and the role of the teacher (Ernest, 1994). Relevant to these key factors, the work of Bourdieu describes the habitus of students, which broadly refers to the embodiment of their cultural background (Bourdieu, 1981; Zevenbergen, 2004). The habitus relates to the experiences that students may or may not have been privy to due to their cultural background, social class, or exposure to specific environments or opportunities, and may be surmised as one’s cultural experience (Bourdieu & Wacquant, 1992). The primary habitus of a child is shaped in the early years, setting up a student either to engage successfully or be excluded within domain specific discourses such as mathematics (Zevenbergen, 2004). In the early childhood years, students who are exposed to environments that are rich with mathematical concepts, language, and structures are developing a primary habitus predisposing them to engage meaningfully with the discourse of school mathematics as they have developed key mathematical foundations (Clarke, Clarke, & Cheesman, 2006; Kilpatrick et al., 2001).

Specifically, important to engaging with mathematics and school discourses is a students’ linguistic habitus, particularly if it is acknowledged that knowledge of specific linguistic discourses is a form of educational capital.

Linguistic capital can influence the ability and way in which a student engages within the school environment as success in school involves the ability to engage with and understand the instructional pedagogic discourse of school (Jorgensen & Grootenboer, 2011). The
pedagogic discourse of school means that students who have had their early habitus shaped by experiences with instructional environments thus exposing them to similar pedagogic discourses will have greater access to knowledge relayed through pedagogic practices (Bourdieu, 1981; Zevenbergen, 2004). Students who have not had strong experiences with instructional environments lack the linguistic capital to understand the pedagogic relay that is occurring (Jorgensen & Grootenboer, 2011). Zevenbergen (2000, 2004) concisely surmises that success in school is about ‘cracking the code’ of the pedagogic discourse, as much as academic success. It is possible to conclude that there is a metaphorical ‘key’ enabling students to access the pathway to begin to engage and become successful at school. Without the necessary linguistic capital, a student, whether intellectually capable or not, or possessing the prerequisite knowledge or not, theoretically, will struggle in a school environment.

Research has found that students of middle-class families are exposed to interactions most similar to the pedagogic discourse of primary classrooms thus shaping their linguistic habitus in a way that prepares them for school (Zevenbergen, 2004). Disparities in linguistic capital of students from different cultural or economic backgrounds have long been noted in the work of Bernstein (2000). One example of this disparity has been noted specifically in relation to exposure to early learning experiences in instructional environments such as swimming lessons. Such experiences have academic benefits for students as they gain capital in relation to instructional environments (similar to school) and early mathematical concepts (Jorgensen & Grootenboer, 2011; Jorgensen, 2012). However, access to such experiences is often limited for working-class families due to the economic investment required.

Students of disadvantaged backgrounds typically associated with working-class families or Indigenous families are less likely to use the formal discourse familiar in schools due to limited experiences rich in instructional discourses and mathematical concepts. Comparatively, these students come to school with less linguistic capital (impoverished language) compared to their middle-class peers. For example, students from disadvantaged backgrounds may possess a linguistic discourse that is lacking many terms common to early mathematics, such as colours, shapes, or numbers (Jorgensen & Grootenboer, 2011). Further, the value placed upon school knowledge may vary across different social and cultural backgrounds. The result of this is that those who have not had a ‘western upbringing’ (Andersen, 2011) are disadvantaged from the very beginning of schooling. What occurs for disadvantaged students is that their early habitus that has been shaped by their early experiences
and home environment is not aligned with a school environment (Bourdieu, 1981). Similarly, Bernstein (2000) argues that making sense of school mathematics is made more difficult for students whose cultures are different to that which is represented in the classroom pedagogy. This is exemplified in the social and cultural context brought to school by Indigenous students which differs from modern, westernised schooling systems (Sarra, 2011a; Schwab & Sutherland, 2001). In other words, it has been suggested that socially disadvantaged students, such as Indigenous students, already lack the predisposition to engage successfully in school interactions. Such a suggestion can be attributed to deficit-based presumptions. These presumptions are divisive in that they imply that, no matter what, students of particular backgrounds unequivocally will struggle at school. Such a view has been avoided in the framing, implementation, and evaluation of this study, however it is unavoidable and important to acknowledge that mathematical learning occurs through engagement with the pedagogic discourse (Bernstein, 2000; Jorgensen & Grootenboer, 2011; Zevenbergen, Mousley & Sullivan, 2004). Therefore, a student who has not been exposed to instructional discourses that prepare them for the discourses of school learning may have difficulties understanding the particular regulatory rules and protocols of the pedagogic discourse of school (Maton & Muller, 2007).

The regulating rules of the school environment and school curriculum have some unchangeable and rigid elements due to the nature of one or two adults facilitating learning in classes of 20 to 30 students, as well as the nature of what knowledge is essential to function as a member of society. The issue of the mismatch between the pedagogic discourse of schools and contrasting cultural or social backgrounds has been approached by conceptualising two solutions in the work of Maton and Muller (2007): either the underlying structuring principles of school itself need to be adjusted, or it needs to be acknowledged that students need support in understanding the instructional discourse in the school context. The former solution is problematic in that particular social groups would be relegated to lower status forms of educational knowledge, which subsequently would be delegating such groups to lower rungs in the labour force (Maton & Muller, 2007). The problem with the latter solution is that it has often been misinterpreted as a deficit view, as to accept such a solution requires the status of different forms of knowledge to be conceded. However such a solution is tangible and feasible and is important to consider.
Examples of how limited experience and understanding of the instructional discourse of school impacts on Indigenous students’ ability to engage and be successful at school have been cited by Indigenous researchers. One example is that the norms of classroom interactions, such as the interaction of teacher questioning, may not be familiar to Indigenous students as there may be different norms at home and in their community (Galloway, 2003). Further, social and behavioural skills that are valued at school may not be possessed by some Indigenous students. For example, Indigenous students may not bring to school common skills related to dealing with conflict at school such as ‘ignore them’, or ‘walk away’, or ‘count backwards from ten’ (Sarra, 2011a). It is understood that behaviour in school impacts on academic performance, therefore the capital that students bring to school in the form of these social and behavioural skills are important to consider (OECD, 2016). Lacking an understanding of common useful ways to engage and participate in the classroom instructional discourse inevitably influences a students’ ability to be successful at school.

2.4.4 Inter-personal domain: School and classroom factors

Quantitative meta-analysis refers to teacher factors as having a greater influence on learning than school factors (i.e., the framework conditions of the school such as funding, class size and so forth) (Hattie, 2008; Hattie & Zierer, 2019), indicating that variation in student achievement is related to factors within schools rather than between schools. However, these analyses emphasise that within school factors extend beyond considering only teachers, and include broader school culture. Research suggests that school leaders are most effective when focusing on instructional strategies that are most effective in supporting students’ achievement (Connell, 1996; Henchey, 2001; Teddlie & Springfield, 1993). Meta-analyses indicate that high expectations, driven by school leaders at a whole school level, have a significant effect on student achievement. Factors associated with high expectations, particularly those related to academic achievement, can operate at both a school level (i.e., whole-school culture in relation to the value of learning and achievement), and the classroom level (i.e., individual classroom teachers’ expectations of learners). In other words, as well as high expectations from school leaders, meta-analyses also indicate that teacher expectations have a high effect size in influencing learning (Hattie, 2008). Therefore, how schools and teachers facilitate high academic expectations through academic engagement, classroom management, and supporting
student attendance will be discussed. Elements of school environment and culture relevant to Indigenous learners that facilitate high expectations will also be considered.

2.4.4.1 Academic engagement

Academic engagement is one factor that is affected by students’ productive dispositions and is a product of the pedagogic discourse of the classroom (Norton, Duy & Thao, 2016). Academic engagement refers to the academic learning time (sometimes referred to as time on task) students spend productively and actively engaged in learning (Gettinger, 1995; Singh, Granville & Dika, 2002). In relation to the importance of engaged learning time, the OECD (2016, p. 3) summarises:

The most effective tools that students, particularly low performers, have at their disposal to develop their skills and make the most of available opportunities are time and attention. Students need to invest enough of their time in learning activities and be more engaged with the task at hand…… Learning time, for instance, can too easily turn into wasted time if teaching practices are not effective.

This statement expresses the sentiment that maximising time spent engaged on learning tasks is important and driven by pedagogical choices. Time spent on task should be purposeful and well directed as some authors cite that the quality of learning activities is more important than the time spent on them alone (Archer & Hughes, 2011; Kohn, 2006; Shernoff, 2010). That is, employing effective pedagogical practices and maximising the time students spend engaged on learning tasks needs to occur to support student achievement. In agreement with these sentiments, Gettinger and Seibert (2002) propose a four-variable model for academic learning time: 1) allocated time, 2) time that is actually used for instruction, 3), engaged time, and 4) academic success and productivity. In this model, allocated time is converted to instructional time, which is converted to engaged (i.e., productive) time learning. Productive time learning is dependent on the pedagogical decisions that teachers make surrounding instruction styles.

Allocated time can be divided in to in-class and out-of-class (i.e., homework etc.) learning experiences. Studies have indicated that out-of-class time spent on homework has a significant influence on academic achievement (Singh, Granville & Dika, 2002). Homework is a form of purposeful rehearsal, and has implications for students’ learning as it supports schema development for later retrieval (Bjork, 2011). Due to the influence of out-of-school learning time, academic engagement is, in part, a manifestation of the community beliefs
surrounding school learning and therefore cannot be controlled completely within the school sphere. Instructional time varies from allocated time due to various losses of time associated with classroom management including student interruptions (i.e., disruptive behaviour) (Hollowood, Sailsbury, Rainforth & Palombaro, 1995). Engaged time where a student is substantively engaged in tasks refers to the time in which students are dedicating sustained and committed focus on tasks. Not all engaged time works to increase students’ academic achievement as it is important that students are experiencing success and that tasks are matched to the learner’s ability level (Gettinger & Seibert, 2002). In relation to in-class engagement, teacher support has been shown to be a significant factor in supporting students’ behavioural, affective, and cognitive engagement (Liu et al., 2018), however, the pedagogical choices teachers make also influence the extent of engaged instructional time (OECD, 2016).

2.4.4.2 Behavioural considerations

With respect to academic engagement in the classroom, it can be seen that classroom management considerations play an important role in influencing academic engagement due to their effect on instructional time (Wang, Haertel, & Walberg, 1993). Meta-analyses indicate that ‘well-managed’ classrooms, where procedures and rules for classroom activities are established and implemented, reported a high effect size, as did classrooms with heightened engagement (Marzano, 2003). The most significant influence on how well a classroom was managed was the teacher’s effectiveness in reducing disruptions by taking quick action on potential behaviour problems with retained emotional objectivity (Marzano, 2003). Other important teacher attributes in managing the classroom climate included teacher-student relationships (Cornelius-White, 2007; Marzano, 2003; Marzano & Marzano, 2003), which involve high dominance (clarity of purpose and strong guidance) and high cooperation (Wubbels, Brekelmans, van Tartwijk, & Admiral, 1999; Wubbels & Levy, 1993).

2.4.4.3 Student attendance

Another factor that impacts on learning and achievement for Indigenous students is attendance due to reported high rates of absenteeism, with Indigenous students more often absent from school than non-Indigenous students (Mellor & Corrigan, 2004; Purdie & Buckley, 2010). This finding was also supported by more recent reports from longitudinal studies carried
out with public school students in Western Australia commissioned by the Department of Education, Employment and Workplace Relations (DEEWR) (Hancock et al., 2013). Correlations between attendance rates and socioeconomic index, and achievement and socioeconomic index were also identified in this report (Hancock et al., 2013). Analysis of 2003 TIMSS results (Thomson et al., 2006) found that 87% of Indigenous students attended schools in which principals reported absenteeism to be a problem (compared to only 34% of non-Indigenous students). For Indigenous students who attended schools where absenteeism was not a problem, TIMSS performance was reported as similar to the international mean. Therefore, this factor is clearly important in terms of educational outcomes. Despite this, the issue remains important as Biddle (2014) found that 20% of reported achievement gaps could be explained through absenteeism.

The primary classroom concern with high absenteeism rates is that teachers are required to design curriculum for students with varying attendance records. This often results in a single class of students varying two to three grade levels. Whilst the label of poor attendance cannot be applied to all Indigenous students, it is an important factor to consider as teaching must account for the different learning needs of students in the class. Any effective program must be created in a manner that allows students access to their current ability level regardless of their year level or attendance. This is important, as the most effective initiatives that improve health, wealth, and happiness are those that address school attendance (Levin, Belfield, Muenning & Rouse, 2007). Educational studies such as the remote numeracy project have identified that effective schools address diversity in their students by streaming students based on attendance and current ability level (Jorgensen, 2015, 2018). Overall, this indicates that, potentially, how a school addresses attendance is important in influencing achievement for Indigenous students.

2.4.4.4 School environment and culture

Factors within schools (rather than differences between schools) have been shown to be most important in influencing students’ achievement, however, research has also highlighted several important considerations for successful schools in Indigenous contexts. These considerations include community partnerships and a culture of high expectations.

Research in Indigenous education and education for low socioeconomic communities, both nationally and internationally, has identified specific features of sustainable partnerships
between schools and communities (Howard, 2001; Howard & Perry, 2007; Lareau & Horvat, 1999; Makuwira, 2007). An important feature of these partnerships includes community ownership (Lowe, 2011). That is, community involvement and sway in school decision making has been shown to be important in sustaining successful programs for Indigenous students. Further, the culture of a school, developed from the organisational culture and leadership within a school, has also been noted as important in addressing deficit models of thinking by the inclusion of particular shared beliefs and assumptions which arise from community partnerships (Sarra, Matthews, Ewing, & Cooper, 2011). It is important that the community and the school share key beliefs related to students’ academic performance including high expectations and commitment to delivering productive and meaningful teaching and learning practices (Sarra, 2003, 2007; Sarra, Matthews, Ewing, & Cooper, 2011). The importance of high student expectations has also been expressed in other Indigenous contexts for example, New Zealand, (McNaughton & Lai, 2008). Sarra critically surmises that “having low expectations of Aboriginal children in schools and tolerating things like absenteeism and bad student behaviour contributed significantly to the process of engineering a negative Aboriginal identity” (2003, p. 15). It is through community engagement and a shared vision with schools that these deficit models of achievement can be addressed.

2.4.5 The socio-historical context for Indigenous learners

The socio-historical domain of learning influences the rules and conduct of interpersonal interaction that facilitates learning at a classroom level. In the conceptual framework for this study, the socio-historical domain has been linked to the inter-personal and intra-individual domains of learning as both domains are regulated by these cultural and social norms and conventions. Therefore, it is important to consider relevant cultural and historical factors that potentially influence learning for Indigenous students. One important factor to consider is the historical relationship that Indigenous people have had with schooling systems as this sheds light on how well the habitus of Indigenous students potentially aligns with instructional discourses. Further elaborating on the influence of the relationships that specific cultural groups have with schools will be discussed by considering the role of parental engagement in school.
2.4.5.1 Indigenous historic experience with formal school

The early period of the occupation of Australia by English colonies has been noted by Martin (2007) to have influenced the relationship between Indigenous and non-Indigenous people. The work of Martin gives voice to important Indigenous perspectives and notes that the declaration of Australia as *terra nullius* by Cook, “rendered us invisible...furthermore, this invisibility served to make us silent” (2007, p. 15). In the early establishment of English colonies in Australia in the late 18th century, chaplain-led conservative educational approaches derived from British systems drove early schooling systems. Authors have noted that the foci within these schooling systems was order, control, and reform (Campbell & Proctor, 2014). Campbell and Proctor (2014) further note that education institutions participated in the idea of breaking cycles of criminality which resulted in the confinement of children mainly from specifically disadvantaged groups (i.e., those deemed by the system to be likely sources of future criminality). Furthermore, authors have noted that this thinking also became the accepted approach for Indigenous children with educational objectives focusing on ‘re-socialising’, ‘re-education’, and ‘civilising’ (Campbell & Proctor, 2014; Haebich, 2000; Scrimgeour, 2006). These approaches resulted in segregated missionary-run schooling attempts for some Indigenous children that were focused on preparation for agricultural labour or domestic service and, as such, the curriculums saw low standards for achievement. Forcible separation of Indigenous children from their families as well as Indigenous servitude were relatively commonplace policy during this period. For some schools in Australia, segregation persisted until the 1950s and authors in this field believe that the influence of separation has had significant consequences for Indigenous children and families into the twenty-first century (Fletcher, 1989a, 1989b).

Historical perspectives of what education looked like for Indigenous youth pre-European contact highlight that the purpose of educational encounters were focused on immediate daily life, and the relevance of putting time and effort into learning particular skills or gaining specific knowledge would be exceptionally clear and logical as it related to the local environment and immediate survival pressures (Campbell & Proctor, 2014). Education was essential for life, and autonomy was highly regarded (Martin, 2007). For this reason, it has been suggested that early disenfranchisement and resistance to a formal model of schooling was a result of incoherence and gaps between what is useful now and what may be useful in the future (Campbell & Proctor, 2014). When considering the types of typical learning
experiences for Indigenous youth, cooperative learning was strongly represented and the learning environments were relaxed. An Aboriginal educator surmised:

Our education isn’t divided up into stages and it doesn’t start at 5 and finish at 15 or whatever. It’s a lifelong, whole-of-life, and holistic process. I’m still learning now – listening to our old people and reading whatever I can get my hands on (Carmichael, 1995, p. 391).

The authors cited above, through their interpretations of historical schooling perspectives in Australia, provide insights into how the approach to schooling developed in the early periods of European occupation. This schooling approach is dichotomous to what was known, established, and important for Indigenous people by these historical records. These historical considerations impact on current Indigenous communities’ relationships with formal schooling.

2.4.5.2 Parental involvement

Despite the importance of parental involvement in facilitating students’ success in school (Brofenbrenner, 1978; Epstein, 2008), there has been a long history of disenfranchisement with parents of culturally disadvantaged groups (Delgado-Gaitan, 1991). Considering historical experiences of Indigenous people with school, it is important to consider the flow-on impacts to parents of students in schools. For many decades, government policy denied Indigenous people access to education in Australia perpetuating educational disadvantage (Gray & Beresford, 2008; Richer, Godfrey, Partington, Harslett & Harrison, 1998). This subsequently becomes an intergenerational issue affecting Indigenous students by creating a gap between parents and schools as parents can be less willing to be involved with schools (Britton, 2000). Just as a students’ linguistic capital precludes or facilitates their successful involvement in school, “absence of appropriate sociocultural knowledge precludes acceptable participation in formal school activities, resulting in isolation for many parents” (Delgado-Gaitan, 1991, p. 21). These factors influence the way parents can advocate for their children in schools as well as parents’ ability to understand their children’s development at school. Because of these factors there exists a power gap. The work of Delgado-Gaitan (1991) focused on how nonconventional forms of parental involvement empowered parents from diverse groups. Other research by Sanders (2008) discussed how parental liaisons can play an important role in increased parental involvement thus reducing the gap between home and
school. Parental encouragement and support in school has been identified as an important factor in influencing Indigenous students’ motivation at school (Martin, 2006). For these reasons, parental engagement is important for Indigenous students (Mellor & Corrigan, 2004; Sarra, 2005; Vass, 2012).

2.4.5.3 Culturally relevant modes of learning

When considering the historical perspectives outlined above in the current context of educational approaches for Indigenous learners, recognising Indigenous diversity means that there can be no single pedagogy for Indigenous learners. However, it has been proposed that there are common pedagogical elements, notwithstanding the existence of socio-geographic and language distinctions (Yunkaporta, 2009). One common element is the importance of language. Yunkaporta describes that “there is a spirit of learning in our words. This is more than just knowledge of what to learn, but knowledge of how we learn it. This is our pedagogy, our way of learning” (Yunkaporta, 2009, p. 1). The implication of this statement is that Indigenous students may have preferred ways of learning. This aligns with an understanding that, historically, Indigenous ways of imparting knowledge have relied heavily on an oral culture (Gorman & Toombs, 2009; Kral, 2009; Nichol & Robinson, 2000). In Indigenous culture, oral communication integrates strongly with non-verbal elements (Yunkaporta, 2009). This highlights the critical nature of face-to-face interactions which integrate both oral and non-verbal elements for Indigenous learners (Warren, Young, & de Vries, 2007). Today, the model of schooling in Australia largely follows the established European tradition in that learning is equated to formal education and the value of learning is largely measured by commercial value, meaning that factual knowledge and job-specific skills are dominant (Hewitt, 2000). The current system predominantly groups learners by age, regardless of other factors (e.g., readiness, motivation, or interest) and relies on formalised means of communicating (i.e., written communication).

It has been proposed that the way individuals make sense of the world is strongly influenced by culture and resistance to schooling can occur when mismatches transpire between cultural background and school (Fogarty & White, 1994; Martin, 2006). This needs careful consideration. Cairney (2003) proposes that the discourses surrounding spoken language from the family for Indigenous Australian children do not match the discourse of school. This mismatch disadvantages students (Dickinson, McCabe, & Essex, 2006; Warren,
Young, & de Vries, 2007). Therefore, potentially, the differences between learning systems and learning styles is important in influencing the way Indigenous learners interact and engage with the instructional discourse of current, formal schooling systems.

2.5 Summary

Both social and psychological conceptualisations of learning were considered in the creation of the conceptual framework for this study. The socio-historical context relevant to Indigenous learners, the inter-personal nature of learning, and cognitive psychology theories relating to intra-personal learning were drawn upon to form a way of conceptualising the domains which influence learning in mathematics and the contributing fields and agents.

The nature of mathematics knowledge has been considered as this has ramifications for how mathematical knowledge is re-located as curriculum, which is then disseminated by the school and implemented by the classroom teacher. Importantly, the vertical, structured nature of mathematical knowledge was noted. An argument was made that mathematical knowledge can be considered as a continuum of sequentially developed skills and concepts moving from basic mathematical reasoning to increasingly abstract concepts.

Consideration of the pedagogic device further developed the conceptual framework for the study by situating the key fields of recontextualisation (the school) and reproduction (the classroom). How these fields are influenced by socio-historical, inter-personal, and intra-individual factors was also described.

Within the intra-individual domain, cognitive load theories described the process of learning and important role of working memory in influencing students’ learning. Metacognitive considerations were also outlined, and the importance of metacognition in supporting problem-solving was discussed. The influence of affective factors on individual students’ learning in mathematics was also detailed, particularly as students’ productive dispositions impact students’ willingness to engage in tasks.

In relation to the inter-personal domain and learning occurring within the classroom, the way in which socio-historical contexts inform the development of the linguistic and mathematical capital of a student was discussed in relation to the facilitation or hindrance of a students’ ability to engage with the instructional discourse of school. Potentially there may be metaphorical roadblocks for Indigenous students to engage successfully in the classroom due
to mismatches between home and instructional discourses. These socio-historical factors can drive whether or not a student has access to the metaphorical ‘key’ to success in school.

Other specific factors explored within the inter-personal domain included academic engagement, and how behavioural considerations can influence academic engagement. It was highlighted that, if students are not attending classes and if behavioural disruptions reduce the available instruction time, the time a student is able to be engaged in learning is reduced. As academic engagement fosters achievement, the effect of a reduction in achievement is a reduction in students’ motivation. In turn, this reduction limits academic engagement in a cyclical way. Furthermore, the importance of schools focusing on attendance, fostering community partnerships, and perpetuating shared high expectations for students was highlighted.

Specific to the socio-historical context for Indigenous learners, considerations surrounding Indigenous people’s historical experiences with school systems was discussed. The contrasts between Indigenous preferred ways of learning and modern, Western schooling systems as well as instances of disenfranchisement with school systems for Indigenous people was noted. Culturally relevant modes of learning were specified to be associated with oral and face-to-face communication.

The conceptual framework for this study emphasises the complex relationship between various influences on Indigenous students’ learning in mathematics. Individual students’ cognition is influenced by teachers’ pedagogy, the classroom discourse, and the school environment. Socio-historical factors inform the norms and conventions of the inter-personal environment. Additionally, cultural and historical factors also influence individual students and their subsequent engagement and achievement at school.
3 LITERATURE REVIEW

3.1 Introduction

This chapter will provide a review of pedagogical approaches and educational reforms and initiatives in mathematics education, both nationally and internationally, within the domain of Indigenous education. Specific approaches to teaching mathematical concepts and skills such as number facts, computations, and problem-solving will also be reviewed in addition to effective professional development considerations.

Section 3.2 will overview different pedagogical approaches in mathematics education, followed by a discussion of various educational reforms and initiatives in Section 3.3. Section 3.4 will overview the teaching and learning of number facts, computations, and problem-solving in primary mathematics education. To conclude this chapter, effective professional development models will be discussed in the context of implementing school-based initiatives (Section 3.5).

Importantly, throughout the literature review, the effectiveness of teaching practices will be evaluated through the lens of human cognitive architecture. A consideration of whether specific practices work to develop students’ conceptual understanding of mathematical topics by reducing demands on students’ working memory and facilitating the development of schema will be considered in the review.

3.2 Pedagogy in mathematics education

Educational research, particularly in the domain of Indigenous education, has examined numerous factors in relation to student learning and achievement including socio-economic factors, school environment factors, and student efficacy. However, when considering effective educational practices, a central question focuses on what impacts on student learning in a meaningful and significant way. Hattie’s (2008) synthesis of educational research meta-analyses provided a list of the most significant factors that were found to influence student achievement. It was found that teachers accounted for approximately 30% of variance in
student achievement as well as being one of the top six sources of variance (e.g., Cornelius-White, 2007; Nye, Konstantopoulos, & Hedges, 2004; Rosenthal & Rubin, 1978). Evidence suggests that teacher effectiveness is an important factor influencing student learning (Hayes et al., 2006; Rowe, 2003; Sanders, Wright, & Horn, 1997). In short, while other variables including attendance, and school and home circumstances are important, quality teaching was found to be a key factor influencing students’ achievement. Therefore, it is useful to explore research on effective teaching practices (pedagogies) and consider their relevance to an Indigenous setting.

3.2.1 Constructivist pedagogies

In recent decades, influential and prevalent pedagogical approaches across many subject areas (including mathematics) have been underpinned by constructivist theories (Farkota, 2005; Klinger, 2009; Richardson, 2003; Rowe, 2006; Sahin, 2010). Emerging from the work of Vygotsky and Piaget, constructivism states that knowledge cannot be transmitted, instead learners actively construct their own understanding (Ainsworth, 2013; Clements & Battista, 1990; Kamii, 1985; Klinger, 2009; Selley, 2013; Splitter, 2009; Sullivan, 2009). Arising from these theories are constructivist approaches to teaching, broadly referring to pedagogical methods that require the student to be active in finding meaning, as opposed to rule finding or memorisation (Ainsworth, 2013; Selley, 2013). Traditional teaching practices were considered to have placed too much emphasis on memorising, and too little attention on the development of deeper understanding. For teachers in all subject domains, these theories give rise to important educational considerations such as the unique construction of knowledge by each student, and the key role of the student in their own learning (Lerman, 1989). This is particularly important in mathematics education as developing conceptual understanding is fundamental to developing numerate students (Geary, 2004).

Constructivist pedagogies can be viewed on a spectrum that ranges from “pure” discovery learning, to teacher-guided models. In primary mathematics, pure discovery learning advocates the loss of ‘rule’ and ‘procedure’ teaching in favour of student exploration, manipulation, and experimentation within their environment to derive information for themselves (Ormrod, 2000; Sullivan, 2009). Students are encouraged to develop their own rules, algorithms, methods, procedures, and strategies. These approaches have also been termed
minimal guidance pedagogies due to the teacher acting as facilitator, rather than structuring students’ approaches to learning tasks (Westwood, 2008). In addition to minimal guidance pedagogies, there are other pedagogies such as problem-based learning that utilise constructivist theory but involve varying levels of teacher guidance and scaffolding (Schmidt, Loyens, van Gog, & Paas, 2007). Problem-based learning is a model of learning that uses authentic, real-life problems and issues that require team-working, research, data collection, and critical thinking to solve (Barrows, 2000; Torp & Sage, 2002; Westwood, 2008). It can be seen from this description that the level of teacher guidance is not stipulated and can be varied in problem-based learning approaches. Overall, although constructivist approaches can be implemented in a variety of ways with varying educational outcomes, approaches involving a more substantive component of teacher guidance are supported empirically (Ausubel, 1964; Engelmann, 2007; Kirschner et al., 2006, 2012; Mayer, 2004; Rowe, 2006; Westwood, 2008).

When considering research from the past 60 years, it has been found that pedagogies in which there is some degree of teacher guidance are more effective than both pure discovery teaching where teacher guidance is minimal, or expository (direct) teaching where intensive teacher guidance may interfere with deeper learning (Engelmann, 2007; Mayer, 2004).

Constructivist pedagogies play an important role in mathematics education as evidenced by the implementation of constructivist principles in national and international learning standards (e.g., NCTM, 1989, 1991). The principles of constructivist pedagogies are also reflected in the rationale for the Australian Curriculum: Mathematics F-10, which currently states that the curriculum “encourages teachers to help students become self-motivated, confident learners through inquiry and active participation in challenging and engaging experiences” (ACARA, 2018d). This statement advocates the use of constructivist approaches to create mathematically autonomous students who are able to access powerful mathematics, an important goal of mathematics education (Clements & Battista, 1990; Cobb, 1988).

Though constructivist pedagogies are commonly recommended in mathematics education research and policies, it is also important to consider that some researchers have raised concerns about adopting minimal guidance pedagogies (e.g., Ausubel, 1964; Donnelly & Wiltshire, 2014; Engelmann, 2007; Hattie, 2008; Kirschner et al., 2006; Mayer, 2004; Rowe, 2006). For example, Donnelly and Wiltshire (2014) reported a review of the Australian Curriculum: Mathematics, and expressed concern that constructivist pedagogies such as
inquiry-based learning, whilst being important, were privileged at the expense of other learning approaches, stating that some essential knowledge requires explicit teaching. These statements suggest that a variety of pedagogical approaches are needed for effective teaching and learning to occur. For example, to develop procedural fluency in mathematics, mastery is difficult to obtain through student-centred models of instruction (Westwood, 2008), particularly when rehearsal is important for schema development (Bjork, 2011). Fluency with mathematics procedures such as computations and number facts is important to reduce the load on working memory. For these reasons, some researchers have stated that concepts and skills associated with hierarchical, esoteric domains such as mathematics require explicit instruction (Sweller, 2016). Klinger (2009) states:

Elementary or foundational concepts, in particular, are not reasonably accessible to exploration and discovery. They need to be learned in essentially the same way as vocabulary and rules of grammar – no matter how they arrive at their ideas, students must know what to write and how to write it in order to reliably record and communicate them (p. 157).

The issue with discovery or inquiry approaches within an esoteric knowledge domain such as mathematics is that the emphasis moves from acquiring domain specific knowledge and skills to placing a large demand on the learner to generate information (Sweller, 2016). Such an emphasis on discovery negates much of what we know about human cognitive structures by placing an unnecessary cognitive load on students. Researchers in this field note that it is accepted that learning requires the construction of schematic knowledge in long-term memory, however it is unnecessary to teach how to construct this knowledge (Sweller, 2016). That is, there is no empirical evidence to support the assertion that what is learnt through discovery is superior to what is learnt through other methods. Furthermore, evidence related to the worked example effect would suggest that approaches involving explicit instruction are more effective than minimal guidance pedagogies (Atkinson et al., 2003; Kirschner et al., 2006; Paas & van Gog, 2006; Sweller, 2016).

Pedagogical variety as well as limiting an overemphasis on unguided approaches are also important to consider as schooling outcomes, even in modern systems, rely on students’ ability to demonstrate learning of particular skills that ultimately require specific processes and procedures for success (Klinger, 2009). For example, student-invented alternative algorithms may be useful with simple sums. However if the educational goal is for students to add large
values efficiently, traditional approaches involving adding and renaming will be required. Alternatively, the computational process can be abdicated to technology. Research suggests that student-invented algorithms compound student errors as suggested by cognitive load theories (Norton, 2012). Sweller (2016) cites the example of completing the question 

\[(a+b)/c=d, \text{ solve for } a\]  

(p. 361), where the best first move is always to multiply out the denominator. There is a domain-specific skill attached to solving this problem that only applies to a finite class of algebraic problems. It is for these reasons that, within the structured discipline of mathematics, teacher guidance plays an important role in mathematics education, as success in mathematics requires manipulation, understanding, and long-term retention of mathematical language and conventions.

3.2.2 Teacher guided pedagogies

As teacher guidance plays an important role in mathematics education, it is important to consider the effectiveness of pedagogies that feature teacher guidance, such as direct (or explicit) instruction.

Evidence from meta-analyses and experimental research design studies indicates direct instruction significantly increases student achievement and self-efficacy (Farkota, 2005; Flores & Kayler, 2007; Grossen & Ewing, 1994; Hattie, 2008; Klahr & Nigam, 2004). A synthesis of empirical research regarding teaching mathematics for low-achieving students found that interventions that led to improvements included the use of direct (explicit) instruction principles to teach concepts and procedures (Baker, Gersten, & Lee, 2002). Overall, direct instruction significantly outperforms problem-based learning when comparing quantitative studies (Hattie, 2008). This demonstrates either a deficit in quantitative studies involving minimal-guidance approaches, or that there is some merit in approaches involving increased levels of teacher guidance. Further, as noted earlier in consideration of the worked example effect, different pedagogies might be appropriate for different stages of learning (e.g., expert versus novice learners) and the gross evaluation used in meta-analyses does not account for these subtleties. Therefore, these findings may require further consideration.

Although these positive empirical findings are important to note, it is also central to consider the concepts, skills, or content foci of the analysed studies as particular elements may be better suited to either direct teaching or to minimally guided models. For example, Snow,
Burns, and Griffin (1998) discussed the concept of the balanced approach, involving both explicit instruction and student-centred activities. This same sentiment was expressed in a review of the Australian Curriculum: Mathematics (Donnelly & Wiltshire, 2014). The different approaches ought not to be seen as a dichotomy, but rather that different learners have different needs and are at different stages of becoming autonomous learners in particular concept domains.

Often direct instruction is criticised incorrectly due to its association with a didactic teacher instructional model. To clarify, it is useful to look at Rosenshine’s (1987) definition of explicit instruction as “a systematic method of teaching with emphasis on proceeding in small steps, checking for student understanding, and achieving active and successful participation by all students” (p. 34). This definition draws attention to the need to consider individual students’ cognition and prior understanding, as is exemplified in constructivist theories, as well as including active learning in the teaching approach. Archer and Hughes (2011), and Hattie (2008) have outlined clear lists of the elements of effective explicit instruction. These elements include sequencing and breaking down skills logically, and focusing lessons with clear learning intentions. The use of lesson goals and success criteria was also noted in key practices in a remote Indigenous numeracy project (Jorgensen, 2015, 2018). A final report on the remote numeracy project noted that “being explicit about the intent of the lessons also helped student to have a clear idea of what they were to learn and how they were to engage with the lesson, again maximising learning time” (Jorgensen, 2018, p. 30). Maximising learning time, and time on task have been noted as important in influencing students’ success (Gettinger & Seibert, 2002; Norton, Duy, & Thao, 2016).

Other elements of explicit instruction include engagement (or a hook), designed to grab students’ attention so that they are receptive and focused in order to maximise academic engagement of students (Hattie, 2008). Following this, the proposed structure of direct instruction lessons include modelling, and checking for understanding prior to guided practice where students work towards mastery whilst being given feedback and individual remediation from the supervising teacher (Archer & Hughes, 2011; Hattie, 2008). There is a balance between guided and supported practice to promote students’ success and confidence. Independent practice allows students to develop mastery further, and to ensure that learning is not forgotten by facilitating the transfer of knowledge from short to long term memory. Individual lessons must also be concluded effectively to enable students to have the opportunity
to organise and consolidate learning as well as to eliminate confusion and frustration by reviewing and clarifying the major points in the lesson. Such lesson models also have been supported positively in the remote numeracy project (Jorgensen, 2015, 2018).

From the practices noted in this section, it can be seen that effective direct instruction does not ignore constructivist theories. There are opportunities for the teacher and students initially to develop critical understanding of concepts, followed by guided practice where students have opportunities to develop understanding, concluded by lesson closures that provide time for the organisation of new information. These elements are one example of the balanced approach advocated by Donnelly and Wiltshire (2014) and Snow et al. (1998).

3.2.3 Other pedagogical approaches: Mastery learning

Another pedagogical approach which has been shown to have a significant influence on student achievement is mastery learning. Mastery learning, an approach identified as effective in meta-analyses, was found to have a positive influence on student attitudes as well as achievement (Kulik, Kulik, & Bangert-Drowns, 1990), which is a highly sought-after educational outcome often difficult to obtain.

Mastery learning initially arose from models such as Bloom’s Learning for Mastery (Bloom, 1968). Bloom’s model focused on individualising instruction depending on student’s needs resulting in uniformly high performance for all. This model differentiated from the conventional models typically seen in some explicit instruction approaches where students receive identical instruction irrespective of individual needs, resulting in normally distributed outcomes within a class (Kulik et al., 1990). Mastery learning is structured around the design of small units of sequenced work developed from pre-testing on unit objects, followed by instruction on objects which are yet to be achieved (Hattie, 2008; Willett, Yamashita, & Anderson, 1983). Timeframes do not constrain the units of work, and the premise of mastery learning is that no student will progress onto further units until mastery of objectives is obtained. This approach includes teacher guidance and teacher structuring whilst also taking into account constructivist theories by acknowledging differences in each students’ learning process.
3.2.4 The importance of feedback

Feedback to students is an important element of effective direct instruction (Hattie, 2008). In meta-analyses conducted by Hattie on the types of influences on students’ learning, feedback was a factor that stood out due to its large effect size and its extensive research base (i.e., 31 meta-analyses and 1,431 individual studies) (Hattie & Zierer, 2019). In summarising what is important about feedback, Hattie notes that it is about creating high expectations and an error-friendly climate in classrooms (Hattie & Zierer, 2019). That is, feedback is driven by identification and interpretation of ‘errors’. This is an important concept to note for mathematics education as mathematics is a field where we are often looking at ‘right’ or ‘wrong’ answers, and analysis of students’ errors has a long history in mathematics education (e.g., Ashlock, 1976; Booker, 2011; Burrows, 1976; Buswell & Judd, 1925; Radatz, 1979; Reisman, 1982). Ashlock (1976, p. 19) noted that “remedial or corrective instruction should be based upon sufficient data to suggest patterns of incorrect and immature procedures”. Rosenshine (2012) further articulates that a teacher’s ability to anticipate errors and warn students of mistakes that are frequently made is an important characteristic of effective teaching. Related to a mastery teaching approach, the process of analysing students’ work to provide feedback enables teachers to understand where students are situated in the course of their learning, and the study of students’ errors enables teachers to plan teaching sequences that are likely to be related directly to the thinking that the students exhibit.

In characterising effective feedback, Hattie describes effective feedback as addressing three questions: Where am I going? (the goals: “feed up”), How am I going? (feedback), Where to next? (feed forward) (Hattie & Zierer, 2019). Feedback can address learning at a task level (i.e., how well the task was understood/performed), the process level (the processes needed to understand/perform tasks), and the self-regulation level (i.e., self-monitoring directs and regulates actions) (Hattie & Zierer, 2019).

3.2.5 Effective pedagogy in Indigenous education

It is important to consider how the aforementioned research concerning effective pedagogies applies to Indigenous education. Many themes consistent with creating safe learning environments apply to Indigenous education with an emphasis on respect, personal sense of agency, and meaningful relationships between teachers and students. However, these
themes often are seen in reform curricula for non-Indigenous learners as well, and they are largely universal educational goals (e.g., the work of Marzano on teacher-student relationships, 2001, 2003).

For Indigenous students, the Longitudinal Literacy and Numeracy Study for Indigenous Students (ILLANS) found that schools with favourable learning environments and teacher-student relations (as rated by students) recorded higher mathematics results (Purdie et al., 2011). However, these goals are what educators should strive for irrespective of students’ backgrounds. Many strategies and characteristics of effective instruction within Indigenous education research apply to the teaching of mathematics and numeracy to all students. Challenging all pupils, and high, achievable expectations and standards are key components of effective mathematics instruction well documented in literature (Askew, Brown, Rhodes, Johnson, & William, 1997; Clarke & Clarke, 2002; Eckermann, 1994; Grouws & Lembke, 1996; Jones, Tanner, & Treadaway, 2000; Jorgensen, 2015; Matthews et al., 2003; Mellor & Corrigan, 2004; Morris, 2014; NCTM, 2000; Reynolds & Muijs, 1999; Russell, 2000; Thomas & Ward, 2002).

Though Indigenous students are not a homogenous group and the practices noted in the preceding paragraph apply to the education of all students, there are some unique factors to consider concerning Indigenous education. One key consideration is that there are possible differences between the knowledge brought to school by some Indigenous and non-Indigenous children. In the same way there may be differences in the knowledge that children from varying socioeconomic backgrounds bring to school (proposed generally by researchers such as Bernstein, 2000, and specifically in mathematics, for example, by Jorgensen, 2012; Jorgensen & Grootenboer, 2011). Some knowledge that is highly culturally valued by Indigenous communities may have little relevance to the Western schooling system which possesses a unique set of social conventions (Batten, Frigo, Hughes, & McNamara, 1998; Dodson, 1994; Heitmeyer, 2001; Hughes 1987; Mellor & Corrigan, 2004). This implies that teachers must carefully consider the knowledge their students bring into the classroom as, in accordance with constructivist theories, this will have implications for student learning. A second key consideration for Indigenous learners is affective factors associated with learning mathematics, such as motivation.
3.2.5.1 Productive dispositions for Indigenous learners

As the way individuals make sense of the world is strongly influenced by culture, the interactions between culture and school must be considered in relation to Indigenous students’ motivation as resistance to schooling can occur when mismatches occur (Fogarty & White, 1994; Groome & Hamilton, 1995; Martin, 2006; Willis, 1983). The implication of this is that it is important to consider the self-efficacy of Indigenous learners, that is, their belief in, and perception of, their ability to succeed and learn successfully (Bandura, 1997). Often, in mathematics education self-efficacy has been conceptualised as a students’ productive disposition, referring to a student’s ability to perceive mathematics as “sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (Kilpatrick et al., 2001, p. 5). There is a cyclical relationship between self-efficacy and motivation which in turn affects performance (Harrison, 2011) as it dominates and drives behaviours (Groom & Hamilton, 1995; Hughes, More & Williams, 2004; Louth, 2012; OECD, 2016). Increasing students’ motivation increases their academic engagement (the time they spend invested in tasks), which is an important consideration for academic success (OECD, 2016). Motivation, a highly relevant factor for educational outcomes, has been identified as a key variable in influencing student learning (Wang, Haertel, & Walberg, 1993).

For Indigenous students, it has been proposed that “some Aboriginal children sometimes subscribe to a stereotypically negative perception of being Aboriginal, thinking that it is not an ‘Aboriginal thing’ to be smart” (Sarra, 2011a, p. 109), which means that Indigenous students are potentially more susceptible to having lower self-efficacy and motivation. The idea of a stereotype threat has been discussed in literature beyond Australian Indigenous studies, which encompasses awareness of negative stereotypes regarding the performance of the social group to which you belong (Steele, 1997; Steele & Aronson, 1995). The idea that a negative self-concept regarding academic achievement may be prevalent in Indigenous students is supported further in literature that suggests that Indigenous students are particularly sensitive to criticism (Groome & Hamilton, 1995; Halse & Robinson, 1999). In colloquial terms, this has been referred to as being ‘shamed’ through academic failure. This brings about considerations that different cultures have different norms with regard to failure and shame as well as how criticism is interpreted. The report into mathematics in Indigenous contexts by the New South Wales (NSW) Board of Studies (Howard, Perry, Lowe, Ziems, & McKnight, 2003) quoting a parental concern that “kids feel that teachers will blame them if they ask questions.
They feel shamed. They won’t put their hands up in maths” (p. 7). The presence of criticism causing shame can reduce students’ motivation (Halse & Robinson, 1999) thus, consideration of the presence of stereotype threats is important as it has been also found to reduce students’ working memory (Beilock, Rydell & McConnell, 2007).

In other literature, Martin (2006) suggested that the result of criticism and a fear of being shamed has resulted in many Indigenous students preferring to “not run the race at all (rather) than run with the prospect of losing” (p. 40). Fear of failure can result in what has been termed a cascading model of failure fearing (Covington & Omelich, 1991; Martin, 2006; Martin & Marsh, 2003; Martin, Marsh, & Debus, 2001). In a cascading model of failure fearing, even those students who initially respond with increased diligence when presented with challenges or failures, eventually can begin to respond in protective manners in the face of continuing failure. However, over an extended time of poor performance, these responses lose their effectiveness and students begin to accept failure, and become disengaged. This disengagement can manifest as active resistance to school (Martin, 2006). Fear of failure is an important educational concern as it is understood that, compared to their non-Indigenous counterparts, Indigenous students are more likely to leave school early (Mellor & Corrigan, 2004; Gray, & Partington, 2012; Partington, Gray, & Byrne, 2006; Ross & Gray, 2005). Research also suggests that Indigenous students’ enthusiasm in school diminishes in junior high school (Gray & Beresford, 2001, 2008) and, potentially, motivational factors may act to explain some of these trends.

In addition to the role motivation may play in reducing Indigenous students’ ability to engage with, and attempt tasks, research has also demonstrated that working memory is affected for students in groups susceptible to stereotype threats (e.g., “it’s not an Aboriginal thing to be smart”). Stereotype threats harm performance when a task places a heavy demand on working memory (Beilock, Kulp, Holt, & Carr, 2004). Therefore, awareness of, or belief in, negative stereotypes associated with one’s own performance may have cognitive impacts that harm students’ performance in a meaningfully way. One proposed way of addressing these issues is utilising positive role models to challenge students’ stereotypes (Beilock & Ramirez, 2011).

Motivation is an important element of a productive disposition as it relates to an individual’s perseverance with a task until completion (Eccles & Wigfield, 2002; Guy et al., 2015). Productive dispositions are particularly significant when considering learners’
mathematics proficiency as the development of the other four cognitively-based strands of mathematical proficiency are dependent on a students’ ability to perceive mathematics as useful and worthwhile, and the envisagement of themselves as an effective and capable learner of mathematics (Kilpatrick et al., 2001; Jacobson & Kilpatrick, 2015; Philip & Siegfried, 2015; Woodward, Beswick, & Oates, 2017). The implications of a fear of failure, or a fear of being shamed, are that productive dispositions may be more significant and influential for Indigenous learners.

3.2.5.2 Oral learning preferences

Another important educational consideration for Indigenous learners is the cultural history of oral learning preferences. When considering the oral culture related to Indigenous ways of learning, studies with early primary students (Prep to Year 3) where Indigenous cultural practices were integrated with Western ways of learning (e.g., the RoleM program; Warren & Miller, 2013) indicated that a focus on an oral language approach to mathematics was one of the actions that most effectively facilitated students’ engagement with mathematics. This is significant considering the importance of motivation for Indigenous learners as discussed in the previous section.

An oral language approach encompasses “comprehending what is being said, understanding the vocabulary being used, and applying this to mathematical contexts” (Warren & Miller, 2013, p. 155). That is, it is important for students to learn the mathematical register made of the semantics and syntax used in mathematics. The need for students to understand the language of mathematics is not unique to Indigenous contexts, with the developmental relationship between language, symbols, and materials noted in the work of Bruner (1966). More specific to the oral language approach espoused in this program was the acknowledgement within the guiding framework of how Indigenous learners are person-orientated (Nichol & Robinson, 2000; Warren & Miller, 2013). This supports elements of Yunkaporta’s (2009) eight ways of learning where personal interactions are valued as a form of non-verbal communication.
3.3 Education reforms and initiatives

When making educational decisions and comparing teaching practices, it is important to consider the broader scope of educational goals, and what effective practice looks like. In other words, it is imperative to consider what it is about particular practices that deem them effective, and how can we measure the effectiveness of these practices. The importance of student achievement as an indicator of effectiveness is summarised by Dinham (2008): “the focus of every school, every educational system… [should be] student learning and achievement” (p. 1). As such, it is appropriate to focus on individual student achievement as a measure of judging the effectiveness of particular educational practices. Therefore, in the following sections pedagogical practices in various educational initiatives and reforms will be evaluated by analysing empirical evidence concerning student achievement.

3.3.1 Mathematics education reforms and initiatives

In the following section, mathematics education reform programs and initiatives that have reported success in raising students’ achievement will be discussed. In particular, Project Follow Through, the Railside reform program, and the Missouri maths effectives project are examined. More importantly, the practices that may have contributed to raising students’ achievement in these projects will be considered in light of cognitive load theories and effective pedagogies discussed in earlier sections.

3.3.1.1 Project Follow Through

Project Follow Through was a large-scale project documenting achievement in various reform schools carried out from 1968 through to the 1970s. This project was the largest educational experiment ever conducted in the United States at the time (Engelmann, 2007). The key feature of this project was the student population as the project worked with communities with high levels of poverty as well as students labelled as disadvantaged, at risk, and low achieving. The students in this project were taught in small groups using a lesson model of short, heavily scripted, and prescribed direct instruction lessons (15-minutes) followed by individual work (30-minutes). The direct instruction component of lessons involved high levels of teacher-student interaction to facilitate feedback (termed reinforcement and corrections in this project), and the series of lessons were sequenced purposefully and
carefully (Engelmann, Becker, Carnine & Gersten, 1988). One line of reasoning behind the scripting of lessons was to enable teacher-aides to also act in the role of teachers, which increased the amount of teaching time for students. The second purpose of scripting was to improve the quality of teacher instruction.

In the mathematical component of this direct instruction project, learning was sequenced around two priorities: the first was teaching addition and subtraction operations in conjunction with their forms in problem-solving contexts (Engelmann et al., 1988); the second priority was developing students’ proficiency with number facts. The results of this project were that the disadvantaged students in direct instruction programs significantly outperformed students in a variety of other US reform programs. The direct instruction program outperformed all other programs in the three measured domains: basic skills (e.g., math facts and computation), cognitive skills (e.g., higher order thinking - problem solving), and affective measures (tests on students’ sense of responsibility and self-esteem). In some instances, the direct instruction program was the only program to have positive outcomes in the three areas measured (e.g., cognitive skills). Overall, the results demonstrated that direct instruction can provide long range, stable, replicable, and positive results with children labelled at risk, low performing, and disadvantaged. It was found that this program was the only program effective for low-performing students. Further analysis of pedagogical practices found that mastery was essential for these students. The critical points were that mastery of foundational knowledge was essential for all learners, but the implications of possessing or not possessing this knowledge was most profound for students from disadvantaged circumstances. The findings from this large-scale project were also supported by earlier reform schools such as the Bereiter Engelmann school in 1964 (Engelmann, 2007).

3.3.1.2 Railside

In recent years, constructivist approaches to teaching and learning mathematics have been presented in mathematics classrooms under the general banner of ‘reform’ programs in an attempt to improve students’ mathematics performance. These reforms have been implemented over past decades, persisting into current education systems both nationally and internationally, as well as in Indigenous settings. A common theme underpinning many reform programs is the use of student-centred pedagogical models. These models predominantly focus
on student autonomy, and authentic learning experiences similar to what is observed in some Australian Indigenous education reforms (e.g., YuMi Deadly math programs).

One such reform was the California Stanford Mathematics Teaching and Learning Study, a longitudinal study conducted across five years, implemented in three schools with 700 students. This study compared a ‘traditional’ school program with a reform program at Railside, an underperforming linguistically, economically, and ethnically diverse urban secondary school. The primary finding was that greater gains in achievement and higher overall achievement were evidenced in the reform school, Railside, compared to other schools using traditional programs in the study (Boaler & Staples, 2008). In addition, the reform school reported reductions in achievement gaps between different ethnic groups. In the reform program, the majority of class time involved students working in groups (72%), with a small component (4%) of lessons dedicated to teacher-led instruction (Boaler & Staples, 2008). In contrast, the traditional classrooms employed a more balanced pedagogical approach, spending a significant amount of time on individual student practice (48%), and teacher-led instruction (36%).

With regard to possible reasons why the Railside reform program was more successful when compared to traditional programs, findings indicated that there were high levels of collaboration among teachers at Railside. Over 10 hours per week were spent designing curricula, discussing teaching, and reflecting to improve practice through the collaborative sharing of ideas. In addition, the program was developed as a response to improving equity in mathematics education, and the recruitment of teachers at Railside focused on hiring like-minded, committed staff to participate in the reform (Boaler & Staples, 2008). The findings regarding teacher collaboration concur with meta-analyses from Hattie which indicated that collective teacher effectiveness is highly influential in facilitating students’ success (Hattie & Zierer, 2019). The idea of committed and like-minded staff is also reiterated in Hattie’s large-scale work. This work discusses the importance of high-quality collaboration with successful processes being typified by intensive discussion and focus on the effectiveness of programs and repeated evaluation resulting in an evidence-based approach and a common vision of teaching quality (Hattie & Zierer, 2019).

In contrast to the traditional schools, the teachers in the reform school were flexible in adapting their curriculum so that significantly longer time could be spent on teaching introductory foundation concepts. The understanding was that, in order to ensure that both
social and socio-mathematical norms were well established, careful and deep teaching was required (Boaler & Staples, 2008; Yackel & Cobb, 1996). Such findings relating to the importance of foundational concepts also concur with what was found in Project Follow Through (Engelmann, 2007). Overall, the findings from Railside highlighted that teachers’ commitment to programs, and the thoughtful planning of mathematics teaching are important for student success. In addition, the careful development of fundamental mathematics concepts also increased student success and provided opportunities to continue with further mathematics studies. These elements are important as some researchers have suggested that minimal-guidance, student-centred approaches, often seen in reform programs, create challenges for lower performing students particularly when emphasis is placed on complex, open-ended problems (as was the case at Railside; Kirby & Williams, 1991; Montague, 1997). This may be because, in the planning of minimally guided approaches, it is assumed at times that learners have adequate prior knowledge, understanding, and skills to engage in tasks where they are expected to be self-directed (Chen et al., 2016a; Kirschner et al., 2006; Kirby & Williams, 1991; Montague, 1997). However, Railside successfully placed an emphasis on complex, open-ended problems by very carefully addressing foundation concepts in their programs, demonstrating that fundamental mathematical knowledge cannot be neglected if students are to negotiate higher-order tasks successfully (Klinger, 2009; Kilpatrick et al., 2001; NCTM, 2000; Rowe, 2006; Woodward, 2006).

3.3.1.3 The Missouri Math Effectiveness Project

An example of a successful reform program based on teacher-guided pedagogies is the 1979 Missouri Math Effectiveness Project. In contrast to the Railside reform, teachers and students in this project developed conceptual understanding together through active questioning instead of via group-based learning. This project aimed to deliver professional training to teachers and introduce them to a structured active teaching method to be applied in Year 4 mathematics in urban, low-income schools. A specific lesson structure was used in the Missouri project (Good, & Grouws, 1979) which was found to improve student achievement. The lessons began with a daily review of concepts and skills, including mental computation exercises, followed by a brief focus on prerequisite skills and concepts leading up to a direct teaching component for new concepts. The direct teaching component had an emphasis on checking for, and developing, conceptual understanding through explanations, demonstrations,
controlled practice, and active questioning. Once concepts were developed, the individual component of the lessons involved uninterrupted, successful student practice with sustained momentum. The individual practice time allocated in each lesson aligns with mastery teaching principles described earlier (Kulik et al., 1990) and effective direct instruction practices (Archer & Hughes, 2011; Hattie, 2008). It is noted that students, at all times, were held accountable for their individual work via teacher checks, and special reviews were also carried out weekly and monthly.

The Missouri Maths Effectiveness Project found that students taught by the treatment teachers outperformed control groups on both standardised and content tests (Good & Grouws, 1979). Overall, this reform exemplifies aspects of a balanced approach with students being neither passive receivers of information, nor unguided in their learning. In this approach, the teacher is viewed as the guide in students’ development (construction) of understanding.

3.3.2 Indigenous education initiatives

Current initiatives in Australia that address Indigenous mathematics education include YuMi Deadly, Stronger Smarter Learning Communities (SSLC), Connected Communities, the remote numeracy project, Make it count, and Good to Great Schools. The primary focus of many of these initiatives has been to address deficit-based practices in schools. Before these initiatives are discussed, international Indigenous education initiatives will be explored in the following sub-section.

3.3.2.1 International Indigenous education initiatives

Initiatives designed to address disparities between groups of students are not unique to Indigenous Australians. Such initiatives are also carried out with Indigenous groups in other countries. These educational initiatives, which centre on addressing deficits and achievement gaps, take place in countries with Indigenous populations including Canada, New Zealand, USA, and Mexico.

An example of an Indigenous initiative in Canada is the culturally responsive pedagogy model designed to support Indigenous students. This model aimed to remove barriers to student achievement and well-being (Ontario Ministry of Education, 2013). Through this model, constructivist approaches (particularly inquiry-based learning) were emphasised to facilitate
culturally responsive teaching (Ontario Ministry of Education, 2013). Other initiatives, such as the Martin Aboriginal Education Initiative arose from the 2011 Canadian census that identified low retention rates for Indigenous Canadians in secondary school. Much like the COAG’s (2016) goal to halve gaps in Indigenous achievement in Australia, the Martin initiative aimed to improve school outcomes for Aboriginal Canadians, particularly for students identified as at-risk. Elements of effective practice that have been identified in a Canadian Indigenous context include community involvement, inclusion of Aboriginal teachers, and using data to support the tracking of improvement (McRae, 2006). More locally, New Zealand projects such as Te Mana (meaning respect, authority, rights) comprise information campaigns designed to raise expectations of Maori achievement, including attendance (McRae, 2006). This focus acknowledges the different values placed upon learning in diverse cultural contexts.

Overall, the low performance and low enrolment in mathematics for Indigenous students on an international scale has been documented but not addressed adequately in current research (e.g., Assembly of Manitoba Chiefs, 1999; Binda & Calliou, 2001; Ezeife, 2006, 2011; Katz & Mccluskey, 2003). There is a need for further empirically supported research focused on exploring ways to overcome cultural, affective, and cognitive barriers.

3.3.2.2 YuMi Deadly

YuMi Deadly maths is a recent project aiming to improve mathematics outcomes for Indigenous students, and students from low-socioeconomic backgrounds (YuMi Deadly Centre, 2020). This program has been implemented in over 200 schools across Australia. It is a prescriptive mathematics program providing details of how to teach specific math concepts. The program is designed for grades F-9 and, typically, is delivered to teachers in professional development sessions facilitated by the YuMi Deadly Centre. This program advocates teaching in a manner that draws from Indigenous conceptualisations of mathematics epistemology (Matthews, 2008) as outlined in Section 2.2. The program cites “mathematics learning is undertaken through active participation – relating kinaesthetic activity to mental models, through a sequence of body → hand → mind” (YuMi Deadly Centre, 2020). Such an approach suggests that Indigenous students may benefit from concrete, tactile forms of pedagogy.

In studies exploring the effectiveness of YuMi Deadly programs, there was “some quantitative evidence of student improvement…[but] the results could be due to maturation or other effects” (YuMi Deadly Centre, 2013, p. 2). There exists evidence of the program
improving students’ achievement as measured by NAPLAN, with findings reporting student improvement of over two years. These findings were above other similar school counterparts (YuMi Deadly Centre, 2017). The focus on professional development for teachers reportedly increased teachers’ pedagogical skills through involvement in the YuMi Deadly program. However, there is some ambiguity in impact evaluation reports from YuMi Deadly programs (2017) as the school samples utilised for these comparisons were not transparent (i.e., the number of compared schools is unspecified), and data were based on voluntary contributions from schools.

3.3.2.3 Stronger Smarter Learning Communities

SSLIC encourages embracing positive Indigenous identity in schools, and centres on the creation of high expectation relationships (Stronger Smarter, 2014). This program highlights the emergence of deficit-based statements in previous educational agendas such as Close the Gap (COAG, 2016). The primary rationale is that quality education is most needed for Indigenous and low socioeconomic students (Stronger Smarter, 2014). The program considered that the most important factor for effective Indigenous education is for schools and educators to evaluate their own practices to establish causes for students’ difficulties (Sarra, 2011b). This program centres on striving for excellence in teaching and continuous reflective practice.

In a summative evaluation of the longitudinal and cross-sectional mixed methods project involving achievement data from 201 participating schools from 2008-2011, there were no positive gains reported in attendance or achievement (as assessed by NAPLAN) as a result of the SSLIC initiatives (Luke et al., 2013). Possible causes for the lack of improvement proposed by authors included a lack of program coherence or whole school curriculum planning (Luke et al., 2013). The suggested policy implications from this report concluded that the program was potentially unsuccessful in raising achievement as there was an increase in principal autonomy without improved curricular leadership (Luke et al., 2013). This demonstrates the importance of practices and pedagogies appropriate to specific domains, and that genericised practices are not sufficient to address achievement disparities. The implication of this is that leadership in schools needs to be able to support and drive domain-specific practices.
3.3.2.4 Connected Communities

Connected Communities is a government initiative in New South Wales with a five-year strategy that commenced in 2013. Connected Communities worked with 15 schools from disadvantaged areas in NSW with the aim of improving education and learning outcomes for Aboriginal children and young people (Centre for Education Statistics & Evaluation, 2015). Its goals included creating school community hubs, increasing staff cultural awareness including Aboriginal language and culture in schooling, creating community partnership executive positions in schools, and personalising learning plans for all students (NSW Department of Education and Communities, 2012). In effect, a key objective was to eliminate a clear boundary between the school and community. Expected outcomes included increased attendance rates, achievement, and post schooling employment and training. The latest report on outcomes from Connected Communities noted that, at the midpoint of the project, there were no gains in academic outcomes, gaps between non-Indigenous samples have not closed (as measured by NAPLAN testing), and senior attendance rates had decreased (Centre for Education Statistics & Evaluation, 2015). No explanations were forthcoming to speculate on the apparent lack of progress.

3.3.2.5 Remote numeracy project

A recent research project focused on remote Indigenous education is the “Success in remote indigenous communities’ project” (Jorgensen, 2015). This project documented the state of numeracy, and current educational practices across 35 remote schools. Although not a specific reform program, this project sought to document, investigate, and celebrate numeracy successes and numeracy practices in remote schools.

Although a variety of teaching practices were noted across the schools (i.e., no common, theoretically supported pedagogical approach), there was a common philosophy behind pedagogical choices, and this was a focus on ascertaining students’ current understanding of concepts (their entry level), followed by strategies targeted to meet the specific student needs (Jorgensen, 2018). In the final report of the project, it was noted that a school-wide approach and a shared vision was important across the schools involved in the study. This finding concurs with findings of other educational reforms such as Railside (Boaler & Staples, 2008). Further, community relationships were also noted to be important in relation
to a shared vision with the community; this speaks to how the schools in remote contexts attempted to establish common values for the learning occurring in the schools.

Utilising middle-leadership and quality professional development to support teachers in enacting envisioned practices was also noted as important across the schools (Jorgensen, 2018). In relation to this, people functioning in the role of numeracy coach (i.e., a specialist numeracy teacher employed to support teachers in numeracy specific practices) was also a feature of many schools. The support from numeracy coaches ranged from in-lesson support, to support in planning, to providing feedback to teachers (Jorgensen, 2018). Employing numeracy coaches in schools is an element of a school’s high expectations for staff in implementing quality mathematics programs. Overall, leadership and transparency were critical in developing a consistent whole-school approach to teaching mathematics. The interim report from the study surmised that the “consistent and transparent framework enabled teachers to know what and how to deliver quality learning” (Jorgensen, 2015, p. 45). This idea is important as a lack of coherence in approaches was previously identified in other reform projects as contributing to a lack of student gains in achievement and vice versa (e.g. Railside [Boaler & Staples, 2008] and SSLC [Luke et al., 2013]).

The reported enacted practices observed in classrooms included differentiation for individual students, high mathematical expectations, a focus on mathematics as a learning priority, and scaffolding. Previously maintaining high expectations has been identified as an important element of effective mathematics education (Clarke, & Clarke, 2002; Eckermann, 1994; Grouws & Lembke, 1996; Jones et al., 2000; Jorgensen, 2015; Matthews et al., 2003; Mellor & Corrigan, 2004; Morris, 2014; NCTM, 2000; Reynolds & Muijs, 1999; Russell, 2000; Thomas & Ward, 2002). Further, culturally responsive practices in relation to employing strategies that were cognisant of issues related to shame were noted in the enacted practices in classrooms (Jorgensen, 2018). In some schools, students were streamed in their classes based on attendance, with those attending regularly placed together, and those with interrupted attendance being placed in classes with greater flexibility and support. Streaming or grouping students in individual classes based on data was also commonplace to cater for the diversity by targeting activities to the needs of the learners.

Mathematics lessons differed across the schools involved in the remote numeracy project, and were at the discretion of individual teachers, however it was observed that teachers were consistent in some aspects. For example, the consistent and unchanging structuring of
mathematics lessons was noted in many schools. The purpose of this was to ensure that students understood the expectations of the lessons, and subsequently increased engagement by reducing any potential confusion. Part of a consistent structure was repetition as many teachers cited the need for repetition due to the differences in numeracy practices at school, and numeracy practices out of school. Stating an explicit intent for lessons was also noted as helping support students to engage with lessons, which acted to maximise learning time. Findings from classroom observations also noted that, in classes where group work was utilised, teachers needed to spend considerable time establishing group work norms; one teacher cited the establishment of these norms to have taken 10 weeks (Jorgensen, 2018).

Some lesson structures discussed as case studies in the interim report (Jorgensen, 2015) demonstrated similarities to those recommended in Hattie’s elements of effective direct instruction, as well as in mastery learning pedagogy (Hattie, 2008; Willett, Yamashita & Anderson, 1983). Most lessons also specifically focused on building proficiency in number, place value, and operations based on an understanding that proficiency in foundation mathematics concepts is important in developing numerate students (Poncy et al., 2010). No quantitative data was provided in the reports concerning students’ achievement in the sample schools.

3.3.2.6 Make it count

Similar to SSLC, the Australian Association of Mathematics Teachers (AAMT) project - Make it Count: Numeracy, mathematics and Indigenous learners, worked with eight diverse groups of schools to develop professional development programs from 2009-2012 in order to increase student engagement, achievement, and attitudes (Morris, 2014). Make it Count focused on the intersection of mathematical pedagogical content knowledge and cultural competency. It addresses culturally responsive mathematics pedagogy aimed at improving learning outcomes of Indigenous learners (Morris & Matthews, 2011). Of the clusters of schools involved in the study, each had different foci and goals in their mathematics programs including contextualisation, developing mathematical resilience, early years, and sustaining engagement in mid-primary years, amongst others.

Reports on achievement gains (as assessed by NAPLAN) were not conclusive in this project due to limited and/or missing achievement data (Forgasz, Leder, & Halliday, 2013). Evaluators did note that the achievement of students involved in the project was similar to
mean scores for other Indigenous students, suggesting that there was no significant measured improvement in achievement from participation in the program (Leder & Forgasz, 2012). However, the program did lead to positive findings concerning Indigenous primary students’ beliefs and attitudes towards mathematics, with no discernible differences in attitudes noted between the Indigenous and non-Indigenous students in the program (Forgasz, Leder & Halliday, 2013). One of the findings pertaining to professional practice arising from the intervention related to the importance of creating learning environments where Indigenous learners feel safe and supported, and part of this encouraged risk taking in mathematics (AAMT, 2011). Overall, the varied results and varied pedagogies and practices enacted in schools involved make it difficult to interpret findings empirically from the Make it Count project.

