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Full Length Article

Exact approximations for skin friction coefficient and convective heat transfer coefficient for a class of power law fluids flow over a semi-infinite plate: Results from similarity solutions

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A B S T R A C T

In the present work, it is aimed to derive exact formulations for skin friction coefficient (C_f) and convective heat transfer coefficient (h) for power law fluids with $0.8 < n < 1.2$ flow over a semi-infinite plate and with the constant wall temperature boundary condition. Similarity analysis was employed to solve the problem for 410,000 cases with different power law indexes (n) and Prandtl numbers ($0.1 < Pr < 1000$). Finally, based on the acquired data from similarity solutions, exact approximations (with R-Square ~ 1) are proposed for C_f and also h. These exact formulations can be considered as replaces for the former ones in the literature for Newtonian fluids ($n = 1$).

1. Introduction

Since many fluids follow non-Newtonian behavior [1–3], it is worth to conduct researches which aim to extend studies on this class of fluids. Among many works in this field, some of them can be found in [4–10]. These works are mainly referred to similarity solution in which the method has been widely employed in the literature for solving boundary layer problems. It must be also cited that similarity procedure has proved itself efficient even for providing solutions for verity of complex fluid flows [11–14].

The goal of the present research is specialized to study a class of power law fluids ($0.8 < n < 1.2$) flow over a semi-infinite plate with constant wall temperature boundary condition in a detailed way.

Here, we mention that among the many models for non-Newtonian fluids, power law model is applicable mainly because of its simplicity, and it only approximately predicts the treatment of a real non-Newtonian fluid. As an example, if the power law index (n) is less than one, the power law predicts that the effective viscosity would decrease with the increase of shear rate indefinitely, which requires a fluid with infinite viscosity at rest and zero viscosity as the shear rate gets close to infinity. But a real fluid has both a minimum and a maximum effective viscosity that rely on the physical chemistry at the molecular level. Therefore, the power law stands as only a suitable description of fluid behavior across the range of shear rates to which the coefficients could be fitted to the experiments [3].

Beside the limitations of power law model, there are many fluids which follow a power law behavior with power law indexes close to unity. This class of fluids can simply include variety of suspensions (Ex. nanofluids (2016) [15]). Furthermore, the value of power law index equal to unity stands as an exception in the nature; so, many assumed Newtonian fluids (especially in higher temperatures) may have power law treatments with values of "n" close to 1 (see (2016) [16,17]). Therefore having specified information for this class of fluids can subsequently benefit us in exact engineering evaluations of this class of fluids.

Two main engineering factors (skin friction coefficient and convective heat transfer) have been targeted in this work. These factors are evaluated and formulated as functions of Reynolds number, Prandtl number and power law index. These formulations are already derived for Newtonian fluids ($n = 1$) in the same conditions; but the need for exact results in engineering problems led us to extend these formulations to include much wider class of fluids.

Using power law model, two classes of Non-Newtonian fluids can be relatively (but not completely) studied (pseudo-plastic and dilatant which is less common). This model simply defines the shear stress as:

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Nomenclature

Symbol	Description (unit)		
Re	Reynolds number	μ	dynamic viscosity (kg/m.s)
T_w	surface temperature (K)	ρ	density (kg/m^3)
υ	kinematic viscosity (m^2/.s)	n	power law index
Pr	Prandtl number	α	diffusivity factor
h	convective heat transfer coefficient (W/m^2.K)	C_f	skin friction coefficient

$$\tau = K\left|\left(\frac{\partial u}{\partial y}\right)\right|^{n-1}\left(\frac{\partial u}{\partial y}\right) \tag{1}$$

which K is the consistency factor and "n" is the power law index. Which n < 1 covers a range of pseudo-plastic fluids and n > 1 includes a class of dilatant fluids [3]. Substituting this model for predicting shear stress leads us to the following governing momentum equations [5]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\upsilon\left(\frac{\partial u}{\partial y}\right)^n\right) = \frac{\partial}{\partial y}\left(\upsilon\left|\left(\frac{\partial u}{\partial y}\right)\right|^{n-1}\frac{\partial u}{\partial y}\right) \tag{3}$$

Note that y-momentum equation can be simply ignored with respect to the order of magnitude.

