Thermomechanical Analysis of Compact High-Performance Electric Swashplate Compressor

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Abstract

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By Mohammad Arqam

This PhD is sponsored by industry and is part of a project to develop and manufacture smart electric compressor for mobile refrigeration and air conditioning applications on commercial and heavy vehicles including industrial machinery. Compact electric compressors are of great value for the future due to the growth of the electric vehicle market. Recent advancements in the field of mobile air conditioning and refrigeration have witnessed extensive use of the swashplate compressor due to its compact structure, continuous operation, small size, light weight and better thermal comfort inside the vehicle. The design of the swashplate compressor is complex so that it requires considerable contributions from different fields of engineering viz. engineering mechanics, heat transfer and fluid dynamics. The estimate of compressor performance through modelling and experiments at the early stages of design and development serves as a useful tool for the designer. The input power, torque, in-cylinder gas pressure and temperature, flow through valves, and volumetric efficiency are important parameters to characterize the compressor performance. In this thesis, a set of practical thermomechanical models are derived and validated against experiments.

An ideal gas based analytical model is developed for a 10 cylinder swashplate compressor with a view to predict its performance in terms of shaft torque and mass flow rate for a given rotational speed requiring minimal computational effort to run. Three sub-models are developed to account for the piston and swashplate kinematics and dynamics through deriving expressions for piston displacement as an explicit function of angle of rotation of swashplate and interactions between forces and moments. The compression process model is formulated to predict in-cylinder temperatures and pressures during one revolution of the swashplate together with refrigerant mass flow rate in and out of the compressor. A complete time-varying model is then developed by combining above three sub-models. Results are obtained in terms of compressor torque and volumetric efficiency and agree well with experiments.
Considering the importance of refrigerant flow through reed valves affecting compressor performance, a real-gas, restricted-flow valve model is also developed and compared with the ideal-gas, ideal-valve model. Real gas properties of R134a are evaluated using NIST standard reference database. A minor-loss discharge coefficient approach is used to determine the refrigerant flow rate through reed valves. The model predicts the discharge temperature, refrigerant mass flow rate and volumetric efficiency accurately as a function of rotational speed. The effect of real gas properties, heat transfer to and from the compressor wall during compression and expansion and the valve model are analyzed. The suction side valve model is found to have the largest influence on the compressor performance as a function of rpm whereas heat transfer model has the least. The key contribution of this study is in assembling a practical combination of models that is capable of capturing the essential physics without being overly complex. To the authors’ knowledge this is the first swashplate study that shows clearly the cyclic variation in thermo-physical properties.

The literature shows the dynamic characteristics of the compressor are well connected with the start-up transients of the swashplate mechanism and the suction and discharge pressures. To evaluate this, an experimentally validated transient swashplate compressor model is developed including mass moment of inertia of the pistons and swashplate to evaluate the motor torque loading during compressor start-up. The effects of essential parameters such as moment of inertia, bearing torque, viscous resistance to the piston motion, suction and discharge pressures on the compressor performance are presented. The actual start-up behavior is tracked using a high-speed data logger capturing phase currents for the BLDC motor, instantaneous power and rotational speed. The suction and discharge pressures are found to have the largest influence on the starting torque whereas rotational mass moment of inertia has the least. The original contribution of this work is in deriving a transient swashplate compressor model that includes the mass moment of inertia of the swashplate mechanism and clarifying the relative importance of line pressures, viscous losses and bearing resistance on the start-up torque.

Since minimizing the size of the compact Brushless DC (BLDC) motor driving the compressor is important, it is worth optimizing the cooling performance of the electric motor. An experimentally validated computational fluid dynamics (CFD) model is developed to investigate the thermal performance of an air-cooled Brushless Direct Current (BLDC) motor driving swashplate compressor. Different fin arrangements on the motor housing are analysed including small protrusions on the fin surface. The findings show greater enhancements can be achieved by adding an extra fin in the cooling flow passage rather than through the inclusion
of grooved walls. Thermographs of the motor housing are found to be in close agreement with the model predictions. The key achievement of this thermal investigation is in demonstrating air-cooling is a practical and effective alternative to refrigerant cooling of compact high performance electric swashplate compressors for mobile refrigeration and air conditioning applications.

The effect of thermal resistance between the windings and stator core of an air-cooled Brushless DC motor is also investigated. Measurements are found to be in close agreement with predictions. The numerical simulations suggest significant benefits of injecting encapsulation material in the stator core to enhance heat transmission from windings to the surrounding electrical steel. To confirm this, an experimental investigation is carried out by adding thermal resin to the winding slots on 2.5 kW and 4 kW brushless DC motors. The findings show that the potting material can reduce the temperature of the windings by 10 °C to 20 °C for electrical power inputs of 2.4 kW to 3.8 kW. The winding temperature is also found to be sensitive to the winding arrangement in the stator slot. With tighter, more compact windings also leading to significant temperature reductions.
Statement of Originality

This work has not been previously submitted for a degree or diploma in any university. To the best of author’s knowledge and belief, this thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

Signed: ------------------------

10/08/2021
Date: ------------------------

Mohammad Arqam
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List of Publications

The following publications have so far been produced as a result of the present research.

Journal Papers:


Confidential Industry Reports:


Conferences:


This thesis is dedicated to my beloved parents for their unconditional sacrifice, endless support, and love.
Acknowledgements

*In the name of Almighty, the Compassionate, the Merciful*

Praise be to *Allah* (S.W.T.) and his blessings be on *Prophet Muhammad* (peace be upon him), his progeny and Companions. First and foremost, I would like to acknowledge my indebtedness and render my warmest thanks to my supervisor, Dr. Peter Woodfield, for the continuous support of my research, for his patience, motivation, enthusiasm and immense knowledge. His friendly guidance and expert advice has been invaluable throughout all stages of the work. I have been extremely lucky to have a supervisor who cared so much about my work and responded to my questions and queries so promptly as and when occasioned. I also wish to express my gratitude to my associate supervisor, Prof. Dzung Dao for extended discussions and valuable suggestions which have contributed greatly to the improvement of the project. The source of inspiration for me is my ideal and role model, a legend who is a versatile man of character and qualities, an exemplary and unique identity, my father Prof. Dr. Kr. Farahim Khan (d.b).

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Lastly, I would love to express my heartfelt gratitude to peers and family for their extended support, motivation and guidance.

Mohammad Arqam

10/08/2021

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<tbody>
<tr>
<td>BLDC</td>
<td>Brushless Direct Current</td>
</tr>
<tr>
<td>VCR</td>
<td>Vapor Compression Refrigeration</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulation</td>
</tr>
<tr>
<td>TDC</td>
<td>Top Dead Centre</td>
</tr>
<tr>
<td>BDC</td>
<td>Bottom Dead Centre</td>
</tr>
<tr>
<td>COP</td>
<td>Coefficient of Performance</td>
</tr>
<tr>
<td>BT</td>
<td>Bearing Torque</td>
</tr>
<tr>
<td>MOI</td>
<td>Moment of Inertia</td>
</tr>
<tr>
<td>VR</td>
<td>Viscous Resistance to Piston Motion</td>
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</table>
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit(s)</th>
</tr>
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<tbody>
<tr>
<td>$A_{cyl}$</td>
<td>area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_d$</td>
<td>nominal flow area of discharge valve</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_{cyl}$</td>
<td>area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A$</td>
<td>cylinder area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$a_i$</td>
<td>acceleration of piston $i$</td>
<td>m.s$^{-2}$</td>
</tr>
<tr>
<td>$C_d$</td>
<td>discharge valve flow coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$C_s$</td>
<td>suction valve flow coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure</td>
<td>J.kg$^{-1}$K$^{-1}$</td>
</tr>
<tr>
<td>$C$</td>
<td>compressor</td>
<td>-</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter of the cylinder</td>
<td>m</td>
</tr>
<tr>
<td>$h_{coeff}$</td>
<td>heat transfer coefficient</td>
<td>W.m$^{-2}$K$^{-1}$</td>
</tr>
<tr>
<td>$h_d$</td>
<td>enthalpy per unit mass at compressor discharge</td>
<td>J.kg$^{-1}$</td>
</tr>
<tr>
<td>$h_s$</td>
<td>enthalpy per unit mass at compressor suction</td>
<td>J.kg$^{-1}$</td>
</tr>
<tr>
<td>$I_z$</td>
<td>rotational mass moment of inertia of swashplate</td>
<td>kg.m$^2$</td>
</tr>
<tr>
<td>$I_{eff}$</td>
<td>effective moment of inertia</td>
<td>kg.m$^2$</td>
</tr>
<tr>
<td>$k$</td>
<td>specific heat ratio</td>
<td>-</td>
</tr>
<tr>
<td>$L$</td>
<td>length</td>
<td>m</td>
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<tr>
<td>$L$</td>
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</tr>
<tr>
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<td>mass of piston</td>
<td>kg</td>
</tr>
<tr>
<td>$M$</td>
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<td>kg</td>
</tr>
<tr>
<td>$M$</td>
<td>motor</td>
<td>-</td>
</tr>
<tr>
<td>$\dot{m}_{out}$</td>
<td>mass flow rate out</td>
<td>kg.s$^{-1}$</td>
</tr>
<tr>
<td>$\dot{m}_{in}$</td>
<td>mass flow rate in</td>
<td>kg.s$^{-1}$</td>
</tr>
<tr>
<td>$n$</td>
<td>number of cylinders</td>
<td>-</td>
</tr>
<tr>
<td>$N$</td>
<td>revolutions per minute</td>
<td>rev.min$^{-1}$</td>
</tr>
<tr>
<td>$P_d$</td>
<td>discharge line pressure</td>
<td>Pa</td>
</tr>
</tbody>
</table>
$P_s$ suction line pressure \hspace{2cm} \text{Pa}

$P_{\text{high}}$ saturation pressure high \hspace{2cm} \text{Pa}

$P_{\text{low}}$ saturation pressure low \hspace{2cm} \text{Pa}

$P_{\text{initial}}$ normalized cylinder pressure \hspace{2cm} \text{Pa}

$P_1$ pressure in cylinder 1 \hspace{2cm} \text{Pa}

$P_2$ pressure in cylinder 2 \hspace{2cm} \text{Pa}

$Q_{\text{high}}$ heat released at condenser \hspace{2cm} \text{W}

$Q_{\text{low}}$ heat received at evaporator \hspace{2cm} \text{W}

$\dot{Q}_{\text{motor}}$ motor heat \hspace{2cm} \text{W}

$R$ radius of swashplate \hspace{2cm} \text{m}

$T_{\text{suc}}$ suction line temperature \hspace{2cm} \text{°C}

$T_{\text{wall}}$ cylinder inside wall temperature \hspace{2cm} \text{°C}

$T_{\text{sat, L}}$ suction line temperature \hspace{2cm} \text{°C}

$T_{\text{sat, H}}$ discharge line temperature \hspace{2cm} \text{°C}

$T_s$ solid temperature \hspace{2cm} \text{°C}

$T_f$ fluid temperature \hspace{2cm} \text{°C}

$u$ internal energy per unit mass \hspace{2cm} \text{Jkg}^{-1}

$V_{\text{suction}}$ suction volume \hspace{2cm} \text{m}^3

$V_{\text{swept}}$ swept volume \hspace{2cm} \text{m}^3

$Vol_{\text{min}}$ minimum volume in cylinder \hspace{2cm} \text{m}^3

$Vol_{\text{prev}}$ previous volume in cylinder \hspace{2cm} \text{m}^3

$Vol_{\text{max}}$ maximum volume in cylinder \hspace{2cm} \text{m}^3

$v_i$ velocity of piston $i$ \hspace{2cm} \text{m.s}^{-1}

$W_{\text{shaft}}$ compressor shaft work \hspace{2cm} \text{W}

$\dot{W}_{\text{electric}}$ electric motor power \hspace{2cm} \text{W}

$\dot{W}_{\text{motor}}$ motor electric power \hspace{2cm} \text{W}

$Z_i$ displacement of piston $i$ \hspace{2cm} \text{m}

$Z$ compressibility \hspace{2cm} -
dt \quad \text{time step} \quad \text{s}

\textbf{Greek}

\theta \quad \text{swashplate angle} \quad \text{rad}

\phi \quad \text{angular position of cylinder} \quad \text{rad}

\rho \quad \text{density} \quad \text{kgm}^{-3}

\tau \quad \text{torque} \quad \text{N.m}

\tau_m \quad \text{motor torque} \quad \text{N.m}

\omega \quad \text{swashplate angular velocity} \quad \text{rad.s}^{-1}

\alpha \quad \text{angular acceleration} \quad \text{rad.s}^{-2}

\delta \quad \text{gap between piston wall and cylinder} \quad \text{m}

\mu \quad \text{dynamic viscosity of lubricant} \quad \text{Pa.s}

\textbf{Subscription}

ac \quad \text{Air-cooled} \quad -

rc \quad \text{Refrigerant cooled} \quad -

ref \quad \text{refrigerant} \quad -

cond \quad \text{condenser} \quad -

evap \quad \text{evaporator} \quad -
Chapter 1

Introduction

This dissertation presents an investigation of electric swashplate compressor to be used on commercial and heavy vehicles including industrial machinery for mobile refrigeration and air-conditioning applications. Electric compressors are being considered as the strong contender for mobile refrigeration in the past few decades as they have several advantages over conventional belt-driven compressors. In this dissertation, an extensive literature review is carried out to study existing numerical models for capturing the thermodynamic behaviour of the compressor during steady and un-steady operation. In the light of the literature, research gaps concerning the need of developing practical and accurate analytical/mathematical models have been identified. To address the gaps, three analytical and two numerical models have been developed to capture the essential physics, start-up characteristics and the overall thermal management of the electric drive. Also, experiments were conducted to provide data to validate the models.

1.1. Background and motivation

Compact electric compressors for mobile applications have several advantages over conventional compressors and are the way of the future for mobile air conditioning and refrigeration. This is because combustion engine vehicles globally are being phased out and replaced by electric vehicles. One current target is that “20% of all road vehicles globally are to be electrically driven as of 2030” (Paris2015 UN Climate Change Conf.). Belt-driven compressors, which are attached to internal combustion engines in conventional vehicles, are not suitable for electric vehicles. As a result of these motivations, the interest in high performance compact electric compressors has increased greatly in recent years due to their small size, light weight and better thermal comfort inside vehicle cabin [1, 2]. Relative to conventional belt-driven compressors, electric compressors are comparatively easy to maintain, can be installed as a compact system and claim low energy consumption [3]. Small, lightweight and high efficiency dc compressors have been developed [4]. Results show that the performance of electric compressors is generally better than that of a conventional compressor driven by a belt connected to the engine.
This project is concerned with developing thermomechanical models for a compact electric swashplate compressor that could be used for refrigeration on commercial and heavy vehicles including industrial machinery. Current compressors for electric vehicles are usually designed for high rotational speed which reduces weight by reducing their swept volumes and thus the car electric power consumption [5]. For developing mathematical/numerical models, many factors need to be considered that have also been the research focus for development of electric motors. These include size, power requirements, output torque, choice of motor design/type and motor cooling strategy. Some types of electric motors such as switched reluctance and induction motors are very well known to maintain their operational characteristics in a wide range of load [6]. Brushless DC motors are the most popular choice [1].

Overheating is one of the key factors limiting the output performance of electric motors [7-11] and therefore is important for high-power electric compressors. The performance of electric motors can be enhanced through the reduction of losses which may cause excessive heat generation that need to be dissipated to the surrounding environment [7]. The losses are of different types; copper losses, iron losses, eddy current losses, mechanical losses because of friction and some other losses due to manufacturing inaccuracy and component assembly. The copper losses contribute around 50% in the total energy loss while others are contributing 15% (core and mechanical) and 20% (additional losses) respectively [8]. The losses are solely responsible for a temperature rise inside the motor and hence lead to considerable thermal load on different components of the motor. Therefore, an important part of this research was designing an efficient cooling jacket. Ultimately, improving the flow path for heat removal will inevitably lead to higher performances.