3.3.2.7 Good to Great Schools Australia

Good to Great Schools Australia [GGSA] was an education initiative in Queensland supporting whole school improvement. GGSA ran a reform program in remote Australian Indigenous schools called the Cape York Academy operating in Aurukun, Coen, and Hope Vale. The key practices were great teachers, effective instruction, and every child counts policies (GGSA, 2017). This project arose from a need to address achievement gaps in disadvantaged, low achieving, and high-risk students, with results indicating significant gains in achievement. For participating schools, students’ NAPLAN results increased every year, with Coen scoring above national averages in numeracy in 2015 (GGSA, 2016). Overall, it was reported that the growth in numeracy achievement was greater than the Australian averages for Years 3-5.

The educational programs in GGSA centred on explicit direct instruction pedagogies, similar to those found to be effective in Project Follow Through (Engelmann, 2007). GGSA defined their pedagogical model, direct instruction, as “an education program that combines explicit instruction pedagogy with a comprehensive curriculum, student assessment and scripted lessons” (GGSA, 2017). Lessons were highly structured and centred on mastery of concepts before moving on, a strategy promoted in mastery learning pedagogies (Hattie, 2008). This approach to mathematics education was selected for this program “based on the principle that there are scientifically established methods of effective instruction (Pearson, 2009, p. 54).
Though this program led to gains in student achievement, reception in the community was mixed and subject to some media controversy. Outcomes from the literacy intervention component of the project indicated that the program had not improved students’ literacy abilities (Guenther & Osborne, 2020). In addition, other research has advocated for curriculum and pedagogical practices that are diverse, socio-politically grounded, critical, and culturally responsive (Brayboy & Castagno, 2009), as exemplified in initiatives such as SSLC, something GGSA did not implement in its explicit program.

3.3.3 Summary of education reforms and initiatives

Analysis of current Australian educational initiatives addressing Indigenous students’ achievement (e.g., YuMi Deadly, SSLC, Connected Communities, the remote numeracy project, Make it count, and GGSA) found variations in the effectiveness of the programs. SSLC, Connected Communities, and Make it Count have not, to date, reported meaningful gains in student achievement as a result of the programs. Possible reasons for the limited change in student achievement included lacking a coherent program and whole school curriculum planning (Luke et al., 2013; Newmann, 1996). Programs that reported gains in achievement employed explicit instruction elements in lessons, supported the use of consistent and clear lesson structures, and a focus on proficiency in foundation mathematics concepts (GGSA, 2017; Jorgensen, 2015).

3.4 Number fact, computation, and problem-solving pedagogy

Factors central to students’ success in mathematics were explored and identified in Chapter 1 (Section 1.3.2). Three elements related to key early number concepts were identified including number facts, computational skills, and problem-solving. Pedagogies and specific initiatives regarding these early number concepts will be explored in the following sections.

3.4.1 Number facts

Efficient and accurate recall of number facts has been found to be crucial for student’s success in primary mathematics and further studies for both Indigenous and non-Indigenous students (Cumming & Elkins, 1999; Isaacs & Carroll, 1999; Poncy et al., 2010; Woodward,
In the following section the nature of learning number facts, particular pedagogical approaches, and current initiatives concerning number facts will be discussed.

3.4.1.1 Pedagogical approaches

Number fact pedagogies and interventions grounded in empirical evidence are crucial for educationally disadvantaged students (Pegg & Graham, 2013). Research has found that high performing students commonly answer mathematics questions through memory retrieval of number facts (Torbeyns, Verschafel, & Ghesquière, 2004). Students who recall number facts from memory are also highly efficient when compared to students who utilise other methods such as counting on or decomposition (Torbeyns et al., 2004). Such a finding may be explained by the reduced cognitive load imposed by computational tasks through the efficient recall of number facts (Woodward, 2006). Drawing from cognitive load theories, number fact difficulties likely contribute to cognitive overload as a student detaches their focus from a computation to solve a needed number fact that is not memorised (i.e., stored in long-term memory for rapid and unlimited retrieval). A student then must hold the unmemorised number fact in their working memory and then return to the computation. Fluid movement between calculating a number fact that is not memorised and completing a computation requires the student to competently and simultaneous store knowledge of the number fact and the stage they are at in the computational process in working memory. A primary aged student may experience cognitive overload from such a process due to the limited capacity of working memory. Thus, facilitating the memorisation process for number facts is an important educational goal due to efficiency benefits, which leads to increases in the capacity to problem solve.

When considering pedagogical approaches best suited to the facilitation of learning of number facts, Poncy et al. (2010) compared different interventions in a general grade 2 classroom and concluded that guided approaches led to sustained improvements and number fact fluency. However, minimally guided interventions failed to deliver sustained improvements and number fact fluency when compared to a control group. A possible explanation for this result is that more highly-guided approaches involved more repetition and time allocated to committing facts to long term memory. This position has been refuted as it has been argued that memorisation methods undermine understanding (Isaacs & Carroll, 1999). This is a valid consideration as the instructional goal must be to develop both procedural
fluency and conceptual understanding (ACARA, 2018; Geary, 2004; Kilpatrick et al., 2001). Isaacs and Carroll’s solution to memorisation methods was the inclusion of strategy discussion related to number facts. The influence of strategy discussion has been explored in interventions comparing the effect of strategy discussion and timed drills, strategy discussion only, or timed drills only (Cumming & Elkins, 1999; Woodward, 2006). It was found that the inclusion of strategy discussion in conjunction with timed drills was more advantageous for students (Woodward, 2006). In summary, if students understand the logic behind number facts, they are more likely to remember the facts compared with pure memorisation detached from meaning and patterning (Norton, 2014).

3.4.1.2 Number fact initiatives: QuickSmart math

QuickSmart (QS) math is an initiative implemented across Australia since 2001 for middle school students. The initiatives in QS target low-achieving students who are still experiencing difficulties with number facts after initial instruction and subsequent differentiation (SiMERR National Research Centre, 2019). QS is a highly successful initiative when comparing empirical measures of achievement to a control group, and was the only initiative mentioned in the 2010 Closing the Gap report. The researched sample in QS consisted of over 10,000 participants and 600 schools with approximately 30% of the sample identifying as Indigenous. As of 2013, QS had worked with half of all schools in the NT (Pegg & Graham, 2013).

Instruction themes in this program include building on students’ pre-existing knowledge to create successful learning experiences and a focus on foundation skills, as well as the involvement of Indigenous teaching assistants as instructors. The pedagogical approach employed in this initiative includes withdrawal of students for three 30-minute periods each week for 30 weeks. This withdrawal entailed long-term instruction concerning explicit strategy knowledge, and the systematic use of timed and deliberate practice activities to overcome basic skill deficits (Pegg & Graham, 2013), an approach similar to Project Follow Through (Engelmann, 2007). In addition to withdrawal sessions, class time includes extended deliberate practice that is individually tailored and informed by formative testing. The overall focus of the initiative is automaticity with the development of students’ fluency and conceptual understanding regarding basic facts. Research informing this study has included studies regarding quality instruction for low-achieving students (e.g., Baker et al., 2002), however,
inclusion of strategy discussion in conjunction with timed drills to teach number facts also concurs with other studies (Cumming & Elkins, 1999; Woodward, 2006). Appropriate teacher and peer modelling are also part of QS programs. These practices are informed by research that has found, for Indigenous students, mathematical language, algorithms, and number facts are best taught through explicit instruction (Howard, Cooke, Lowe, & Perry, 2011). Overall though, the premise of lessons is to maintain interest and increase students’ intrinsic motivation through frequent experiences of success.

3.4.2 Computational skills

A key goal in primary education, both nationally and internationally, is to create numerate students and this includes being able to carry out the four operations (addition, subtraction, multiplication, and division) skilfully (efficiently and accurately) (ACARA, 2018b; NCTM, 2016; Thomson et al., 2012b). It has been proposed that “fluent operation within the symbolic domain is at the heart of the mathematics discipline” (Leong, Ho, & Cheng, 2015, p. 7). Ideally, the aim is to develop both procedural and conceptual understanding regarding these operations as they are essential to problem solving (Geary, 2004). Achievement data indicating Indigenous students’ low performance in foundational mathematics concepts calls for careful consideration of pedagogy concerning computations. In the following section, various pedagogical approaches concerning computational concepts and algorithms will be discussed and evaluated.

3.4.2.1 The development of addition, subtraction, multiplication, and division concepts

In considering pedagogy surrounding the teaching of addition, subtraction, multiplication, and division concepts and associated procedures (e.g., algorithms), it is important to understand how students’ conceptual understanding of each of the four operations is best developed. In particular, it is useful to consider students’ implicit, intuitive models for each operation as this can inform effective pedagogy (Fischbein, Deri, Nello & Marino, 1985).

Studies exploring students’ development of addition and subtraction concepts and skills have found that students’ solutions to problem-solving questions (i.e., word problems) reflects the semantic structure of the problem (Carpenter & Moser., 1983; Hiebert, 1982). Carpenter, Hiebert and Moser (1983) classify the semantic structure of simple addition and subtraction
problems types as either change, combine, compare, or equalise. Change problems relate to situations requiring a direct action such as joining (addition) or separating (i.e., takeaway – subtraction). Combine problems do not contain a direct action, and represent problems where the quantities could be either considered individually or as parts of the whole. Compare problems relate to comparison between two quantities, and these problems can either be modelled as a missing part (e.g., \( 2 + ? = 12 \)), or subtraction (\( 12 - 2 = ? \)). Similarly, equalise problems can include characteristics of compare problems (e.g., they can also involve missing part contexts), or change problems. In equalise problems, there is a direct action given, but comparison is also involved (Carpenter et al., 1983). Carpenter and Moser (1984) identified that students’ intuitively solve change problems using counting on or back strategies, and that students’ solve missing part problems (e.g., \( 2 + ? = 12 \)) by starting with the smaller quantity and counting up to construct the larger at first. That is, students’ early solutions to compare problems are indicative of a matching process involving one-to-one correspondence between two sets. The development of students’ strategies to solve addition and subtraction problems holistically develops from direct modelling strategies, to more sophisticated strategies using counting skills and facts (Carpenter & Moser, 1984).

Nunes, Bryant, Evans and Bell (2010) advocate that multiplicative and division concepts should be developed by building on students’ informal knowledge relating to their use of one-to-many correspondence, rather than repeated addition. Whilst counting or addition may be procedures students’ intuitively (i.e., prior to instruction) use to solve multiplication or division problems, students intuitively rely on one-to-many correspondence as the conceptual model to represent the relations in multiplicative problems (Kouba, 1989). Several studies support the idea that students’ have an understanding of one-to-many correspondence when starting school (e.g., Becker, 1993; Carpenter, Ansell, Franke, Fennema, and Weisbeck, 1993; Frydman and Bryant, 1988). The effectiveness of instruction focusing on one-to-many correspondence when developing the multiplicative concept has been supported in experimental studies where students’ receiving instruction focused on one-to-many correspondence obtained greater gains in achievement than those receiving instruction focused on repeated addition (Park & Nunes, 2001).
3.4.2.2 Pedagogical approaches

Developed from the work of Bruner (1966), one instructional model used to teach algorithms to carry out computations is the concrete-representational-abstract (CRA) teaching sequence. In some literature, this sequence is also termed concrete-pictorial-abstract (CPA), however, the meanings of the terminology in each case is equivalent. CRA is an instructional approach designed to promote conceptual understanding and involves new concepts to be first introduced through concrete, manipulative materials (e.g., using base-10 blocks to model addition). The concrete phase of the sequence is supported by theories surrounding the role of manipulatives in mathematics (e.g., Reisman, 1982). Once students obtain mastery using concrete materials, appropriate drawings, pictures, and representations are used to solve problems (e.g., drawing images of base-10 blocks or using place value charts to guide solutions). Once mastery using representations is obtained, more abstract forms can be explored (e.g., only symbolic representations of sums; Mancl, Miller & Kennedy, 2012). Within this model, it is important to note that the stages within the CRA sequence are not always distinctly separated or linear, and that many of the stages are developed alongside other stages (Bruner, 1966). For example, symbolic representations of computations are developed alongside the use of concrete materials or appropriate representations.

There has been a long history of evidence supporting the effectiveness of the CRA teaching sequence for students with difficulties in mathematics, specifically with mathematics computations (Jordan, Miller, & Mercer, 1999; Morin & Miller, 1998; Sousa, 2008). Specifically, CRA instruction has been found to be effective for students who struggle with addition, subtraction, multiplication, and division facts as well as multi-digit problems (Flores, 2009, 2010; Harris, Miller, & Mercer, 1995; Mancl et al., 2012; Miller & Kaffar, 2011; Miller & Mercer, 1993; Morin & Miller, 1998). Empirical studies have also found that when students with learning difficulties were taught place-value skills through CRA instruction, they outperformed a control group that received only abstract instruction (Peterson, Mercer, & O'Shea, 1988). In addition, Indigenous education research has suggested that teaching must involve concrete activities centred on making links between symbols and their meaning (Matthews et al., 2003).
3.4.2.3 Alternative algorithms in math education

Currently in mathematics education, there is an increase in pedagogical approaches advocating the use of alternative algorithms to carry out addition, subtraction, multiplication, and division computations (Boaler & Staples, 2008; Groth, 2007; Norton, 2012; YuMi Deadly Centre, 2020). Alternative algorithms are often applied to reform programs in contexts involving disadvantaged or struggling students (e.g., YuMi Deadly). The agenda for alternative algorithms originates from Dutch studies and programs that focused on methods for adding and subtracting values up to 100, based on student-created methods and mental strategies (Thompson, 2007a). In these methods, addition and subtraction involves splitting the numbers and adding or subtracting the tens and ones separately, then recombining to obtain the final answer. Dutch methods contrast to traditional or standard algorithms which typically start adding or subtracting values in a systematic manner beginning from the ones, and moving up to the tens. Interpretation and application of these Dutch programs to international classrooms has resulted in many educators and educational researchers promoting these strategies as superior to traditional algorithms due to the success of Dutch mathematics programs. This has resulted in avoidance of the teaching of traditional methods despite Dutch students still learning these algorithms from 3rd grade onwards (Beishuizen, 1993).

3.4.2.4 YuMi Deadly strategies and alternative algorithms

In the YuMi Deadly math program, many of the methods for teaching algorithms stem from Dutch methods. Three methods for addition and subtraction are taught in this program: separation, sequencing, and compensation. The first method, separation, involves splitting numbers into parts based on place value and recombination to obtain the final answer (YuMi Deadly Centre, 2011a, b). The second method, sequencing, involves leaving one value as a whole then adding or subtracting parts of the other number. The third method, compensation, involves leaving both numbers as whole and looking for changes to make the calculation easy, then compensating for the change. For two-digit operations, these methods seem logical, however, in separation and sequencing methods, students are required to add partial products meaning that they are required to add twice. This is an added cognitive load for students.

The criticism of these alternative methods is that, whilst it is possible, potentially, to teach students these methods for two digits values, it is impractical and ineffective to use these
methods with larger values. In the case of the three methods of adding described earlier, and for many other alternative algorithms, they are abstractions of mental models for computation. The blurring of mental calculation strategies and traditional, vertical algorithms is problematic. Demanding the recording of mental calculation models places an unnecessary cognitive demand on students, and these methods are less generalisable (Plunkett, 1979). Comparatively, traditional algorithms are standardised, efficient, and generalisable. The lack of generalisability of alternative algorithms to large numbers means that the “traditional” algorithm would still have to be taught to students. Presenting three possible methods of addition and subtraction is also an unnecessary added cognitive load for students. While there is a place for mental strategies in the teaching of operations, care must be taken in formalising and overusing them in inappropriate situations.

3.4.3 Traditional algorithms

The value of traditional algorithms is that they are a reliable mental structure that can be applied to any set of numbers. This supports what is understood about human cognitive architecture as the mental structure of algorithms can be remembered more efficiently, and accessed more easily (Bruner 1960; Hiebert & Lefevre, 1986; Hiebert & Wearne, 1996). When first introducing the concept of each operation (addition, subtraction, multiplication, or division), it is important to consider carefully the use of materials as well as the use of concise language in order to provide meaning and develop conceptual understanding (Booker et al., 2014; Norton, 2014). Hiebert and Wearne (1996) state that “conceptual understanding facilitates the acquisition process” (p. 252).

Following conceptual understanding of the operation, the vertical algorithm for larger numbers in initial instruction with an emphasis on place value can be developed (Booker et al., 2014). The central understanding being developed in early instruction is that of renaming - renaming 10 ones as 1 ten, and 10 tens as 1 hundred etc. (Booker et al., 2014; Norton, 2014). In this way, the recording and language of addition remains the same from preparatory years through to upper primary. The foundations of addition and subtraction support students’ learning of, and proficiency with multiplication and division algorithms.

Traditional algorithms have received some criticism in the literature (Kamii, 1985; Randolph & Sherman, 2001; Thompson, 2007a, b, 2008a, b). A common concern is the
prevalence of errors. Difficulties that are found with traditional algorithms centre on incorrect renaming, misconceptions concerning zero, or confusion with recording (Booker et al., 2014; Norton, 2012, 2014). However, when students using alternative algorithms are compared to those employing traditional methods, students were more successful at completing addition, subtraction, multiplication or division problems with traditional methods (Norton, 2012).

A second potential criticism relating to traditional algorithms relates to their applicability to all types of computations. Whilst change problems (either joining or separating), and combine problems can be considered suited to traditional algorithms without adaptation or variation, this is not always the case for other problem structures. Due to the structure of missing part problems originating from either compare or equalise contexts (e.g., $2 + ? = 12$), it could be considered that problem solving contexts involving these problems are less efficiently solved via traditional algorithms. However, given that missing part problems can be modelled as subtraction, these types of problems can still be, procedurally, effectively solved via traditional algorithms. In exploring early primary (Year 2) students’ strategies to solve a range of these problem types, Carpenter and Moser (1983) noted that initially, prior to any instruction, students’ solved compare and equalise problems using a type of counting up strategy (e.g., to solve $2 + ? = 12$, students would add counters to 2 until they reached 12) but, after instruction (involving several lessons including problem solving in situations involving joining, separating, combining, and comparing with concrete materials), students most prevalent strategy shifted to solving the problem as a subtraction. This type of finding supports the efficacy of solving missing addend problems using traditional algorithms, especially as they can be applied to larger values.

Concern has also been expressed regarding the usefulness of traditional algorithms. For example, Girling (1977) states that algorithms should only be taught “as part of the armoury of techniques that we have to help an understanding of number and not because they are useful” (p. 13). Whilst algorithms are not the only important part of mathematics, statements such as these potentially have led to the undervaluing of ‘pencil-and-paper’ calculations. Students with limited mastery of the four operations are systematically limited as they progress through their schooling years (Bryant et al., 2008; Calhoon et al., 2007; Cawley & Miller, 1989; Cawley et al., 1979; Fleischner et al., 1982; Gersten et al., 2005; National Mathematics Advisory Panel, 2008; Warner et al., 1980). Whilst it is easy to hand children calculators when calculating at a primary level, operating with algebraic expressions and understanding concepts behind these
manipulations are built off a foundation concerning these traditional algorithms (Kotsopoulos, 2007; Woodward, 2006). For example, if one were to teach binomial product expansion, the traditional multiplication algorithm is useful. Whilst it can be taught procedurally or learnt by rote, it does not add to a deep understanding of the algebraic manipulations carried out. Similarly, fractional operations can often become complicated with the use of calculators, or impossible if they also include algebra. Overall, the simplest purpose for maintaining the role of traditional methods in classrooms is that they are applicable for all values and do not add to students’ cognitive load unnecessarily.

Consistency and clarity in the pedagogical approach to teaching algorithms is important as initiatives such as SSLC noted that a lack of improvement in student achievement may have been a result of a lack of program coherence (Luke et al., 2013; Newmann, 1996). Programs that did implement consistent approaches to teaching programs found that students benefited from the unchanging, and clear structure (Jorgensen, 2015).

3.4.4 Problem solving

Historically, problem-solving in mathematics has received much attention in curriculum reforms resulting in an emphasis in problem-solving orientations in instructional programs (e.g., ACARA, 2018b; NCTM, 1989). Problem-solving requires the application of mathematical knowledge where strategies must be chosen purposely in order to find a solution. In this way, problem-solving is distinct from specific mathematical knowledge such as computational procedures (Hiebert et al., 1996). Earlier the importance of problem-solving heuristics to support metacognition in conjunction with strategic competence were discussed. An example of a heuristic problem-solving model from Polya (1988) is outlined in Figure 5.
Strategic competence refers to a student’s ability to formulate and represent mathematical problems appropriately (Kilpatrick et al., 2001). This ability can be considered as the use of metacognitive skills, which involves the understanding of cognitive processes and products (Flavell, 1976). The strategy of asking effective questions during the problem-solving process has been shown to develop metacognitive skills and abilities (Hacker & Dunlosky, 2003). Developing metacognition capabilities is an important consideration as students with higher metacognitive skills are more successful problem-solvers (Desoete, Roeyers & Buysse, 2001).

Research as highlighted how positive attitudes towards mathematics are linked to less careless errors when problem-solving for Indigenous students (Clarkson, 1983). This highlights the importance of the affective strand of mathematical proficiency in influencing the cognitive strands (Kilpatrick et al., 2001). Indigenous Papua New Guinea students have been found to make errors most frequently that are related to comprehending the task (i.e., identifying what the problem-solving question is asking them to do), and this is associated with strategic competence (Clarkson, 1991).

3.4.5 Summary of number fact, computation, and problem-solving pedagogy

In evaluating pedagogical approaches for number facts in mathematics, the importance of strategy discussion in conjunction with timed tasks has been highlighted (Cumming & Elkins, 1999; Woodward, 2006). An argument has also been made in literature that traditional algorithms, as well as the CRA teaching sequence, support students’ conceptual development and align with cognitive load theories (Mancl, Miller & Kennedy, 2012; Norton, 2014). For
problem-solving, research highlights that metacognitive skills and problem-solving heuristics are important in supporting students’ strategic competence (Desoete et al., 2001).

3.5 Professional development

There is consensus in educational research concerning the importance of teachers’ professional development in supporting student outcomes (Akiba & Liang, 2016; Coldwell, 2017; Desimone, Smith & Phillips, 2013; Kutaka et al., 2017; Villegas-Reimers, 2003; Yoon, Duncan, Lee, Scarloss, & Shapley, 2007). Little (1987) describes professional development as “any activity that is intended partly or primarily to prepare paid staff members for improved performance in present or future roles in the school districts” (p. 491). The definition of professional development provided by Little immediately highlights the potential for highly diverse professional development experiences. Scholars in the field suggest that it is more meaningful to focus on the critical features of the professional development activity rather than the type of activity (Desimone, Porter, Garet, Yoon & Birman, 2002; Garet, Porter, Desimone, Birman & Yoon, 2001). A focus on features is also supported by research which indicates that changes in teachers’ practices as a result of professional development activity are explained by the features of the activity (Desimone et al., 2002). Desimone (2009) argues that there can be consensus in the core features of effective professional development (professional development that has resulted in changes in practice, or resulted in increasing teacher knowledge and skills). These features are: a content focus, active learning, coherence, duration, and collective participation.

In evaluation of these features, it is proposed that a content focus, that is activities that focus on subject matter content and how students learn that content, is potentially the most important feature (Banilower, Heck & Weiss, 2005; Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Cohen, 1990; Cohen & Hill, 2001; Desimone et al., 2002; Garet et al., 2001). The feature of active learning highlights the importance of teachers being engaged in the learning experiences. Coherence refers to consistency between the activity and teachers’ knowledge and beliefs, as well as coherence between school policies. In relation to the duration of professional development, research has identified that activities must be sufficient in duration, including the length of the individual activity and spread of activities over a particular duration (a semester is recommended), to bring about intellectual and pedagogical change (Cohen & Hill, 2001).
The feature of collective participation requires interaction and discourse between teachers as a form of learning.

To describe the action of implementing effective professional development in a given context, Desimone (2009, p. 184) proposes the following steps:

1. Teachers experience effective professional development.
2. The professional development increases teachers’ knowledge and skill and/or changes their attitudes and beliefs.
3. Teachers use their new knowledge and skills, attitudes, and beliefs to improve the content of their instruction or their approach to pedagogy, or both.
4. The instruction changes foster increased student learning.

As a part of effective professional development, mentoring initiatives have been shown to enhance teachers’ pedagogy by preparing teachers to adopt new practices (Onchwari & Keengwe, 2008; Weaver, 2004). The value of mentoring has been noted to be related to enabling continual access to professional development activities (Tugel, 2004). The nature of mentoring that is ongoing and consistent also supports Desimone’s (2009) features of effective professional development as it features an extended duration, active learning, and provides coherence to an initiative. The one-on-one nature of mentoring facilitates collegiality (Dantoni, 2001) and means the relevance, context, and content focus can be tailored for each teacher.

Another value of mentoring is that it provides an effective means of addressing teachers’ potential resistance to change (Onchwari & Keengwe, 2008). This is because the core of teachers’ issues can be ascertained as the mentor can relate to the teacher’s experiences or challenges. Onchwari and Keengwe (2008) discuss how fellow teachers as mentors can increase teachers’ comfort to share concerns. This professional development process is indicative of the fostering of relationships through this type of mentoring. It is through these relationships that mentors can begin to shift teachers’ attitudes, making way for changes in practice.

3.6 Summary

This chapter has provided a review of effective practice recommendations in the field of mathematics and Indigenous education. The lens through which effective practice was
evaluated utilised cognitive load theories and knowledge of human cognitive architecture. When employing these theories, the goal of instruction is to lessen the load on working memory and facilitate learning, meaning that information has been transferred to long-term memory (Atkinson & Shiffrin, 1968). With respect to cognitive load theories, a central factor in analysing teaching approaches was the level of guidance provided by the teacher throughout instruction (Chen et al., 2016a).

As teachers have a significant impact on student achievement (Cornelius-White, 2007; Hattie, 2008; Hayes et al., 2006; Nye et al., 2004; Rosenthal & Rubin, 1978; Rowe, 2003; Sanders et al., 1997), pedagogical approaches in mathematics education were explored. Overall, empirical studies have found that approaches employing a component of teacher guidance, and mastery learning practices were effective in raising student achievement (Ausubel, 1964; Baker et al., 2002; Clarke & Clarke, 2006; Engelmann, 2007; Farkota, 2005; Flores & Kayler, 2007; Grossen & Ewing, 1994; Hattie, 2008; Kirschner et al., 2006, 2012; Klahr & Nigam, 2004; Kulik et al., 1990; Rowe, 2006; Westwood, 2008). Specific to Indigenous learners, the important role of motivation in facilitating student achievement was also highlighted in literature as well as the importance of inter-personal, oral learning.

In the evaluation of current educational initiatives addressing Indigenous students’ achievement in Australia, it was identified that the effectiveness of programs was varied. In some instances, further data was needed to evaluate conclusively the impact of many programs currently operating in Australia. This indicated that there exists a gap in current research and practice in implementing effective Indigenous education initiatives, and evaluating these initiatives. Overall, practices that were noted as effective or important in initiatives included a need for school and program coherence (Luke et al., 2013; Newmann, 1996), as well as explicit instruction elements, clear lesson structures, and a focus on mastery of mathematics concepts (GGSA, 2017; Jorgensen, 2015, 2018).

Number facts, computational skills, and problem solving were identified in Chapter 1 as critical foundational concepts that need addressing in future studies (ACARA, 2018b; Cumming & Elkins, 1999; Geary, 2004; Isaacs & Carroll, 1999; NCTM, 2016; Poncy et al., 2010; Thomson et al., 2012b; Woodward, 2006). Research indicated that number facts are taught most successfully through strategy discussion in conjunction with timed drills (Cumming & Elkins, 1999; Pegg & Graham, 2013; Woodward, 2006). For teaching algorithms, the CRA teaching sequence was indicated to be effective in supporting students’ proficiency
(Flores, 2009, 2010; Harris, Miller, & Mercer, 1995; Manel et al., 2012; Miller & Kaffar, 2011; Miller & Mercer, 1993; Morin & Miller, 1998). The importance of metacognitive skills and problem-solving heuristics in supporting students’ problem-solving proficiency was also found in research (Desoete et al., 2001; Hacker & Dunlosky, 2003).

In relation to developing and evaluating effective professional development that was implemented in this study, Desimone’s (2009) framework highlighted several key features of professional development. These elements were utilised in the methodological design of the study to increase the likelihood of up-take of recommended practices. Further, these elements will be drawn upon in the analysis of the effectiveness of the professional development in changing teaching practice as well as the consideration of the subsequent impact on student achievement.

In conclusion, many initiatives concerning number in the domain of Indigenous mathematics education currently are having little impact on student achievement. There exists a gap in literature for an initiative that involves the elements of effective practice as identified in this literature review. This gap in literature has led to the creation of the research questions for this study concerning improving students’ mathematics achievement in an Indigenous community school. The aim of this study is to implement a mathematics initiative to improve Indigenous students’ mathematical proficiency. Therefore, the study will evaluate how teachers implement the practices, how students’ mathematical proficiency changed as a result of the initiative and what factors were influential in changing students’ proficiency.

Research Question 1: How do teachers implement a mathematics initiative in an Indigenous community school?

Research Question 2a: What is primary students’ mathematics proficiency prior to a mathematics initiative in an Indigenous community school?

Research Question 2b: What is primary students’ mathematics proficiency after a mathematics initiative in an Indigenous community school?

Research question 3: What factors related to the implemented initiative influenced primary students’ mathematics proficiency in an Indigenous community school?
4 METHODOLOGY

4.1 Overview

This chapter provides an overview of the methodology used in this study. Section 4.2 outlines the research design for the study. Following this, Section 4.3 details the context of the study, Section 4.4 describes the role of the researcher, and Section 4.5 describes the participants in the study. The implementation of the initiative is outlined in Section 4.6, followed by description of the research process in Section 4.7. Details and justification of the data sources used in the study, in addition to the data collection procedures are outlined in Section 4.8. Following this, the data analysis and reporting processes are described (Section 4.9). A consideration of the measures taken to address validity throughout the study (Section 4.10), and ethical considerations of the research (Section 4.11) are presented, and the chapter concluded with a summary of the methodology (Section 4.12).

4.2 Research design

A mixed method research design was chosen as the methodological approach for this study, aligning with pragmatist philosophies. Pragmatism is typified by the research question playing a central role in methodological and design considerations thus moving beyond philosophical issues (in mixed methods research) concerning the incompatibility thesis (Tashakkori & Teddlie, 2010). That is, the research question is more important than methods or the researcher's worldview underlying method selection (Biesta, 2010). Pragmatism operates under the tenet of selecting the philosophical or methodological approach that is best suited to the studied research problem (Howe, 1988). Greene (2007) has termed this the substantive theory stance. Pragmatism involves both an objective and subjective point of view. Viewing these two epistemological stances as lying on a continuum, pragmatism accepts movement between either stance as appropriate (Tashakkori & Teddlie, 1998). That is, at times it is easy for the researcher and subjects to stand apart, and other times it is crucial for them to be interactive. This dialectic stance is useful due to the complexity of human phenomena, and the
use of multiple perspectives provides a better understanding of these complexities (Greene & Hall, 2010). This means the worldview adopted in the design of this study was problem-centred and pluralistic, as well as real-world practice oriented (Creswell, 2012).

Therefore, tenets of pragmatism such as the methodological eclecticism and integration of both quantitative and qualitative techniques requiring paradigm pluralism (Johnson, Onwuegbuzie & Turner, 2007; Onwuegbuzie & Leech, 2005; Tashakkori & Teddlie, 2010) are drawn upon in the methodological approach in this study. As the centrality of the research questions is a key characteristic of mixed methods research, the nature of the research questions guided the selection of data collection methods (Cohen, Manion, & Morrison, 2011a; Onwuegbuzie & Leech, 2005; Tashakkori & Teddlie, 2010).

Mixed methods research involves the collection and interpretation of both quantitative and qualitative data in a single study, providing breadth and depth of data to explain the complex picture of a social phenomenon (Creswell, 2012; Creswell & Plano Clark, 2018; Cohen, Manion, & Morrison, 2011b; Denscombe, 2008; Gay, Mills & Airasian, 2006; Greene & Caracelli, 2003; Johnson et al., 2007; Sieber, 1973). This methodological approach was selected because applying quantitative or qualitative methods alone would not have been sufficient to capture and explain factors associated with the teaching and learning of mathematics in the school, nor effectively answer the research questions. The mixing of methods has been proposed as a way of conducting research that supports understanding of Indigenous ways of knowing, thus transcending what qualitative methods alone can provide (Botha, 2011).

This study followed an explanatory mixed methods design (Creswell & Plano Clark, 2018) with quantitative data concerning students’ proficiency being collected first, followed by qualitative observations and interviews to explain the initial quantitative findings. Similarly, quantitative data concerning students’ proficiency at the end of the initiative was connected to, and explained by, the qualitative observations and interviews carried out throughout the initiative. Next, data from the three stages of the study were analysed, and factors influencing students’ proficiency identified. The sequence of data collection is outlined in Figure 6.
4.3 Context

The sample school in this study was a community run school for Indigenous students in southeast Queensland. The school caters for Prep-12 students, and had a population of approximately 200 students with 93% identifying as Indigenous (ACARA, 2016). Following Government guidelines, all classes were taught in English and there was no policy requirement to teach mathematics in Indigenous dialects.

Being community run, this school presented a unique educational opportunity for Indigenous students. The ethos of the school was to provide a culturally-safe learning environment that had the dual purpose of both valuing and promoting Indigenous identity, and expecting high educational standards. The seminal work of Bin-Sallik (2003) defined a culturally-safe environment as one that facilitates an emotionally and physically safe environment in which there is shared respect and no denial of identity. Indigenous ownership for the school meant that there was an understanding of students’ cultural experiences, with the expectation that teaching could build from these experiences, making instruction culturally-responsive and culturally-inclusive. These experiences included utilising familiar contexts in teaching, increasing motivation by understanding the potential value of learning in students’ lives, and tailoring teaching by understanding the degree to which students are familiar with particular concepts and content (Enyedy & Mukhopadhyay, 2007).
ownership by Indigenous communities has been identified as a key factor in improving schooling outcomes (Hunter & Schwab, 2003; Lowe, 2011). In addition, students benefited by not facing many of the issues presented to some Indigenous people in traditional Western schools, including (but not limited to) cultural conflict, cross-cultural miscommunication, and racism (Hunter & Schwab, 2003).

Historically, in the school, primary-aged students had performed, on average, consistently below the achievement standards stipulated in the Australian Curriculum: Mathematics F-10 (2018b) for their respective year levels. This meant modified mathematics programs were implemented routinely in the primary school and in the junior secondary classes. Consequently, this presented barriers for students when entering senior studies in Year 11 and 12 as they were limited to studying prevocational mathematics. Prevocational mathematics is a strand of mathematics tailored to vocational applications, which does not contribute to entrance to tertiary studies. Generally, prevocational mathematics is undertaken by students who have not experienced success in junior secondary mathematics. This pathway acts to limit the opportunities of students to engage in more advanced studies (e.g. tertiary studies in science, and technology fields). Though the school historically performed above schools with statistically similar backgrounds (similar socio-educational backgrounds such as family education history, school location, and proportions of Indigenous students) in national testing, differences existed between the school’s results and national averages, with the school performing substantially below these averages from 2008-2015 for Year 5 numeracy (ACARA, 2016). The school’s performance in the Year 5 numeracy NAPLAN test from 2008-2015 is detailed in Figure 7 and Figure 8.
Figure 7. Year 5 NAPLAN numeracy results - comparison of school (green, diamond shape) and similar schools (yellow, circle shape). Results are reported as “substantially above” the comparison schools in 2008, 2010, 2012, and 2013 and “above” in 2009, and 2011.

Figure 8. Year 5, NAPLAN numeracy results - comparison of school (red, diamond shape) to all other schools (purple, square shape). Results are reported as “substantially below” in all reporting years.

As stated previously, these comparisons indicate that students in the sample school consistently have performed above statistically similar schools (Figure 7), however, the sample school has performed well below Australian averages (Figure 8). In Figure 7, it can also be noted that the sample school reported a larger spread of scores than statistically similar schools, indicating large differences in students’ achievement within the same year level. The data from NAPLAN confirms the wide spread of student abilities within the school which was below what is expected for the given year levels.

The impact of students’ achievement in mathematics across the school had an influence on the mathematics programs that were enacted in primary classes prior to the initiative. For
example, due many students’ having a perceived or recorded achievement standard below the expected national standards as outlined in the Australian Curriculum: Mathematics (ACARA, 2018b), teaching staff had identified that proficiency with core mathematical skills such as place value and number concepts were an urgent priority. This meant that problem-solving was not typically emphasised in primary mathematics classes. The previous foci of mathematics programs prior to the initiative is important in establishing the context within the school at the start of the initiative.

4.4 The role of the researcher

As the researcher does not play a neutral role in the collection and interpretation of data, or the delivery of the initiative, it is important to consider the researcher’s perspectives and experiences relevant to the study. The researcher is a mathematics teacher who had a long history of working with individual or small groups of students to diagnose and remediate students’ mathematical difficulties. This history and experience with error diagnosis and remediation came from both the researcher’s previous studies involving analysis of mathematical errors in high school mathematics, and five years of working in the sample school as a classroom teacher. This means that the researcher brought previous experience to the study regarding why students make particular errors as well as the thinking that manifested these errors resulting from considerable experience in remediation work with students.

The researcher’s role was highly interactive with teachers throughout the initiative. The initiative was guided by the researcher by supporting classroom teachers in the planning and proposed delivery of their mathematics program. The researcher also had a significant role in developing and delivering professional development sessions to the teachers involved in the study. Mathematics lessons in the school were implemented by a classroom teacher, supported by a teaching assistant. Changes to normal classroom routines were minimised when collecting testing data, field notes, or conducting student interviews.

The researcher’s role was also highly interactive with students throughout the initiative. The researcher’s experience working in the school meant that the researcher had developed relationships with students participating in the study. That is, at some point during this time, the researcher had taught all students involved. Having an established relationship with students had positive implications for students’ willingness to engage in testing with the researcher throughout the initiative. In studies with Indigenous students from Papua New
Guinea, Clarkson (1983) noted that students’ “attitudes” were important as “if a student does not like you, he/she will not work so as to punish you” (p. 356). Given the experience at the school, the researcher also concurs with such a statement for students in the context of this study. That is, if students do not like you, they will avoid you in an active and physical manner, or refuse to do work with you. This might manifest as students putting their head down on the desk to ignore you, leaving the room, refusing to go with you to do work, or staring off into the distance while not completing the work. However, disliking the person working them is not always the only cause for such behaviours (i.e., such responses could also be elicited when work was perceived as “too hard”). Therefore, the researcher worked to avoid such compounding influences related to students’ seeking to avoid working with researchers by establishing positive relationships with the students.

4.4.1 Professional development

Professional development was designed to include the core features of effective professional development identified by Desimone (2009). These included a content focus, active learning, coherence, duration, and collective participation. There were two types of professional development that occurred over the course of the initiative. The first type of professional development sessions were collegial meetings facilitated by the researcher. These were held on a weekly basis in after-school hours over the course of the initiative. During these meetings, teachers broke into small focus groups and spent time collaboratively developing the mathematics programs for their classes.

The second type of professional development was structured sessions delivered by the researcher. There were two structured professional development sessions run over the course of the initiative and the timeline for these sessions is outlined in Table 2. The purpose of these sessions was to facilitate the implementation of the initiative successfully.
Table 2
Timeline of professional development activities.

<table>
<thead>
<tr>
<th>School Term</th>
<th>Researchers interaction with teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Professional development session 1 (after school): January: Initial meeting to discuss effective practice recommendations. All teachers in attendance (Helen, Diane, Jane, and Paul).</td>
</tr>
<tr>
<td>2</td>
<td>Nil</td>
</tr>
<tr>
<td>3</td>
<td>Professional development session 2 (during school time): Mid-term (May): Focus on reviewing the diagnostic data, recommendations from the data, problem-solving data. All teachers excluding Year 2/3 teacher (Helen) in attendance. Professional development session 3 (during school time): End of term (June): Further information on study purpose, and underlying theories for the effective practice recommendations from literature. All teachers excluding Year 2/3 teacher (Helen) in attendance.</td>
</tr>
</tbody>
</table>

The first of these structured sessions focused on presenting teachers with effective practice recommendations stemming from the review of literature. This session involved the researcher and the teachers meeting to discuss the summary of literature findings at the beginning of the school year. A focus on content relating to relevant pedagogical practices in mathematics education for teachers in this school was employed as it was identified as a critical feature of effective professional development (Desimone, 2009). This session was conducted prior to commencing the initiative and majority of the first school term was considered an introduction to the initiative as data was not formally collected. However, during the first school term, data from the administration of the pre-initiative PAT-M in order to provide a baseline understanding of students’ proficiency as measured by this test item was collected.

Following this, the second professional development session was run approximately half-way through the 7-month initiative during school hours, with the school facilitating the release of teachers from classes to participate in the sessions. This session addressed the initial findings from the pre-initiative diagnostic and problem-solving tests, as well as recommendations from these findings. A month later a third professional development session was conducted. The intention of this session was to build on discussions from the second session, and also to increase and advance teachers’ understanding of the literature recommendations that had been provided throughout the initiative by outlining the underlying theories in greater detail. For example, the purpose of the research study, human cognitive architecture, cognitive load theories, and constructivist theories were discussed within the
context of effective mathematics education. The aim of this session was to increase teachers’ understanding of underlying theories, which highlighted the relevance and importance of the literature recommendations with the intention of maximising the chances that particular practices might be adopted as a result.

4.5 Sample and participants

In this study, the sample was four classes comprising Grade 2/3 (n=12), Grade 3/4 (n=12), Grade 4/5 (n=12) and Grade 5/6 (n=14). The population was approximately 76 students with a final sample size of 50 students. The number of students in each class, and the distribution by gender and year level is outlined in Table 3.

Table 3
Number of students in each grade and numbers of female and male students in each class

<table>
<thead>
<tr>
<th>Year</th>
<th>Total n</th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2/3</td>
<td>12</td>
<td>n (Year 2) = 6</td>
<td>n (Year 3) = 6</td>
</tr>
<tr>
<td>Year 3/4</td>
<td>12</td>
<td>n (Year 3) = 4</td>
<td>n (Year 4) = 8</td>
</tr>
<tr>
<td>Year 4/5</td>
<td>12</td>
<td>n (Year 4) = 3</td>
<td>n (Year 5) = 9</td>
</tr>
<tr>
<td>Year 5/6</td>
<td>14</td>
<td>n (Year 5) = 6</td>
<td>n (Year 6) = 8</td>
</tr>
</tbody>
</table>

The total population of students across the four classes can only be estimated due to students commencing and leaving the school throughout the initiative. The final sample is smaller than the total population of all classes due to students leaving the school throughout the initiative resulting in incomplete data for some students. As well, movement of students between classes meant that achievement could not be tracked and attributed to the teaching and learning occurring in a single classroom. Therefore, the final sample comprised the students who had stayed in the same class throughout the intervention (7-months).

The school utilised multiple composite classes in the primary years to balance student numbers, and also to provide opportunities for students who have not met educational goals to remain in a lower grade or for those that are excelling to be further advanced in a higher grade. The use of composite classes allows for greater flexibility in grouping students in terms of ability, allowing instruction to be targeted more appropriately to students’ abilities. Effectively, composite classes attempted to address the highly varied achievement of students as evidenced by the NAPLAN data discussed earlier.
Students from Year 2 to Year 6 were selected for the initiative as they represented the formative years in which computations (addition, subtraction, multiplication, and division) and number facts (particularly multiplication facts) are introduced, and consolidated according to both national and international guidelines and syllabus documents (e.g., Australian Curriculum: Mathematics F-10 [ACARA, 2018b], Common Core State Standards [NCTM, 2017], and the Hong Kong mathematics curriculum [The Government of Hong Kong, 2007]). Foundation and Year 1 classes were not included in this study as typically symbolic computation procedures (such as algorithms) are not formally covered in these years. As well, secondary classes (Years 7-12) were excluded because the curriculum moves beyond basic number to focus on algebra and other advanced mathematical concepts.

Four classroom teachers were involved in this study. The demographics of the teachers were varied, and outlined in Table 4. In Chapter 5, Section 5.3, further contextual details concerning teacher demographics are outlined in the discussion of the enactment of the initiative by each teacher.

<table>
<thead>
<tr>
<th>Table 4 Teacher demographics</th>
<th>Age (years)</th>
<th>Total teaching experience (years)</th>
<th>Teaching experience at sample school (years)</th>
<th>Other information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helen (Year 2/3)</td>
<td>50-60</td>
<td>12</td>
<td>0 (new to school)</td>
<td>-</td>
</tr>
<tr>
<td>Diane (Year 3/4)</td>
<td>40-50</td>
<td>15</td>
<td>12</td>
<td>Identified as Aboriginal &amp;/or Torres Strait Islander</td>
</tr>
<tr>
<td>Jane (Year 4/5)</td>
<td>50-60</td>
<td>15</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Paul (Year 5/6)</td>
<td>30-40</td>
<td>14</td>
<td>14</td>
<td>Identified as Aboriginal</td>
</tr>
</tbody>
</table>

Most notable in relation to the teacher demographics was the duration of experience at the sample school. Two of the teachers (Helen and Jane) had two years or less experience in the sample school, compared to Diane and Paul who each had over ten years of experience in the sample school. Despite these variations, the overall teaching experience was relatively similar across the sample, ranging from 12 years to 15 years, indicating that all teachers were considered to be experienced. Each classroom teacher had a teacher aide to assist on a daily basis. Consistent with Australian qualification standards, teacher aides possessed a variety of
qualifications but did not hold a Bachelor or Master’s degree, or associated Diploma in teaching (excluding the teacher aide in Year 2/3 who was a special education teacher).

4.6 The initiative

The mathematics initiative in this study was conducted over a seven-month period from March to October of the school year (in Australia, the school year typically commences in late January and concludes in mid-December). Guided by current literature, reflective practice and teacher collaboration in the reflexive design and implementation of the teaching program throughout the initiative was deemed important. In the planning of the initiative, the intention was for the mathematics programs to focus centrally on structuring appropriate opportunities in order to develop foundation mathematics concepts thoughtfully in each class. The components of the initiative practice recommendations are summarised in Table 5.

<table>
<thead>
<tr>
<th>Initiative elements</th>
<th>Components of effective mathematics education from literature</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Development and implementation of the initiative</td>
<td>A shared, school-wide vision and approach for the program.</td>
<td>(Jorgensen, 2018)</td>
</tr>
<tr>
<td></td>
<td>Focus of the initiative mathematics programs:</td>
<td>(Boaler &amp; Staples, 2008)</td>
</tr>
<tr>
<td></td>
<td>- Structured program to provide appropriate opportunities to thoughtfully develop foundation mathematical concepts.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High levels of teacher collaboration concerning:</td>
<td>(Boaler &amp; Staples, 2008)</td>
</tr>
<tr>
<td></td>
<td>- Designing mathematics programs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Planning teaching practices</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Reflecting on teaching practices</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sustained involvement of the researcher to act as a mentor/expert mathematics teacher (i.e., a numeracy coach).</td>
<td>(Jorgensen, 2018)</td>
</tr>
<tr>
<td></td>
<td>Utilisation of the elements of effective professional development:</td>
<td>(Desimone, 2009)</td>
</tr>
<tr>
<td></td>
<td>- Content focus</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Active learning</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Coherence</td>
<td></td>
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<td></td>
<td>- Duration</td>
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<td></td>
<td>- Collective participation</td>
<td></td>
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<tr>
<td>Approach to developing mathematics units</td>
<td>Utilisation of a mastery approach:</td>
<td>(Archer &amp; Hughes, 2011; Engelmann, 2007; Kulik, Kulik, &amp; Bangert-Drowns, 1990)</td>
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<tr>
<td>-----------------------------------------</td>
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<td>--------------------------------------------------------------------------</td>
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<tr>
<td></td>
<td>- Small units of sequenced work</td>
<td>(Engelmann, 2007; Kulik, Kulik, &amp; Bangert-Drowns, 1990; Pegg &amp; Graham, 2013)</td>
</tr>
<tr>
<td></td>
<td>- Pretesting followed by instruction</td>
<td>(Archer &amp; Hughes, 2011; Engelmann, 2007; Jorgensen, 2015, 2018; Kulik, Kulik, &amp; Bangert-Drowns, 1990)</td>
</tr>
<tr>
<td></td>
<td>- Differentiation of learning for individual students</td>
<td>(Archer &amp; Hughes, 2011; Engelmann, 2007; Jorgensen, 2015, 2018; Kulik, Kulik, &amp; Bangert-Drowns, 1990)</td>
</tr>
<tr>
<td></td>
<td>- Not bound by timeframes – work continues until mastery of objectives is obtained. This may involve repetition of concepts over several lessons.</td>
<td>(Engelmann, 2007; Kulik, Kulik, &amp; Bangert-Drowns, 1990)</td>
</tr>
<tr>
<td></td>
<td>High mathematical expectations</td>
<td>(Jorgensen, 2015, 2018)</td>
</tr>
<tr>
<td></td>
<td>Focus on number (place value and operations) as a priority</td>
<td>(Jorgensen, 2015, 2018)</td>
</tr>
<tr>
<td>Lesson elements</td>
<td>A consistent and transparent lesson structure.</td>
<td>(Jorgensen, 2018; Pegg &amp; Graham, 2013)</td>
</tr>
<tr>
<td></td>
<td>Clear lesson goals and success criteria</td>
<td>(Hattie, 2008; Jorgensen, 2015, 2018)</td>
</tr>
<tr>
<td>Lessons should include:</td>
<td>- Consolidation and revision of concepts to develop fluency (fast paced), including mental computation exercises</td>
<td>(Archer &amp; Hughes, 2011; Good &amp; Grouws, 1979; Jorgensen, 2018)</td>
</tr>
<tr>
<td></td>
<td>- Explicit teaching of new mathematics concepts: focusing on developing meaning and student understanding through explanation, demonstration, illustrations etc. This phase also involves student input and checking for understanding.</td>
<td>(Archer &amp; Hughes, 2011; Baker, Gersten &amp; Lee, 2002; Good &amp; Grouws, 1979; Hattie, 2008; Jorgensen, 2018; Pegg &amp; Graham, 2013)</td>
</tr>
<tr>
<td></td>
<td>- Individual, independent practice</td>
<td>(Archer &amp; Hughes, 2011; Hattie, 2008; Good &amp; Grouws, 1979; Pegg &amp; Graham, 2013)</td>
</tr>
<tr>
<td></td>
<td>Providing feedback data to students</td>
<td>(Archer &amp; Hughes, 2011; Baker, Gersten &amp; Lee, 2002)</td>
</tr>
<tr>
<td>Approach to teaching number facts</td>
<td>Strategy discussion in conjunction with timed drills.</td>
<td>(Cumming &amp; Elkins, 1999; Pegg &amp; Graham, 2013)</td>
</tr>
</tbody>
</table>
From the synthesis of literature, it was recommended that math fact fluency should be taught using a combination of strategy discussion and timed practice of learned facts. The recommended practices also outlined that algorithms should be taught first by using concrete materials, followed by representations of materials, and then abstract (symbolic) models (a concrete, representational, abstract [CRA] teaching sequence). The intent of these practices and strategies was to focus on developing students’ conceptual understanding of the procedures being carried out. In addition, to improve fluency with, and understanding of, the four operations, it was recommended that teachers use a consistent approach to the teaching of the four operations by using the same standard algorithm and model of language across all grades when teaching addition, subtraction, multiplication, and division. An example of the recommended traditional algorithm with consistent place value language is outlined in Figure 9 below.

![Figure 9: Example of vertical addition algorithm with place value language.](image-url)
Problem solving recommendations included the integration of problem-solving heuristics proposed by Polya (1988) in classroom teaching to facilitate metacognition. This included the recommendation of using a classroom poster outlined in Figure 10 to support problem-solving teaching and learning.

![Figure 10: Classroom problem-solving poster utilising Polya's (1988) problem-solving heuristics](image)

Compared to the specificity of practices surrounding number facts and computations, recommended practices surrounding problem-solving were substantially less prescriptive. This decision was made in response to the school context prior to the initiative as problem-solving was not a key focus in the primary mathematics programs prior to the initiative. This meant that the implementation of any practices surrounding problem-solving were relatively new for most teachers, therefore the recommended practices were not as highly detailed to allow for problem-solving programs to develop in a responsive, authentic, and needs-based manner throughout the initiative.

From a consideration of literature concerning models of effective mathematics programs, it was recommended that lessons include teacher-guided components as well as individual, guided practice. Furthermore, pre-testing and weekly class testing throughout the term was recommended to inform the guided practice components of lessons. These practices align with a mastery learning model of teaching.

In each of the classes, the implemented curriculum was the Australian Curriculum F-10: Mathematics (ACARA, 2018b). Each class planned to focus on curriculum topics covering concepts in the lowest year of the composite class with the inclusion of concepts from the highest year level also included. It was intended that the content focus of mathematics units and lessons focused substantially on number facts, computational proficiency, and problem-solving. The envisioned structure of lessons delivering this curriculum for the teaching period
of the initiative in each class was ascertained through individual meetings with each classroom teacher and the researcher.

4.7 Research process

Due to the pragmatic approach of this study, the data sources and analysis methods employed were driven by the three research questions. Figure 11 outlines how the selected data sources acted to answer the research questions.

This study firstly ascertained the current state of students’ mathematical proficiency, and concluded by confirming whether the initiative had acted to change students’ proficiency, which addressed the research question:

Research Question 2a: What is primary students’ mathematics proficiency prior to a mathematics initiative in an Indigenous community school?

Research Question 2b: What is primary students’ mathematics proficiency after a mathematics initiative in an Indigenous community school?

The data sources that were used to assess students’ mathematical proficiency included the Progressive Achievement Test – Mathematics (PAT-M), and classroom diagnostic tests.
The PAT-M tests were multiple choice, written tests that were progressive in terms of difficulty and are aligned with the Australian Curriculum: Mathematics F-10. The test items covered the six mathematics strands: Number, Algebra, Geometry, Measurement, Statistics, and Probability. The purpose of these tests was to provide a standardised measure of achievement for all aspects of mathematics defined by the curriculum. These tests are scored using scaled measures to enable tests for different year levels to be compared. The possible impact of the initiative on students’ mathematics proficiency in all areas of mathematics was ascertained from these tests. Whether gains in students’ achievement were above what would be expected from ‘normal’ (non-intervention) schooling time was determined through the calculation of effect sizes. Effect sizes have been considered meaningful measures of educational gain, and have been used in similar initiatives such as QuickSmart (Hattie, 2008; Pegg & Graham, 2013).

In addition to the PAT-M test, a set of classroom diagnostic tests was used to probe students’ achievement, procedural fluency, and conceptual understanding of specific mathematical concepts such as place value, addition, subtraction, multiplication, and division. The diagnostic tests were from a set of classroom screening tests developed by Booker (2011). The use of diagnostic tests served several purposes in this study. Firstly, the test contributed to answering research question 2a and 2b as it provided quantitative measures of achievement for the concepts tested. Secondly, diagnostic tests allowed for identification of students’ errors and computation strategies therefore enabling a detailed analysis of why students were or were not successful with the tested concepts. This information contributed to answering the third research question by identifying factors related to the initiative that influenced students’ mathematics proficiency. Thirdly, the diagnostic tests informed the initiative by allowing a diagnostic teaching cycle to be employed over the course of the study. A diagnostic teaching cycle involves identification of students’ capabilities (current achievement), hypothesising potential reasons for students’ difficulties, formulating objectives to structure remediation of difficulties, and employing corrective remedial procedures in a cycle of ongoing evaluation (see Glennon, 1963; Mager & Peatt, 1962, Popham & Baker, 1970; Reisman, 1977; Reisman, 1982). Initial data analysis of diagnostic test findings was disseminated to teachers in professional development sessions partway through the initiative allowing for the teachers to consider teaching practices in light of the diagnostic data. Finally, the diagnostic tests were used to initiate Newman interviews with students. These interviews probed students’ problem-solving processes to identify sources of difficulty.
Problem-solving interviews have been used in other studies to probe Indigenous students’ problem-solving approaches, and to identify and classify errors (Clarkson, 1983, 1991). The value of task-based, one-on-one interviews has been noted in research to give better insight into Indigenous students’ knowledge comparative to written forms (Grootenboer & Sullivan, 2013). Leder and Forgasz (2006) also note the need for research to obtain rich data beyond performance tests to ensure that the reflected behaviours adequately reflect what happens in a classroom setting. Newman interviews provide a mechanism of providing such data.

Following an explanatory sequential design, classroom observations and teacher interviews were carried out to elaborate on, and refine the quantitative results to address the following research question.

Research Question 1: How do teachers implement a mathematics initiative in an Indigenous community school?

Classroom observations were collected in the form of field notes comprising detailed diary entries from the researcher. This data provided a rich description of the initiative and allowed for the emergence of unexpected observations and findings impacting on students’ mathematics proficiency. Teacher interviews at the beginning of the initiative ascertained teachers’ envisioned practices enabling the teacher’s intentions to be cross-checked with the enacted practices at a later time. Teacher interviews at the end of the initiative were used as a form of member checking to confirm that the researchers’ classroom observations aligned with what the teachers considered their practices to be.

4.8 Data sources and data collection procedures

In the following section, the data sources used in this study will be described in detail. A variety of data sources were used including test instruments, interviews, and observations. In the process of selecting data sources related to tests of mathematical proficiency, equity issues for Indigenous students regarding assessment, particularly standardised assessments (as discussed in Stobart, 2005) were considered. To guide the selection of data sources, Klenowski (2009) discusses how alternative assessments give useful insight when striving to gain a deeper understanding of students’ processes when solving complex problems. To attempt to address
these considerations in this study, the use of multiple data sources (assessments) served as alternatives to standardised tests alone.

4.8.1 Standardised mathematics tests

In order to ascertain students’ mathematical proficiency and the impact of the initiative on students’ proficiency, standardised mathematics tests such as PAT-M were utilised. The set of PAT-M tests provided a measure of students’ skills in, and understanding of, school mathematics (Stephanou & Lindsey, 2013). The PAT-M is a widely used achievement test in Australia and New Zealand developed and published by the Australian Council for Educational Research and the New Zealand Council for Educational Research. These tests have been designed for three specific uses: (1) to provide a snapshot of current achievement in mathematics, (2) to monitor student improvement over time, and (3) to assist in setting realistic goals and planning effective work programs (Stephanou & Lindsey, 2013). The intended uses of these tests aligned with this study, which aimed to assess students’ current mathematics proficiency, monitor the impact of an initiative over time, and to inform the development of the initiative. There are 10 PAT-M tests, one for each school year (Years 1 to 10), making the assessment pieces progressive in difficulty and appropriate to concepts covered in the relevant school years. Each test consists of 30-40 multiple choice items with at least five items for each of the mathematics strands (number, algebra, geometry, measurement, statistics, and probability). An example of the test items for the Year 1 assessment is provided in Appendix A. Student achievement in PAT-M tests is measured quantitatively via test scores.

The procedure for the PAT-M tests in the study involved the researcher administering the test to small groups of 2-5 students in a quiet, small group working space in the school (e.g., separate small room at the back of a classroom). The students were asked to complete the set questions to the best of their ability. According to test instructions, all tests require 40 minutes of testing time and there is an expectation that they are completed without calculators. As data-driven teaching had been identified previously as a quality teaching practice (Jorgensen, 2015), the results of the PAT-M tests were provided to classroom teachers at the beginning of the initiative.

To provide a further comparison on changes to students’ mathematical proficiency as measured by standardised mathematics tests, whole school findings from the National
Assessment Program - Literacy and Numeracy (NAPLAN) were evaluated as well. NAPLAN is an Australian annual, national assessment for students in Years 3, 5, 7 and 9. This standardised test provides a measure of students’ achievement compared to identified standards. The NAPLAN numeracy test measures students’ achievement in numeracy for the relevant year level (National Assessment Program, 2016).

4.8.2 Mathematics diagnostic tests

In addition to a standardised mathematics test, a set of diagnostic tests was used throughout this initiative to address the second research question. Diagnostic testing has long been recognised as an important component of teaching in mathematics, forming a cyclical approach to teaching and learning (Ashlock, 1976). The importance of including a diagnostic tool is that, in addition to determining achievement levels (e.g., what students know), it is valuable to also determine students’ understanding of concepts (e.g., how students know), and diagnostic assessment fulfils this role (Booker, 2011; Brown & Burton, 1978). Using a standardised test such as PAT-M alone would not provide the type of detailed insight into students’ understanding of specific concepts. In addition, PAT-M questions involved multiple choice formats, which does not allow for the probing of students’ understanding.

The diagnostic tests used in this study were from the Booker diagnostic tests (Booker, 2011). The Booker tests are designed for use in the classroom to diagnose error patterns in the domains of number, addition, subtraction, multiplication, and division. There are five individual test papers assessing each of these domains. Each test consists of two forms, an “A” and “B” test to allow for pre- and post-testing on the same mathematical concepts. Findings from these tests assisted in answering the second research question by ascertaining students’ current proficiency with tested concepts, and tracking students’ progress in tested areas over the course of the initiative.

The first Booker diagnostic test, numeration, examines students’ ability to identify and sequence numbers, understand place value up to hundred-thousands, and round numbers. Overall, this test assessed students’ understanding of basic number, including place value. The next four tests (addition, subtraction, multiplication, and division) begin by testing students’ ability to recall facts associated with each of the four operations. The tests then proceed, progressively, to assess students’ ability to carry out computations for each of the operations,
which allows for clear identification of students’ difficulties. For example, in the addition test, students’ ability to add two-digit numbers with and without renaming is tested first. Following these questions, two more questions test the same concepts with three-digit numbers. The distinction between these four questions determines whether a student understands two-digit addition, can rename, and whether they can rename for three-digit values as well as two-digit values. Questions progress in difficulty up to adding five-digit values. Problem-solving for each operation is tested similarly in a progressive manner, probing different elements of problem-solving questions (such as added information or questions where interpretation is required). Problem-solving questions are mixed with algorithm questions in these tests to prevent fatigue. An overview of the test items for each diagnostic test is detailed in Appendix B.

Overall, the sequencing and level of student working required in the diagnostic tests enabled rich data to be obtained. It is for this reason that the Booker tests were included in the study, to guide the initiative and explain the results of the standardised PAT-M tests by identifying specific difficulties with particular fundamental number concepts through error analysis. This error analysis assisted in answering the third research question as it allowed for identification of students’ root mathematical difficulties, and confirmed how teaching practices may have increased or decreased the prevalence of these difficulties.

The tests were completed in small groups (2-5 students) in a quiet small group working space at the school. In total, the diagnostic tests typically take approximately 60 minutes to complete, however, students were allowed to continue until they had completed testing. As well, students were permitted to return for multiple sessions to complete the test, either during a single day or across sequential days to help prevent fatigue influencing results. This decision was made reflexively during the administration of these tests due to student need; it was found that many students were unable to attempt all five diagnostic tests in a single sitting. Facilitated by the researcher, the students were informed to complete the set questions to the best of their ability, and show all working. For this test, students were not permitted to have questions read to them as a component of later interviews was to identify if reading or comprehension difficulties were the cause of problem-solving difficulties. These interviews determined whether problem solving difficulties were rooted in literacy difficulties, or mathematical difficulties. The results and initial analysis of the diagnostic tests were also disseminated to
classroom teachers via professional development throughout the study to inform teaching. This was part of the diagnostic teaching cycle that was implemented in the initiative.

4.8.3 Newman interviews

As described earlier, the impact of the initiative on students’ mathematics fluency and understanding was ascertained through pre- and post-testing instruments. In order to explain students’ results on problem-solving questions in the diagnostic tests further, Newman’s error analysis interviews were carried out. These interviews also informed the initiative by identifying specifically what was impeding students in problem-solving, thus potentially enabling teachers to target teaching accordingly. Repeating these interviews during, and at the conclusion of the initiative, contributed to answering the second research question as changes in students’ problem-solving proficiency were able to be tracked.

Newman’s error analysis interviews were conducted individually for all students in the study. Newman interviews enabled the identification of places where a student was making errors in a problem-solving question from a list of five steps (reading, comprehension, transformation, process skills, or encoding). Newman identified that a student must work effectively through the five stages detailed in Table 6 to complete a written mathematics problem successfully. These interviews built on standardised mathematics tests (PAT-M tests) and diagnostic tests conducted earlier in the study as they aided in identifying reasons behind students’ errors or difficulties.
Table 6
The five Newman questions (Adapted from the State of New South Wales Department of Education and Communities, 2011; White, 2005)

<table>
<thead>
<tr>
<th>Example interview question</th>
<th>Steps to problem solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Please read the question to me. If you don’t know a word, leave it out.</td>
<td>Reading (decoding)</td>
</tr>
<tr>
<td>“An error would be classified as READING if the child could not read a key word or symbol in the written problem to the extent that this prevented him/her from proceeding further along an appropriate problem-solving path.” (White, 2005, p. 17)</td>
<td></td>
</tr>
<tr>
<td>2. Tell me, what is the question asking you to do?</td>
<td>Comprehension</td>
</tr>
<tr>
<td>“The child had been able to read all the words in the question, but had not grasped the overall meaning of the words and, therefore, was unable to proceed further along an appropriate problem-solving path.” (White, 2005, p. 17)</td>
<td></td>
</tr>
<tr>
<td>3. Tell me how you are going to find the answer</td>
<td>Transformation</td>
</tr>
<tr>
<td>“The child had understood what the questions wanted him/her to find out but was unable to identify the operation, or sequence of operations, needed to solve the problem.” (White, 2005, p. 17)</td>
<td></td>
</tr>
<tr>
<td>4. Show me how you get your answer, and “talk aloud” as you do it, so that I can understand how you are thinking.</td>
<td>Process skills</td>
</tr>
<tr>
<td>“The child identified an appropriate operation, or sequence of operations, but did not know the procedures necessary to carry out these operations accurately.” (White, 2005, p. 17)</td>
<td></td>
</tr>
<tr>
<td>5. Now, write down your actual answer.</td>
<td>Encoding</td>
</tr>
<tr>
<td>“The child correctly worked out the solution to a problem, but could not express this solution in an acceptable written form.” (White, 2005, p. 17)</td>
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</table>

This testing was carried out at the beginning of the initiative after the PAT-M and diagnostic tests had been administered. The Newman interviews were re-conducted at the completion of the study to ascertain the overall progress made throughout the initiative. These interviews were conducted with individual students by the researcher in a quiet small group working area at the school. The interviews took between 10-15 minutes. The procedure for conducting a Newman interview involved making the student comfortable by informing them that the purpose of the interview was to help them with their mathematics (White, 2005). Then the student was provided with new copies of the first addition, subtraction, and multiplication problem-solving question that they answered incorrectly on the diagnostic tests. The student was then asked the Newman interview questions outlined in Table 6 in sequential order. If a
student had answered all of the problem-solving tasks correctly on the diagnostic test then the interviews were not conducted.