Moreover using $\alpha_t = \alpha\left|\frac{\partial u}{\partial y}\right|^{n-1}$ as the thermal diffusivity for power law fluids [5], energy equation can be written as:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha\left|\left(\frac{\partial u}{\partial y}\right)\right|^{n-1}\frac{\partial T}{\partial y}\right) \tag{4}$$

Eqs. (2)–(4) are transformed into two ODEs by defining a similarity parameter. Among the classic methodologies of solving these ODEs (some of them are employed in [18]), Shooting Technique procedure with a simple forward discrete scheme was employed in the present research. After validating the code for the classic Blasius case, it has been applied for numerous values of 0.8 < n < 1.2 and Pr > 0.1 (410,000 cases of different values of "n" and "Pr" were solved in this research).

It is well-known that for Newtonian fluids (n = 1), $C_{fx} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$ and $h_x = 0.332\frac{k}{x}\mathrm{Re}_x^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}$; but there are not yet the same formulations (approximations) for power law fluids. So, power law forms of these equations are introduced in here for 0.8 < n < 1.2.

Applying the transformed ODEs for 410,000 cases of 0.8 < n < 1.2 and 0.1 < Pr < 1000 and using Least Square Regression Techniques, skin friction coefficient and convective heat transfer are accurately formulated (R-Square ∼ 1). As previously mentioned, these new formulations can be considered as replaces for evaluation of these two important factors in the literature.

2. Deriving the transformed ODEs for power law fluids

Using order of magnitude technique, the governing PDEs for power law fluids flow over a semi-infinite plate can be obtained as Eqs. (2) and (3):

In which $\upsilon = \frac{K}{\rho}$.

As the classic Blasius solution, here we assume that the x-component of velocity is self-similar. So, this component of velocity can be written as:

$$\frac{u}{U_0} = F(\eta) \tag{5}$$

By assuming similarity parameter as $\eta = ayx^b$ (a > 0), stream function can be obtained as follows:

$$\psi = \int u dy$$
$$\psi = \int U_0 F(\eta) dy \tag{6}$$
$$\psi = \int \frac{U_0 F(\eta)}{ax^b} d\eta = \frac{U_0}{ax^b} f(\eta) : f = f(\eta)$$

By definition of stream function, the continuity equation (Eq. (2)) will be automatically satisfied. Furthermore, the velocity components can be obtained as:

$$u = \frac{\partial \psi}{\partial y} = U_0 f' \tag{7}$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{1}{a}x^{-b}f(-bx^{-1}U_0) - U_0 x^{-1} ybf' \tag{8}$$

The derivatives of x-component velocity are then obtained as:

$$\frac{\partial u}{\partial x} = U_0 f''(aybx^{b-1}) \tag{9}$$

$$\frac{\partial u}{\partial y} = U_0 f'' ax^b \tag{10}$$

$$\frac{\partial^2 u}{\partial y^2} = U_0 a^2 x^{2b} f''' \tag{11}$$

Substituting Eqs. (7)–(11) into x-momentum equation (Eq. (3)) results in:

$$ff''\left(\frac{bU_0^2}{x}\right) = \upsilon na^2 U_0 x^{2b} f'''(U_0 f'' ax^b)^{n-1} \tag{12}$$

Eq. (11) can be re-written as:

$$-ff'' = f''' f''^{(n-1)}\left(-\frac{\upsilon na^{n+1}U_0^{n-2}x^{b(n+1)+1}}{b}\right) \tag{13}$$

The similarity solution exists if only $b(n+1)+1 = 0$.

So, $b = -\frac{1}{n+1}$. By this definition of b, the multiplying factor in the right-hand side of Eq. (13) would be a constant. For simplification, this constant can be assumed as to be unit in value. Therefore:

$$-\frac{\upsilon na^{n+1}U_0^{n-2}}{b} = 1 \tag{14}$$

Note that in the left hand side of Eq. (14), all the parameters except b, is positive; so the right hand side of Eq. (14) must be either positive. Moreover, selecting the right hand side of Eq. (14) equal to 1, is just a simplifying assumption. Therefore, mathematically, it may take any value arbitrarily.

From Eq. (14), the constant factor of "a" can be obtained as:

$$a = \left(\frac{U_0^{2-n}}{n(n+1)\upsilon}\right)^{\frac{1}{n+1}} \tag{15}$$

Finally the governing PDEs for continuity and momentum will be reduced into the following ODE:

$$f'''f''^{(n-1)} + ff'' = 0 \tag{16}$$

The above equation is a third order ODE; so it requires three conditions to be solved. These conditions are just as those for classic Blasius equation which are denoted as:

$$\begin{aligned} &f(0) = 0 \\ &f'(0) = 0 \\ &f'(\infty) = 1 \end{aligned} \tag{17}$$

Continuing the same procedure for Eq. (4) and with the definition of $\theta = \frac{T - T_\infty}{T_w - T_\infty}$, the final transformed ODE for energy equation can be obtained as:

$$\theta'' f''^{n-1} + \theta'((n-1)f''^{n-2} + n\Pr f) = 0 \tag{18}$$

In Eq. (18), $\Pr = \frac{\upsilon}{\alpha}$.