For state-of-the-art refrigeration circuits, it is desirable to have compressors with adjustable refrigerant flow rates and low refrigerant leakage [12]. The swashplate electric compressor considered here is provided with a power converter that allows it to work at different speeds with the aid of improving overall refrigeration and air conditioning performance. The key feature of this compressor is, there is a lower risk of refrigerant leakage which will eventually become a considerable factor while dealing with the global warming perspective of the refrigerants (according to ASHRAE) used in car refrigeration and air conditioning application.
1.2. Target product and application

Fig. 1-1 shows the design of the electric swashplate compressor. The design is such that a 10 cylinder swashplate compressor is coupled to a 600V Brushless DC current (BLDC) motor. An electronic controller sits at the top of the unit to control over the current supplied to the motor and hence the torque. The design shown in Fig. 1-1 is part of a project “develop and manufacture smart electric compressor for mobile refrigeration on heavy electric vehicle” co-funded by the Innovative Manufacturing CRC (IMCRC) and SuperCool Asia Pacific (this PhD’s industry sponsor) in collaboration with Griffith University. The product is currently being tested and ready to hit the market as early as end of 2021.

Figure 1-1 Electric Swash-plate compressor for mobile refrigeration currently under development at Supercool (This PhD’s industry sponsor) in collaboration with Griffith University

1.3. Problem statement

This industry-sponsored PhD is concerned with developing computational heat transfer and fluid dynamics models for a smart high-powered compact electric compressor that is being developed through collaboration between Griffith University and a refrigeration technology company, SuperCool Asia Pacific. The thrust of this PhD was to develop computationally efficient, practical analytical and numerical models to account for the compressor mechanics, thermodynamics, and transient characteristics during start-up and cooling performance. The research also includes experiments to compare against predictions.
1.4. Research gaps and objectives

Based on the research background and detailed literature review which will be presented in chapter two, some key research gaps are identified. It is evident from the literature that thermodynamics models of swashplate compressor exist but need more development. Literature prior to this thesis showed a need to develop an analytical model to evaluate motor torque loading during compressor start-up under different operating conditions. Also, design and development of electric swashplate compressors for air-conditioning in commercial and heavy vehicles including industrial machinery were unexplored. How do such devices perform in terms of cooling and other performance parameters? The literature also shows that experiments concerning compressor performance in terms of volumetric efficiency and three phase current measurements during start-up are not common among researchers. Therefore, the specific objectives of this research are:

- Development of a thermodynamic model for electric swashplate compressor
- Development of analytical models to capture start-up behaviour of the compressor
- Experimental-validation of thermodynamic and analytical models
- Development of a cooling model for electric swashplate compressor
- Experimental validation of computational multi-physics fluid dynamics model

1.5. Research method and dissertation outline

1.5.1 Research method

To achieve the objectives outlined above, the research approach was divided in five stages viz. literature review, analytical/numerical modelling, experiments, data analysis and interpretation, computational fluid dynamics (CFD) modelling and validation. In the first stage, an extensive literature review was carried out to develop the basic understanding of the subject matter and to identify research gaps and objectives. The review is also integrated where appropriate throughout this thesis. The second stage was about formulating and deriving mathematical equations and solve to be used as analytical tools for this type of electric compressors. In the third stage, experiments were performed at SuperCool Asia Pacific testing facilities to collect data for model comparison. The fourth stage involved data analysis and
comparison with predictions to test the accuracy of mechanical models. In the final stage, CFD models were developed for the electric drive cooling jacket and compared with experiments.

**1.5.2 Thesis outline**

**Chapter 1** presents the background and motivation, research gaps and objectives of the project. **Chapter 2** captures a comprehensive literature review about the different mathematical models reported concerning thermodynamics, start-up characteristics, and thermal management of the compressor electric drive. It is a combined form of the literature associated with each thesis chapter.

**Chapter 3** discusses an analytical model based on ideal gas for predicting compressor performance under steady-state operations. The chapter includes a published paper explaining the model.

**Chapter 4** presents an analytical model based on real gas to capture the essential physics, and performance of the compressor in terms of torque and volumetric efficiency. The chapter is presented as a published paper.

**Chapter 5** provides a mathematical model for the compressor transient start-up behaviour under different starting conditions. The chapter is presented as a published paper.

**Chapter 6** demonstrates the cooling jacket design for the swashplate compressor electric drive and the benefits of using air-cooling over refrigerant cooling through computational modelling. The chapter is presented as unpublished work – waiting for IP clearance from industry partner.

**Chapter 7** describes an experiment performed to check thermal resistance between windings and the stator material and potential solutions for a more effective heat transmission. A numerical model was then developed to match the experiment. The chapter is presented as unpublished work – waiting for IP clearance from industry partner.

**Chapter 8** describes an experiment performed to see the effectiveness of thermal potting resin in the stator winding slots on 2.5 kW and 4 kW Brushless DC motors driving electric swashplate compressor. The chapter is presented as unpublished work – waiting for IP clearance from industry partner.
Chapter 9 summarises the research outcomes and their significance to the field. It also, sheds some light on future directions/recommendations that might be worth considering for further research in the area concerned.
Chapter 2

Literature review

2.1. Introduction

This chapter includes literature related to the swashplate compressor mechanics, thermodynamics, start-up characteristics, different cooling strategies used to enhance thermal performance of the electric motor driving swashplate compressor, importance of thermal resistance between the windings and the stator material and the effective treatments for the winding hotspots using encapsulation materials. The structure is such that the work related to every heading will be discussed in detail in the following chapters.

2.2. Mechanical analytical models

The existing literature on the modelling of swash-plate compressors can be broadly classified into two categories – mechanical design and thermo-mechanical performance. The first deals with the modelling of individual components of the compressor such as valve dynamics, friction in reciprocating parts and compressor mounting brackets. The second category includes models that aim towards predicting compressor performance such as kinematic and dynamic characteristics and evaluation of compressor performance including shaft torque and volumetric efficiency.

The dynamic analysis of a swash-plate was presented by [2] together with a mathematical model for stress analysis, bearing load calculation and sizing and aligning counterweights for a good running balance. The findings conclude through a complete mathematical model, it is possible to analyze/predict the dynamic behavior of the swashplate mechanism and optimize running balances and piston loading. An approximate sinusoidal motion for the piston was observed. An optimization technique could be employed in case the number of loading points does not match the number of masses on the swashplate. Liu et al. [3] investigated dynamic characteristics of the wobble-plate compressor using a geometric description of the wobble plate. The results suggest piston motion of the wobble plate compressor follows cosine curve and the inertia force may have been the cause for the machine vibration. Also, the gas pressure
force is responsible for the total resistance moment and it keeps changing with the wobble plate rotation. The kinematic and dynamic characteristics of a six-cylinder variable-displacement wobble plate compressor were thoroughly analyzed by [4] to control the cooling capacity. They concluded inertia and friction forces significantly affect the cooling capacity, quiet operation and the durability of the compressor parts while operating at higher rpm. An analytical model for the wobble plate compressor was developed by [5] and applied to a sample variable displacement swash-plate compressor for some practical design guides. The inertia force, gas torque fluctuations, nutation motion of the wobble plate and the differential pressures were calculated. As suggested, the results can be used to predict the effect of different relevant parameters on the compressor kinematics. Tian et al. [6] developed a steady-state mathematical/analytical model for a variable displacement swash-plate compressor by combining sub-models for piston and swash-plate dynamics, compression process and flow control valve. An analytical expression based on experimental data was presented for the compression model.

The literature related to the dynamic behavior of the swashplate compressor suggests that suction gas temperature, clearance volume and pressure losses across valves are the factors that may contribute in reducing volumetric efficiency of the compressor [1]. Incorporating all the phenomenon responsible for volumetric inefficiencies in a model is a tedious job. Most of the existing literature focuses at modelling volumetric efficiency of hermetic compressors. An analytical formula for volumetric efficiency of hermetic compressors was expressed by [7] that includes the effect of suction gas temperature. The formula takes most important factors into consideration which have a significant effect on the compressor refrigeration. It is found that an accurate prediction of suction gas pressure and the dead volume is essential to get the best out of model when compared with experiments. Moreover, modelling of the compressor work and capacity would give a decent idea of the compressor performance and may help designers at the very early stages of compressor design. Schreiner et al. [8] presented simulation results on volumetric inefficiencies associated to the compression process of the compressor used for household refrigeration. The simulation was carried out adopting a small capacity refrigeration compressor through a numerical model to evaluate all the important parameters concerning the compressor operation and performance. The parameters such as in-cylinder gas pressure, flow through valves, leakage through clearance volume, refrigerant thermophysical properties and pulsating gas behavior in mufflers affecting volumetric efficiencies were assessed. A steady-state mathematical model for a reciprocating compressor for automotive air-conditioning
applications was presented by [9]. An analytical expression for isentropic efficiency based on experimental data was presented. The equations were derived based on simple mass-energy balance and the measured experimental data. The data collected to compare the model against was from a test facility that was using R134a refrigerant. The model was proved to predict refrigerant discharge enthalpy, mass flow rate and the compressor power with reasonable accuracy.

A study by [2] provides an example of a dynamic analysis of a swash-plate compressor using a mathematical model to predict bearing loads and stresses. The swashplate-piston motion was found to be close enough to a sinusoidal approximation. [3-5] carried out similar studies for wobble-plate compressors. Tojo et al. [5] adapted wobble-plate models for application to a swash-plate compressor and developed some practical design guides. A practical compromise has to be made between balances, space and weight increase as the inertia of the piston and swashplate directly affects controllability of the piston displacement in the high speed range/region. Similarly, [6] developed an analytical model for a variable displacement swashplate compressor including piston sub-models, swashplate dynamics and a flow control valve model. The results showed that four modes of operation (viz. constant rotary speed and constant piston stroke length, variable rotary speed and constant piston stroke length, constant rotary speed and variable piston stroke length, variable rotary speed and variable piston stroke length) also exist for the variable displacement swashplate compressor like the variable displacement wobbleplate compressor. Multi-valued relationships between compressor parameters were also proposed when the piston stroke length changes.

Reliable predictions of variation in compressor performance with rotational speed are of significant interest. Variable-speed electric motors offer the new freedom to select and optimize the speed rather than simply rotating synchronously with the internal combustion engine using traditional belt drives. A practical compressor model for detailed analysis should be able to predict power requirements, torque fluctuations and variations in volumetric efficiency with rotational speed.

For the compression thermodynamics, a popular approach is to treat the refrigerant as an ideal gas with constant specific heats [10]. By further assuming isentropic compression and expansion, this set of assumptions leads to very convenient equations relating temperatures and pressures between different stages to volume ratios:
\[
\left( \frac{T_2}{T_1} \right)_{s=\text{const.}} = \left( \frac{P_2}{P_1} \right)^{(k-1)/k} = \left( \frac{V_1}{V_2} \right)^{k-1}
\]  

(2.1)

Where \( k \) is the ratio of specific heats. If Eq. (2.1) is combined with a reed-valve model that offers no resistance to flow once the cylinder pressure matches the external pressure of the refrigerant, then an easy-to-implement model can be derived (as pointed out by [11]):

\[
\dot{m} = \rho_{\text{suc}} V_{\text{swept}} \left( 1 - \frac{V_{\text{min}}}{V_{\text{swept}}} \left( \frac{P_d}{P_s} \right)^\frac{1}{k} - 1 \right) f n 
\]

(2.2)

\[
\dot{W}_{\text{ideal}} = \dot{m} c_p T_{\text{suc}} \left( \frac{P_d}{P_s} \right)^{(k-1)/k} - 1 
\]

(2.3)

Where \( \dot{m} \) is the average mass flow rate of refrigerant, \( \dot{W}_{\text{ideal}} \) is the power required to drive an ideal isentropic compressor, \( f \) is the compressor frequency and \( n \) is the number of cylinders. This approach has two major shortcomings. Firstly, the model predictions for refrigerant flow are simple linear relations with respect to rotational speed (which tends to overestimate performance at high speed) and secondly, the ideal-gas treatment is only a rough approximation for refrigerant equation of state [12].

Volumetric efficiency is a key parameter for predicting refrigerant mass flow rate as a function of rotational speed of the compressor. It provides a simple link between the suction side density, compressor volume, rotational speed and mass flow rate. The volumetric efficiency is given (e.g. [13]) as follows:

\[
\eta_{\text{vol}} = \frac{V_{\text{suction}}}{V_{\text{swept}}} = \frac{\left( \frac{m_{\text{min}}}{\rho_{\text{suc}}} \right)}{V_{\text{swept}} \times \frac{N}{60} \times n} 
\]

(2.4)

Where \( n \) is the number of cylinders and \( N \) is the revolutions per minute. Grolier et al. [14] proposed an analytical formula for predicting volumetric efficiency including the effect of suction vapour temperature. Simulations were done by [15] predicting volumetric inefficiencies for a domestic refrigerator compressor while [16] presented a steady-state empirical model for volumetric efficiency. Comparing Eq. (2.4) with Eq. (2.2), the bracketed factor in Eq. (2) represents the ideal volumetric efficiency.
The major factors cited as responsible for reduction in volumetric efficiency from the ideal value are in-cylinder superheating, losses and backflow through valves, leakage past piston rings, clearance volume, suction valve throttling, suction gas heating and clearance volume gas expansion [1, 17]. A detailed approach to estimate the volumetric efficiency is to capture the effect of each factor by using appropriate mathematical/analytical models and then summing up the individual effects. According to [17] volumetric efficiency of the variable speed compressor varies with the pressure ratio and the compressor speed due to fluid acceleration effects and cross correlation to system pressure ratio and valve pressure drop. While the factors affecting volumetric efficiency have been identified, the relative importance of the various parameters appears to be somewhat unclear to the best of the authors’ knowledge in the open literature.

For convenient, practical calculations, several authors have used experimental data to develop empirical expressions for the volumetric efficiency as a function of rpm ($N$) for particular compressors. These are listed in Table 2.1 and plotted in Fig. 2-1. Generally, the experimental data show a massive decrease in volumetric efficiency with increasing rpm. The behaviour is typically non-linear and for some cases, the volumetric efficiency initially increases up to a maximum limit/value and then decreases with increasing rpm.

<table>
<thead>
<tr>
<th>Source</th>
<th>Curve fitting</th>
<th>Equation</th>
<th>Const.</th>
<th>Compressor type</th>
<th>$P_{\text{in}}$ (kPa)</th>
<th>$P_{\text{dis}}$ (kPa)</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darr et al. [16]</td>
<td>Linear</td>
<td>$\eta_v = aN + b$</td>
<td>$a = -0.0001$</td>
<td>Swashplate</td>
<td>183-341</td>
<td>1075-2716</td>
<td>Automotive air-conditioning</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b = 0.8889$</td>
<td></td>
<td></td>
<td></td>
<td>Mobile refrigeration</td>
</tr>
<tr>
<td>Li et al. [17]</td>
<td>Linear</td>
<td>$\eta_v = aN + b$</td>
<td>$a = -6 \times 10^{-05}$</td>
<td>Reciprocating</td>
<td>167-372</td>
<td>1075-2718</td>
<td>Automotive air-conditioning</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b = 0.6968$</td>
<td></td>
<td></td>
<td></td>
<td>Mobile refrigeration</td>
</tr>
<tr>
<td>Srivastava et al. [1]</td>
<td>Polynomial</td>
<td>$\eta_v = aN^2 + bN + c$</td>
<td>$a = -8 \times 10^{-08}$</td>
<td>Swashplate</td>
<td>250</td>
<td>1580</td>
<td>Automotive air-conditioning</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b = 0.0003$</td>
<td></td>
<td></td>
<td></td>
<td>Mobile refrigeration</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$c = 0.5049$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tian et al. [6]</td>
<td>Polynomial</td>
<td>$\eta_v = aN^2 + bN + c$</td>
<td>$a = -2 \times 10^{-08}$</td>
<td>Swashplate</td>
<td>250</td>
<td>1500</td>
<td>Automotive air-conditioning</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b = 0.0001$</td>
<td></td>
<td></td>
<td></td>
<td>Mobile refrigeration</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$c = 0.4463$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturer of 200 cc</td>
<td>Polynomial</td>
<td>$\eta_v = aN^2 - bN + c$</td>
<td>$a = -7 \times 10^{-09}$</td>
<td>Swashplate</td>
<td>180</td>
<td>1600</td>
<td>Automotive air-conditioning</td>
</tr>
<tr>
<td>compressor used in this</td>
<td></td>
<td></td>
<td>$b = -2 \times 10^{-05}$</td>
<td></td>
<td></td>
<td></td>
<td>Mobile refrigeration</td>
</tr>
<tr>
<td>study</td>
<td></td>
<td></td>
<td>$c = 0.7664$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In contrast with the findings for practical compressors shown in Fig. 2-1, the ideal-gas ideal-valve compressor model does not predict a change in volumetric efficiency with rpm (see bracketed factor in Eq. (2.2) which represents volumetric efficiency of an ideal compressor). Moreover, while the experimentally determined relations in Table 2.1 are practical for a given compressor operating under a given set of conditions, simply expressing the volumetric efficiency as a function of rpm does not account for any of the causes of the behaviour and therefore is less portable to new situations.