Student responses to each question were transcribed by the researcher during the interview and students’ written work during the interview was also collected. The purpose of the Newman interviews was to find the first breakdown point where the student answered incorrectly. However it was recommended by White (2005) to ask one question beyond the breakdown point because it may be that the student is unable to express their knowledge of a particular step clearly (Clements, 2004). If the student was able to answer the question correctly in the interview when it was previously answered incorrectly in the written test, the initial error in the written test was considered a careless error made by the student (White, 2005). As a result of the student’s responses, at the completion of the interview, the error classification was decided by observing the first point where students answered the Newman interview question incorrectly.

4.8.4 Classroom observations and teacher interviews

To answer the first research question and identify how teachers implemented the initiative, classroom observations were carried out throughout the initiative. Classroom observations focused on qualitatively recording 1) lesson structures, 2) pedagogical approach/es (particularly focusing on how computations, number facts, and problem-solving were taught), 3) frequency and type of teacher interaction with students, and 4) student time on task. These observations were recorded by the researcher in the form of extended diary entries. Throughout the initiative, approximately two mathematics lessons each week were observed in each class. Field notes were collected continually over the course of the initiative. One purpose of the field notes was to record the classroom teachers’ envisioned enactment of the initiative in the classroom, including the intended content, lesson structures, unit plans, teaching strategies, and modes of delivery for mathematics lessons. Throughout the initiative, observations of mathematics lessons were conducted by the researcher to document classroom teachers’ enacted practices. This allowed for the teaching practices used in the initiative to be described in detail and linked to student proficiency. Observations of the classes during mathematics lessons also allowed for identification of classroom factors and other relevant occurrences in students’ learning environments that may have impacted on teaching and learning. While most of the data sources used in this study relate to
student performance on mathematical tasks, it is important to be able to describe the teaching process and classroom dynamics that facilitated or hindered individual learning clearly, and field notes facilitated satisfying this aim.

Additional data which focused on how teachers implemented the initiative was collected in the form of informal, unstructured interviews with the teachers. These interviews were firstly conducted early in the initiative after the first professional development session, and were focused on ascertaining teachers envisioned practices. The interviews were then repeated at the end of the initiative to member check classroom observations relating to lesson structures and pedagogical approaches with teachers. Distinguishing between the envisioned and enacted practices was important in identifying practices that teachers wanted to, but did not, implement thus shedding light on the practicalities of using the recommended practices informed by literature.

4.9 Data analysis and reporting

Due to the range of qualitative and quantitative data being collected in this study, data sources were analysed and reported in several ways. The use of multiple data sources allowed for triangulation of findings in order to increase the validity of findings by ensuring they are consistent and dependable (Cohen, et al., 2011a; Creswell, 2012; Gay et al., 2006). The data sources, data analyses, and links to how the evidence from this process address each research question is outlined in Table 7. Details of analysis for each data source are discussed in the following sections.
### Table 7
Data sources and data analysis techniques to answer the research questions

<table>
<thead>
<tr>
<th>Data source</th>
<th>Data analysis</th>
<th>Research question</th>
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</table>
| Classroom observations & teacher interviews | - Case reports documenting teaching practices  
- Analysis of prevalence of teaching practices, reported as no, low, moderate, or high implementation  
- Member checking with teachers concerning teaching practices  
- Comparisons made between envisioned and enacted practices  
- Integration of findings in light of student data to isolate themes of factors influencing students’ proficiency by searching for patterns in findings. | Research Question 1: Identification of teaching practices.  
& Research Question 3: Teaching practices subsequently linked to changes in students’ mathematical proficiency. |
| Standardised mathematics test: PAT-M | - Mean scaled score for each class  
- Effect size for each class, comparative to the norming sample  
- Mean scaled score for each age-group | Research Question 2a/2b: Identification of how students’ mathematical proficiency changed throughout the initiative. |
| Diagnostic tests                     | - Mean achievement for each class for each test  
- Analysis of changes to individual student scores in each class  
- Analysis of changes to the frequency of errors for each class, and across the sample | Research Question 3: Identification of the barriers to students’ proficiency, and how this changed throughout the initiative.  
Changes to error types subsequently linked to teaching practices. |
| Newman interviews                    | - Analysis of changes to the frequency of difficulties (using Newman’s 5 stages) for each class |                                                                                  |

### 4.9.1 Standardised mathematics tests

The PAT-M instruments provide data concerning students’ proficiency with the tested mathematics concepts. The data from PAT-M provided a mechanism to document the starting proficiency of students, and to ascertain the change in students’ mathematics proficiency as a result of the initiative.

The results of the PAT-M were measured quantitatively on a numerical Rasch measurement scale. A Rasch measurement scale allows different tests to be measured on the same scale for comparison purposes. This has been implemented in the PAT-M tests by the test authors by assuming that the achievement of a student and the difficulty of an individual
test question can be captured and placed on the same scale (Andrich, 1988). Student achievement reflects the level of understanding demonstrated by the student, and the question difficulty reflects the level of understanding required to answer the individual question correctly. Each test question is assigned a numerical score on the Rasch scale based on its difficulty. As well, overall student achievement is assigned a value on the Rasch scale.

Over the course of the initiative, students completed two sittings of the PAT-M test and each student received an individual score on the Rasch scale for each sitting. The mean Rasch scores for each cohort were calculated, and changes in each cohort’s mean score from the beginning of the initiative to the end of the initiative were calculated. This provided a numerical measure of students’ growth in each class. The mean scores for each cohort were then compared to the national norming sample. As the norming sample data for the PAT-M contained the mean achievement for each year level for testing carried out in October, achievement gains over each 12-month period (from year to year) was averaged to give an expected achievement for each month of each school year. This gave a linear progression of expected achievement levels according to the norming sample data. This data from the norming sample is outlined in Figure 12.

In addition to comparing mean scores for each cohort, the mean scores for students of the same year level (age) were calculated within each composite class. For example, within Year 2/3, the mean score of the Year 3 students in this cohort was calculated for comparison

![Figure 12: PAT-M norming sample data](image-url)
with the Year 3 students in the Year 3/4 cohort. This analysis facilitated more dependable comparisons between each cohort due to the comparison of similar aged students.

To analyse each cohort’s progress over the course of the initiative, Cohen effect sizes \((d)\) were also calculated. Effect sizes were calculated using the formula in Figure 13.

\[
d = \frac{\bar{X}_e - \bar{X}_s}{s_{pooled}}
\]

\(d\) = Cohen’s \(d\) effect size  
\(\bar{X}\) = mean  
\(s\) = standard deviation  
Subscripts: \(s\) refers to the beginning of the initiative and \(e\) refers to the end of the initiative.

*Figure 13: Effect size calculations (Thalheimer & Cook, 2002)*

Effect sizes allowed for further comparison to be made between pre- and post-initiative achievement gains. Effect sizes are an empirical measure that answers the question “how does the effect of an initiative compare to a typical year of growth for a given target population of students?” (Hill, Bloom, Black, & Lipsey, 2008, p. 173). Other large-scale initiatives such as QuickSmart have utilised effect sizes to evaluate gains in student achievement (Pegg & Graham, 2013). Studies have found that for any student at any school, effect sizes of 0.20 typically are considered small, 0.50 considered medium, and 0.80 considered large (Cohen, 1988). However, the values for small, medium, and large effect sizes are not absolute (Cohen, 1988; Salkind, 2010) and, in analysis of major longitudinal studies (e.g., Progress in International Reading Literacy Study [PIRLS], TIMSS, NAPLAN), 0.40 was found to be the average effect size (growth) for a school year (Hattie, 2012). Therefore, an effect size above 0.40 can be classified as more-than-expected growth over a year, and lies in the zone of desired effects for effective teaching (Hattie, 2012). For more specific comparison purposes, the effect sizes for the norming sample were also calculated; this gave a measure of what type of effect sizes were typical for this test.

As well as providing a quantitative summary of overall student progress throughout the initiative, results from the PAT-M test were analysed within each tested mathematics domain (number, algebra, geometry, measurement, statistics, and probability). This was reported as the average percentage of questions answered correctly for each class in each of the six domains. This analysis provided information concerning how achievement in specific domains were influenced over the course of the initiative. This detailed analysis of PAT-M results revealed whether changes in students’ achievement occurred in specific mathematics areas, or across all
areas. This analysis was also provided to teachers in addition to individual student’s scores to aid in informing teaching by identifying particular areas of need.

4.9.2 Mathematics diagnostic tests

For the diagnostic tests, mean achievement for each domain (place value, addition, subtraction, multiplication, and division) was reported for each cohort, and gains in achievement throughout the initiative were calculated. In addition to analysing the mean improvement for each class, changes to individual students in each cohort were analysed for trends. For example, how the initiative impacted on students that reported lower levels of achievement (comparative to others in the class) at the beginning of the initiative was observed. Analysis of changes to individual student achievement within the class also included observing changes to the spread of scores within each class calculated as a standard deviation. This formed the major part of the quantitative data analysis related to the mathematics diagnostic tests.

As well as analysing quantitative achievement, an analysis of the types of student errors on each incorrectly answered question was also carried out by the researcher. Analysis of students’ scripts with respect to whole number algorithms has a long history (e.g. Ashlock, 1976; Booker, 2011; Burrows, 1976; Buswell & Judd, 1925; Radatz, 1979; Reisman, 1982). In this study, error analysis was a process of identifying students’ proficiency gap by classifying whether a particular error was caused by a lack of conceptual understanding of the topic, or whether it may have been caused by a mistake associated with fluent application of procedures (i.e., procedural fluency). Kilpatrick et al. (2001) propose that if students understand a method, meaning they possess conceptual understanding of the method, they are unlikely to forget it. Conceptual understanding supports retention. Therefore, if a method of solving a particular problem on the diagnostic test was incorrect, or misapplied, it was considered that a student lacked conceptual understanding of the associated concept. Other conceptual errors also relate to place value difficulties as conceptual understand of multi-digit arithmetic is exemplified by fluid and flexible understanding of place value manipulations (Hiebert & Wearne, 1996).

Earlier discussion on distinctions between conceptual understanding and procedural fluency identified that delineations between strands of proficiency are difficult due to their interrelations. In many ways, conceptual understanding facilitates procedural fluency in a
cyclical manner. Further, correct application of a procedure to solve an operation does not always imply that there was conceptual understanding. The intent of the error analysis carried out in this study was not to speculate on students’ thinking, but to draw from literature and the researcher’s experience to suggest in a reasonable manner what errors may have contributed to the incorrect answers. The small group administration of the test also meant that the researcher could observe the process of students attempts, which also shed light on the error analysis. This was facilitated by the researcher’s recording of testing observations throughout the initiative, which also included the recording of any pertinent considerations, such as student behaviour during testing, which may have impacted on students’ attempts. A detailed example of the error analysis process is outlined in Figure 14.

The student started this problem by calculating 3x6 (observed during testing as the first step completed by the student). The student correctly obtained the answer 18 for 3x6, and correctly renamed the value as 1 ten and 8 ones. This renaming was correctly recorded (with the 1 renamed ten denoted in the working space above the sum).

The remainder of the answer recorded (31 tens) suggests that the student then proceeded to calculate 6 tens x 5 tens, plus the renamed ten (=31 tens). The number facts recorded are correct in this step, however the student has skipped several stages of the computation which were not attempted at any later point. That is, the student did not calculate 6 ones x 6 tens or 5 tens x 3 ones. Skipping these steps meant that the final answer was incorrect.

This attempt suggested the critical error for this student was lacking an understanding of the 2x2 digit multiplication algorithm, indicated by the skipped steps. This error was classified as lacking conceptual understanding of the 2x2 digit multiplication algorithm.

Figure 14: Example of error analysis

Once errors were classified, categories of errors were coded following an emergent design as error categories were not predetermined. The frequency of error categories was also reported and trends in prevalent errors, and changes to error frequencies throughout the initiative were analysed. Analysis of trends in error patterns helped explain students’ achievement on the diagnostic tests. Further, these trends were then linked to the classroom observations concerning teaching practices as well as other pertinent observations from testing. For example, whether a student was highly reluctant or disengaged in completing the diagnostic tests was drawn upon to facilitate explanation of the trends in errors.
4.9.3 Newman interviews

The qualitative data collected from Newman interviews provided a mechanism to further explore students’ problem-solving proficiency. These interviews were used to isolate the specific barriers preventing students from completing problem-solving questions answered incorrectly on the diagnostic test as well as further exploring student errors and computation strategies. These interviews were carried out at the beginning and end of the initiative enabling changes in problem-solving proficiency to be tracked.

Newman interviews were analysed by recording the breakdown point for each student (i.e., reading, comprehension, transformation, process skills, or encoding). This analysis at the beginning of the initiative allowed for the identification of each individual’s barriers, and this information was disseminated to classroom teachers to guide the initiative. The frequency of each breakdown point was then analysed for each class allowing for the identification of trends across classes. This determined whether there were any common difficulties across the sample, or within classes. Whether the frequency of breakdown points changed throughout the initiative was also analysed, and relevant trends in these changes identified. The comparisons between the pre- and post-initiative difficulties determined whether students progressed through the five steps of problem-solving (see Table 6). Examples of each Newman error (from the outlined five steps of problem solving) are outlined here:

i) Reading

The reading stage of problem-solving involves being unable to read the words in the passage of text associated with the problem-solving task fluently. Figure 15 outlines an interview response from one Year 4 student who was working towards being able to read the problem-solving question.
Question 1: Jackie had 138 marbles. She played with Cathy and Kylie and won 65 more marbles. How many marbles does she have now?

<table>
<thead>
<tr>
<th>Reading</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Please read the question to me. If you don’t know a word, leave it out.</td>
<td>3. Tell me how you are going to find the answer</td>
</tr>
<tr>
<td>Student attempt to read the question. Student pointed to the word “question” and said I don’t know that word. The student also said “minutes” for “marbles”. The student also said “Carey” for “Cathy”.</td>
<td>The student shrugged and said “I don’t know”</td>
</tr>
<tr>
<td>“I don’t know.”</td>
<td>The interview was ceased at this stage.</td>
</tr>
<tr>
<td>The researcher then read the question to the student and re-asked Q2 to see if the student was able to comprehend the task.</td>
<td>“Find out how much marbles”</td>
</tr>
</tbody>
</table>

**Figure 15:** Sample of analysis of a student’s interview responses demonstrating that they are working towards being able to read the question.

The student’s ability to attempt to comprehend the task appeared to be inhibited by reading difficulties. This was supported further by the student being unable to comprehend the task requirements when the question was read to them.

**ii) Comprehension**

The comprehension stage of problem-solving involves articulating the requirements of the question. In Figure 16 the interview responses from one Year 6 student’s attempt at a subtraction problem-solving question is outlined. On the written diagnostic test this student did not attempt this question but was willing to attempt the question during an individual interview.
Question: Russell and Jason sold 356 cans of lemonade at the swimming club by 12:00pm. They also sold 167 bottles of water by this time. The club had 480 cans of lemonade and 360 bottles of water available to sell. How many more cans of lemonade can they sell before it is all gone?

Reading:
1. Please read the question to me. If you don’t know a word, leave it out. The student correctly read the question.

Comprehension:
2. Tell me, what is the question asking you to do? “Sell bottles?”

Transformation:
3. Tell me how you are going to find the answer “Umm, you takeaway?”

Process Skills:
4. Show me how you get your answer, and “talk aloud” as you do it, so that I can understand how you are thinking. “Umm, I don’t know”

Encoding
5. Now, write down your actual answer. No final answer given

Figure 16: Sample of analysis of a student’s interview responses demonstrating that they are working towards being able to comprehend the question.

In this instance, the student was able to read the question correctly, and is working towards being able to identify that the problem is asking them to subtract the number of sold lemonade cans from the initial total. This is indicative of a student working towards the comprehension stage of problem-solving. The written script alone provided no information about the students’ ability to complete a problem of this form, but the Newman interview yielded critical information about the students thinking and problem-solving proficiency.

Other students’ work manifested more complex issues relating to comprehension. Figure 17 is an example of a Year 6 student’s attempt at solving a subtraction problem-solving question on the written diagnostic test.

Figure 17: Sample of analysis of a student’s written solution demonstrating difficulties identifying the correct values to operate on.
Figure 17 is an example of a student using “number grabbing” methods when solving problem-solving questions. The student had not comprehended what the question was asking them to calculate and inappropriately “grabbed” all values from the question and operated on them hoping that it would yield the correct answer. During Newman interviews, the student was re-administered this question. The five Newman questions and the student’s responses to each question are outlined in Figure 18.

<table>
<thead>
<tr>
<th>Question</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading:</td>
<td>The student correctly read the question.</td>
</tr>
<tr>
<td>1. Please read the question to me. If you</td>
<td></td>
</tr>
<tr>
<td>don’t know a word, leave it out.</td>
<td></td>
</tr>
<tr>
<td>Comprehension:</td>
<td>“Add the chocolate and strawberry”</td>
</tr>
<tr>
<td>2. Tell me, what is the question asking</td>
<td>“Add them”</td>
</tr>
<tr>
<td>you to do?</td>
<td></td>
</tr>
<tr>
<td>Transformation:</td>
<td></td>
</tr>
<tr>
<td>3. Tell me how you are going to find the</td>
<td></td>
</tr>
<tr>
<td>answer</td>
<td></td>
</tr>
<tr>
<td>Process Skills:</td>
<td>Wrote 19 + 47</td>
</tr>
<tr>
<td>4. Show me how you get your answer, and</td>
<td></td>
</tr>
<tr>
<td>“talk aloud” as you do it, so that I can</td>
<td></td>
</tr>
<tr>
<td>understand how you are thinking.</td>
<td></td>
</tr>
<tr>
<td>Encoding</td>
<td>Answered 66</td>
</tr>
<tr>
<td>5. Now, write down your actual answer.</td>
<td></td>
</tr>
</tbody>
</table>

Figure 18: Sample of analysis of a student’s interview responses demonstrating difficulties with comprehending the task, and subsequently transforming the task into the correct computation.

Despite being able to read the question correctly in a literal sense, the student was unable to comprehend what the question was asking in terms of the calculation. This shed further light on the student’s ability to problem-solve. Though the written script suggested that the student did comprehend that the question was subtraction, the results of the Newman interview suggested that this was more of a guess than an understanding of the problem as the student switched to thinking the problem required addition. Further, despite the student demonstrating number grabbing on the written script, the Newman interviews suggested that the student understood that chocolate and strawberry were the key elements of the problem. However, the student did not understand the requirement to find the difference between the values or which operation to select to do so. Thus, the student exhibited some comprehension of the task, but was unable to connect this to the appropriate numerical operation. In other words, the student lacked an understanding of the contexts of subtraction (take away, compare, and missing part). In this instance the problem was of the compare structure (“how many more chocolate than strawberry”). This student’s comprehension and subsequent transformation difficulties acted as a critical barrier to the student’s problem-solving success.
iii) Transformation

The transformation stage of problem-solving involves identifying the correct mathematical operation, or forming the correct mathematical computation to answer a problem-solving question. For one Year 6 student in the sample, a random value within the question was selected as an answer to the first subtraction question on the diagnostic test. Such a response did not provide valuable information about the student’s thinking or problem-solving capabilities. This student’s response to the Newman interview questions are outlined in Figure 19.

| On Saturday, Ruby and Dora sold ice-creams. They sold 47 chocolate, 32 vanilla and 19 strawberry ice-creams. How many more chocolate than strawberry ice-creams did they sell? |
| Reading: 1. Please read the question to me. If you don’t know a word, leave it out. The student correctly read the question. |
| Comprehension: 2. Tell me, what is the question asking you to do? “How many more chocolate than strawberry” |
| Transformation: 3. Tell me how you are going to find the answer “Answer for what” Interviewer responded “the question here” “Add it up?” |
| Process Skills: 4. Show me how you get your answer, and “talk aloud” as you do it, so that I can understand how you are thinking. Wrote down computation with all values added together. |
| Encoding 5. Now, write down your actual answer. Answered 98 |

*Figure 19: Sample of analysis of a student’s interview responses demonstrating that they are working towards transforming the task into the correct computation.*

The student’s responses in the Newman interview indicated that, supposedly the student comprehended that they needed to find out how many more chocolate ice-creams were sold in comparison to strawberry ones, however the student was still working towards being able to identify what operation would correctly answer the problem. As a result, the student tentatively resorted to addition, exhibiting that they were developing the capability to successfully transform the problem into the appropriate operation.

This script is an example of the complexities of classifying the stage a student is working towards using a Newman interview. It also exemplifies how the above classification requires assumptions regarding a students’ true comprehension. For example, the student response in Figure 19 may have been a result of the student repeating the written question...
rather than completely comprehending the task. Therefore, these analyses highlighted how, potentially, it is useful to view the comprehension and transformation stages of problem-solving together for analytical purposes. Comprehending a problem, and subsequently transforming it into the appropriate computation is related to strategic competence (Kilpatrick et al., 2001). It is possibly more useful for analysis to focus on evidence of strategic competence due to the issues in distinguishing between comprehension and transformation stages, as indicated in this example.

iv) Process skills

The process skills stage of problem solving requires a student to carry out the mathematical computation/s required to complete the problem-solving task correctly. A student working towards the process skill stage will have been able to comprehend the task and transform the task into the correct computation successfully, but may still be developing their proficiency in completing the computation itself. A student working towards this stage may be developing their procedural fluency with the operation, or conceptual understanding of the computation itself.

During the written test, one Year 6 student answered the addition question (outlined in the interview in Figure 20) as “204”, as opposed to the correct answer of “203”. No working was shown for this problem therefore it might be assumed that the error was a careless mistake regarding number facts. However, during the Newman interview, it was clear that the student was still developing their proficiency with carrying out procedures associated with addition. The interview responses from the student are outlined in Figure 20.
Jackie had 138 marbles. She played with Cathy and Kylie and won 65 more marbles. How many marbles does she have now?

Reading:
1. Please read the question to me. If you don’t know a word, leave it out. The student correctly read the question.

Comprehension:
2. Tell me, what is the question asking you to do? “Add 138 add 65.”

Transformation:
3. Tell me how you are going to find the answer “Add 138 add 65.”

Process Skills:
4. Show me how you get your answer, and “talk aloud” as you do it, so that I can understand how you are thinking. Wrote down the sum 135 + 65 but lined up values under incorrect place values.

Encoding
5. Now, write down your actual answer. “788.” The student made no comment that the answer was unreasonable.

Figure 20: Sample of analysis of a student’s interview responses demonstrating that they are working towards fluently applying the correct process skills associated with the addition computation.

From the student’s interview script (Figure 20), it can be seen that the student successfully comprehends what the task is requiring them to do, and has transformed the question into an appropriate sum. However, the student was still working towards representing and recording the sum appropriately, demonstrating that they were working towards completing the process skills stage of problem-solving successfully.

v) Encoding

Encoding relates to articulating the correct solution to a problem after having solved the task procedurally. In this study, no students were impeded by the encoding stage of problem-solving.
Another possible outcome from a Newman interview is that the student answers the question correctly within the interview itself. Figure 21 denotes the interview from a Year 6 student who was able to answer the question successfully during an interview.

Jackie had 138 marbles. She played with Cathy and Kylie and won 65 more marbles. How many marbles does she have now?

**Students written test attempt:**

Here the student has formed the correct computation, and has only forgotten to add the renamed hundreds place value.

**Reading:**
1. Please read the question to me. If you don’t know a word, leave it out.

**The student correctly read the question.**

**Comprehension:**
2. Tell me, what is the question asking you to do?  

**“Plus 65 and 138”**

**Transformation:**
3. Tell me how you are going to find the answer

**“Plus 65 and 138”**

**Wrote correct sum on paper and calculated incorrect answer before researcher could ask Question 4.**

**Process skills:**
4. Show me how you get your answer, and “talk aloud” as you do it, so that I can understand how you are thinking.

**Researcher asked Question 4. When student started to talk through the attempt the student stated:**

**“Wait, that’s not right”**

**Student rewrote sum and recalculated.**

**Encoding:**
5. Now, write down your actual answer.

**“203”**

*Figure 21:* Sample of analysis of a student’s written solution and interview demonstrating that they were supported in obtaining the correct answer during the interview through self-checking.

In this example, the student had been scaffolded by the interview questions to re-check their answer, and realised the original attempt during the interview where they obtained the answer of 165 was unreasonable. This contrasts with the diagnostic test attempt, in which the
student obtained a similarly unreasonable answer (103), but did not check the answer as evidenced by no reattempt made on the task. In this instance, the Newman interview supported the student in correctly completing the problem-solving tasks. This phenomenon also occurred on the only other incorrect problem-solving response readministered to this student during Newman interviews.

Figure 22 contains a second example of a Year 4 student correctly answering the problem-solving task during the Newman interview.

<table>
<thead>
<tr>
<th>On Saturday, Ruby and Dora sold ice-creams. They sold 47 chocolate, 32 vanilla and 19 strawberry ice-creams. How many more chocolate than strawberry ice-creams did they sell?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students written test attempt:</td>
</tr>
<tr>
<td>The student originally selected all values in the question and attempted to subtract them. When this did not yield a viable computation, the student has added all three values instead of subtracting, and obtained the final answer of “98”.</td>
</tr>
</tbody>
</table>

Reading:
1. Please read the question to me. If you don’t know a word, leave it out. The student correctly read the question.

Comprehension:
2. Tell me, what is the question asking you to do? “See which one, umm, how many more did she sell out of chocolate and strawberry.”

Transformation:
3. Tell me how you are going to find the answer “Start from 19 and count to 47”

Process skills:
4. Show me how you get your answer, and “talk aloud” as you do it, so that I can understand how you are thinking.

Encoding:
5. Now, write down your actual answer. “28”

*Figure 22: Sample of analysis of a student’s written solution and interview demonstrating that they are were able to obtain the correct answer during an interview.

In the original attempt during the written test, the student used “number grabbing” methods to attempt to solve the question. This indicated that the student was working towards comprehending what the question was asking them to calculate, and had ‘grabbed’ all values from the question and operated on them hoping that it will yield the correct answer. When the student was re-administered the question during interviews, the student was able to identify the
values that needed to be operated on correctly, and selected an appropriate method of solving the problem. The interviews supported the student in obtaining the correct answer.

4.9.4 Classroom observations and teacher interviews

Classroom observations focused on documenting 1) lesson structures, 2) pedagogical approach/es (particularly focusing on how computations, number facts, and problem-solving were taught), 3) frequency and type of teacher interaction with students, and 4) student time on task. Data analysis involved the dividing of the data into these units of meaning as advocated by Wellington (2015). These qualitative observations were analysed by the creation of case reports for each class. The degree of implementation of particular practices (nil, low, moderate, or high levels of implementation) were analysed and reported in a graphical format. Low level of implementation was classified as the use of the practice at some point throughout the initiative, however, it was not employed consistently throughout each week of teaching. Moderate level of implementation was classified as frequent use of the practice, but not on an everyday basis. High level of implementation was classified as use of the practice daily. Teacher interviews focused on member checking the researcher’s interpretation of the observed elements of classroom practice. They also formed a way of analysing differences between teachers envisioned and enacted practices.

The qualitative data from both teacher interviews and classroom observations was analysed using Wellington’s (2015) stages for interpreting qualitative data. This process is derived partly from the constant comparative method of analysing qualitative data (e.g., Delamont, 1992; Glaser & Strauss, 1967; Guba & Lincoln, 1985). This model was drawn upon in data analysis in answering the third research question which required triangulation by integrating findings from other data sources in conjunction with classroom observations and teaching interviews. That is, integration of the quantitative and qualitative data needed to be carried out as the quantitative findings were explained by the qualitative findings. Changes to students’ proficiency (RQ 2a/2b) were linked to teaching practices (RQ 1) to explain what factors associated with the initiative influenced students’ achievement (RQ 3). Table 8 below outlines the analysis process for classroom observation and teacher interview data, in conjunction with the diagnostic and Newman interview data sources as findings were integrated and links drawn during analysis between teaching practices and student errors.
Table 8
Qualitative data analysis stages

<table>
<thead>
<tr>
<th>Qualitative analysis stage (Wellington, 2015)</th>
<th>Analysis of classroom observations and teacher interviews</th>
<th>Analysis of data concerning student errors (from diagnostic tests &amp; Newman interviews)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1 &amp; 2: Immersion in the data &amp; reflecting</td>
<td>The researchers sustained and interactive role throughout the initiative meant that the researchers had extensive prior knowledge and familiarity with the data relating to teacher’s implementation of the initiative. That is, mandated by the observational nature of data collection, the researcher was immersed in the data throughout the study. Data was transcribed throughout data collection in the form of diary entries. Teacher interviews were recorded and transcribed for analysis. This occurred with concurrent immersion in the students’ data through the coding of student errors.</td>
<td>The process of collection student data was also highly immersive due to the nature of the researcher administering tests in small groups.</td>
</tr>
<tr>
<td>Stage 3: Analysing, taking apart the data</td>
<td>Moving from familiarisation with the data, each of the four observed features (lesson structures, pedagogical approach/es, frequency and type of teacher interaction with students, student time on task) was coded qualitatively as none, limited, moderate, or high. In this way, coding was data driven.</td>
<td>Moving from marking students’ responses on the diagnostic tests as either correct or incorrect, student errors were then described and coded.</td>
</tr>
<tr>
<td>Stage 4 &amp; 5: Recombining/synthesising data &amp; locating the data: relating back to other work</td>
<td>The intention of this stage is to examine and refine categories by looking for patterns and relationships. This involved recombining the decontextualized data from stage 3 for integration. This was achieved by creating case reports of practices in each class and looking for potential links between patterns in student achievement in each class and teaching practices. This involved interpretation of the influence of practice considering student data and literature. To support this synthesis, case reports for students were also carried out to shed light on illuminating or typical cases in each class in the effort to explain achievement patterns in a more meaningful way. Patterns between all data sources were sought by looking for themes, regularities, irregularities, and contrasts as suggested by Delamont (1992). The synthesis of data was located continually with relevant literature to support the final justifications for the identification of factors that influenced students’ proficiency. The findings from this integration are reported mainly in the discussion chapter as overarching themes of factors associated with the initiative that influenced students’ proficiency.</td>
<td></td>
</tr>
</tbody>
</table>
4.10 Validity

Mixed methods research is characterised by the purposeful use of a range of techniques that are both qualitative and quantitative in nature to best answer research questions. To determine the validity of assertions and conclusions resulting from data analysis, trustworthiness criteria were applied (Guba, 1981; Guba & Lincoln, 1985).

The first trustworthiness criterion is associated with the credibility of the findings in a study, and is often termed internal validity in positivist criteria (Guba, 1981). Credibility is associated particularly with determining if the interpretation of participants’ reality is accurate (Guba & Lincoln, 1985). The credibility and confidence in the findings arising from this study was increased by ensuring prolonged engagement with participants to overcome any potential misinterpretations by the researcher throughout the study (Guba, 1981; Tashakkori & Teddlie, 2009). Also, due to the researcher’s pre-established role in the school, the context and culture of the school was well understood, and rapport was already established with students and staff, thus increasing the credibility of findings by providing cultural sensitivity (Gay et al., 2006).

Secondly, persistent, systematic, and continual observations were made over the course of the initiative to allow for clear identification of atypical or pervasive qualities and results observed (Gay et al., 2006; Guba, 1981; Tashakkori & Teddlie, 2009). Furthermore, credibility was enhanced through the use of multiple data sources and several methods of analysis.

A further check on the authenticity of the data and analysis was conducted via member checking with teachers through debriefing sessions which allowed for participating staff to affirm whether descriptions and field notes were reflective of their experiences and perspectives (Guba & Lincoln, 1985). The use of multiple data forms also allowed for a type of member checking with students as it identified any potential misinterpretations in observations of errors or strategies as multiple sources of data regarding students’ work was collected throughout the initiative.

The second trustworthiness criterion is transferability (external validity in positivist criterion), which refers to the degree to which findings are applicable to other contexts with other participants (Guba, 1981). By providing rich descriptions of the study through specification of the study context as well as description of the initiative, readers can determine the applicability of findings in other contexts depending on the similarly in contexts (Gay et al., 2006; Guba, 1981). As such, the transferability of any assertions to other contexts will be the responsibility of the reader.
Dependability (corresponding to reliability in positivist criterion) is the third trustworthiness criterion and is associated with determining the consistency of findings, particularly whether they are replicable in similar contexts, or with similar participants (Guba, 1981). To determine the consistency and dependability of any findings, triangulation methods were used in this study by collecting multiple data sources (Cohen, et al., 2011a; Creswell, 2012; Gay et al., 2006). The use of mixed methods including multiple sources of qualitative and quantitative data allows for confidence in findings as they will be supported with many pieces of evidence.

The last trustworthiness criteria, confirmability (or objectivity by positivist criterion), refers to the degree to which the findings of the study are neutral and free of biases, motivations, interests, and the perspectives of the researcher (Guba, 1981). Due to the interactive role of the researcher in this study, considering this role was important due to the potential biases that may result from the researcher’s unique background, experiences, and perspectives (Creswell, 2012). Triangulation through integration of both qualitative and quantitative data sources contributed to establishing the trustworthiness of this study as quantitative findings were supported by qualitative observations. The original sources of data that gave rise to particular assertions were made traceable by including original samples of students’ work and verbatim quotes from participants in the reporting of results where relevant. Though it is impossible to eliminate all researcher bias, the thorough documentation and multiple data collection opportunities assisted in recognising and minimising potential biases (Creswell, 2012).

4.11 Ethics

Full ethical clearance was obtained for this study (GU Ref No: 2017/264). It was important that the planning, conduct, and dissemination of results from this study was conducted in an ethical manner, particularly as this study involved children and Indigenous people. Throughout this study, the core values important to Indigenous people were respected. These values are reciprocity, respect, equality, responsibility, survival and protection, and spirit and integrity (NHMRC, 2015). To achieve this, the researcher was accountable to the Indigenous school community and had first access to research findings (Denzin, Lincoln & Smith, 2008). This was achieved by ensuring results were disseminated to teachers and the school in a timely manner throughout the study so they had tangible use for the education of
the tested students. Findings for the study were not kept separate from the day-to-day endeavour of teaching at the school.

In this study, to ensure that the research was conducted in a respectful and appropriate manner that was beneficial to the school, a reference group was created at the school. This group consisted of Indigenous school teachers, school board members, and auxiliary staff, including an educational psychologist. This reference group was consulted regarding the design and proposed implementation of the initiative. A relationship was maintained with the reference group throughout the study in the effort to ensure that practices were beneficial and appropriate for students, staff, and the local community. The progress of the study was reported regularly to this group. Consultation with the reference group also acted as a mechanism in which potential risks could be identified throughout the study.

To deidentify participants, pseudonyms were assigned to teachers and students when reported on individually. Students were assigned numbers in the whole class analysis of data. The physical data from the study was stored and archived in a secure location on site as well as at Griffith University. To ensure that participation in this study was not contrary to the best interests of students, parent/guardian consent was sought in order for students to participate in the study and this participation was voluntary in nature. The consent forms and information package that was provided to students, parents, and teachers is detailed in Appendix C. In addition, if there were any instances of discomfort as a result of testing or interviews associated with this study, the student and/or parent/guardian was able to choose for the student to cease participation at any point in the study. It was also important to consider whether the mathematics initiative would meet the educational needs of all students (as not doing so would not be in the best interest of students) in the design of the study. To increase the likelihood of the program being appropriate for all students, the classroom teachers involved in this study participated in whole school planning to develop the model of practice for the initiative which was informed by evidence-based recommendations provided by the researcher. That is, how the initiative was implemented was up to the individual classroom teacher, and the initiative recommendations served as recommendations only.
4.12 Summary

In summary, a mathematics initiative was implemented and evaluated in an Indigenous community school in southeast Queensland. Aligning with pragmatic philosophies, the research design for the study was an explanatory sequential mixed methods design.

The context of the study was an urban Indigenous community school, and the sample included four classes of students from Grades 2 to 6 (n= 50). The students in the school were achieving below their year levels as stipulated by the Australian Curriculum: Mathematics F-10 (ACARA, 2018b).

The researcher had an established role in the school, and had an established positive relationship with the teachers and students involved in the study. The researcher’s role was interactive throughout the initiative by providing professional development, and supporting teachers’ implementation of the initiative.

The mixed method design of this study involved the collection of a range of quantitative and qualitative data using a variety of data sources. Following a sequential explanatory design, quantitative data was firstly collected, followed by supporting qualitative data. Through this process, quantitative student achievement data was linked to specific factors associated with the initiative that influenced students’ proficiency.

To determine how teachers implemented the initiative, the collected data sources included classroom observations and informal interviews with teachers. These data sources addressed the first research question. Data source utilised to evaluate students’ mathematical proficiency included a standardised mathematics achievement test (the PAT-M), a suite of mathematics diagnostic tests, and Newman interviews. These data sources addressed the second research question.

Data analysis was carried out in several ways to enable the triangulation of findings by integrating findings across sources. Classroom observations and teacher interviews were analysed by creating case reports documenting the prevalence of particular teaching practices. Quantitative analysis of changes to students’ mathematical proficiency was carried out for the PAT-M, and diagnostic tests. To answer the third research question, analyses of teaching practices, and classroom observations were integrated and linked to student data to isolate themes of factors influencing students’ proficiency by searching for patterns in findings. Analysis of student errors on diagnostic tests, in conjunction with Newman interviews, also
contributed to answering the third research question as changes in students’ errors and problem-solving proficiency were linked to teaching practices in each class.

The validity of assertions and conclusions in the study was supported by considering the trustworthiness criteria of credibility, transferability, dependability, and confirmability (Guba, 1981; Guba & Lincoln, 1985). The use of multiple data sources, and the triangulation of findings across data sources, particularly in relation to the use of several measures of students’ mathematical proficiency, helped to support the validity of findings. Further validation of findings was provided via member checking and an established understanding of the school context. Prolonged engagement and persistent, systematic, and continual observations at the site ensured the credibility of the findings. The ethical conduct of research was considered, particularly in relation to the core values of ethical research with Indigenous people (NHMRC, 2015).
5 FINDINGS

5.1 Introduction

This chapter will present the findings of the study answering Research Questions 1 and 2. The purpose of this chapter is to present the analysis of students’ mathematical proficiency pre- and post-initiative, and the analysis of teachers’ implementation of the initiative. Section 5.2 will provide contextual information about the school to help inform the findings, followed by an analysis of how the initiative was enacted by teachers in each class (Section 5.3). Demographic information of the students will be presented in Section 5.4, and Section 5.5 will present the findings related to students’ mathematics proficiency over the course of the initiative. Section 5.6 will present case reports from individual students, and Section 5.7 will summarise the findings from all data sources and case reports thus facilitating the identification of specific factors related to the initiative that acted to influence students’ mathematics proficiency.

5.2 School support throughout the initiative

The community owned school where this study was conducted was supportive of the educational initiative implemented in the study. Previously, the school had identified the gap between students’ levels of achievement and the expected standards in mathematics (according to national curriculum, ACARA, 2018b). Therefore, from an administrative perspective, the school was supportive of both the initiative and the researcher maintaining the role of mentor and expert teacher throughout the initiative. This was important as several professional development sessions were required to begin and maintain the initiative.

At a whole school level, there were attendance policies facilitated by a funded family support unit who worked directly with families to support student attendance. Student attendance was a priority in the school. It was considered of greater importance to have students attend school than be suspended or expelled due to behavioural reasons. This condition had implications for the accepted standard of behaviour within the school.
A culture of high academic standards was purported at the whole school level as reflected in the school ethos. However, the school was aware that students were working below their current year level standards and, commonly, modified curriculum programs were implemented across the primary school. Modified curriculums involved tailoring programs on curriculum standards from earlier year levels. Therefore, in composite classes modified curriculums often meant basing the program off the standard of the lowest year level. For example, the Year 4/5 program would be mainly derived from Year 4 content. One teacher, Helen, commented that this was a challenge at the school:

The biggest challenge I suppose from what I know in this school [is] that … modified maths programs happen… I think that’s a shame. I think that the challenge is that we get the kids on their expected levels. From what I can see in this class that I’ve had; I think that’s doable. But I think you have to have a plan where you’re going to start early with it.

With respect to the academic expectations of the mathematics programs, there was little discourse between the school administration and the community. That is, there was no consistent, transparent reporting of what achievement level (according to the national curriculum, ACARA, 2018b) a student was working at. There was also no frequent, formal parent-teacher communication such as parent-teacher interviews.

5.3 Enactment of the initiative

The following section will present the findings regarding how each of the classroom teachers implemented the mathematics initiative aimed at raising students’ mathematics proficiency thus answering the first research question. Firstly, teachers’ responses to professional development throughout the initiative will be discussed. Following this, how the recommended practices were enacted will be outlined in case reports for each class. The purpose of this is to understand the construction of the classroom discourse by the teacher in order to understand changes to students’ achievement.

5.3.1 Teachers’ responses to professional development

Throughout the initiative, the researcher’s role was interactive with teachers during professional development sessions, planning time, and classroom observations. This scenario
allowed for several key findings to emerge during observations. These interactions were where
the proposed initiative was presented and explained to the teachers, and where discussion
concerning the implementation of the initiative occurred between the teachers and researcher.

Over the course of the initiative it was observed that changing teachers’ practice
required consistent and sustained engagement in professional development. The first
professional development session took place at the commencement of the study in January
(2017). Throughout classroom observations during this first school term (prior to the formal
commencement of the initiative with students), it was observed that many of the practices
recommended in the initial professional development session had not been implemented into
classrooms, and teachers expressed limited interest in adopting them. As a consequence, two
further professional development sessions were implemented partway through the initiative to
support teachers in understanding and implementing the recommended practices.

A finding from the professional development sessions was that teachers’ responses to
professional development sessions varied depending on the focus and timing of the session.
During the first professional development session conducted after school, it was observed that
teachers’ engagement was limited. This trend was also observed in the collegial meetings run
throughout the school year in after-school time. In after-school sessions, many teachers were
often late, and attendance was inconsistent. There was limited discussion and collaboration
during these sessions. All teachers displayed general passive behaviours towards receiving the
information regarding the initiative. These behaviours were demonstrated via teachers not
asking questions, late arrivals, and seeking to leave at the earliest opportunity. Suggestions to
increase collegial collaboration were met with resistance during meetings. For example, during
the first term of weekly collegial meetings, it was proposed by some teachers that they would
like to participate in a “watch each other work” system where they visited classes to observe
other teacher’s mathematics lessons. The intent of this suggestion was to increase collaboration
and gain ideas for improving current mathematics programs. When this was proposed, teachers
were reluctant to participate with the primary concerns cited as being judged, and potentially
receiving negative feedback on their practice.

In comparison, the sessions that were conducted during school time were received
differently by teachers. During these sessions, teacher attitudes were positive and they appeared
interested in the topics covered. There was collaboration and high levels of discussion with and
between teachers regarding teaching practice. Other than the difference in the timing of the
sessions, another critical difference in the second and third sessions conducted during school time was that the information presented was based on data concerning each teacher’s current students. These recommendations directly linked to the data and their current practice. These findings indicate that teachers found the content of the sessions of greater relevance to their everyday practice.

During the professional development session run after school at the beginning of the year, it was found that the presentation of literature recommending the inclusion of strategy discussion when teaching number facts was received passively and without comment. In contrast, when data concerning mean scores from individual classes on number facts from diagnostic testing was presented in the second session, teachers participated in extended dialogue comparing their specific pedagogies when teaching number facts. This type of contextualisation allowed for greater depth in conversations concerning why strategy discussion was a recommended practice from literature. The types of discussions that were observed indicated that the meaningfulness of the literature recommendations was increased when the recommendations were contextualised with relevant classroom data. When asked to comment specifically on which parts of the professional development were useful, a teacher (Diane) noted that:

> It was very interesting hearing what other teachers do because that to me is more practical. I find the practical side of it way more useful … Even though we read it, even though we talk about it, but seeing it in action from somebody else’s point of view, or [how] somebody else teaches a concept, I find that more beneficial.

Similarly, another teacher (Paul) also noted: “if it’s not relevant to what we’re doing here at the school then I think it’s pointless us doing the PD [professional development]”. Further, Jane also noted that the benefit of the sessions was that “it was based on hard evidence and not general practice”. These three statements demonstrate consensus among the three teachers who participated in the professional development sessions, and reinforce the idea that professional development was most useful for teachers when it was put in context. When teachers were asked to reflect on the during-school sessions, all teachers reported that the sessions were a productive use of professional development time. Teachers noted that the relevant feedback to assist with class and individual needs for planning, hearing what other
teachers do, and discussion of teaching strategies were the most valuable aspects of the sessions. These reflections are also supported by the observations of the researcher.

5.3.1.1 Teachers responses to the initiative and mentoring

In this study, teachers were involved in a sustained initiative for 7-months involving continual engagement with the researcher who acted in the role of mentor to support the implementation of the initiative. At the conclusion of the initiative, teachers were asked to reflect on their experiences and perspectives regarding the value of the initiative and the researcher’s mentor support. All teachers involved in the study expressed that they felt being involved in the initiative was worthwhile and, when asked, all said that they would recommend other teachers to be involved in a similar initiative. In relation to being involved in the initiative, teachers reflected that it was valuable in providing consistency. Diane stated that the reason she would recommend other teachers to be involved in a similar initiative was because “it brings more uniformity because we’re all working towards a common goal”. Similarly, Paul stated: “I think it's just beneficial and it keeps all the teachers on track. Plus, it also keeps challenging the teachers as well. It just makes it better and it makes it a better environment for our kids”.

When reflecting on the role of the researcher as mentor, teachers also expressed that it was valuable to their practice as the researcher acted as an expert source of advice that was readily available. Teachers also noted that they felt that access to the researcher brought about consistency in teaching approaches within the school. When discussing working with the researcher over the duration of the initiative, Jane stated:

I've quite enjoyed it. Because sometimes I'm not overly confident, so I know you're there and then I'll go, oh how do you do this, teach this, [Researcher]? You know, so we're on the same page. Because I think it's important that all the classes are using the same language and teaching the same

Paul shared similar sentiments to Jane and reflected specifically on how the immediacy of the advice provided by the researcher as a mentor was valuable, stating:

I think you've been really, really good for me…If ever I've needed advice in any topic you've always been there. I've never had to wait for advice, you've always given it to me on the spot. Which has been really good, so I thank you for that.
These reflections from teachers indicate that the relationship with the researcher, and availability of the researcher was important in shaping teachers’ experience with the initiative. The sharing of ideas was considered of central importance to teachers, as Helen summarised:

I always like talking to other teachers, people like you just to get ideas. I think it’s really important that we share that and get advice from each other, suggestions from each other that I’ve tried this before, and it’s worked. That’s what I really value. Because you can’t actually teach alone.

Therefore, teachers felt that both the initiative and the support of the researcher were valuable. What made the experience with the initiative impactful for teachers included the availability of the researcher for support, the consistency in teaching approaches provided by the initiative, as well as the sharing of ideas to support each other’s teaching.

5.3.2 Year 2/3: Helen

Information concerning Helen’s demographics, perceptions, and teaching approaches is presented in the following sections.

5.3.2.1 Demographics

The Year 2/3 classroom teacher, Helen, was a female teacher who did not identify as Indigenous. Helen’s qualifications were a Bachelor of Education from an Australian institution. She had been working as a teacher for twelve years. Helen began working at the sample school at the beginning of the initiative, so was a new teacher to the school.

5.3.2.2 Perceptions of the school, students, and mathematics

When asked to comment on how the sample school was different to other schools she had taught in, Helen commented that the primary difference was the high instances of challenging behaviour and stated: “it does affect teaching then because you have to be on your toes all the time...you’ve just got to engage the children and that’s a challenge every day...”. When asked to comment on her perception of the academic capabilities of students in her class, Helen noted:
Many of them are very capable and quite capable. So, I actually think - and they have proved it this year - that some can certainly be above the expected achievement standard for their year level. I think we have to recognise that and foster that.

This indicated that Helen held positive views about students’ capabilities to achieve in mathematics in her class. In relation to how Helen felt about teaching mathematics, Helen stated:

I love teaching maths. Love it. Especially with little people when you’re still learning those concepts and you’re still using the concrete materials. I love it when their eyes light up because they suddenly get it, and they see it and they can understand it.

Her perceived strengths in teaching mathematics related to use of the CRA teaching sequence, and when commenting on her strength, Helen noted: “I think I understand the importance of teaching the concepts using concrete materials, especially for this age level, and making them able to see it, to understand it… I understand that process to move from the concrete to the symbolic and that sort of thing”. When asked to comment on her perceived weaknesses in teaching mathematics, Helen reflected on differentiation: “I think pushing the ones who need pushing and differentiating, I suppose, that you are targeting everybody. You’ve got to just make sure you’re always targeting everybody and where they’re at. Sometimes you don’t always pick that up”. These insights from Helen can be linked to the pedagogical choices made in her classroom.

5.3.2.3 Overview of teaching approach

In terms of Helen’s approach to teaching the Year 2/3 class during mathematics, Helen was a teacher who was observed to be highly interactive with students as evidenced by the high level of teacher time with students during lessons. This concurs with observations that Helen was an engaged and student-focused teacher. In terms of the amount of time students were engaged academically in the Year 2/3 class (i.e., time on task), it was observed that this class reported high levels of time on task during mathematics lessons. Furthermore, Helen was proficient in working towards limiting the amount of time behavioural incidences disrupted learning and teaching time.
Consultation with Helen at the beginning of the initiative revealed that the envisioned structure for mathematics lessons involved:

Beginning with 15 minutes of practice of number facts with concrete support (e.g., number lines, number ladders, counters, hundreds boards), followed by:

Rotational group work for the remainder of the mathematics lesson, which focused on weekly topics.

In Year 2/3, it was observed throughout the initiative that small group work was the major pedagogical strategy utilised in this class. Helen confirmed these strategies in discussion describing her pedagogical approach throughout the initiative:

What we decided to do was to split the class into our year 2 students and our year 3 students and so that we can target their year level and give every child an opportunity then to achieve their standard for their year level.

The group work component formed the main body of the lesson covering the concept that Helen was focusing on in mathematics for a particular day. Group work generally involved 4-5 students working on an assigned task (e.g., iPad mathematics game, worksheet, or working with the teacher completing questions on a concept), with the intention of each group working on a different task. Groups of students would then rotate through a variety of tasks within a lesson. Helen describes her enacted practices as:

I always start out maths with a bit of drill and practice, definitely some number work, number facts work we would do. On different days it might change, like I might do time one day and money another, just to touch on different things. Then whatever we had planned that lesson, so whatever concept we’re working on we were going to that, and then at the end we’d finish with some revision or something we’d been doing through the week or from the previous week; just so that we’re covering things.

So, 15, 20 minutes at the start [was] drill and practice. Maybe then half an hour on [the] whole teaching focus, and … 15 minutes or so at the end to go over other stuff that you know that you’ve worked out you needed. Or something I realised I needed to target.

The group work that occurred was structured by capability as measured by data collected by the classroom teacher, with the lowest group often being withdrawn to work with the teacher aide (special education teacher). As classroom observations continued throughout
the school year, it was observed that the pedagogical focus on small group work became the primary focus of all mathematics lessons, and the number of worksheet-based tasks increased. The result was a loss in time dedicated to explicit instruction in mathematics lessons. This consequence was contrary to the recommendation to have a primary pedagogical approach focusing on explicit instruction techniques. However, there was some use of several of the recommended practices within this class including lesson goals, concrete materials when introducing concepts, strategy discussion, and mastery teaching cycles. A final summary of how frequently particular pedagogical factors were utilised is outlined in Figure 23.

Figure 23: Level of implementation of pedagogical practices in Year 2/3. Note: 0 refers to no implementation of the specific practice, 1 refers to limited implementation, 2 refers to moderate levels of implementation, and 3 refers to high levels of implementation.

Figure 23 summarises Helen’s practice. Helen included moderate levels of the use of lesson goals, and timed number fact tasks in conjunction with strategy discussion aligning with the recommended practices. Helen was observed to maintain moderately high academic standards with work during mathematics as evidenced primarily through the continual marking of students work. The continual marking provided feedback for students that could be corrected
by students or used as future learning tasks. Related to the mathematics program in Year 2/3 and its alignment to the Australian curriculum, Helen noted:

There was certainly year 2s in here at quite capable and more than capable of doing their curriculum and moving forward from that; and the year 3s needed exposure to it so that you could - to their year level. So that they could show what they can do and what they can’t do.

This indicates that Helen was focused decidedly on delivering content at the intended year level as outlined in the Australian Curriculum, rather than a modified program that works at lower year level standards.

5.3.2.4 Pedagogical approaches for number facts, computations, and problem-solving

In relation to Helen’s pedagogical approaches surrounding number facts, after the dissemination of the initial analysis of the pre-initiative findings, it was observed that Helen began to include strategy discussion in lessons when teaching number facts. This occurred after she became aware of the pre-initiative findings that indicated fluency with number facts was the differentiating factor between her mathematics groups that were scaled by capability (i.e., low, middle, and high mathematics groups). Helen’s instruction surrounding number facts included short discussion of number fact strategies (e.g., developing students’ understanding of +9 facts by discussion how they are one less than the +10 facts), followed by a short, timed worksheet task where students answered ~20 fact questions. Overall, strategy discussion and timed number fact tasks were not a feature of every mathematics lesson as intended in Helen’s envisioned practices. This was the major difference between Helen’s envisioned and enacted practices.

In relation to teaching computations, observations indicated that Helen’s instruction did not feature a strong focus on conceptually developing students’ understanding of what addition, subtraction, multiplication, or division where. However, instruction did focus on utilising concrete resources such as multibase arithmetic blocks to conceptually develop students’ understanding of renaming, and the associated algorithms (see Figure 9). In the Year 2/3 class, addition and subtraction were the main focus of computation instruction (lessons on multiplication or division concepts were not observed to have been included in this classes mathematics program). Helen taught students the traditional, vertical algorithms for addition
and subtraction using consistent place value language, aligning with the recommended practices (see Booker et al., 2014 for description of the language, symbols, and materials related to teaching vertical algorithms utilised by Helen). The nature of the four types of addition and subtraction problem structures (change, compare, combine, equalise) noted by Carpenter et al. (1983) were not explicitly discussed in instruction, nor were other problem structures for multiplication or division.

In relation to problem-solving, at the beginning of the initiative Helen’s envisioned practices included focusing on problem-solving in one mathematics lesson of each week (e.g. Friday mathematics lesson were intended to be problem-solving lessons). What was included in these lessons was unspecified when planning the shape of the Year 2/3 mathematics program. In the enacted practices for Year 2/3, it was observed that no lessons were dedicated to problem-solving throughout the initiative, and no formal instruction on problem-solving was noted.

5.3.2.5 Alignment of teaching approach to recommendations

Though there was a heavy emphasis on group work in Helen’s teaching approach and frequent use of worksheets, some mathematics lessons or particular sections of lessons featured structured and explicit instruction regarding mathematical concepts (e.g., teaching addition). Helen utilised concrete materials and the recommended algorithm pedagogies to conceptually develop new concepts during the initial explicit instruction phase. Mastery teaching involving weekly diagnostic, formative assessment was also a feature of Helen’s regular pedagogy.

Academic learning time was maximised in the class through high levels of teacher engagement and the effective management of behavioural disruptions. Helen also maintained high academic standards through the frequent checking of student work, and the reporting back of achievement to students (by providing them with marked work). This feedback fostered a learning environment where learners were observed to expect to be working towards success, as opposed to a “done is good enough” attitude.

5.3.3 Year 3/4: Diane

Information concerning Diane’s demographics, perceptions, and teaching approaches is presented in the following sections.
5.3.3.1 Demographics

The teacher for Year 3/4, Diane, was a female teacher who identified as Indigenous and was qualified with a Diploma of Early Childhood Teaching from an Australian institution. Diane had been working as a teacher for approximately 15 years, and had been teaching at the sample school for 12 years.

5.3.3.2 Perceptions of the school, students, and mathematics

When asked to comment on the difference between the school and other schools, Diane noted:

The obstacles are greater…the obstacles that we have to face are multiplied as compared to what’s in, I guess, a mainstream school…the biggest challenge I think is trying to teach when there’s so much else going on. Because there are so many other major issues, like behaviour-wise that children might have.

However, she also noted that the positive of the school for her was the fact that it was predominantly Indigenous: “on the flipside, what’s different is it’s good having a predominantly Indigenous school because the Indigenous children for once are in the majority, they’re not the minority. So that’s a positive impact as well. Positive difference”.

When asked to comment on her perception of the academic capabilities of students in her class, Diane noted:

I think they’re more than capable, yes. A lot of students are. There are obviously students that will struggle and will continue as they go through years. But most of the children in the class are capable if they have the right support and opportunities in place.

This comment indicated that Diane held positive views about students’ academic capabilities despite some students needing further support. In relation to teaching mathematics, Diane felt confident, but did not have positive experiences with mathematics herself at school.

In the end, I hated maths. But when I first started high school I was really, really good at maths. But I had a bad experience with a teacher, and ever since then I hated maths… I think he humiliated me about homework once in front of the whole class. Then from then on, because he was the maths teacher, whenever I would turn up, I was just completely turned off.
In discussion about Diane’s strengths in teaching mathematics, Diane self-identified that her strength was sticking to a consistent classroom routine. Diane commented that “I stick to routine quite stringently … I think that is a strength because the children know what the expectation is for the day, for the week, or whatever”. When asked to discuss her perceived weaknesses, Diane discussed the difficulties of catering to diverse student needs, making the work interesting, and drawing on concrete resources.

Sometimes I do fear that I’m not helping everyone. It’s really hard to cater to every individual need. That is my weakness, I guess. Also, I guess a weakness is making it interesting for the child. I can be quite regimental and just quite - not so much boring, but I guess some people might look at it as boring. So, my weakness is I don’t tend to use a lot of resources that I probably should, like concrete materials and things like that. I kind of stick to the same process or the same routine, yeah.

These findings from Diane give valuable insight into her perceptions of the school and students, and teaching mathematics. As an Indigenous teacher, Diane’s comments also shed insight into her relationship with mathematics, and how interactions with teacher’s have shaped her perceptions.

5.3.3.3 Overview of teaching approach

Diane’s teaching approach in the Year 3/4 class involved mathematics lessons that followed a consistent daily structure. The intention was that students in the class would also learn the structure and become self-sufficient with understanding the lesson organisation (i.e., knowing what type of activities were coming next to get appropriately prepared, without added teacher instruction). This classroom organisation related to Diane’s self-identified strength relating to consistent classroom routines. In addition, Diane prioritised high behavioural standards during classes. She was observed to minimise behavioural interruptions consistently that otherwise may have caused disruption without her supervision and guidance. Consequently, efficient use of time spent on task during mathematics lessons resulted.

Consultation with Diane at the beginning of the initiative revealed that the envisioned structure for mathematics lesson involved the following:

1. 30 minutes: Answering number fact questions, copying out a set of specific times tables, and independent learning of times tables.
2. 30 minutes: “Skills lesson” - explicit instruction component of lesson, involving the teaching and modelling of a concept.

3. Weekly testing of times tables at the end of the week.

Diane confirmed her lesson structure in discussion about her pedagogical approach throughout the initiative:

Well when we come in, we always do our mental computations, which we call beat the bomb. Then that’s always followed by times tables and a different way that I do times tables is I just, I individually cater to the children with their times tables. Because I think every child learns at a different pace, so they have the right to go well ahead of others in they know those times tables that we’re learning as a whole class. I don’t do that at the beginning of the year, because I think we ought just to get the basics. We all start on the same timetables, but usually in term two I just let them power on at their own pace.

After that we might have a skills lesson. It’s all based on number, and then for the last half an hour usually, if not 20 minutes, is the actual maths lesson. Sometimes I’ll incorporate a game in there if there’s time. Most times not. But probably the first 20 minutes is usually designed so that the children just work through it at their own pace... They know what they have to do. That routine is set. So, it’s not so stringent. The children are kind of like in charge of their own learning for that process, or that time.

Observations throughout the initiative demonstrated that Diane’s enacted structure matched her envisioned structure in mathematics lessons. Diane’s pedagogic approach did not alter throughout the initiative.

Several other recommended practices were also observed in Diane’s approach to teaching mathematics, including high academic expectations. High academic standards and marking of students’ work was a major feature of Diane’s teaching approach in mathematics. She regularly consulted with students one-on-one during mathematics lessons by marking their work and providing feedback on what needed to be fixed. Her expectation was that students would return immediately to fix any errors independently. Then, the consultation cycle would be repeated. Essentially, what Diane was implementing, was a method of maintaining high academic standards through short diagnostic teaching cycles. These cycles comprised a small number of set tasks requiring accuracy in answers to be completed by students then checked by a teacher.

When asked to reflect on the changes to her practice throughout the initiative, Diane noted that she felt she was on track. She did not feel that she changed her practice as a result
of the professional development sessions. Diane’s use of specific practices is outlined in Figure 24.

![Figure 24: Level of implementation of pedagogical practices in Year 3/4. Note: 0 refers to no implementation of the specific practice, 1 refers to limited implementation, 2 refers to moderate levels of implementation, and 3 refers to high levels of implementation.](image)

In relation to the lack of change in Diane’s practice, it was observed that Diane did not change her practice of requiring students to copy out a set of multiplication facts for an extended period of time at the beginning of a lesson. Diane continued this pedagogy despite both the practice recommendations from literature, and discussion of data findings indicating that a shift in practice may provide better academic gains for students. When asked to comment on how Diane utilised the knowledge of the student data provided from the initial analysis of findings from this study, Diane noted that she used it “probably not as much as I should have. I did look at it, I did glance over it...I guess I was stuck on sticking to my program”.
In relation to teaching number facts, Diane began every mathematics lesson with number fact tasks. These tasks included students’ copying a specific times table fact out four times (e.g., writing out the 7 times tables four times), and answering ~30 number fact questions. Diane implemented a sequenced approach for students’ studies of times tables, with students first copying, practicing, and learning their 1, 2, 4, 5, and 10 times tables, followed by the 3, 6, 8, 9, and 12 times tables. Students were tested on the relevant set of times tables that they were learning for the week on Friday to determine if they were ready to move on to the next set of times table facts. In relation to the number fact questions, these were levelled sets of questions beginning with addition and subtraction mental sums, moving to more complex sums and the inclusion of multiplication and division mental sums. These tasks were not timed, and the teacher marked students’ responses to the set of questions at the conclusion of every day and, if all questions were correct, the student would be moved to the next level of questions in the following lesson. For students who did not answer all of the questions correctly, the teacher would consult with students in the following lesson to discuss which problems were answered incorrectly and to support students’ in overcoming their difficulties by briefly discussing strategies. The number fact component of each mathematics lesson constituted a major portion of the learning time (~30 minutes of a 60 minute lesson). This approach to teaching number facts remained consistent throughout the duration of the initiative.

Diane’s approach to teaching computations included explicit instruction on the traditional, vertical algorithms for addition, subtraction, multiplication, and division computations. This approach aligned with the recommended pedagogies, and Diane used consistent place value language within this approach (see Booker et al., 2014). The development of students’ understanding of computations, renaming, and the associated algorithms was not developed with the use of concrete materials. In terms of allocating practice questions to students, Diane allocated small quantities of problems and focused on accuracy in students’ answers. Often the allocated tasks were followed by students bringing their work to the teacher for conferencing where the teacher would mark and discuss their attempts, or Diane would carry out whole class modelling of the responses focusing on checking for student understanding throughout. Similar to Year 2/3, the nature of the four types of addition and subtraction problem structures, or other problem structures for multiplication or division were not included in Diane’s instruction.
Within the Year 3/4 class, there was no envisioned or enacted specific focus on problem-solving tasks other than utilising Polya problem solving posters (see Figure 10) to discuss the steps of solving a problem. However, it was noted that students were introduced to some strategies such as circling the key information in questions, and discussion of question types to explore what information might provide insight into which operation was appropriate to solve the task.

5.3.3.5 Alignment of teaching approach to recommendations

As discussed above, Diane’s pedagogic approach focused on explicit instruction. Group work was not a feature of instruction. In terms of the quantity of work set, Diane set small quantities of work (i.e., questions) for students to complete as part of a mastery teaching approach implemented as diagnostic teaching cycles. Similar to the Year 2/3 class, the continual checking of student work and continuous feedback cycle with the teacher was seen to foster a learning environment where most students strived for accuracy. Diane did follow recommendations concerning the need to develop ideas conceptually at the introductory phase of learning following the recommended pedagogy and language for teaching algorithms. Overall, the combination of Diane’s consistent lesson structure, high behavioural standards, and high academic standards was observed to result in high levels of time on task for students in Year 3/4.

Though these elements of practice in Year 3/4 aligned with recommendations, elements such as extensive untimed number facts tasks were also observed and were not changed throughout the initiative, despite the recommendations. Further, concrete resources were not regularly drawn upon to develop place concepts or understanding of algorithms.

5.3.4 Year 4/5: Jane

Information concerning Jane’s demographics, perceptions, and teaching approaches is presented in the following sections.
5.3.4.1 Demographics

The teacher from Year 4/5, Jane, was a female teacher who did not identify as Indigenous. Jane’s qualifications consisted of a Bachelor of Education. Jane had been teaching for the past 15 years and had worked at the sample school for two years.

5.3.4.2 Perceptions of the school, students, and mathematics

Discussions with Jane indicated that she did not feel that there were tangible differences between the sample school and other schools where she had taught, as these schools primarily consisted of students from low-socioeconomic backgrounds. However, Jane did identify that behaviour was the biggest challenge in teaching mathematics at the sample school. In relation to Jane’s perceptions of the academic capabilities of students in her class, Jane commented:

Yeah, most of them. Except for, like any school, where you've got some children that will really struggle to get where they should be. But academically there's nothing really wrong there. Yeah, there's no reason why they shouldn't succeed.

This comment indicated a positive view of student capabilities. However, Jane included conditions in her comment signifying that it may not be all students who, potentially, are capable in mathematics. When asked whether she was confident in teaching mathematics, Jane stated: “not really, I'm more confident teaching English or HASS [History and Social Science] or something [other] than maths… I wouldn't just get the book open and feel confident teaching it without going over it first”.

Jane felt that group work was her strength in teaching mathematics despite not utilising this practice throughout the initiative. When asked to reflect on her weaknesses in teaching mathematics, Jane stated: “probably just knowledge without going over it first. Like if someone was to ask me something and I hadn't gone over that thing, I might not know the answer depending on what grade level”. This evidence indicates that Jane lacked confidence in relation to her mathematical content knowledge.

5.3.4.3 Overview of teaching approach

Jane’s approach to teaching the Year 4/5 class during mathematics was exemplified by use of routine copying tasks (i.e., copying out times tables), as well as high quantities of
mathematics questions for students to complete individually. Jane was not observed to be highly interactive with individual students during mathematics lessons. Consultation at the beginning of the initiative revealed that Jane’s envisioned structure for mathematics lessons involved:

1. Timed number fact tasks, followed by whole class strategy discussion for number facts (mainly focusing on multiplication facts).
2. Place value questions given on whiteboard to be completed individually by students, followed by whole class discussion of answers.
3. Explicit instruction on the focus concept for the lesson (e.g., “I do, You do, We do” modelling by the teacher on the whiteboard).
4. Independent work focusing on answering questions related to the focus concept: Approximately 10-20 computation questions assigned on the classroom whiteboard.