Since Eq. (18) is a second order ODE for θ, so it requires two thermal conditions. As the classic Blasius case, these conditions are:

$$\begin{aligned} &\theta(0) = 1 \\ &\theta(\infty) = 0 \end{aligned} \tag{19}$$

In the next section, the applied methodology for solving Eqs. (16) and (18) is presented.

3. Solving methodology

Eq. (16) can be converted into the algebraic form by a forward discrete scheme as follows:

Using Taylor expansion, the second and the third derivatives of similarity function can be defined as:

$$f''(i) = \frac{f(i+2) - 2f(i+1) + f(i)}{\Delta\eta^2} \tag{20}$$

$$f'''(i) = \frac{f(i+3) - 3f(i+2) + 3f(i+1) - f(i)}{\Delta\eta^3} \tag{21}$$

Substituting Eqs. (20) and (21) into Eq. (16) results in the final algebraic form of:

$$f(i+3) = \frac{-f(i)f''(i)}{f''(i)^{n-1}}\Delta\eta^3 + 3f(i+2) - 3f(i+1) + f(i) \tag{22}$$

Coming to the boundary conditions, we have $f(0)$ known; so in the algebraic form $f(i=1) = 0$. Also, $f'(0) = 0$ yields that $f(i=2) = 0$. For starting the numerical procedure, the value of $f(i=3)$ must be known as well. Here we applied Shooting Technique to reach the exact value of $f(i=3)$. In this research $\eta = \infty$ was set as $\eta = 10$ (which is most applicable for similarity solutions of Newtonian fluids).

Using 1000 points in the numerical procedure yields that:

$$\Delta\eta = \frac{10}{1000 - 1} \approx 0.01 \tag{23}$$

As for the terminal boundary condition we have $f'(n) = 1$, in the numerical procedure, the error relation can be simply defined as:

$$Error = |(f(n) - f(n-1) - \Delta\eta)| \tag{24}$$

Therefore, setting the above relation to be less than 10^{-5} as the breaking condition in the numerical procedure, corresponds to this that $f'(n) = 0.999$ is assumed as a favorable correspondence of $f'(n) = 1$. This means that the absolute error has been set to be 10^{-3}.

In the present work, an iterative procedure has been done in order to find the best guess of $f(i=3)$ which meets the condition of the above-defined error. Based on the outcome numerical results (which are based on applying the forward discrete scheme, 1000 points and the above-mentioned error scheme), this simple and user-friendly developed code is highly accurate for all the values of 0.8 < n < 1.2. This code is provided in the appendix section of this paper.

When, the solutions are reached for Eq. (16), the transformed ODE for energy equation can be re-written as:

$$\theta(\eta) = \int_0^\eta \theta'(0) e^{\int_0^\eta -\frac{((n-1)f''^{n-2} + n\Pr f)}{f''^{n-1}} d\eta} d\eta + 1 \tag{25}$$

In which $\theta'(0)$ is expressed as:

$$\theta'(0) = -\frac{1}{\int_0^\infty e^{\int_0^\eta -\frac{((n-1)f''^{n-2} + n\Pr f)}{f''^{n-1}} d\eta} d\eta} \tag{26}$$

Eqs. (25) and (26) can be simply solved by a numerical integrating procedure.

In this research, the variation of power law index was set to be between 0.8 and 1.2 with the change step of 0.01. Also, Prandtl number was changed between 0.1 and 1000 with the change step of 0.1. This means that the final formulations are based on the results of 410,000 similarity solutions.

Formulating Skin Friction Coefficient and Convective Heat Transfer Coefficient for 0.8 < n < 1.2

According to the previous discussions, the variation of power law index has been assumed to be between 0.8 and 1.2. The developed code has been first validated for the classic Blasius case (Newtonian fluids). In this case, $\eta|_{0.99}$ was calculated to be 3.5207. Because of using a different similarity constant (compared to the classic Blasius solution), this value should be multiplied by $\sqrt{2}$ in order to be corresponded to the former Blasius results. So, $\sqrt{2} \times 3.5207$ falls into about 0.42% error with the historical value of 5 as $\eta|_{0.99}$. This validation is shown in Fig. 1. Some results for the solutions of Eq. (16) for 0.8 < n < 1.2 are shown in Fig. 2. It can be understood from Fig. 2 that by having lower values of "n" as the power law index, the second derivative of similarity function gets closer to zero and besides, $\eta|_{0.99}$ rises to greater numbers.