### 2.3. Transient start-up of swashplate compressor:

The literature published on compressor transients can be broadly divided into two categories. The first deals with the overall refrigeration system transients which have time scales that are much larger than the compressor start-up and shut-down. The second is related to the compressor dynamics, predicting velocities, accelerations and displacements within the compressor mechanism itself during start-up and shut-down.

Modelling transients in refrigeration and air conditioning systems were first initiated by [18-20]. Following this, [21], [22] developed analytical/mathematical models to investigate the dynamic characteristics of household refrigerators through using mass and energy conservation equations to illustrate the interaction between the various components of the refrigeration system. The gas/vapour compression in the cylinder and mass flow rate were modelled through fitting curves to experimental data. The results for suction and discharge pressure variation and transients of different system parameters were presented using relatively long timescales (6
min to 5hr). The model predictions for pressure variation in the evaporator and condenser were found to be in a close agreement with experiment. Borges et al. [23] presented an analytical model to investigate the dynamic response of a household refrigerator. Through some experimental validation it was found that the model can predict the system’s energy consumption within ±2%. Moreover, some energy saving opportunities were also explored. A mathematical model based on lumped parameter analysis was developed by [24] to study the transient behaviour of a domestic refrigerator. The model could capture the refrigerant (evaporator) and compartments (freezer and fresh food) temperature, initial compressor pull down, compressor on-off duration, and compressor on-off cycle based on freezer temperature.

A few researchers have analysed the transient characteristics of a vapour compression refrigeration system by taking different components of the circuit into consideration. Koury et al. [25] developed a mathematical model to predict the dynamic behaviour of the refrigeration system with a variable speed control. The model includes small control volumes for the condenser and evaporator defined by a system of mass, momentum, and energy balance equations. The mass flow rate through the system was obtained by using a volumetric efficiency curve as a function of clearance volume and saturation pressures. The compression process was assumed to be isentropic and 70% compression efficiency was considered to evaluate the energy consumption of the compressor. Browne et al. [26] proposed a transient model for vapour compression based liquid chillers to evaluate the dynamic characteristics of the system. The compressor mechanism dynamics were not included but rather a steady-state compression process model was adopted whereby assuming the compressor attained full operating speed instantly. The volumetric efficiency curve was used to evaluate mass flow rate and compressor energy consumption. Results were in good agreement with the experimental data, yet some empirical relations were also required to precisely predict the start-up behaviour of the system. Willatzen et al. [27] developed a mathematical model to simulate the dynamic characteristics of the heat exchangers in the vapour compression refrigeration circuit. A set of ordinary differential equations for the mass and energy conservation was formulated. The heat exchanger was modelled considering three zones (phase-region) for the refrigerant flow inside the heat exchanger viz. liquid, vapour, and two-phase. The model was then applied to an evaporator and a few cases of transient behaviour were investigated by [28].

Simple transient models for vapour compression refrigeration circuits using different refrigerants can be found in the open literature. A transient model for the vapour compression refrigeration circuit was developed by [29] which includes component models for the
compressor, condenser, expansion valve and evaporator. The refrigerant in each component was modelled using lumped parameter analysis. Mass and energy conservation were applied on each component of the refrigeration circuit to capture the change in temperature with time. A set of standard assumptions for the compression/expansion processes was taken into consideration. An experimentally validated mathematical model was developed by [30] to capture the transient behaviour of a split air conditioning system using R22 and R410a refrigerants. The sub-models for the condenser and evaporator were based on mass, momentum and energy conservation equations using the implicit finite difference approach. The condensing temperature and pressure drop for R410a were found to be lower than that of R22 refrigerant. The transient losses during start-up were found to be almost similar for both refrigerants.

A few articles have been published on the compressor start-up behaviour concerning compressor body vibrations. Gerhold et al. [31] developed a mathematical model for a single cylinder reciprocating compressor to analyse the vibrations produced during start-up and shutdown processes. The compression process model was determined from experimental data fitted to a polytropic line. The motor torque loading came from the torque-speed characteristics curve of a shunt wound motor. An experimentally validated numerical model was developed by [32] to study the start-up transients of a reciprocating compressor to understand the compressor displacement components. The results showed that during start-up, the model can fairly estimate the compressor displacement. Erol et al. [33] also studied the compressor start-up transients through developing a non-linear, experimentally validated model. The motor torque was presented as a function of the compressor speed. Polytropic compression and expansion with constant suction and discharge pressures were assumed. Porkhial et al. [34] developed an empirical model based on the experimental data to investigate the start-up characteristics of a reciprocating compressor under different operating conditions. The analysis revealed more energy consumption during the start-up than that of steady-state operation. Kim et al. [35] studied the transients in a reciprocating compressor by employing suspension springs to suppress the vibrations. The gas force on the piston surface was determined using experimental data. A method to estimate the motor start-up speed profile for different speed-torque characteristics curve was proposed by [36]. The speed-torque characteristics curve was considered essential for obtaining accurate results.

Performance of reciprocating compressors has been investigated by a number of researchers during start-up and shut down transients. Link and Deschamps [37] developed an
experimentally validated numerical model to investigate the performance of reciprocating compressor during start-up and shut-down. In-cylinder pressure, piston motion, valve displacement, and resistive torques were investigated to see their effects on the compressor performance during start-up and shut-down processes. The results showed that the reed valve dynamics were highly affected by the transient load. Also, the model was used to predict the minimum voltage needed for the compressor start-up as a function of equalized pressure and coil actuation time. Dutra and Deschamps [38] developed a simulation model for hermetic reciprocating compressors including three sub-models (compression model based on mass and energy conservation equations, heat transfer model, and single phase induction motor model) to investigate the compressor performance. Mean compressor speed and motor slip were found to have considerable effect on the compressor efficiency. Some experimental validation was performed to check the model in terms of temperature distribution, compressor efficiency, and motor performance. Motor temperature dependence on the input voltage was also investigated through parametric analysis.

Numerical and experimental investigation of rotary compressor start-up characteristics under different operating conditions have been reported by a few people. Wu et al. [39] performed an experimental analysis to investigate the cold start-up behaviour of the rotary compressor of an air conditioning system using an environmentally friendly, highly efficient R290 refrigerant under cold ambient conditions. The compressor pressure and temperatures, mixture (oil & refrigerant) viscosity, and the level of oil in oil sump were the characteristics under investigation. The start-up time for pressures and temperatures were found to be considerably longer than that of R22 and R410a systems. Another notable finding was that after system start-up, refrigerant/oil mixture viscosity and oil level in the compressor oil sump were within an adequate range to assure steady start-up of the system. Wu et al. [40] conducted an experimental investigation of a rotary compressor (connected to the heat pump system) start-up characteristics under low ambient heating conditions. The characteristics include the system and in-cylinder pressures and temperatures. It was found that the suction pressure during a low ambient warm start-up condition is higher than that of a cold start-up condition. Moreover, the time needed for R290 heat pump system to reach steady-state pressure was much longer than that of R410a refrigeration system. Also, at the beginning of cold and warm start-ups, a small amount of liquid was produced in the compressor cylinder under a low ambient heating condition. Lin et al. [41] experimentally investigated the dynamic characteristics of an R290 rotary compressor during warm start-up under cooling conditions. The analysis of pressures
and temperatures of the system and compressor, oil/refrigerant mixture viscosity and the oil level in the sump was carried out. The outcomes showed that the minimum suction pressure during the cold start-up was lower than that of the warm start-up. The liquid level and viscosity ranges for the cold start-up were higher than that of the warm start-up under the cooling conditions. This led to an important conclusion that the warm start-up kept the compressor well lubricated.

**Table 2.2** summarises the published works concerning transient behaviour of the refrigeration circuit including compressor dynamic characteristics during start-up and shut-down operations. Most of the studies [37], [18-30] focus on single cylinder reciprocating compressors and the relatively slow transient behaviour of pressures and temperatures rising to a steady state in the refrigeration circuit. Studies concerned with vibration [32-35] include the mechanism inertia which is neglected in many other studies. A small number of studies consider transient behaviour of rotary and swash-plate compressors [39-43]. To the best of the authors’ knowledge, no studies include the inertia of the swashplate mechanism.
Table 2. 2 Studies published on compressor transients

<table>
<thead>
<tr>
<th>Authors</th>
<th>Circuit type/application type</th>
<th>Refrigerant type</th>
<th>Modelling</th>
<th>Transient refrigeration mechanism</th>
<th>Transient compressor mechanism</th>
<th>Experimental technique</th>
<th>Analysis type</th>
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<tr>
<td>Link and Deschamps [37]</td>
<td>Vapour compression refrigeration</td>
<td>R600a</td>
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<td>✓</td>
<td>✓</td>
<td>Pressures, valve dynamics, piston motion</td>
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<td>Compressor power and temperatures</td>
<td>Mathematical</td>
</tr>
<tr>
<td>Authors</td>
<td>System Type</td>
<td>Refrigerant(s)</td>
<td>Measured Parameters</td>
<td>Other Parameters</td>
<td>Solution Method</td>
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<td>Browne et al. [26]</td>
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<td>✓</td>
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<td>Household refrigerators</td>
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2.4. Thermal management of swashplate electric drive

2.4.1. Cooling jacket design

While only a few studies have considered air-cooled electric drives for refrigeration compressors [44], a significant body of literature is available on general thermal management of electric motors. For example, a numerical analysis was performed by [45] to check for the efficient cooling jacket designs for a Brushless DC (BLDC) motor to reduce the surface temperature. They observed a correlation between cooling flow rate and the number of flow channels. The study was considered successful in identifying important parameters affecting temperature profile of the motor including winding hot spots. Kwon et al. [46] numerically investigated the thermal performance of 600 W BLDC motors to be used on electric vehicles. They found that the fin/extended surfaces can significantly enhance the heat transfer rate from the motor housing to the surrounding environment. The stator and magnet temperatures were found to be 62.3 °C and 32.2 °C respectively. Our work builds on these studies by focusing on keeping the cooling jacket compact which is highly desired for mobile applications.

Alternative cooling systems for realizing compact high-power electric motors are also under investigation in the literature. Davin et al. [47] conducted an experiment that showed that an oil cooling system with even a small amount of oil has potential to assist better thermal management for electric motors. The effect of the oil flow rate, rpm and oil temperature was investigated. Results suggest that oil flow rate has much influence on the overall cooling performance of the motor whereas the effect of rpm was found to be less important. Zahang et al. [48] examined a number of thermal models for water cooled synchronous electric machines to be used on electric vehicles. They built a prototype for a water-cooled permanent magnet machine and found that the heat transfer between rotor and stator core is controlled by the air gap. Acquaviva et al. [49] also proposed a solution for the cooling of brushless DC machines through using water cooling jackets. They carried out a transient analysis to clearly see the temperature profile and optimised it to achieve better thermal performance. The L-shaped flat heat pipe potential in motor cooling applications was explored by [50]. It was concluded that it can significantly reduce the motor surface temperature through providing better thermal management. Huang et al. [51] analysed a hybrid cooling system (air flow, coolant flow and heat pipe) for electric motors and suggested that the combined system (coolant flow and heat pipe) works well as far as energy saving is concerned. Thus, compact cooling systems for electric motors is a key point of interest for mobile applications.
Some studies have compared alternative cooling systems for general mobile applications with a focus on electric vehicles. Three methods used for electric motor cooling namely air cooling, water cooling and oil cooling were compared by [52]. They pointed out that air cooling is mainly utilised in train motors and is considered superior to other cooling methods/techniques in relation to cost. Oil cooling shows a better thermal/cooling performance since the thermal properties of liquids are generally better than gases, however oil lacks cost effectiveness and there are issues with cooling uniformity as suggested by [47]. Water cooling on the motor is also acceptable among EV manufacturers since it provides considerably better thermal/cooling uniformity as compared to that of oil cooling. Refrigerant cooling was considered by [9] for cooling of electric vehicle motors and the results showed better thermal/cooling performance along with the cooling uniformity if compared with water cooling.

There are a number of analytical and numerical thermal techniques used to evaluate the temperature distribution within the motor, motor performance, cooling system and potential of using end-turn encapsulation material for the motor performance improvement [53-57]. The most common are Lumped parameter, FEA and CFD based techniques [58]. Thermal analysis of Totally Enclosed Fan-Cooled (TEFC) induction motors was performed by [59] using a commercial software (MotorCAD) followed by an experimental validation. An uncertainty of ± 5º C was achieved on the winding temperature which could be considered acceptable for most of the industry applications. The thermal performance of a permanent magnet (PM) motor at steady state was investigated by [60] analytically and numerically using thermal networks and a commercial FEM software. They concluded that the rotor temperature in their design was not high enough to cause a demagnetisation problem. CFD models for thermal performance analysis of electric motors are also presented by several authors [60, 61]. Most of them assumed that the heat sources are evenly distributed within the generating components. The effect of thermal contact between windings and the stator core has not been explored yet using CFD. Analytical thermal models are being commonly used by researchers due to their compactness, simplicity and computational cost. However, these models are considered to be inaccurate if not validated by experiments. Experimental validation is essential as there are a number of factors (manufacture-dependent) that can only be obtained and optimised through experiments.
2.4.2. Stator windings

Overheating is one of the key factors limiting the output performance/capacity of electric motors [62-66]. The performance of BLDC motors can be enhanced through the reduction of losses which need to be dissipated to the surrounding environment [62]. The losses are of different types; copper losses, iron losses, mechanical losses because of friction and some other losses due to manufacturing inaccuracy and component assembly. The copper losses contribute around 65% in the total energy loss, core/iron losses 27%, bearing losses 6% and 2% other losses [63]. The losses are solely responsible for the temperature rise inside the motor and hence lead to considerable thermal load on different components of the motor. A temperature rise of 10 °C has been estimated to lead to a 50% reduction in the life of the winding insulation [67]. The durability of the winding insulation system depends on the stator winding temperature while the permanent magnet (PM) efficiency is affected by the rotor temperature [67]. Therefore, thermal management of the windings is a key consideration for motor design.