The actual enacted lesson structure for the Year 4/5 class during the initiative was similar to the intended structure. However, the use of timed number fact tasks and the inclusion of place value questions were implemented in daily mathematics lessons only sporadically. Jane described her classroom routine throughout the initiative as follows:

We usually do - the first 10 minutes, say, would be maybe times tables. Then the next we might spend five minutes - or vice versa - on place value. Then basically get into whatever strand we're learning. Then we usually mark it as a class and go over it as a class or group. If I've got a group and I'm by myself, I'll rush and put something on a little whiteboard for the real slow ones, yeah, and then they'll do that separately. Then we usually just mark it as a class.

What Jane outlined in her description of her practices aligns with the first two steps of the envisioned lesson structure. When Jane discussed rushing to put tasks on another whiteboard for slower students, she was referring to providing alternative questions to students who may struggle to complete the other set place value questions.

Jane’s lessons consistently included students copying out times tables at the beginning of the lesson for 10 to 15 minutes. She persisted with this practice despite the recommendations
from literature and discussion of data findings indicating that academic gains were not being realised through this pedagogic strategy. In relation to this strategy, Jane identified:

I was doing writing, but I think that's a bit of a waste of time sometimes because only certain ones will write it... Yeah, only certain ones write it and the rest go, oh I did it, but really, they did it yesterday or something.

It was observed that Jane was reluctant to abandon this practice as it was a key behaviour management technique that was used to settle the class at the beginning of lessons. It was also a pedagogic strategy that did not require planning, or set up time, so was simple to implement for the teacher. Jane commented: “some of that was just to settle the class, but it was really just a waste of time”.

Another consistently observed element of Jane’s enacted pedagogy was the limited checking of individual student work. Additionally, there was very limited engagement between the teacher and individual students during the lesson. Jane’s use of specific practices throughout the initiative is outlined in Figure 25.

Figure 25: Level of implementation of pedagogical practices in Year 4/5
Note: 0 refers to no implementation of the specific practice, 1 refers to limited implementation, 2 refers to moderate levels of implementation, and 3 refers to high levels of implementation.
Although Jane did draw from explicit instruction practices in attempting to develop students’ understanding of algorithms conceptually, often this teacher’s modelling strategies were very brief and procedurally focused. After modelling a mathematical procedure, Jane usually followed up with approximately 10-20 computations on the classroom board assigned for students to complete individually. For the most part, students were expected to complete these tasks without assistance, and frequently answers were not checked or corrected.

5.3.4.4 Pedagogical approaches for number facts, computations, and problem-solving

Jane’s approach to teaching number facts in Year 4/5 consisted of beginning majority of mathematics lessons by having students’ copy a specific times table (e.g., 7 times tables) four times. This task was usually bound by a certain time limit for students to complete it in (e.g., 10 minutes) with the expectation that students would complete the task independently and silently. The allocation of the times tables to be copied that day was not specific, and generally students focused on one set of times tables for the duration of the week. Strategy discussion was encouraged in the recommended practices at the beginning of the initiative, and during the professional development sessions partway through the initiative. The researcher also provided modelling in Year 4/5 of how to include strategy discussion in lessons, however strategy discussion was not integrated as a regular part of mathematics lessons in Year 4/5 throughout the initiative. There was only irregular inclusion of timed number fact tasks, which consisted of 50 sequentially levelled single-digit multiplication fact questions.

In relation to teaching computations, Jane did focus on explicit instruction to develop students’ proficiency with the traditional, vertical algorithm for all four operations. Whilst Jane did attempt to follow the recommended practice of consistent place value language when completing the algorithms (as outlined by Booker et al., 2014), this was sometimes substituted by “short cut” methods, focusing on the procedural aspects of the algorithm. For example, an acronym for the steps of the division algorithm (DMSB, meaning Divide, Multiply, Subtract, then Bring down) was taught to students. Following any teacher modelling, Jane would then assign various computation problems on the class whiteboard (e.g., 4567+3217=?, 7894-1239=? etc.). This featured approximately 10-20 questions. Jane did not regularly mark students’ attempts to assigned computation problems. When many students were struggling to solve any of the computation questions, Jane would model the answers to the whole class. The development of students’ understanding of computations, renaming, and the associated
algorithms was not developed with the use of concrete materials. There was no instruction on the types of problem structures for addition, subtraction, multiplication or division. Similar to Year 3/4, there was no envisioned or enacted specific focus on problem-solving tasks. Problem-solving was not noted as a consistent feature of any mathematics lessons other than utilising Polya problem solving posters (see Figure 10) to discuss the steps of solving a problem.

5.3.4.5 Alignment of the teaching approach to recommendations

Several of Jane’s teaching practices in mathematics did not align with recommended pedagogies. In particular, the limited inclusion of strategy discussion in relation to times tables (number facts) was prevalent throughout the initiative. Whilst Jane did utilise the recommended algorithm pedagogy and language moderately, and she did attempt to develop students’ conceptual understanding at the introductory stage, there was no use of concrete materials and limited use of a mastery teaching approach (i.e., utilising weekly testing to inform teaching). Students in this class did spend a moderate amount of time on task. However, primarily, this time was focused on routine tasks such as copying times tables or answering computation sums which were not checked by the teacher.

5.3.5 Year 5/6: Paul

Information concerning Paul’s demographics, perceptions, and teaching approaches is presented in the following sections.

5.3.5.1 Demographics

Paul, the teacher of Year 5/6, was a male teacher who identified as Aboriginal. Paul possessed a Bachelor’s degree in secondary education in health and physical education from an Australian institution. Paul had been teaching for fourteen years and had competed all of his teaching at the sample school.

5.3.5.2 Perceptions of the school, students, and mathematics

Paul stated that the primary difference between this school and others was the culture of the school.
I think that it's the whole laidback culture of the school. I'm not saying it's like really, really relaxed. But I'm saying that the way that we communicate with each other, especially with staff, especially in how we communicate with our kids, I think it's a whole lot different to what you would do in a Queensland or mainstream school.

This statement is perhaps suggestive of the culturally safe environment provided by the nature of the school being community owned. In relation to students’ capabilities to achieve in mathematics, Paul reflected:

I'd love for them to achieve as highly as each individual could. So, I don't expect some kids in my class to achieve A's, because I know for a fact they're not going to get there. I just want them to improve their own knowledge and just make huge improvements within themselves…Our kids come from these really bad backgrounds and the biggest thing that I kind of look for in kids is improvement…We don't expect them to be scholars, but we look for improvement. I think that's one of the biggest things here at the [redacted] school that is a great thing that we have. Of course, I do have high expectations of them, but sometimes giving them high expectations is very, very challenging.

In relation to students’ capabilities to meet standards prescribed by the national curriculum, Paul stated: “I think some kids will never get there. But I think there's a lot of kids here that are very capable, but to get there would be a lot of effort and a lot of support”. These statements from Paul suggests that he holds a mixed view on student capabilities with the suggestion that the hurdles for some students to achieve national standards are potentially too great.

When commenting on how he felt about teaching mathematics, Paul discussed how he felt confident with basic mathematics such as operations. However, he was not confident with concepts associated with Year 5 and 6 mathematics standards. Paul stated:

I'm confident with the basics of maths, like with the whole operations and stuff like that. When it comes into the whole fractions and all that type of stuff, I have to seek support to teach it… Because I didn't like it at school, probably because I didn't have an understanding of them at school.

He self-identified his strength in teaching mathematics as making mathematics fun with the way he communicates with children by making it “more relevant to their type of language that they’re used to”. In describing his weakness, Paul stated that it was “definitely the planning part… probably not having enough knowledge of the certain topics that I need to teach”.
5.3.5.3 Overview of teaching approach

Paul’s approach to teaching during mathematics was similar to the Year 4/5 teacher’s (Jane’s) approach. Classes featured some use of copying tasks (i.e., copying out times tables), and high quantities of mathematics questions given on the class board for students to complete individually. Paul was not highly interactive with individual students during mathematics lessons. Solutions to set questions were provided sporadically, and Paul did not check through solutions with the class. As well, individual student’s work was not checked regularly. Consultation at the beginning of the initiative revealed that Paul’s envisioned structure for a typical mathematics lesson involved:

1. Strategy discussion regarding multiplication facts (5 minutes)
2. Timed number fact tasks focused on multiplication facts (5 minutes)
3. Place value questions centred on focus areas revealed in pre-testing data – questions given on board to be individually completed by students, followed by whole class discussion of answers (10 minutes)
4. Explicit instruction on the focus concept for the lesson, following a “I do, we do, you, do” structure (10 minutes)
5. Independent student work – questions (mainly focused on the four operations) given on the class board to be completed individually by students. Teacher to work with a small group, or check on individual student’s understanding during this time (30 minutes)

It can be observed from the envisioned structure that Paul planned to structure mathematics lessons that focused predominantly on independent, individual student work. During observations, Paul’s enacted lesson structure near the beginning of the initiative typically followed the structure:

1. Copying out of times tables (10 minutes)
2. Timed number fact task focus on multiplication facts (5 minutes)
3. Place value questions (10 minutes)
4. Independent student work completing questions given on the board.
During observations, one of the most significant differences between Paul’s envisioned and enacted pedagogy was the limited interaction with students during independent work time. During lessons he had little interaction with students, did not check individual work, and only occasionally worked with individual students. In most lessons the students worked independently while Paul remained at his desk working on other tasks. In lessons where mental math questions were not given, the transition between copying out multiplication facts and beginning board work was often non-specific, meaning that students were often directed to begin the questions on the board only when they were finished copying out the set facts. It was observed that this often meant some students were still copying out facts 20-30 minutes into a lesson while others had finished all of the board work. Paul’s interaction with students often only involved brief statements such as “come on, let’s go, keep doing your work”. In one instance, when a student was experiencing difficulties saying “I don’t get it, my answer doesn’t make sense, this isn’t working”, Paul did not come over to offer any assistance initially and stated (in reference to a particular problem-solving question) “what don’t you get? This is an everyday situation, figure it out”. During this particular lesson, half an hour of the lesson had passed before Paul interacted with any individual students.

At the beginning of the initiative, Paul also had limited inclusion of strategy discussion concerning multiplication facts, despite this being part of the envisioned lesson structure. Another key difference was the inclusion of routine copying out of times tables tasks to begin the lesson, despite the practice recommendations. Similar to Jane, Paul also was reluctant to abandon this practice throughout the initiative as it was a key behaviour management technique that was used to settle the class at the beginning of lessons. It was also a pedagogic strategy that did not require planning, or set-up time so was simple to implement for the teacher. Paul was observed to use the time students spent copying out times tables to write up the questions that students were to complete during independent work time on the class board. Additionally, Paul intended to use weekly testing to inform teaching in his envisioned mathematics structure. However, this was not observed as a part of his enacted weekly routine. Paul’s use of specific practices throughout the initiative is outlined in Figure 26.
It can be seen that Paul did utilise timed number fact tasks throughout the initiative, but this was not done in conjunction with regular strategy discussion. Teacher modelling through explicit instruction to develop students' conceptual understanding was also limited in lessons.

5.3.5.4 Pedagogical approaches for number facts, computations, and problem-solving

Similar to Year 4/5, Paul’s approach to teaching number facts in Year 5/6 regularly involved beginning mathematics lessons by having students copy a specific times table (e.g., 7 times tables) four times with a particular time limit (e.g., 10 minutes). The allocation of the times tables to be copied that day was also not specific, and generally students focused on one set of times tables for the duration of the week. Strategy discussion was not included in lessons.
despite practice recommendations and modelling provided by the researcher. In relation to the recommendation to include timed number fact tasks, mathematics lessons in Year 5/6 included infrequent inclusion of such tasks, consisting of 50 sequentially levelled single-digit multiplication fact questions.

In relation to teaching computations, there was infrequent teacher modelling of addition, subtraction, multiplication, or division computations however the teacher did attempt to follow the recommended practices related to computations (as outlined by Booker et al., 2014), including use of consistent place value language when teaching the traditional vertical algorithm. After the number fact tasks (copying times tables), students were typically assigned computation questions on the class whiteboard (~20 questions) to complete independently. Students’ solutions to these problems were not checked, and answers were infrequently provided or modelled. There was no instruction on the types of problem structures for addition, subtraction, multiplication or division. Similar to Year 3/4 and 4/5, there was no envisioned or enacted specific focus on problem-solving tasks. Problem-solving was not noted as a consistent feature of any mathematics lessons other than utilising Polya problem solving posters (see Figure 10) to discuss the steps of solving a problem.

5.3.5.5 Alignment of teaching approach to recommendations

Paul was observed to have little implementation of many recommended pedagogies throughout the initiative. Similar to the Year 4/5 class, students in the Year 5/6 class spent a moderate amount of time on task, however this was primarily focused on routine tasks such as copying times tables or answering computation sums. These tasks were not checked by the teacher.

A mastery teaching approach facilitated by weekly testing, though part of the envisioned lesson structure, was not enacted. This practice, in conjunction with the limited explicit instruction and teacher modelling, suggested that Paul either had a reluctance to, or was unable to, scaffold students’ conceptual understanding of mathematical concepts. Also, concrete materials were not drawn upon to support students’ understanding in mathematics, and the majority of lessons focused on routine computations.
5.3.6 Summary

A critical finding during the enactment of the initiative was that the way in which professional development time was organised was essential for teachers’ engagement in the sessions. All teachers involved in the professional development identified that it was useful. However, after-school professional development resulted in limited engagement and interest by the teachers. In contrast, when the school facilitated professional development time during school hours, teachers were highly engaged and enthusiastic about the sessions. Further, there was consensus that worthwhile professional development sessions needed to be aligned to the context of individual classes, and utilise real data. Nevertheless, despite teachers identifying that these sessions were useful and important, the extent to which teachers implemented the recommended pedagogies was varied.

The findings indicated that there are two potential explanations for teachers’ resistance to implement recommended pedagogies in their classrooms. Firstly, changing teaching practice is not a simple process that is achievable by only conveying key information regarding effective pedagogical practices. Communications with all teachers in the study revealed that the prevailing attitude was that there had to be a clear “pay-off” to stimulate them to adopt a new practice, or change to an alternative practice. Primarily, what this “pay-off” meant was that the new or alternative practice essentially had to be easier to implement in the classroom than the previous practice. It was observed that if a currently utilised practice was easier to implement than an alternative recommended practice from literature, the recommended practice was unlikely to be adopted in the classroom. For example, using pre-made worksheets available online or in textbooks for the large portion of mathematics lessons is much easier to implement in terms of preparation time and instructional effort compared to explicit instruction. This observational evidence suggested that teachers would take the path of least resistance when choosing their pedagogical approaches to teaching mathematics.

The second potential reason of teachers not adopting specific practices was found to be related to behaviour management in the classroom. For example, some practices that were not recommended had a significant pay-off in terms of student behaviour, or student compliance during mathematics lessons. For example, setting the task of routinely copying out multiplication facts multiple times for 10 to 30 minutes was observed to have the effect of immediately settling the class and reducing noise levels. This condition provided teachers with time to complete other tasks as students did not need immediate assistance. The approach of
copying multiplication facts was discussed during the second professional development session when the data indicated that students’ proficiency with multiplication facts was below what would be expected for all year levels by national standards (ACARA, 2018b). This was despite the Year 3/4, 4/5, and 5/6 classes having adopted the practice of copying multiplication facts for over a term. Considering the substantial time that copying these facts took out of mathematics lessons, alternative practices which had a higher expected pay-off for students’ learning, as indicated by critical literature, were suggested. However, teachers in the Year 3/4, 4/5, and 5/6 class were observed to continue to employ the practice of copying tables.

A key element that contributed towards the initiative being received positively by the teaching staff was the researcher’s established and sustained role in the school. The role of the researcher as an insider to the school meant that a professional rapport had been established with teachers, and teachers were willing to work productively and engage with the researcher. In addition, the regular presence of the researcher in the school enabled a flexibly delivery of the initiative where adjustments could be applied when the need arose (e.g., the subsequent professional development sessions that were not anticipated originally).

The school’s support of the initiative by the Principal was also important particularly in relation to backing the goals and purpose of the initiative to teaching staff by reiterating that the school administration (i.e., the Principal) believed that the initiative was a worthwhile and valued undertaking. The school also facilitated the professional development during class time, with this professional development implementation being well received in comparison to after-school sessions. Without these sessions, it is possible that reception and uptake of the recommendations guiding the initiative would have been different to what was observed. Overall, creating change in teaching practice was found to be a gradual, long-term pursuit that needed constant drive, evaluation, and re-evaluation.
5.4 Student demographic data

Demographic information of the students in each class is outlined in Table 9, including the breakdown of the number of students in each grade level in each composite class.

Table 9
Number of students in each grade and numbers of female and male students in each class

<table>
<thead>
<tr>
<th>Total n</th>
<th>Year 2/3</th>
<th>n (Year 2) = 6</th>
<th>n (Year 3) = 6</th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td></td>
<td></td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Year 3/4</td>
<td>12</td>
<td>n (Year 3) = 4</td>
<td>n (Year 4) = 8</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Year 4/5</td>
<td>12</td>
<td>n (Year 4) = 3</td>
<td>n (Year 5) = 9</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Year 5/6</td>
<td>14</td>
<td>n (Year 5) = 6</td>
<td>n (Year 6) = 8</td>
<td>9</td>
<td>5</td>
</tr>
</tbody>
</table>

Excluding Year 2/3 where the split of students’ ages is equal between Year 2 and Year 3, all other composite classes contain more students in the higher-grade level than the lower.

Students’ participation in school mathematics throughout the course of the initiative was ascertained by observing attendance rates from January to September, therefore accounting for the first three terms of the school year. Table 10 describes the average attendance rates for each cohort. The average attendance rate for all students was 87%. The 2019 Close the Gap report (Department of the Prime Minister & Cabinet, 2019) reported that, in 2018, attendance rates for Indigenous students in Australia were approximately 82%. Therefore, the sample classes in this study reported a higher average attendance rate than National Indigenous rates, however, still fell below the average attendance rate of 93% for non-Indigenous students (Department of the Prime Minister & Cabinet, 2019).

Table 10
Attendance data: Average attendance rates as percentages for each cohort

<table>
<thead>
<tr>
<th></th>
<th>Average attendance rate for Term 1 to Term 3 (%)</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2/3, n=12</td>
<td>80.7</td>
<td>21.1</td>
</tr>
<tr>
<td>Year 3/4, n=12</td>
<td>92.8</td>
<td>4.3</td>
</tr>
<tr>
<td>Year 4/5, n=12</td>
<td>86.7</td>
<td>10.7</td>
</tr>
<tr>
<td>Year 5/6, n=14</td>
<td>87.6</td>
<td>12.2</td>
</tr>
<tr>
<td>Total average, n=50</td>
<td>86.9</td>
<td></td>
</tr>
</tbody>
</table>
The Year 4/5 and 5/6 classes reported similar attendance rates from Term 1 to 3, varying by less than 1%. The Year 2/3 cohort reported the lowest attendance rate with the highest standard deviation, and was the only class that reported an average attendance rate below the average for Indigenous students in Australia. The Year 3/4 cohort reported the highest attendance rate with the smallest standard deviation which was close to equal to the average national attendance rate for non-Indigenous students.

5.5 Changes in students’ mathematics proficiency

In the following section, changes in students’ mathematics proficiency throughout the mathematics initiative as measured by the PAT-M, NAPLAN, diagnostic tests, and Newman interviews will be presented. Each of these data sources examines students’ mathematical proficiency at different depths. The PAT-M provides a broad, standardised measure of students’ achievement. NAPLAN findings provide further support to verify PAT-M findings. The diagnostic tests further explore students’ proficiency with specific mathematical concepts (i.e., place value, computations, and problem-solving). Error analysis of students’ responses on diagnostic tests, in conjunction with Newman interviews, provides detailed information on causes of students’ mathematical difficulties. The findings from this analysis provide answers to Research Questions 2a and 2b. These findings also guided the identification of relevant factors associated with the initiative that may have influenced students’ proficiency, which is relevant to answering Research Question 3.

5.5.1 PAT-M findings: Overview

The following section will outline an overview of the results from the PAT-M for each class. Additionally, an analysis of how achievement changed in each of the four tested mathematics strands (number, geometry, measurement, and statistics) is provided. The pre- and post-initiative results for each class will be reported and compared to the national norming sample expected means. Due to the composite age classes, comparisons of students from the same year level will also be made between the classes (e.g., Year 3 students in Year 2/3 and Year 3/4 will be compared). As outlined in the previous chapter, making comparisons between classes in this study is difficult due to the composite classes. Furthermore, comparisons of students in the same year level assists in making more accurate comparisons as these students
will be the same age, and typically will have completed the same amount of years in school. Also, effect sizes will be calculated and related to effect size comparisons in literature. Cohen (1988) classified effect sizes of 0.20 as small, 0.50 as medium, and 0.80 as large. Other literature has found 0.40 to be the average effect size expected during a school year (Hattie, 2012).

Detailed reporting of the PAT-M findings is reported in Appendix D, E and F. At the beginning of the initiative, the mean score for each class was below the norming sample mean. By the end of the initiative, all classes reported positive gains in mean scores. For the Year 2/3, 3/4, and 5/6 classes, the gap between the classes’ mean score and the norming sample reduced over the course of the initiative. The Year 3/4 class reported the largest reduction in the distance from the norming sample mean with the gap reducing by over quadruple the difference reported at the beginning of the initiative.

5.5.1.1 PAT-M results: Year 2/3

Figure 27 graphically outlines the change in scores throughout the initiative compared to the norming sample for Year 2/3. The Year 2/3 cohort experienced a similar growth rate to that of the national norming sample.

![Graph of the growth in PAT-M achievement for Year 2/3 compared to the norming sample, n=11.](image-url)
At the beginning of the initiative, the Year 2/3 cohort was performing below the standard of a late Year 1 student (i.e., below the norming samples October mean for Year 1). By the end of the initiative, the Year 2/3 cohort was performing at an equivalent standard to an early Year 2 student from the norming sample (i.e., similar expected score to that of a Year 2 student in March). This equated to over 6 months improvement within the timeframe of the 7-month initiative (the exact improvement in terms of time is difficult to identify as October of Year 1 is the first administration of the PAT-M to the norming sample). The standard deviation for the Year 2/3 cohort also decreased during the initiative meaning that there was a reduction in the spread of scores for this class. Overall, the Year 2/3 cohort had an effect size of 1.26 for the initiative, and the comparative norming sample effect size was 0.38. Comparing the effect size found for Year 2/3 to Hattie’s (2012) classification, the high effect size of 1.26 was above the zone of desired effects for an educational intervention.

Of the four tested mathematics strands (number, geometry, measurement, and statistics), further analysis revealed that, on the pre-test, the Year 2/3 cohort performed highest in geometry. However, this was a similar pattern observed in the norming sample so may be attributed to the ease of the specific geometry test items. On the post-test, the Year 2/3 class only experienced improvement in the geometry strand with declines in achievement for all other areas. The improvement of 20% in geometry was well above (16% above) the comparative improvement recorded in the norming sample.

Changes to individual students’ scores throughout the course of the initiative were also analysed. When observing the changes to individual student’s achievement, it was found that the three of the lowest achieving students reported the largest gains in achievement. The lowest achieving student reported the largest gains (equivalent to more than 9-months improvement within the timeframe of the 7-month initiative), and scored above the class mean by the end of the initiative. These findings indicated that the initially lowest achieving students in Year 2/3 substantially improved throughout the initiative, and the result was a halving of the class standard deviation.

5.5.1.2 PAT-M results: Year 3/4

Figure 28 graphically outlines the change in scores throughout the initiative compared to the norming sample for Year 3/4. The Year 3/4 cohort experienced a more substantial
improvement in a shorter timeframe than the national norming sample. In a relatively short timeframe, the gap in achievement for this cohort had nearly closed.

The pre-test findings found that the Year 3/4 cohort achieved a standard equivalent to an early Year 2 student from the norming sample (February of Year 2). At the end of the initiative, the Year 3/4 cohort achieved a similar standard to that of a mid-Year 3 student (August of Year 3). This equated to an improvement of approximately 1 year and 2 months within the 7-month initiative, twice the expected gain, indicating that this cohort had closed the gap in achievement with the national norming sample throughout the course of the initiative. On average, Year 3 students in the sample were only 2 months behind the age appropriate standard by the end of the initiative, and Year 4 students were 14 months behind the age appropriate standard (as opposed to 2-years and 7-months at the beginning of the initiative). Overall, the Year 3/4 cohort reported a high effect size of 1.36, which is above the zone of desired effects, and was the largest of any class group in the sample. Comparatively, the norming sample effect size was 0.32. Comparing the effect size reported for Year 3/4 and the equivalent effect size for the norming sample indicates a significant effect for this class.

Figure 28: Graph of the growth in PAT-M achievement for Year 3/4 compared to the norming sample, n=11.
Detailed analyses of which strands of mathematics accounted for the substantial growth in achievement revealed that the Year 3/4 cohort improved in all strands (number, geometry, measurement, and statistics) by 12% or more over the course of the initiative. Comparing this to the norming sample, there was an increase in achievement equivalent to or above that of the norming sample for all strands. Overall, similar to the Year 2/3 cohort, the Year 3/4 students improved most markedly in geometry. This class was the only cohort that experienced no decline in improvement for any tested strands.

When observing the changes to individual students’ scores in Year 3/4, it was found that there was a large spread of scores at the beginning of the initiative with students ranging in ability from below a late Year 1 standard to an early Year 3 standard. By the end of the initiative, the two lowest achieving students in Year 3 reported the largest gains in achievement. Similar to the Year 2/3 cohort, this indicated that the lowest achieving students in Year 3 experienced the greatest improvement. One of the initially lower achieving Year 3 students met the Year 3 national norming sample standard by the end of the initiative, equating to a gain of 2 years over the course of the 7-month initiative. Overall, 75% of the Year 3 students (3 students out of 4) were meeting the Year 3 expected standard by the end of the initiative, compared to only 25% meeting the expected standard at the beginning of the initiative. The highest achieving student doubled the amount by which they exceeded the national norming sample mean by the end of the initiative (equating to two years improvement) indicating that the highest achieving students in this group also experienced significant gains over the course of the initiative. Overall, these findings demonstrate that both the lower and higher achieving students in this group experienced substantial improvements over the course of the initiative.

5.5.1.3 PAT-M results: Year 4/5

Figure 29 graphically outlines the change in scores for Year 4/5 throughout the initiative compared to the norming sample. The Year 4/5 cohort experienced a similar growth rate to that of the national norming sample.
Figure 29: Graph of the growth in PAT-M achievement for Year 4/5 compared to the norming sample, n=12.

At the beginning of the initiative, the Year 4/5 cohort was performing at a level similar to that of an early Year 3 student (February of Year 3) when compared to the national norming sample. By the end of the initiative, the Year 4/5 cohort was performing at an equivalent standard to that of a mid-Year 3 student (approximately July of Year 3). This equated to 6-months improvement, which is less than the timeframe of the 7-month initiative. The standard deviation for this cohort remained consistent, changing from 6.86 to 6.21 throughout the initiative. This standard deviation means that approximately 68% of the students’ scores varied by over one year in achievement, and this did not change substantially throughout the initiative. Overall, the Year 4/5 cohort reported a medium effect size of 0.48 for the initiative, compared to the norming sample effect size of 0.27. This effect size is similar to what would be expected during an intervention as indicated by literature, and was within the zone of desired effects.

Further analysis of students’ achievement by mathematics strand (number, geometry, measurement, and statistics) found that the Year 4/5 students performed below the late Year 3 national norming sample students in each tested strand at the beginning of the initiative with the greatest achievement being reported in geometry. The post-test findings indicated that the Year 4/5 cohort declined or experienced no improvement in achievement for each tested strand over the course of the initiative. Overall, the gap in achievement between the Year 4/5 cohort and the norming sample also increased for the number, geometry, and statistics strands.
However, the Year 4/5 students experienced the greatest positive change in achievement in the measurement strand over the course of the initiative.

Analysis of individual student achievement for this class revealed that there was a large spread of scores in this class at the beginning of the initiative. None of the students in the Year 4 or 5 groups achieved scores equivalent to or above the national norming sample means at the beginning of the initiative. The pre-initiative findings demonstrate that, within the Year 5 group alone from this class, there was a spread of two years in achievement. On the post-test PAT-M, it was found that the lowest achieving student from Year 4 and Year 5 on the pre-testing experienced the largest gains in improvement (gains of 9 months and 16 months respectively). However, it was also found that not all students from the Year 4/5 cohort experienced gains in achievement. This data indicates mixed success in raising students’ proficiency in this class with a small number of students improving substantially during the initiative, and other students reporting little improvement, or declines in achievement. When considering the whole class, this result demonstrates that, other than the two lowest achieving students in Years 4 and 5, students in this class did not increase their proficiency substantially over the course of the initiative.

5.5.1.4 PAT-M results: Year 5/6

Figure 30 graphically outlines the change in scores throughout the initiative compared to the norming sample for Year 5/6. The Year 5/6 cohort experienced more substantial improvement than the national norming sample during the initiative. However, this group still achieved below the expected standard for the age group by the end of the initiative.
For the Year 5/6 cohort, the pre-testing findings reported that they achieved a standard equivalent to a late Year 3 student from the norming sample (November of Year 3). At the end of the initiative, the Year 5/6 cohort achieved a similar standard to that of a mid-Year 4 student from the norming sample (August of Year 4). This equated to an improvement of approximately 10 months within the 7-month initiative. The standard deviation for this class increased throughout the initiative, moving from 12.16 to 15.26 during the course of the initiative, meaning that the spread of students’ scores in the class increased. This was equivalent to the spread of scores at the beginning of the initiative varying by approximately 3 years and 6 months, and the spread of scores varying by over four years by the end of the initiative. Overall, the Year 5/6 cohort had a small effect size of 0.37, the second smallest effect size reported in the study. Comparing this to the norming sample effect size of 0.24 confirms that the sample students did report a more substantial improvement during the timeframe than the norming sample. However, the class effect size was not within the zone of desired effects (≥0.40), as reported in literature.

Analysis of the results by mathematics strand for the pre-test indicated that the Year 5/6 cohorts average scores were below the reported averages for the norming sample (late Year 3) for each strand. Analysis of the post-test indicated that the Year 5/6 cohort declined in average achievement in the number, geometry, and statistics strands. However, the gap between the class and the norming sample reduced for all strands excluding statistics. Overall, this class only experienced positive improvement in the measurement strand.
When observing individual scores, it was found that many Year 5 students were achieving slightly below the national mean at the beginning of the initiative. The Year 6 students in this cohort had a particularly large spread of scores with the lowest performing student in Year 6 achieving a score below that of a late Year 1 student from the norming sample, and the highest achieving student above a late Year 6 standard, equating to over five years difference. The post-test findings indicated that the lowest performing student in the Year 5 group experienced the largest gains of 2 years and 7 months over the course of the initiative, however fewer Year 5 students were meeting the norming sample standard by the end of the initiative (one student only). One high achieving student from the Year 5 group declined in achievement by a small amount over the course of the initiative, however, all other students in this cohort experienced positive gains in achievement. In the Year 6 group, the lowest achieving student reported a significant decline in achievement, and one other Year 6 student also reported a small decline in achievement over the course of the initiative. Conversely, another low achieving student from Year 6 experienced the greatest improvement of 3 years and 2 months during the initiative. Overall, this data again indicates mixed success in raising students’ achievement, as supported by the increase in standard deviation, similar to the pattern of findings observed for the Year 4/5 class. Substantial improvements in proficiency were only found for a small number of students, and the large spread of scores was not reduced over the course of the initiative.

5.5.1.5 Comparisons across cohorts

Comparisons of students of the same ages across each of the classes were made to compare the influence of individual teachers on students’ achievement accurately throughout the initiative. Detailed information regarding the comparisons across each age group is outlined in Appendix G.

When comparing the achievement of the Year 3 students in Year 2/3 and Year 3/4, it was found that the Year 3 students in Year 3/4 scored above the Year 3 students in Year 2/3 at the beginning of the initiative. By the end of the initiative, the difference in achievement between the two groups of Year 3 students more than doubled. This indicated a substantial difference in the way in which students’ proficiency was changed over the course of the initiative in the two classes, with the Year 3/4 class outperforming the age-equivalent peers in Year 2/3.
When comparing the achievement of the Year 4 students in Year 3/4 and Year 4/5, it was found that the Year 4 students in Year 4/5 performed above the Year 4 students in Year 3/4 at the beginning of the initiative. By the end of the initiative this changed, as the Year 4 students in Year 3/4 performed substantially higher than the Year 4 students in Year 4/5. The findings indicated that the Year 4 students in Year 4/5 reported minimal changes in proficiency (less than 1-month improvement) over the course of the initiative. Comparatively, the Year 4 students in Year 4/5 reported an increase in achievement equivalent to 18-months, over double the expected achievement gain. Similar to the comparison of achievement for Year 3 students across the Year 2/3 and 3/4 classes, these findings indicated a substantial difference in the way in which students’ proficiency was changed over the course of the initiative in the two classes. Results revealed the Year 3/4 class again outperforming the age equivalent peers in Year 4/5. This supports the whole class findings from analysis of the PAT-M that found that the Year 3/4 cohort reported the largest effect size and the greatest gains in achievement over the course of the initiative.

Comparisons between the Year 5 students from Year 4/5 and 5/6 demonstrated that the Year 5 students from Year 5/6 experienced greater increases in achievement over the course of the initiative, and achieved above the Year 5 students in Year 4/5 on the pre- and post-initiative PAT-M. However, these differences were not as large as the differences in the achievement for Year 3 and 4 students across the composite classes.

These analyses serve to highlight further the differing extent to which mathematical proficiency, as measured by achievement on the PAT-M, was influenced in each of the four classes. Comparing students of similar ages highlighted that the Year 3/4 cohort experienced substantially higher increases in achievement compared to peers in Year 2/3 and Year 4/5. Whilst the Year 5 students in Year 5/6 achieved greater achievement compared to peers in Year 4/5, the differences were small compared to the differences in the other cohort comparisons suggesting that there were similar changes to achievement in Year 4/5 and 5/6.

5.5.1.6 Linking PAT-M findings to NAPLAN outcomes

A further standardised test carried out in Australian schools includes the NAPLAN numeracy test. It is useful to observe the findings from NAPLAN prior to, during, and after the conclusion of this initiative to see if these align to what was found from the PAT-M. The
numeracy NAPLAN test findings outlined in Figure 31 and Figure 32 describe students’ achievement compared to the Australian averages. The initiative was conducted during 2017.

**Figure 31**: Year 3 numeracy NAPLAN data from the sample school (ACARA, 2020)

**Figure 32**: Year 5 numeracy NAPLAN data from the sample school (ACARA, 2020)

Looking broadly at these findings, it can be seen that prior to the initiative (2017 and before), students’ achievement on NAPLAN had reduced from 2015 to 2017 for Year 3, and reduced from 2015-2016 for Year 5. The NAPLAN findings from 2017 (partway through the year that the initiative was conducted) indicated increases in students’ achievement in Year 5. However, this trend did not continue into 2019 once the initiative was not continued in the school (and long-term tracking of this data is difficult as it is likely that the student body has substantially changed two years after the initiative in 2017). Similarly, the Year 3 data indicates
an increasing trend in achievement from 2017 onwards. The notable trend from NAPLAN was that achievement continued to increase in 2018 for both year levels after students had experienced the entirety of the initiative. This supports the findings from PAT-M which indicated increases in achievement as well as a reduction in the gap between students in the sample school and non-Indigenous Australian students throughout the initiative.

5.5.1.7 Summary of PAT-M findings

Findings from this study indicated that the majority of students experienced improvements as measured by PAT-M, however, the extent of these improvements varied across the cohorts. In the Year 2/3 cohort, the majority of students experienced improvements as measured by PAT-M with the lowest Year 2 student reporting the largest gains in achievement. Similarly, in Year 3/4 all students reported improvements, with the lowest Year 3 and highest Year 4 students reporting the biggest changes in achievement. Nearly all Year 3 students from this cohort were also meeting the Year 3 national mean by the end of the initiative. In Year 4/5 and Year 5/6, the results were varied with not all students reporting gains in achievement. However, it was found that some of the lowest achieving students reported the largest improvements. When comparing students of the same age across classes, it was found that students in Year 3/4 reported larger gains in achievement compared to peers of the same year level in Year 2/3 and 4/5. Overall, these comparisons support the whole class findings from analysis of the PAT-M that indicated that the Year 3/4 class was, comparatively, the most successful in raising students’ proficiency.

5.5.2 Diagnostic test findings: Overview

In the following section, the findings from the set of diagnostic pre- and post-initiative tests will be reported. Firstly, the numeration diagnostic test will be analysed and reported, followed by the findings from the addition, subtraction, multiplication, and division computation diagnostic tests. The problem-solving findings also will be reported. The critical findings, as indicated by the diagnostic test data, are summarised in Table 11.
Table 11
Summary of written test findings

<table>
<thead>
<tr>
<th>Key finding</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Critical difficulties with place value concepts persisted throughout the initiative.</td>
</tr>
<tr>
<td>2) Number fact proficiency was below what was expected for the year level in all classes throughout the initiative.</td>
</tr>
<tr>
<td>a) A correlation between engagement in ability-based class mathematics groups and number fact proficiency (Year 2/3) was found.</td>
</tr>
<tr>
<td>3) There was limited change in proficiency with addition, subtraction, multiplication, and division computations for most classes.</td>
</tr>
<tr>
<td>4) Critical errors hindering computational proficiency were associated with:</td>
</tr>
<tr>
<td>a) Productive dispositions</td>
</tr>
<tr>
<td>b) Conceptual understanding of renaming processes</td>
</tr>
<tr>
<td>c) Procedural fluency with number facts</td>
</tr>
<tr>
<td>5) Written test findings – problem-solving difficulties were associated with:</td>
</tr>
<tr>
<td>a) Productive dispositions (not answering questions)</td>
</tr>
<tr>
<td>b) Strategic competence:</td>
</tr>
<tr>
<td>i. Difficulties identifying the correct operation</td>
</tr>
<tr>
<td>ii. Difficulties selecting the correct values from the question</td>
</tr>
<tr>
<td>Issues with procedures associated with computations were not a critical barrier to problem-solving success, suggesting that proficiency with strategic competence and productive dispositions are firstly required when problem-solving</td>
</tr>
<tr>
<td>6) These themes in findings indicated that, whilst changes to mathematical proficiency as measured by the PAT-M were substantial, changes in proficiency were varied and not as large as measured by the diagnostic tests. To describe students’ proficiency in the subsequent sections, the following terminology is used: 1) no proficiency, 2) limited proficiency, 3) developing proficiency, and 4) proficient. No proficiency refers to student/s who have experienced virtually no success with the tested questions, reflected by scores of 0, or very near 0. The rationale for this classification is that a student scoring 0 is unable to answer any questions, so does not have any proficiency with the tested concept. Limited proficiency refers to student/s who have demonstrated the ability to answer some tested questions, but success with these questions is limited. Limited proficiency is reflected by a success rate less than 50% constituting a common boundary of a “passing grade”. Developing proficiency refers to student/s who are nearing proficiency with the skills and concepts tested in questions, however</td>
</tr>
</tbody>
</table>

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complete proficiency has not yet been achieved as reflected by success rates between 50% and 80%. *Proficient* students (or students who have demonstrated proficiency) refer to students who have demonstrated near perfect results (i.e., success rates of 80% or above) when answering questions on specific skills and concepts. The rationale for developing these criteria was to have consistency in language when discussing achievement levels.

5.5.2.1 Numeration

The findings from the numeration diagnostic test revealed that students had difficulties with key place value concepts. The numeration test examined students’ proficiency with place value and renaming concepts. Numeration proficiency is necessary for computational proficiency due to the critical role place value and renaming has in mathematical computation procedures for addition, subtraction, multiplication, and division. These concepts were examined in 20 questions which were arranged by difficulty with questions beginning with 2-digit concepts moving through to 6-digit values. Examples of test questions are outlined in Table 12 below.

<table>
<thead>
<tr>
<th>Example question</th>
<th>Concept examined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write in words the number that is shown:</td>
<td>Naming numbers from base-ten materials.</td>
</tr>
<tr>
<td><img src="image.png" alt="Image" /></td>
<td></td>
</tr>
<tr>
<td>(Answer: 320)</td>
<td></td>
</tr>
<tr>
<td>Write the missing numbers: 42, 43, ___, ___, 46, ___, 48</td>
<td>Completing ascending or descending number patterns counting by one (i.e., ordering and sequencing numbers).</td>
</tr>
<tr>
<td>What does the 4 stand for in 5472?</td>
<td>Identifying place values.</td>
</tr>
<tr>
<td>(Answer: 4 hundreds or 400)</td>
<td></td>
</tr>
<tr>
<td>How many tens are in 809?</td>
<td>Identifying place values, particularly with internal zeros.</td>
</tr>
<tr>
<td>(Answer: 80 tens)</td>
<td></td>
</tr>
<tr>
<td>Round 46 to the nearest ten.</td>
<td>Rounding to the nearest ten.</td>
</tr>
<tr>
<td>(Answer: 50)</td>
<td></td>
</tr>
<tr>
<td>Write the number with: 5 hundreds, 4 thousands, 2 ones, and 6 tens</td>
<td>Place values.</td>
</tr>
<tr>
<td>(Answer: 4562)</td>
<td></td>
</tr>
<tr>
<td>Write the number eighteen thousand and forty-two.</td>
<td>Naming numbers, specifically including internal zeros.</td>
</tr>
<tr>
<td>(Answer: 18,042)</td>
<td></td>
</tr>
</tbody>
</table>
Table 13 outlines the mean scores for each cohort on the pre- and post- numeration tests. The change in mean is also detailed.

<table>
<thead>
<tr>
<th>Total questions administered</th>
<th>Pre-test: March</th>
<th>Post-test: October</th>
<th>Change in Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2/3, n=12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>2.75</td>
<td>5.25</td>
<td>+2.5</td>
</tr>
<tr>
<td>Year 3/4, n=12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>7.17</td>
<td>10.67</td>
<td>+3.5</td>
</tr>
<tr>
<td>Year 4/5, n=11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>9.45</td>
<td>12.63</td>
<td>+3.18</td>
</tr>
<tr>
<td>Year 5/6, n=14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>11.14</td>
<td>13.86</td>
<td>+2.72</td>
</tr>
</tbody>
</table>

*Note: The Year 2/3 class was only administered questions 1 to 9 on the numeration diagnostic test as the remainder of the test questions covered concepts that were beyond what is expected in the national standards for Year 2 or Year 3 students (ACARA, 2018b). During the administration of this test, the Year 2/3 students were offered the opportunity to proceed with the remainder of the questions by the researcher. None of the students were able to attempt any of the questions beyond question 9.

When observing the findings from the pre-initiative numeration diagnostic test, it was found that none of the classes demonstrated consistent proficiency with the set numeration tasks. Developing proficiency was observed in upper grades, such as Year 5/6, where students are expected to have achieved proficiency with the content tested in the numeration test according to the Australian Curriculum (ACARA, 2018b). Further, the most substantial changes in proficiency were observed in the lower classes, such as Year 2/3, which approximately doubled the mean numeration score throughout the initiative. Given the context that the Year 2/3 class was not administered all test items, and is not expected to have knowledge of many of these test items by Year 2 or 3 curriculum standards, this demonstrated notable gains in achievement. By the end of the initiative it was found that all classes reported an increase in their mean score on the numeration diagnostic test. The standard distribution for each class remained similar for both rounds of testing, however, most classes standard distribution decreased except for Year 5/6. Whether the improvements were related to an increase in achievement on individual test questions was further analysed and outlined in Appendix I.

Observation of questions where overall achievement increased throughout the initiative revealed that there were four types of questions where achievement increased by at least 20% or more. These questions included rounding, naming most two and three-digit values from materials (e.g., multi-base arithmetic blocks or bundling sticks), and ordering and sequencing
values. Other questions where achievement significantly increased included the place value question “what does the 8 stand for in 480?”.

Observations on trends in achievement on the pre- and post-initiative numeration test indicated that when asked specific place value/renaming questions students’ proficiency began to decline. When students were asked place value questions relating to 6-digit values (e.g., “how many ten thousands are there in 308 621?”), only three students in the Year 5/6 cohort could answer correctly. Specific questions where achievement decreased over the course of the initiative were related to place value knowledge (namely, a 3-digit value with internal zeros when represented with multi-base arithmetic blocks). The evidence suggested that place value knowledge involving internal zeros was specifically difficult for students, as students’ success rate naming other values represented by multi-base arithmetic blocks increased over the course of the initiative. Related to a similar, but more advanced, understanding of place value were place value questions such as “write the number that has 25 hundreds and 3 ones”. Such questions also produced a large decline in success rates. Gains in achievement were also limited on test questions associated with identifying the amount of a particular place value in a number (e.g., how many tens are in 6203). Overall, this evidence is suggestive of little improvement, or declines in improvement, in key place value knowledge across the sample of students over the course of the initiative.

Additionally, analysis of the change in the spread of test scores for each class was carried out for the numeration diagnostic test. The detailed analysis is located in Appendix I. For the Year 2/3, 3/4, and 4/5 cohorts, the spread of scores reduced throughout the initiative. Most notably, the Year 3/4 cohort reported the largest improvement in the minimum reported scores throughout the initiative, indicating significant improvements for the lowest achieving students in this group. Overall, analysing the changes in the spread of scores for each cohort indicated that all classes had improved in the minimum scores achieved on the test. This result supports the findings in the analysis of improvements in mean achievement for each class. However, most classes still reported a large range of abilities in relation to numeration.

Overall, the findings of the numeration diagnostic test are indicative of students displaying limited understanding of tested place value knowledge, and this remained largely unchanged over the course of the initiative.
5.5.2.2 Number facts

To examine students’ proficiency with number facts, 10 number fact questions were given to students orally for each of the four computations. The addition and multiplication number fact questions were single digit questions while the subtraction and division number fact questions were 2x1-digit sums (i.e., 17–4 and 15÷5). Firstly, the mean score on the ten number fact questions on the post-test will be presented in conjunction with the change in the mean score. Table 14 outlines the average scores on the number facts post-test, and the change in the mean.

Table 14
Analysis of diagnostic tests: Number facts

<table>
<thead>
<tr>
<th></th>
<th>Addition Number Facts</th>
<th>Subtraction Number Facts</th>
<th>Multiplication Number Facts</th>
<th>Division Number Facts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M (/ 10)</td>
<td>Change in M</td>
<td>M (/ 10)</td>
<td>Change in M</td>
</tr>
<tr>
<td>Year 2/3, n=12</td>
<td>8.1</td>
<td>+2.3</td>
<td>5.4</td>
<td>+2.8</td>
</tr>
<tr>
<td>Year 3/4, n=12</td>
<td>9.7</td>
<td>+1.2</td>
<td>8.1</td>
<td>+0.1</td>
</tr>
<tr>
<td>Year 4/5, n=11</td>
<td>9.1</td>
<td>+0.4</td>
<td>7.9</td>
<td>-0.5</td>
</tr>
<tr>
<td>Year 5/6, n=14</td>
<td>9.5</td>
<td>+0.2</td>
<td>9.6</td>
<td>+0.1</td>
</tr>
</tbody>
</table>

5.5.2.2.1 Addition number facts

Overall, it was found that all classes increased in proficiency with addition number facts. Year 2/3 students increased their mean score from 6 to 8 out of 10 questions, indicating that students’ achievement had moved from the developing level to nearly proficient. Year 3/4, 4/5, and 5/6 students’ mean scores on the addition number fact test indicated that most students were proficient in this domain.

5.5.2.2.2 Subtraction number facts

When observing the changes in subtraction number fact results, the Year 2/3 cohort made the most significant improvement moving from a mean score of 3 to 5.4. This is indicative of developing proficiency by the end of the initiative. This improvement is positive but the desired outcome of proficient mastery had not yet been achieved. In addition, whilst there were gains in achievement, the spread in individual scores in the Year 2/3 class did not reduce over the course of the initiative. This indicates that the initiative had not acted to
facilitate the Year 2/3 students becoming proficient in subtraction fact recall, a standard expected by national curriculum standards (ACARA, 2018b) for this year level. The average score on the subtraction number fact test for Years 3/4 and 5/6 did not increase by a large amount over the course of the initiative, however, the average scores suggested that majority of students were proficient, or nearly proficient in this domain. However, similar to the Year 2/3 class, the spread of scores in Year 3/4 was still large at the end of the initiative. Whilst the spread of scores remained unchanged in Year 3/4, half of these students scored 100% on the post-initiative subtraction fact test. This evidence indicated that the initiative did act to increase Year 3/4 students’ subtraction proficiency. The Year 4/5 cohort was the only cohort that experienced a decline in subtraction fact proficiency, and an increase in the spread of scores across the class throughout the initiative. In particular, analysis of individual Year 4/5 students’ scores found that three students who scored highly on the pre-test (scores of 9 or 10 out of 10) reported a reduction of 10% to 20% in achievement over the course of the initiative.

5.5.2.2.3 Multiplication and division number facts

All tested cohorts reported an increase in achievement for the multiplication number facts with the Year 3/4 cohort again reporting the largest gain in achievement. Previously, the Year 3/4 cohort indicated limited proficiency with multiplication facts on the pre-test, however, students’ proficiency was developing with an average score of 5.4 on the post-test. Overall, whilst the Year 4/5 and 5/6 cohorts did increase in proficiency on the multiplication fact test, students’ proficiency was still developing in these year levels, and is below what is expected by national curriculum standards (ACARA, 2018b). On the division number fact test, all tested cohorts reported an increase in achievement. The Year 5/6 cohort reported the largest increase in achievement, however proficiency was limited in this grade. Although Year 3/4 and 4/5 students also reported positive gains in achievement on the division number fact test, proficiency remained very limited in these groups.

5.5.2.2.4 Correlations between number fact proficiency and within-class ability grouping

For the Year 2/3 cohort it was found that there was a relationship between the mathematics group in which students participated and their number fact scores. The teacher-
identified mathematics groups (low, middle, and high) had large differences in the average number fact score for each as outlined in Table 15.

Table 15
Average number fact score for each mathematics group in Year 2/3, n=16

<table>
<thead>
<tr>
<th>Group</th>
<th>Average Number Fact Score (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1 - Top Group</td>
<td>88.6</td>
</tr>
<tr>
<td>Group 2 - Middle Group</td>
<td>54</td>
</tr>
<tr>
<td>Group 3 - Low Group</td>
<td>27.5</td>
</tr>
</tbody>
</table>

5.5.2.2.5 Summary

In summary, the majority of the cohorts reported improvements in the mean scores for each fact test. However, the extent of the improvements was varied. The mean scores for all cohorts indicated that students were proficient in the recall of addition number facts from Year 2 to Year 6. With subtraction number facts, whilst most grades reported means that were indicative of a developing proficiency, analysis of the spread of scores indicated a large range of proficiencies for Year 2/3, and Year 3/4. This finding was similar for the multiplication number facts test with all tested cohorts reporting a large range of scores. Whilst more students were able to answer division fact questions on the post-initiative test compared to the pre-test findings, proficiency was limited across the sample. Trends observed in the whole class findings discussed earlier were supported by analysis of the change in the spread of scores in each class (Appendix J).

5.5.2.3 Addition computation

The addition computation diagnostic test assessed students’ ability to answer 10 questions of progressive difficulty. Examples of the questions are outlined in Table 16.

Table 16
Addition computation question examples

<table>
<thead>
<tr>
<th>1) 73 + 37</th>
<th>2) 50 + 48</th>
<th>3) 537 + 256</th>
<th>4) 574 + 395</th>
<th>5) 326 + 578</th>
</tr>
</thead>
<tbody>
<tr>
<td>6) 567 + 908</td>
<td>7) 6 + 9 + 4 + 8 + 5</td>
<td>8) 798 + 826 + 134 + 957</td>
<td>9) 7654 + 4396</td>
<td>10) 39286 + 56906</td>
</tr>
</tbody>
</table>

*Note: Questions were formatted vertically on the diagnostic test paper.*
The following section will outline the changes in mean scores for each class on the computation component of the addition diagnostic pre- and post-initiative tests. The change in scores for each class on individual test questions was also analysed for each class. To observe the types of errors made on the addition diagnostic test further, error analysis was also conducted. Therefore, the changes in the types and frequency of errors made will be discussed in this section.

Table 17 outlines the overall mean scores for each grade on the pre- and post-initiative addition computations test as well as the change in mean.

Table 17  
Addition diagnostic test summary statistics: change in mean scores for each grade

<table>
<thead>
<tr>
<th>Year</th>
<th>Total questions administered</th>
<th>Pre-test: March</th>
<th>Post-test: October</th>
<th>Change in M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>Year 2/3, n=12</td>
<td>4</td>
<td>1.00</td>
<td>0.90</td>
<td>1.17</td>
</tr>
<tr>
<td>Year 3/4, n=12</td>
<td>10</td>
<td>5.40</td>
<td>3.33</td>
<td>6.33</td>
</tr>
<tr>
<td>Year 4/5, n=11</td>
<td>10</td>
<td>6.65</td>
<td>2.03</td>
<td>7.27</td>
</tr>
<tr>
<td>Year 5/6, n=14</td>
<td>10</td>
<td>8.18</td>
<td>2.04</td>
<td>8.71</td>
</tr>
</tbody>
</table>

Findings indicated a small positive change in each classes’ mean scores on the addition test with all classes reporting an improvement of less than 10%. The development in proficiency for addition computations in each class is described in Table 18.

Table 18  
Description of changes in each class’s proficiency with addition computations

<table>
<thead>
<tr>
<th>Year</th>
<th>Pre-test: March</th>
<th>Post-test: October</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2/3, n=12</td>
<td>Limited proficiency</td>
<td>Limited proficiency</td>
</tr>
<tr>
<td>Year 3/4, n=12</td>
<td>Developing proficiency</td>
<td>Developing proficiency</td>
</tr>
<tr>
<td>Year 4/5, n=11</td>
<td>Developing proficiency</td>
<td>Developing proficiency</td>
</tr>
<tr>
<td>Year 5/6, n=14</td>
<td>Proficient</td>
<td>Proficient</td>
</tr>
</tbody>
</table>

*Note: Limited proficiency is classified as <50% achievement, developing proficiency is classed as 50-80% achievement, and proficient is classed as >80%.

Observing the changes in each classes’ mean addition computation score indicated that there was limited change in each classes’ proficiency with addition computations throughout the initiative, despite this being a teaching focus during the initiative. In Year 2/3, the findings indicated there was no substantial understanding or proficiency with addition computations in this class. In Year 3/4, the class that reported the largest achievement gains, there existed a
large range in proficiency which remained unchanged throughout the initiative, and five students (42%) reported a reduction in addition computation scores over the course of the initiative. Similarly, in Year 4/5 the large range of proficiency within the class remained unchanged during the initiative. In Year 5/6, there were overall positive gains in proficiency for some of the most complex questions. There was also a reduction in the range of proficiency reported within the class with the proportion of students reporting scores of 80% and above (indicative of proficiency with addition computations) increasing. However, a proportion of students in this class (four students) also reported reductions in their overall computation score throughout the initiative. Overall, this evidence suggests that the initiative did little to enact any substantive changes to students’ proficiency with addition computations in each class. Detailed analysis of achievement for individual test questions, the changes in students’ spread of scores, and error frequencies for each cohort is detailed in Appendix K.

5.5.2.3.1 Error analysis for addition

Observing the frequency and types of errors on the addition diagnostic tests helped to explain findings. Whether the frequency of errors changed over the course of the initiative helped in understanding how students’ proficiency with addition computation had developed. Analysis of the pre-initiative test revealed that the most frequent errors were related to procedural fluency with number facts, multi-digit sum errors, not attempting questions relating to productive dispositions, and conceptual and procedural errors relating to renaming. The types of prevalent errors did not change substantially over the course of the initiative.

Analysis revealed that number fact errors were the most prevalent error on both the pre- and post-initiative tests with no change in the frequency reported over the course of the initiative. An example of a number fact error is outlined in Figure 33.

![Figure 33: Example of a number fact error in computation questions](image)

Here the student has incorrectly added 6 ones and 8 ones to obtain 13 ones, instead of 14 ones.

The remainder of the solution indicates proficiency with the addition concept and renaming procedures.

In the example outlined in Figure 33, the Year 6 student has added the two values incorrectly and has exhibited a number fact error despite the written script indicating proficiency with the concept of addition and the associated renaming procedures. Despite all
cohorts demonstrating proficiency with addition number facts, number fact errors were the most prevalent error students exhibited when solving addition computation problems. Throughout the initiative, the prevalence of number fact errors increased in the youngest classes (Year 2/3 and Year 3/4). This is perhaps attributed to the increase in students attempting all test questions, particularly in Year 2/3. Predominantly, however, the written test did not reveal why these findings may have occurred.

Other prevalent errors were associated with renaming. Renaming errors included forgetting to record or add on renamed values, and instances where students did not rename and simply recorded double-digit answers in a single place value. Throughout the initiative, the instances of students not renaming, and violating renaming conventions by recording double-digit answers in a single place value increased significantly. An example of this type of error is outlined in Figure 34. This error is indicative of students either lacking a conceptual understanding of the renaming process, in that renaming needed to occur in these situations, or lacking procedural fluency associated with the addition algorithm because they did not know where or how to record renamed values. The fact that students were not attempting to rename in any fashion suggested that this type of error may have been more conceptual in nature. The prevalence of this type of error greatly increased over the course of the initiative. This aligns with the findings of the numeration diagnostic test which found that critical place value knowledge was still limited for students by the end of the initiative.

Here the student has added each place value and recorded the answers without attempting to rename. The student has also incorrectly added 4 ones and 5 ones, demonstrating an addition number fact error.

Figure 34: Example of a renaming error in addition computation questions

Other prevalent errors were associated with multi-digits sums, and not attempting the question. These error patterns were largely consistent upon analysis of both the pre- and post-initiative test, however, the prevalence of multi-digit sum errors slightly reduced over the course of the initiative. It appears reasonable to expect difficulties with multi-digit sums considering students’ difficulties with addition computation questions involving adding only two values as well as the compounding number fact difficulties found in the analysis of written test scripts. Potentially, not attempting questions is explained by two causes; one is that the student does not know how to complete the task, that is, they do not possess conceptual
understanding to begin attempting the question, and the second potential cause is that they choose not to complete the question. Choosing not to complete the question is indicative of a student lacking a *productive disposition* as they are not motivated to attempt the task, and this manifests as avoidance (Kilpatrick et al., 2001).

In Table 19, changes in prevalence of instances of specific errors throughout the initiative is summarised. Only errors with a frequency of occurrence of five instances or more are reported. The denoted order of errors in Table 19 is indicative of the prevalence of each error type on pre-initiative analysis. Further detailed analysis of error frequencies and changes to error frequencies for each class is outlined in Appendix K.

<table>
<thead>
<tr>
<th>Error</th>
<th>Change in prevalence (frequency pre-test → frequency post-test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number fact error</td>
<td>No change (33 → 33)</td>
</tr>
<tr>
<td>Multi-digit sum error</td>
<td>Decrease (27 → 20)</td>
</tr>
<tr>
<td>No attempt</td>
<td>Increase (18 → 20)</td>
</tr>
<tr>
<td>Forgot to add on a recorded renamed value</td>
<td>Increase (12 → 14)</td>
</tr>
<tr>
<td>Forgot to record renamed value</td>
<td>Decrease (11 → 9)</td>
</tr>
<tr>
<td>Confused with subtraction</td>
<td>No change (8 → 8)</td>
</tr>
<tr>
<td>Attempt appears random</td>
<td>No change (5 → 5)</td>
</tr>
<tr>
<td>Did not rename, recorded double-digit answers in a single place value</td>
<td>Increase (5 → 22)</td>
</tr>
</tbody>
</table>

Specific to the Year 2/3 cohort, the pre-test highlighted that majority of errors were associated with random attempts to complete questions, or no attempts to complete questions. This scenario suggested that students had little understanding of addition computations. On the post-test, the instances of students not attempting the question significantly reduced (to zero occurrences) as did the number of random attempts. This may suggest that students’ understanding of the addition algorithm, and confidence in attempting addition computation questions had increased. However, the mean score on addition computation questions did not increase for this grade, so the reduction in these types of errors is not reflective of an increase in proficiency. Increasing students’ productive dispositions towards the addition tasks, reflected via more students attempting tasks, did not increase their proficiency. In Year 3/4, the pre-test revealed that the frequency of not attempting questions increased for this cohort,
suggesting that there was no change in students’ productive dispositions. For the Year 4/5 cohort, the pre-test indicated that the most common errors were number fact errors, despite students being proficient with the recall of addition number facts. The frequency of this type of error reduced on the post-test, however it was still the most prevalent error for this cohort. The evidence from the number fact and computation tests indicated that this error is perhaps associated with carelessness as Year 4/5 students demonstrated proficiency with addition number facts. Another plausible explanation is that students had obtained proficiency in addition facts without developing their abilities to transfer this knowledge to computations or problem-solving tasks. In Year 5/6, the most prevalent errors on the pre-test were associated with multi-digit addition sum errors, number facts, and forgetting to add on record renamed values. The frequency of number fact errors, and forgetting to add on renamed values reduced on the post-test indicating that students’ procedural fluency had increased over the course of the initiative.

5.5.2.4 Subtraction computation

Examples of the 10 subtraction computation questions are outlined in Table 20.

| Table 20 |
| Subtraction computation question examples |
| 1) 87 – 38  | 2) 70 – 56  | 3) 685 – 368  | 4) 846 - 467  | 5) 621 – 573  |
| 6) 603 – 424  | 7) 7806 – 4653  | 8) 9005 – 7896  | 9) 5000 – 2812  | 10) 80348 - 49689  |

The following section will outline the findings from the computation component of the subtraction diagnostic pre- and post-initiative tests. Table 21 outlines the overall mean scores for each grade on the pre- and post-initiative subtraction computations test, as well as the change in mean.
Table 21
Subtraction diagnostic test summary statistics: change in mean scores for each grade

<table>
<thead>
<tr>
<th>Total questions administered</th>
<th>Pre-test: March</th>
<th>Post-test: October</th>
<th>Change in M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>Year 2/3, n=12</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Year 3/4, n=12</td>
<td>10</td>
<td>3.12</td>
<td>4.00</td>
</tr>
<tr>
<td>Year 4/5, n=11</td>
<td>10</td>
<td>4.74</td>
<td>3.23</td>
</tr>
<tr>
<td>Year 5/6, n=14</td>
<td>10</td>
<td>6.04</td>
<td>3.21</td>
</tr>
</tbody>
</table>

Findings indicated that for the Year 2/3, 4/5, and 5/6 classes there was nil or limited reported change in students’ achievement with subtraction computation tasks. The development in proficiency for subtraction computations in each class is described in Table 22.

Table 22
Description of changes in each class’s proficiency with subtraction computations

<table>
<thead>
<tr>
<th>Pre-test: March</th>
<th>Post-test: October</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2/3, n=12</td>
<td>No proficiency</td>
</tr>
<tr>
<td>Year 3/4, n=12</td>
<td>Limited proficiency</td>
</tr>
<tr>
<td>Year 4/5, n=11</td>
<td>Limited proficiency</td>
</tr>
<tr>
<td>Year 5/6, n=14</td>
<td>Developing proficiency</td>
</tr>
</tbody>
</table>

*Note: Limited proficiency is classified as <50% achievement, developing proficiency is classed as 50-80% achievement, and proficient is classed as >80%.

The qualitative description of changes in each classes proficiency with subtraction computations reflects similar findings from the addition computation test. There were relatively minimal changes to each class’s proficiency with subtraction computations throughout the initiative, despite this being a teaching focus during the initiative. In the Year 2/3 class, no students were able to answer any subtraction computation questions by the end of the initiative, and whole class improvements was 7% or less in Year 4/5 and 5/6. This level of proficiency is below what is expected by curriculum standards (ACARA, 2018b). Overall, the Year 3/4 class reported the largest achievement gains which were substantially above what was reported in other classes. By the end of the initiative, this cohort reported comparative achievement to the Year 4/5 cohort in this component of the diagnostic test. However, the post-initiative findings indicated that there remained a large portion of the Year 3/4 cohort who were still unable to answer any subtraction computation questions (42%). Detailed analysis of the change in
achievement for individual test questions, the changes in students spread of scores, and error frequencies for each cohort is detailed in Appendix L.

5.5.2.4.1 Error analysis for subtraction

To further explain the findings reported earlier, the overall changes in frequency of errors for the subtraction computation test were analysed. At the beginning of the initiative, it was found that prevalent errors were associated with limited conceptual understanding of the subtraction process and renaming. Students commonly found the difference between values rather than renaming, or answered ‘0’ where renaming was required. Students’ finding the difference between values was indicative of a procedural understanding of the subtraction process. Students lacked conceptual understanding because they assumed that subtraction is commutative without understanding that this is not true for subtraction. An example of this renaming error is outlined in Figure 35.

![Figure 35: Example of a conceptual renaming error (finding the difference between values rather than renaming) in computation questions](image)

Here the student has found the difference between 7 ones and 8 ones rather than renaming.

Students answering ‘0’ where renaming was required potentially is indicative of students possessing a higher level of understanding than those who only found the difference (as outlined in Figure 36). These students may have been identifying that subtracting a larger value from a smaller value was not possible (when not dealing with negative integers). However, this thinking still demonstrates a lack of conceptual understanding and procedural fluency with the renaming process.

![Figure 36: Example of a conceptual renaming error (answering “0” instead of renaming) in computation questions](image)

Here the student has potentially realised that 8 ones cannot be subtracted from 5 ones, and has recorded “0” as their answer instead of renaming.

Errors associated with renaming were equally prevalent across Year 3/4, 4/5, and 5/6. Errors associated with renaming concur with the findings of the numeration diagnostic test. Such errors identified that students had difficulties answering place value questions,
particularly associated with renaming (such as identifying how many tens were in a given value, a key prerequisite understanding associated with the renaming process when completing subtraction sums).

Similar to the findings from the addition diagnostic test, the third most common error on the subtraction pre-test was not attempting the question relating to students’ productive dispositions. Following this, number fact errors were the next most prevalent error on the subtraction test, again similar to the findings of the addition diagnostic test. Table 23 denotes the changes in prevalence of instances of specific errors throughout the initiative. Only errors with a frequency of occurrence of five instances or more are reported. The order of errors in Table 23 is indicative of the prevalence of each error type on pre-initiative analysis, however, this order did not change throughout the initiative. Further detailed analysis of all error frequencies and changes to error frequencies for each class is outlined in Appendix L.