Here it is worth to note that as the power law index of "n" gets closer to zero, the parameter of "a" goes to infinity (see Eq. (15)). Infinite values of "a" simply makes infinite values of the similarity parameter as well. Which means the assumption of self-similarity would be invalid. Besides by having very small values of "n", the fluid tends to express an invisid behavior with no shear stress on the wall. On the other hand, as the similarity parameter goes to infinite values for $n \sim 0$,the assumption of $\eta = 10$ as a correspondence of $\eta \sim \infty$ will be no longer valid. And so, the terminal boundary layer condition must be corresponded to very large values of η. This means that there will be no exact numerical solutions for $n \sim 0$. As there is neither numerical nor analytical solution in this case, one cannot proceed in solving the governing ODEs in this region. Fig. 3 indicates a validation for temperature distribution in different Prandtl numbers. According to Fig. 3, $\eta|_{0.99}$ was found to be 3.506 for Pr = 1 and n = 1. This means that $\frac{\delta}{\delta_t} \approx 1$; which was already expected according to the former approximation for Newtonian fluids ($\frac{\delta}{\delta_t} \approx \Pr^{\frac{1}{3}}$).

Skin friction coefficient can be defined as:

$$C_f(x) = \frac{\tau_w}{\frac{1}{2}\rho U_0^2} \tag{27}$$

Using the following definition for Generalized Reynolds number,

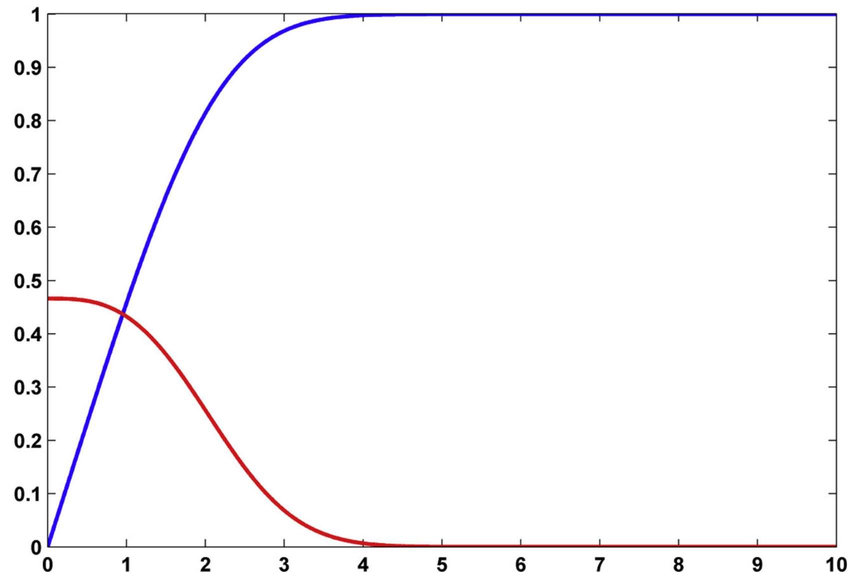

Fig. 1. Distribution of $f'(\eta)$ and $f''(\eta)$ as functions of η for Newtonian case (n = 1). Red line is for $f''(\eta)$ and blue line is for $f'(\eta)$: A zoomed view.