Thermal management of electric motors has been investigated and reported by many researchers. Improved thermal design/management can be achieved at the expense of increasing total weight and size of the motor for a required torque and power [68]. Size reduction, on the other hand, is desired for mobile applications and can save cost as the amount of copper wire, magnets and lamination steel needed is less. The recent developments in thermodynamic modelling of electric motors include Finite element method (FEM), Computational fluid dynamics (CFD) and lumped capacitance parameter circuits [69-71]. Wallerand and Laurent [69] analysed the thermal behaviour of a permanent magnet motor by employing a commercial FEM software with a thermal network approach. They found that under the maximum load condition, the rotor temperature was not high enough to create demagnetization problems. Boglietti et al. [72] investigated the thermal performance of induction motors using another commercially available software (MotorCAD). The model on comparing with experiment showed an acceptable accuracy of ± 5 °C for the winding temperature. Thermal network models are widely accepted because of their simplicity and low computational cost. However, they are considered less accurate when it comes to optimization of thermal parameters such as contact resistance between winding and the stator, heat sink and the fan geometries [73]. Lee et al. [65] performed a numerical analysis to investigate the thermal reliability of a BLDC motor. A high-speed fan was employed to control the airflow to provide the desired cooling. The lifespan equation for the Printed Circuit Board (PCB) connected with the lifetime equation was proposed using PCB space temperature and intake
air temperature as inputs. Kwon et al. [66] investigated thermal characteristics (temperature distribution and internal thermal resistance) of a BLDC motor through conducting an experimentally-validated numerical analysis. They concluded that it is easily possible to manufacture electric motors with better thermal management to achieve higher power outputs. Many methods of heat transfer enhancement from the motor active parts are described in the open literature. Review papers discussing cooling technologies, heat transfer analysis and heat transfer enhancements for the permanent magnet motors were presented by [74, 75]. One of the most efficient ways cited in the literature to dissipate heat from the windings effectively is to spray liquid on the winding/coil surface. The idea was used and tested numerically as well as experimentally by [75, 76]. Other approaches to enhance the thermal performance of electric motors include use of a Totally Enclosed Fan (TEF) mounted on the motor housing and the customised Open Ventilated Fans (OVF) to force the air directly to the hotspots/active parts of the motor [77]. In low power electric motors, the use of the above active ways of heat transfer enhancement could become a complex issue however the passive ways might be more suitable [78]. Therefore, a phase change material/medium in the form of heat tubes was also investigated for better heat dissipation from the windings of a low power electric motor [79]. Applications that require high-speed motors (vacuum within the housing), heat transfer via thermal radiation could be a way to dissipate heat losses in the rotor through providing improved surface emissivity for the motor components [79].

2.4.3. Thermal potting resin:

Direct liquid cooling inside an electric motor can be challenging and may result in oil degradation at winding hotspots and can trigger poor heat transmission/exchange [80-82]. Forced air-cooling requires low thermal resistance between components in areas where the flow of air is restricted [83-85]. These two methods can be supplemented using thermal resins with high thermal conductivity to meet the design and operation requirements [86-88]. Efficient electromagnetic design at the design stage can help reduce the size and weight of the motor for a required torque and power output. The motor cost can be saved through reducing the amount of copper, steel and magnet material [89, 90]. Improving heat transmission while designing electric motors enables better power output for a given size and extends motor lifetime.

When the designed cooling system (forced air or water)/thermal management strategy is insufficient to carry out the heat generated inside the motor, application of encapsulation material (epoxy resin) with higher thermal conductivity in the stator-winding air gap could be
the way of heat transfer enhancement as reported by [91-94]. Thermal analysis of segmented stator winding design was carried out by [95] using high thermal conductivity end-turn encapsulation material (epoxy) to check for the motor performance improvement. The calculation results show good agreements with the experiment. A 3-D FEA thermal model and experiment were presented by [91] to demonstrate the performance improvement of the electrical machine using two different high thermal conductivity end-turn encapsulation materials. The results show that the use of high thermal conductivity encapsulants can potentially reduce the end-turn temperature rise (36%) and increase the motor output torque (13%). Most studies focus on the effects of using encapsulation material on the end windings rather than in the stator slot for the windings.

High thermal conductivity resins have been demonstrated to enable high power density and low winding temperatures. In a number of studies, it has been shown that potting resins provide better thermal contact between the windings and motor stator and thus reduce winding temperature and increase motor power output. Nategh et al. [96] used three different winding insulation schemes (viz. standard varnish (k = 0.2 W/m.K), standard epoxy (Epoxylite, k = 0.9 W/m.K) and a silicon material (CoolTherm SC-320, k = 3.2 W/m.K)) to demonstrate experimental thermal analysis of an oil-cooled induction motor. The high thermal conductivity silicon material was found to be the most effective providing 50 °C lower temperature compared to a ‘varnish only’ motor. Li et al. [97] performed an experimental study followed by numerical simulations for totally enclosed air-cooled, highly efficient, off the shelf induction motors potted with two different thermal resins (1.2 W/m.K epoxy and 3.2 W/m.K silicon). The silicon potted motor was found to be 35 °C cooler than the ‘varnish only’ motor which eventually allowed an increased 16% motor power output.
8. Schreiner, Joao Ernesto; Deschamps, Cesar J.; and Barbosa, Jader R., "Theoretical Analysis of the Volumetric Efficiency Reduction in Reciprocating Compressors due to In-Cylinder Thermodynamics" International Compressor Engineering Conference, 2010.


Chapter 3

Ideal Gas Model

Statement of contribution to co-authored published paper

This chapter includes a co-authored paper. The bibliographic details of the co-authored paper, including all authors, are:


My contribution to the paper involved: literature review, analytical modelling, experiment, writing and editing manuscript.

Signed: ------- Date: 10/08/2021

PhD Candidate (corresponding author of paper): Mohammad Arqam

Countersigned: ------- Date: 10/08/2021

Principal Supervisor (co-author): Dr. Peter Woodfield

Countersigned: ------- Date: 10 August 2021

Co-supervisor (co-author): Dr. Dzung Viet Dao
3.1 Abstract

Recent advancements in the field of mobile air conditioning and refrigeration have witnessed an extensive use of the swash plate compressor due to its compact structure, continuous operation, small size, light weight and better thermal comfort inside the vehicle. The design of the swash plate compressor is complex so that it requires considerable contributions from different fields of engineering viz. engineering mechanics, heat transfer and fluid dynamics. An estimate of compressor performance through an analytical/mathematical model at the early stages of design and development serves as a useful tool for the designer. The input power, refrigerant mass flow rate, compression ratio and volumetric efficiency are important parameters to characterize the compressor performance. This paper presents an analytical/mathematical model for a 10-cylinder swash plate compressor with the emphasis on predicting its performance in terms of shaft torque and mass flow rate for a given rpm. A kinematic model is developed to obtain the piston displacement as an explicit function of angle of rotation of the swash plate. The model of piston and swash plate dynamics is developed then by analyzing the interactions between forces and moments. The compression process model is formulated to determine the temperature and pressure inside the cylinder during one revolution of the swash plate along with the total mass flow rate in and out of the compressor. A time-varying model for the compressor is developed by combining the above three sub-models. Some experimental validation comparing predicted and measured drive torque have been done to verify the analytical/mathematical model. The predicted torque is in close agreement with the measured value.

3.2. Introduction

Swash-plate compressors are widely employed in mobile refrigeration and air conditioning applications due to their compact structure, light weight and small size [1]. The design of such compressors is complex and requires an estimate of performance to be provided by the designer. Analytical/Mathematical models serve as useful tools during the early stages of design. The power and volumetric efficiency are important parameters for characterizing the compressor performance. Estimation of shaft torque is important for design of the driving motor and sizing of the shaft and other mechanical components.

This chapter presents an analytical/mathematical model for a 10-cylinder fixed-displacement swash-plate compressor with the emphasis on predicting its performance in terms of shaft
torque. A kinematic model is developed to obtain the piston displacement as an explicit function of angle of rotation of the swashplate. The model of piston and swashplate dynamics is developed then by analyzing the interactions between forces and moments. The compression process model is formulated to determine the temperature and pressure inside the cylinder during one revolution of the swashplate along with the total mass flow rate in and out of the compressor. A steady-operation model for the compressor is developed by combining the above three sub-models. Results are presented for a 10 cylinder fixed-displacement swash-plate compressor.

3.3. Analytical model for investigation of swash plate electric compressor

The present study considers a fixed-displacement swash-plate compressor with 10 cylinders. The pistons are mounted along the periphery of the swash-plate. The rotating motion of the swash-plate (a tilted disc mounted at a constant angle from the shaft axis) is converted to reciprocating motion of piston inside the cylinder. Cylinders are aligned on both sides of the swash-plate in such a way that compression in one cylinder causes suction in the symmetrically opposite cylinder. Pressure actuated reed valves are employed for both suction and discharge. Fig. 3-1 gives an overview of the geometrical configuration of the compressor. Only one cylinder is shown for clarity. $Z$ is the displacement of the top of the piston from its lowest point and $s$ is the circumferential distance through which the swash-plate has rotated. $\phi$ is the angular location of the cylinder with respect to a datum line and $\theta$ is the angle through which the swash-plate has rotated. $\omega$ is the angular velocity (rad/s) and $\alpha$ is the angular acceleration (rad/s$^2$).
3.3.1. Kinematics:

Assuming the top surface of the swash-plate is a single plane and \( \mathbf{n} \) is a unit normal vector to that plane, then \( \mathbf{n} \) is a function of \( \theta \), the angular position of the lowest point on the plate:

\[
\mathbf{n} = \sin(\psi) \cos(\theta) \mathbf{i} + \sin(\psi) \sin(\theta) \mathbf{j} + \cos(\psi) \mathbf{k}
\]  
(3.1)

Consider a point, \( p \) on the plate in contact with the push-rod (connecting rod) for one of the pistons located at an angle \( \phi \) shown in Fig. 3-1. The position vector of \( p \) for the coordinate system shown in Fig. 3-1 will be:

\[
\mathbf{r}_p = R \cos(\phi) \mathbf{i} + R \sin(\phi) \mathbf{j} + z_p \mathbf{k}
\]  
(3.2)

Because \( \mathbf{r}_p \) is in the plane for the plate, it will be at 90° to the normal and the dot product \( \mathbf{r}_p \cdot \mathbf{n} = 0 \). This gives:

\[
z_p = -R \tan(\psi)(\cos\phi \cos\theta + \sin\phi \sin\theta) = -R \tan(\psi) \cos(\phi - \theta)
\]  
(3.3)
Therefore for the ith piston located at angle $\phi_i$, the displacement of the piston from bottom dead centre is given by:

$$Z_i = R\tan(\psi)(1 - \cos(\phi_i - \theta))$$  \hspace{1cm} (3.4)

The velocity of the piston is related to the angular velocity by differentiating Eq. (3.4) with respect to time to get:

$$v_i = -R\tan(\psi)\omega \sin (\phi_i - \theta)$$  \hspace{1cm} (3.5)

### 3.3.2. Dynamics

Fig. 3-2 shows a free-body diagrams of the swash plate and piston. It is assumed that force from each cylinder acts in the direction perpendicular to the plate. There are also some reaction forces at the bearings.

![Free-body diagram](image)

Figure 3-2 Free-body diagram (a) swash plate (b) piston

For rotation of the rigid body about the z-axis, Newton’s 2nd Law gives:

$$\sum M_z = I_z \alpha$$  \hspace{1cm} (3.6)

Where $I_z$ is the rotational moment of inertia about the z-axis and $\alpha$ is the angular acceleration. To solve Eq. (3.6) the torque due to each of the reaction forces from the pistons shown in Fig. 3-2 is needed. The moment vector of the force from the ith piston is given by the cross product:
Substituting Eq. (3.1) and Eq. (3.2) into Eq. (3.7) and evaluating the component in the $z$-direction gives the torque about the axis exerted by the force from the $i$th piston is:

$$\tau_i = F_i R \sin(\psi) \sin(\phi_i - \theta)$$  \hspace{1cm} (3.8)

Substituting into Eq. (3.6) gives:

$$\tau_m - R \sin(\psi) \sum_{i=1}^{N} F_i \sin(\phi_i - \theta) - \tau_{bearing} = l_2 \alpha$$ \hspace{1cm} (3.9)

Where $N$ is the total number of pistons.

Newton’s 2\textsuperscript{nd} Law applied in the $z$-direction for the piston gives:

$$F_i \cos(\psi) - P_i \frac{\pi D^2}{4} - \mu \frac{\pi DL}{\delta} v_i = m a_i$$ \hspace{1cm} (3.10)

Where $P_i$ is the pressure in the cylinder, $D$ is the diameter of the cylinder, $L$ is the length of the piston in contact with the wall of the cylinder, $\delta$ is the gap between the piston and the cylinder, $\mu$ is the dynamic viscosity of the lubricant, $v_i$ is the velocity of the piston, $m$ is the mass of the piston and $a_i$ is the acceleration of the piston. Eqs (3.9) and (3.10) with (3.4) and (3.5) needed to be solved.

Neglecting the mass of the cylinder and viscous force in Eq. (3.10), the pressure in the cylinder is connected to the force applied by the swashplate using:

$$F_i = \frac{P_i \pi D^2}{4 \cos \psi}$$ \hspace{1cm} (3.11)

For any angular position, if the mass of the swashplate and bearing resistance torque is neglected, the torque that needs to be supplied to the swashplate is:

$$\tau = R \sin(\psi) \sum_{i=1}^{N} F_i \sin(\phi_i - \theta)$$ \hspace{1cm} (3.12)
3.4 Swashplate compressor model – ideal gas with constant specific heat

This study considers the steady state case (constant rpm), ideal gas, isentropic compression, massless piston, and massless swashplate.

3.4.1 Mathematical Formulation

To complete the model, pressures inside the cylinder as a function of the piston position and geometry are needed. The pressure in the cylinder $P_i$, needed in Eq. (3.10) is quite complicated. To a first approximation, the adiabatic processes are modelled using isentropic equations for an ideal gas with constant specific heats (Eq. (3.13)). A more accurate approach would be to use tables of data for the refrigerant.

$$P = P_o \left( \frac{Vol_o}{Vol_l} \right)^k$$

(3.13)

In Eq. (3.13) $k$ is the ratio of specific heats for the refrigerant vapor. Based on some data from NIST chemistry webbook [2], $k \approx 1.23$ might be a reasonable estimate for R134a vapor.

The simplest model to account for opening and closing of the valves is isentropic compression and expansion while the reed valves are closed and constant pressures $P_{\text{high}}$ and $P_{\text{low}}$ when the outlet and inlet valves are open respectively. For an ideal gas with constant specific heats this gives:

$$P_i = \text{MIN} \left( P_{\text{high}}, P_{\text{low}} \left( \frac{Vol_{\text{max}}}{Vol_i} \right)^k \right)$$

(3.14)

When the piston is going up and

$$P_i = \text{MAX} \left( P_{\text{low}}, P_{\text{high}} \left( \frac{Vol_{\text{min}}}{Vol_i} \right)^k \right)$$

(3.15)

When the piston is going down.

If it is assumed that the temperature of the vapor in the cylinder at the beginning of compression is the temperature of the superheated vapor in the suction line then the mass in the cylinder during compression while reed valves are closed can be found using the ideal gas equation:

$$m = \frac{P_{\text{low}}Vol_{\text{max}}}{RT_{\text{suction}}}$$

(3.16)
With this information, the temperature at any point during compression is given by the ideal gas equation:

\[ T_i = \frac{P_i Vol_i}{mR} \]  

(3.17)

Where \( R \) is the ideal gas constant:

\[ R = \frac{R_{univ}}{M_{refrigerant}} \]  

(3.18)

Where \( M_{refrigerant} = 102.03 \text{ kg/kmol for R134a.} \) When the reed valve is open it is simplest to assume that the temperature stays constant. When the reed valves are closed, the mass in the cylinder stays constant. From this, the mass flow rate depends on the velocity of the pistons and whether the valves are open or closed. For example, the contribution to the mass flow rate out when the outlet valve is open is given by:

\[ m_i = \rho v_i \pi \frac{D^2}{4} \]  

(3.19)

Where the density \( \rho \) is calculated using the ideal gas equation with \( P_{\text{high}} \) and \( T_{\text{max}} \).

### 3.4.2 Calculation Results

The case of 200 cc compressor with 10 cylinders was considered. Tables 3.1 & 3.2 give assumed geometry and conditions. Pressures were selected to correspond to those of an experimental test run.

#### Table 3.1 Assumed geometry

<table>
<thead>
<tr>
<th>No. Cyl.</th>
<th>Tot swept vol. (cm³/ft³)</th>
<th>Clear. vol. (cm³/ft³)</th>
<th>D (cm/ft)</th>
<th>R (cm/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>200/0.007</td>
<td>1/0.000035</td>
<td>3.1/0.10</td>
<td>3.5/0.11</td>
</tr>
</tbody>
</table>

#### Table 3.2 Assumed conditions

<table>
<thead>
<tr>
<th>( P_{\text{suction}} ) (Bar/kPa)</th>
<th>( P_{\text{discharge}} ) (Bar/kPa)</th>
<th>( T_{\text{suction}} ) (°C/°F)</th>
<th>( C_p ) (kJ/kg.K)/(Btu/lb·°F)</th>
<th>( C_v ) (kJ/kg.K)/(Btu/lb·°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.8/380</td>
<td>21.32/2132</td>
<td>31.70/89.06</td>
<td>1.032/0.24</td>
<td>0.837/0.19</td>
</tr>
</tbody>
</table>

Fig. 3-3 (a) shows the instantaneous total mass flow rate in and out of the compressor during
one revolution. The pulses from the individual cylinders are clearly evident. Fig. 3-3 (b) shows the pressure and temperature inside one of the cylinders during a complete revolution of the swash-plate.