Table 23
Frequency of subtraction computation errors

<table>
<thead>
<tr>
<th>Error</th>
<th>Change in prevalence (frequency pre-test → frequency post-test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Did not rename, found the difference between values</td>
<td>Decrease (78 → 75)</td>
</tr>
<tr>
<td>Did not rename, answered ‘0’ where renaming was required</td>
<td>Decrease (35 → 31)</td>
</tr>
<tr>
<td>No attempt</td>
<td>Increase (29 → 31)</td>
</tr>
<tr>
<td>Number fact error</td>
<td>Increase (25 → 26)</td>
</tr>
<tr>
<td>Renaming error: incorrect renaming process with internal 0’s</td>
<td>Decrease (22 → 19)</td>
</tr>
<tr>
<td>Attempt appears random</td>
<td>Increase (14 → 17)</td>
</tr>
<tr>
<td>Incomplete question (partially completed)</td>
<td>Decrease (11 → 8)</td>
</tr>
<tr>
<td>Started sum from the left</td>
<td>Decrease (7 → 0)</td>
</tr>
<tr>
<td>Unnecessarily renamed</td>
<td>Decrease (5 → 4)</td>
</tr>
</tbody>
</table>

When observing changes to the frequency of common subtraction computation errors, it was found that errors associated with renaming all reduced in occurrence throughout the initiative, however, the reductions were minimal, and the instances of the errors were comparatively large. Similar to what was observed with the addition computation test, instances of students not attempting questions also increased throughout the initiative. On the addition computation test, it was found that there was no change to number fact errors, and on the subtraction computation test there was an increase in the instances of this error.
Overall, the only substantial gains in subtraction computation proficiency were reported in Year 3/4, and this was accompanied by a reduction in number fact errors and some types of renaming errors. This evidence suggests that the initiative acted to increase students conceptual understanding of critical renaming processes positively in Year 3/4. This outcome was reflected by an improvement in achievement with subtraction computations. For the classes that reported little to no change in subtraction proficiency (e.g., Year 2/3, 4/5, and 5/6), the prevalent barriers to success remained similar when comparing pre- and post-test errors, and were mainly associated with renaming processes, not attempting questions, and number facts. The trends in the evidence above also suggested that increasing students’ confidence to attempt test questions did not equate to significant gains in proficiency. These trends were similar to what was observed in the analysis of the addition computation test.

5.5.2.5 Multiplication computation

Table 24 outlines examples of the 10 multiplication computation questions.

<table>
<thead>
<tr>
<th>Multiplication computation question examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 49 x 3  2) 74 x 5  3) 874 x 4  4) 487 x 7  5) 63 x 56</td>
</tr>
<tr>
<td>6) 34 x 18  7) 70 x 58  8) 46 x 58  9) 543 x 47  10) 749 x 80</td>
</tr>
</tbody>
</table>

The following section will delineate the changes in mean scores for each class on the computation component of the multiplication diagnostic pre- and post-initiative tests. Table 25 outlines the overall mean scores for each grade on the pre- and post-initiative multiplication computations test, as well as the change in mean. The Year 2/3 class was not administered the multiplication diagnostic test due to students’ difficulties on the addition and subtraction diagnostic tests, and also because none of the students in the sample could recall any multiplication facts. This situation indicated that students in this class had no understanding of the multiplication concept. The pre-test found that all classes had little proficiency with multiplication computations.
Findings indicated that for the Year 2/3, 4/5, and 5/6 classes there was limited change in students’ proficiency with multiplication computation tasks. The development in proficiency for multiplication computations in each class is described in Table 26.

Table 26
Description of changes in each class’s proficiency with multiplication computations

<table>
<thead>
<tr>
<th>Year</th>
<th>n</th>
<th>Pre-test: March</th>
<th>Post-test: October</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 3/4, n=12</td>
<td>10</td>
<td>No proficiency</td>
<td>No proficiency</td>
</tr>
<tr>
<td>Year 4/5, n=11</td>
<td>10</td>
<td>No proficiency</td>
<td>No proficiency</td>
</tr>
<tr>
<td>Year 5/6, n=14</td>
<td>10</td>
<td>Limited proficiency</td>
<td>Limited proficiency</td>
</tr>
</tbody>
</table>

*Note: Limited proficiency is classified as <50% achievement, developing proficiency is classed as 50-80% achievement, and proficient is classed as >80%.

Over the course of the initiative, all classes experienced small positive changes in the mean score for multiplication computations, but these increases were less than 6%. The Year 3/4 and 4/5 cohorts essentially had no proficiency with multiplication computations. The Year 5/6 cohort had limited proficiency with multiplication computations, which was well below the proficiency standard expected for these year levels.

In Year 3/4, only two students in the class could answer any multiplication computation questions at the beginning of the initiative. More than 83% of the class could not answer any multiplication computation questions. Overall, 65% of the cohort were still unable to answer any multiplication computation questions by the end of the initiative. Students’ lack of proficiency with multiplication computations is supported by the findings of the number fact test, which indicated that students in this class had limited proficiency with multiplication facts.

At the beginning of the initiative, only half of Year 4/5 students were able to answer the simplest 2x1 digit multiplication questions. Over the course of the initiative, Year 4/5 students reported a decline in achievement for these questions. Less than 25% of the cohort...
could answer 38x4 at the end of the initiative. In Year 5/6, nearly all students in the class demonstrated proficiency with simple 2x1 and 3x1 digit multiplication computations at the beginning of the initiative, however, success rates declined once students reached 2x2 digit multiplication. No students in Year 5/6 were able to answer any 3x2 digit multiplication questions. By the end of the initiative, Year 5/6 students also reported a decline on some test questions including some of the simplest computation items. The limited achievement with multiplication computations in Year 4/5 and 5/6 is supported by the findings of the number fact test, which indicated that students in these grades had only developing proficiency with multiplication facts. Detailed analysis of the change in achievement for individual test questions, the changes in students spread of scores, and error frequencies for each cohort is located in Appendix M.

5.5.2.5.1 Error analysis for multiplication

The pre-test found that the most frequent error on the multiplication computation test was not attempting the question, followed by number fact errors. The frequency of non-attempts on the pre-test was nearly double that of the next most frequent error. Not attempting the test question occurs for two potential reasons: students do not know the how to attempt multiplication computation questions or, students are displaying avoidance behaviours when presented with multiplication computation questions. The first potential cause is related to students lacking conceptual understanding and/or procedural fluency with multiplication computations, while the second is related to lacking productive dispositions. By the conclusion of the initiative, the overall number of students not attempting questions increased from 101 instances to 128 instances. Further analysis found that this error increased in prevalence for all classes.

The second most common error, excluding the questions that students were not administered, was number fact errors. This error relates to students’ procedural fluency. This finding concurs with the results of the multiplication fact test, which indicated that students had little to no understanding of multiplication facts (i.e., they had not yet reached proficiency with recalling multiplication facts). The frequency of number fact errors did not change significantly over the course of the initiative, and this is also supported by the minimal changes to students’ proficiency with multiplication facts over the course of the initiative.
The next most common errors were associated with incorrect attempts to apply the algorithm. This again suggests that students’ difficulties were associated with limited *procedural fluency* and/or *conceptual understanding*. The number of students attempting to complete 2x2 digit algorithms using a similar pattern to solving 2x1 digit algorithms increased significantly over the course of the initiative. An example of this error is outlined in Figure 37.

Here the student has correctly multiplied 3 ones and 6 ones to obtain 18 ones, and this value has been correctly renamed. Rather than proceeding to multiply 3 ones by 5 tens, the student has moved on to multiply the tens values, forgetting to multiply other values in the sum.

*Figure 37*: Example of a procedural/conceptual error where the student has attempted to apply the 2x1 digit multiplication algorithm to a 2x2 digit multiplication sum.

The student has moved through the computation in Figure 37 by calculating *ones x ones* and *tens x tens*, neglecting to multiply the remaining components of the sum. This type of error is indicative of an extension of the procedural application of the pattern followed when multiplying 2x1 digit sums. The student has failed either to understand the 2x2 digit computation conceptually, or that the final answer is unreasonable for the given sum.

Table 27 denotes the changes in prevalence of instances of specific errors throughout the initiative. Only errors with a frequency of occurrence of five instances or more are reported. The order of errors in Table 27 is indicative of the prevalence of each error type on pre-initiative analysis. Further detailed analysis of all error frequencies and changes to error frequencies for each class is outlined in Appendix M.

<table>
<thead>
<tr>
<th>Error</th>
<th>Change in prevalence (frequency pre-test ➔ frequency post-test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No attempt</td>
<td>Increase (101 ➔ 128)</td>
</tr>
<tr>
<td>Number fact error</td>
<td>Increase (46 ➔ 47)</td>
</tr>
<tr>
<td>Applied the multiplication algorithm incorrectly (see Figure 37)</td>
<td>Increase (40 ➔ 63)</td>
</tr>
<tr>
<td>Attempt appears random</td>
<td>Decrease (20 ➔ 5)</td>
</tr>
<tr>
<td>Forgot a step when completing the 2x2-digit multiplication algorithm</td>
<td>Decrease (13 ➔ 3)</td>
</tr>
<tr>
<td>Incomplete question (partially completed)</td>
<td>Increase (5 ➔ 6)</td>
</tr>
<tr>
<td>Forgot to add on a recorded renamed value</td>
<td>Increase (6 ➔ 8)</td>
</tr>
</tbody>
</table>
Overall, the initiative did not influence students’ proficiency with multiplication computations meaningfully, as indicated by the small changes in proficiency. Critical errors were attributed either to not attempting questions, or a lack of procedural fluency with multiplication facts, with the number of students not attempting questions increasing in all classes over the course of the initiative. This evidence indicates that the classes tested above had little understanding of, or fluency with, the multiplication computation.

5.5.2.6 Division computation

Table 28 outlines examples of the eight division computation questions on the diagnostic test.

Table 28
Division computation question examples

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64 ÷ 4</td>
<td>2</td>
<td>485 ÷ 7</td>
</tr>
<tr>
<td>3</td>
<td>928 ÷ 3</td>
<td>4</td>
<td>643 ÷ 4</td>
</tr>
<tr>
<td>5</td>
<td>6657 ÷ 9</td>
<td>6</td>
<td>7236 ÷ 8</td>
</tr>
<tr>
<td>7</td>
<td>49356 ÷ 7</td>
<td>8</td>
<td>45701 ÷ 14</td>
</tr>
</tbody>
</table>

The following section will outline the changes in mean scores for each class on the computation component of the division diagnostic pre- and post-initiative tests. Table 29 summarises the overall mean scores for each grade on the pre- and post-initiative division computations test as well as the change in mean. The division diagnostic test was not administered to the Year 2/3, Year 3/4, and Year 4/5 classes at the beginning of the initiative for several reasons. Firstly, division computations without remainders are first introduced in the Australian Curriculum: Mathematics in Year 4 (ACARA, 2018b), therefore, it is not expected that Year 2/3 students would be able to complete division problems of this standard. The decision also was made not to administer the tests to the Year 3/4 and Year 4/5 classes due to their difficulties on the multiplication diagnostic test. In Year 3/4 and Year 4/5, the average score on single-digit multiplication fact questions was less than 50%. The findings from the multiplication computations questions also demonstrated that no Year 3/4 or Year 4/5 students were able to complete 2x2 digit multiplication sums, and few were able to complete 2x1 digit sums. During pre-initiative testing, all Year 3/4 and 4/5 students were offered the opportunity to complete the division computation test by the researcher engaging them in discussion with some simple division facts (e.g., “do you know what 8 divided by 2 is?”). This discussion was an attempt to gauge students understanding of the concept of division and division.
computations. This informal questioning confirmed that none of the Year 3/4 or 4/5 students were able to engage with the division diagnostic test at the beginning of the initiative.

Table 29
Division diagnostic test summary statistics: change in mean scores for each grade

<table>
<thead>
<tr>
<th>Total questions administered</th>
<th>Pre-test: March</th>
<th>Post-test: October</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Year 2/3, n=12</td>
<td>0</td>
<td>NA</td>
</tr>
<tr>
<td>Year 3/4, n=12</td>
<td>8</td>
<td>NA</td>
</tr>
<tr>
<td>Year 4/5, n=11</td>
<td>8</td>
<td>NA</td>
</tr>
<tr>
<td>Year 5/6, n=14</td>
<td>8</td>
<td>1.24</td>
</tr>
</tbody>
</table>

The pre-initiative findings indicated that the Year 5/6 class had limited proficiency with division computations. Over the course of the initiative, the Year 5/6 cohorts mean score on the division computation test increased, but not substantially (<9%). By the end of the initiative, the Year 3/4 and 4/5 classes reported means scores on the division diagnostic that were reflective of no proficiency with division.

In the Year 3/4 cohort, the mean scores are reflective of only two students being able to answer any division computation questions. Approximately 83% of this class has no proficiency with division computations. Similarly, in Year 4/5 only one student could answer one division computation question correctly by the end of the initiative, meaning that approximately 91% of this class has no proficiency with division computations. The Year 5/6 cohort reported positive gains in achievement for all the tested division computation questions. At the beginning of the initiative, only one student in Year 5/6 could answer any division computation questions. This is below the expected national achievement standard for Year 5 and 6 students as fluency with single digit division (which pertains to all test questions excluding Question 8) is expected in Year 5 (ACARA, 2018b). By the end of the initiative, five students (reflective of 36% of the class) from Year 5/6 could answer some division computation questions successfully. One student in this class demonstrated proficiency with division computations, scoring 8 out of 10 on the post-test. Further detail of the division diagnostic test findings is located in Appendix N.
5.5.2.6.1 Error analysis for division

When observing errors that inhibited students’ success with division computations, it was found that the most frequent errors were not attempting the question and not being administered the test questions. The majority of the Year 5/6 cohort were not administered the test questions on the pre-test. This decision was made by the researcher during the administration of the diagnostic test based on a student’s performance on the preceding diagnostic tests as well as the student’s indication of whether they were able to attempt the test questions. Students were asked if they knew how to complete division computation questions and urged to attempt them if possible. For example, one student indicated that they did not know how to start any of the algorithms and, given that they were unable to answer any division number fact questions, the division test questions were not administered. Ultimately, the decision to not administer the division computation questions was mainly due to the inability of students to answer any division number fact questions correctly. The frequency of Year 5/6 students who were not administered the test substantially decreased on the post-test, however, the number of students not attempting test questions increased significantly. Overall, students’ proficiency with division was limited for all tested classes. This concurred with earlier findings relating to students’ difficulties with subtraction, multiplication, and recalling multiplication and division facts.

5.5.2.7 Problem-solving

The following section will outline the findings from the problem-solving component of each diagnostic test. Examples of the problem-solving questions administered to students is outlined in Table 30.
Table 30
Example problem-solving questions

<table>
<thead>
<tr>
<th>Problem-solving questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
</tr>
<tr>
<td>Q1. Jackie had 138 marbles. She played with Cathy and Kylie and won 65 more marbles. How many marbles does she have now?</td>
</tr>
<tr>
<td>Q2. Gabriel’s parrot lost 7 black feathers and 6 blue feathers. How many feathers did his parrot lose?</td>
</tr>
<tr>
<td>Q3. At the zoo, the giraffes each had 768 bales of hay. The seals eat 392 buckets of fish. The elephants eat 695 bales of hay. How many bales of hay were eaten?</td>
</tr>
<tr>
<td><strong>Subtraction</strong></td>
</tr>
<tr>
<td>Q1. On Saturday, Ruby and Dora sold ice-creams. They sold 47 chocolate, 32 vanilla and 19 strawberry ice-creams. How many more chocolate than strawberry ice-creams did they sell?</td>
</tr>
<tr>
<td>Q2. Sam sells milkshakes. He has a 12 litre milk container. He puts 5 litres of milk into it. How many more litres of milk does he need to fill his container?</td>
</tr>
<tr>
<td>Q3. Russell and Jason sold 356 cans of lemonade at the swimming club by 12:00pm. They also sold 167 bottles of water by this time. The club had 480 cans of lemonade and 360 bottles of water available to sell. How many more cans of lemonade can they sell before it is all gone?</td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
</tr>
<tr>
<td>Q1. Simone planted 4 rows of strawberries in the morning and 3 rows in the afternoon. Each row had 48 plants. How many strawberry plants did she put in her garden?</td>
</tr>
<tr>
<td>Q2. A tap leaks 3 litres of water each day. How much water will leak in 1 week?</td>
</tr>
<tr>
<td>Q3. Ali worked at a nursery. During the morning he added fertiliser to 46 pot plants and watered 34 rows of plants. In the afternoon, he watered 52 rows of plants and fertilised 27 pot plants. If there are 68 plants in each row, how many plants did he water?</td>
</tr>
<tr>
<td><strong>Division</strong></td>
</tr>
<tr>
<td>Q1. A farmer dug 672 potatoes over 6 days. If she has 8 sacks and she puts the same number of potatoes in each one, how many potatoes will be in each sack?</td>
</tr>
<tr>
<td>Q2. The greengrocer has 40 mangoes to display in 5 baskets. If he wants the same number in each basket, how many mangoes will each basket hold?</td>
</tr>
<tr>
<td>Q3. During the harvesting seasons, the sugar cane farm sent 4580 tonnes of cane by rain to the sugar mill. If each cane bin can take 7 tonnes of sugar cane, how many bins were needed to transport the cane to the mill?</td>
</tr>
</tbody>
</table>

The change in mean score for each class was analysed as well as each classes’ achievement on individual problem-solving questions. The types and change in frequency of errors made for each operation was also evaluated. The findings from the problem-solving component of the diagnostic test indicated that, across the cohorts, success was below what would be expected by national curriculum standards. Detailed analysis of the mean scores on individual test questions and frequency of errors for each class is located in Appendix O.

Analysis of trends in errors for problem-solving tasks found that not attempting the question was the most prevalent error. Potentially, this trend is related to students’ still developing the necessary conceptual understanding and/or procedural fluency (i.e., the student does not know or understand how to attempt the task), or is related to productive dispositions (i.e., work avoidance behaviours). Observing error patterns for the addition problem-solving
tasks indicated that not attempting questions is most likely not a result of limited procedural fluency as the students who did not attempt the question did not record the sum that needed to be solved (regardless of whether they were capable of solving it). Further, questions where students experienced difficulties also included those that were procedurally simple (i.e., Gabriel’s parrot lost 7 black feathers and 6 blue feathers. How many feathers did his parrot lose?). Further, the computation tests indicated that the cohorts such as Year 5/6 were proficient with addition computations, however the frequency of non-attempts on the addition problem-solving tasks remained prevalent for this class. By the end of the initiative, the Year 5/6 cohort reported more non-attempts on the addition problem-solving questions than all other grades. This trend was similar for the subtraction problem-solving questions.

Another common error was carrying out the question using an incorrect operation (e.g., completing the addition questions using subtraction). Potentially, this error is suggestive of a lack of conceptual understanding as students struggled to identify the correct operation that needed to be employed. An example of this error is outlined in Figure 38.

In this example, the student has initially selected the correct operation (addition) and correct values, however decided this was incorrect and crossed out this calculation without proceeding.

The student then selected the correct values, but the incorrect operation and attempted to solve the problem using subtraction.

The subtraction attempt also indicates a lack of understanding of subtraction and renaming as the student found the difference between the tens values rather than renaming.

**Figure 38**: Example of an error associated with identifying the correct operation when problem-solving from a Year 4 student.

Other frequent errors included not isolating the correct values from the questions that had additional, unnecessary information. The cause of this is potentially related to ‘number grabbing’ techniques and is associated with strategic competence. A student resorting to ‘number grabbing’ means that the meaning of the question has not been obtained and random values from the question were selected and operated upon. An example of this error is outlined in Figure 39.
This student has struggled with identifying which operation to utilise in this question as indicated by the crossed-out attempts. However, the student managed to eventually identify that the problem should be solved using addition.

The student has then proceeded to operate on all of the values given in the question and was unable to identify that this was incorrect. This is suggestive of “number grabbing”.

Figure 39: Example of an error associated with number grabbing when problem-solving from a Year 5 student.

Overall, errors associated with procedural fluency were not common. Rather, proficiency in the strands of productive dispositions, conceptual understanding, and strategic competence preceded any procedural difficulties. That is, it was found that errors associated with the procedures associated with the algorithms (e.g., renaming) were limited and not prevalent across the sample on problem-solving tasks (i.e., students’ difficulties were not associated with issues carrying out the various computations). This suggested that, potentially, proficiency with conceptual understanding, strategic competence, and a productive disposition must be first obtained before focusing on procedural fluency.

Mean proficiency with problem-solving questions increased for most computations in most classes over the course of the initiative, and the increases in achievement were accompanied by a decrease in the number of students not attempting questions. As well, a reduction in errors is associated with selecting the wrong values or wrong operation. This is indicative of an increase in students’ strategic competence, their ability to formulate and appropriately represent problems. This means that students’ comprehension of test questions had increased over the course of the initiative.

5.5.2.8 Testing behaviour

It is important to note students’ responses to being administered written diagnostic tests, and to discuss general behaviour during mathematics classes because these conditions potentially influence students’ achievement as measured in testing instruments. Generally, in the lower grades (i.e., Year 2/3), there was no major instances of students’ expressing negative feelings towards completing the written test tasks.

In Year 3/4, there were few instances of students declining to attempt some tasks in the diagnostic tests (three students across the cohort). The questions that students refused to
attempt were generally questions where the student did not feel comfortable in their abilities. As the diagnostic tests were progressive in difficulty throughout each item, the tasks that students declined to attempt were generally the latter questions. In one instance in Year 3/4, one student wrote “skip” for each multiplication test question involving 7 or 8 times tables, and all 2x2 and 3x2 digit sums. In Year 4/5, instances of avoidance behaviours were also observed. During pre-testing, one student wrote “bored” on a testing page, and appeared to record random values for the computations on that page of the test.

In Year 5/6, there were several instances of disruptive student behaviours that were aligned with task avoidance during testing, particularly during the post-initiative testing. Analysis of how students’ achievement was influenced by occurrences of avoidance behaviours indicated that students in this cohort who reported declines in testing scores throughout the initiative typically manifested disruptive behaviours or work avoidance behaviours during the post-initiative testing. Observation of changes in addition computation scores alone revealed no incidences of note occurring with the seven students in Year 5/6 that reported positive gains in achievement throughout the initiative. For many of these seven students, observational records indicated that they were focused, and worked diligently throughout the testing.

Six students in this cohort (43%) declined in their achievement on addition computation tasks throughout the initiative. Work avoidance behaviours were exhibited by these students. During the post-initiative testing, two of the students were argumentative and rude to the researcher when asked to begin their testing. Initially both students would not attempt the tests, and one of the students swore at the researcher during testing. On the problem-solving questions, when one of these students was asked why they had left a question mark (“?”) as the answer to one question, they said that they didn’t know how to do them. When prompted to try and give them a go the student refused. This discussion concluded the student’s addition test, and when given the subtraction test following this discussion, the student marked every test question with a question mark (“?”) and said they were done, refusing to complete any more work in this session (first session when attempting the test). In the second session, the student remained unwilling and would not complete the multiplication or division tests. The student walked round the room saying they didn’t know how to do them and disrupted other students. The subsequent impact of this behaviour on achievement was that the student answered 1/10 of the multiplication computation questions correctly at the end of the initiative despite scoring
5/10 in other testing rounds. For the multiplication problem-solving questions, the student recorded the correct sum for the questions but would not complete them. This suggested that the student understood the problem but chose to leave the question incomplete. When asked to complete the division questions, the student responded that they were dumb and couldn’t do it.

Another Year 5/6 student who reported a decline in achievement on addition problem-solving tasks throughout the initiative also exhibited similar behaviours. Whilst this student completed the first components of the post-initiative diagnostic testing without incident (e.g., the addition test), the student refused to complete the multiplication or division test. When taken to work on the tests, the student refused to complete any of the work. The student was disruptive during testing walking around the room turning the lights on and off to gain attention and to avoid the set work. The decline in diagnostic tests scores was consistent across each computation as the student scored 8/10 for multiplication computations earlier in the year but then scored 0/10 at the end of the initiative. Similar to other students in this class, this student repeatedly expressed “this is dumb” during testing.

From the same group of students in Year 5/6 where achievement declined, another student continually asked the researcher if they could skip particular questions such as the multi-digit addition questions. This is an example of negotiating task demands to avoid work. In the multiplication test, the other student repeatedly said they couldn’t do the test because they didn’t have a multiplication fact sheet. This student was very resistant to attempting many of the questions and required much gentle encouragement to get through the tests. Overall, these findings indicated that work avoidance behaviours directly influenced students’ achievement.

5.5.2.9 Summary of diagnostic test findings

The six key findings obtained from the written tests administered during the initiative (PAT-M and diagnostic tests) are summarised in Figure 40. As demonstrated in Figure 40, many on the findings are supported by aligned findings from other data sources. The alignment between findings increases the validity of any assertions as the credibility, authenticity, dependability, and confirmability is ensured through the triangulation of findings from multiple data sources (Guba & Lincoln, 1985).
The first key finding was that proficiency with place value concepts and number facts were important in influencing students’ achievement on the subsequent addition, subtraction, multiplication, and division diagnostic tests. Limited changes to students’ proficiency with place value concepts and number facts potentially explains why changes to computational proficiency were limited as evidenced by frequent errors associated with renaming processes. The second key finding was that difficulties with number fact proficiency inhibited students’ success as evidenced by the prevalence of number fact errors.

Across the diagnostic tests, the third key finding indicated that students in most classes reported little change in computation proficiency. Explaining this finding, the fourth key finding was that students’ productive dispositions were found to have impacted on students’ achievement across multiple test items (e.g., both computation and problem-solving questions) consistently. Conceptual understanding and procedural fluency difficulties also inhibited
students’ success with solving computations concurring with the students’ observed difficulties completing place value questions on the numeration test, and number fact difficulties.

Further, the fifth key finding in relation to students’ problem-solving proficiency was that strategic competence preceded the development of procedural fluency, that is, students struggled to understand the problems and formulate the correct computations to be carried out. As this condition impeded students’ formulation of correct operations, procedural difficulties were not common on problem-solving tasks. Therefore, the sixth key finding was that strategic competence and productive dispositions first need to be established to support students’ in problem-solving in this context successfully.

5.5.3 Newman interview findings

In the following section, the findings from the Newman interviews will be reported for each class. The specific purpose of the Newman interviews was to isolate where a student was making errors throughout the process of solving a problem-solving question. Newman (1996) identified that errors in problem solving can be attributed to issues with reading, comprehension, transformation, process skills, or encoding. These interviews were conducted individually with all students from Years 3/4, 4/5, and 5/6. Students in Year 2/3 were not interviewed as they were not administered any of the problem-solving questions from the diagnostic tests. On each diagnostic test (addition, subtraction, multiplication, and division), the first question answered incorrectly by an individual student was re-administered in the interview. The procedure of the interview, and the specific interview questions are outlined in Section 4.8.3, Table 6. Detailed analysis of Newman interview findings is located in Appendix P. Trends in the findings from Newman interviews were consistent across the three classes.

5.5.3.1 Newman interview findings: Year 3/4

Table 31 outlines the change in frequency of each error type for the Year 3/4 class for the addition and subtraction problem-solving questions. This class was only administered problem-solving questions for addition and subtraction during the Newman interviews. The multiplication problem-solving questions were not used for the Newman interviews as only two students from the class could answer any multiplication problem-solving questions. It was determined that the addition and subtraction problem-solving questions would yield enough
information to shed light on the types of difficulties encountered by this cohort when solving problems.

Table 31
Newman interviews: Changes to frequency of errors for Year 3/4 (addition and subtraction problem solving), n=12

<table>
<thead>
<tr>
<th></th>
<th>Change in frequency of error occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Add</td>
</tr>
<tr>
<td>Reading</td>
<td>↓1</td>
</tr>
<tr>
<td>Comprehension</td>
<td>↓5</td>
</tr>
<tr>
<td>Transformation</td>
<td>↑2</td>
</tr>
<tr>
<td>Process skills</td>
<td>0</td>
</tr>
<tr>
<td>Encoding</td>
<td>0</td>
</tr>
<tr>
<td>Correct</td>
<td>↑1</td>
</tr>
</tbody>
</table>

The findings of the pre-initiative Newman interviews indicated that comprehension difficulties were the most prevalent error in the Year 3/4 class, followed by transformation difficulties. Comprehension involves students answering the question: “Tell me, what is the question asking you to do?” This is the first hurdle for students when problem-solving after they have read the question successfully. Analysis of students’ verbal responses to problem-solving questions during interviews revealed that being unable to identify what the question was asking them to do was often the cause of students choosing the incorrect operation or selecting the incorrect values. In such circumstances students resorted to ‘number grabbing’ strategies when they could not comprehend what the question was asking them to do. Difficulties selecting the correct operation and selecting the incorrect values from problems were also found to be common errors on written problem-solving test scripts. Evidence of students resorting to number grabbing strategies in the face of limited comprehension is outlined in the interview transcript in Figure 41.
Question: On Saturday, Ruby and Dora sold ice-creams. They sold 47 chocolate, 32 vanilla and 19 strawberry ice-creams. How many more chocolate than strawberry ice-creams did they sell?

Reading:
1. Please read the question to me. If you don’t know a word, leave it out. Student correctly read the question.

Comprehension:
2. Tell me, what is the question asking you to do? “I don’t get this. Take 47, 32, and 19?”

Transformation:
3. Tell me how you are going to find the answer Student wrote sum 47-32-19.

Process skills:
4. Show me how you get your answer, and “talk aloud” as you do it, so that I can understand how you are thinking. Student would not verbally express thinking.

Encoding:
5. Now, write down your actual answer. 13

Figure 41: Interview transcript from Year 4 student demonstrating number grabbing strategies.

The interview transcript in Figure 41 is an example of a student resorting to number grabbing strategies when they were unable to comprehend what the question was asking them to do. The student is unable to express what the question was asking them to do, but then opted to subtract all of the values in the question despite that being an inappropriate sum. Utilising number grabbing techniques was also observed on this student’s test scripts for other problem-solving tasks as outlined in Figure 42.

Question: On Friday, Russell and Jason sold drinks at their swimming club. They sold 53 lemon drinks, 58 bottles of water and 26 orange drinks. How many more lemon than orange drinks did the they sell?

Figure 42: Diagnostic test script from Year 4 student demonstrating difficulties identifying the correct values and resorting to number grabbing techniques.

For this question, the student has attempted to solve the problem by subtracting all the values. The student, in this instance, did not attempt to find an answer to the task. The interview transcript in Figure 43 is an example of a student being unable to comprehend what the question was asking them to do, and as a result the student constructed computations that were not possible, reflecting a lack of number sense as well.
Question: On Saturday, Ruby and Dora sold ice-creams. They sold 47 chocolate, 32 vanilla and 19 strawberry ice-creams. How many more chocolate than strawberry ice-creams did they sell?

Reading:
1. Please read the question to me. If you don’t know a word, leave it out.
   Student correctly read the question.
   Student wrote down the values from the question as she read.

Comprehension:
2. Tell me, what is the question asking you to do?
   “Oh that’s hard.”
   Wrote down 47 and 19. Interviewer asked why she wrote that.
   “I don’t know.”
   Student had a long pause and re-read the question
   “This is very hard miss.”
   Interviewer re-asked question 2. Student re-read the question to herself again and was thinking for a long time.
   “Do I have to add it up or take it away? Or what about this one (wrote division symbol).”
   Interview was ceased at this point.

*Figure 43:* Interview script from Year 4 student demonstrating difficulties identifying the correct values.

As indicated in the interview transcript in Figure 41 and Figure 43, lacking comprehension manifested difficulties in transformation, that is, forming the correct mathematical computation to solve the problem. However, in some instances, students who supposedly correctly comprehended the tasks, but manifested transformation difficulties, also resorted to number grabbing strategies. An example of this is outlined in Figure 44.
Question: On Saturday, Ruby and Dora sold ice-creams. They sold 47 chocolate, 32 vanilla and 19 strawberry ice-creams. How many more chocolate than strawberry ice-creams did they sell?

Reading:
1. Please read the question to me. If you don’t know a word, leave it out.
   The student correctly read the question

Comprehension:
2. Tell me, what is the question asking you to do?
   “How many more…”
   (student groaned in frustration)
   “I don’t get that”
   (student re-read the question aloud)
   “So it’s asking me how many more ice-creams from chocolate than strawberry ice-creams are they going to sell?”

Transformation:
3. Tell me how you are going to find the answer
   “I’m going to get all the numbers”
   (Student wrote down all the values from the question)
   “So I have to figure out how many more they’re going to sell? Could they sell 19? I think they’re going to sell 32 … the same amount as that (student pointed to chocolate value - 47) so they get the same amount of chocolate.”
   Interview was ceased at this point.

Figure 44: Interview script from Year 4 student demonstrating number grabbing strategies.

These findings indicated that when students could not comprehend the question or transform the question into the correct mathematical operation, the common errors that occurred included selection of the incorrect operation or selection of the incorrect values from the question. These findings concur with the findings from the analysis of the written problem-solving test scripts. These errors are associated with difficulties in the strands of mathematical proficiency relating to conceptual understanding and strategic competence.

The post-initiative findings indicated that errors associated with comprehension reduced substantially in Year 3/4. The reduction in comprehension errors over the course of the initiative concur with the findings of the analysis of the written problem-solving questions, which indicated a reduction in the prevalence of errors associated with students not selecting correct values or operations. Students’ strategic competence in relation to their comprehension of the questions increased over the course of the initiative. By the end of the initiative, transformation errors increased in prevalence for the Year 3/4 cohort. The reduction in comprehension errors and the increase in transformation errors suggested that students had progressed in the process of problem-solving and were now at the next step of this process. That is, comprehension issues had been addressed and now transformation issues needed to be
addressed. Furthermore, it was found that the prevalence of correct answers increased throughout the initiative. Instances of correct answers indicated that students were able to solve the problem correctly during the interview that they were previously unable to correctly answer during the diagnostic testing.

As Newman interviews were not conducted for the multiplication problem-solving questions on the pre-test due to students’ difficulties in Year 3/4, and as the division problem-solving questions on the pre-test were not administered, comparisons could not be made concerning changes in the types of errors exhibited by students. However, analysis of written scripts from the multiplication and division post-test problem-solving questions demonstrated that transformation errors were also the primary barrier to success on multiplication and division questions in Year 3/4.

5.5.3.2 Newman interview findings: Year 4/5

Table 32 outlines the change in frequency of each error type for the Year 4/5 class for the addition, subtraction, and multiplication problem-solving questions.

<table>
<thead>
<tr>
<th></th>
<th>Change in frequency of error occurrence</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Addition</td>
<td>Subtraction</td>
</tr>
<tr>
<td>Reading</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Comprehension</td>
<td>↓ 2</td>
<td>↓ 1</td>
<td>↓ 6</td>
</tr>
<tr>
<td>Transformation</td>
<td>0</td>
<td>0</td>
<td>↑ 5</td>
</tr>
<tr>
<td>Process skills</td>
<td>↑ 1</td>
<td>↓ 1</td>
<td>0</td>
</tr>
<tr>
<td>Encoding</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Correct</td>
<td>0</td>
<td>↑ 1</td>
<td>↑ 1</td>
</tr>
</tbody>
</table>

At the beginning of the initiative, the most frequent difficulty for the Year 4/5 class was also comprehension, followed by transformation. Similar to the Year 3/4 cohort, the Year 4/5 cohort reported a decrease in comprehension errors over the course of the initiative, and an increase in transformation errors for multiplication problem-solving questions. For the division problem-solving questions, comparisons cannot be made as they were not administered at the beginning of the initiative, however, transformation errors were the most prevalent error at the
end of the initiative, similar to what was found for Year 3/4. It was also found that many students in the Year 3/4 and 4/5 classes required reassurance when reading people’s names in questions. Often students would not proceed in reading until the researcher had confirmed they had read the name correctly.

5.5.3.3 Newman interview findings: Year 5/6

Table 33 outlines the change in frequency of each error type for the Year 5/6 class for the addition, subtraction, and multiplication problem-solving questions.

| Table 33 Newman interviews: Changes to frequency of errors for Year 5/6 (addition, subtraction, and multiplication problem solving), n=12 |
| Change in frequency of error occurrence |
| Addition test | Subtraction | Multiplication |
| Reading | 0 | 0 | 0 |
| Comprehension | 0 | ↓ 3 | ↓ 3 |
| Transformation | ↑ 2 | ↑ 2 | ↑ 2 |
| Process skills | ↓ 2 | ↓ 1 | ↑ 1 |
| Encoding | 0 | 0 | 0 |
| Correct | ↑ 2 | ↑ 3 | ↑ 4 |

At the beginning of the initiative for the Year 5/6 class the most prevalent finding during Newman interviews where students were readministered incorrectly answered problem-solving questions was that students were answering the question correctly. Following this, similar to the Year 3/4 and 4/5 class, the most prevalent barriers were comprehension and transformation. Also following the trend observed in Year 3/4 and 4/5, the prevalence of comprehension errors decreased by the end of the initiative, and the instances of transformation errors increased. The number of students answering questions correctly also increased substantially for this cohort over the course of the initiative.

Though the findings of the division Newman interviews for Year 5/6 had no pre-initiative comparison, at the end of the initiative it was found that transformation errors were the most frequent error, and there were also high instances of students answering the question correctly. These findings concur with the findings from the analysis of the addition, subtraction, and multiplication interviews.
5.5.3.4 Testing behaviour

It was observed during interviews that many of the students in the Year 5/6 cohort were now attempting problem-solving questions which they previously would not attempt in the written test. All students attempted all problem-solving questions presented in the Newman interviews. Observational records noted that, in particular, some students were concerned about answering questions correctly in the interview setting. It appeared that the human interaction involved in the interviews increased the stakes of the task for students, and students wanted to please the researcher and get the correct answer. Behaviours observed to be indicative of this positioning included students restarting attempts, making multiple attempts, and self-correcting previous interview question responses. This was found to be the case for students in all classes. For example, Alice, a student in Year 4/5, was concerned that she could not answer the multiplication and division problem-solving tasks and was very persistent in her attempts. Alice sat and reread the questions to herself multiple times, not wanting to give up. Alice did not move on from each task until she obtained a final answer, stating that she did not want her teacher to see her wrong answers. These types of frequent self-correcting behaviours were not observed during the administration of the written diagnostic test.

As another example, another student in Year 6, Suzie, expressed embarrassment and reluctance when unable to answer a multiplication problem-solving task correctly during the interview questioning. When presented with the next problem-solving question involving division, the student asked the interviewer “if I get this wrong, does it matter?” in order to re-confirm the stakes of the interview. It appeared that the student wanted to ascertain whether success on the question “mattered” before attempting. The student indicated that they were happy to attempt the question on the condition that the stakes were not high and it didn’t “matter” if, consequently, the answer was incorrect.

The Year 3/4 teacher, Diane, commented on the differences in students’ ability to work in mathematics in a one-on-one setting. In relation to commenting on difficulties with working with students in the school, Diane noted:

Another big thing I find, is catering to those children that just cannot cope in the classroom situation...there are some children that just refuse to do work no matter what. No matter what you do, no matter what you present to them. But if you sit with them one-on-one then they will do something. But that is just not possible in a classroom.
Difficulties for students working individually were found on the diagnostic tests with frequent reported non-attempts. However, the positive findings in the one-on-one Newman interview concurred with Diane’s views expressed above. Evidence suggested that students worked more effectively in these one-on-one settings.

Important trends related to reading were also found in the administration of Newman interviews despite reading not being a prevalent barrier to students’ problem-solving success. During interviews, when not highly confident with a word (particularly people’s names), students would often turn immediately to the researcher to seek some form of verbal or non-verbal confirmation that they had read the word correctly before they would attempt to proceed with reading the problem-solving task. What this meant in practicality was that often students required a small “nod” from the researcher to confirm they were proceeding correctly, which they often were. It appeared that, if the student did not have an adult or more capable peer sitting next to them to reassure them that they were reading correctly, the name or word would have acted as a road block to continuing to read, and subsequently, would inhibit the process of completing the question. Whilst not inhibiting students’ success during Newman interviews, this finding may explain some of the prevalent non-attempts during the written test.

5.5.3.5 Summary of Newman interview findings

Overall, there were similar patterns observed across the cohorts in the changes of error frequencies using Newman interview methods. Students’ abilities to complete problem-solving tasks progressed throughout the initiative as comprehension difficulties decreased in prevalence. The by-product of this decrease in comprehension difficulties resulted in an increase in transformation errors. These changes are indicative of progression in students’ problem-solving capabilities throughout the initiative, however students still possessed critical problem-solving difficulties related to understanding and transforming problem-solving tasks. A key finding from the Newman interviews was that the nature of Newman interviews acted to influence students’ problem-solving capabilities positively.

5.6 Case reports

The following section will outline five case reports comprising students across each of the classes involved in the initiative. These case studies serve as telling cases of particular kinds
of phenomena found. They provide observations, and analysis at a finer level. Analysis of individual cases sheds light on why particular trends were found in the data as discussed in the previous sections.

5.6.1 Daisy, Year 2

The Year 2 student, Daisy, who reported the largest gains in proficiency as measured by the PAT-M, is a particularly illuminating case to observe from the Year 2/3 cohort. Daisy’s case demonstrates how proficiency with computations and problem-solving is inhibited when conceptual understanding of renaming procedures and the nature of the operations themselves (i.e., what is 12-7, can you tell me a story for 12-7) is not well established. Further, Daisy’s case also demonstrates that achievement on the PAT-M does not equate to proficiency with mathematical concepts such as addition and subtraction.

At the beginning of the initiative, Daisy reported a PAT-M achievement well below that of a late Year 1 student when compared to norming samples. By the end of the initiative, Daisy achieved above the class mean and was achieving a PAT-M score equivalent to a mid-Year 2 student. This achievement equated to over 9-months improvement over the course of the 7-month initiative. On the diagnostic tests, Daisy reported a 44% improvement on the numeration test which is well above the class mean improvement (28%). The gains in numeration proficiency potentially act to explain Daisy’s substantial improvements on the PAT-M test.

Daisy’s proficiency with addition and subtraction computations did not change throughout the initiative. By the end of the initiative, Daisy was only able to answer one of the 2-digit addition questions involving no renaming. Limited proficiency with addition computations was not atypical for her class. Daisy did not possess or gain an understanding of or fluency with addition computations throughout the initiative and reported to recording seemingly random values. An example of Daisy’s attempts at 3-digit addition question involving renaming is outlined in Figure 45.

Here the student has appeared to record random values in place values where renaming was required.

*Figure 45: Example of Daisy’s errors (recording random values where renaming was required) in addition computation questions.*
A potential explanation for Daisy’s difficulties with addition computations is her developing proficiency with addition number facts (50% achievement pre- and post-initiative). However, number facts do not appear to have impeded Daisy’s developing abilities to complete addition computations unless renaming was involved. This finding is consistent with what was found when observing the data for the Year 2/3 cohort which indicated that the class had reasonable proficiency with addition number facts, however, addition computation proficiency was limited for the entire cohort. The findings indicate that limited understanding of renaming concepts and processes critically inhibited students’ proficiency with computations. This conclusion is also supported by analysis of the errors made on the addition computation test which indicated that conceptual understanding of the renaming process was a prevalent error.

In relation to subtraction, Daisy remained unable to answer any computation questions by the end of the initiative. This is consistent with the whole class findings which indicated that none of the Year 2/3 students were able to answer any subtraction computation questions by the end of the initiative despite this being a focus of teaching instruction throughout the initiative. The written test scripts from Daisy indicate that the root difficulties with subtraction constituted a lack of conceptual understanding with majority of the attempts (both pre- and post-initiative) indicating random answers, a convolution of addition procedures, or to a lesser degree, finding the differences between values rather than correctly subtracting. Such difficulties were consistent across the Year 2/3 cohort indicating that the instructional practices employed in this class had not developed students’ conceptual understanding of subtraction in a meaningful way. When assessing whether students in this cohort were able to give a contextual story for the sum “12 – 7”, it was found that none of the students in this class were able to do this at the beginning nor at the end of the initiative. This is indicative of a lack of conceptual understanding related to the subtraction concept. Potentially, this finding acts to explain why Daisy, and the remainder of the students in the Year 2/3 cohort, possessed significant difficulties with subtraction despite the focused initiative.

5.6.2 Kevin, Year 3

Kevin was a student from the Year 3/4 cohort who was the only Year 3 student who was not yet meeting the Year 3 expected standard (as measured by the PAT-M) by the end of the initiative. Kevin’s case demonstrates how lacking foundational place value knowledge, and
fluency with number facts impacts negatively on students’ dispositions towards mathematics, which manifested as number grabbing strategies, and frustration toward tasks. Kevin’s case also demonstrates how instruction that does not conceptually develop students’ understanding of operations following a CRA sequence negatively impacts on students’ achievement. Further, his case illustrates how some problem-solving strategies related to Polya’s problem-solving heuristics positively influence problem-solving proficiency.

Kevin was a student who was sometimes disruptive, and struggled to engage in tasks in mathematics lessons. Often, Kevin’s learning was impeded by behavioural factors, consistent with many other students in this study. At the beginning of the initiative, Kevin achieved a score on PAT-M that was below the Year 1 standard when compared to the norming sample, the lowest reported score for this class. Over the course of the initiative, Kevin reported the third largest gain in score for the Year 3/4 class. This gain was above the average growth for the class and above the growth of the comparative norming sample.

For the diagnostic tests, Kevin reported an improvement of 50% on the numeration diagnostic test, answering 70% of the test items correctly by the end of the initiative. However, Kevin was one of the five students in this cohort who reported a reduction in addition computation proficiency (reducing from 20% to 10%) during the initiative. Further, despite demonstrating 100% accuracy with number facts at the beginning of this initiative, Kevin’s accuracy reduced to 80% over the course of the initiative. Similarly, Kevin remained unable to answer any subtraction computation questions by the end of the initiative, and his subtraction fact fluency reduced from 60% to 40%. During the post-initiative testing, Kevin was reluctant to complete oral subtraction facts stating: “I’m not good with takeaway”. Kevin was observed to lose track during calculations repeatedly and become frustrated. Consequently, after the fourth subtraction fact question, Kevin started guessing answers. In earlier attempts to complete the fact questions, Kevin resorted to writing the facts out as vertical sums to work them out.

When observing the types of errors exhibited by Kevin, the addition computation test revealed that the primary error was failing to rename, and the frequency of this error remained consistent on the pre- and post-initiative tests (error occurred on 40% of answered questions). For subtraction computations, answers were either random (50% of questions), or were incorrect application of the subtraction algorithm as the difference in values was found (50% of questions). By the end of the initiative, Kevin did not submit any random answers to questions, but his attempts revealed that he was confusing the subtraction algorithm with
addition processes, and was still finding the difference in values rather than correctly subtracting (70% of questions). Kevin exemplifies an illuminating case to observe as he was the only Year 3 student in the Year 3/4 cohort who did not benefit substantially from the initiative. His accuracy with computations and number facts remained unchanged or reduced over the course of the initiative, and he was the only Year 3 student in this class not meeting Year 3 standards as measured by PAT-M.

When evaluating why Kevin’s results were unique to this cohort, Kevin’s removal to work in a small focused group with a teacher aide during mathematics lessons is important to note. As Kevin was identified as one of the lower achieving students in the Year 3/4 class, the teacher chose to dedicate the additional teaching support provided to each of the classes (i.e., a teacher aide) to working with a small group of 3-4 lower-achieving students during mathematics lessons. The evidence from the data in this study indicates that withdrawal to this small group did not act to influence Kevin’s proficiency positively in comparison to the remainder of the cohort who were instructed by the classroom teacher. The difference between the instruction received in the whole class group compared to the withdrawn group was the difference between the teacher and the teacher aide’s detailed pedagogical content knowledge. The teacher aide’s approach to teaching place value and computations was dissimilar to what was suggested in the effective practice recommendations, which specifically related to the CRA teaching sequence and consistent language approach to teaching algorithms in the withdrawn group. In essence, the teacher aide did not have the detailed mathematical pedagogical content knowledge to instruct the small group of students proficiently with regard to the mathematics content. Similar findings were found for other students who formed the withdrawn group that worked with the teacher aide. On the post-initiative testing, 42% (five students) from Year 3/4 remained unable to answer any subtraction computation questions, and four of these five students were part of the withdrawn group.

In addition to this finding, Kevin experienced difficulties with subtraction number facts as evidenced by him resorting to the strategy of writing out the subtraction facts as vertical sums in order to work them out rather than relying on mental computations during testing. Despite this strategy, Kevin still answered majority of the oral facts incorrectly. Kevin was observed to often lose track while trying to figure out subtraction number facts and repeatedly got frustrated. He resorted to answering with random numbers after the fourth oral fact question. This evidence explains Kevin’s observed difficulties answering subtraction
computation questions as the observations from testing indicated that Kevin was experiencing cognitive overload with subtraction due to his lack of fluency with related number facts.

In relation to problem-solving, Kevin reported an improvement from the beginning of the initiative. He progressed from answering none of the addition or subtraction problem-solving questions correctly to answering 1 out of 3 of the questions correctly for both operations by the end of the initiative. When observing how Kevin’s proficiency developed using Newman interviews, it was found that, for both operations, Kevin’s difficulties moved from comprehension difficulties to process skill errors. At the beginning of the initiative, Kevin’s comprehension issues with mathematical problem-solving tasks were significant. For the tested addition problem, Kevin was unable to comprehend that the task was related to addition as outlined in the interview script in Figure 46.

| Question: Jackie had 138 marbles. She played with Cathy and Kylie and won 65 more marbles. How many marbles does she have now? |
| Reading:  |
| 1. Please read the question to me. If you don’t know a word, leave it out. | The student correctly read the question  |
| Comprehension:  |
| 2. Tell me, what is the question asking you to do? | “Figure out how much marbles she has. She had 138 she won 65 and they’re asking me how much she had left.” |
| Transformation:  |
| 3. Tell me how you are going to find the answer | “Takeaway some stuff.” |
| Process skills:  |
| 4. Show me how you get your answer, and “talk aloud” as you do it, so that I can understand how you are thinking. | The student wrote horizontally 138 – 65, then put 8 up on his fingers then quickly answered “70” |
| Encoding:  |
| 5. Now, write down your actual answer. | The student recorded 70 |

Figure 46: Newman interview script from Kevin indicating a comprehension error (addition problem-solving).

The evidence in Kevin’s interview script demonstrates Kevin’s lack of comprehension of the task, and confusion with subtraction. Going beyond comprehension, it was also evident that Kevin was unable to represent or complete subtraction computations correctly (i.e., did not
possess strategic competence). Neither could he demonstrate the adaptive reasoning to check whether the answer was reasonable.

By the end of the initiative, the nature of Kevin’s difficulties changed, and he demonstrated increased comprehension of the tasks as outlined in Figure 47. When readministered a similar addition problem-solving question as described earlier, process skills were the primary inhibiter of success. This was consistent with Kevin’s results on the computation test that indicated he had no proficiency with subtraction.

<table>
<thead>
<tr>
<th>Question: Connor collected 147 shells at the beach. His friends George and Christos gave him 56 shells they had collected. How many shells does he have now?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading: 1. Please read the question to me. If you don’t know a word, leave it out.</td>
</tr>
<tr>
<td>Comprehension: 2. Tell me, what is the question asking you to do?</td>
</tr>
<tr>
<td>Transformation: 3. Tell me how you are going to find the answer</td>
</tr>
<tr>
<td>The student circled the values in the question.</td>
</tr>
<tr>
<td>Process skills: 4. Show me how you get your answer, and “talk aloud” as you do it, so that I can understand how you are thinking.</td>
</tr>
<tr>
<td>Encoding: 5. Now, write down your actual answer.</td>
</tr>
</tbody>
</table>

*Figure 47: Newman interview script from Kevin indicating a process-skill error (addition problem-solving).*

The script above demonstrates the new strategies that Kevin utilised to progress further in the problem-solving task. Kevin had been taught by the teacher aide to circle the key information in problem solving tasks. This is consistent with heuristics proposed by Polya as this strategy focuses on supporting the “understand the problem” and the “devise a plan” stages in a tangible way. The interview in Figure 47 suggests that, after instruction, Kevin was able to comprehend questions with greater fluency, and correctly identify the necessary operation. Kevin’s primary difficulty was in carrying out the addition computation. This outcome was consistent with the finding that Kevin had limited proficiency with addition computations. Overall, this suggests that the teacher aide was able to enact change in Kevin’s proficiency in some aspects, but was unable to in others.
5.6.3 Lana, Year 3

Lana, a Year 3 student from the Year 3/4 cohort, was the highest achieving student in this group. Lana’s case demonstrates how proficiency with place value concepts and renaming processes as well as fluency with number facts facilitates proficiency with computations and problem-solving. Her case illustrates the concept of mathematical knowledge being hierarchical in that proficiency with one concept or skill leads to further proficiency, as evidenced by Lana’s results.

During the first three terms of the school year, Lana reported an attendance rate of 94.7%, above the average attendance rate for her class, and both Indigenous and non-Indigenous students nationally. At the beginning of the initiative, Lana reported a PAT-M score equivalent to a late Year 3 student when compared to the national norming sample. This result was above the standard expected for her age. By the end of the initiative, Lana reported a PAT-M score equivalent to a late Year 5 student equating to two years growth during the 7-month initiative. Comparatively, the Year 3/4 class reported mean growth of 14 months over the course of the initiative. Lana was achieving well above what typically would be expected for her age according to this data.

When evaluating why a student such as Lana could benefit from the initiative substantially more than other peers, one possible explanation is Lana’s high proficiency with critical place value concepts and renaming. On the numeration diagnostic test, Lana achieved scores over 1 standard deviation above the class’s mean score, and reported improvement above the mean growth for the class. For the addition diagnostic test, Lana reported 100% achievement for addition number facts and addition computations at the beginning and end of the initiative. For subtraction, Lana reported number fact and computation achievements of 90% and 100% for the pre- and post-initiative tests respectively indicating proficiency with this concept. Lana’s addition and subtraction computation scores were well above the class mean (Year 3/4 computation means were 63% and 52% respectively). Similarly, for multiplication computations, Lana reported an achievement of 40% on both the pre- and post-initiative tests, 30% above the post-initiative class mean. Comparatively, 65% of this cohort were unable to answer any multiplication computation questions by the end of the initiative. Potentially, proficiency with number facts explains this finding. Lana reported a multiplication fact score of 100% by the end of the initiative, which was substantially above the class mean of 54%. She was also one of only two students in the Year 3/4 cohort who was able to answer
any division computation questions, reporting a score of 40% by the end of the initiative which was above the reported class means for all cohorts.

At the beginning and end of the initiative, Lana answered all addition and subtraction problem solving questions correctly. At the end of the initiative, Lana participated in Newman interviews in relation to a multiplication and division problem-solving question that were answered incorrectly on the written test. During the interviews, Lana answered both questions correctly.

Overall, the impact of proficiency with key place value concepts and number facts seems to have had a positive effect on Lana’s ability to complete computation and problem-solving tasks. Findings from Lana’s case supports the hierarchical conceptualisation of mathematical knowledge because proficiency with foundational concepts supported further proficiency with other concepts and skills.

5.6.4 Lisa, Year 6

From the Year 5/6 cohort, the Year 6 student Lisa was of interest due to her observed decline in proficiency with mathematical concepts such as addition computation. Lisa’s case demonstrates the impact of a students’ productive disposition towards mathematics being influenced negatively.

The pre-initiative PAT-M test indicated that Lisa reported an achievement level equivalent to an early Year 3 student, over 3 years below the expected standard. Throughout the initiative, Lisa reported the third largest gain in achievement within the Year 5/6 cohort as measured by the PAT-M with a score equivalent to a mid-Year 5 student when compared to the norming sample. Despite Lisa’s substantial gains in achievement as measured by the PAT-M, she experienced difficulties throughout the initiative.

Throughout the initiative, Lisa’s achievement on the number fact and computation diagnostic tests declined. For example, Lisa’s achievement on the addition computation test declined from 90% at the beginning of the initiative to 40% when readministered the test partway through the initiative. A reduction in achievement was also reported for addition number facts with Lisa reporting a pre-initiative score of 100%, and an achievement of 40% partway through the initiative. Similarly, Lisa’s subtraction number fact achievement declined from 100% at the beginning of the initiative to 30% partway through. When observing test
scripts from Lisa, it was found that many questions were answered with random values as outlined in Figure 48.

![Figure 48: Example of seemingly random answers by Lisa.](image)

The recording of random values is potentially an example of work avoidance behaviours as the student was attempting to appease the researchers request to complete the tasks. Answering in such a manner was an attempt to demonstrate that the questions are “done”, without completing the work. Answering with random values (e.g., answering 300 for $7 + 5$) was also observed on the number fact component of the addition and subtraction diagnostic test. These findings relating to work avoidance behaviours were also found on analysis of Lisa’s other computation test scripts and were still demonstrated by Lisa in cases where she was progressing in her proficiency with the tested concepts. An example of this included subtraction computations where Lisa’s achievement increased from 0% to 50% throughout the initiative. Analysis of Lisa’s achievement on this test demonstrated that Lisa’s conceptual understanding and procedural fluency with subtraction and renaming had improved as a result of the initiative. Despite these improvements, Lisa still answered many of the subtraction computations with answers that appeared random. In addition to random answers, Lisa also recorded several times that she did not know the how to answer the question, and that it was difficult for her as outlined in Figure 49. The presence of random answers and non-attempts are again potentially indicative of issues with productive dispositions towards the tasks.
For this subtraction computation task, Lisa has recorded seemingly random values for the answer.

In this sample of work, Lisa has correctly identified that renaming needed to occur and has renamed 70 tens to 69 tens, but has failed to record the renamed ten in the ones place value. Lisa has then not proceeded with the task and has recorded “I don’t know”, despite having indicated an understanding of the initial steps to solving the computation task.

When presented with a subtraction problem-solving task, Lisa has not attempted the task at all.

*Figure 49:* Examples of Lisa recording random answers, or recording “I don’t know” for subtraction computation and problem-solving tasks.

The persistent nature of Lisa’s work avoidance was observed further on the remainder of Lisa’s subtraction and multiplication diagnostic tests as outlined in Figure 50.

*Figure 50:* Example of Lisa not attempting the multiplication computation tasks.

The example of Lisa’s work in Figure 50 indicated that Lisa was able to answer two of the multiplication computation questions correctly, which further supports the assertion that,
most likely, the responses were related to Lisa’s motivation towards completing the task. This evidence suggested that the responses of “I don’t know” and “I hate this” are likely indicative of unwillingness to attempt the tasks, rather than a lack of understanding regarding multiplication computations. Also, this trend was observed on the division diagnostic test.

During class lessons it was observed that Lisa was disengaged regularly and off-task. Lisa commonly yelled out answers or unrelated comments during class teaching time whilst the teacher or other students were trying to speak. Lisa was disengaged with the diagnostic testing in the second attempt repeatedly stating: “This is dumb”. During the Newman interview testing, when Lisa was asked to describe how they were going to find the answer, Lisa responded as follows:

Lisa: I don’t get none of this Miss. I’m not that smart.
Researcher: Why do you think that?
Lisa: Cause
Researcher: Why do you think you’re not smart at math?
Lisa: Because math is hard.
Researcher: Why is it hard?
Lisa: Because there are things I can’t do, and some I kinda can. That’s why it’s hard. And you have to learn and my brain hurts every time I learn.

This conversation sheds light on the thinking that may be occurring for students in this class. When presented with questions that are challenging, this student perceives them as things that they can’t do and then concludes that they are not smart, and that they can’t do mathematics. This type of thinking can become cyclical: the work is hard so they feel that they can’t do it resulting in the task being avoided. The outcome of this thinking is that the work remains ‘too hard’ as the challenge is never addressed and overcome. Lisa’s case is an example of a student who has a poor self-concept of their own academic ability exhibiting work avoidance behaviours. The findings related to Lisa’s reduction in achievement demonstrate the impact of productive dispositions on achievement.

During interim testing partway through the initiative, it was found that the trends observed with Lisa’s achievement were consistent across the class. For example, it was found that 6 out of the 14 Year 5/6 students declined in addition computation proficiency, and five declined in subtraction computation proficiency. Further, five students declined in their ability to answer both multiplication fact and multiplication computation questions. A decline in
achievement in concepts such as addition and subtraction for Year 5 and 6 students is atypical throughout a school year. Potential reasons for these changes such as critical barriers relating to productive dispositions towards the tasks have been explored in Lisa’s case.

5.7 Summary of all findings

In summary, the initiative reported positive gains in mathematical achievement as measured by PAT-M for each class. When observing trends in individual student achievement, it was found that across all classes, generally the initially lowest achieving students benefited the most from the initiative. When comparing performance between classes, it was found that the Year 3/4 cohort reported the largest gains in achievement on the PAT-M test. The observed pedagogies that influenced students’ achievement in Year 3/4 included explicit instruction in relation to developing students’ conceptual understanding of new mathematical concepts, short diagnostic teaching cycles, consistent lesson structures, and high expectations. Pedagogical practices such as copying out times tables tasks possibly act to explain why changes in proficiency with number facts were limited in most classes throughout the initiative. Practices associated with removal of small groups of lower achieving students to work with teacher aides were also shown to affect students’ achievement negatively. Further, it was found that achievement on PAT-M was not contingent on proficiency with mathematical operations or number facts.

Analysis of the findings from the computation diagnostic tests revealed that, overall throughout the initiative, changes to students’ proficiency were limited. It was important to observe why this was the case particularly as the finding does not correlate to what was found for the PAT-M. Error analysis revealed that critical errors that inhibited students’ success included factors related to productive dispositions (i.e., not attempting tasks), conceptual understanding of the operations and renaming processes, and procedural fluency with number facts. Relating to problem-solving, students’ proficiency with problem-solving increased for most computations in most classes throughout the initiative. Key errors observed on the written test influencing students’ success included difficulties selecting the correct operations, difficulties isolated the correct values from questions, and not attempting tasks. These findings indicate that students first need to develop strategic competence as a priority to be successful in problem-solving, and these issues precede procedural fluency issues (associated with process skills). Newman interviews confirmed that comprehension difficulties were prevalent across
the sample at the beginning of the initiative, however, concurring with the written test findings, students’ problem-solving proficiency increased during the initiative as reflected by a reduction in computation errors. The consequence of a reduction in computation errors meant that the prevalence of transformation errors increased. In other words, students were progressing through the five stages of problem-solving, but difficulties with transforming the questions to appropriate computations still existed.

Newman interviews also revealed that the nature of the interviews acted to influence students’ problem-solving proficiency positively, particularly in the older cohorts (i.e., Year 5/6). Many students who previously had answered problem-solving tasks incorrectly on the diagnostic test were able to answer the questions correctly during an interview with the researcher. The potential reason for this finding was observed to be linked to the influence of the personal interaction involved in the interview, and the role of the metacognitive scaffolding provided by the interview questions.

In addition to the findings related to students, the way in which teachers responded to the initiative and professional development was also contingent on factors such as the timing of the professional development sessions. These factors were intertwined with school factors, including overarching standards and expectations. How well this aligned with community expectations was also important. There is a relationship between cultural learning preferences for Indigenous communities as the findings from this study relating to students’ learning preferences. The summary of the relationships between findings is outlined in Figure 51. These factors will be discussed in the following chapter.
Figure 51: Summary of alignment between findings from all data sources
6 DISCUSSION

6.1 Introduction

This chapter critically analyses the findings of the study, and aims to identify factors that influenced students’ mathematics proficiency from the primary mathematics initiative implemented in an Indigenous community school. This analysis will provide evidence to address the third research question:

*What factors related to the implemented initiative influenced primary students’ mathematics proficiency in an Indigenous community school?*

Factors influencing students’ proficiency will be explored within three domains of learning: the intra-individual, the inter-personal, and the socio-historical. Exploration of these factors is necessary to: 1) understand variations in findings related to changes in students’ mathematical proficiency across data sources; 2) understand the underlying reasons for the limited mathematical proficiency of students at the beginning of the initiative; 3) understand variations in changes to students’ mathematical proficiency between cohorts; 4) justify pedagogical recommendations developed from findings from the initiative; and 5) understand the role of cultural factors in influencing students’ mathematical proficiency.

Within the intra-individual domain, factors related to individual students will be discussed including the role of place value and number fact knowledge, motivational factors, the role of metacognition, and behavioural factors. The influence of access as well as teachers’ attitudes towards professional development will be considered within the inter-personal domain. Further related to teachers, insights on changing teaching practices gained from the initiative, and the influence of specific pedagogies on student achievement across each of the classes will be deliberated. Also, within the inter-personal domain, school factors will be explored. The socio-historical domain will summarise critical factors related to teaching and learning for Indigenous students.
6.2 Explaining changes to students’ mathematical proficiency

Students’ mathematical proficiency changed in a variety of ways across the duration of the initiative. Why particular measures of mathematical proficiency reported positive improvement (e.g., whole class PAT-M gains in achievement of up to 14-months, and individual gains up to 3-years or more) whilst other measures reported relatively limited improvement (e.g., number fact and computational proficiency remained below the expected achievement standard for each year level) may be explained by considering the hierarchical structure of mathematics knowledge.

As an esoteric knowledge form, mathematics is structured hierarchically in that knowledge must be developed in a particular sequence (Freudenthal, 1973; Sfard & Linchevski, 1994). The relationship between the hierarchical structure of mathematics and the findings from this study is outlined in Figure 52.

![Figure 52: Depiction of a hierarchical knowledge structure, with integration of the knowledge tested by particular data sources.](image)

In a hierarchically organised subject, prerequisite knowledge is a critical consideration (Muller & Taylor, 1995). For example, problem-solving is dependent on computational proficiency, which is subsequently reliant on number fact fluency and understanding of place value concepts (Geary, 2004). Cognitive load theories help to explain why prerequisite knowledge needs to be retained as the ability to fluently recall critical prerequisite knowledge such as number facts and place value concepts reduces working memory load allowing for more effective problem solving (Baxter, Olson, & Woodward, 2001; Woodward, 2006). Therefore, the limited computation proficiency of students by the end of the initiative may be
explained by students’ critical difficulties with place value concepts (as indicated by the numeration diagnostic test), and limited number fact proficiency that persisted throughout the initiative. The findings from this study indicated that difficulties with place value concepts and limited number fact proficiency remained into the upper primary years. The implication is that this critical knowledge was not developed in the early years, and the gap in knowledge persisted for the remainder of students’ primary schooling years. While such findings are not unique to Indigenous learners, the challenges were particularly persistent for students in this context.

Consideration of the hierarchical nature of mathematical knowledge helps to explain possible reasons why proficiency as measured by the PAT-M increased and effect sizes were generally large. These increases were reported despite students’ difficulties with prerequisite mathematical concepts and skills (i.e., place value and number facts), and subsequent difficulties with computations. When examining the nature of the types of questions delivered on the PAT-M test, it was found that a substantial portion of the test questions related to numeracy as opposed to application of specific mathematical skills or procedures. That is, questions were largely examples of horizontal mathematisation (Freudenthal, 1991) as they related to mathematics derived from real-world tasks. This is also the common definition of numeracy (e.g., OECD, 2001). For example, on the Year 3 PAT-M test, the proportion of test questions are distributed across the strands of mathematics as:

1. Number: ~51.4% of test items
2. Geometry and measurement: ~34.3% of test items
3. Statistics: ~14.3% of test items

This distribution indicates that a substantial portion (over one third) of the test items related to geometry or measurement. Further analysis of the geometry and measurement questions revealed that the majority of the questions were numeracy-type tasks and were answerable through horizontal mathematisation techniques. Proficiency in answering these questions is developed as mundane knowledge forms are learnt, through common-sense exposure to applications in real-world contexts. For example, specific test items on the PAT-M asked what units would be used to measure how much milk was in a big jug (i.e., litres, metres, kilograms, centimetres), or what block would fit in a specific hole depicted in an image. Answering these questions is not contingent on possessing any specialised mathematical
knowledge or understanding developed solely from school instruction. For example, a person with spatial awareness and experience with everyday measurement most likely would be able to develop the necessary conceptualisations to answer these questions. These questions are examples of horizontal mathematisation using Freudenthal’s definitions (1991). The finding that PAT-M and the diagnostic tests were measuring vastly different aspects of mathematics was supported by the examination of specific cases which revealed that improved achievement on PAT-M was not contingent on computational proficiency. This was supported further by analysis of findings from the PAT-M that revealed that geometry and measurement were the strands in which majority of the classes reported the greatest improvements. Likewise, findings from the numeracy test carried out by NAPLAN support these conclusions as students’ numeracy achievement also increased as a result of the initiative in the sample school. In short, the increase in proficiency with mundane knowledge forms was not equivalent to, or accompanied by, the development of a deeper esoteric understanding of number concepts.