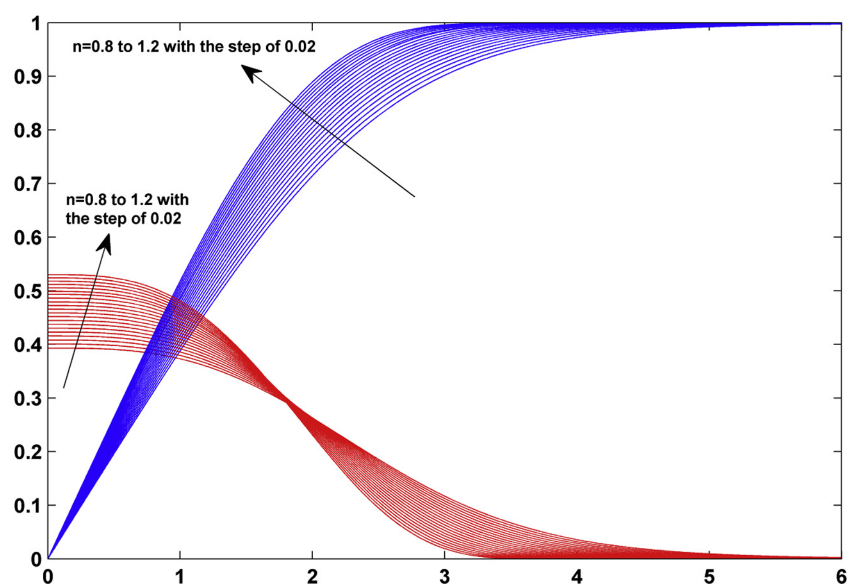

Fig. 2. Distribution of $f'\eta$ and $f''\eta$ as functions of η for 0.8 < n < 1.2. X-axis is η. Red line is for $f''(\eta)$ and blue line is for $f'(\eta)$: A zoomed- in view.

$$\mathrm{Re}(x) = \frac{\rho x^n U_0^{2-n}}{K} \quad (28)$$

And with assistance of Eq. (10), one can reach the following form for skin friction coefficient:

$$C_{fx} = 2[n(n+1)]^{\frac{-n}{n+1}}(f''|_{\eta=0})\mathrm{Re}_x^{\frac{-1}{n+1}} \quad (29)$$

Therefore, it is seen that if $f''|_{\eta=0}$ can be written as a function of "n", Eq. (29) will become an explicit formulation as a function of known parameters.

Reminding the previous discussions, we know that for $n \sim 0$ the fluid tends to be invisid with no shear rate. Therefore, assuming $f''|_{\eta=0}$ as to be in the form of an^b, has its physical correspondence.

Using Least Square regression, it was found that $f''|_{\eta=0}$ can be written as:

$$f''|_{\eta=0} = 0.469n^{0.739} \quad (30)$$

R-Square for Eq. (30) was calculated to be 0.9999.

The validation for Eq. (30) is provided in Fig. 4.

Coming to convective heat transfer coefficient, subject to thermal boundary layer (constant wall temperature condition) we have:

$$\frac{q}{A} = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = h_x(T_w - T_\infty) \quad (31)$$

In which, "k" is thermal conductivity. In the non-dimensional form of Eq. (31), convective heat transfer coefficient can be written as:

$$h_x = -k\frac{\partial \theta}{\partial y}\bigg|_{y=0} \quad (32)$$

Or

$$h_x = -ka\theta'|_{\eta=0}x^b \quad (33)$$

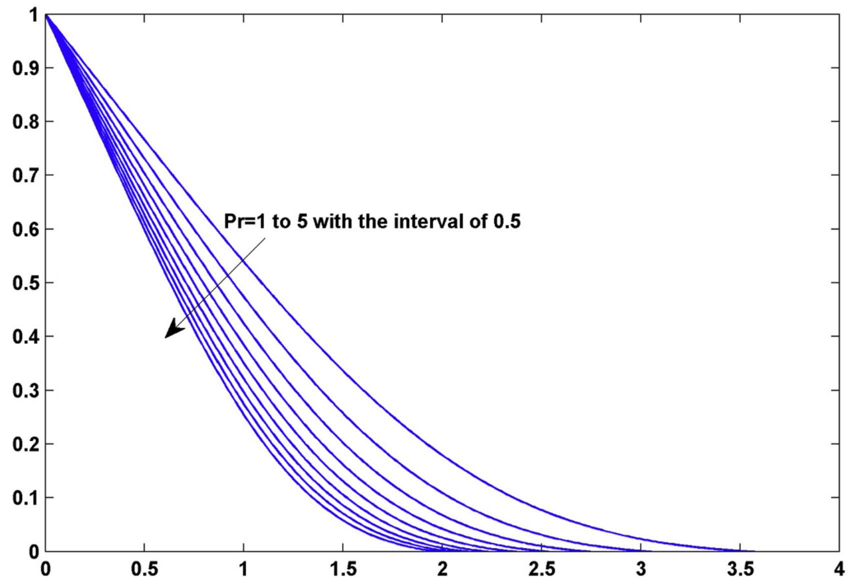

Fig. 3. Distribution of $\theta(\eta)$ as a function of η for n = 1 X-axis is η and Y-axis is. θ (η). A zoomed-in view.