![Graph](attachment:image.png)

Figure 3-3 (a) Mass flow rate of refrigerant for 10 cylinders during one revolution of swashplate (b) Pressure and temperature inside one cylinder during one revolution of swash-plate

3.5. Experimental validation

The experiment was performed at SuperCool Asia Pacific testing facilities. The setup consists of a variable capacity swash-plate compressor (200 cc/rev), a condenser (capacity performance of 7.6 kW), a universal refrigerator (cooling capacity of 9.4 kW), and a thermal expansion valve (TXV). It also contains a drier and an oil separator as auxiliary equipment.

A 30 kW induction motor controlled by a variable frequency drive (frequency inverter) was connected to the compressor to control the rotational speed to investigate a range of speeds at which the compressor works in an actual vehicle, and provide the related data such as torque (Nm), power (kW) and the speed (RPM) of the compressor. Fig. 3-4 shows the schematic diagram of the experimental setup. 15kW and 10kW electric heaters were placed in the condenser and evaporator rooms to supply heated air, and a heater exchanger unit to remove hot air was also located in the condenser room. T-type air thermocouples and K-type probes are mounted on the system to obtain the refrigerant temperatures and pressures. The sensors are connected to two data loggers to send the information to a computer. Investigation of the collected data was carried out numerically in a spreadsheet.
The measured average torque at different revolutions per minute (rpm) is outlined below in Table 3.3. The conditions correspond to the initial stages of operation – not long after starting up. Hence the suction temperatures are still high. These conditions are of interest for design since the torque is higher than it is for steady operation of the air conditioner.

Table 3.3 Average torque (22 °C/71.6 °F ambient)

<table>
<thead>
<tr>
<th>RPM</th>
<th>Suction P (Bar/kPa)</th>
<th>Suction T (°C/°F)</th>
<th>Discharge P (Bar/kPa)</th>
<th>Discharge T (°C/°F)</th>
<th>Av. Torque (N.m/ft.lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1778</td>
<td>4.10/410</td>
<td>29.5/85.1</td>
<td>20.00/2000</td>
<td>49.85/121.71</td>
<td>16.42/12.11</td>
</tr>
<tr>
<td>2135</td>
<td>3.80/380</td>
<td>31.70/89.06</td>
<td>21.32/2132</td>
<td>56.05/132.89</td>
<td>17.85/13.16</td>
</tr>
<tr>
<td>2370</td>
<td>3.80/380</td>
<td>30.75/87.35</td>
<td>17.99/1799</td>
<td>41.95/107.51</td>
<td>19.08/14.07</td>
</tr>
<tr>
<td>2670</td>
<td>3.70/370</td>
<td>30.90/87.62</td>
<td>18.89/1889</td>
<td>46.00/114.8</td>
<td>19.22/14.17</td>
</tr>
<tr>
<td>2974</td>
<td>3.60/360</td>
<td>30.75/87.35</td>
<td>19.20/1920</td>
<td>45.10/113.18</td>
<td>20.42/15.06</td>
</tr>
</tbody>
</table>

The validation is done for a 10 cylinder fixed displacement swash-plate compressor at 2135 rpm as depicted in Fig. 3-5 (a). The predicted values include the fluctuation from the cylinders while the time resolution of the measured value is insufficient to do this, so it appears as a horizontal line in Fig. 3-5 (a). The predicted torque is certainly in the range of the experimental data shown in Fig. 3-5. The trend with rpm shown in Fig. 3-5 (b) is different for the experiment.
compared with the simulation. The experimental data show a steady increase with increasing rpm while the simulation is almost independent of rpm except for the case at 2135 rpm. The higher value can be attributed to the higher discharge pressure shown in Table 3.3 for 2135 rpm, which is used as a model input.

From the results shown in Fig. 3-5, it is reasonable to conclude that the main contribution to the torque can be explained by the compression of the gas. It is conceivable that the extra resistance from the friction of the piston could bring the predictions closer to the experimental values concerning the trend of increasing torque with rpm. The isentropic efficiency is 100% as the compression process assumed to be isentropic. The variation of volumetric efficiency with clearance values and rpm is shown in Fig. 3-6. The model overpredicts volumetric efficiency. The predicted volumetric efficiency is constant because it only accounts for the clearance volume in the model. The volumetric efficiency is usually defined as the ratio of the actual volumetric flow rate at the inlet and the maximum volumetric flow rate. The temperature during compression is high (Fig. 3-3 (b)). Non-ideal gas properties should help. Including the masses/inertia of the pistons and swash plate will make the model more complete. Including friction, may give the model prediction an upward trend with rpm like experimental data.

Figure 3-5 Predicted and measured torque at (a) 2135 rpm (b) different rpm
Figure 3- 6 Variation of volumetric efficiency with (a) clearance values (b) rpm at 11% of clearance

3.6. Summary

This chapter presents an analytical/mathematical model for a 10-cylinder swash plate compressor with the emphasis on predicting its performance in terms of shaft torque. A kinematic model is developed to obtain the piston displacement as an explicit function of angle of rotation of the swashplate. The model of piston and swashplate dynamics is developed then by analyzing the interactions between forces and moments. The compression process model is formulated to determine the temperature and pressure inside the cylinder during one revolution of the swashplate along with the total mass flow rate in and out of the compressor. Some experimental validation comparing predicted and measured drive torque have been done to verify the analytical/mathematical model. The predicted and measured torque are in reasonable agreement with each other. The shaft torque can further be used for design and selection of the shaft at early stages of the design. It is reasonable to conclude that the main contribution to the torque can be explained by the compression of the gas. It is conceivable that the real gas properties can bring predictions closer to the experimental values since friction may move the predictions further from the experiment. Including the masses/inertia of the pistons and swash plate will make the model more complete. A more complete model is discussed in the following chapters and as expected, the main contribution to the torque can be attributed to the higher discharge pressure.
Bibliography


Chapter 4

Real Gas Model

Statement of contribution to co-authored published paper

This chapter includes a co-authored paper. The bibliographic details of the co-authored paper, including all authors, are:


My contribution to the paper involved: literature review, analytical modelling, experiment, writing and editing manuscript.

Signed: ------- Date: 10/08/2021

PhD Candidate (corresponding author of paper): Mohammad Arqam

Countersigned: ------- Date: 10/08/2021

Principal Supervisor (co-author): Dr. Peter Woodfield

Countersigned: ------- Date: 10 August 2021

Co-supervisor (co-author): Dr. Dzung Viet Dao
4.1 Abstract

A real-gas, restricted-flow valve model is compared with an ideal-gas, ideal-valve model for a 10-cylinder swashplate refrigeration compressor. Real gas properties of R134a are evaluated using the NIST standard reference database. A minor-loss discharge-coefficient approach is used to model the refrigerant flow rate through reed valves while the ideal-valve model requires no pressure difference to open the valve. In contrast with the ideal model, the discharge temperature, refrigerant mass flow rate and volumetric efficiency as a function of rotational speed are predicted well by including real-gas properties and flow restriction on the inlet valve. The ideal-gas model significantly overpredicts the discharge temperature and shows no dependence on rpm. Heat transfer to and from the cylinder wall during compression and expansion is found to have only a small effect on predictions of compressor performance. The valve model for the suction side has the largest influence on compressor performance predictions as a function of rpm.

4.2. Introduction

With the rise of electric vehicles and the desire to transfer mechanical-drive mobile refrigeration technology into the electrical drive context, it is valuable to revisit the theory and dynamics of swash-plate refrigeration compressors. Swash-plate compressors are a good choice for this investigation since they are popular for mobile refrigeration and air conditioning thanks to their high capacity and compact size [1]. The goal of this chapter is to propose a practical model that can explore the effects of non-ideal gas behavior, heat transfer in the cylinder and pressure loss across the reed valves. The focus is on rotational speed dependent performance measures for swash-plate compressors and the key contribution is demonstrating the relative importance of the studied input parameters.

4.3. Methodology

Fig. 4-1 defines the geometrical parameters and gives an overview of the model. The model links the torque, angular position and angular velocity of the swashplate with the position, pressure, and temperature of the refrigerant in the cylinder and with the flow rates through the reed valves. An overall assumption for the model is that the suction and discharge pressures always remain constant. The compressor considered in this study has five double-ended pistons (i.e. 10-cylinder compressor) but only one is shown in Fig. 4-1 for clarity.
4.3.1 Kinematics and solid dynamics

From the geometry of the mechanism, the displacement $Z_i$ from Bottom Dead Centre (BDC) of the $i$th piston is given by

$$Z_i = R \tan(\psi)(1 - \cos(\phi_i - \theta))$$  \hspace{1cm} (4.1)

where $R$ is the radial position of the point of contact of the piston on the swashplate, $\theta$ is the angular position of the lowest point on the swashplate (i.e. point closest to the electric motor) and $\phi_i$ is the angular location of the $i$th piston measured using the same coordinate system. Thus when $\theta = \phi_i$, the $i$th piston is at bottom dead center and $Z_i = 0$.

By differentiating Eq. (4.1) with respect to time, the velocity of the piston is related to the angular velocity as:

$$v_i = -R \tan(\psi) \omega \sin(\phi_i - \theta)$$  \hspace{1cm} (4.2)

Neglecting the mass of the cylinder and viscous/friction forces, Newton’s second law applied
to the piston reduces to a simple force balance where the pressure \( P_i \) in the cylinder is related to the force \( F_i \) applied by the swashplate (normal to the swashplate surface) as:

\[
F_i = \frac{P_i \pi D^2}{4 \cos \psi}
\]  

(4.3)

For any angular position, if the mass of the swashplate and bearing resistance torque is neglected, the rotational torque that needs to be supplied to the swashplate is:

\[
\tau = R \sin(\psi) \sum_i F_i \sin(\phi_i - \theta)
\]  

(4.4)

Note that because the pistons are double-ended it does not matter whether \( P_i \) is gauge pressure or absolute pressure once substituted into Eq. (4.4). This occurs because the difference in pressure between the opposite ends of the double piston provides the net force to the swashplate so atmospheric pressure cancels out. Note also that the sign of the velocity and force needs to be opposite for pistons on the other side of the swashplate to the one shown in Fig. 4-1 to get the direction of the force correct.

### 4.3.2 Thermodynamics

By neglecting kinetic energy and potential energy, the law of conservation of energy can be written as:

\[
(m_{in} h_s - m_{out} h_d) dt - h_{coeff}. A_{cyl} (T_{cyl} - T_{wall}) dt + P_{cyl} \frac{\pi}{4} D_{cyl}^2 dz
\]

\[
= m_2 u_2 - m_1 u_1
\]  

(4.5)

Where \( dt \) is an incremental change in time and \( dz \) is the corresponding change in piston position. The subscripts 1 and 2 denote before and after the incremental time change, respectively. The first term on the left-hand side of Eq. (4.5) represents energy transfer by mass, the second term, heat transfer to the cylinder wall and the third term, boundary work due to compression/expansion.

Similarly, conservation of mass is generally given by:

\[
\frac{dm_{cyl}}{dt} = m_{in} - m_{out}
\]  

(4.6)
where $\dot{m}_{in}$ is the mass flow coming into the cylinder, $\dot{m}_{out}$ is the flow escaping through discharge valve and $dm_{cyl}$ is the change of mass inside the cylinder over the time period $dt$.

Equations (4.5) and (4.6) can be simplified for each of the four processes (suction, compression, discharge and expansion).

### 4.3.2.1 Compression

The compression process starts when the piston makes displacements (infinitesimal) towards Top Dead Centre (TDC) by compressing the refrigerant inside the cylinder, and as a result the in-cylinder pressure and temperature increase. The mass balance leads to a simple term for a closed system:

$$\frac{dm}{dt} = 0 \quad (4.7)$$

Further, the energy balance simplifies to:

$$u_2 = u_1 + \frac{1}{m} \left\{ (P_{cyl} \frac{\pi}{4} D_{cyl}^2 dz - h_{coeff}.A_{cyl}(T_{cyl} - T_{wall})dt \right\} \quad (4.8)$$

### 4.3.2.2 Discharge

The discharging process starts when the in-cylinder pressure exceeds the discharge pressure. Regarding the in-cylinder gas volume and neglecting backflow the continuity equation may be written as follows:

$$\frac{dm}{dt} = -\dot{m}_{out} \quad (4.9)$$

If the in-cylinder pressure exceeds the discharge pressure, the valve opens but flow is restricted through a minor loss coefficient. Eq. (4.10) shows the relation, which is appropriate for relating flow rates to pressure losses of fluids flowing through an orifice and is often used for reciprocating compressors [2-5]:

$$\frac{dm}{dt} = C_d \rho A_d \sqrt{\frac{2(P - P_d)}{\rho}} \quad (4.10)$$

$A_d$ is the flow area through discharge valve and $C_d$ is a flow resistance coefficient. $P$ and $\rho$ are in-cylinder gas pressure and density. While various empirical relations for estimating the flow
rate can be found (e.g. [6]), Eq. (4.10) has the merit that if flow resistance coefficient is taken to be constant, the same relation can be used for different fluids. Rearrangements of Eq. (4.10) have also been proposed [7, 12].

Finally, during discharge, the first law of thermodynamics (Eq. 4.5) simplifies to:

\[
\frac{m_1}{m_2} u_1 + \frac{1}{m_2} \left\{ P_{cyl} \frac{\pi}{4} D_{cyl}^2 dz - \dot{m}_{out} h_d dt - h_{coefficient} A_{cyl} (T_{cyl} - T_{wall}) dt \right\} \quad (4.11)
\]

### 4.3.2.3 Expansion

After the piston changes direction, a certain amount of refrigerant is present in the clearance volume at high pressure and expands as the piston moves away from TDC. The procedure and equations of the expansion are similar to the compression step. Continuity leads to:

\[
\frac{dm}{dt} = 0 \quad (4.12)
\]

The energy balance is given by:

\[
u_2 = u_1 + \frac{1}{m} \left\{ (P_{cyl} \frac{\pi}{4} D_{cyl}^2 dz) - h_{coefficient} A_{cyl} (T_{cyl} - T_{wall}) dt \right\} \quad (4.13)
\]

### 4.3.2.4 Suction

The suction process starts when the pressure in the cylinder falls below the suction pressure as the piston goes down to the Bottom Dead Centre (BDC). The suction model follows the same concept as the discharging model and leads to the following equations.

\[
\frac{dm}{dt} = \dot{m}_{in} \quad (4.14)
\]

The valve model is like Eq. (14) except that the flow is driven from outside.

\[
\frac{dm}{dt} = c_5 \rho_s A_s \sqrt{\frac{2(P_s - P)}{\rho_s}} \quad (4.15)
\]

Conservation of energy leads to

\[
u_2 = \frac{m_1}{m_2} u_1 + \frac{1}{m_2} \left\{ P_{cyl} \frac{\pi}{4} D_{cyl}^2 dz + \dot{m}_{in} h_s dt - h_{coefficient} A_{cyl} (T_{cyl} - T_{wall}) dt \right\} \quad (4.16)
\]
The pressure and temperature of the refrigerant in the cylinder are calculated when the piston is located at the BDC and this is the beginning of the compression process.

4.3.3 Real gas properties

To calculate properties, two independent thermodynamic properties are required, and then other properties such as temperature, enthalpy and pressure can be determined via equations of state. The two key properties are density and internal energy which are calculated and updated from conservation of mass and first law of thermodynamics as outlined above. Equations of state for R134a are conveniently implemented using the NIST property database software, REFPROP [10] which uses the equation of state by [8] for R134a.