These findings suggest that improving students’ proficiency at a numeracy level precedes, or is easier to achieve, than changing students’ proficiency with specific, and more abstract, mathematical structures and concepts. This finding is important because proficiency with critical structures and concepts such as number facts and computation processes becomes increasingly necessary as mathematics learning progresses through the year levels. Aligning this with the work of Bernstein (1999, 2000), who visualises esoteric knowledge forms such as mathematics as a triangle as outlined in Figure 52, numeracy can be considered as preceding proficiency with increasingly abstract mathematics knowledge, such as computational structures (i.e., algorithms), which are dependent on proficiency with prerequisite understanding of place value, and number facts. This critical analysis of the varied findings from data sources explains the sequential and hierarchical way in which mathematical proficiency is developed. Possible reasons for particular trends in students’ proficiency will be explained further in subsequent sections.

6.3 Factors influencing students’ mathematics proficiency

The probing of students’ proficiency in this study moved from a general view of students’ achievement on standardised mathematics tests (PAT-M), to a more focused investigation of specific concepts (diagnostic tests), to an analysis of students’ difficulties (through error analysis of written scripts in conjunction with Newman interviews). The depth
of analysis facilitated by the range of data sources in conjunction with observational data identified a range of critical factors that influenced students’ mathematical proficiency. These factors pertain to the intra-individual domain as they relate to an individual’s cognitive process and affective attributes (such as productive dispositions). In addition, it is also understood that the intra-individual domain is influenced by the inter-personal environment and regulated by the socio-historical regulating norms and conventions (see Figure 53). How these domains impact on students will be further explored in subsequent sections.

Figure 53: The influence of the socio-historic and inter-personal domain on the intra-individual domain.

6.3.1 The intra-individual domain: Student-specific factors

The following section considers the intra-individual domain of learning. This includes consideration of individual student’s learning processes, and individual student’s understanding of, and fluency with, specific concepts and skills that impact on their
mathematical proficiency. Figure 54 summarises the intra-individual domain related to student learning.

![Intra-Personal Domain](image)

*Figure 54: Factors related to students’ proficiency, within the intra-individual domain*

The findings from this study suggested that proficiency with critical prerequisite concepts and skills, such as place value and number facts, potentially explained students’ difficulties with computations as assessed by diagnostic tests. Further, it was also found that student motivation and productive dispositions towards mathematics played a critical role in influencing students’ success in mathematics throughout the initiative in the way that it influenced academic engagement. Analysis of the problem-solving findings yielded information concerning the critical role of metacognition and the importance of oral, face-to-face learning for Indigenous learners in this context. The way in which behaviour impacted on students’ ability to engage in the instructional discourse of the classroom will also be discussed.

6.3.1.1 The critical role of place value and number fact knowledge

Understanding and fluency of place value concepts and number facts is necessary for students’ success with computations. This finding was supported consistently by correlations across data sources that indicated that students’ proficiency on number and place value tasks (from the numeration diagnostic test) was limited both at the beginning of the initiative, and throughout the initiative. As well, renaming and number fact errors were a prevalent barrier to students’ success on computation tasks. Despite variation in teaching practices across the four classes, these issues were prevalent throughout the initiative in each class. Cognitive load
theory may help to explain why place value and number facts are critical to students’ mathematical proficiency.

In the analysis of the diagnostic test findings, it was found that a dominant error inhibiting students’ success on computation tasks was conceptual understanding and procedural fluency associated with renaming procedures. This was highlighted in the subtraction diagnostic test findings where the most common errors were related to renaming (e.g., finding the difference between values rather than correctly renaming and subtracting, or answering ‘0’ where renaming was required). Renaming processes are related to a conceptual understanding of place value.

Factors that influenced students’ proficiency at an individual student level also related to proficiency with number facts. At the beginning of the initiative, each cohort’s achievement on the number fact component of the diagnostic tests was below what is typically expected for each year level according to national curriculum documents (ACARA, 2018b). Throughout the initiative, there was varied improvement in number fact proficiency for each class, however, proficiency remained below expected standards. For example, by the end of the initiative, Year 3/4, 4/5, and 5/6 students were still demonstrating developing proficiency with multiplication facts. These findings explain why number fact errors were a prevalent error on diagnostic computation tests. It was also found that, in Year 2/3, students’ results on the pre-initiative number fact test were a predictor of whether students were able to participate in the high, middle, or low streamed mathematics groups in class. That is, it was number fact proficiency alone that facilitated students’ ability to engage with increasingly complex mathematical tasks. Such findings support the model of mathematical knowledge presented in Figure 52 and are consistent with the depiction of mathematics as essentially a hierarchical discipline (Freudenthal, 1973; Sfard & Linchevski, 1994).

Cognitive load theory discusses how it is important to consider the cognitive load imposed by a task as the working memory is limited in capacity and duration. If a task places large demands on working memory, then cognitive overload can occur which subsequently inhibits learning (Pass, Renkl & Sweller, 2004). When performing a computation, a student must draw on their understanding of renaming concepts and procedures as well as recall the necessary number facts. Cognitive load theory explains that when fluency and understanding of place value and number facts is not established, it is likely that students will experience cognitive overload. That is, it is highly likely that a student who is not fluent with number facts
and is not able to rename between place values proficiently and flexibly, most likely will struggle with computation tasks that require proficiency with these skills and concepts. For example, to calculate a sum such as “603 – 48”, renaming is required and a student must realise than in 603 there are not zero tens, but rather 60 tens (that is, 6 hundreds can be renamed as 60 tens). As well as considering place value, the cognitive load imposed by computational tasks increases if the recall of number facts is not efficient (Woodward, 2006). The findings from this study support the assertion that proficiency with place value concepts and number fact fluency is a prerequisite of computational work as possessing this prerequisite knowledge lessens the demand on student’s working memory (Sweller, 2011, 2016).

These findings reaffirm that place value and number fact fluency are critical components of mathematics education, and place value and number fact proficiency directly influence students’ success with other mathematics concepts and tasks. The importance of focusing on developing proficiency with number concepts has also been noted in other Indigenous contexts, and within the Remote Numeracy Project (Jorgensen, 2018). Notable teaching practices in this project included emphasising and spending a considerable amount of lesson time on developing competency, confidence, and fluency with “the big ideas of number” which include place value and operations. The Remote Numeracy Project cites that the critical number concepts noted were seen to be the “key” to developing numeracy. The importance of foundational proficiency with place value and number facts supports the conceptualisation of mathematical knowledge as hierarchical in nature, as discussed in Section 6.2. Proficiency with particular mathematical concepts is dependent on proficiency with foundational concepts as evidenced in the case of place value and number facts in supporting computational proficiency.

6.3.1.2 Prioritising the development of strategic competence for problem-solving proficiency

In relation to students’ problem-solving proficiency, it is important that instruction focuses on developing students’ strategic competence by increasing their familiarity and competency in understanding problem structures. Firstly, strategic competence needs to be developed prior to focusing on computational proficiency within the context of the problem-solving tasks. Findings indicated that students’ problem-solving proficiency was hindered by underlying comprehension difficulties, which were indicative of a lack of strategic competence. The mathematical elements of the problem (e.g., the computations) were not the
primary difficulty for students. Rather, comprehending and transforming the worded task into the appropriate mathematical computation was the critical difficulty. A priority on developing strategic competence is supported by the hierarchy of steps proposed in Newman’s error analysis (White, 2005), which places comprehension and transformation prior to process skills.

Strategic competence relates to a students’ ability to formulate and represent mathematical problems appropriately (Kilpatrick et al., 2001), which is what students are required to do in the comprehension and transformation stages in a Newman interview. Students across each cohort had various difficulties with problem-solving tasks which were related to strategic competence (i.e., comprehending and transforming the task into an appropriate computation) as measured by diagnostic tests and Newman interviews. Prevalent errors were associated with the selection of the incorrect operation or the selection of the incorrect values on which to operate. These findings indicate that students’ comprehension difficulties manifest as students’ employing “number grabbing” techniques in an attempt to solve the problem without understanding the task, or with students seemingly selecting a random incorrect operation in an attempt to complete the task (as observed in analysis of the diagnostic test scripts). Students’ key problem-solving difficulties surrounding strategic competence found in this study highlight the importance of exploring and discussing problem structures with students. For example, working with students to identify what within the problem signifies that it is subtraction, and exploring strategies to identify correctly the values on which to operate.

These findings are contextually important for Indigenous learners considering that literacy difficulties, as measured by standardised tests (e.g., PISA), for Indigenous learners are commonly cited. For example, the 2012 PISA findings noted a two-and-a-half-year gap between Indigenous learners and non-Indigenous peers in reading (Dreise & Thomson, 2014). However, this study sheds further light on this matter and goes beyond identifying broad reading difficulties. The findings from Newman interviews found that it was not the mechanics of reading the problems that was the difficulty for learners, but it was the ability to comprehend tasks. This finding has implications for directing teaching and intervention focuses for learners with problem-solving difficulties in this context.
Effective questioning facilitates metacognition for problem-solving proficiency

Scaffolded questions supporting the problem-solving process increase students’ problem-solving proficiency as they provide support for students’ metacognitive processes. Specifically, Newman interviews supported students’ strategic metacognitive skills by providing the scaffolded problem-solving heuristics. This was evidenced by findings from Newman interviews that indicated that the nature of these interviews acted to increase students’ problem-solving proficiency. That is, it is proposed that the Newman interviews acted as a problem-solving heuristic. The similarities between the Newman interview questions and Polya’s problem-solving heuristic are outlined in Table 34. The importance of problem-solving heuristics, strategic metacognitive skills, and effective questioning has been highlighted in studies focused on students’ problem-solving capabilities (e.g., Flavell, 1976; Hacker & Dunlosky, 2003; Ozsoy & Ataman, 2009).

<table>
<thead>
<tr>
<th>Polya’s four phases of problem-solving (Polya, 1988)</th>
<th>Newman interview questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand the problem</td>
<td>Question 1 (Reading): Please read the question to me?</td>
</tr>
<tr>
<td></td>
<td>Question 2 (Comprehension): Tell me, what is the question asking you to do?</td>
</tr>
<tr>
<td>Devise a plan</td>
<td>Question 3 (Transformation): Tell me how you are going to find the answer?</td>
</tr>
<tr>
<td>Carry out the plan</td>
<td>Question 4 (Process skills): Show me how you are going to get your answer.</td>
</tr>
<tr>
<td></td>
<td>Question 5 (Encoding): Now, write down your actual answer.</td>
</tr>
</tbody>
</table>

Whilst the “look back” phase of Polya’s stages is not related directly to a specific Newman interview question, it was found that in a Newman interview students were generally reflective of their answers (i.e., “looking back”) throughout all stages of solving the problem.

Administration of the Newman interviews revealed that many students, particularly in Year 5/6, were able to answer problem-solving tasks correctly during the interview when previously they were answered incorrectly on the independently completed diagnostic test. It was found that the process of conducting Newman interviews acted to slow students down (i.e., increase the time spent solving each question). In many instances, students were observed to
restart attempts, make multiple attempts, and self-correct throughout the interview process. These were all behaviours that were not observed during the administration of the written diagnostic test. The nature of the interview and the questioning stages encouraged students to self-regulate and not rush through attempts.

These findings evidenced the Newman interviews acting as metacognitive scaffolding as a result of the questioning provided throughout the interview. That is, the interview supported students in articulating what they needed to do to solve the task as well as to consider the approaches they were going to take. Comparatively, during completion of a written test it is possible to proceed with attempting the questions without considering an approach to the task, and without employing any metacognitive strategies. These findings support earlier literature which indicated that the ability to ask effective questions during the problem-solving process acts both to develop and to activate metacognitive skills (Desoete et al., 2001; Hacker & Dunlosky, 2003) and, as observed in the findings, fosters mathematical problem-solving. In this study, Newman interviews acted to reduce incidences of strategic competence errors via the metacognitive scaffolding provided by the questions.

6.3.1.4 The complex influence of Indigenous students’ productive dispositions on mathematical proficiency

Indigenous students’ mathematical proficiency can be hindered by their productive dispositions towards tasks. When Indigenous students do not envision themselves as capable learners of mathematics, the negative influence on achievement is profound as it is exacerbated by complex cultural factors such as “shame” and stereotype threats (Beilock et al., 2004; Beilock et al., 2007; Groome & Hamilton, 1995; Halse & Robinson, 1999; Martin, 2006). The importance of productive dispositions (related to self-efficacy; Bandura, 1997) as an essential strand of mathematical proficiency (Kilpatrick et al., 2001) has been highlighted in this study’s findings. Positive attitudes towards mathematics have been linked to Indigenous students making fewer careless errors in problem-solving (Clarkson, 1983).

Throughout the diagnostic tests, a frequent trend was that the most prevalent barriers to students’ success with computation tasks was related to not attempting test questions. The prevalence of students not attempting test questions persisted in all classes despite the types of instructional practices implemented. Throughout the initiative, instances of students not attempting questions did not decrease substantially and, in fact, increased in many instances
There are two potential reasons for students not attempting items on the written test. The first potential cause is related to students lacking conceptual understanding and/or procedural fluency with the particular computation being assessed. That is, the student simply does not know how to do addition, subtraction, multiplication, or division computations so does not attempt the test items. This is a potentially valid explanation for such findings as the frequency of students not attempting tasks for the addition and subtraction diagnostic tests decreased as the year level increased (i.e., typically the most instances of non-attempts were in Year 2/3, and the least instances were in Year 5/6). Other evidence from numeration tests (assessing place value concepts) and number fact tests support this explanation as students did not possess proficiency with the foundational concepts necessary to attempt, or be successful with, computations.

The second potential reason for the high frequency of non-attempts on the diagnostic test is associated with students’ productive dispositions because lacking procedural fluency and conceptual understanding of the operation or computation does not explain all findings. In Year 5/6, non-attempts were prevalent for the multiplication computation diagnostic test and instances of non-attempts increased throughout the initiative. Proficiency with computations was limited in this cohort despite Year 5/6 students demonstrating developing proficiency with multiplication facts, which indicated some level of understanding of the concept of multiplication. Further, students in this cohort had substantial exposure to multiplication computations throughout the school year as a portion of mathematics teaching time was focused on this topic. Therefore, limited conceptual understanding cannot explain these findings alone, and it is possible that students were not attempting substantial portions of test items as they had not developed a productive disposition towards mathematics and did not possess the motivation to complete the tasks. Similar avoidance of tasks have been linked to students’ productive dispositions in other studies (Woodward, Beswick, & Oates, 2017).

The case report from Lisa in Year 5/6 confirmed the presence of work avoidance behaviours and a lack of a productive disposition which were also observed in other students. For example, Lisa’s comments such as “I hate this”, “my brain hurts”, “this is dumb” and recording “hard” or “I don’t know” on questions is indicative of a lack of a productive disposition towards mathematics. Lacking a productive disposition coupled with teaching practices that were contrary to recommended practices (i.e., poor remediation of students’ difficulties with concepts during mathematics lessons) acted to influence students’ motivation.
and subsequent proficiency as measured by the diagnostic tests significantly. However, due to the complex inter-relationships between proficiency strands, it is possible that students’ difficulties related to productive dispositions and conceptual understanding both acted to influence their proficiency.

Lisa’s responses during testing also exemplified negative stereotypes surrounding her capability (i.e., “I’m not that smart”), and these attitudes were not unique to Lisa’s case. At the school, “that’s shame” (in reference to getting something wrong that was perceived as something you should have been able to do) was part of students’ daily colloquial language. Shame was an internalised part of students’ everyday behaviour. These findings align with research that discusses Indigenous students’ increased fear of failure and preference to not give things a go at all, rather than fail (Martin, 2006). This supports the potential link between students not attempting test questions and their productive dispositions in that it was preferable for the students not to attempt the questions rather than risk attempting the questions and potentially getting them wrong. Specifically, Indigenous researchers have noted that Indigenous students can prescribe to negative beliefs surrounding academic capability (Sarra, 2011a). However, stereotype threats are not unique to Indigenous students (i.e., Beilock et al., 2007; Kilpatrick et al., 2001) and discussion of the influence of these beliefs potentially has broader applicability to other vulnerable groups.

The relationship between students’ productive dispositions (or self-efficacy) and motivation to complete tasks has been noted to impact on students’ performance in other research (e.g., Harrison, 2011). One potential reason for the behavioural manifestations of students’ limited productive dispositions influencing performance is that frequent behavioural disruptions to learning reduces academic learning time. In Gettnger and Seibert’s (2002) four variable model on academic learning time, behavioural disruptions reduce the engaged time in learning. The way in which students’ productive dispositions has driven students’ behaviour in this study has been well documented in the findings, particularly in relation to testing behaviour which was characteristic of work avoidance. These findings concur with wider research (Groom & Hamilton, 1995; Hughes, More & Williams, 2004; Louth, 2012). The impact of students’ conceptualisation of themselves as capable learners of mathematics has been documented in the administration of testing in this study, resulting in subsequent work avoidance behaviours. Work avoidance impacts on students’ learning time during class. That is, it is likely that these attitudes and behaviours exhibited by students is not unique to when
they were completing tests, and that they were also avoiding work during class. The relationship between students’ motivation to complete tasks and invest in their learning manifests as their academic time on task, which has been shown to influence academic success (OECD, 2016). Students’ limited productive dispositions have been documented in the findings from this study via low motivation to spend concentrated time on task learning mathematics concepts during class. That is, some students had limited investment in their learning.

The cyclical relationship between academic achievement, students’ productive dispositions, and subsequent academic time on task has been reported in other studies (e.g., Ma, 1997; Ma & Xu, 2004). It is understood that students’ motivation, fostered by a productive disposition, affects their willingness to engage with and spend time on tasks (OECD, 2016). A students’ perseverance facilitates academic performance and achievement as students invest greater effort in their learning (Duckworth & Seligman, 2006). The work of Ma and Xu (2004) outlines that the relationship between self-efficacy and achievement is not reciprocal, but achievement in mathematics has causal dominance over productive dispositions (i.e., attitudes towards mathematics). The cyclical relationship between productive dispositions and achievement potentially explains why students in this study were working well below expected standards, and why it is difficult to influence students’ productive dispositions as it is contingent on increases in achievement. When students are under-achieving, their beliefs in their capability to achieve may be influenced negatively which reduces their motivation and decreases their invested time on task. Subsequently, this only makes it harder to increase achievement and the negative cycle perpetuates itself. This also potentially explains why achievement gaps become greater throughout the grades as the spiral of declining achievement and reduced self-efficacy continues to increase students’ difficulties with the mathematics work. The likelihood of students investing academic engagement in their learning decreases throughout the years through repeated experience with difficulties meeting achievement standards, resulting in lowering their productive dispositions. Avoidance behaviours negatively influence students’ academic achievement in a cyclical way (Harrison, 2011; Ma, 1997; Ma & Xu, 2004). How teacher practices in classes influenced students’ productive dispositions will be discussed later in this chapter.
6.3.1.5 The difficulties for Indigenous students’ working within the school discourse

To support Indigenous students’ achievement and success at school meaningfully, the way in which Indigenous students’ backgrounds may impact on their success in engaging in the school discourse needs to be considered. Essentially, access to engaging in school can become limited for Indigenous students due to mismatches between home discourses and school discourses, as well as limited exposure to early instructional discourses. Findings related to students’ behaviour and investment in mathematical tasks in this study indicated that Indigenous students struggle to engage productively in mathematics when they are not able to engage in the discourse of the school environment. The influence of students having limited productive dispositions towards mathematics has been discussed in the previous section. It is important to consider further the way in which the behavioural manifestations associated with work avoidance relate to mismatches between the school discourse and the capital that students bring to school.

It is understood that the habitus of a student and the linguistic capital that they bring to school is influenced by a student’s cultural background (Bourdieu, 1981; Bourdieu & Wacquant, 1992; Zevenbergen, 2004), and that Indigenous students bring to school a cultural background that is potentially not aligned with a school environment. Due to cultural factors, Indigenous students may have had minimal exposure to the types of instructional discourses relied upon in schools (Zevenbergen, 2004), and this places Indigenous students at a disadvantage. The work of Sarra (2011) discussed how Indigenous students may not bring to school skills deemed necessary in the school environment that are expected, generally, to have been developed prior to entering school. Consideration of the findings related to behavioural occurrences throughout this initiative align with assertions that the capital these students brought to school placed them at a disadvantage as evidenced by difficulties engaging with, and sometimes the desire to avoid, particular tasks. In this study, frequent disengaged and disruptive behaviours observed during mathematics lessons indicated that some students struggled to engage in the instructional discourse of school. These findings align with literature that suggests that the capital of students influences the way in which they engage with the school environment, and engagement with the pedagogic discourse of school is needed for success in school (Jorgensen & Grootenboer, 2011).

Throughout the initiative, several instances of student behaviour were noted to have impacted on students’ achievement in mathematics. These include work refusal during
administration of the mathematics tests as well as other disengaged or disruptive behaviours during testing or mathematics class time. In the development of the initiative, observation and analysis of behavioural factors was not an intended focus of the study, therefore no formalised data was collected on in-class behaviour during mathematics lessons. However, field observations throughout the initiative revealed that some students demonstrated work refusal behaviour, or other behaviours that were disruptive to learning. The manifestation of such behaviours was observed in a variety of ways including being late to class, refusing to come to class, refusing to commence work, talking to peers whilst instruction was delivered, putting their head down on the desk and attempting to sleep, not complying with instructional directions, crawling under tables, walking around the room, leaving the room, purposefully disrupting the teacher, and engaging in conflict with peers. Similarly, records of behavioural instances occurring during testing also highlighted instances of work refusal and disruption. These instances were prevalent particularly in the Year 5/6 cohort. The evidence from these occurrences indicates that work avoidance behaviours directly influenced students’ achievement. These assertions are supported by findings related to increased student achievement when there is consideration of the discourses in which Indigenous students are more familiar with. That is, when tasks are delivered in oral, face-to-face, and individual settings, Indigenous students’ motivation to complete tasks is supported, and their academic achievement increases.

6.3.1.6 The importance of oral communication and face-to-face interactions for Indigenous students’

Individual, oral, and face-to-face communication can influence Indigenous students’ engagement in completing mathematics tasks, and subsequently support increases in their mathematical proficiency. Oral learning and face-to-face interactions have a central role in learning mathematics for Indigenous learners. Face-to-face interaction with the researcher played a critical role in nurturing students’ motivation and reducing avoidance behaviours as indicated by the Newman interview findings. The motivational impact of positive and timely affirmation of students’ thinking has been well documented (e.g., Engelmann, 2007; Hattie, 2008). Yunkaporta (2009) concisely surmises that “in our world the deepest knowledge is not in words. It is in the meaning behind the words, in the spaces between them, in gestures or looks, in meaningful silences” (p. 2). The findings of this study support this assertion.
In this study it was found that no students declined to complete the problem-solving questions during the one-on-one interview. This indicated that the nature of the format of an interview had a positive impact on students’ investment in completing the questions, and students were more willing to persevere with the tasks. The finding that no students were displaying task avoidance during interviews is important considering that not attempting the questions was the most predominant barrier to success for students on the written problem-solving diagnostic tests. During interviews it appeared that the students did not want to disappoint the researcher administering the question and, as a result, perceived that not giving an answer was not an option. The personal nature of the interviews increased the students’ perceived accountability associated with the task. Further, the way in which students displayed interest in the researcher’s body language evidenced the importance of the non-verbal communication that occurred during the interview process.

In addition to the inter-personal nature of Newman interviews increasing students’ investment in completing the tasks, it was also found that, though, commonly, reading-related difficulties did not impede students’ problem-solving proficiency, there were notable findings surrounding students’ ability to proceed with reading questions when they came across difficult words in questions, particularly people’s names. Often students required confirmation from the researcher administering the test that they had read a word or name correctly before they would proceed in reading the question or attempt to solve the problem. This finding potentially explains why students may not have attempted problem-solving tasks during administration of a diagnostic test, and indicates that the personal nature of the Newman interviews was influential in facilitating students’ problem-solving achievement. That is, students may not have attempted problem-solving questions on the diagnostic tests because they were “stuck” on a word that they would not proceed past, and a very small gesture of reassurance from the researcher enabled them to overcome this obstacle. Overall, these findings have important implications for Indigenous learners when considering cultural factors, such as the importance of oral communication and non-verbal communication for Indigenous learners as noted in previous research (e.g., Gorman & Toombs, 2009; Nichol & Robinson, 2000; Warren & Miller, 2013; Yunkaporta, 2009).
6.3.2 The inter-personal domain: Teacher-specific factors

In the following section, factors related to the teacher, the facilitator of the inter-personal classroom environment, will be discussed. It is important to explore differences in practices between each classroom due to the reported differences in the changes to students’ proficiency in each cohort. Figure 55 summarises this relationship.

![Diagram showing the inter-personal domain relationship between students and teachers.](image)

*Figure 55: Factors associated with teachers that influence students' mathematical proficiency*

Changing teachers’ practices was found to be a difficult endeavour that was dependent on the sustained involvement of the researcher in the role of a mentor. That is, the relationships fostered between the researcher and teachers through a mentoring professional development model were important in supporting teachers’ willingness to consider implementing new pedagogical practices. The types of considerations teachers take into account when choosing whether or not to employ new recommended practices was also highlighted in findings from this study. In relation to influencing students’ mathematical proficiency, pedagogical choices in each class, and how they influenced student achievement, depended on how they met the cognitive and affective needs of the students. This will be presented as a synthesis of the
effectiveness of pedagogies in influencing students’ mathematical proficiency in the context of this study. Pedagogies that positively developed Indigenous students’ productive dispositions towards mathematics, and proficiency in mathematics will be the focus.

6.3.2.1 The path of least resistance: Teachers considerations regarding practice choices

In terms of evaluating the impact and effectiveness of this initiative, it is important to explore why teachers were making decisions to employ, or not to employ, recommended practices. Firstly, access to professional development potentially influenced the uptake of recommended practices. Further, when teachers made choices regarding the implementation of new practices, they considered the ease of implementing the new practices, and the “pay-off” of the new practices in relation to student behaviour. Teachers capability and confidence in implementing new practices is also important in influencing pedagogical choices. These choices have implications for students’ learning in mathematics.

Access to professional development is important in influencing teachers’ implementation of recommended practices as evidenced by the findings from the Year 2/3 class teacher, Helen. Helen was a teacher who was engaged in the initiative throughout the year. She exhibited interest in employing effective teaching practices, and committed to maintaining high academic expectations for her students. However, Helen was not able to be released during class time to participate in the two professional development sessions that were conducted throughout the initiative. This potentially influenced the uptake of recommended practices in Year 2/3 as she was observed to increasingly use small group work focused on worksheet tasks throughout the initiative. Prevalent use of worksheets meant that classroom instruction time was not focused on the conceptual development of critical mathematics concepts and skills through explicit instruction. Lack of conceptual development of these concepts and skills was predominant particularly when worksheets were being used in a group-work setting, which meant that most students were working through tasks independently without teacher guidance. Further, use of worksheets also replaced the critical role of using concrete materials to support conceptual development in the early phases of learning concepts. This process is a critical stage of the CRA sequence (Bruner, 1966; Mancl et al., 2012; Matthews et al., 2003; Reisman, 1982).

Rotational group work also limits a teacher’s capability to provide scaffolding during instruction. Cognitive load theories highlight the importance of scaffolding support for novice learners (Atkinson et al., 2003; Chen et al., 2016a; Hattie, 2008; Kalyuga et al., 2003; Renkl et al.,
The example of Helen’s practices serves to highlight how not having the opportunity to participate in professional development sessions potentially affected Helen’s chance to further consider her teaching approaches in light of the effective practice recommendations and preliminary data analysis. However, access to professional development was shown not to be the sole factor influencing teachers’ uptake of suggested practices.

The findings from this study indicate that changing teaching practice was a complex endeavour in this setting. Some teachers lacked confidence in teaching mathematics, and had limited pedagogical content knowledge in mathematics. These difficulties manifested as limited uptake of recommended practices despite participation in professional development opportunities (see Section 5.3). This finding was exemplified in the Year 4/5 and 5/6 classes where several recommended practices were not implemented even when teachers identified that students may not be benefiting from current practices (which were contrary to recommendations). Examples of practices that persisted through the initiative included limited use of concrete resources, and utilisation of routine copying tasks to teach number facts in some cohorts.

Utilisation of concrete materials is a recognised element of effective practice associated with CRA teaching sequences in mathematics education (e.g., Bruner, 1966; Mancl, Miller & Kennedy, 2012; Matthews et al., 2003; Reisman, 1982). In Year 4/5 and 5/6, concrete materials were not utilised at any time during the teaching sequence even when the use of concrete materials for the development of mathematical understanding of key upper primary concepts such as multiplication and division were recommended practices. The reasons provided for this were twofold. Firstly, the use of concrete materials creates organisational demands on the classroom teacher. That is, to utilise concrete materials successfully during lessons, there are several logistical considerations such as the sharing of materials, and appropriate use of concrete manipulatives by groups of students. It was observed that such logistics posed difficulties for teachers in relation to managing on-task student behaviour because poorly managed use of materials creates a distraction for students (i.e., something to fidget with on the table distracting from the teaching instruction). Managing concrete materials required a shared understanding between teachers and students regarding their appropriate use, and these routines were not established in these classes. Even in classes where behavioural disruptions were minimised through the establishment of known routines, such as those exemplified in Year 3/4, the use of concrete materials was not prevalent due to the concern that they were a
potential distraction which would impact on students’ ability to engage with the teaching instruction occurring. Secondly, effective utilisation of concrete materials requires the teacher to possess the necessary pedagogical content knowledge to do so. It is understood that use of concrete materials in mathematics is important for Indigenous students (Matthews et al., 2003) or students who have difficulties (Mancl et al., 2012) so that students can make links between symbols and their meaning.

These findings highlight the need for developing pedagogical content knowledge regarding the effective use of concrete materials in primary mathematics classes. The effective use of materials requires teachers to have a deep understanding of how to use the materials to enhance conceptualisation. If teachers do not possess strong pedagogical content knowledge surrounding the use of resources, then the logistical difficulties become too overwhelming for them to be used, as exemplified in some classes in this study. Beyond the scope of the observed sample of classes in this study, limited or no use of concrete resource was common in classes throughout the school, even in early years. As such, from the very first years of primary school there was not systematic development of students’ conceptual understanding of mathematical concepts by creating links between mathematical language, mathematical representations (i.e., the concrete materials), and mathematical symbols.

The trend of not employing practices due to behavioural or organisational considerations was also observed for other recommended practices. Despite indications of students’ persistent difficulties with number facts presented in professional development sessions, it was observed that Diane (Year 3/4), Jane (Year 4/5), and Paul (Year 5/6) were reluctant to abandon the practice of requiring students to copy out multiplication facts. Routine copying was implemented in these classes throughout the initiative despite pre-initiative findings indicating that current practices were not facilitating students’ proficiency with number facts, and despite teachers identifying that the task was “a waste of time” which did not contribute to students’ learning. Teachers indicated their reluctance to abandon these tasks in classes due to their key role in managing behaviour. The task of copying times tables acted to settle a class by occupying them from the very beginning of the mathematics lessons. It is likely that substitute activities or practices were not implemented as setting a copying task did not require any pre-organisation or teacher interaction throughout. Furthermore, this task was accessible to all students regardless of mathematics ability level. It could be argued that, in these instances, the goal of the teacher was centred on order, compliance, and ease of
implementation, rather than effective learning. In the field of reproduction, the students and teachers had negotiated a ‘truce’, a settlement in which easy-to-complete mathematical tasks were the reward for compliant behaviour.

These examples of pedagogical choices provide insight into how many of the practices chosen (or avoided) by teachers in this study were driven by behavioural considerations, and teachers’ capability. There was considerable resistance to change to alternative practices as the current practices were easier to implement, and/or had behavioural payoffs. This sheds light on understanding some impeding barriers in implementing effective practice recommendations in this context.

6.3.2.2 Teachers’ engagement in professional development

Considering the timing of professional development sessions is critical in influencing teachers’ participation and engagement in sessions. In this study, staff engagement was potentially contingent on the timing of meetings. Further, the importance of professional development being highly relevant to the current classroom practice of teachers was also highlighted when observing teachers’ responses to various professional development opportunities. These findings concur with Desimone’s (2009) features of effective professional development: a content focus, active learning, coherence, duration, and collective participation. This study contributes further to Desimone’s five features and suggests that the timing is also important in influencing the effectiveness of professional development sessions. In addition, how the professional development matches teachers’ level of pedagogical content knowledge needs to be considered.

During the initiative, professional development sessions were run in two formats: during school hours and after school hours. The during school professional development was supported by school administration through the release of teachers from classroom teaching duties during the sessions. Teachers were observed to attend these sessions consistently. Conversely, teachers generally responded to the after school professional development sessions with inconsistent attendance, which indicates that either attendance at these sessions was not seen as a priority, or that the timing was impractical for teaching staff. During these sessions, teachers were not highly engaged as evidenced by the limited discussion and collaboration. They acted as passive receivers of information. Teacher attitudes during the sessions were also generally negative as indicated by the negative responses to suggestions from co-workers.
Overall, the limited engagement demonstrated by staff during the professional development meetings in the early stages of the initiative reduced the effectiveness of the meetings and potentially explains the limited uptake of some of the recommended pedagogies at the beginning of the initiative. Desimone (2009) proposed that, to change teachers’ practice (to see them use new knowledge and skills to improve their approach to pedagogy), teachers need to experience effective professional development to increase their knowledge and skills, or change their attitudes and beliefs. These early sessions demonstrated that the professional development was not effective for teachers despite the content focus of the session. The ineffectiveness of the sessions was apparent in the limited collective participation, and lack willingness to participate in active learning.

In addition to timing of the sessions, another potential factor influencing teachers’ engagement may have been related to the content of the session. The first session focused on delivering the summary of effective practice recommendations to teachers. These recommendations were presented in a way that initially aimed to support teachers’ agency by presenting the recommendations under the presumption that teachers had the capacity to implement the practices as they saw fit. After limited uptake of practices was observed in the early phases of the initiative, the researcher reflexively reframed the professional development sessions to more specifically support teachers. This was enacted by establishing a context for the sessions based on student data for each teacher’s class. The sessions then focused on working with the teachers to model specifically how to use materials to develop algorithms conceptually, and discussing specific ways in which practices could be implemented (e.g., specifically discussing how strategy discussion could become part of a mathematics lesson) in light of the student data. In some instances, the modelling of specific mathematics lessons was included. The subsequent professional development sessions, and informal support by the researcher as a mentor, focused predominantly on supporting the development of teachers’ pedagogical content knowledge in mathematics as this was found to be necessary.

Consequently, because of the changes to the focus and timing of the sessions, teachers perceived the professional development sessions held during school hours to be effective and productive uses of their time. Teachers were observed to be highly engaged, positive, and interested as evidenced by the high levels of collaboration and discussion throughout the meetings as well as the self-reported satisfaction in the sessions by teachers. Teachers valued the collegial and collaborative discussions that resulted from the meetings, particularly related
to the sharing of practices. This indicated that the professional development was effective as it featured active learning and collective participation (Desimone, 2009). What teachers valued from these sessions, and what made them effective was the practical nature of the sessions. The sessions were highly contextualised, and practice discussions were based on evidence (class data). Therefore, these sessions were highly relevant to each classroom teacher. This aligns with the effective professional development feature of an appropriately contextualised content focus (Desimone, 2009). The communal sharing of ideas related to each class’s teaching practices was valued more by teachers than sessions where they acted as passive receivers of information regarding practices.

As teachers responded to the two types of professional development held during school hours, and after school hours in such a dichotomous manner, the provision of release time from teaching duties for professional development opportunities can be viewed as critical to increasing teachers’ engagement in such sessions. This has implications for recommendations regarding effective professional development in this setting. However, despite these professional development opportunities, changes to teaching practices that were envisioned and recommended were still not fully realised in the context of this study. This serves to highlight how professional development experiences centred on developing teachers’ pedagogical content knowledge capacity need to be extensive and sustained (beyond the scope of the 7-month initiative). This has implications for teacher training and professional development as some teachers in this study had been practising primary teachers for many years without deep understanding of how to develop and structure effective and high-quality learning experiences within mathematics. In this context, it was insufficient to assume that teachers had the capacity to implement effective teaching recommendations without extensive support.

6.3.2.3 The sustained role of the researcher as mentor was critical to the initiative

Supporting teachers in implementing effective practices required constant drive, evaluation, and re-evaluation. Having an expert teacher act in the role of a mentor is one method valued by teachers, and is vital to the success of educational initiatives such as the present initiative. Throughout the initiative, it was observed that the role, sustained presence, and involvement of the researcher was important in facilitating successful implementation of the initiative. Professional development delivered through mentoring is effective in enhancing
teaching practices and supporting teachers in adopting new practices (Onchwari & Keengwe, 2008; Weaver, 2004). It has been noted that the value of mentoring lies in the relationship between the mentor and teacher, and the mentor’s ability to relate to the teacher’s context and challenges. It is through this relationship that mentors can begin to shift teachers’ attitudes and begin to change practice (Onchwari & Keengwe, 2008), and this was found in the evaluation of teacher’s perspectives surrounding the initiative and the mentoring provided to them in this study.

In previous sections, the challenges associated with changing teaching practices have been discussed and observations throughout the initiative indicated that, without support from the researcher, it is likely that fewer changes to practice would have been observed. The researcher acted to support changes in teaching practice through the delivery of professional development sessions, and by acting in the role of mentor or expert teacher throughout the initiative. For example, the researcher’s role was important in providing teachers with the skills to support the implementation of specialised pedagogy such as the pedagogy associated with teaching computations and algorithms. The sustained presence of the researcher meant that an expert resource was available to upskill teachers and support the implementation of practices in a reflexive manner on an ‘as needed’ basis. As discussed earlier, teachers cited that access to the researcher was important in supporting their practice throughout the initiative.

Continual presence of the researcher in the school throughout the initiative also meant that the researcher could assist in providing direction in relation to the initiative, which provided opportunities for practices to be changed. For example, part-way through the initiative the researcher was able to model requested mathematics lessons for classroom teachers to assist in the implementation of practices. It is proposed that, if the researcher had delivered the effective practice recommendations at the beginning of the initiative and not returned until the end to collect final data, it is likely that the initiative would not have maintained the intended direction for the duration of the initiative. The presence of practices that were contrary to recommendations at the end of the initiative served to highlight that changing teaching practice is a gradual, long-term pursuit that requires sustained timeframes beyond the 7-month initiative. It also demonstrates that practising and experienced teachers still benefited from, and needed support in implementing high quality pedagogy in mathematics.
6.3.2.4 Synthesis of the effectiveness of pedagogies in influencing students’ proficiency

Pedagogical practices’ alignment with Indigenous students’ cognitive and affective needs influences their effectiveness. It was found that there were variations in the way students’ proficiency was influenced between each class, indicating that different factors influenced students’ learning within the school (Hattie, 2008; Hattie & Zierer, 2019). The pedagogic choices made by individual teachers in each class influenced students’ mathematical proficiency in different ways as these choices influence students’ cognitive processes associated with learning (e.g., Sweller, 2016), affective factors associated with students’ productive dispositions, and the extent of engaged instruction time (OECD, 2016). Instruction that focused on explicit guidance and scaffolding such as explicit instructional practices and strategy discussion were shown to influence student learning positively. Further, mastery teaching approaches that facilitated a diagnostic teaching cycle were also found to support students’ cognitive processes and addressed students’ affective needs. These practices will be further explored in detail in the following sub-sections.

6.3.2.4.1 The importance of explicit instruction: Guiding and scaffolding student learning

Conceptual development of concepts and skills using scaffolded and guided teaching practices is critical in supporting students’ mathematical proficiency. Effective pedagogies that focus on developing concepts in structured and meaningful ways increase academic learning time (Kohn, 2006; Shernoff, 2010; OECD, 2016), and facilitate the transfer of learning to long-term memory and schema development (Chen et al., 2016a; Van Merrienboer & Paas, 1990).

Meaningful and productive academic learning time was reduced in some classes by teaching practices that were neither effective nor high quality (Kohn, 2006; Shernoff, 2010; OECD, 2016) as exemplified in the focus on worksheet tasks in some classes. Pedagogical practices in Year 2/3 implemented by the teacher Helen featured a range of recommended practices (e.g., recommended algorithm pedagogy and language, use of concrete materials, use of lesson goals etc.), but also included others which were not recommended (e.g., non-facilitated group work, frequent use of worksheets). The reasons for these choices have been discussed in earlier sections, and the influence of these practices on students’ achievement will now be considered.
Helen’s practice featured confident use of concrete resources as part of the CRA teaching sequence (Bruner, 1966), however, students’ proficiency with computations such as addition and subtraction did not change throughout the initiative despite the use of this recommended practice. The reasons for this can potentially be linked to further observation of Helen’s practice which was characterised by group work and prevalent use of worksheets. The use of these instructional practices meant that time dedicated to explicit instruction associated with developing students’ conceptual understanding of concepts was lost. Therefore, whilst there was quality teaching occurring using concrete materials in the class, there was not enough time for students to develop their conceptual understanding with the topic appropriately, and this reflected in poor gains in achievement on the diagnostic tests results. This relates to human cognitive architecture because students are not supported in developing new schema and moving new knowledge from the short term to long term memory (Chen et al., 2016a; Van Merrienboer & Paas, 1990). Utilising worksheets in group formats where students are working independently also does not follow the worked example effect where guidance and scaffolding in the learning process is critical.

Explicit instruction, defined as “a systematic method of teaching with emphasis on proceeding in small steps, checking for student understanding, and achieving active and successful participation by all students” (Rosenshine, 1987, p. 34), is considered necessary when establishing students’ understanding of computations. As the teacher establishes consistent mathematics language and follows a sequence moving from concrete resources to representations to abstract (symbolic) examples (Mancl et al., 2012), students’ conceptual understanding of the operation and renaming processes develops. Therefore, it is possible that Helen’s use of concrete resources was not sufficient to facilitate students’ mathematics proficiency as they were not moving towards the next level of the CRA process by connecting representations and concrete materials to symbols.

### Strategy discussion supports students’ mathematical proficiency

Teacher guided practices involving strategy discussion in conjunction with timed tasks is effective in enhancing students’ proficiency with number facts (Cumming & Elkins, 1999; Pegg & Graham, 2013; Poncy et al., 2010; Woodward, 2006). Strategy discussion plays a critical role as it helps students understand the logic behind the facts and gives meaning to the patterns, increasing the likelihood that students will remember them (Norton, 2014). Within
the Year 2/3 class, a pedagogical practice that Helen did implement in alignment with the recommendations was moderate use of strategy discussion and timed tasks when teaching number facts. The use of these practices was prevalent in Year 3/4 (Diane) also, but was not a common practice in Year 4/5 (Jane) or 5/6 (Paul). Overall, Year 2/3 reported the largest gains in addition and subtraction number fact proficiency (over double the other cohorts), and this can be explained potentially by Helen’s use of strategy discussion and timed tasks to teach addition and subtraction number facts in Year 2/3.

Similarly, Year 3/4 reported the largest gains in multiplication number fact proficiency (over double the other cohorts) as well, and Diane’s practice focused on strategy discussion and timed testing at the end of each week to facilitate students’ learning of multiplication facts. Comparatively, the Year 4/5 and 5/6 cohorts reported minimal changes in proficiency where teachers were implementing the practice of copying out times tables. Copying out number facts again relates to limited productive use of teaching time, and spending considerable components of lessons on this task reduces effective academic learning time. Findings from this study indicate that copying out times tables was not an adequate style of rehearsal to facilitate the movement of new concepts or skills from the working memory to the long-term memory (Bjork, 2011). Therefore, practices that increase meaningful learning time, where students’ conceptual understanding is developed in a structured and scaffolded way, have a positive impact on students’ mathematical proficiency.

6.3.2.4.3 Positive pedagogies: The importance of a diagnostic, mastery teaching approach and appropriate feedback

Mastery teaching approaches positively influence students’ mathematical proficiency and productive dispositions toward mathematics. A mastery approach facilitates a practical method of providing meaningful affective feedback to students whilst also facilitating a continuous diagnostic teaching cycle. Feedback is a critical element of effective explicit instruction (Hattie, 2008), and positively supports students’ achievement (Hattie & Clarke, 2019; Hattie & Zierer, 2019).

The Year 3/4 cohort reported growth in mathematical proficiency above other classes throughout the initiative. The teacher, Diane, implemented a mastery teaching approach by utilising explicit instruction followed by small volumes of set work (mathematical problems), followed by assessment of students attempts (through individual consultation with students).
This sequence was not time bound as students were individually directed and then required to reattempt tasks if their original answers were incorrect. Through effective student consultation, Diane maintained high academic expectations in Year 3/4 through a diagnostic teaching cycle as accuracy of attempts was expected. In the consultation with students, Diane provided valuable, real-time diagnosis of individual difficulties or misconceptions through her feedback which could be actioned by students immediately. This demonstrates a mastery learning approach (Hattie, 2008; Kulik, Kulik, & Bangert-Drowns, 1990; Willet, Yamashita, & Anderson, 1983). During the short daily consultations, Diane would mark student’s work and provide cognitive and affective feedback. The expectation was that students would follow the consultation by fixing any identified errors immediately and independently, as outlined in Figure 56. This consultation and feedback process was evidenced as a continuous cycle in the classroom routine (see Section 5.3.3).

As noted in earlier chapters, error analysis has a long history in mathematics education, (e.g. Ashlock, 1976; Booker, 2011; Burrows, 1976; Buswell & Judd, 1925; Radatz, 1979; Reisman, 1982) and, as noted earlier, is critical in supporting a mastery approach. By providing an expectation that students’ attempts must be correct and then providing adequate, targeted support, the diagnostic mastery approach increased students’ experiences with success on mathematical tasks in the observed class. Feedback through the diagnostic cycle achieved a meaningful and practical way for the teacher to ascertain what students understand and what mistakes they are making. Through this process, teaching and instruction can be tailored accurately. By doing this, teaching is never “missing the mark” of where students are at in their learning process, and instruction can be tailored to ensure it is highly relevant and appropriate to students’ ability and learning needs. Diagnostic feedback provided by Diane exemplified the
three questions of Hattie’s (Hattie & Zierer, 2019) effective feedback: “where am I going, how am I going, where to next?”. Critically, the feedback also answered the “why” of how the students were progressing (i.e., why was that mistake made? why is that not the right process? why is this incorrect). By understanding the “why”, students can increase self-regulation in their learning as they understand flaws in previous thinking, and have strategies to move forward. These practices had tangible, positive results on students’ achievement in this class, as outlined in Section 5.5.

In addition to providing cognitive guidance in relation to their mathematics learning, it was found that the feedback cycle implemented by Diane (Figure 56) also worked to develop students’ affective dispositions (self-efficacy) in a cyclical way, as outlined in Figure 57. The affective feedback provided by the teacher included affirmations of achievement and effort as well as specific reference to improvement, often as a reminder to individual students about how far they were progressing. This method worked to maintain high academic standards in the classroom by establishing norms surrounding the expectation that students’ re-attempt problems until correct answers had been obtained, and by providing students with the necessary motivation and guidance to persist with the task until completion. Research indicates that increasing mathematics achievement positively influences students’ attitudes towards mathematics (i.e., productive dispositions), which in turn facilitates the development of a student’s motivation (Ma & Xu, 2004). As identified earlier, improving motivation increases a student’s engagement in, and time spent on, mathematics tasks (OECD, 2016). This, in conjunction with the appropriate cognitive tools, provided by classroom instruction, supports further development of student achievement.

The affective cycle outlined in Figure 57 also highlights how the diagnostic teaching cycle employed in Year 3/4 (that facilitated increased mathematics achievement for students), acts to increase academic learning time as students are more likely to engage in mathematics tasks as their productive dispositions towards mathematics are developed.
Academic learning time is important in facilitating students’ achievement (Gettinger & Seibert, 2002). Therefore, a mastery approach fosters students’ academic achievement and positively influences students’ attitudes (Kulik et al., 1990). It fosters academic engagement and maximises effective academic learning time as the cyclical diagnostic approach allows students to have frequent experience with success on tasks, and learning is appropriate to the learners’ ability level (Gettinger & Seibert, 2002).

To further understand the influence of feedback and a diagnostic teaching cycle, it is useful to look at the Year 4/5 and 5/6 classes who reported smaller gains in achievement than the Year 3/4 cohort. In some instances, the Year 3/4 cohort outperformed the Year 4/5 cohort by the end of the initiative. In Year 4/5 and 5/6, there was lower teacher engagement with students as illustrated by limited checking of student work. This influenced the maintenance of high academic standards in the classroom as students quickly understood their work would not be checked. When student work wasn’t checked their motivation to complete the task correctly was lowered, and it established the standard that obtaining a correct answer was not important or valued. Teacher support is significant in supporting students’ engagement in class (Liu et al., 2018).

Checking student work and forming a diagnostic teaching cycle is facilitated by the teacher possessing a certain depth of knowledge surrounding place value concepts and computations. Therefore, one potential reason for teachers such as Jane and Paul not adopting these cycles relates to teacher capacity to diagnose student errors. Considering that Jane (Year 4/5) and Paul (Year 5/6) both cited their difficulties with mathematics, and their need for support in teaching mathematics, it is possible that they were not able to diagnose students’

Figure 57: Description of the cyclical relationship between achievement and students’ affective dispositions.
mathematical difficulties and errors competently. Not implementing this diagnosis impacted on student achievement as evidenced by the disparate results between Year 3/4, and Year 4/5 and 5/6. This is also evidenced by differences in students’ achievement when working with a teacher aide in Year 3/4 compared to those in the remainder of the class. As exemplified in the case of Kevin, students who received instruction from the teacher aide did not develop any proficiency with some operations throughout the initiative. This is potentially because the teacher aide did not have the detailed pedagogical content knowledge to instruct students effectively, and diagnose student errors, similar to what occurred in Year 4/5 and 5/6.

Further, when a teacher is not modelling the answering of questions to students, the benefits of the worked example effect that are well understood in the field of cognitive psychology (Chen et al., 2016b) are not realised. The concept of a worked example can be achieved through teacher modelling of responses to questions. Consistent modelling of language and procedures, as advocated in the recommended practices associated with teaching algorithms, supports students in developing schema so the processes and procedures can be transferred to long-term memory for later retrieval and application (Chen et al., 2016a; Van Merrienboer & Paas, 1990). For these reasons, worked examples are important for novice learners (Chen et al., 2016b; Kirschner et al., 2006). The reason why teacher modelling can be equated to the effect of worked examples is because the critical elements that worked examples (in the written form) are providing for students are high levels of guidance and scaffolding. High levels of guidance and scaffolding, as provided by a worked example, are provided by teacher modelling more appropriately, particularly in the earlier primary years. This is a potential reason why classes where teachers had little interaction with students, and teachers did not model answers frequently (as exemplified in Year 4/5 and 5/6), experienced lower gains in achievement compared to classes where there was a high frequency of students’ work being checked (through individual consultation) and high levels of modelling (as exemplified in Year 3/4). Overall, high levels of teacher engagement with students, a continual consultation cycle forming a diagnostic teaching cycle, explicit instruction, effective feedback, teacher modelling, and high academic and behavioural expectations positively supported the development of students’ mathematical proficiency in this setting.
6.3.2.4.4 *Positive pedagogies: Developing students’ productive dispositions*

Teachers play an important role in developing students’ productive dispositions towards mathematics. How pedagogy influences students’ productive dispositions is an important consideration for Indigenous students because work avoidance, as a result of limited productive dispositions towards mathematics, is a significant factor in inhibiting students’ mathematical proficiency. Just as increased self-efficacy drives a students’ motivation and increases their engagement and achievement (Chen et al., 2016a; Ma, 1997; Ma & Xu, 2004), the reciprocal relationship also occurs, as observed in Year 5/6 (see Figure 58).

![Diagram showing the negative cyclical relationship between achievement and students' affective dispositions.](image-url)

*Figure 58: Description of the negative cyclical relationship between achievement and students’ affective dispositions.*

The converse cycle between achievement, and student motivation and engagement outlined in Figure 58 describes the case where a student experiencing continual difficulties has a diminished positive attitude towards mathematics. This results in decreased motivation, and manifests as active avoidance of the task, rather than engagement in the task. Consequently, a decrease in achievement further diminishes a students’ attitude towards mathematics.

When teachers do not foster students’ productive dispositions actively, students may display work avoidance behaviour (Woodward, Beswick, & Oates, 2017), and this impacts on academic achievement as demonstrated in some classes in this study. As discussed earlier, it was found that avoidance behaviours were particularly prevalent in the higher grades (i.e., Year 5/6). Whether this was linked to factors relating to the specific classroom teacher in Year 5/6, or whether it was linked to the age of the students, or some combination of both, cannot be determined empirically within the scope of this study. However, when considering the teaching practices of the Year 5/6 teacher, it can be seen that Paul’s practices and classroom environment...
were not focused on increasing students’ motivation and confidence in mathematics. For example, Paul’s limited interaction with students during mathematics classes indicates that developing students’ productive dispositions towards mathematics was not a primary focus or objective within this classroom.

It is proposed that pedagogic choices made in Year 5/6 such as the implementation of high volumes of set questions limited teacher interaction and support. A dominant focus on routine tasks (including copying tasks) acted negatively to influence students’ productive disposition towards mathematics. This indicates that Indigenous students, or at least students in this class, were sensitive to continual and consistent mathematical challenges that remained unaddressed by the teacher. Students’ difficulties with core concepts were not being addressed in the classroom, which potentially led to frustration and avoidance behaviour. Overall, this means that affective factors impacted on students’ mathematical proficiency in this class because students’ positive motivation towards mathematics was not facilitated by the enacted teaching practices. Students’ self-efficacy (termed productive dispositions in mathematics) was not being developed actively in this cohort. Because the development of the cognitive strands of mathematical proficiency is contingent on students’ productive dispositions (Kilpatrick et al., 2001), when students do not possess a productive disposition toward mathematics the subsequent limited academic engagement and limited motivation reduces achievement outcomes.

6.3.2.4.5 Consistent routines increase academic learning time

Consistent routines support students’ academic achievement in mathematics as the predictable structure of lessons increases academic learning time. The teaching approach of the Year 3/4 teacher, Diane, involved mathematics lessons that followed a consistent daily structure with the intention that students in the class would also learn the structure and become self-sufficient with lesson routines (i.e., students knew what activities were coming next and could get appropriately prepared without added teacher instruction). Research in Indigenous settings, including the Success in Remote Indigenous Contexts project (Jorgensen, 2018), has proposed that consistent lesson structures reduce student confusion, which subsequently enables students to focus on the tasks, rather than guessing the expectations of the teacher. A consistent lesson structure is an important element of effective explicit instruction (e.g., Archer & Hughes, 2011; Rosenshine, 1987; Hattie, 2008).
This finding was also supported in this study as a predictable and consistent lesson structure fostered students’ independence in their learning, and reduced classroom and/or behaviour-related disruptions related to students being off-task, or needing to ask what was required of them in Year 3/4. The result is that the total time students spend on task is increased as consistent routines increase engagement time (Archer & Hughes, 2011; Gettinger & Seibert, 2002), and time is not lost to unnecessary disruptions as students adjust to the expectations of the assigned task or activity. Considering that it has been proposed that some Indigenous students may also enter school without familiarity with the necessary skills to function in the school environment (Sarra, 2011a), consistent routines potentially are facilitating some of the necessary supports for students to learn how to operate effectively within a classroom discourse.

6.3.3 The inter-personal domain: School-specific factors

The following section will outline school factors that acted to influence students’ proficiency over the course of the initiative. Figure 59 summarises the critical factors within the inter-personal domain related to the school as the agent within the field of recontextualization.
Community ownership and influence within the school was important in fostering a culturally appropriate learning environment at a whole school level. School support of the initiative, and the researcher as a mentor, was important in enabling successful implementation of the initiative. Further, the way in which the school supports student attendance and expectations surrounding academic standards potentially plays a role in supporting students’ mathematical proficiency. These factors possibly explain why many students in this study
benefited from their schooling above and beyond the average Australian student, as indicated by substantial increases in proficiency as measured by standardised tests throughout the initiative. However, the findings also indicated that in most classes, students were still working below the expected achievement standards stipulated in the national curriculum (ACARA, 2018b) with little change to students’ proficiency as measured by diagnostic tests that focused on the assessment of changes in esoteric knowledge forms. Therefore, the role of school factors is also important to consider in explaining why students’ difficulties existed, and why some difficulties remained pervasive, despite the initiative. Within the school domain, these elements primarily relate to the chosen priorities of the school (e.g., attendance, or consistent academic and behaviour standards).

6.3.3.1 Community ownership supports a culturally safe learning environment

Community ownership and involvement in schools is valuable for Indigenous students. Community involvement through partnerships and ownership in schools is important in successfully sustaining programs aimed at improving school outcomes for Indigenous students (Howard, 2001; Howard & Perry, 2007; Hunter & Schwab, 2003; Lareau & Horvat, 1999; Lowe, 2011; Makuwira, 2007). Whilst this was not examined directly in this study, the nature of the learning environment at a whole school level is important to consider to contextualise the findings for this study. It is important to acknowledge that students were in a learning environment that was culturally safe, and culturally accepting. Bin-Sallik’s (2003) summation of cultural safety was exemplified in the school:

Cultural safety extends beyond cultural awareness and cultural sensitivity. It empowers individuals and enables them to contribute to the achievement of positive outcomes. It encompasses a reflection on individual cultural identity and recognition of the impact of personal culture on professional practice (p. 21).

The nature of the learning environment in this Indigenous community-operated school meant that students were not facing issues such as cultural conflict, cross-cultural miscommunication, and racism which some groups of Indigenous people have experienced in other schooling settings (Hunter & Schwab, 2003). The Indigenous ownership and governance for the school meant that students’ cultural identity was not only acknowledged, but was understood and facilitated.
6.3.3.2 Focusing on student attendance as a priority

Attendance at school is vital to success, however, this focus had implications for other focus areas related to behavioural and academic expectations. The importance of focusing on student attendance has been highlighted as critical in key national agendas such as Closing the Gap (COAG, 2016). In this study, in addition to providing a culturally safe learning environment, the school also focused on encouraging student attendance. This is important due to disparities between Indigenous and non-Indigenous students’ attendance rates (DEEWR; Hancock, Shepherd, Lawrence, & Zubrick, 2013; Mellor & Corrigan, 2004; Levin, Belfield, Muenning & Rouse, 2007; Purdie & Buckley, 2010; Thomson et al., 2006), and that achievement gaps are partially accounted for by absenteeism (Biddle, 2014). Whether schools are successful in closing the gap in school attendance is important as the COAGs 2018 goal of closing the gap in attendance was not realised with an average difference in attendance rates of 11% (or 8% in primary school specifically) existing between Indigenous and non-Indigenous Australian students in 2018 (Department of the Prime Minister & Cabinet, 2019). At a national level, attendance rates for non-Indigenous students have not improved since 2014 despite these government initiatives.

The sample school supported student attendance through a variety of mechanisms including specifically-funded family support units that addressed attendance at the local level with families. The findings of this study indicated that, on average, the students in this school reported a higher average attendance rate compared to National Indigenous student attendance rates, however still fell below the average for non-Indigenous students. Further examination of attendance rates revealed the class that was most successful in closing the gap in achievement (the Year 3/4 as measured by the PAT-M) reported the highest attendance rates of all classes involved in the study, and were the only cohort that had an attendance rate similar to the National average for non-Indigenous students. However, despite these positive attendance rates, the primary issue caused by absenteeism was not negated in the sample school. That is, teachers were still attempting to account for a wide variety of ability levels within each class. This was addressed at a school level through the creation of modified curriculums where the planning and teaching was aimed at year levels below the year level of the students in the class (e.g., Year 2/3 program focused on Year 2 content from the Australian Curriculum).

A focus on attendance is important, however it does have wider implications associated with the standards and expectations relating to students’ participation and behaviour in school.
In the sample school, attendance was valued more than behavioural considerations. This meant that students who displayed disruptive behaviours such as starting physical fights with students and teachers, or refusing to attend or participate in class, were still supported in remaining at school. Such a goal may be very important, and there are many implications surrounding negative outcomes of exclusion or expulsion practices, however the consequences of this are influential on the learning that can occur in the classroom. When these behaviours become permitted and normalised, the quality of teaching that can be delivered in the classroom is impacted mainly through the loss of academic learning time due to disruptions. There is no easy solution to these considerations, as encouraging attendance in culturally safe places such as this school is essential for Indigenous students, and such decisions made by the school were appreciated by students and the community.

6.3.3.3 A whole school focus on academic standards

Maintaining approaches focused on high academic standards within a school that are similarly shared by the wider community is important for Indigenous students’ academic success at school. It was found that the school’s intention was to attempt to promote high academic standards, but this was difficult and required some sacrifices due to the key focus on attendance. It is important that there is a shared understanding relating to academic standards between the school and community, and that this is driven by a commitment to delivering effective teaching practices (Sarra, 2003, 2007; Sarra, Matthews, Ewing, & Cooper, 2011). There are complex implications for not having high expectations of Indigenous students as it perpetuates stereotype threats and negative connotations to cultural identities (Sarra, 2003). Avoiding such stereotypes is recommended.

For this school, high academic expectations were manifested as the delivery of the Australian curriculum despite many students not yet meeting national standards for their ages (as evidenced by students achieving below the national norming sample mean on PAT-M). In some instances, this meant that classrooms delivered content from earlier year levels. However, the choice was still made to deliver an academic program. The goal of the academic programs, and the goal of instruction in each classroom within the school was to meet the Australian curriculum achievement standards (ACARA, 2018b). In essence, the school attempted to focus on not delivering a purposefully ‘watered-down’ curriculum, and attempted to maintain a learning environment that enabled students to develop a deep understanding of mathematics.
(Jorgensen, 2014). However, developing academic standards is not a simple endeavour as evidenced by the schools perceived need to draw from modified curriculums. This indicates that in some ways, the goal of delivering an academic program was not realised within the school. Beyond the school’s intentions, achieving a whole school focus on high academic standards involves shared valuing of high academic expectation between the school and the community (Sarra, 2003, 2007; Sarra, Matthews, Ewing, & Cooper, 2011).

6.3.3.4 School support makes effective initiatives possible

A mentoring model of professional development was highly valued by teachers in the study, and implementing these models required school support. Mentoring as professional development is effective in supporting teachers to adopt new practices in educational interventions (Onchwari & Keengwe, 2008). The value of mentoring as professional development is that it enables continual professional development (Tugel, 2004). Due to the continual nature of such professional development, school support of these mentoring opportunities is necessary. In this context, the school supported the inclusion of mentor teacher support for classroom teachers. That is, beyond the scope of this initiative, the school held paid positions for a literacy coordinator, history and social science coordinator, and mathematics coordinator. These coordinators acted as mentors and expert teacher resources for staff. Allocating school funding for this type of support indicates a purposeful choice at a whole school level to deliver high quality programs to students.

Though mentor support was included for other subjects beyond mathematics, the mentor support system facilitated by the school made the specific mathematics initiative in this study possible. The researchers’ established role in the school as a mentor teacher, and the schools support of the mathematics initiative indicates that the school was motivated to change enacted classroom practice by providing a framework and support for teaching mathematics. The mentor provided valuable subject and content specific knowledge in real-time for teachers on a day-to-day basis. Through implementing this initiative, it was clear that beginning and maintaining the changes to practice required as part of this initiative would not have been possible without the continual support of the mentor. Mentor support also gave teachers access to valuable professional development opportunities over the course of the initiative. In short, senior administration enabled and supported the attempts to increase the quality of mathematics teaching and learning within the school in tangible ways.
As well as provisions for mentor support, the school also supported specific professional development opportunities associated with the initiative. Professional development is recognised as important in changing teaching practice (Coldwell, 2017), and these activities were essential in influencing changes in teacher practice over the course of the initiative. The school supported the dedication of time for professional development both within, and outside of, teaching hours. Without the support of the school in providing mentors and professional development time, the initiative may not have been successful, or even possible. In summary, the school administration was committed to improving academic outcomes.

6.3.4 The socio-historical domain: Culture-specific factors

During the discussion of critical student and teacher factors that influenced students’ mathematical proficiency throughout this initiative, particular factors were intertwined with considerations related to the culture of the Indigenous students. Figure 60 summarises the critical factors within the socio-historical domain related to culture specific impacting factors.

Factors driven by socio-cultural considerations include the importance of oral and face-to-face learning, and motivational factors for Indigenous students, as well as the way in which the community holds schools accountable for students’ learning.
Figure 60: Factors associated with the socio-cultural domain that influence students' mathematical proficiency
6.3.4.1 Summary on the implications of cultural learning preferences

Sociocultural congruency between students’ familiar discourses and school discourses can support students’ mathematical proficiency. As discussed in Section 6.3.1.6, it was found that oral, face-to-face settings positively influenced Indigenous students’ mathematics proficiency when compared to written tests. This has important implications for considering modes of teaching and assessment for Indigenous learners as the findings from this study indicate a strong link between motivation to complete the task, the value of the task, and oral, face-to-face administration of test items.

It is understood that Indigenous students can experience a mismatch between home discourses and school discourses (Cairney, 2003; Dickinson, McCabe & Essex, 2006; Maton & Muller, 2007; Sarra, 2011a; Warren, Young & Devries, 2007). This has important considerations for the way Indigenous students are taught and assessed. The instructional discourse of school requires achievement and understanding to be evaluated largely in written form, especially in mathematics (after the very early years of primary schooling). For Indigenous students, there is potentially a difference between the expectations surrounding the formal instructional discourse of the schooling environment, and home and community discourses with which they are familiar. If students do not understand, or are not familiar with the regulatory rules and protocols of the pedagogic discourse in school, this has implications for their learning and the way achievement is assessed.

As exemplified in this study, students can demonstrate greater proficiency with mathematical tasks with an oral and one-on-one situation, and they are more motivated to achieve in this setting. Maton and Muller (2007) discussed a potential solution for mismatches between school and home discourses in that greater support must be provided for students to understand the instructional discourse of school. Perhaps a more conscientious and responsible position which does not place blame back on students and community (Sarra, 2003, 2011; Sarra, Matthews, Ewing, & Cooper, 2001; Warren, Young & Devries, 2007) is to consider the main formats in which we teach and assess students understanding, or how we prepare students in the early years to work within these formats.
6.3.4.2 Summary on motivational implications for Indigenous students

Motivation considerations are a critical cultural consideration for Indigenous learners in mathematics, and the teacher has an important role in influencing this. Teacher support is significant in influencing students’ behavioural, affective, and cognitive engagement in school (Liu et al., 2018). Section 6.3.1.4 discussed how a fear of failure, or a fear of ‘shame’ had a negative impact on Indigenous students’ motivation in this study and resulted in work avoidance behaviours. Indigenous teachers at this school, when asked to comment on their own mathematics experiences at school, provided further insight into the complex influence of a fear of failure in mathematics. An Indigenous teacher, Diane, discussed how a humiliating experience with a teacher was enough to make her “turn off” mathematics. The influence of being ‘shamed’ in mathematics was articulated clearly by Diane, and the sentiments expressed by Diane relate to what other Indigenous parents have noted (Howard et al., 2003). The outcome of fear of failure related to students’ motivation to complete tasks is work avoidance, as found in this study. Work avoidance as a result of these complex issues has implications for learning because work avoidance behaviours reduce academic learning time which cyclically affects students’ achievement.