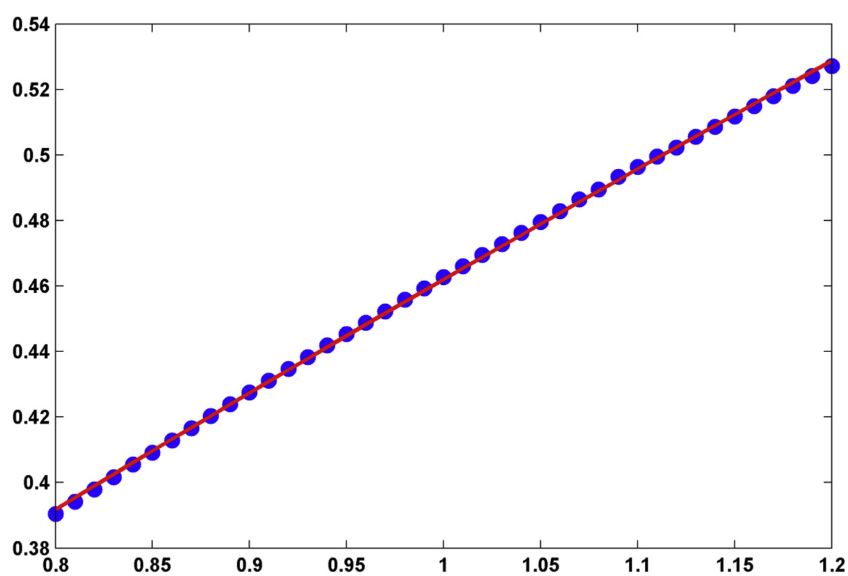

Fig. 4. $f''|_{\eta=0}$ (Y-axis) as a function of "n" (X-axis). Blue dots are from the similarity solutions and Red line is the correlation (Eq. (30)).

In which, "a" and "b" are the previous-defined factors of similarity parameter. So, from Eq. (33), if $\theta'|_{\eta=0}$ can be found as a function of power law index and Prandtl number, convective heat transfer coefficient will be an explicit function of known parameters. Some results of similarity solutions for $\theta'|_{\eta=0}$ as a function of Prandtl number and power law index are shown in Fig. 5. It seems that in certain values of "n", $\theta'|_{\eta=0}$ follows a power form of Prandtl. Also, by increasing "n", the values of $\theta'|_{\eta=0}$, decreases for all the Prandtl numbers.Similar to the previous procedure used for formulating skin friction coefficient, here we assumed the following form for formulating $\theta'|_{\eta=0}$:

$$\theta'|_{\eta=0} = an^b \mathrm{Pr}^{cn^d+e} \tag{34}$$

Using the data from all the studied cases, the constant factors of Eq. (34) obtained and finally $\theta'|_{\eta=0}$ formulated as:

$$\theta'|_{\eta=0} = (-0.47n^2)\mathrm{Pr}^{-0.102n^{1.29}+0.432} \tag{35}$$

In Eq. (35), "n" is the power law index. This equation falls within about 0.001% error with the acquired data On the other hand, R-Square for Eq. (35) was the same as for Eq. (30).

Now, using the definition of generalized Reynolds number, the final form of convective heat transfer coefficient can be achieved as:

$$h_x = 0.47n^2(n(n+1))^{-\frac{1}{n+1}}\frac{k}{x}\mathrm{Re}_x^{\frac{1}{n+1}}\mathrm{Pr}^{-0.102n^{1.29}+0.432} \tag{36}$$

Eq. (36) can be simply considered as a replacement for the former formulation of $h_x = 0.332\frac{k}{x}\mathrm{Re}_x^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}$. Note that for n = 1, Eq. (36) is accurately the same as $h_x = 0.332\frac{k}{x}\mathrm{Re}_x^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}$.