4.3.4 Clearance volume, flow resistance coefficient and heat transfer coefficient

The above model requires geometrical details and two coefficients. For this study, these parameters were estimated to obtain realistic values and then the model sensitivity to each parameter was investigated. The values used are given in Table 4.1. The heat transfer coefficient is assumed to be a constant estimated from empirical correlations reported by [9]. More elaborate approaches for in-cylinder heat transfer can also be found in a recent review by [9]. The values of $C_d$ and $C_s$ for use in Eqs. (14) and (19) are quite uncertain, given that self-actuating valves are considerably more complex than the minimum viable flow model under consideration here (e.g. see [4], [11-13]) for more comprehensive approaches). $A_d$ and $A_s$ are nominal flow areas due to ambiguity associated with the complicated geometry.

![Table 4.1 Key Model Input Data (Geometry corresponds to the tested compressor)](image)

- **Compressor capacity**: 200 cc
- **No. cylinders**: 10
- **Clearance volume**: 1 cc
- **Heat transfer coefficient**: 80 Wm$^{-2}$K$^{-1}$
- **$C_d$**: 0.5
- **$C_s$**: 0.5
- **$A_d$**: 0.5 cm$^2$
- **$A_s$**: 0.5 cm$^2$

4.3.5 Numerical implementation

The model was coded up in Visual Basic and implemented as a macro function in an Excel spreadsheet. The NIST property database software was dynamically linked to the spreadsheet so that equation of state functions could be called directly and automatically updated. Equations (4.7) to (4.16) were solved using an explicit time-marching scheme. The differential $dt$ was replaced with a finite time step and the updated properties were calculated depending on the stage of the compressor cycle. During suction and discharge, the continuity equation was
solved first to obtain the new mass, \( m_2 \). Following this, Eq. (4.8), (4.11), (4.13) or (4.16) was solved to obtain \( u_2 \) which is the specific internal energy. With the new mass (via Eq. (4.10) or (4.15)) and volume determined via Eq. (4.1), the state was defined using density and internal energy. The discharge stage of the compression cycle was found to be the most sensitive to the size of the time step. To resolve this issue, during discharging 20 to 40 times the number of time steps when compared with the other stages were found sufficient for time-step size independence. The temperature and pressure (\( T_{\text{cyl}} \) and \( P_{\text{cyl}} \)) are assumed to be constant for the duration of each time step (when using Eqs. (4.8), (4.11), (4.13) and (4.16)) and then properties are updated using the real-gas model.

### 4.3.6 Experiment:

To validate the model, a series of experiments were carried out at SuperCool Asia Pacific testing facilities at Ormeau, Australia. Fig. 4-2 shows a schematic diagram of the experimental setup. The key components are a fixed-displacement swash-plate compressor Unicla UP150 (150 cc rev\(^{-1}\)) or UP200 (200 cc rev\(^{-1}\)), a condenser KYSOR Westran KC90 (capacity performance of 7.6 kW), and a universal refrigerator microbus BEU-848L-100 (cooling capacity of 9.4 kW) and a thermal expansion valve (TXV). A drier and an oil separator were also present as auxiliary equipment. A 30 kW induction motor with a variable frequency drive was used to control the rotational speed of the compressor. A volumetric flow meter located at the exit to the evaporator was used to monitor the flow rate of refrigerant through the circuit. The components used in the experiment were connected through hoses to circulate refrigerant between three separate insulated rooms – compressor room, evaporator room and the condenser room. 10kW and 15kW electric heaters were set to supply heated air in the condenser and evaporator rooms.

The parameters controlled were the compressor speed, temperature, condensing pressure and air conditioning load. The compressor was driven by a variable frequency motor and the speed was adjusted by changing the power frequency. The temperatures were adjusted through controlling heat in the condenser and evaporator rooms. The condensing pressure can be adjusted by controlling the air flow rate through the condenser. The air conditioning load can be adjusted through controlling electric heating power in the evaporator room. The parameters measured were the compressor speed, refrigerant mass flow rate, the refrigerant temperatures and pressures at suction and discharge of the compressor. The compressor speed was measured by scaling the electric motor drive speed via the pulley diameter ratio, the mass flow rate was
measured with a Coriolis flow meter. The refrigerant temperature and pressure were measured with K-type thermocouples and electronic pressure sensors were connected to data loggers to send the information to a computer. The tests were carried out at fixed evaporator and condenser temperatures of -1 °C and 58 °C with a variable compressor speed from 750 to 5000 rpm.

![Schematic diagram of the experimental setup](image)

**Figure 4-2** Schematic diagram of the experimental setup

### 4.3.7 Uncertainty analysis of the experimental data

The measurement equipment have the following accuracies: k type thermocouple ±0.5 °C, refrigerant pressure ±2% of transducer full scale, refrigerant volume flow rate ±5%, compressor speed ±1% for 750 to 4000 RPM shown in Table 4.2. The effect of each variable on the volumetric efficiency has been investigated/estimated. The error propagation was also computed using the 2σ standard approach. The volume flow rate of refrigerant has the maximum effect on the volumetric efficiency however the refrigerant temperature and pressure have least.
Table 4. 2 Uncertainty analysis (2σ standard uncertainties)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty</th>
<th>Effect on volumetric efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coriolis flow meter</td>
<td>± 5%</td>
<td>5</td>
</tr>
<tr>
<td>Compressor rotational speed</td>
<td>± 1%</td>
<td>1</td>
</tr>
<tr>
<td>Total swept volume/capacity</td>
<td>1 cc</td>
<td>0.7</td>
</tr>
<tr>
<td>K-type thermocouple</td>
<td>± 0.5 °C</td>
<td>0.0</td>
</tr>
<tr>
<td>Pressure sensor</td>
<td>± 2% (Full scale)</td>
<td>2</td>
</tr>
<tr>
<td>Motor torque measurement</td>
<td>± 1%</td>
<td>-</td>
</tr>
<tr>
<td>Combined uncertainty</td>
<td></td>
<td>5.5%</td>
</tr>
</tbody>
</table>

4.4. Results and discussion

A series of experiments were conducted using 150cc and 200cc compressors for model validation. The model was then used to gain insight into the importance of thermodynamic properties, valve resistance and heat transfer.

4.4.1 Model validation

![Figure 4-3](image)

Figure 4-3 Comparison with experiment for 150 cc and 200cc compressor. a) volumetric efficiency, b) mass flow rate. The solid lines in (b) use the experimentally measured pressure ratio for each data point while the dashed line assumes a fixed pressure ratio.

Fig. 4-3 compares the model predictions with experimental data for volumetric efficiency and mass flow rate. For volumetric efficiency (Fig. 4-3 (a)), predictions are in reasonable agreement with measurements for the cases considered. The variation in volumetric efficiency in Fig. 4-3 (a) is predominantly due to changing the rpm with higher rpm corresponding to lower volumetric efficiency. Other variables such as temperature and fluid type also had some
effect. Clearly the real-gas, real-valve model is in better agreement with experiment than the ideal model for **Fig. 4-3 (b)**. The maximum deviation between the calculated and measured discharge temperature (not shown in Fig. 4-3) for the real-gas is less than 13%. The calculated values of mass flow rates are a little more than measured values and the maximum deviation between them is less than 0.005 kg.s\(^{-1}\). For the experiment, the inlet conditions and hence the pressure ratio changed with rpm. As a result, the ideal gas predictions in **Fig. 4-3 (b)** do not follow straight lines. To highlight that this is due to changes in experimental conditions rather than rpm, the ideal-gas, ideal-valve case with a constant pressure ratio is also plotted in **Fig. 4-3 (b)** as a dashed red line. As expected from Eqs. (2.1) and (2.2), ideal predictions are straight lines if the only condition varied is the rotational speed.

### 4.4.2 Effect of real gas properties on compressor performance

**Fig. 4-3** shows that the ideal gas model overpredicts the mass flow rate at high rpm. Maximum temperatures were also over predicted. This can be explained in part by looking at the variation of the key properties during the cycle, as shown in **Fig. 4-4**. It is quite evident that the real gas properties for both R134a and R1234yf are considerably different from those of the ideal gas which are assumed constant. **Fig. 4-4 (a)** shows that the specific heat capacity is lower for the suction side than for the discharge side with a variation of around 35% for the cycle for R134a and 40% for R1234yf. Moreover, it is higher during the compression stage than it is for expansion and refilling. Similarly, a 15% variation can be found for the ratio of specific heats, \(k\), for R134a and 20% for R1234yf. The compressibility, \(Z\) is 8 to 25% lower than the ideal gas value of 1. As shown in **Fig. 4-4 (c)**, the compressibility is lowest for small volumes (i.e. high temperatures and pressures).
Figure 4-4 Cyclic variation of R134a and R1234yf refrigerant properties for different rpms (200 cc compressor)

The low compressibility relative to an ideal gas shown in Fig. 4-4 (c), helps explain the difference between ideal gas and the real-gas model shown in Fig. 4-5 (a). During the compression stage, the lower compressibility means that a smaller volume can be reached before the pressure is high enough to open the discharge valve. Hence the compression curves in Fig. 4-5 (a) are shifted towards smaller volumes when compared with the ideal gas case. The low compressibility relative to ideal gas is also largely responsible for the higher discharge temperature prediction by the ideal gas model shown in Fig. 4-5 (b).

Fig. 4-5 also shows the effect of changing the rotational speed of the compressor. As the rpm is increased, the suction side in-cylinder pressure becomes lower and the discharge in-cylinder pressure becomes higher. This is resulting from the flow restriction due to the valves. At higher rpm there is less time for mass to flow through the inlet valve and hence the in-cylinder density is lower during suction, the pressure is lower and the energy balance (Eq. (4.16)) predicts the temperature is lower (Fig. 4-5 (b)). Likewise, on the discharge side, at high rpm, there is less time for the refrigerant to escape and the pressure becomes higher. The variations in temperature and pressure shown in Fig. 4-5 are the cause of the changes in $k$, $c_p$ and $Z$ with rpm shown in Fig. 4-4.
In-cylinder variation of pressure and temperature for different rpms (200 cc compressor)

4.4.3 Effect of heat transfer model

The heat transfer from the refrigerant to the cylinder walls via Newton’s law of cooling in all four stages of the compression cycle (Eqs. 4.8, 4.11, 4.13, 4.16) has a surprisingly small effect. For these simulations, the wall temperature was taken to be 55°C which is higher than the refrigerant temperature for more than half of the cycle (Fig. 4-5 (b)) so overall, a net heat transfer from the wall to the refrigerant might be expected. The form of the energy equation suggests that as rotation becomes faster the heat transferred per cycle to the wall will be smaller due to the smaller time interval. There is some indication of this in Fig. 4-6 (b) where the temperature difference between the cases of $h_{\text{coeff}} = 80$ Wm$^{-2}$K$^{-1}$ and 160 Wm$^{-2}$K$^{-1}$ are larger at lower rpm. However, overall Fig. 4-6 shows that the effect of heat transfer on torque and discharge temperature is small for heat transfer coefficients in the range from 0 to 160 Wm$^{-2}$K$^{-1}$). Volumetric efficiency was also found to be not affected greatly by heat transfer to the cylinder wall for heat transfer coefficients in the range from 0 to 160 Wm$^{-2}$K$^{-1}$).

Figure 4-6 Low sensitivity to in-cylinder heat transfer for R134a (200 cc compressor)
4.4.4 Effect of valve model

Variation of volumetric efficiency with rpm is found to be almost entirely due to the valve sub-model in this study (when the pressure ratio remains constant). Fig. 4-7 examines the effect of the clearance volume and the flow coefficients for the valve model. As a reference point, the simulation results are compared with the experimental correlation in Table 4.1 for this compressor. As expected, the clearance volume (Fig. 4-7 (a)) is important for the volumetric efficiency. It has the effect of uniformly increasing the volumetric efficiency with reduction in clearance volume without being influenced by rpm. The discharge valve flow coefficient, $C_d$ (Eq. 4.10), has negligible influence on the volumetric efficiency, as shown in Fig. 4-7 (b). The influence of the discharge valve is related to how much refrigerant remains when the piston reaches top dead centre. Fig. 4-5 suggests that at top dead center (end of the discharge process), similar temperature and pressures for a range of different rotational speeds and therefore the mass remaining in the cylinder is similar. On the other hand, the volumetric efficiency is quite sensitive to the suction valve flow coefficient, $C_s$ (Eq. 4.15) as shown in Fig. 4-7 (c). This is because any flow resistance for the suction valve influences how much gas enters the cylinder with less time to enter at higher rpm. The lower pressures and temperatures for higher rpm at bottom dead center shown in Fig. 4-5 are consistent with the sensitivity to suction valve conditions apparent in Fig. 4-7 (c).

![Effect of clearance volume](image1)

![Effect of discharge valve flow coefficient](image2)
4.4.5 Key contributions of this study and future work

The key contribution of this study is in assembling a practical model for a swashplate refrigeration compressor that accounts for the kinematics of the swashplate mechanism, valve flow restriction, real-gas properties and heat transfer, all together in one analytical model. While important studies have been published on swashplate compressors, to the authors’ knowledge this is the first swashplate study in the open literature that shows clearly the cyclic variation in thermophysical properties (Figs. 4-4 and 4-5 – including the effect of RPM and refrigerant type) and the dominating influence of the suction side inlet flow restriction on the sensitivity of the volumetric efficiency of a swashplate compressor to RPM (Fig. 4-7). For each of the sub-models considered, there are alternatives available in the literature of different levels of complexity. Here, a practical combination of models has been proposed and demonstrated against experimental data to be capable of capturing the essential physics without being overly complex. The model in this work could be extended in future studies to couple an electric drive and include rotational mass moment of inertia to capture the transient start up for investigation of torque loading during start conditions.

4.5 Summary

A real-gas, restricted-flow valve model is compared with an ideal-gas, ideal-valve model for a 10-cylinder swashplate refrigeration compressor. Real gas properties of R134a are evaluated using the NIST standard reference database. A minor-loss discharge-coefficient approach is
used to model the refrigerant flow rate through reed valves. This chapter has shown that by using a combination of real-gas properties for the refrigerant and a relatively simple flow model for the valves it is possible to achieve reasonably good volumetric efficiency predictions as a function of rotational speed for a swashplate compressor. Heat transfer from the refrigerant to the cylinder and the pressure drop across the discharge valve were found to be of lesser importance than the other parameters considered. The ideal gas model performed poorest in terms of predicting the discharge temperature. This may be attributed to the compressibility of the R134a being lower than unity. Generally, the real-gas model performed better than the ideal gas model. The most important consideration is the pressure drop across the suction valve in relation to volumetric efficiency prediction. The volumetric efficiency was found to be quite sensitive to the suction valve flow coefficient because any flow resistance for the suction valve influences how much gas enters the cylinder with less time to enter at higher rpm. The model could be enhanced further in future studies through consideration of the inertia of pistons and swashplate and the motor start-up torque.
Bibliography


Chapter 5

Transient Start-up

Statement of contribution to co-authored published paper

This chapter includes a co-authored paper. The bibliographic details of the co-authored paper, including all authors, are:


My contribution to the paper involved: literature review, analytical modelling, experiment, writing and editing manuscript.

Signed: -------  10/08/2021  Date: --------------------

PhD Candidate (corresponding author of paper): Mohammad Arqam

Countersigned:-------  10/08/2021  Date: --------------------

Principal Supervisor (co-author): Dr. Peter Woodfield

Countersigned:-------  10 August 2021  Date: --------------------

Co-supervisor (co-author): Dr. Dzung Viet Dao
5.1 Abstract:
The design of compact, high performance electric swashplate refrigeration compressors demands a clear understanding of different physical phenomena and their interactions taking place inside the compressor. The dynamic characteristics of the compressor are associated with the start-up transients of the swash-plate mechanism and the time variation of suction and discharge pressures. An experimentally validated, easy to implement transient swashplate compressor model has been developed that can capture the essential physics, including inertia of the pistons and swashplate to evaluate the electric motor torque loading during compressor start-up. The effects of moment of inertia, bearing torque, viscous resistance to piston motion, and suction and discharge pressures on the torque and compressor output power are investigated. For model validation, the start-up behavior is tracked experimentally using a high-speed data logger to monitor the changing phase currents of the brushless DC motor, capturing both the instantaneous power and rotational speed. Rotational mass moment of inertia is found to have only a small effect on the compressor torque and power output and can be made negligible by changing settings in the start-up algorithm for the electric motor controller. Suction and discharge pressures during start-up are found to have the largest influence on the required starting torque. More than 95% of torque is found to be because of the line pressures. Predictions are in good agreement with measurements and show that depending on the starting refrigerant pressures in the supply lines, the starting torque can be lower than the operating torque for the compressor. The original contribution of this work is in deriving a transient swash-plate compressor model that includes the inertia of the swash-plate mechanism and in clarifying the relative importance of inertia, line pressures, viscous losses and bearing resistance on the required start-up torque for this type of compressor.