6.3.4.3 Community expectations and school accountability

Community expectations regarding students’ academic engagement with school, and school accountability play a role in supporting or hindering students’ success at school. Earlier discussions have highlighted the importance of maximising students’ academic engagement, and ensuring that academic learning time is productive and affective (Norton, Duy & Thao, 2016; Gettinger, 1995; Gettinger & Seibert, 2002; Singh, Graville & Dika, 2002). The way in which academic engagement is a product of the pedagogic discourse of the classroom created by each teacher has been discussed in consideration of teachers’ pedagogy earlier in this chapter. However, learning time also includes both in-class and out-of-class learning experiences. Homework (work completed out-of-class) can have an influence on students’ academic achievement as students are increasing their academic learning time beyond school (Singh, Graville & Dika, 2002). A precursor to out-of-class learning time commences in schools by setting expectations around completing homework. This was not seen in the school in this study as homework was not set. Whilst the school plays a role in setting out-of-class
tasks, this cannot be controlled within the school as it is driven by community and parent beliefs as well as expectations surrounding school learning. These expectations are a manifestation of cultural ideologies surrounding the value of learning and mathematical knowledge. Completing school tasks also has privilege considerations, as generally students require the appropriate resources as well as space to complete homework, and this is not always available to some students from disadvantaged backgrounds. Homework not being set and checked has potentially serious implications for the learning of a hierarchical discipline such as mathematics where a great deal of knowledge must be stored in long-term memory (Sweller, 2016). Moving from short to long-term memory requires rehearsal, and homework plays an important part in facilitating this rehearsal (Bjork, 1994, 2011).

Further relating to this is the way in which the community and parents hold the school accountable for student learning. Parental involvement is important in supporting students’ success at school (Brofenbrenner, 1978; Epstein, 2008; Mellor & Corrigan, 2004; Sarra, 2005; Vass, 2012), as is shared understanding and valuing of the importance of academic standards between school and community (Sarra, 2003, 2007; Sarra, Matthews, Ewing, & Cooper, 2011). Despite being a community school, minimal reporting of students’ achievement levels and the modified curriculums in the school indicated that there was little accountability from the community in relation to what students were learning, how they were progressing, and what program was being delivered (see Section 5.2). Part of this may be associated with Indigenous parents’ disenfranchisement with school, and isolation from participating in school due to power gaps perpetuated by social disadvantage (Britton, 2000; Gray & Beresford, 2008; Richer et al., 1998).

6.4 Summary

A students’ ability to engage successfully in the pedagogic discourse of school is driven by the capital that students bring to school, and their familiarity working within instructional discourse of school. This relationship is summarised in Figure 61.
The early habitus of students and their experience with early mathematical concepts is shaped by socio-historical considerations particularly related to culturally specific factors for Indigenous students (Bourdieu, 1981; Bourdieu & Wacquant, 1992; Zevenbergen, 2004). A student’s ability to engage with mathematics successfully at school depends on how the home discourse of a student, which shapes their early habitus, has prepared them to engage in the instructional discourse of school, and develop the critical prerequisite mathematical knowledge (Jorgensen & Grootenboer, 2011). The role of prerequisite knowledge is also important beyond
the early years of school as limited proficiency with prerequisite knowledge in later years will have a cyclical negative effect on students’ ability to engage with the content of the given year level (Geary, 2004). If there is a mismatch between the capital students bring to school and what is necessary to operate in the instructional discourse of school, then a roadblock to school occurs. This block to successfully engaging academically at school operates just as preventing physical access to schools does.

It was found that students’ mathematical proficiency was influenced by how well teaching practices were aligned with principles of cognition, and positively influenced students’ productive dispositions. Teaching practices that positively influenced Indigenous students’ mathematical proficiency included consistent classroom routines, high academic and behavioural expectations, a mastery teaching approach delivered through a diagnostic teaching cycle, and explicit instruction.

At a whole school level, the overarching culturally safe environment is potentially important in facilitating Indigenous students’ success at school. The way in which whole school practices pertaining to behavioural, academic, and attendance standards was noted as an important influence on students’ learning in this chapter. School support of educational initiatives such as the current study was fundamental in the success of the program. Within the school, it was also found that the teacher plays an important role in facilitating students’ learning through the field of reproduction, the classroom.

The process of shifting teaching practice to align with effective practices was found to be a complex process regulated by access to professional development, and the nature of the professional development. The findings from the study add to research related to effective professional development (e.g., Desimone, 2009). Teacher decisions regarding pedagogy were driven by practical considerations about what was easiest to implement, and what had the highest benefits for student compliance. This highlighted how important it was to consider the realities of asking teachers to change their teaching practices. Ongoing mentoring was an important part of facilitating teachers in implementing an educational initiative focused on implementing effective practice recommendations from the literature.

For individual students, proficiency with some critical concepts and skills acted to facilitate or hinder students’ success in mathematics. These included understanding of place value concepts, and fluency with number facts. To support students’ mathematical proficiency, providing metacognitive support through effective questioning was found to be vital. For
Indigenous students, assessment of proficiency in oral, face-to-face, and one-on-one formats facilitated their success with mathematical tasks indicating that the way we examine Indigenous students’ proficiency is critical.

Consideration of all domains of learning for Indigenous students creates a holistic picture of the complex interplay between factors influencing Indigenous students’ proficiency in mathematics. It is apparent that the relationships are not simple, and all are intertwined and interdependent. Each cannot be considered in isolation. A student cannot learn without a teacher, and a teacher must operate with a school. The sociocultural environment in which we all operate is also important in shaping learning. Despite this complex interplay of factors, influencing Indigenous students’ mathematical proficiency in a positive way is a worthwhile and feasible endeavour, as exemplified in this study.
7 CONCLUSION

7.1 Introduction

The purpose of the study was to implement and evaluate the effectiveness of a mathematics initiative designed to improve students’ mathematical proficiency in an Indigenous community school. This study answered three research questions. Firstly, how teachers implemented a mathematics initiative in an Indigenous community school was ascertained. Following this, the second research questions determined primary students’ mathematics proficiency prior to, and after the mathematics initiative. The third research question identified factors related to the implemented initiative that influenced primary students’ mathematics proficiency.

In this chapter, a summary of the thesis will be outlined (Section 7.2), followed by recommendations stemming from the study (Section 7.3). In Section 7.4, the limitations of the study will be discussed. The conclusion of the study is provided in Section 7.5.

7.2 Summary of the thesis

The Introduction chapter of the thesis outlined the importance of Indigenous education research focusing on equitable educational experiences to support student outcomes. Key background information related to Indigenous education and mathematics education as well as the research approach adopted in this study for Indigenous education was highlighted. The need to focus on strengthening students’ proficiency with key factors relevant to success in school mathematics, including number facts, computations, and problem-solving, was also detailed. The chapter concluded with a section outlining the significance of the study for its contribution to effective educational practices in mathematics education that support achievement for Indigenous students.

The second chapter detailed the innovative guiding conceptual framework for this study, which integrated perspectives from sociology and psychology to identify and consider influences on student learning. Within this framework, factors influencing learning were
considered within the intra-individual domain, inter-personal domain, and socio-historical domain. The intra-individual domain considered how individual student’s construct understanding, and cognitive psychology underpinned the evaluation of these considerations. The social interactions occurring within the classroom facilitated by the teacher and school were considered within the inter-personal domain. Socio-historical considerations related to parallels or differences between home discourses and the school discourse, historical experiences, and the cultural or communal valuing of knowledge forms. Also, the esoteric and hierarchical nature of mathematics knowledge was highlighted to inform the evaluation of findings in this study due to its influence on the establishment and delivery of, and learning within, the pedagogic discourse.

The Literature Review chapter reviewed pedagogical approaches and educational initiatives in mathematics education and Indigenous education. Cognitive load theory was used as a theoretical lens to evaluate pedagogical approaches in mathematics. Various initiatives in both mathematics education, and Indigenous mathematics education were explored. Due to variability in findings, the review highlighted the need for research to continue to explore effective educational initiatives for Indigenous students. Specific approaches to teaching number facts, computations, and problem-solving were also discussed. The review of pedagogy and educational initiatives informed the development of the recommended practices included in the initiative implemented in this study.

Within the Methodology chapter, the research design for this study, the research process, data sources, and data analysis were detailed. Following an explanatory mixed methods design, the quantitative arm of the design related to students’ mathematical proficiency was measured using a standardised mathematics test (Progressive Achievement Test – Mathematics) as well as adapted classroom diagnostic tests which measured proficiency with number facts, all four operations, and related problem-solving. This was followed by the qualitative arm of the design which included Newman problem-solving interviews with students along with qualitative coding and analysis of students’ errors from the diagnostic test. Newman interviews facilitated further exploration of, and aided in explanation of, students’ proficiency in mathematics. In addition, classroom observations and informal teacher interviews were collected throughout the study to document teaching practices and other relevant occurrences within each classroom. Data analysis involved triangulation of findings from multiple data sources to increase the validity of findings.
The following chapter outlined how findings from each data source were synthesised and triangulated to identify factors related to the initiative that influenced students’ mathematical proficiency. At the beginning of the initiative, the mathematical proficiency of students in the sample school were below national means, and below what is expected by national curriculum standards. The initiative acted to influence students’ proficiency in a positive manner, and closed gaps in achievement by some measures (i.e., on standardised tests) with some classes reporting gains in achievement over double the expected gains (i.e., 1 year and 2 months within the 7-month initiative). However, throughout the initiative students’ proficiency with specific mathematical concepts such as place value, all four operations, and number facts did not change substantially.

The Discussion chapter provided a possible explanation for the findings reported in the previous chapter related to mixed changes in students’ mathematics proficiency, by drawing on the hierarchical knowledge structure of mathematics. Factors impacting on students’ mathematical proficiency in the initiative were discussed within the intra-individual domain related to student-specific factors, the inter-personal domain related to teacher-specific factors and school-specific factors, and the socio-historical domain related to culture-specific factors. The importance of supporting Indigenous students to engage in the pedagogic discourse of schools effectively was highlighted, as was the importance of developing students’ proficiency with critical foundational mathematics concepts and skills. Furthermore, pedagogy that supports students’ cognitive and affective needs was found to be of significant importance. Finally, the manner in which schoolwide standards support students’ learning was discussed.

7.3 Recommendations stemming from the study

Supporting Indigenous students’ mathematical proficiency is an intricate and complex process. The findings are significant for its contribution to understanding factors that influence Indigenous students’ mathematical proficiency in this context. Specifically, the findings of the study have implications for practice and future research which are considered in the following sections, namely: 1) Several recommendations for effective mathematics education for Indigenous learners; 2) Implementation of a mathematics initiative with teachers through professional development experiences; and 3) Further research in this field.
7.3.1 Recommendations for effective mathematics education for Indigenous learners

Indigenous students’ mathematical proficiency is influenced by interrelated factors beyond the classroom and school, however, it is important that accountability for this is not placed back on the community or students (Sarra, 2003, 2011; Sarra, Matthews, Ewing, & Cooper, 2001). It is important to ensure that the responsibility of providing high-quality mathematics learning experiences for Indigenous students lies with education systems. It is not adequate to offer an impoverished mathematics curriculum based on language, culture, or social background. In fact, it is vital to provide opportunities for students to develop deep understanding of mathematics (Jorgensen, 2014). This concept is important as two solutions to mismatches between the pedagogic discourse of school and students’ cultural or social backgrounds (Dickinson et al., 2006; Warren, Young & Devries, 2007) are to either change what is taught at school, or to provide support for students in understanding the instructional discourse of school (Maton & Muller, 2007). It is clear that the former solution is not effective for Indigenous students as it delegates learning to lower status forms of educational knowledge. Therefore, it is critical to identify what effective mathematics education for Indigenous students looks like in schools and classrooms. The findings from this study have led to eight recommendations.

7.3.1.1 Community accountability

It is important for schools to build meaningful community relationships concerning mathematics learning and student achievement. These relationships are needed to build alignment and transparency between school and community standards as well as expectations surrounding attendance, achievement, and behaviour. This transparency is particularly relevant considering that there is potential disenfranchisement between school and Indigenous communities and families (Gray & Beresford, 2008; Richer et al., 1998). It is important to build parental and community involvement as it supports students’ success and motivation in school (Brofenbrenner, 1978; Epstein, 2008; Martin, 2006; Mellor & Corrigan, 2004; Sarra, 2005; Vass, 2012). Without addressing these community and parent relationships, and actively supporting involvement and engagement with the school, the intergenerational gap between parents and schools continues (Britton, 2000). One potential solution for these issues is to
consider unconventional, empowering forms of supporting parental involvement (Delgado-Gaitan, 1991; Sanders, 2008).

Further, when the community holds the school accountable for student learning, this has the potential to lift standards within the school. In this study, the school was held to little responsibility by the community for students’ gap in achievement and expected standards according to the national curriculum. Without questioning why modified curriculums were delivered, and demanding better outcomes, potentially, schools can proceed with practices which are not in the best interests of students’ achievement at school. Therefore, there is potential for the community to play a vital role in supporting students’ mathematics proficiency. As well, schools need to be responsible for the ways in which they foster parental and community engagement within the school. Schools have a responsibility to begin these conversations and invite transparency and honesty into the educational process. This responsibility is particularly vital for disadvantaged communities where parents and members of the community can feel excluded from participation in schools through lack of sociocultural knowledge.

7.3.1.2 Supporting teachers in changing practices

Changing teaching practice requires sustained support over an extended period of time. Mentor teachers can be effective in delivering continuous professional development and supporting changes in teaching practices (Onchwari & Keengwe, 2008; Weaver, 2004). A mentor who is familiar with the context in which teachers are working has unique insight into the school, the students, and the classrooms. This context allows continuous professional development delivered by a mentor teacher to be effective as it addresses the core of teaching issues. It is important that the mentor teacher has strong pedagogical content knowledge for the given subject and how students learn that content (PCK), as this (i.e., a content focus) is one of most important features of effective professional development (Banilower, Heck & Weiss, 2005; Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Cohen, 1990; Cohen & Hill, 2001; Desimone et al., 2002; Garet et al., 2001). This study found that experienced teachers still required support in developing the necessary pedagogical content knowledge to implement recommended practices in classrooms effectively.

The relationship that can be fostered between a mentor and teachers is also important in the process of shifting teachers’ attitudes (Onchwari & Keengwe, 2008), which has been
shown to be gradual in the case of this study. Mentoring as professional development is effective as it facilitates collegiality through the relationships that are formed (Dantoni, 2001). A mentor initiative for effective professional development also addresses the feature related to the duration of effective professional development as it is an efficient way to have sustained and connected professional development activities in a school setting (Tugel, 2004). Mentoring has the capability to be reflexive as the relevance, context, and content focus can be tailored for each teacher. Teachers also valued the immediacy of help provided by a mentor in the context of this study.

7.3.1.3 Features of effective professional development

Effective professional development needs to be as relevant as possible to teacher’s current classroom environment, and the timing of professional development sessions can influence teacher’s engagement. Professional development is important in assisting teachers in enhancing practices, and consequently supporting student outcomes (Coldwell, 2017). Desimone (2009) identified core features of effective professional development as a content focus, active learning, coherence, duration, and collective participation. This study builds on these features by specifically identifying the need to consider the timing of professional development sessions as this influences teachers’ engagement and subsequent effectiveness of the session. Teachers were most engaged in professional development when it was during regular work hours rather than after school teaching hours. Further, teachers valued professional development the most when it was tied to real data and related to their classroom contexts specifically. They also valued opportunities to share practice. Teachers need to see immediately why the professional development is relevant to what they do in their classrooms.

7.3.1.4 Appropriate and accurate assessment for Indigenous students

The way Indigenous students are assessed influences their mathematical proficiency, and assessment considerations are an issue of equity for Indigenous students. Oral, face-to-face, individual interviews positively influence Indigenous students’ productive dispositions towards tasks and facilitate self-regulation, which can lead to improving achievement. Language plays an important role in Indigenous cultures with oral modes of learning integrated strongly with non-verbal elements (Gorman & Toombs, 2009; Nichol & Robinson, 2000;
The importance of oral and non-verbal elements of communication highlight the importance of person-to-person interaction for Indigenous students. This study exemplified instances of Indigenous students having little motivation to complete written mathematics tests. However, during an oral interview, none declined to participate and complete the tasks. Students’ achievement improved through this mode of assessment. This has important implications for schools as well as national and international standardised testing considering that written tests are a commonly used mode of assessing students’ capabilities and achievement. Perhaps the mode of testing Indigenous students explains consistently reported poor performance on standardised tests of achievement (Klenowski, 2009). Therefore, it is recommended that assessment practices utilise oral, individual modes of assessment built on an established relationship with the assessor in order to capture Indigenous students’ achievement accurately.

7.3.1.5 Supporting skills for success at school

Attention needs to be given to developing Indigenous student’s familiarity with the school discourse, and capability to engage successfully with the school discourse. From the very start of formal schooling, making sense of school mathematics is made more difficult for Indigenous students who may come from a cultural context which is not represented in the classroom pedagogy (Andersen, 2011; Bernstein, 2000). Further, Jorgensen and Grootenboer (2011) discussed the impact of inequitable experiences with early pedagogic discourses prior to school. Access to early pedagogic discourses shape students’ understanding of early mathematical concepts and familiarise students with the discourse of school. These experiences have equity connotations as students from diverse cultural and economic backgrounds often do not have the same access to these pre-school instructional discourses (Bernstein, 2000; Jorgensen & Grootenboer, 2011; Jorgensen, 2012; Zevenbergen, 2000). This means students of Indigenous backgrounds potentially can bring less linguistic capital to school. That is, Indigenous students’ early habitus may not have been shaped in a manner that allows them to engage with school mathematics successfully (Bourdieu; 1981; Bourdieu & Wacquant, 1992; Clarke et al., 2006; Kilpatrick et al., 2001).

There has been attention in research to the realisation that some Indigenous students may not possess the necessary skills, or be familiar with essential skills, required for successful functioning in a classroom (e.g., Cairney, 2003; Galloway, 2003; Sarra, 2011a). A potential
solution to this is that we need to give attention to developing the necessary skills related to operating within an instructional discourse for Indigenous students in the very early years of primary school (Maton & Muller, 2007). Therefore, it is recommended to focus on developing Indigenous students’ capacity and self-efficacy related to becoming competent learners within the school discourse in the early years of school. A wider recommendation would be to consider policies and programs which support access to pre-schooling programs for Indigenous students.

7.3.1.6 Effective mathematics pedagogy

Effective pedagogy in mathematics features practices which work to maximise academic learning time, support the development of students’ productive dispositions, and support students’ cognitive development. Explicit instructional practices are important in scaffolding novice learners in developing proficiency with mathematical concepts. A diagnostic teaching cycle fostering a mastery teaching approach, meaningful and effective feedback to students, consistent lesson structures, and high academic and behavioural standards has positive outcomes for students’ mathematical proficiency.

It is important that, for hierarchical subjects such as mathematics, foundational concepts are taught through explicit practices (Klinger, 2009). Explicit instruction is a way of effectively drawing on what is understood about human cognitive architecture by ensuring that practices do not add to learners’ cognitive load unnecessarily (Sweller, 2016). This recommendation aligns with research supporting guided approaches involving explicit instruction (e.g., Archer & Hughes, 2011; Atkinson et al., 2003; Baker et al., 2002; Hattie, 2008; Kirschner et al., 2006; Paas & van Gog, 2006). Part of effective explicit instruction includes breaking down complex skills and strategies into smaller instructional units (Archer & Hughes, 2011) and this aligns with a mastery teaching approach (Hattie, 2008; Kulik et al., 1990).

Small units of sequenced work in the format of a diagnostic teaching cycle were found to be particularly effective in supporting students’ mathematical proficiency in this study. The benefit of a diagnostic teaching cycle is that it supports effective feedback (Hattie & Zierer, 2019). Individual consultation with a teacher is a powerful way of meeting both students’ affective and cognitive needs. The teacher can support students’ mathematics achievement within the cycle directly by increasing the frequency of students’ experiences with success in mathematics. As exemplified in Figure 62, increased achievement cyclically improves
students’ engagement with tasks, which is critical for students’ success (OECD, 2016). Through a diagnostic teaching approach and explicit instruction, the focus in class is on engaged time on quality learning activities (Kohn, 2006; Shernoff, 2010).

Maximising students’ learning time is important to consider as it has implications for the amount of time students spend engaged in mathematics tasks, which in turn supports increased mathematics achievement (Archer & Hughes, 2011; Duckworth and Seligman, 2006; Gettinger & Seibert, 2002). Minimising behaviour or organisational disruptions by maintaining high classroom standards and implementing consistent routines maximises learning time.

For these reasons, the advocated practices support students’ cognitive and affective development in mathematics. It is important that teaching practices employ effective approaches which cater to students needs in mathematics. The findings from this study illustrate how the practices outlined earlier have met the needs of Indigenous students in this study effectively.

7.3.1.7 Structuring mathematics programs

Mathematics programs need to prioritise developing proficiency with foundational mathematical concepts and skills such as place value and number facts, particularly in the early primary school years. Further, it is important in later primary years to assess whether students possess proficiency with these foundational concepts and skills and to remediate any issues as
a matter or priority. It is essential to remedy these issues prior to commencing the study of further concepts as success with each advanced concept is contingent on the foundational content.

The hierarchical nature of mathematics (Bernstein, 1999; Freudenthal, 1973; Muller & Taylor, 1995; Sfard & Linchevski, 1994) exemplifies why it is critical to address foundational mathematical concepts. For example, proficiency with number facts supports students in carrying out computations (Woodward, 2006). Without fluent recall of number facts, students’ working memory may become overloaded (Baxter, Olson, & Woodward, 2001) thus hindering success with further tasks. Proficiency with place value concepts works in the same way as it supports students in renaming procedures which are necessary when carrying out computations (Geary, 2004). A focus on the big ideas of mathematics (i.e., place value, and fluency with number facts) has been noted in other Indigenous contexts (Jorgensen, 2018). Proficiency with these concepts and skills acts as a gatekeeper for students’ future success in mathematics.

7.3.1.8 Teaching problem-solving

Using problem-solving heuristics such as those provided in Newman interviews is effective in supporting students’ strategic competence when problem-solving by providing metacognitive scaffolding. Across the stages of the problem-solving process, it is important that students develop adequate strategic competence (i.e., ability to comprehend the question and transform it into an appropriate operation) as a priority. With problem-solving, strategic competence firstly needs to be developed prior to focusing on developing process skills (i.e., computational skills).

Problem-solving heuristics supports students in employing metacognitive strategies (Ozsoy & Ataman, 2009; Weinsten & Mayer, 1983). The heuristics provided by Polya’s (1988) four stages are very similar to the questions proposed in Newman interviews (White, 2005). These heuristics, and the subsequent metacognitive support, facilitate students’ strategic competence. Students then are able to regulate themselves effectively through the problem-solving process and are better able to comprehend problem-solving questions, transform the questions into the appropriate operation, and calculate the correct answer. Therefore, it is recommended that teachers teach students how to use prompts similar to those in the Newman interviews effectively and independently. It is recommended that teachers draw from the
Newman interview questions when talking through problems to model the process of effective questioning when problem-solving.

7.3.2 Recommendations for further research

Further research in Indigenous education is essential to shaping future educational policy, and studies such as the present one provide insights into how schools and teachers can support Indigenous students’ success in mathematics effectively. Recommendations for further research pertain to strengthening the breadth and depth of data collected as well as collecting further data on teaching practices, students’ dispositions towards mathematics, and school and community factors.

7.3.2.1 Extending the timeframe of the study

A notable future direction for further studies would be to extend the timeframe for tracking student achievement in mathematics. Findings from this study highlighted that perhaps changing students’ achievement with mathematical concepts requiring greater abstraction and specificity with mathematical structures (i.e., computations and associated algorithms) require an extended time period. Future studies are needed to provide support or otherwise to this finding, and whether a successful approach to address gaps in achievement for Indigenous students is to have an extensive, consistent long-term initiative such as the current initiative.

Future research on a similar initiative over a greater timeframe would also provide insight into how teachers’ enacted practices change over a longer period. It was proposed that changing teacher’s practice was a long-term endeavour that required constant drive, evaluation, and re-evaluation by the researcher. Extending the timeframe of the initiative to take a longitudinal view at supporting teachers to implement effective practice recommendations would yield insightful evidence surrounding the implementation of effective educational initiatives. Given that it was found that teachers needed support in developing their pedagogical content knowledge as well as gaining confidence in teaching mathematics, an extended timeframe to address these needs may be effective.
7.3.2.2 Extending data collected

A further recommendation for future studies is to deepen the breadth of data collected in the rich setting of an Indigenous community school. This study set out to focus on student achievement (i.e., proficiency in mathematics) and, as such, primarily measured this achievement through a variety of mechanisms. Future studies can extend on findings from this study by collecting further empirical data concerning teaching practices and changing teaching practices in similar settings, students’ dispositions, and the influence of school and community factors. This would strengthen and further support findings surrounding the school and community influence on students’ mathematics proficiency.

A suitable follow up study would be to measure and quantify changes in teachers’ beliefs and attitudes throughout a similar educational initiative. In relation to affective factors for students, such as students’ productive dispositions towards mathematics, implementing a data collection procedure surrounding this factor would provide insights in future studies. For example, Likert-scale measures are a common approach for examining students’ attitudes and beliefs (Leder & Forgasz, 2006). Engaging with community, such as parents and the broader community, also would provide valuable insight pertaining to expectations of students, community valuing of mathematical knowledge, and engagement with the school as these have been proposed as important factors for supporting Indigenous students’ mathematical proficiency in this study.

7.4 Limitations of the study

This study examined students from a single school that was unique in its location and context. As such, generalisability of the findings is limited. However, the findings are suggestive of trends that may be common to Indigenous students in Australia as well as students from other disadvantaged groups. Further, findings surrounding effective pedagogy can be linked to general best practice in education particularly for students who experience difficulties with mathematics. Many issues addressed in this study are not unique to Indigenous students alone, but rather relate to quality and equitable educational practices.

There are also limitations relating to the choice of mathematics tests used as data sources in this study. Across the wide variety of available mathematics tests, there is inevitable variability between what is examined and what is prioritised across different tests. This study
has attempted to use several types of tests for mathematical proficiency to address this issue, however it is still possible that different findings may have been possible if other data sources were used. It is for the reader to assess generalisability of the findings from each data source from the explanations of what is examined in each source.

7.5 Conclusion

This study utilised an innovative conceptual framework (Figure 63) to facilitate the identification of factors that influenced Indigenous students’ mathematical proficiency in an Indigenous community school. Within this framework, the intra-individual, inter-personal, and socio-historical domains of learning were considered. There are interactions between the domains, in that learning occurring within the intra-individual domain is influenced by the social nature of the learning environment. The regulating norms and conventions established within socio-historical domain influences the very nature of the inter-personal learning environment that is constructed within a school. At the porous boundaries of each of these domains, the fields of recontextualisation and reproduction are influenced by agents such as the school and teachers. The broader cultural community within the regulating socio-historical domain plays a role in the valuing of mathematical knowledge. The findings from this study have exemplified the complex interrelationships between these domains of learning at the sites of the school and classroom.
Figure 63: Socio-psychological framework

Closing the gap in achievement for Indigenous students, by some measures, was achieved in this study. The findings from this study exemplify the potential for Indigenous students to grow in their mathematical proficiency well beyond what is expected if the learning environment supports their cognition and self-efficacy towards mathematics. However, the findings also highlight that it is more difficult to change students’ proficiency with specific mathematical concepts that are at a higher level of abstraction and require accurate application of procedures as well as a conceptual understanding of numerous foundational concepts. As such, to enact change in relation to students’ proficiency with specific computations, it is recommended that more time is afforded to instruction, particularly due to the requirement for proficiency with place-value and number facts. This exemplifies why it is critical to ensure that students develop proficiency with place-value and number facts in early primary years, or that these concepts and skills are remediated prior to embarking on the study of computations and related problem-solving.
For Indigenous students, oral and face-to-face communication has a positive influence on their mathematical proficiency by supporting their productive dispositions. This highlights the complex link between culturally preferred modes of communicating and success with school mathematics. This has implications for how we teach and assess Indigenous students’ mathematical proficiency. The differences between home and school discourses has an impact on Indigenous students’ mathematical proficiency. The findings from this study highlight the need to consider how schools can support Indigenous students to engage meaningfully in the pedagogic discourse of school.

Creating substantial change in teaching beliefs and practice is a gradual, long-term pursuit that requires constant drive, evaluation, and re-evaluation. Teachers made choices surrounding teaching practices that were based on the ease of implementation of specific practices, and what pay-offs practices had for student behaviour. Therefore, these elements are important to consider when attempting to bring about changes in teaching practice. Professional development opportunities were critical in assisting teachers in implementing effective practices, however engagement in professional development was contingent on its alignment with the features of effective professional development. The timing and relevance of professional development was particularly important in influencing teachers’ engagement in sessions.

Teachers’ pedagogical choices were found to impact upon students’ mathematics proficiency, particularly those related to the ways in which teaching practices supported students’ cognition and productive dispositions towards mathematics. Teaching needs to develop students’ conceptual understanding of mathematical ideas using explicit, scaffolded instruction that is appropriate to the student’s current ability level. Effective practices for Indigenous students in this context related to consistency in classroom routines, and diagnostic teaching cycles. Diagnostic teaching cycles formed an effective mastery teaching approach which encompasses tenants of effective student feedback. However, implementation of a diagnostic teaching cycle in mathematics is contingent on teachers’ pedagogical content knowledge as less confident teachers of mathematics and teacher aides did not implement these practices. Effective pedagogical choices in this study related to how they effectively maximised students’ engaged academic learning time and were also related to classroom and school expectations of behaviour.
The manner in which teachers influenced students’ productive dispositions towards mathematics was found to have an impact on students’ mathematical proficiency. Classes utilising effective student feedback in diagnostic teaching cycles where students had frequent experiences of success were often successful in addressing this issue. However, regardless of teaching practices, work avoidance and motivation to complete tasks were persistent issues for Indigenous students throughout the study. Other examples of teaching practices in classes highlight how it is easy to erode students’ productive dispositions towards mathematics in this context. Negative conceptions on ability, and a preference to avoid work rather than risk failure results in a cyclical spiral of declining achievement.

In this study, the school played an important role in supporting the educational initiative. The school was a culturally safe environment for students negating many potential issues that Indigenous students face in other educational settings. Whilst the school attempted to support academic standards and high behavioural expectations, modified curriculums were utilised and attendance at school was prioritised over behavioural considerations. These may or may not be unavoidable in this context and student attendance is a vital goal. However, the prioritising of this goal influences the standards and expectations by setting a tone concerning students’ capabilities and what is permissible at school. These considerations interrelate with community factors surrounding how the community holds the school accountable for students’ learning, and the value of mathematics knowledge.

Overall, Indigenous students encounter a complex, interrelated set of factors which influence their mathematical proficiency. Knowledge of these factors and relationships between factors means that responsible actions can be undertaken by schools and teachers to support students’ learning effectively. This study is significant for its contribution of knowledge concerning these factors to ensure that we continue to strive collectively for equity in outcomes for Indigenous students.
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## APPENDIX A: PAT-M TEST ITEM EXAMPLE

### Test 1 Item descriptors and curriculum links

<table>
<thead>
<tr>
<th>Item</th>
<th>Australian national curriculum</th>
<th>PATMaths achievement level</th>
<th>Item difficulty (out of 100)</th>
<th>Item description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>N 1 ACMNA006 Fractions and decimals</td>
<td>4 87.7</td>
<td>Recognise a half of a group</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>N 1 ACMNA004 Number and place value</td>
<td>4 94.0</td>
<td>Recognise the place values of a two-digit number</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>N 1 ACMNA015 Number and place value</td>
<td>5 95.0</td>
<td>Find the missing number in a simple addition</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>N 1 ACMNA013 Number and place value</td>
<td>5 96.0</td>
<td>Put one-digit and two-digit numbers in order</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>N 2 ACMNA034 Money and financial mathematics</td>
<td>5 95.8</td>
<td>Recognise the coins that give a small total amount of money ($1)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>N 1 ACMNA013 Number and place value</td>
<td>5 96.7</td>
<td>Recognise the number that lies between two two-digit numbers</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>N 2 ACMNA026 Number and place value</td>
<td>5 97.9</td>
<td>Recognise counting by fours with two-digit numbers</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>N 1 ACMNA001 Number and place value</td>
<td>5 99.7</td>
<td>Count by naming numbers in sequence</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>N 1 ACMNA028 Number and place value</td>
<td>5 99.3</td>
<td>Find the fourth number in a sequence</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>N 1 ACMNA023 Number and place value</td>
<td>5 99.8</td>
<td>Identify and order representations of numbers</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>N 1 ACMNA005 Patterns and algebra</td>
<td>5 96.5</td>
<td>Locate an object in given groupings</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>N 1 ACMNA018 Fractions and decimals</td>
<td>5 109.9</td>
<td>Calculate half of a small even number</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>N 1 ACMNA015 Number and place value</td>
<td>5 102.7</td>
<td>Solve a simple word problem by subtracting two-digit numbers</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>N 1 ACMNA015 Number and place value</td>
<td>6 107.1</td>
<td>Solve a simple subtraction problem</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>N 1 ACMNA015 Number and place value</td>
<td>6 108.3</td>
<td>Interpret a given representation of numbers</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>G 1 ACMG022 Shape</td>
<td>3 84.4</td>
<td>Recognise a triangle from a description of its sides and corners</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>G 1 ACMG022 Shape</td>
<td>4 91.2</td>
<td>Recognise the 3D object that looks like a cylinder</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>G 1 ACMG009 Shape</td>
<td>5 95.1</td>
<td>Recognise a shape among other 2D shapes</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>G 1 ACMG009 Shape</td>
<td>4 93.9</td>
<td>Recognise a sphere among other 3D objects</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>G 1 ACMG023 Location and transformation</td>
<td>5 99.1</td>
<td>Follow a path using left, right turn</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>M 1 ACMG020 Using units of measurement</td>
<td>3 83.3</td>
<td>Read a digital clock</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>M 2 ACMG041 Using units of measurement</td>
<td>5 100.0</td>
<td>Use a calendar to read a day of the week for a given date</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>M 1 ACMG020 Using units of measurement</td>
<td>5 100.1</td>
<td>Read time on a clock face to the half hour</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>M 2 ACMG007 Using units of measurement</td>
<td>5 104.0</td>
<td>Choose the shape that covers the fewest grid squares</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>M 1 ACMG021 Using units of measurement</td>
<td>6 116.1</td>
<td>Describe duration using hours</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>S 1 ACMSP263 Data representation and interpretation</td>
<td>5 96.2</td>
<td>Understand one-to-one correspondence when reading displays</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>S 1 ACMSP263 Data representation and interpretation</td>
<td>5 96.9</td>
<td>Find the item named most often in a given list</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>S 1 ACMSP262 Data representation and interpretation</td>
<td>5 99.3</td>
<td>Use a simple data display to find the number of items in a category</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>S 1 ACMSP263/ ACMSP950 Data representation and interpretation</td>
<td>5 101.7</td>
<td>Read a pictograph where the symbol stands for one unit</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>S 1 ACMSP262 Data representation and interpretation</td>
<td>6 108.1</td>
<td>Decide the best question to ask for a simple survey</td>
<td></td>
</tr>
</tbody>
</table>
## Appendix B: Diagnostic Test Question Topics

<table>
<thead>
<tr>
<th>Question</th>
<th>Place value diagnostic test</th>
<th>Addition diagnostic test</th>
<th>Subtraction diagnostic test</th>
<th>Multiplication diagnostic test</th>
<th>Division diagnostic test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 digit number, materials → symbol</td>
<td>10 facts given orally</td>
<td>10 facts given orally</td>
<td>10 facts given orally</td>
<td>10 facts given orally</td>
</tr>
<tr>
<td>2</td>
<td>Teen number, language → symbol</td>
<td>Reading an addition sum and writing a story to match</td>
<td>Reading a subtraction sum and writing a story to match</td>
<td>Reading a multiplication sum and writing a story to match</td>
<td>Reading a division sum and writing a story to match</td>
</tr>
<tr>
<td>3</td>
<td>2 digit number, ordering</td>
<td>2 digit add 2 digit number, renaming, zero in answer</td>
<td>2 digit subtract 2 digit numbers, renaming</td>
<td>1 digit by 2 digit numbers, renaming</td>
<td>Write a story to match</td>
</tr>
<tr>
<td>4</td>
<td>2 digit number, sequencing</td>
<td>2 digit add 2 digit numbers, no renaming, zero fact</td>
<td>2 digit subtract 2 digit numbers, zero fact</td>
<td>1 digit by 2 digit numbers, renaming, zero fact</td>
<td>2 digit number divided by 1 digit, renaming, remainder</td>
</tr>
<tr>
<td>5</td>
<td>3 digit number, materials → symbol</td>
<td>3 digit add 3 digit numbers, rename ones</td>
<td>3 digit subtract 3 digit numbers, rename ones</td>
<td>1 digit by 3 digit numbers, renaming</td>
<td>3 digit number divided by 1 digit, renaming, remainder</td>
</tr>
<tr>
<td>6</td>
<td>3 digit number, materials → symbol, internal zero</td>
<td>3 digit add 3 digit numbers, rename tens</td>
<td>3 digit subtract 3 digit numbers, rename ones and tens</td>
<td>1 digit by 3 digit numbers, renaming, zero in answer</td>
<td>3 digit number divided by 1 digit, internal zero, remainder</td>
</tr>
<tr>
<td>7</td>
<td>3 digit number, place value</td>
<td>Problem solving: 3 digit add 2 digit numbers, too much information</td>
<td>Problem solving: 2 digit subtract 2 digit numbers, too much information</td>
<td>Problem solving: 1 digit by 2 digit numbers, interpret information</td>
<td>Problem solving: 3 digit number divided by 1 digit, internal zero, remainder</td>
</tr>
<tr>
<td>8</td>
<td>3 digit number, renaming</td>
<td>Problem solving: Addition concept, interpret meaning</td>
<td>Problem solving: Subtraction concept, missing part</td>
<td>Problem solving: Multiplication concept, interpret meaning</td>
<td>Problem solving: 3 digit number divided by 1 digit, interpret information</td>
</tr>
<tr>
<td>9</td>
<td>3 digit number, ordering</td>
<td>3 digit add 3 digit numbers, rename ones and tens, zero in answer</td>
<td>3 digit subtract 3 digit numbers, rename ones and tens</td>
<td>2 digit by 2 digit numbers, renaming</td>
<td>Problem solving: Division concept, interpret meaning</td>
</tr>
<tr>
<td>10</td>
<td>2 digit number, rounding</td>
<td>3 digit add 3 digit numbers, renaming, zero fact</td>
<td>3 digit subtract 3 digit numbers, internal zero</td>
<td>2 digit by 2 digit numbers, renaming</td>
<td>4 digit number divided by 1 digit, renaming, remainder</td>
</tr>
<tr>
<td>11</td>
<td>3 digit number, rounding</td>
<td>Add several 1 digit numbers</td>
<td>4 digit subtract 4 digit numbers, internal zero</td>
<td>2 digit by 2 digit numbers, renaming, zero fact</td>
<td>4 digit number divided by 1 digit, internal zero, no remainder</td>
</tr>
<tr>
<td>12</td>
<td>4 digit number, place value</td>
<td>Add several 3 digit numbers, renaming</td>
<td>4 digit subtract 4 digit numbers, internal zeros</td>
<td>2 digit by 2 digit numbers, renaming, zero in tens</td>
<td>5 digit number divided by 1 digit, internal zeros, remainder</td>
</tr>
<tr>
<td>13</td>
<td>4 digit number, ordering</td>
<td>4 digit add 4 digit numbers, renaming, zero in answer</td>
<td>4 digit subtract 4 digit numbers, no ones, tens, hundreds</td>
<td>2 digit by 3 digit numbers, renaming</td>
<td>5 digit number divided by 2 digit, renaming, remainder</td>
</tr>
<tr>
<td>14</td>
<td>4 digit number, renaming</td>
<td>5 digit add 5 digit numbers, renaming, zero fact</td>
<td>5 digit subtract 5 digit numbers, internal zero</td>
<td>2 digit by 3 digit numbers, renaming, zero ones</td>
<td>Problem solving:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4 digit number divided by 1 digit, interpret question</td>
</tr>
<tr>
<td>15</td>
<td>4 digit number, ordering</td>
<td>Problem solving: 3 digit add 3 digit numbers, interpret question</td>
<td>Problem solving: 3 digit subtract 3 digit numbers, too much information, interpret question</td>
<td>Problem solving: 2 digit by 2 digit numbers, interpret question</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>4 digit number, rounding</td>
<td>5 digit number, language symbol</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>5 digit number, comparing</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>6 digit number, place value</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>6 digit number, renaming</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX C: ETHICS PACKAGES

Mathematics Education – Research Project
SCHOOL INFORMATION SHEET

Who is conducting the research

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Griffith student researcher:
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Why is the research being conducted?
The purpose of this study is to evaluate the effectiveness of a mathematics educational program designed to improve students’ mathematical achievement in your school. By investigating the impact of particular educational practices that influence students’ mathematics achievement, the findings of this study can contribute to an understanding of effective mathematics education. The importance of understanding effective practice in the domain of mathematics is crucial in developing numerate students who are able to use mathematics in a wide range of real-life situations. This is particularly important due to current equity issues as when any students, including Indigenous students, are precluded from achievement in mathematics, and future educational and employment opportunities become limited. This project is being carried out to fulfil the requirements of a Doctor of Philosophy for the student researcher.

What you will be asked to do
This project will involve the participation of students from Grade 2/3, Grade 3/4, Grade 4/5, and Grade 6. In these grades, teachers will be asked to allow the researcher to be involved in the planning of lessons and observe the delivery of mathematics classes over the course of a school year. This will include providing teachers at the school with information and support regarding effective mathematics education practices.

Throughout the year, students’ mathematics achievement will be tracked to document their progress. This will involve the collection of achievement data from an achievement test called PAT-M (Progressive Achievement Test – Mathematics), and classroom diagnostic tests carried out over the course of the school year. Samples of students’ mathematics work demonstrating their understanding of mathematical concepts will also be collected throughout the year. As part of the program, researcher will also conduct interviews with students’ to explore their problem solving capabilities. The goal of these interviews is to identify individual barriers to mathematics understanding and tailor the program to improve individual students’ problem solving capabilities and mathematics outcomes.

The basis by which participants will be selected or screened
This study will focus on students’ capabilities with computations (addition, subtraction, multiplication, and division), and recall number facts (particularly multiplication facts). Students that will be included in this project will be those who have returned informed parental/guardian consent.
The expected benefits of the research
It is expected that all students involved will benefit from this study through participation in a sustained program where highly effective teaching and learning will be occurring. The diagnostic testing and interviews are designed to give classroom teachers and the research team detailed and insightful information about student’s mathematics capabilities. This data will inform the mathematics program. Teaching will be highly responsive to individual students’ needs. The participation of the teachers in the program is likely to enhance their professional learning by modelling best practice mathematics teaching.

Risks to you
The research team does not foresee that there will be any risks to students during the conduct of this research as it should be a very similar experience to a regular mathematics lesson. If there are any signs of distress as a result of testing or interviews associated with this project, the student may choose to not participate or the researcher will terminate the process. The aim of the project is to enhance the children’s confidence and mathematical capability.

Your confidentiality
Throughout the collection of data for this project, students will be identifiable to the research team for comparison purposes but, once data collection is complete, students will be de-identified. The school and students will not be identified in the reporting of results for this project. No identifiable data will be retained and all participants will remain anonymous.

Your participation is voluntary
Participation of students in this project is voluntary. Parents will be asked to provide signed, informed consent prior to students being involved in the project. Participation in this project is also voluntary for students who may choose not to participate in testing or interviews. Students or parents may withdraw from the study at any time.

Questions / further information
For any additional information or questions concerning, members of the research team may be contacted.

The ethical conduct of this research
Griffith University conducts research in accordance with the National Statement on Ethical Conduct in Human Research. If you have any concerns or complaints about the ethical conduct of the research project contact the Manager, Research Ethics on 3735 4375 or research.ethics@griffith.edu.au.

Feedback to you
The results of this project will be communicated throughout the course of the program to participating teachers to inform teaching practices. Parents will also be advised of students’ progress in mathematics through regular report cards at the end of each school term.

Privacy Statement – non disclosure
The conduct of this research involves the collection, access and/or use of your identified personal information. The information collected is confidential and will not be disclosed to third parties without your consent, except to meet government, legal or other regulatory authority requirements. A de-identified copy of this data may be used for other research purposes. However, your anonymity will at all times be safeguarded. For further information consult the University’s Privacy Plan at http://www.griffith.edu.au/about-griffith/plans-publications/griffith-university-privacy-plan or telephone (07) 3735 4375.
# Mathematics Education – Research Project

**SCHOOL CONSENT FORM**

**Research team**

<table>
<thead>
<tr>
<th>Dr Christine McDonald</th>
<th>Dr Stephen Norton</th>
</tr>
</thead>
<tbody>
<tr>
<td>School of EPS</td>
<td>School of EPS Studies</td>
</tr>
<tr>
<td>373 55831</td>
<td>373 55792</td>
</tr>
<tr>
<td><a href="mailto:c.mcdonald@griffith.edu.au">c.mcdonald@griffith.edu.au</a></td>
<td><a href="mailto:s.norton@griffith.edu.au">s.norton@griffith.edu.au</a></td>
</tr>
</tbody>
</table>

**Griffith student researcher:**

<table>
<thead>
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</tr>
<tr>
<td><a href="mailto:bronwyn.reidoconnor@griffithuni.edu.au">bronwyn.reidoconnor@griffithuni.edu.au</a></td>
</tr>
</tbody>
</table>

By signing below, I confirm that I have read and understood the information package and in particular have noted that:

- I understand that my involvement in this research will include the participation of classroom teachers in a mathematics program. For students, this will involve the collection of mathematics tests, and student work and interviews with the research team.
- I have had any questions answered to my satisfaction;
- I understand the risks involved;
- I understand the potential benefits to me from my participation in this research;
- I understand that my participation in this research is voluntary;
- I understand that if I have any additional questions I can contact the research team;
- I understand that I am free to withdraw at any time, without explanation or penalty;
- I understand that I can contact the Manager, Research Ethics, at Griffith University Human Research Ethics Committee on 3735 4375 (or research.ethics@griffith.edu.au) if I have any concerns about the ethical conduct of the project; and

☐ I agree to participate in the project.

<table>
<thead>
<tr>
<th>Name</th>
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<tr>
<th>Date</th>
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</table>
Mathematics Education – Research Project
PARENT/GUARDIAN INFORMATION SHEET

Who is conducting the research

Senior Investigators:
Dr Christine McDonald
School of Education and Professional Studies
373 55831
c.mcdonald@griffith.edu.au

Dr Stephen Norton
School of Education and Professional Studies
373 55792
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Griffith student researcher:
Bronwyn Reid O’Connor
School of Education and Professional Studies
0431648708
bronwyn.reidocconnor@griffithuni.edu.au

Why is the research being conducted?
The purpose of this study is to evaluate a math program designed to improve students’ achievement in your school. The findings of this study will be used by researchers to better understanding effective mathematics education, especially for Indigenous children. This project is being carried out to fulfil the requirements of a Doctor of Philosophy for the student researcher.

What you will be asked to do
This project will involve the participation of your child. Grade 2/3, Grade 3/4, Grade 4/5, and Grade 6 students are being asked to participate in this study. Your child will participate in completing mathematics tasks and short interviews, designed to give their teacher detailed information about your child’s mathematics level. This will allow for teachers to better help your child in mathematics.

Your child will be included in this project only if you have returned this consent form. Participation in this project is voluntary. You or your child may withdraw from this study at any time.

The expected benefits
It is expected that your child will benefit from this study as they are part of a math program created from leading education research. This program will specifically work on improving your child’s ability to add, subtract, multiply, divide, and problem solve.
Risks to you
The research team does not foresee that there will be any risks to your child during the conduct of this research as it should be a very similar experience to a regular mathematics lesson. If there are any instances of discomfort as a result of testing or interviews associated with this project, your child may choose to not participate or the researcher will stop the research involving your child. The aim of the research is to improve your child’s confidence and achievement in mathematics.

Your confidentiality
Your child’s work and information will be identifiable to the research team for comparison purposes but, once data collection is complete, students will be no longer be identified. The school and students will not be identified in the reporting of results for this project. Complete confidentiality of your child’s data will be maintained.

Questions / further information
For any additional information or questions concerning, members of the research team may be contacted.

The ethical conduct of this research
Griffith University conducts research in accordance with the National Statement on Ethical Conduct in Human Research. If you have any concerns or complaints about the ethical conduct of the research project contact the Manager, Research Ethics on 3735 4375 or research-ethics@griffith.edu.au.

Feedback to you
You will be advised of your child’s progress in mathematics through regular report cards at the end of each school term and though a final project report.

Privacy Statement – non disclosure
The conduct of this research involves the collection, access and/or use of your identified personal information. The information collected is confidential and will not be disclosed to third parties without your consent, except to meet government, legal or other regulatory authority requirements. A de-identified copy of this data may be used for other research purposes. However, your anonymity will at all times be safeguarded. For further information consult the University’s Privacy Plan at http://www.griffith.edu.au/about-griffith/plans-publications/griffith-university-privacy-plan or telephone (07) 3735 4375.
Mathematics Education – Research Project
PARENT CONSENT FORM

Research team
Senior Investigators:
Dr Christine McDonald
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School of Education and Professional Studies
0431648708
bronwyn.reidocconnor@griffithuni.edu.au

By signing below, I confirm that I have read and understood the information package and in particular have noted that:

• I understand that my involvement in this research will include my child’s participation in interviews with the research team about mathematics learning, and the collection of my child’s mathematics tests, and mathematics work.
• I have had any questions answered to my satisfaction;
• I understand the processes involved;
• I understand the potential benefits to me and my child from participation in this research with respect to mathematics learning;
• I understand that my child’s participation in this research is voluntary (your decision will in no way impacting upon the service your child receives from the school or your child’s grades);
• I understand that if I have any additional questions I can contact the research team;
• I understand that I am free to withdraw at any time, without explanation or penalty;
• I understand that I can contact the Manager, Research Ethics, at Griffith University Human Research Ethics Committee on 3735 4375 (or research-ethics@griffith.edu.au) if I have any concerns about the ethical conduct of the project; and

☐ I agree to participate in the project.

<table>
<thead>
<tr>
<th>Name</th>
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</table>

<table>
<thead>
<tr>
<th>Signature</th>
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<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date</th>
<th></th>
</tr>
</thead>
</table>
Appendix D: PAT-M Results by Class

Table 1 outlines the pre- and post-test mean Rasch scaled scores for each class compared to the national norming sample. This enabled analysis of each class’s growth in scores throughout the initiative compared to the expected growth from the norming sample. Table 2 outlines the changes in each classes distance from the national norming sample means throughout the initiative.

Table 1
Mean results from PAT-M pre- and post-test for each class

<table>
<thead>
<tr>
<th>Year 2/3, n=11</th>
<th>M</th>
<th>SD</th>
<th>Year 3/4, n=11</th>
<th>M</th>
<th>SD</th>
<th>Year 4/5, n=12</th>
<th>M</th>
<th>SD</th>
<th>Year 5/6, n=14</th>
<th>M</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test: March</td>
<td>Sample OARS norming sample</td>
<td></td>
<td></td>
<td>Pre-test: March</td>
<td>Sample OARS norming sample</td>
<td></td>
<td></td>
<td>Pre-test: March</td>
<td>Sample OARS norming sample</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Year 2/3, n=11</td>
<td>90.97</td>
<td>6.02</td>
<td>97.28 (Yr2)</td>
<td>14.90</td>
<td>97.60</td>
<td>4.33</td>
<td>103.00 (Yr2)</td>
<td>14.90</td>
<td>1.26</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>Year 3/4, n=11</td>
<td>96.57</td>
<td>9.28</td>
<td>106.29 (Yr3)</td>
<td>14.60</td>
<td>109.88</td>
<td>10.31</td>
<td>110.90 (Yr3)</td>
<td>14.60</td>
<td>1.36</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>Year 4/5, n=12</td>
<td>105.98</td>
<td>6.86</td>
<td>113.61 (Yr4)</td>
<td>14.20</td>
<td>109.12</td>
<td>6.21</td>
<td>117.40 (Yr4)</td>
<td>14.20</td>
<td>0.48</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>Year 5/6, n=14</td>
<td>111.39</td>
<td>12.16</td>
<td>119.61 (Yr5)</td>
<td>12.80</td>
<td>116.49</td>
<td>15.26</td>
<td>122.70 (Yr5)</td>
<td>12.80</td>
<td>0.37</td>
<td>0.24</td>
<td></td>
</tr>
</tbody>
</table>

*Note: The norming sample mean reported is equivalent to the lowest year level for the composite class (e.g., the Year 2/3 cohort is compared to the Year 2 norming sample mean).

Table 2
Growth in distance from the norming sample mean from pre- to post-test by class

<table>
<thead>
<tr>
<th>Year 2/3, n=11</th>
<th>Pre-test</th>
<th>Post-Test</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 3/4, n=11</td>
<td>-9.72</td>
<td>-1.02</td>
<td>8.70</td>
</tr>
<tr>
<td>Year 4/5, n=12</td>
<td>-7.63</td>
<td>-8.28</td>
<td>-0.65</td>
</tr>
<tr>
<td>Year 5/6, n=14</td>
<td>-8.22</td>
<td>-6.21</td>
<td>2.02</td>
</tr>
</tbody>
</table>

*Note: The norming sample mean reported is equivalent to the lowest year level for the composite class (e.g., the Year 2/3 cohort is compared to the Year 2 norming sample mean).
APPENDIX E: ANALYSIS OF PAT-M RESULTS BY STRAND

Year 2/3 Findings

Table 3 outlines the PAT-M results by mathematical strand for the Year 2/3 cohort at the beginning of the initiative.

Table 3
Analysis of PAT-M test results for Year 2/3 by strand

<table>
<thead>
<tr>
<th>Average percentage of correct answers (%)</th>
<th>Year 2/3, n=10</th>
<th>OARS norming sample (Test 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number, n=15</td>
<td>31</td>
<td>28.9</td>
</tr>
<tr>
<td>Geometry, n=5</td>
<td>46</td>
<td>41.1</td>
</tr>
<tr>
<td>Measurement n=5</td>
<td>34</td>
<td>27.6</td>
</tr>
<tr>
<td>Statistics, n=5</td>
<td>38</td>
<td>26.7</td>
</tr>
</tbody>
</table>

The findings demonstrate that the Year 2/3 cohort scored, on average, higher in each tested strand than the Year 1 norming sample. It is important to remember that the Year 2/3 cohort is being compared to a younger norming cohort in this comparison, as the Year 2/3 students were administered the end of Year 1 equivalent PAT-M. Whilst direct comparison here is difficult due to the differences in age, how the Year 2/3 cohort progresses over the course of a school year in comparison to the norming sample can be observed after post-testing.

Table 4 outlines the PAT-M results by mathematical strand for the Year 2/3 cohort at the end of the initiative.

Table 4
Analysis of the improvement in PAT-M test results for Year 2/3 by strand

<table>
<thead>
<tr>
<th>Improvement in average percentage of correct answers (%) from March to October</th>
<th>Year 2/3, n=10</th>
<th>OARS norming sample (Test 1: Yr1 to Test 2: Yr2)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>-9.3</td>
<td>8.6</td>
<td>-18.0</td>
</tr>
<tr>
<td>Geometry</td>
<td>20.0</td>
<td>3.6</td>
<td>16.4</td>
</tr>
<tr>
<td>Measurement</td>
<td>-2.0</td>
<td>12.0</td>
<td>-14.1</td>
</tr>
<tr>
<td>Statistics</td>
<td>-4.0</td>
<td>14.0</td>
<td>-17.5</td>
</tr>
</tbody>
</table>
Year 3/4 Findings

The Year 3/4 cohort was administered the end of Year 2 equivalent test, and their average percentage results for each tested strand is outlined in Table 5 and compared to the Year 2 norming sample.

Table 5
Analysis of PAT-M test results for Year 3/4 by strand

<table>
<thead>
<tr>
<th></th>
<th>Average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 3/4, n=11</td>
</tr>
<tr>
<td>Number, n=15</td>
<td>30.9</td>
</tr>
<tr>
<td>Geometry, n=5</td>
<td>32.7</td>
</tr>
<tr>
<td>Measurement n=5</td>
<td>25.5</td>
</tr>
<tr>
<td>Statistics, n=5</td>
<td>45.5</td>
</tr>
</tbody>
</table>

Table 6 outlines the PAT-M results by mathematical strand for the Year 3/4 cohort at the end of the initiative.

Table 6
Analysis of the improvement in PAT-M test results for Year 3/4 by strand

<table>
<thead>
<tr>
<th></th>
<th>Improvement in average percentage of correct answers (%) from March to October</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 3/4, n=11</td>
</tr>
<tr>
<td>Number</td>
<td>25.6</td>
</tr>
<tr>
<td>Geometry</td>
<td>27.9</td>
</tr>
<tr>
<td>Measurement</td>
<td>20.0</td>
</tr>
<tr>
<td>Statistics</td>
<td>12.7</td>
</tr>
</tbody>
</table>
Year 4/5 Findings

Table 7 outlines the average improvement for each tested strand for the Year 4/5 cohort compared to the national norming sample.

Table 7
Analysis of PAT-M test results for Year 4/5 by strand

<table>
<thead>
<tr>
<th></th>
<th>Average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 4/5, n=12</td>
</tr>
<tr>
<td>Number, n=18</td>
<td>47.2</td>
</tr>
<tr>
<td>Geometry, n=6</td>
<td>66.7</td>
</tr>
<tr>
<td>Measurement n=6</td>
<td>40.3</td>
</tr>
<tr>
<td>Statistics, n=5</td>
<td>41.7</td>
</tr>
</tbody>
</table>

The pre-test found that the Year 4/5 students performed below the national norming sample in all strands, however geometry was the strand in which students were most successful.

Table 8 outlines the PAT-M results by mathematical strand for the Year 4/5 cohort at the end of the initiative.

Table 8
Analysis of the improvement in PAT-M test results for Year 4/5 by strand

<table>
<thead>
<tr>
<th></th>
<th>Improvement in average percentage of correct answers (%) from March to October</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 4/5, n=12</td>
</tr>
<tr>
<td>Number</td>
<td>-24.0</td>
</tr>
<tr>
<td>Geometry</td>
<td>-33.4</td>
</tr>
<tr>
<td>Measurement</td>
<td>-0.0</td>
</tr>
<tr>
<td>Statistics</td>
<td>-21.7</td>
</tr>
</tbody>
</table>
Year 5/6 Findings

Table 9 outlines the average improvement for each tested strand for the Year 5/6 cohort compared to the national norming sample. The Year 5/6 cohort also completed the test designed for end of Year 3 students (Test 3). This test was administered as it was the most appropriate for the class based on their current abilities in mathematics.

Table 9
Analysis of PAT-M test results for Year 5/6 by strand

<table>
<thead>
<tr>
<th></th>
<th>Average percentage of correct answers (%)</th>
<th>Year 4/5, n=12</th>
<th>OARS norming sample (Test 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number, n=18</td>
<td></td>
<td>61.9</td>
<td>71.8</td>
</tr>
<tr>
<td>Geometry, n=6</td>
<td></td>
<td>59.5</td>
<td>77.2</td>
</tr>
<tr>
<td>Measurement n=6</td>
<td></td>
<td>45.2</td>
<td>68.2</td>
</tr>
<tr>
<td>Statistics, n=5</td>
<td></td>
<td>54.3</td>
<td>71.1</td>
</tr>
</tbody>
</table>

Table 10 outlines the average improvement for each tested strand for the Year 5/6 cohort compared to the national norming sample. Whilst the Year 5/6 cohort performed below the national norming sample in all tested strands on the pre-test, it was found that this class achieved the highest in the number strand.

Table 10
Analysis of the improvement in PAT-M test results for Year 5/6 by strand

<table>
<thead>
<tr>
<th></th>
<th>Improvement in average percentage of correct answers (%) from March to October</th>
<th>Year 5/6, n=14</th>
<th>OARS norming sample (Test 3: Yr3 to Test 4: Yr4)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>-7.8</td>
<td>-15.8</td>
<td>8.0</td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td>-18.1</td>
<td>-24.3</td>
<td>6.3</td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td>17.9</td>
<td>-19.9</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Statistics</td>
<td>-5.7</td>
<td>-3.0</td>
<td>-2.8</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX F: ANALYSIS OF INDIVIDUAL STUDENT PAT-M RESULTS

Year 2/3 Findings

Figure 1 outlines the pre- and post-initiative PAT-M scores for individual students from the Year 2/3 cohort.

Figure 1: Individual scores on PAT-M pre and post-test: Year 2/3, n=10.

Year 3/4 Findings

Figure 2 outlines the pre- and post-initiative PAT-M scores for individual students from the Year 3/4 cohort.

Figure 2: Individual scores on PAT-M pre and post-test: Year 3/4, n=11
Year 4/5 Findings

Figure 3 outlines the pre- and post-initiative PAT-M scores for individual students from the Year 4/5 cohort.

![Figure 3: Individual scores on PAT-M pre and post-test: Year 4/5, n=12.](image)

Year 5/6 Findings

Figure 4 outlines the pre- and post-initiative PAT-M scores for individual students from the Year 5/6 cohort.

![Figure 4: Individual scores on PAT-M pre and post-test: Year 5/6, n=14.](image)
APPENDIX G: PAT-M RESULTS BY YEAR LEVEL

In Table 11, the mean score for each year level is reported. The comparison of each year level to the national norming sample provides an accurate comparison between the sample students and national averages, as composite classes do not need to be taken into consideration.

Table 11
Mean results from PAT-M pre-test conducted in March for each year level

<table>
<thead>
<tr>
<th></th>
<th>Pre-test: March</th>
<th>Distance from expected mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample M</td>
<td>SD</td>
</tr>
<tr>
<td>Year 2, n=4</td>
<td>91.88</td>
<td>9.61</td>
</tr>
<tr>
<td>Year 3, n=17</td>
<td>95.54</td>
<td>8.50</td>
</tr>
<tr>
<td>Year 4, n=12</td>
<td>96.39</td>
<td>7.99</td>
</tr>
<tr>
<td>Year 5, n=15</td>
<td>109.47</td>
<td>6.67</td>
</tr>
<tr>
<td>Year 6, n=8</td>
<td>110.66</td>
<td>14.96</td>
</tr>
</tbody>
</table>

It was found that Year 2 students achieved a mean score closest to that of the norming sample, however, they were still achieving at a level below that of a late Year 1 student. The gap between the norming sample and each year level increased by over double the gap reported for Year 2 for all other year levels in the sample. This suggests that gaps in achievement increase significantly after Year 2. The Year 3 and Year 4 cohorts both achieved mean scores equivalent to an early Year 2 student. Year 5 students were performing at a level approximate to that of a mid-Year 3 student, meaning that they were two year levels behind the expected standard. Similarly, the Year 6 cohort mean scores were equivalent to that of a late Year 3 student, three year levels behind the expected standard.

Table 12 outlines the changes in each year levels distance from the national norming sample means throughout the initiative.

Table 12
Growth in distance from the norming sample mean from pre- to post-test by year level

<table>
<thead>
<tr>
<th></th>
<th>Distance from norming sample mean (measured by Rasch scaled score)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-test</td>
</tr>
<tr>
<td>Year 2, n=4</td>
<td>-5.40</td>
</tr>
<tr>
<td>Year 3, n=17</td>
<td>-10.75</td>
</tr>
<tr>
<td>Year 4, n=12</td>
<td>-17.22</td>
</tr>
<tr>
<td>Year 5, n=15</td>
<td>-10.14</td>
</tr>
<tr>
<td>Year 6, n=8</td>
<td>-13.83</td>
</tr>
</tbody>
</table>
APPENDIX H: PAT-M COMPARISONS ACROSS YEAR LEVELS

Comparison of the mean scores for Year 3 students is outlined in Table 13.

<table>
<thead>
<tr>
<th>Year 3 students from</th>
<th>Mean pre-initiative PAT-M Score</th>
<th>Mean post-initiative PAT-M Score</th>
<th>Average Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2/3, n=6</td>
<td>90.4</td>
<td>96.1</td>
<td>5.7</td>
</tr>
<tr>
<td>Year 3/4, n=4</td>
<td>98.1</td>
<td>111.8</td>
<td>13.7</td>
</tr>
</tbody>
</table>

Comparison of the mean scores for Year 4 students is outlined in Table 14.

<table>
<thead>
<tr>
<th>Year 4 students from:</th>
<th>Mean pre-initiative PAT-M Score</th>
<th>Mean post-initiative PAT-M Score</th>
<th>Average Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 3/4, n=7</td>
<td>95.7</td>
<td>108.8</td>
<td>13.1</td>
</tr>
<tr>
<td>Year 4/5, n=3</td>
<td>101.3</td>
<td>101.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Comparison of the mean scores for Year 5 students is outlined in Table 15.

<table>
<thead>
<tr>
<th>Year 5 students from:</th>
<th>Mean pre-initiative PAT-M Score</th>
<th>Mean post-initiative PAT-M Score</th>
<th>Average Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 4/5, n=9</td>
<td>107.5</td>
<td>111.7</td>
<td>4.1</td>
</tr>
<tr>
<td>Year 5/6, n=6</td>
<td>112.3</td>
<td>118.3</td>
<td>6.0</td>
</tr>
</tbody>
</table>
APPENDIX I: ANALYSIS OF NUMERATION DIAGNOSTIC TEST

Pre-initiative findings for the numeration diagnostic test

To observe further why proficiency was low on the numeration pre-initiative test, the percentage of correct answers for each test item was observed for each class, and these findings are outlined in Table 16. Particular questions where the average percentage of correct answers was particularly low (approximately 20% or less) are highlighted in bold text in Table 16.