4. Summarizing the new results obtained by the present research

For steady state, two dimensional and incompressible power law fluids ($0.8 \leqslant n \leqslant 1.2$) flow over a semi-infinite plate with constant temperature boundary condition ($0.1 \leqslant \mathrm{Pr} \leqslant 1000$), we have:

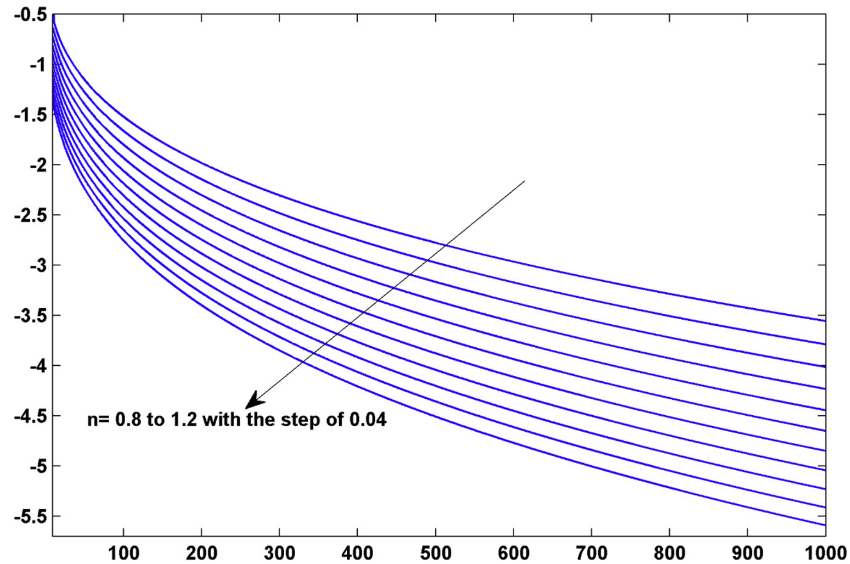

Fig. 5. $\theta'|_{\eta=0}$ (Y-axis) vs. "Pr" (X-axis) in different power law indexes.

$$C_{fx} = 2[n(n+1)]^{\frac{-n}{n+1}}(0.469n^{0.739})^n \mathrm{Re}_x^{\frac{-1}{n+1}} \quad (\textbf{R-Square} \sim \mathbf{1}) \tag{37}$$

The average skin friction coefficient can be subsequently obtained as:

$$C_f = 2(n+1)[n(n+1)]^{\frac{-n}{n+1}}(0.469n^{0.739})^n \mathrm{Re}^{\frac{-1}{n+1}} \tag{38}$$

And,

$$h_x = 0.47n^2(n(n+1))^{-\frac{1}{n+1}}\frac{k}{x}\mathrm{Re}_x^{\frac{1}{n+1}}\mathrm{Pr}^{-0.102n^{1.29}+0.432} \quad (\textbf{R-Square} \sim \mathbf{1}) \tag{39}$$

The average convective heat transfer coefficient can be subsequently obtained as:

$$h = 0.47(n(n+1))^{\frac{n}{n+1}}\frac{k}{L}\mathrm{Re}^{\frac{1}{n+1}}\mathrm{Pr}^{-0.102n^{1.29}+0.432} \tag{40}$$

In which, $\mathbf{Pr} = \frac{\upsilon}{\alpha}$ (α is the constant in definition of thermal diffusivity for power law fluids).

$\alpha_t = \alpha\left(\frac{\partial u}{\partial y}\right)^{n-1}$ (α_t is the thermal diffusivity for power law fluids).

$\upsilon = \frac{K}{\rho}$ (K is the consistency factor for power law fluids).

Table 1
Deviations of skin friction coefficient in different power law indexes and Reynolds numbers from Newtonian fluid.

Power Law Index	Reynolds Number			
	10 Error%	100 Error%	1000 Error%	10000 Error%
0.8	*2.884*	*14.5454*	*24.8066*	*33.8356*
0.85	*2.1008*	*10.8259*	*18.7734*	*26.0126*
0.9	*1.3708*	*7.1697*	*12.6276*	*17.7647*
0.95	*0.6758*	*3.565*	*6.3702*	*9.0938*
1	*0*	*0*	*0*	*0*
1.05	*0.6706*	*3.5375*	*6.4861*	*9.5186*
1.1	*1.3481*	*7.0595*	*13.0928*	*19.466*
1.15	*2.0434*	*10.578*	*19.8264*	*29.8483*
1.2	*2.7659*	*14.1047*	*26.6946*	*40.6736*

Table 2
Deviations of convective heat transfer coefficient in different power law indexes, Reynolds numbers and Prandtl = 1 from Newtonian fluid.