5.2 Introduction
With the increase of environmental issues and sustainability, there is great value in developing the next generation of high-performance electric-vehicle refrigeration compressors such that they are more compact and more reliable, with higher efficiency, lower noise levels and reasonable cost. For electric drives, the maximum torque required during start-up and/or continuous running will decide the required size for the electric motor. On the mechanical side, compressor transient start-up is often related to the noise level, energy consumption and reliability of the compressor. During start-up, the operating conditions are critical since the equalized system pressure forces valves to perform poorly due to higher mass flow rates of the
refrigerant. Another drawback associated with lower speed is that a fully hydrodynamic lubrication layer may not be established in the bearings during start-up which may cause wear and increase mechanical losses [1].

In the light of the literature reviewed, it is found that almost no attempt has been made to investigate the start-up behaviour of an electric swashplate compressor taking into account the inertia of the pistons and swashplate, bearing torque, viscous resistance to piston motion, and suction and discharge pressures. This chapter deals with developing an experimentally validated, simple, easy to implement mathematical model to simulate start-up transients in reciprocating electric swashplate compressors. Since the initial few seconds are important for considering the effect of mechanical inertia as the mechanism accelerates, this study focuses on the first 1.4 s of time to investigate compressor start-up behaviour. The model is used to gain insight into the importance of inertia, bearing torque, viscous resistance to piston motion and the suction and discharge pressures.

5.2 The model

Fig. 5-1 shows the schematic model of the electric compressor and its geometrical parameters. The model links the torque, angular position and angular velocity of the swashplate with the position, pressure, and temperature of the refrigerant in the cylinder and with the flow rates through the reed valves. An overall assumption for the model is that the suction and discharge pressures remain constant or change slowly with time as a model input rather than a prediction. The compressor considered in this study has five double-ended pistons (i.e. 10-cylinder compressor) but only one is shown here for clarity.
5.2.1. Kinematics

Assuming the top surface of the swash plate is a single plane and \( \hat{n} \) is a unit normal vector to that plane, then \( \hat{n} \) is a function of \( \theta \), the angular position of the lowest point on the plate:

\[
\hat{n} = \sin(\psi) \cos(\theta) \hat{i} + \sin(\psi) \sin(\theta) \hat{j} + \cos(\psi) \hat{k}
\]  
(5.1)

Consider a point \( p \) on the plate in contact with the swash plate for one of the pistons located at an angle \( \phi \) shown in Fig. 5-1. The position vector of \( p \) for the coordinate system shown in Fig. 5-1 will be:

\[
\vec{r}_p = R \cos(\phi) \hat{i} + R \sin(\phi) \hat{j} + z_p \hat{k}
\]  
(5.2)

Because \( \vec{r}_p \) is in the plane for the plate, it will be at 90° to the normal and the dot product \( \vec{r}_p \cdot \hat{n} = 0 \). This gives:

\[
z_p = -R \tan(\psi) \left( \cos \phi \cos \theta + \sin \phi \sin \theta \right) = -R \tan(\psi) \cos(\phi - \theta)
\]  
(5.3)
Therefore, for the $i$th piston located at angle $\phi_i$ the displacement of the piston from bottom dead centre is given by:

$$Z_i = R \tan(\psi) \left(1 - \cos(\phi_i - \theta)\right)$$  \hspace{1cm} (5.4)

The velocity of the piston is related to the angular velocity by differentiating Eq. (5.4) with respect to time to get:

$$v_i = -R \tan(\psi) \omega \sin(\phi_i - \theta)$$  \hspace{1cm} (5.5)

Acceleration of the $i$th piston can be related to angular acceleration by differentiating again:

$$a_i = R \tan(\psi) \omega^2 \cos(\phi_i - \theta) - R \tan(\psi) \omega \alpha \sin(\phi_i - \theta)$$  \hspace{1cm} (5.6)

5.2.2. Dynamics

Fig. 5-2 shows a free-body diagram of the swash plate and shaft. We have assumed that force from each cylinder acts in the direction perpendicular to the plate. There are also some reaction forces at the bearings. Neglecting the tangential (friction) component of the force from the swashplate to the piston will cause a small under prediction of the torque required to drive the compressor.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{swash_plate_diagram.png}
\caption{Free-body diagram of swash plate.}
\end{figure}

Figure 5-2 Free-body diagram of swash plate.

For a rotation of a rigid body about the $z$-axis, Newton’s 2nd Law gives:

$$\sum M_z = I_z \alpha$$  \hspace{1cm} (5.7)
where $I_z$ is the rotational moment of inertia about the $z$-axis and $\alpha$ is the angular acceleration. $I_z$ includes the rotor of the electric motor and the swashplate. To solve Eq. (5.7) we need to find the torque given by each of the reaction forces from the pistons shown in Fig. 5-2. The moment vector of the force from the $i$th piston is given by the cross product:

$$\overline{M}_i = \overline{r}_i \times \overline{F}_i = -\overline{r}_p \times \hat{n}$$  \hspace{1cm} (5.8)

Substituting Eq. (5.1) and Eq. (5.2) into Eq. (5.8) and evaluating the component in the $z$-direction gives the torque about the axis exerted by the Force from the $i$th piston is:

$$\tau_i = -F_i R \sin(\psi) \sin(\phi_i - \theta)$$  \hspace{1cm} (5.9)

Substituting into Eq. (5.7) gives:

$$\tau_m - R \sin(\psi) \sum_{i=1}^{N} F_i \sin(\phi_i - \theta) - \tau_{bearing} = I_z \alpha$$  \hspace{1cm} (5.10)

Where $N$ is the total number of pistons.

Fig. 5-3 shows the free-body diagram for the piston:

![Figure 5-3 Free-body diagram of piston.](image)

Newton’s 2nd Law applied in the $z$-direction for the piston gives:

$$F_i \cos(\psi) - (P_1 - P_2)_i \frac{\pi D^2}{4} - \mu \frac{\pi DL}{8} v_i = m a_i$$  \hspace{1cm} (5.11)

where $P_1$ is the pressure in the cylinder, $D$ is the diameter of the cylinder, $L$ is the length of the double-ended piston in contact with the wall of the cylinder, $\delta$ is the gap between the piston
and the cylinder, $\mu$ is the dynamic viscosity of the lubricant, $v_i$ is the velocity of the piston, $m$ is the mass of the piston and $a_i$ is the acceleration of the piston. To get the angular acceleration, Eqs. (5.10) and (5.11) were solved together with Eqs. (5.4), (5.5) and (5.6).

5.2.3. Pressure inside the cylinder

The pressure in the cylinder $P_i$, needed in Eq. (5.11) is quite complicated. A simplified model adopted for this study:

1. Adiabatic compression until pressure equals the high-pressure side of the refrigeration circuit as the piston goes up.
2. Constant pressure as the piston pushes refrigerant through the valve (piston going up)
3. Short period of adiabatic expansion until pressure inside the cylinder reaches the low-pressure side of the refrigeration cycle as piston goes down. This will depend on $Vol_{\text{min}}$ in Fig. 5-1.
4. Constant pressure as new refrigerant enters the cylinder until the bottom of the stroke.

This was implemented in an excel spreadsheet using MAX and MIN functions. To a first approximation, the adiabatic processes could be modelled using isentropic equations for an ideal gas with constant specific heats (Eq. (5.12)). A more accurate approach would be to use tables of data for the refrigerant.

$$P = P_0 \left( \frac{Vol_0}{Vol} \right)^k$$ (5.12)

In Eq. (5.12), $k$ is the ratio of specific heats for the refrigerant vapour. Based on data from NIST chemistry webbook, $k \approx 1.23$ is a reasonable estimate for R134a vapour.

For any position of the piston, pressure inside the cylinder is a function of suction and discharge pressures and can be calculated as:

$$P = \text{MAX} \left( P_{\text{suction}}, \text{MIN} \left( P_{\text{discharge}}, P_{\text{prev}} \left( \frac{Vol_{\text{prev}}}{Vol} \right)^k \right) \right)$$ (5.13)
Where $P_{\text{prev}}$ and $Vol_{\text{prev}}$ are the pressure and cylinder volume from the previous numerical time step in the computation.

### 5.2.4. Viscous resistance to motion

This will be a function of temperature. Fig. 5-4 shows an example of the viscosity of lubricant used by the compressor manufacturer of the compressor used for the experimental work. The third term on the left-hand side of Eq. (5.11) will increase in proportion to the viscosity and therefore will be higher at low temperatures.

![Dynamic viscosity of lubricants](image)

**Figure 5- 4 Dynamic viscosity of lubricants (as an example of its temperature dependence)**

To a reasonable approximation, the temperature dependence of liquid viscosities can be modelled using:

$$\mu = C e^{b/T}$$  \hspace{1cm} (5.14)

Where $b$ and $C$ are empirical constants for a particular liquid. For the analysis, lubricant PAG 46 HD was considered.

### 5.2.5. Initial and boundary conditions

The differential equations are 2\textsuperscript{nd} order due to the acceleration terms. Therefore two initial conditions are required for each equation. These are at time zero, every rigid body starts from rest and initial positions correspond to those predicted by the mechanism when $\theta$ is zero. Since we do not have a thermal model, we only need to express mechanical boundary conditions. Many of these are decided by the geometry (e.g. the cylinder is rigid and bearing surfaces are also rigid). Viscous friction is experienced in the cylinder and in the bearing a constant resistance torque is assumed. The pressure in the cylinder provides the most important mechanical boundary condition for the model. The range of pressures considered as mechanical
boundary conditions are listed in Table. 2. The suction pressure ranges from 100 to 540 kPa whereas the discharge pressure ranges from 590 to 1190 kPa. The initial/equalized pressure was assumed to be 460 kPa. The operating conditions considered here correspond to actual measurement.

5.3 Solving dynamic model for electric compressor

An initial condition for \( \theta \) and \( \omega \) was set so that Eq. (10) may be solved together with Eq. (5.11) to obtain \( \theta \) and \( \omega \) as a function of time. By rearranging terms, motor torque can be formulated as Eq. 19. Eqs. (5.4) – (5.6) were used to relate \( Z, v \) and \( a \) to \( \theta, \omega \) and \( \alpha \). This time-dependent system of equations was solved using a numerical scheme. Different terms were included or omitted to see their effect on the transient response of the compressor.

Noting it can be shown mathematically that

\[
\sum_{i=1}^{N} \sin\left(\frac{2\pi}{N} i - \theta\right) = \frac{N}{2} \quad (5.15)
\]

and

\[
\sum_{i=1}^{N} \sin\left(\frac{2\pi}{N} i - \theta\right) \cos\left(\frac{2\pi}{N} i - \theta\right) = 0 \quad (5.16)
\]

The required torque supplied by the motor can be evaluated using Eq. 5.17:

\[
\tau_m = a I_{eff} + \omega C_\omega + \tau_b - C_{cyl} \sum_{i=1}^{N} (P_1 - P_2) i \sin(\varphi_i - \theta) \quad (5.17)
\]

Where

\[
I_{eff} = I_z + \frac{N}{2} m R^2 \tan^2(\psi) \quad (5.18)
\]

\[
C_\omega = \mu \frac{N \pi D L}{2 \delta} R^2 \tan^2(\psi) \quad (5.19)
\]

\[
C_{cyl} = R \tan(\psi) \frac{\pi D^2}{4} \quad (5.20)
\]
5.3.1 Numerical Implementation

Assuming the angular acceleration is effectively constant over a small change in angle $\Delta \theta$ gives:

$$\omega \approx \omega_{prev} + \frac{\alpha}{\omega_{prev}} \Delta \theta$$  \hfil (5.21)

The angular position of the piston pair can be calculated as:

$$\varphi_i = \frac{2\pi}{5} \times i \quad (i = 1, 5)$$ \hfil (5.22)

The maximum volume inside the cylinder can be estimated using dead volume and swept volume as:

$$Vol_{\text{max}} = Vol_{\text{clear}} + R \tan(\psi) \frac{\pi D^2}{4}$$ \hfil (5.23)

Volume inside the cylinder at any time can be estimated using:

$$Vol_1 = Vol_{\text{max}} - Z_i \frac{\pi D^2}{4}$$ \hfil (5.24)

$$Vol_2 = Vol_{\text{clear}} + Z_i \frac{\pi D^2}{4}$$ \hfil (5.25)

Where superscripts 1 and 2 refer to symmetrically opposite cylinders.

The model was set-up in an excel spreadsheet to calculate the motor torque loading (Eq. 5.17) during the compressor start-up under two different conditions.

The effective moment of inertia due to piston and swashplate masses was calculated using Eq. 5.18. The second term on the right-hand side of Eq. 5.17 is the contribution of viscous resistance to the piston motion and its effect on the motor torque and was calculated using Eq. 5.19. The third term, bearing torque was assumed constant for the current model. The last term on the right-hand side of Eq. 5.17 is the pressure term. Pressures in the symmetrically opposite cylinders were calculated using Eq. 5.13. The discharge and suction pressures were assumed to be constant or varying with time as model inputs. The model input parameters are listed below in Table 5.1.
Table 5.1 Key model input data (Geometry corresponds to the tested compressor)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor capacity</td>
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<td>cc</td>
</tr>
<tr>
<td>No. Cylinders</td>
<td>10</td>
<td>--</td>
</tr>
<tr>
<td>Clearance volume</td>
<td>1</td>
<td>cc</td>
</tr>
<tr>
<td>Swashplate mass</td>
<td>0.21</td>
<td>kg</td>
</tr>
<tr>
<td>Swashplate radius</td>
<td>0.05</td>
<td>m</td>
</tr>
<tr>
<td>Piston mass</td>
<td>0.11</td>
<td>kg</td>
</tr>
<tr>
<td>Suction Pressure</td>
<td>100-540</td>
<td>kPa</td>
</tr>
<tr>
<td>Discharge Pressure</td>
<td>590-1190</td>
<td>kPa</td>
</tr>
<tr>
<td>Cylinder Diameter</td>
<td>0.031</td>
<td>m</td>
</tr>
<tr>
<td>Sp. Heat Ratio (R134a)</td>
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<td>--</td>
</tr>
<tr>
<td>Piston length</td>
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<td>m</td>
</tr>
<tr>
<td>δ</td>
<td>0.00009</td>
<td>m</td>
</tr>
<tr>
<td>Bearing torque</td>
<td>0.2</td>
<td>Nm</td>
</tr>
<tr>
<td>$P_{\text{initial}}$</td>
<td>460</td>
<td>kPa</td>
</tr>
</tbody>
</table>

5.4 Experiment

To investigate the compressor performance during transient start-up and to obtain data for model validation, an experiment was performed at SuperCool Asia Pacific testing facilities at Ormeau, Australia. Measurements included voltage supplied to the motor and high-speed sampling of electric motor phase currents. The purpose of the experiment was to capture the start-up transients in the swashplate compressor while limiting the attention to just a few seconds. Fig. 5-5 shows a schematic diagram of the experimental setup. The setup can be divided in three sections viz. motor controller power board, refrigeration circuit and electric drive (assembly of motor and compressor). The key components are a fixed-displacement swash-plate compressor Unicla UP200 (200 ccrev⁻¹), a condenser KYSOR Westran KC90 (capacity performance of 7.6 kW), and a universal refrigerator microbus BEU-848L-100 (cooling capacity of 9.4 kW) and a thermal expansion valve (TXV). A drier and an oil separator were also present as auxiliary equipment. A 4 kW brushless DC motor with a variable frequency drive was used to control the rotational speed of the compressor. The motor controller power board is made up of three pairs of power IGBTs arranged in a bridge structure. Each pair governs the switching of one phase of the motor. Both high and low-side IGBTs are controlled using Pulse-width modulation (PWM) to convert the dc voltage into a driving modulated voltage. The use of PWM allows the start-up current to be limited and offers precise
control over speed and torque. An amplifier is present to sense the current and send it to the micro-controller unit which is connected to a high speed data logger to supply the required data to a computer.