Table 16
Analysis of numeration diagnostic test by question

<table>
<thead>
<tr>
<th>Question</th>
<th>Numeration test question</th>
<th>Year 2/3</th>
<th>Year 3/4</th>
<th>Year 4/5</th>
<th>Year 5/6</th>
<th>Overall average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>n=12</td>
<td>n=12</td>
<td>n=11</td>
<td>n=14</td>
<td>n=49</td>
</tr>
<tr>
<td>1</td>
<td>2 digit number, materials → symbol</td>
<td>58</td>
<td>75</td>
<td>82</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>2</td>
<td>Teen number, language → symbol</td>
<td>67</td>
<td>83</td>
<td>91</td>
<td>100</td>
<td>86</td>
</tr>
<tr>
<td>3</td>
<td>2 digit number, ordering</td>
<td>25</td>
<td>58</td>
<td>91</td>
<td>79</td>
<td>63</td>
</tr>
<tr>
<td>4</td>
<td>2 digit number, sequencing</td>
<td>17</td>
<td>67</td>
<td>55</td>
<td>79</td>
<td>55</td>
</tr>
<tr>
<td>5</td>
<td>3 digit number, materials → symbol</td>
<td>42</td>
<td>67</td>
<td>73</td>
<td>71</td>
<td>63</td>
</tr>
<tr>
<td>6</td>
<td>3 digit number, materials → symbol</td>
<td>42</td>
<td>75</td>
<td>91</td>
<td>86</td>
<td>73</td>
</tr>
<tr>
<td>7</td>
<td>3 digit number, place value</td>
<td>0</td>
<td>17</td>
<td>27</td>
<td>36</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>3 digit number, renaming</td>
<td>17</td>
<td>8</td>
<td>9</td>
<td>21</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>3 digit number, ordering</td>
<td>8</td>
<td>33</td>
<td>45</td>
<td>50</td>
<td>35</td>
</tr>
<tr>
<td>10</td>
<td>2 digit number, rounding</td>
<td>17</td>
<td>55</td>
<td>57</td>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>11</td>
<td>3 digit number, rounding</td>
<td>17</td>
<td>27</td>
<td>43</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>4 digit number, place value</td>
<td>42</td>
<td>55</td>
<td>64</td>
<td>54</td>
<td>54</td>
</tr>
<tr>
<td>13</td>
<td>4 digit number, ordering</td>
<td>0</td>
<td>18</td>
<td>29</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>14</td>
<td>4 digit number, renaming</td>
<td>8.3</td>
<td>9.1</td>
<td>36</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>15</td>
<td>4 digit number, ordering</td>
<td>33</td>
<td>45</td>
<td>71</td>
<td>51</td>
<td>51</td>
</tr>
<tr>
<td>16</td>
<td>4 digit number, rounding</td>
<td>Not administered</td>
<td>8.3</td>
<td>0</td>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>17</td>
<td>5 digit number, language → symbol</td>
<td>25</td>
<td>55</td>
<td>64</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td>18</td>
<td>5 digit number, comparing</td>
<td>83</td>
<td>82</td>
<td>79</td>
<td>81</td>
<td>81</td>
</tr>
<tr>
<td>19</td>
<td>6 digit number, place value</td>
<td>0</td>
<td>36</td>
<td>43</td>
<td>27</td>
<td>27</td>
</tr>
<tr>
<td>20</td>
<td>6 digit number, renaming</td>
<td>0</td>
<td>0</td>
<td>21</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

The specific questions with which students had difficulties were questions 8, 13, 14, 16, and 20. Primarily these questions focused on place value knowledge. Three of these questions required students to identify the value of a particular digit in a number, and how many of a particular place value were in a number. The other questions involved rounding to the nearest hundred, and identifying what value was ten less than 7003. Also, there are several questions (question 1, 3, 6, and 18) where the Year 4/5 class performed better than the Year 5/6 class.
Post-initiative findings for the numeration diagnostic test

Particular questions where achievement has increased significantly are highlighted in bold text in Table 17.

Table 17
Analysis of numeration diagnostic test by question

<table>
<thead>
<tr>
<th>Q</th>
<th>Numeration test question</th>
<th>Change in average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Year 2/3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n=12</td>
</tr>
<tr>
<td>1</td>
<td>2 digit number, materials (\rightarrow) symbol</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>Teen number, language (\rightarrow) symbol</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>2 digit number, ordering</td>
<td>58</td>
</tr>
<tr>
<td>4</td>
<td>2 digit number, sequencing</td>
<td>75</td>
</tr>
<tr>
<td>5</td>
<td>3 digit number, materials (\rightarrow) symbol</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>3 digit number, materials (\rightarrow) symbol</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>3 digit number, place value</td>
<td>25</td>
</tr>
<tr>
<td>8</td>
<td>3 digit number, renaming</td>
<td>-8</td>
</tr>
<tr>
<td>9</td>
<td>3 digit number, ordering</td>
<td>33</td>
</tr>
<tr>
<td>10</td>
<td>2 digit number, rounding</td>
<td>50</td>
</tr>
<tr>
<td>11</td>
<td>3 digit number, rounding</td>
<td>17</td>
</tr>
<tr>
<td>12</td>
<td>4 digit number, place value</td>
<td>-17</td>
</tr>
<tr>
<td>13</td>
<td>4 digit number, ordering</td>
<td>50</td>
</tr>
<tr>
<td>14</td>
<td>4 digit number, renaming</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>4 digit number, ordering</td>
<td>8</td>
</tr>
<tr>
<td>16</td>
<td>4 digit number, rounding</td>
<td>Not administered</td>
</tr>
<tr>
<td>17</td>
<td>5 digit number, language (\rightarrow) symbol</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>5 digit number, comparing</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>6 digit number, place value</td>
<td>8</td>
</tr>
<tr>
<td>20</td>
<td>6 digit number, renaming</td>
<td>8</td>
</tr>
</tbody>
</table>

Numeration diagnostic test – analysis of spread of scores by class

Following the analysis of the results on individual numeration test items, the change in the spread of scores for each class was observed. For each class, the frequency of each test score (from 0 to 20) was noted and graphed for both the pre and post-test. Figure 5 describes the spread of scores for the Year 2/3 class.
For the Year 2/3 cohort, the spread of scores has reduced throughout the course of the initiative with all students now scoring 3 or more on the numeration test. The majority of this cohort was achieving scores of 5 or 6 out of 9 compared to the majority of the cohort scoring 1 out of 9 on the pre-test.

Figure 6 outlines the change in the spread of numeration scores for the Year 3/4 students. On the pre-test, the mean score on the numeracy diagnostic test was approximately 7 out of 20 set questions but this has increased to 10 out of 20 on the post-test.

From Figure 6 it can be seen that the spread of scores for the Year 3/4 class also reduced with all students scoring 7 or above. Previously there was a core group of students who demonstrated limited achievement on the numeration test, however this lower group is now not so distinct from the remainder of the class. There appears to be a group of students achieving scores between 7 to 11 out of 20 and a higher group achieving scores of 14 to 16 out of 20. Overall, the number of students achieving higher on the numeration test has increased throughout the course of the initiative.
Figure 7 outlines the change in the spread of numeration scores for the Year 4/5 students.

Overall, it appears that the spread of scores for this cohort has also improved, with all students scoring 8 or more on the post-test.

Figure 8 outlines the change in the spread of numeration scores for the Year 5/6 students.

The pre-test findings indicated that this class had the largest spread of scores, and the post-test findings indicate that this has remained largely unchanged. There are still a small number of students from this cohort achieving low scores on the numeration test while the remainder of the class has scores ranging from 11 to 20 out of 20.
APPENDIX J: NUMBER FACT TEST – ANALYSIS OF SPREAD OF SCORES BY CLASS

Year 2/3 Findings

Figures 9 and 10 outline the spread of scores on the pre and post addition and subtraction fact tests respectively for Year 2/3.

Figure 9. Spread of scores on the addition number fact oral test: Year 2/3, n=12, total questions administered=10

On the pre-test, the Year 2/3 cohort reported a large spread in scores on the addition number fact test. On the post-test, it can be seen that the spread of students’ scores significantly reduced with all students scoring 5 out of 10 or above. Majority of the cohort were now achieving scores of 8, 9, or 10 out of 10 indicating proficiency with addition facts as observed above in the analysis of mean scores.

Figure 10. Spread of scores on the subtraction number fact oral test: Year 2/3, n=12, total questions administered=10

For the subtraction number fact test, the large spread in students’ scores remained on the post-test. The spread of students’ scores indicates that there was a large range of abilities.
in this cohort with some students achieving scores of 1 or 2 out of 10, and others achieving 8 or 9 out of 10. Overall, as observed above, students’ proficiency with subtraction was still developing in this cohort.

**Year 3/4 Findings**

Figures 11, 12 and 13 outline the spread of scores on the pre and post addition, subtraction, and multiplication fact tests respectively for Year 3/4.

![Addition Fact Score](image1)

*Figure 11. Spread of scores on the addition number fact oral test: Year 3/4, n=12, total questions administered=10*

As discussed above, the mean score for Year 3/4 indicated that students in this cohort were proficient with addition number facts. Observing the change in the spread of scores indicates that the range of scores on the addition number fact test reduced for this class over the course of the initiative with majority of students scoring 10 out of 10 on the test.

![Subtraction Fact Score](image2)

*Figure 12. Spread of scores on the subtraction number fact oral test: Year 3/4, n=12, total questions administered=10*
On the subtraction fact test, it can be seen that the spread of scores had not similarly reduced over the course of the initiative. The findings on the post-test indicate that there was a group of proficient students in this cohort scoring 9 or 10 out of 10, however there were still some students with scores between 3 and 7 out of 10 indicating a developing proficiency. The four students who indicated a developing proficiency had additional behaviour needs, and mainly participated in the removed mathematics group with the teacher aide.

Figure 13. Spread of scores on the multiplication number fact oral test: Year 3/4, n=12, total questions administered=10

Similar to the subtraction test, the spread of students’ scores on the multiplication fact test in Year 3/4 also did not reduce over the course of the initiative. The spread of students’ scores indicates that there is a large range of abilities in this cohort. For the division number fact test, whilst the classes mean has improved over the course of the initiative, this is reflective of only two students being able to answer any division fact questions correctly.

Year 4/5 Findings

Figures 14, 15 and 16 outline the spread of scores on the pre and post addition, subtraction, and multiplication fact tests respectively for Year 4/5.

Figure 14. Spread of scores on the addition number fact oral test: Year 4/5, n=11, total questions administered=10
The findings of the post-initiative addition number fact test indicated that all students in the Year 4/5 cohort were achieving 100%, which is a reduction in the spread of reported scores over the course of the initiative. The one student that reported a score of 0 was because of an incomplete administration of the test due to absences.

Similarly, the spread of students’ scores has reduced for the Year 4/5 cohort on the subtraction number fact test. Again, the one student reporting a score of 0 on the post-test was due to absences and an incomplete administration of the test. The majority of students in this cohort were reporting scores between 7 and 10 out of 10 on the post-test, indicating that proficiency had been achieved or is nearly achieved.

On the multiplication number fact test, students in Year 4/5 again reported a large range of scores on the post test. The one student reporting a score of 0 on the post-test was due to absences and an incomplete administration of the test. There appeared to be few students in this group who were proficient in the recall of multiplication facts with only one student scoring 10 out of 10. Nearly half of the cohort reported a score of 5 out of 10, indicating a developing proficiency with multiplication facts. These findings may act to explain difficulties in
completing multiplication computation questions. For the division number fact test, whilst the classes mean improved over the course of the initiative, this was reflective of only three students being able to answer any division fact questions correctly.

**Year 5/6 Findings**

For Year 5/6, the post-test findings indicated that students achieved an average score of 9.5 and 9.6 of the addition and subtraction number fact tests respectively. Proficiency with addition and subtraction facts is expected for this year level and is essential knowledge for the topics and content covered in Year 5 and 6 mathematics. Two students in the sample reported a decline in score from 10 to 9 on the addition number facts test, however this may be attributed to a careless error. Similarly, two students reported a decline from 10 to 8 and 9 on the subtraction number facts test. All other students in the sample reported an increase or no change in addition and subtraction number fact scores. All students scored between 7 and 10, and 8 to 10 respectively on the addition and subtraction number fact post-tests.

Figures 17 and 18 outlines the spread of scores on the pre and post multiplication and division fact tests respectively for Year 5/6.

![Figure 17. Spread of scores on the multiplication number fact oral test: Year 5/6, n=13, total questions administered=10](image)

Analysis of the post-test multiplication fact test for Year 5/6 indicated that the spread of students’ scores reduced with all scoring 4 or above. However, these findings indicate that there was a portion of this class that is demonstrating developing proficiency with multiplication facts. Whether this has implications for students’ ability to complete multiplication computation questions was observed in later analysis.
On the division number fact test, the post-test analysis found that only two students from the Year 5/6 cohort could complete any division fact questions, however the post-test findings indicated that 8 students, over half of the sample, can now answer division fact questions.

*Figure 18. Spread of scores on the division number fact oral test: Year 5/6, n=14, total questions administered=10*
APPENDIX K: ANALYSIS OF ADDITION COMPUTATION TEST

Analysis of scores by test question

Table 18 outlines the change in average scores for each individual computation question across each class.

Table 18
Analysis of addition diagnostic test: Computation

<table>
<thead>
<tr>
<th>Example Question</th>
<th>Change in average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 2/3, n=12</td>
</tr>
<tr>
<td>1. 73 + 37</td>
<td>-8</td>
</tr>
<tr>
<td>2. 50 + 48</td>
<td>42</td>
</tr>
<tr>
<td>3. 537 + 256</td>
<td>8</td>
</tr>
<tr>
<td>4. 574 + 395</td>
<td>17</td>
</tr>
<tr>
<td>5. 326 + 578</td>
<td>25</td>
</tr>
<tr>
<td>6. 567 + 908</td>
<td>-17</td>
</tr>
<tr>
<td>7. 6 + 9 + 4 + 8 + 5</td>
<td>-8</td>
</tr>
<tr>
<td>8. 798+826+134+957</td>
<td>0</td>
</tr>
<tr>
<td>9. 7654 + 4396</td>
<td>8</td>
</tr>
<tr>
<td>10. 39286 + 56906</td>
<td>17</td>
</tr>
</tbody>
</table>

Analysis of spread of scores in each class

Figure 19 outlines the spread of scores out of the four administered questions on the addition computation portion of the diagnostic pre- and post-test for Year 2/3 students. On the pre-initiative test, half of the cohort was unable to answer any computation questions correctly.

Figure 19: Spread of scores on the addition computation test: Year 2/3, n=12, total questions administered=4
The post-initiative test findings indicate that the majority of the Year 2/3 cohort can answer at least one simple addition computation question (e.g., 2 or 3-digit addition questions) correctly. Overall, success with addition computations remains limited for this cohort throughout the initiative. This class indicated on the number fact tests that students had a reasonable level of proficiency with addition number facts so presumably the root cause of students’ difficulties with addition computations stemmed from a lack of conceptual understanding concerning the algorithm and the renaming process.

Figure 20 outlines the spread of scores on the addition computation portion of the diagnostic pre and post-test for Year 3/4 students. On the pre-initiative test, students in this cohort reported a large spread of scores.

![Figure 20: Spread of scores on the addition computation test: Year 3/4, n=12, total questions administered=10](image)

Analysis of the post-initiative test indicates that this cohort still contained a large range of abilities in relation to addition computations, however majority of the cohort are scoring 5 or above. This is suggestive of a developing proficiency with addition computations. Five students in this cohort reported a reduction in their overall achievement on the addition computation test.

Figure 21 outlines the spread of scores on the addition computation portion of the diagnostic pre and post-test for Year 4/5 students.
Overall, it can be seen that the spread of scores remained largely unchanged throughout the initiative. Three students from this cohort reported a reduction in their overall computation score throughout the course of the initiative.

Figure 22 outlines the spread of scores on the addition computation portion of the diagnostic pre- and post-test for Year 5/6 students. On the pre-initiative test, it was found that majority of the students in this cohort were scoring between 8 and 10 out of 10, with a small number of outlier students demonstrating low proficiency with any addition computations. The students that scored 0 and 3 out of 10 on the pre-test did so despite demonstrating proficiency with addition number facts as they confused the questions with subtraction or did not attempt the questions.

The proportion of students who scored between 8 and 10 on the addition computation test increased on the post-initiative test for the Year 5/6 cohort. The lower proportion of students who struggled with addition computation is not observed on the post-test findings. Four students from this cohort reported a reduction in their overall computation score throughout the course of the initiative.
**Analysis of error frequency**

Table 19 outlines the overall total occurrence of each error on the pre- and post-initiative tests, and also reports the change in frequency for each error type. The Year 2/3 cohort have not been reported in this sample due to the small number of questions that were able to be attempted by students in the cohort.

Table 19
Description of frequency of each error type from the addition diagnostic test

<table>
<thead>
<tr>
<th>Error</th>
<th>Total, all computation questions</th>
<th>Sample: Year 3/4, 4/5, 5/6</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-test</td>
<td>Post-test</td>
<td></td>
</tr>
<tr>
<td>Number fact error</td>
<td>33</td>
<td>33</td>
<td>0</td>
</tr>
<tr>
<td>Multi-digit addition sum question, unsure of error cause</td>
<td>27</td>
<td>20</td>
<td>-7</td>
</tr>
<tr>
<td>No attempt</td>
<td>18</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>Forgot to add on a recorded renamed value</td>
<td>12</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td>Forgot to record renamed value</td>
<td>11</td>
<td>9</td>
<td>-2</td>
</tr>
<tr>
<td>Question not administered - too difficult for student</td>
<td>10</td>
<td>0</td>
<td>-10</td>
</tr>
<tr>
<td>Carried out sum as if it were subtraction/confused the addition algorithm with subtraction algorithm</td>
<td>8</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Student does not understand how to complete questions, but is attempting the test. Most answers appear random - causes are not explainable.</td>
<td>5</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Did not rename values, squished double-digit answers into a single place value column</td>
<td>5</td>
<td>22</td>
<td>17</td>
</tr>
<tr>
<td>Renamed incorrectly where zero was involved in the answer, did not record renamed value</td>
<td>3</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>Unsure of error cause</td>
<td>6</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Incomplete question: question is only partially completed</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Recorded incorrect renamed value</td>
<td>2</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>Answered '1' in instances where renaming was required</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>Recorded renamed values backwards</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Recorded unnecessary renamed value</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>Started sum from the left</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Crossed off bottom value in ones column. Unsure of reason</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Did not record last value, left as a renamed value</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 20 below outlines the frequency for each type of error, and the change in frequency on the computation test questions broken down by individual grade.
Table 20
Description of frequency of each error type by class from the addition diagnostic test

<table>
<thead>
<tr>
<th>Error</th>
<th>Year 2/3, computations Q1-4 only n=12 Pre/post</th>
<th>Year 3/4, computations (all questions) n=12 Pre/post</th>
<th>Year 4/5, computations (all questions) n=11 Pre/post</th>
<th>Year 5/6, computations (all questions) n=14 Pre/post</th>
</tr>
</thead>
<tbody>
<tr>
<td>No attempt</td>
<td>14 / 0</td>
<td>7 / 13</td>
<td>6 / 4</td>
<td>5 / 3</td>
</tr>
<tr>
<td>Number fact error</td>
<td>2 / 9</td>
<td>8 / 11</td>
<td>17 / 11</td>
<td>8 / 2</td>
</tr>
<tr>
<td>Multi-digit addition sum question, unsure of error cause</td>
<td>0 / 0</td>
<td>8 / 10</td>
<td>10 / 4</td>
<td>9 / 6</td>
</tr>
<tr>
<td>Question not administered - too difficult for student</td>
<td>0 / 0</td>
<td>10 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Forgot to add on a renamed value</td>
<td>0 / 5</td>
<td>1 / 1</td>
<td>3 / 3</td>
<td>8 / 5</td>
</tr>
<tr>
<td>Forgot to record renamed value</td>
<td>2 / 4</td>
<td>8 / 1</td>
<td>3 / 3</td>
<td>0 / 1</td>
</tr>
<tr>
<td>Unsure of error cause</td>
<td>0 / 1</td>
<td>4 / 4</td>
<td>1 / 1</td>
<td>1 / 0</td>
</tr>
<tr>
<td>Carried out sum as if it were subtraction/confused the addition algorithm with subtraction algorithm</td>
<td>0 / 2</td>
<td>4 / 4</td>
<td>0 / 2</td>
<td>4 / 0</td>
</tr>
<tr>
<td>Student does not understand how to complete questions, but is attempting the test. Most answers appear random - causes are not explainable.</td>
<td>21 / 4</td>
<td>5 / 0</td>
<td>0 / 0</td>
<td>0 / 1</td>
</tr>
<tr>
<td>Did not rename values, squished double-digit answers into a single place value column</td>
<td>3 / 15</td>
<td>5 / 7</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Renamed incorrectly where zero was involved in the answer, did not record renamed value</td>
<td>0 / 0</td>
<td>3 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Incomplete question: question is only partially completed</td>
<td>0 / 2</td>
<td>1 / 0</td>
<td>1 / 1</td>
<td>0 / 1</td>
</tr>
<tr>
<td>Recorded incorrect renamed value</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>2 / 0</td>
</tr>
<tr>
<td>Answered '1' in instances where renaming was required</td>
<td>0 / 0</td>
<td>1 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Recorded renamed values backwards</td>
<td>0 / 1</td>
<td>0 / 2</td>
<td>0 / 0</td>
<td>1 / 0</td>
</tr>
<tr>
<td>Recorded unnecessary renamed value</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>1 / 0</td>
</tr>
<tr>
<td>Started sum from the left</td>
<td>10 / 1</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Crossed off bottom value in ones column. Unsure of reason</td>
<td>0 / 2</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Did not record last value, left as a renamed value</td>
<td>0 / 3</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
</tbody>
</table>
APPENDIX L: ANALYSIS OF SUBTRACTION COMPUTATION TEST

Analysis of scores by test question

Table 21 outlines the change in average scores for each individual computation question across each class.

Table 21
Analysis of subtraction diagnostic test: Computation

<table>
<thead>
<tr>
<th>Q</th>
<th>Change in average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 2/3, n=12 Year 3/4, n=12 Year 4/5, n=11 Year 5/6, n=14</td>
</tr>
<tr>
<td>1. 87 - 38</td>
<td>0 25 9 7.1</td>
</tr>
<tr>
<td>2. 70 - 56</td>
<td>0 16.7 -36.3 7.2</td>
</tr>
<tr>
<td>3. 685 - 368</td>
<td>0 25 -18.2 21.4</td>
</tr>
<tr>
<td>4. 846 - 467</td>
<td>0 33.3 -9.1 14.3</td>
</tr>
<tr>
<td>5. 621 - 573</td>
<td>33.3 27.2 21.5</td>
</tr>
<tr>
<td>6. 603 - 424</td>
<td>25 0 -21.4</td>
</tr>
<tr>
<td>7. 7806-4653</td>
<td>33.3 45.4 0</td>
</tr>
<tr>
<td>8. 9005-7896</td>
<td>50 18.2 7.2</td>
</tr>
<tr>
<td>9. 5000-2812</td>
<td>33.3 36.3 21.4</td>
</tr>
<tr>
<td>10. 80348-49689</td>
<td>Not administered 41.7 9 7.2</td>
</tr>
</tbody>
</table>

*Note: the values in each question varied between the pre- and post-initiative tests.

Analysis of spread of scores in each class

Figure 23 outlines the spread of scores on the subtraction computation portion of the diagnostic test for Year 3/4 students.

Figure 23: Spread of scores on the subtraction computation test: Year 3/4, n=12, total questions administered=10
Figure 24 outlines the spread of scores on the ten administered questions on the subtraction computation portion of the diagnostic test for Year 4/5 students.

Figure 24: Spread of scores on the subtraction computation test: Year 4/5, n=11, total questions administered=10

Figure 25 outlines the spread of scores on the ten administered questions on the subtraction computation portion of the diagnostic test for Year 5/6 students.

Figure 25: Spread of scores on the subtraction computation test: Year 5/6, n=14, total questions administered=10

**Analysis of error frequency**

Table 22 outlines the overall total occurrence of each error on the pre- and post-initiative tests, and also reports the change in frequency for each error type.
Table 22
Description of frequency of each error type from the subtraction diagnostic test

<table>
<thead>
<tr>
<th>Error</th>
<th>Total, all computation questions</th>
<th>Sample: Year 2/3, 3/4, 4/5, and 5/6, n=37</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-test</td>
<td>Post-test</td>
<td></td>
</tr>
<tr>
<td>Did not rename, just found difference</td>
<td>78</td>
<td>75</td>
<td>-3</td>
</tr>
<tr>
<td>Did not rename, answered '0' where renaming was required</td>
<td>35</td>
<td>31</td>
<td>-4</td>
</tr>
<tr>
<td>No attempt</td>
<td>29</td>
<td>31</td>
<td>2</td>
</tr>
<tr>
<td>Number fact error</td>
<td>25</td>
<td>26</td>
<td>1</td>
</tr>
<tr>
<td>Renaming error: incorrect renaming process when internal 0's were involved</td>
<td>22</td>
<td>19</td>
<td>-3</td>
</tr>
<tr>
<td>Student does not understand how to complete questions, but is attempting the test. Most answers appear random - causes are not explainable.</td>
<td>14</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>Incomplete question: question is only partially completed</td>
<td>11</td>
<td>8</td>
<td>-3</td>
</tr>
<tr>
<td>Question not administered - too difficult for student</td>
<td>10</td>
<td>0</td>
<td>-10</td>
</tr>
<tr>
<td>Started sum from the left</td>
<td>7</td>
<td>0</td>
<td>-7</td>
</tr>
<tr>
<td>Unnecessarily renamed</td>
<td>5</td>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>Renaming error: added on a value when renaming, instead of taking it away</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Confused question with addition/confused subtraction algorithm with addition algorithm</td>
<td>3</td>
<td>21</td>
<td>18</td>
</tr>
<tr>
<td>Attempted to rename, but attempt does not preserve the place value of the renamed values. E.g. takes 1 ten, and renames as as 1 one.</td>
<td>2</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>Unsure of error cause due to messy working out</td>
<td>2</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>Attempted to rename, but attempt is incorrect. Renamed to seemingly random values</td>
<td>2</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Renamed, but didn't record the value that was taken away</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Attempted to rename, but attempt is incorrect. Renamed “1” to “10” instead of “11”</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>Took away 2 when renaming, instead of 1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Unsure of thinking behind attempt/unsure of the cause of the error</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Added an unnecessary and incorrect renamed value</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

Table 23 outlines the frequency for each type of error on the computation test questions broken down by individual grade for the pre- and post-initiative tests.
<table>
<thead>
<tr>
<th>Error</th>
<th>Year 2/3, computations Q1-4 only n=12 Pre/post</th>
<th>Year 3/4, computations (all questions) n=12 Pre/post</th>
<th>Year 4/5, computations (all questions) n=11 Pre/post</th>
<th>Year 5/6, computations (all questions) n=14 Pre/post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Did not rename, just found difference</td>
<td>6 / 16</td>
<td>35 / 25</td>
<td>18 / 19</td>
<td>25 / 20</td>
</tr>
<tr>
<td>Did not rename, answered ‘0’ where renaming was required</td>
<td>5 / 14</td>
<td>34 / 15</td>
<td>0 / 0</td>
<td>1 / 2</td>
</tr>
<tr>
<td>Number fact error</td>
<td>2 / 5</td>
<td>9 / 4</td>
<td>10 / 11</td>
<td>6 / 6</td>
</tr>
<tr>
<td>No attempt</td>
<td>18 / 6</td>
<td>12 / 18</td>
<td>13 / 5</td>
<td>4 / 7</td>
</tr>
<tr>
<td>Renaming error: incorrect renaming process when internal 0’s were involved</td>
<td>0 / 0</td>
<td>4 / 0</td>
<td>5 / 8</td>
<td>13 / 11</td>
</tr>
<tr>
<td>Partially complete question</td>
<td>0 / 1</td>
<td>4 / 0</td>
<td>0 / 6</td>
<td>7 / 1</td>
</tr>
<tr>
<td>Question not administered – too difficult for student</td>
<td>0 / 0</td>
<td>10 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Student does not understand how to complete questions, but is attempting the test. Most answers appear random – causes are not explainable.</td>
<td>21 / 11</td>
<td>6 / 4</td>
<td>6 / 0</td>
<td>2 / 2</td>
</tr>
<tr>
<td>Started sum from the left</td>
<td>12 / 0</td>
<td>7 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Confused question with addition/confused subtraction algorithm with addition algorithm</td>
<td>0 / 12</td>
<td>1 / 7</td>
<td>0 / 1</td>
<td>2 / 1</td>
</tr>
<tr>
<td>Attempted to rename, but attempt is incorrect. Renamed to seemingly random values</td>
<td>0 / 4</td>
<td>2 / 0</td>
<td>0 / 0</td>
<td>0 / 4</td>
</tr>
<tr>
<td>Unnecessarily renamed</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>3 / 2</td>
<td>2 / 2</td>
</tr>
<tr>
<td>Renaming error: added on a value when renaming, instead of taking it away</td>
<td>0 / 0</td>
<td>1 / 0</td>
<td>1 / 3</td>
<td>1 / 0</td>
</tr>
<tr>
<td>Unclassified: unsure of thinking behind attempt/unsure of the cause of the error</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>0 / 1</td>
<td>1 / 0</td>
</tr>
<tr>
<td>Unsure of error cause due to messy working out</td>
<td>0 / 0</td>
<td>2 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Attempted to rename, but attempt does not preserve the place value of the renamed values. E.g. takes 1 ten, and renames as 1 one.</td>
<td>0 / 0</td>
<td>2 / 9</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Renamed, but didn’t record the value that was taken away</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>2 / 2</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Took away 2 when renaming, instead of 1</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>1 / 1</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Attempted to rename, but attempt is incorrect. Renamed “1” to “10” instead of “11”</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>1 / 0</td>
</tr>
<tr>
<td>Added an unnecessary and incorrect renamed value</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>1 / 0</td>
<td>0 / 0</td>
</tr>
</tbody>
</table>
APPENDIX M: ANALYSIS OF MULTIPLICATION DIAGNOSTIC TEST

Analysis of scores by test question

Table 24 outlines the change in average scores for each individual computation question across each class.

Table 24
Analysis of multiplication diagnostic test: Computation

<table>
<thead>
<tr>
<th>Q</th>
<th>Year 2/3, n=12</th>
<th>Year 3/4, n=12</th>
<th>Year 4/5, n=11</th>
<th>Year 5/6, n=14</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>49x3</td>
<td>8.4</td>
<td>-27.2</td>
<td>-14.3</td>
</tr>
<tr>
<td>2.</td>
<td>74x5</td>
<td>16.7</td>
<td>-18.2</td>
<td>0</td>
</tr>
<tr>
<td>3.</td>
<td>874x4</td>
<td>8.3</td>
<td>54.6</td>
<td>7.2</td>
</tr>
<tr>
<td>4.</td>
<td>487x7</td>
<td>16.7</td>
<td>0</td>
<td>7.2</td>
</tr>
<tr>
<td>5.</td>
<td>63x56</td>
<td>0</td>
<td>0</td>
<td>-14.3</td>
</tr>
<tr>
<td>6.</td>
<td>34x18</td>
<td>0</td>
<td>9.1</td>
<td>35.7</td>
</tr>
<tr>
<td>7.</td>
<td>70x58</td>
<td>0</td>
<td>0</td>
<td>21.5</td>
</tr>
<tr>
<td>8.</td>
<td>46x58</td>
<td>0</td>
<td>0</td>
<td>28.6</td>
</tr>
<tr>
<td>9.</td>
<td>543x47</td>
<td>0</td>
<td>0</td>
<td>14.3</td>
</tr>
<tr>
<td>10.</td>
<td>749x80</td>
<td>0</td>
<td>0</td>
<td>21.4</td>
</tr>
</tbody>
</table>

*Note: the values in each question varied between the pre- and post-initiative tests.

Analysis of spread of scores in each class

Figure 26 outlines the spread of scores out of the ten administered questions on the multiplication computation portion of the diagnostic test for Year 3/4 students.

*Figure 26: Spread of scores on the multiplication computation test: Year 3/4, n=12, total questions administered=10*
Figure 27 outlines the spread of scores out of the ten administered questions on the multiplication computation portion of the diagnostic test for Year 4/5 students.

![Graph showing the spread of scores](image1)

*Figure 27: Spread of scores on the multiplication computation test: Year 4/5, n=11, total questions administered=10*

Figure 28 outlines the spread of scores out of the ten administered questions on the multiplication computation portion of the diagnostic test for Year 5/6 students.

![Graph showing the spread of scores](image2)

*Figure 28: Spread of scores on the multiplication computation test: Year 5/6, n=14, total questions administered=10*

**Analysis of error frequency**

Table 25 outlines the change in error frequency overall for the Year 3/4, 4/5, and 5/6 groups. Again, the Year 2/3 cohort was again not included in this analysis as this test was not administered to this group.
Table 25
Description of frequency of each error type from the multiplication diagnostic test

<table>
<thead>
<tr>
<th>Error</th>
<th>Total, all computation questions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample: Year 3/4, 4/5, and 5/6 n=37</td>
</tr>
<tr>
<td></td>
<td>Pre-test</td>
</tr>
<tr>
<td>No attempt</td>
<td>101</td>
</tr>
<tr>
<td>Question not administered - too difficult for student</td>
<td>63</td>
</tr>
<tr>
<td>Number fact error</td>
<td>46</td>
</tr>
<tr>
<td>Applies the multiplication algorithm incorrectly</td>
<td>40</td>
</tr>
<tr>
<td>Student does not understand how to complete questions, but is attempting the test. Most answers appear random - causes are not explainable.</td>
<td>20</td>
</tr>
<tr>
<td>Forgot one of the steps when completing the 2x2 digit multiplication algorithm</td>
<td>14</td>
</tr>
<tr>
<td>Unclassified: unsure of thinking behind attempt/unsure of the cause of the error</td>
<td>12</td>
</tr>
<tr>
<td>Incomplete question: question is only partially completed</td>
<td>5</td>
</tr>
<tr>
<td>Forgot to add on a renamed value</td>
<td>6</td>
</tr>
<tr>
<td>Recorded renamed values incorrectly</td>
<td>3</td>
</tr>
<tr>
<td>Recorded renamed values backwards</td>
<td>3</td>
</tr>
<tr>
<td>Did not rename, recorded answers in the next place value</td>
<td>3</td>
</tr>
<tr>
<td>2x2 digit sums: Recorded answer to ones x tens as a renamed value, rather than recording the answer</td>
<td>2</td>
</tr>
<tr>
<td>Error caused by using immature strategies</td>
<td>1</td>
</tr>
<tr>
<td>Used alternative algorithms that did not successfully lead to the correct answer</td>
<td>1</td>
</tr>
<tr>
<td>Unsure of error cause due to messy working out</td>
<td>1</td>
</tr>
<tr>
<td>Forgot to record renamed value</td>
<td>1</td>
</tr>
<tr>
<td>2x1: Did ones x ones, but then just brought down the tens value, rather than multiplying by it as well</td>
<td>0</td>
</tr>
<tr>
<td>Added values instead of multiplying</td>
<td>0</td>
</tr>
<tr>
<td>2x2 multiplication: recorded a value in the wrong place value</td>
<td>0</td>
</tr>
<tr>
<td>Did not rename values, squished double-digit answers into a single place value column</td>
<td>0</td>
</tr>
<tr>
<td>Forgot to record zero when multiplying by tens (second line of working)</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 26 outlines the frequency for each type of error on the multiplication computation test for each grade for the pre and post-tests.
Table 26
Description of frequency of each error type by class from the multiplication diagnostic test

<table>
<thead>
<tr>
<th>Error</th>
<th>Year 3/4, computations (all questions)</th>
<th>Year 4/5, computations (all questions)</th>
<th>Year 5/6, computations (all questions)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n=12 Pre/post</td>
<td>n=11 Pre/post</td>
<td>n=14 Pre/post</td>
</tr>
<tr>
<td>No attempt</td>
<td>39 / 54</td>
<td>36 / 39</td>
<td>26 / 35</td>
</tr>
<tr>
<td>Question not administered - too difficult for student</td>
<td>30 / 10</td>
<td>13</td>
<td>20 / 8</td>
</tr>
<tr>
<td>Number fact error</td>
<td>4 / 6</td>
<td>27 / 24</td>
<td>15 / 17</td>
</tr>
<tr>
<td>Applies the multiplication algorithm incorrectly</td>
<td>0 / 0</td>
<td>13 / 3</td>
<td>8 / 0</td>
</tr>
<tr>
<td>Student does not understand how to complete questions, but is attempting the test. Most answers appear random - causes are not explainable.</td>
<td>10 / 1</td>
<td>10 / 3</td>
<td>0 / 1</td>
</tr>
<tr>
<td>Forgot one of the steps when completing the 2x2 digit multiplication algorithm</td>
<td>0 / 1</td>
<td>0 / 0</td>
<td>14 / 2</td>
</tr>
<tr>
<td>Unclassified: unsure of thinking behind attempt/unsure of the cause of the error</td>
<td>11 / 1</td>
<td>1 / 0</td>
<td>0 / 1</td>
</tr>
<tr>
<td>Carried out algorithm incorrectly (3x2 digit)</td>
<td>0 / 0</td>
<td>6 / 8</td>
<td>4 / 5</td>
</tr>
<tr>
<td>Did 2x2 algorithm like a 2x1 digit algorithm</td>
<td>6 / 22</td>
<td>0 / 19</td>
<td>3 / 6</td>
</tr>
<tr>
<td>Incomplete question: question is only partially completed</td>
<td>1 / 2</td>
<td>1 / 1</td>
<td>3 / 3</td>
</tr>
<tr>
<td>Forgot to add on a renamed value</td>
<td>0 / 0</td>
<td>4 / 1</td>
<td>2 / 7</td>
</tr>
<tr>
<td>Recorded renamed values incorrectly</td>
<td>0 / 0</td>
<td>1 / 0</td>
<td>2 / 0</td>
</tr>
<tr>
<td>Recorded renamed values backwards</td>
<td>0 / 0</td>
<td>1 / 0</td>
<td>2 / 0</td>
</tr>
<tr>
<td>Did not rename, recorded answers in the next place value</td>
<td>0 / 0</td>
<td>1 / 0</td>
<td>2 / 2</td>
</tr>
<tr>
<td>2x2 digit sums: Recorded answer to ones x tens as a renamed value, rather than recording the answer</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>2 / 1</td>
</tr>
<tr>
<td>Unsure of error cause due to messy working out</td>
<td>0 / 0</td>
<td>0 / 2</td>
<td>1 / 0</td>
</tr>
<tr>
<td>Forgot to record renamed value</td>
<td>0 / 0</td>
<td>1 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Error caused by using immature strategies</td>
<td>1 / 1</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Used alternative algorithms that did not successfully lead to the correct answer</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>1 / 0</td>
</tr>
<tr>
<td>2x1 did sums: Did ones x ones, but then just brought down the tens value, rather than multiplying by it as well</td>
<td>1 / 3</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Added values instead of multiplying</td>
<td>11 / 3</td>
<td>0 / 0</td>
<td>2</td>
</tr>
<tr>
<td>Recorded a value in the wrong place value</td>
<td>0 / 4</td>
<td>0 / 0</td>
<td>0</td>
</tr>
<tr>
<td>Did not rename values, squished double-digit answers into a single place value column</td>
<td>0 / 9</td>
<td>0 / 1</td>
<td>0</td>
</tr>
<tr>
<td>Multiplication: Forgot to record zero when multiplying by tens (second line of working)</td>
<td>0 / 0</td>
<td>0 / 1</td>
<td>0 / 1</td>
</tr>
</tbody>
</table>
APPENDIX N: ANALYSIS OF DIVISION DIAGNOSTIC TEST

Analysis of scores by test question

Table 27 outlines the average scores for each individual computation question for each class.

Table 27
Analysis of division diagnostic test: Computation

<table>
<thead>
<tr>
<th>Q</th>
<th>Year 2/3, n=12</th>
<th>Year 3/4, n=12</th>
<th>Year 4/5, n=11</th>
<th>Year 5/6, n=14</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.  67 ÷ 4</td>
<td>16.7</td>
<td>0</td>
<td>35.7</td>
<td></td>
</tr>
<tr>
<td>2.  485 ÷ 7</td>
<td>8.3</td>
<td>0</td>
<td>14.3</td>
<td></td>
</tr>
<tr>
<td>3.  928 ÷ 3</td>
<td>0</td>
<td>0</td>
<td>28.6</td>
<td></td>
</tr>
<tr>
<td>4.  643 ÷ 4</td>
<td>8.3</td>
<td>9.1</td>
<td>28.6</td>
<td></td>
</tr>
<tr>
<td>5.  6657 ÷ 9</td>
<td>8.3</td>
<td>0</td>
<td>7.2</td>
<td></td>
</tr>
<tr>
<td>6.  7236 ÷ 8</td>
<td>0</td>
<td>0</td>
<td>21.5</td>
<td></td>
</tr>
<tr>
<td>7.  49356 ÷ 7</td>
<td>0</td>
<td>0</td>
<td>7.2</td>
<td></td>
</tr>
<tr>
<td>8.  45701 ÷ 14</td>
<td>8.3</td>
<td>0</td>
<td>21.4</td>
<td></td>
</tr>
</tbody>
</table>

*Note: the values in each question varied between the pre- and post-initiative tests.

Analysis of spread of scores in each class

Figure 29 outlines the spread of scores achieved on the division computation test for the Year 3/4 cohort.

![Figure 29: Spread of scores on the division computation test: Year 3/4, n=12, total questions administered=8](image)

Figure 30 outlines the spread of scores achieved on the division computation test for the Year 4/5 cohort.
Figure 30: Spread of scores on the division computation test: Year 4/5, n=11, total questions administered=8

Figure 31 outlines the spread of scores achieved on the division computation test for the Year 5/6 cohort.

Figure 31: Spread of scores on the division computation test: Year 5/6, n=14, total questions administered=8

**Analysis of error frequency**

Table 28 outlines the change in error frequency of errors the Year 3/4, 4/5, and 5/6 groups. Overall errors are not reported for the division computation test as only the Year 5/6 cohort was administered the pre-test.
Table 28
Description of frequency of each error type by class from the division diagnostic test

<table>
<thead>
<tr>
<th>Error</th>
<th>Year 3/4, computations (all questions) n=12 Pre/post</th>
<th>Year 4/5, computations (all questions) n=11 Pre/post</th>
<th>Year 5/6, computations (all questions) n=14 Pre/post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question not administered - too difficult for student</td>
<td>NA / 72</td>
<td>NA / 8</td>
<td>87 / 16</td>
</tr>
<tr>
<td>No attempt</td>
<td>NA / 12</td>
<td>NA / 61</td>
<td>13 / 45</td>
</tr>
<tr>
<td>Division: Student attempts to carry out correct algorithm, but makes procedural errors</td>
<td>NA / 1</td>
<td>NA / 1</td>
<td>3 / 7</td>
</tr>
<tr>
<td>Incomplete question: question is only partially completed</td>
<td>NA / 0</td>
<td>NA / 0</td>
<td>2 / 5</td>
</tr>
<tr>
<td>Unclassified: unsure of thinking behind attempt/unsure of the cause of the error</td>
<td>NA</td>
<td>NA / 9</td>
<td>2 / 5</td>
</tr>
<tr>
<td>Division: Answer was correct, but no remainder was recorded</td>
<td>NA</td>
<td>NA / 1</td>
<td>1 / 1</td>
</tr>
<tr>
<td>Number fact error</td>
<td>NA / 1</td>
<td>NA / 6</td>
<td>0 / 2</td>
</tr>
<tr>
<td>Carried out division correctly, but recorded wrong values in the answer</td>
<td>0</td>
<td>0</td>
<td>0 / 2</td>
</tr>
<tr>
<td>Issues with reversing the division calculations. Students thinking is 7/5 instead of 5/7</td>
<td>0</td>
<td>0 / 1</td>
<td>0 / 3</td>
</tr>
<tr>
<td>Didn't record '0' answers</td>
<td>0 / 4</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
APPENDIX O: ANALYSIS OF PROBLEM-SOLVING DIAGNOSTIC TEST

Analysis of achievement by class

Table 29 outlines the overall mean scores (out of 3) for each operation for each class on the pre- and post-initiative test.

Table 29
Problem-solving diagnostic test summary statistics: mean scores for each grade

<table>
<thead>
<tr>
<th></th>
<th>Year 3/4, n=12</th>
<th></th>
<th>Year 4/5, n=11</th>
<th></th>
<th>Year 5/6, n=14</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre</td>
<td>Post</td>
<td>Pre</td>
<td>Post</td>
<td>Pre</td>
<td>Post</td>
</tr>
<tr>
<td>Addition</td>
<td>0.67</td>
<td>1.75</td>
<td>0.82</td>
<td>1.45</td>
<td>1.57</td>
<td>2.07</td>
</tr>
<tr>
<td>Subtraction</td>
<td>0.42</td>
<td>1.08</td>
<td>0.27</td>
<td>0.82</td>
<td>1.07</td>
<td>1.14</td>
</tr>
<tr>
<td>Multiplication</td>
<td>0.17</td>
<td>0.58</td>
<td>0.45</td>
<td>0.36</td>
<td>0.57</td>
<td>0.86</td>
</tr>
<tr>
<td>Division</td>
<td>0</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
<td>0.21</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Addition problem-solving

The specific achievement on each addition problem-solving questions at the beginning of the initiative will be presented below in Table 30.

Table 30
Analysis of diagnostic test: Addition problem-solving

<table>
<thead>
<tr>
<th></th>
<th>Average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 2/3, n=12</td>
</tr>
<tr>
<td>Q1. 3-digit add 2-digit numbers</td>
<td>Not administered</td>
</tr>
<tr>
<td>Q2. 1-digit add 1-digit, identify question as addition and interpret meaning.</td>
<td>Not administered</td>
</tr>
<tr>
<td>Q3. 3-digit add 3-digit numbers, extra information</td>
<td>Not administered</td>
</tr>
</tbody>
</table>

The specific changes in achievement on individual addition problem-solving test questions at the end of the initiative are outlined in Table 31.
Table 31
Analysis of diagnostic test: Addition problem-solving

<table>
<thead>
<tr>
<th>Question Description</th>
<th>Year 2/3, n=12</th>
<th>Year 3/4, n=12</th>
<th>Year 4/5, n=11</th>
<th>Year 5/6, n=14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1. 3-digit add 2-digit numbers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q2. 1-digit add 1-digit, identify question as addition and interpret meaning.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q3. 3-digit add 3-digit numbers, extra information</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>16.6</td>
<td>9.1</td>
<td>28.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>33.3</td>
<td>18.2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>58.4</td>
<td>36.4</td>
<td>21.4</td>
<td></td>
</tr>
</tbody>
</table>

From Year 5/6, 36% could not answer the question: “Gabriel’s parrot lost 7 black feathers and 6 blue feathers. How many feathers did his parrot lose?” Difficulties with the second question potentially are indicative of a lack of conceptual understanding relating to identifying the question as an addition as procedurally this question is simple to solve. The rate of success for this question in Year 5/6 did not change throughout the initiative. In Year 4/5, only two students from the cohort could answer the first subtraction problem-solving question, and only one could answer the second at the beginning of the initiative. The success for these questions did increase in this cohort throughout the initiative with three students in the class being able to answer each question. Less than half of Year 4/5 could answer “A tap leaks 3 litres of water each day. How much water will leak in 1 week?” at the beginning of the initiative, and less students were able to correctly answer this question by the end of the initiative (decrease in achievement from 45.5% to 36.4%, n=11). No students in Year 3/4 could answer this question at the beginning of the initiative, and only one student could answer this question by the end of the initiative.

The frequency and types of errors for the addition problem-solving pre- and post-initiative test are outlined in Table 32.
Table 32
Description of frequency of error by class for addition problem solving

<table>
<thead>
<tr>
<th>Error</th>
<th>Year 3/4 problem-solving n=12 Pre/post</th>
<th>Year 4/5, problem-solving n= 11 Pre/post</th>
<th>Year 5/6, problem-solving n=14 Pre/post</th>
</tr>
</thead>
<tbody>
<tr>
<td>No attempt</td>
<td>8 / 4</td>
<td>7 / 4</td>
<td>6 / 7</td>
</tr>
<tr>
<td>Number fact error</td>
<td>0 / 2</td>
<td>2 / 0</td>
<td>0 / 1</td>
</tr>
<tr>
<td>Carried out question using subtraction</td>
<td>7 / 4</td>
<td>9 / 5</td>
<td>5 / 1</td>
</tr>
<tr>
<td>Did not isolate correct values</td>
<td>5 / 1</td>
<td>9 / 5</td>
<td>3 / 3</td>
</tr>
<tr>
<td>Question not administered - too difficult for student</td>
<td>3 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Forgot to add on a renamed value</td>
<td>0 / 0</td>
<td>0 / 1</td>
<td>1 / 0</td>
</tr>
<tr>
<td>Unsure of error cause, no working shown</td>
<td>5 / 3</td>
<td>0 / 0</td>
<td>5 / 0</td>
</tr>
<tr>
<td>Forgot to record renamed value</td>
<td>1 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Student does not understand how to complete questions, but is attempting the test. Most answers appear random - causes are not explainable.</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>1 / 0</td>
</tr>
<tr>
<td>Copied values from the question wrong</td>
<td>1 / 0</td>
<td>1 / 0</td>
<td>1 / 1</td>
</tr>
<tr>
<td>Did not line up place values correctly</td>
<td>1 / 0</td>
<td>0 / 1</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Partially complete question</td>
<td>0 / 1</td>
<td>0 / 1</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Did not rename values, squished double-digit answers into a single place value column</td>
<td>0 / 2</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
</tbody>
</table>

At the beginning of the initiative, the findings indicated that students in each class had difficulties with addition problem-solving tasks. For example, the achievement of the Year 3/4 and 5/6 cohorts on the first problem-solving task is below what is expected by curriculum standards particularly for Year 5 and 6. Further, in most classes students difficulties were exacerbated on the second problem-solving task. Difficulties with the second question are indicative of a lack of conceptual understanding relating to identifying the question as an addition sum, as procedurally this question is simple to solve. Once additional unnecessary information was added into the problem-solving tasks, as was done in the third question, success rates further declined for all classes.

By the end of the initiative, average success rates on each addition test question increased across all year levels, excluding Question 2 for Year 5/6 which reported no improvement. The post-test findings indicated that students in Year 4/5 and 5/6 were proficient with the first addition problem-solving question. For the Year 3/4 and 4/5 cohorts, success rates on Question 2 and 3 significantly increased over the course of the initiative. By the end of the
initiative, the Year 3/4 cohort reported higher mean achievement for Question 3 compared to Year 4/5 and 5/6.

**Subtraction problem-solving**

Achievement on the subtraction problem-solving questions for each class at the beginning of the initiative is outlined in Table 33.

Table 33  
Analysis of diagnostic test: Subtraction problem-solving

<table>
<thead>
<tr>
<th></th>
<th>Average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 2/3, ( n=12 )</td>
</tr>
<tr>
<td>Q1. 2-digit subtract 2-digit numbers, too much information</td>
<td>Not administered</td>
</tr>
<tr>
<td>Q2. 2-digit subtract 1-digit, subtraction concept, missing part.</td>
<td>16.7</td>
</tr>
<tr>
<td>Q3. 3-digit subtract 3-digit numbers, too much information, interpret question.</td>
<td>8.3</td>
</tr>
</tbody>
</table>

The changes in achievement on individual subtraction problem-solving test questions at the end of the initiative are outlined in Table 34.

Table 34  
Analysis of diagnostic test: Subtraction problem-solving

<table>
<thead>
<tr>
<th></th>
<th>Change in average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 2/3, ( n=12 )</td>
</tr>
<tr>
<td>Q1. 2-digit subtract 2-digit numbers, too much information</td>
<td>Not administered</td>
</tr>
<tr>
<td>Q2. 2-digit subtract 1-digit, subtraction concept, missing part.</td>
<td>25</td>
</tr>
<tr>
<td>Q3. 3-digit subtract 3-digit numbers, too much information, interpret question.</td>
<td>16.7</td>
</tr>
</tbody>
</table>

At the beginning of the initiative in Year 3/4 and 4/5, it was found that achievement on all subtraction problem-solving questions was limited. For Question 1, both classes scores were less than 20%. Only two students from Year 3/4 could answer the first two questions. In Year 4/5, only two students from the cohort could answer Question 1 and only one student could answer Question 2. Similar to the addition problem-solving set of questions, limited success on the second question indicated difficulties with conceptually understanding the question and
selecting the appropriate operation as the underlying calculation (12 – 5) is not procedurally complex. For the Year 5/6 cohort, success with Question 2 was again much higher than the other grades indicating that conceptual understanding was potentially further developed in this grade.

By the end of the initiative average success rates on each subtraction test question increased across all year levels, excluding Question 3 for Year 4/5, and 5/6. The post-test findings indicated that approximately 40% of all cohorts could correctly answer the first subtraction problem-solving question. The Year 3/4 cohort was achieving a success rate comparable to the Year 4/5 and 5/6 cohorts for the first problem-solving question (the scores were 41.7%, 45.5%, and 42.9% respectively across each grade). Similar patterns were observed for Question 2, with the Year 3/4 cohort achieving a higher success rate than the 4/5 cohort (41.7% and 36.4% respectively. The Year 5/6 cohort made little improvement in relation to solving Question 1 and 2 over the course the initiative. Overall, the Year 3/4 cohort reported the highest achievement for Question 3, with 25% of the cohort successfully answering the question, compared to 0% and 7% of the Year 4/5 and 5/6 cohorts respectively. These findings support what was found on the analysis of the subtraction computation test, with the Year 3/4 cohort experiencing the largest improvement of all cohorts and reported a mean similar to that of the Year 4/5 cohort by the end of the initiative.

The frequency and types of errors for the subtraction problem-solving pre- and post-initiative test are outlined in Table 35.
Table 35
Description of frequency of error by class for subtraction problem-solving

<table>
<thead>
<tr>
<th>Error</th>
<th>Year 3/4 problem-solving (n=12) Pre/post</th>
<th>Year 4/5, problem-solving (n=11) Pre/post</th>
<th>Year 5/6, problem-solving (n=14) Pre/post</th>
</tr>
</thead>
<tbody>
<tr>
<td>No attempt</td>
<td>13 / 9</td>
<td>11 / 9</td>
<td>13 / 15</td>
</tr>
<tr>
<td>Did not isolate correct values</td>
<td>7 / 3</td>
<td>13 / 9</td>
<td>3 / 3</td>
</tr>
<tr>
<td>Unsure of error cause, no working shown</td>
<td>3 / 0</td>
<td>2 / 0</td>
<td>9 / 0</td>
</tr>
<tr>
<td>Carried out question as an addition</td>
<td>4 / 3</td>
<td>3 / 4</td>
<td>0 / 1</td>
</tr>
<tr>
<td>Did not rename, just found difference</td>
<td>2 / 0</td>
<td>2 / 1</td>
<td>0 / 1</td>
</tr>
<tr>
<td>Number fact error</td>
<td>0 / 0</td>
<td>3 / 0</td>
<td>1 / 2</td>
</tr>
<tr>
<td>Partially complete question</td>
<td>0 / 1</td>
<td>4 / 2</td>
<td>0 / 3</td>
</tr>
<tr>
<td>Question not administered - too difficult for student</td>
<td>3 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Problem solving: Did not line up place values correctly</td>
<td>1 / 0</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Unsure of error cause/no working shown</td>
<td>0 / 7</td>
<td>1 / 0</td>
<td>0 / 1</td>
</tr>
<tr>
<td>Error caused by using immature strategies</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>1 / 1</td>
</tr>
<tr>
<td>Quoted seemingly random values where renaming was required</td>
<td>0 / 2</td>
<td>0 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Student does not understand how to complete questions, but is attempting the test. Most answers appear random - causes are not explainable.</td>
<td>0 / 0</td>
<td>0 / 1</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Wrote sum the wrong way around (e.g., 275-360 instead of 360-275)</td>
<td>0 / 0</td>
<td>0 / 2</td>
<td>0 / 1</td>
</tr>
</tbody>
</table>

The types of errors that were prevalent on the subtraction problem-solving pre-test were similar to the findings from the addition problem-solving test. One of the most prevalent errors on the set of subtraction problem-solving questions was associated with not attempting the question associated with productive dispositions. Also, it was found that students again struggled to isolate the correct values indicating difficulties with strategic competence. Very few errors were attributed to procedural difficulties associated with solving subtraction computations. Over the course of the initiative, the number of students who were not attempting the question reduced in Year 3/4 and 4/5 but increased in Year 5/6. The frequency of students not isolating the correct values also reduced in Year 3/4 and 4/5 but did not change in Year 5/6.
Multiplication problem-solving

Achievement on the multiplication problem-solving questions for each class at the beginning of the initiative is outlined in Table 36.

Table 36
Analysis of diagnostic test: Multiplication problem-solving

<table>
<thead>
<tr>
<th></th>
<th>Average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 2/3, n=12</td>
</tr>
<tr>
<td>Q1. 1-digit by 2-digit numbers, interpret information, multi-step problem.</td>
<td>0.0</td>
</tr>
<tr>
<td>Q2. 1-digit by 1-digit, multiplication concept, interpret meaning</td>
<td>Not administered</td>
</tr>
<tr>
<td>Q3. 2-digit by 2-digit numbers, interpret question, extra information, multi-step problem.</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The changes in achievement on individual multiplication problem-solving test questions by the end of the initiative are outlined in Table 37.

Table 37
Analysis of diagnostic test: Multiplication problem-solving

<table>
<thead>
<tr>
<th></th>
<th>Change in average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 2/3, n=12</td>
</tr>
<tr>
<td>Q1. 1-digit by 2-digit numbers, interpret information, multi-step problem.</td>
<td>8.3</td>
</tr>
<tr>
<td>Q2. 1-digit by 1-digit, multiplication concept, interpret meaning</td>
<td>Not administered</td>
</tr>
<tr>
<td>Q3. 2-digit by 2-digit numbers, interpret question, extra information, multi-step problem.</td>
<td>0</td>
</tr>
</tbody>
</table>

Overall, performance on the multiplication problem-solving test was limited for all grades. Only two students in Year 3/4 could answer any multiplication problem-solving questions at the beginning of the initiative and these students were only able to answer Question 2. Less than half of Year 4/5 students could answer Question 2, and this was similarly the only question students in this class were able to answer. In Year 5/6, two students could answer the first problem-solving question at the beginning of the initiative, less than half could answer the second question, and none could answer the third. The proportion of students in Year 4/5 who could answer the second question successfully was greater than that reported in Year 5/6. Students difficulties with multiplication tasks concur with the findings from the multiplication computation test.
By the end of the initiative for Question 1, only the Year 3/4 cohort reported an increase in achievement, and the increase in achievement is reflective of only one student being able to correctly answer the question. Overall, achievement remained limited for all cohorts on this test question (only 3 students from the sample could correctly answer Question 1 at the conclusion of the initiative). The Year 3/4 and 5/6 cohorts reported an increase in achievement for Question 2, however Year 4/5 declined in achievement. The Year 3/4 cohort significantly improved their success rates for Question 2, and 50% of all Year 3/4, and 64% of all Year 5/6 students could answer the question on the post-test correctly. For Question 3, there were no gains in achievement for Year 3/4 or 4/5, and no students from either cohort could successfully answer the question. Only one student from Year 5/6 could answer Question 3 correctly. Overall, there were largely no gains in achievement or small gains in achievement reported across each cohort for the multiplication problem-solving tasks.

The frequency and types of errors for the multiplication problem-solving pre- and post-initiative test is outlined in Table 38.

<table>
<thead>
<tr>
<th>Error</th>
<th>Year 3/4 problem-solving n=12 Pre/post</th>
<th>Year 4/5, problem-solving n=11 Pre/post</th>
<th>Year 5/6, problem-solving n=14 Pre/post</th>
</tr>
</thead>
<tbody>
<tr>
<td>No attempt</td>
<td>17 / 19</td>
<td>16 / 15</td>
<td>16 / 12</td>
</tr>
<tr>
<td>Question not administered - too difficult for student</td>
<td>9 / 3</td>
<td>4 / 0</td>
<td>7 / 3</td>
</tr>
<tr>
<td>Did not isolate correct values</td>
<td>4 / 4</td>
<td>7 / 7</td>
<td>3 / 4</td>
</tr>
<tr>
<td>Carried out as addition</td>
<td>4 / 3</td>
<td>4 / 2</td>
<td>1 / 0</td>
</tr>
<tr>
<td>Unsure of error cause, no working shown</td>
<td>2 / 2</td>
<td>0 / 1</td>
<td>5 / 2</td>
</tr>
<tr>
<td>Incomplete question: question is only partially completed</td>
<td>0 / 1</td>
<td>2 / 5</td>
<td>1 / 1</td>
</tr>
<tr>
<td>Error caused by using immature strategies</td>
<td>1 / 0</td>
<td>0 / 0</td>
<td>0 / 1</td>
</tr>
<tr>
<td>Student does not understand how to complete questions, but is attempting the test. Most answers appear random - causes are not explainable.</td>
<td>0 / 0</td>
<td>1 / 0</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Number fact error</td>
<td>0 / 0</td>
<td>0 / 1</td>
<td>1 / 1</td>
</tr>
<tr>
<td>Used alternative algorithms that did not successfully lead to the correct answer</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>1 / 0</td>
</tr>
<tr>
<td>Multi-step problem solving: Did not carry out correct operations with correct values</td>
<td>0 / 0</td>
<td>0 / 1</td>
<td>0 / 2</td>
</tr>
<tr>
<td>Multi-step problem solving: Incomplete, only solved part of the problem</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>0 / 3</td>
</tr>
<tr>
<td>Forgot to add on a renamed value</td>
<td>0 / 0</td>
<td>0 / 1</td>
<td>0 / 0</td>
</tr>
</tbody>
</table>
When observing the types of errors that inhibited students’ success when solving multiplication problem-solving questions, it was found that not attempting the question was again the most frequent error for all classes. Also, similar to the findings of the addition and subtraction problem-solving tests, failure to isolate the correct values was also a prevalent error. Analysis of the types of errors present on the post-initiative multiplication problem-solving test reveals that there is little change in the frequencies of any particular errors excluding the number of questions that were not administered to students which decreased for each cohort.

**Division problem-solving**

The specific achievement on individual division problem-solving questions at the beginning of the initiative will be presented below in Table 39.

**Table 39**
Analysis of diagnostic test: Division problem-solving

<table>
<thead>
<tr>
<th>Division</th>
<th>Average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 2/3, n=12</td>
</tr>
<tr>
<td>Q1. 3-digit number divided by 1-digit, extra information</td>
<td></td>
</tr>
<tr>
<td>Q2. 2-digit number divided by 1-digit, division concept, interpret meaning</td>
<td></td>
</tr>
<tr>
<td>Q3. 4-digit number divided by 1-digit</td>
<td></td>
</tr>
</tbody>
</table>

The changes in achievement on individual division problem-solving test questions by the end of the initiative are outlined in Table 40.

**Table 40**
Analysis of diagnostic test: Division problem-solving

<table>
<thead>
<tr>
<th>Division</th>
<th>Change in average percentage of correct answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year 2/3, n=12</td>
</tr>
<tr>
<td>Q1. 3-digit number divided by 1-digit, extra information</td>
<td>Not administered</td>
</tr>
<tr>
<td>Q2. 2-digit number divided by 1-digit, division concept, interpret meaning</td>
<td>Not administered</td>
</tr>
<tr>
<td>Q3. 4-digit number divided by 1-digit</td>
<td></td>
</tr>
</tbody>
</table>

The frequency and types of errors for the division problem-solving pre- and post-initiative test are outlined in Table 41.
Table 41
Description of frequency of error by class for division problem-solving

<table>
<thead>
<tr>
<th>Error</th>
<th>Year 3/4 problem-solving n=12</th>
<th>Year 4/5, problem-solving n= 11</th>
<th>Year 5/6, problem-solving n=14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question not administered - too difficult for student</td>
<td>NA / 27 Pre/post</td>
<td>NA / 3 Pre/post</td>
<td>33 / 6 Pre/post</td>
</tr>
<tr>
<td>No attempt</td>
<td>NA / 6 Pre/post</td>
<td>NA / 18 Pre/post</td>
<td>6 / 24 Pre/post</td>
</tr>
<tr>
<td>Did not isolate correct values</td>
<td>NA / 0 Pre/post</td>
<td>NA / 1 Pre/post</td>
<td>0 / 0 Pre/post</td>
</tr>
<tr>
<td>Error caused by using immature strategies</td>
<td>NA / 0 Pre/post</td>
<td>NA / 4 Pre/post</td>
<td>0 / 0 Pre/post</td>
</tr>
<tr>
<td>Number fact error</td>
<td>NA / 0 Pre/post</td>
<td>NA / 1 Pre/post</td>
<td>0 / 0 Pre/post</td>
</tr>
<tr>
<td>Subtracted values</td>
<td>NA / 0 Pre/post</td>
<td>NA / 2 Pre/post</td>
<td>0 / 0 Pre/post</td>
</tr>
<tr>
<td>Added values</td>
<td>NA / 0 Pre/post</td>
<td>NA / 2 Pre/post</td>
<td>0 / 0 Pre/post</td>
</tr>
<tr>
<td>Unsure of error cause/no working shown</td>
<td>NA / 0 Pre/post</td>
<td>NA / 3 Pre/post</td>
<td>0 / 2 Pre/post</td>
</tr>
<tr>
<td>Incomplete question: question is only partially completed</td>
<td>NA / 0 Pre/post</td>
<td>NA / 0 Pre/post</td>
<td>0 / 4 Pre/post</td>
</tr>
</tbody>
</table>

It was found that only two students in Year 5/6 could answer any of the division problem-solving questions at the beginning of the initiative. For each individual question, only one student was able to correctly answer the question. Average success rates for the set of division problem-solving questions did not increase significantly for any cohort over the course of the initiative. Analysis of the post-test revealed that there was a small improvement for Questions 1 and 2 for the Year 3/4 cohort, however this is reflective of only one and two students respectively being able to correctly answer these questions. In Year 5/6, on the post-test four students were able to answer any division problem-solving questions. Whilst this is a positive improvement, it is still reflective of a small portion of this cohort that was able to answer any questions of this form.

Analysis of errors made on the set of division problem-solving questions at the beginning of the initiative demonstrates that the majority of errors made by the Year 3/4 cohort were students either not attempting the question or not being administered the question as it was too difficult. For the Year 4/5 cohort, the majority of the errors were students not attempting the question. Not attempting the question is associated with a productive disposition. Analysis of errors for the Year 5/6 cohort revealed that the number of questions that were not administered to this cohort significantly reduced over the course of the initiative. However, the frequency of students not attempting the question also significantly increased.
APPENDIX P: NEWMAN INTERVIEW FINDINGS

Year 3/4 Findings

Figure 32 outlines the frequency of each error type for the Year 3/4 class at the beginning of the initiative.

![Graph showing frequency of errors for Year 3/4 class](image)

*Note: “NA” means that the student did not answer any of the assigned problem-solving questions incorrectly, and was not administered the Newman interview for that operation.

Figure 33 outlines the frequency of particular errors on the multiplication and division post-test problem-solving questions.

![Graph showing frequency of errors for multiplication and division](image)

*Note: Majority of 3/4 cohort was not administered the Newman interview for division as they did not attempt any of the division test questions.
Year 4/5 Findings

Figure 3 outlines the frequency of each error type for the Year 4/5 class at the beginning of the initiative.

Figure 3: Frequency of each Newman interview error for Year 4/5, n=9, pre-initiative.

For Year 4/5, the frequency of particular errors on the division post-test problem-solving questions is outlined in Figure 35.

Figure 35: Frequency of each Newman interview error for Year 4/5 (division problem solving), n=9, post-initiative.
Year 5/6 Findings

Figure 36 outlines the frequency of each error type for the Year 5/6 class at the beginning of the initiative.

![Figure 36: Frequency of each Newman interview error for Year 5/6, n=12, pre-initiative.](image)

For Year 5/6, the frequency of particular errors on the division post-test problem-solving questions is outlined in Figure 37.

![Figure 37: Frequency of each Newman interview error for Year 5/6 (division problem solving), n=12, post-initiative.](image)