Power Law Index	Reynolds Number (Prandtl Number = 1)			
	10 Error%	100 Error%	1000 Error%	10000 Error%
0.8	*5.5017*	*7.3939*	*22.0492*	*38.7045*
0.85	*4.4235*	*4.928*	*15.1946*	*26.4657*
0.9	*3.1349*	*2.9161*	*9.345*	*16.1756*
0.95	*1.6551*	*1.2913*	*4.3261*	*7.4517*
1	*0*	*0*	*0*	*0*
1.05	*1.8173*	*1.002*	*3.7432*	*6.4085*
1.1	*3.786*	*1.7507*	*6.9921*	*11.9539*
1.15	*5.897*	*2.2763*	*9.8188*	*16.7791*
1.2	*8.1425*	*2.6039*	*12.2823*	*20.999*

Table 3
Deviations of convective heat transfer coefficient in different power law indexes, Reynolds numbers and Prandtl = 10 from Newtonian fluid.

Power Law Index	Reynolds Number (Prandtl Number = 10)			
	10 Error%	100 Error%	1000 Error%	10000 Error%
0.8	*0.216*	*13.8919*	*29.434*	*47.097*
0.85	*0.0823*	*9.694*	*20.4269*	*32.2099*
0.9	*0.2002*	*6.0341*	*12.6578*	*19.6953*
0.95	*0.1651*	*2.826*	*5.9067*	*9.0797*
1	*0*	*0*	*0*	*0*
1.05	*0.2757*	*2.5009*	*5.2007*	*7.8256*
1.1	*0.6455*	*4.7237*	*9.8065*	*14.6181*
1.15	*1.0954*	*6.7073*	*13.9077*	*20.5525*
1.2	*1.6133*	*8.4842*	*17.5783*	*25.7688*

Table 4
Deviations of convective heat transfer coefficient in different power law indexes, Reynolds numbers and Prandtl = 100 from Newtonian fluid.

Power Law Index	Reynolds Number (Prandtl Number = 100)			
	10 Error%	100 Error%	1000 Error%	10000 Error%
0.8	*6.2797*	*20.783*	*37.2655*	*55.9973*
0.85	*4.4561*	*14.6765*	*25.8969*	*38.2151*
0.9	*2.8235*	*9.2467*	*16.0711*	*23.3218*
0.95	*1.3476*	*4.384*	*7.5113*	*10.7324*
1	*0*	*0*	*0*	*0*
1.05	*1.2426*	*3.9772*	*6.6361*	*9.2213*
1.1	*2.4*	*7.6067*	*12.5357*	*17.2017*
1.15	*3.4884*	*10.9373*	*17.8113*	*24.1547*
1.2	*4.5217*	*14.0096*	*22.5546*	*30.2505*

Table 5
Deviations of convective heat transfer coefficient in different power law indexes, Reynolds numbers and Prandtl = 1000 from Newtonian fluid.

Power Law Index	Reynolds Number (Prandtl Number = 1000)			
	10 Error%	100 Error%	1000 Error%	10000 Error%
0.8	*12.7103*	*28.0912*	*45.571*	*65.4361*
0.85	*9.2007*	*19.8853*	*31.6153*	*44.493*
0.9	*5.9387*	*12.5565*	*19.5877*	*27.0581*
0.95	*2.8831*	*5.9655*	*9.1403*	*12.4102*
1	*0*	*0*	*0*	*0*
1.05	*2.738*	*5.4311*	*8.0497*	*10.5958*
1.1	*5.3533*	*10.4025*	*15.1823*	*19.7071*
1.15	*7.8644*	*14.9755*	*21.5378*	*27.5937*
1.2	*10.2863*	*19.2013*	*27.2305*	*34.4617*

Although the deviation of power law index was at the most 0.2 from Newtonian fluids, but according to Eqs. (38) and (40), the deviations of skin friction coefficient and also convective heat transfer coefficient from those of Newtonian fluids were highly considerable especially in higher Reynolds and Prandtl numbers. These deviations are calculated for some power law indexes, Reynolds and Prandtl numbers and collected in Tables 1–5.

5. Conclusions

Skin friction coefficient and convective heat transfer coefficient are formulated based on the acquired data from 410,000 analytical solutions for power law fluids with the power law indexes between 0.8 and 1.2. As expected, these formulations stand as functions of power law index (n), Reynolds and Prandtl numbers and fall within about 0% error with the data of similarity solutions. This research was done because there are many fluids that have a power law behavior with the power law indexes close to unity and consequently, we need exact results for related engineering problems. By this research, it is shown that even slight changes in the power law index, may result in significant deviations in C_f and h from Newtonian assumption of fluids. For the best of your knowledge, these formulations have not yet been introduced in the literature and can be simply considered as replacements for former ones which are currently available for Newtonian fluids.

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