Electronic pressure transducers were selected to measure the pressure in suction and discharge lines. The refrigerant temperatures were measured with negative temperature coefficient (NTC) thermistors and k-type thermocouples. The components used in the experiment were connected through hoses to circulate refrigerant between compressor, evaporator and the condenser. At the beginning of the experiment, the compressor and pipelines were set to an adequate vacuum condition to remove humidity, air and other contaminants that might affect the overall performance of the system. And then the system was charged with an appropriate amount of R134a refrigerant. By adjusting the refrigerant amount and control valve settings, desired stabilized operating conditions could be achieved. The compressor was then switched on when the pressures in the supply and discharge lines equalized. The compressor was tested in 20 °C room temperature.

Figure 5- 5 Schematic diagram of the experimental setup.
5.4.1 Current measurements

The algorithm for the start-up of the brushless DC electric motor operates in three stages – alignment, open-loop and then closed loop. This allows the motor to start without the need for position sensing such as hall sensors. The three stages can be seen in Fig. 6 (a) which shows the measured phase currents. During the alignment phase the stator windings are energized and the rotor magnets pull the rotor into a known position. The magnetic field produced by the stator is then rotated without feedback during the open-loop stage. Once the motor has reached a preset speed, the algorithm transitions to closed-loop where feedback enters the algorithm via high-speed measurements of the phase currents.

Fig. 5-6 (a)-(c) show that there is a clear distinction between electric current patterns for closed-loop and open-loop operation. In Fig. 5-6 (a), closed loop setting performs better than open loop in terms of peak current limit. In the open loop setting, the peak current is 14 A however in the closed loop setting it is only 10 A. The pattern is different for open and closed loop also as depicted in Fig. 5-6 (b) and (c) where the open-loop case looks more sinusoidal. For both stages, measuring the time between peaks makes it possible to determine the instantaneous rotational speed of the rotor as a function of time.
Figure 5-6 Measured currents for compressor start-up. (a) 3-phase current, (b) open loop (100 ms), (c) closed loop (20 ms), (d) PWM pattern (2 ms)
5.4.2 Power measurements

The measured net output current from the stator windings was used to calculate the input power supplied to the motor. To do this calculation, the supply voltage and net current entering the motor windings is required. In Fig. 5-5, from Kirchoff's law it is apparent that the phase currents can be added together to determine the instantaneous net current into the motor. Fig. 5-6 (d) shows the Pulse-width modulation (PWM) pattern for a period of 2 ms under the closed loop setting. The PWM frequency was set at 10000 Hz. The instantaneous total current and hence the instantaneous power is obtained through adding the phases together and multiplying by the supply voltage. Fig. 5-7 shows measured electrical input power peaks for 1 ms in the open loop and closed loop settings. 600 DC voltage was supplied to the motor for the entire start-up process. At the beginning of the start-up the motor draws low current and hence the average power is low (530 W) during the first 1 ms as shown in Fig. 5-7 (a). As the rotational speed ramps up, motor starts to draw more power with an average of 1000 W as depicted in Fig. 5-7 (b). Under the closed loop operation, motor draws even more power with an average of 2500 W as can be seen in Fig. 5-7 (c). Another point to note here is the pulses become smoother as the controller switches from open to closed loop settings.
5.4.3 Uncertainty analysis of the experimental data

The measurement equipment has the following accuracies: k type thermocouple ±0.5 °C, refrigerant pressure ±2% of transducer full scale, compressor speed ±1% for 500 to 1800 RPM shown in Table 5.2. The high-speed current measurement uncertainty is largely due to uncertainty in the shunt resistors (1%). The DC power supply has an uncertainty estimated at ±2 V for the 600 V setting.
Table 5. 2 Measurement uncertainty

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Variable</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current transformer</td>
<td>Current measurement</td>
<td>&lt; 1% or 0.05A</td>
</tr>
<tr>
<td>DC power supply</td>
<td>Voltage measurement</td>
<td>&lt; 0.5% or 2V</td>
</tr>
<tr>
<td>K-type thermocouple</td>
<td>Temperature measurement</td>
<td>0.2% or ± 0.5 °C</td>
</tr>
<tr>
<td>Electronic pressure sensor</td>
<td>Pressure measurement</td>
<td>± 2% (Full scale)</td>
</tr>
</tbody>
</table>

5.5 Results and discussion

An experiment was conducted using a 200cc compressor for model validation. The model was then used to gain insight into the importance of rotational mass moment of inertia, bearing torque, viscous resistance to the piston motion, suction and discharge pressures respectively. Fig. 5-8 illustrates measured pressure transients in the suction and discharge lines. At the beginning of the start-up, the pressures change almost linearly until 5 seconds beyond which they settle at relatively steady values. The lines in Fig. 5-8 represent the model input pressures. Note that the rate at which the discharge pressure rises and suction pressure falls may be faster than those shown in Fig. 5-8 due to the slow response of the electronic pressure transducers. Sensitivity to the pressure rise rate is considered in Section 5.6. The initial pressure was taken to be constant at 4.6 bar (460 kPa) based on Fig. 5-8. For ‘hot start’ simulation, pressure values at the 6th second were taken as model inputs to predict the motor torque and compressor power output. This condition was used for most simulations since the phase currents showed that steady power output was reached much before 5 seconds. Other values were also used as model inputs to investigate their effects on the torque and power output.

Figure 5-8 Measured suction and discharge line pressures for two start-up cases.
5.5.1 Model validation

The angular speed of the motor as a function of time is an input to the model. Fig. 5-9 shows the model inputs used together with measured data for the rotational speed of the 200 cc swashplate electric compressor. As can be seen here, model inputs are in close agreement with measurements. Two cases were considered to illustrate the importance of compressor start-up in a considerably warm condition. Case 1 (Fig. 5-9 (a)) illustrates a quick start-up where compressor reaches maximum speed of operation in about 1.2 s while case 2 (Fig. 5-9 (b)) shows comparatively a slow start-up where compressor does not reach maximum operating speed in the same time. The difference here in both cases is predominantly due to the electronic motor controller setting. In the open loop setting, compressor rotational speed varies linearly in both cases. However, in closed loop, it ramps up quickly and then stabilized to the preset value for operation.

Figure 5-9 Revs per minute for the 200 cc swashplate compressor. (a) Case 1, (b) Case 2

Fig. 5-10 compares the measured input and predicted output power for the 200 cc swashplate electric compressor in two start-up scenarios. Figs. 5-10 (a) and (b) demonstrate input and output power in open and closed loop settings. At zero time (rotor stationary) the compressor draws approx. 500 W of input electrical power to aid for the rotor alignment. Like rotational speed, input and output powers vary linearly in the open loop stage and reach almost 1000 W in about 1.01 s and then when the controller starts working under the closed loop setting, the power shoots up to 2700 W in only 100 ms as depicted in Fig. 5-10 (a). The pulses get closer when the compressor speeds up and become stable under closed loop operation. In Fig. 5-10 (b) the input power goes up to 1400 W in the same time as in case 1 and this is due to compressor attaining maximum speed of operation in about 2 s (not shown here). The
efficiency of the compressor was also calculated as shown in Fig. 5-11. As expected, open loop settings are not efficient for both cases. It was read out to be more than 80% in closed loop setting for both cases. Case 2 is more efficient than case 1 in the closed loop setting. From Figs. 5-9, 5-10 and 5-11, it is quite evident that the model predictions are reasonable and the results are in close agreement with measurements.

The efficiency of the electric motor on steady operation was measured separately and found to be consistent with the steady running condition for Fig. 10. The efficiency shown represents the efficiency of the electric motor calculated by instantaneous mechanical input to the compressor divided by the instantaneous electrical power input to the electric motor.

\[ \eta = \frac{\text{Compressor mechanical input power}}{\text{Motor electrical input power}} \]  

(5.26)

Figure 5-10 Comparison with experiment for 200 cc compressor. (a) Case 1, (b) Case 2.
5.5.2 Fast Fourier Transform (FFT) Analysis

To estimate the frequency of operation, a FFT analysis was performed using 100 ms time span of closed loop measured input and predicted output power for the case 1. Fig. 5-12 (a) shows a section of Fig. 5-10 (a) under the closed loop setting. For the measured input power, 24 small pulses and 4 big pulses correspond to 10 pulses of the predicted output power that makes a complete revolution of the swashplate. The small pulses in the measured electrical input power are due to the 24 stator slots in the motor and the big ones refers to the 4 pairs of magnets mounted on the rotor. The pulses in the predicted output power are because of the 10 cylinders in the compressor. Figs. 5-12 (b) and (c) show the frequency predictions for the measured input and predicted output powers. Two peaks in Fig. 5-12 (a) correspond to rotor poles and stator slots however just one peak in Fig. 5-12 (c) depicts the mechanical frequency of the compressor.
Figure 5-12 FFT analysis for the compressor power. (a) Power (100 ms), (b) FFT (experiment), (c) FFT (model).

The absence of the mechanical frequency modes in the electrical power reading (Fig. 5-12b) shows that transients in the power input from the electrical side are dominated by the geometry of electric motor and are little influenced by the multiple cylinders on the compressor.

5.5.3 Effect of mass moment of inertia (MOI)

The first term on the right hand side of Eq. 5.17 is the contribution of effective mass moment of inertia from pistons and swashplate to the torque supplied by the motor and the output mechanical power of the compressor as demonstrated in Fig. 5-13. Two different values of angular acceleration ($\alpha = 1.5, 2 \text{ rad/s}^2$) was tested to see when the moment of inertia starts to have some influence on the compressor start-up performance. The torque (Fig. 5-13(a)) supplied by the motor is found to have been influenced by the mass moment of inertia, especially in the region when the angular acceleration is doubled. Fig. 5-13(b) shows simulations of the effect of mass moment of inertia on the compressor mechanical output power. Curves with and without the mass moment of inertia are nearly superimposed on each
other. Thus it is evident that the mass moment of inertia, even at different angular accelerations, does not have much effect on the compressor start-up performance in terms of torque or power output. There is a slight difference apparent in the region where the compressor speeding up the fastest (just after time = 1 s). As should be expected the difference in power due to including the mechanism inertia is connected to the angular acceleration. The pulses also become closer with the increase of angular acceleration.

![Figure 5-13 Effect of mass moment of inertia on (a) torque, (b) power](image)

**5.5.4 Effect of bearing torque (BT)**

The third term on the right-hand side of Eq. 5.17 is the contribution of bearing torque to the torque supplied by the motor and the output mechanical power of the compressor as demonstrated in Fig. 5-14. The bearing torque seems to have a little effect on the torque and power. Different values for the bearing torque ranging from 0.2 to 0.6 Nm were tested to see their influence. The bearing torque found to have a small effect on the supplied motor torque and compressor power output as depicted in Figs. 5-14 (a) and (b). There is a difference of 0.6 Nm observed when the bearing torque was assumed to be 0.6 Nm at the very beginning of the compressor start-up. For all other calculations besides those in Fig. 5-14, the bearing torque was assumed to be 0.2 Nm.
5.5.5 Effect of viscous resistance (VR)

The second term on the right hand side of Eq. 5.17 is the contribution of viscous resistance to piston motion. It influences the torque supplied by the motor and the output mechanical power of the compressor as demonstrated in Fig. 5-15. Similar to mass moment of inertia and bearing torque, viscous resistance also does not have much influence on the compressor start-up performance. Figs. 5-15 (a) and (b) show the required motor torque and the compressor power at different viscosity values depending on the lubricant (oil) temperature. It was found that the viscous resistance to piston motion becomes important when the lubricant viscosity is high (at the lower temperature (-10 °C) in this case). It was also noted from Figs. 5-15 (a) and (b), the viscous resistance comes into effect with rotational speed of the compressor as it seems to be of less importance until 0.5s of the compressor start-up. Afterwards, it has much influence in the closed loop operating zone with a difference of approximately 2 Nm torque and 400 W of compressor mechanical output power. In conclusion, viscous resistance is only effective when the compressor is exposed to a cold start-up scenario if the motor speed becomes high before the lubricant warms up sufficiently.
Figure 5-15 Effect of viscous resistance on (a) torque, (b) power

5.5.6 Effect of suction pressure

The terms involving $P_1$ and $P_2$ on the right hand side of Eq. 5.17 represent the contribution of suction pressure (and discharge pressure) to the torque supplied by the motor and the output mechanical power of the compressor as demonstrated in Fig. 5-16. For the calculations, the initial pressure (460 kPa) was assumed to be constant as listed in Table 5.1 and corresponds to measurements. The effect of suction pressure has been investigated by keeping the discharge pressure constant on the motor torque as shown by the simulations in Fig. 5-16 (a). It is clear that the suction pressure has a significant effect on the motor torque required by the compressor. At low suction pressures, compressor does not require high torque to aid the start-up process as it only needs 8 Nm of torque to be supplied at 100 kPa. However, as the compressor starts to draw more refrigerant, the torque needed would be higher as depicted in Fig. 5-16 (a). The torque required to start the compressor is in the range from 13 to 15 Nm for suction pressures ranging from 260 to 540 kPa. One more interesting thing to note here in Fig. 5-16 (a) is when the suction pressure goes above the initial pressure inside the cylinders, the pulses flatten towards the top under the open loop setting as demonstrated by yellow and blue lines in Fig. 5-16 (a). The output mechanical power from the compressor follows the same trend as the
torque supplied by the motor and it increases with the increase of refrigerant entering via the suction line as shown in Fig. 5-16 (b). The requirement of torque supplied from the motor is reduced when the amount of refrigerant in the suction line is controlled through the expansion valve in the evaporator.

![Figure 5-16 Effect of suction pressure on (a) torque, (b) power](image)

5.5.7 Effect of discharge pressure

The terms involving $P_1$ and $P_2$ on the right hand side of Eq. 5.17 also represent the contribution of discharge pressure to the torque supplied by the motor and the output mechanical power of the compressor as illustrated in Fig. 5-17. For these calculations, the initial pressure (460 kPa) corresponds to measured initial pressure as listed in Table 5.1. Effect of discharge pressure was considered by keeping the suction pressure constant on the motor torque as shown in Fig. 5-17 (a). Like suction pressure, discharge pressure is found to have significant influence on the required motor torque. At low discharge pressures, the torque required to start the compressor is only 6 Nm and 8 Nm with a lift of 1 Nm during the start of the closed loop operating zone. The output mechanical power from the compressor follows the same trend as the torque supplied by the motor.
5.5.8 Effect of Transient pressures

As shown in Fig. 5-8, due to slow response of pressure transducers it took 5 s for the suction and discharge pressure readings to reach the maximum pressure. The time to reach steady operation will depend on the volume of the tubes in the condenser and evaporator and the initial conditions. Therefore, it is worth considering this variation in pressures on the startup torque and power requirements. As depicted in Fig. 5-18 (a), the initial torque is dramatically affected by the transient pressure in the suction and discharge lines. This effect is much greater than any of the other parameters considered in this study. For cold start conditions the torque required is much lower than that required for steady operation. Therefore, when designing an electric drive for a swashplate compressor, it is sufficient to size the motor based on steady operation without consideration of the mechanism inertia.
Figure 5-18 Effect of transient pressures on (a) torque, (b) power (dP/dt corresponds to the rate of change of discharge pressure)

5.6 Summary

An experimentally validated, transient compressor model has been developed that can capture the essential physics of a refrigeration swashplate compressor, including inertia of the pistons and swashplate, viscous resistance and bearing resistance. The electric drive was a Brushless DC motor with a 3-phase variable speed controller which allowed access to shunt resistors for high-speed measurement of phase currents. The phase currents were used to obtain direct measurements of instantaneous rotational speed and instantaneous current draw and power during the first few seconds of startup. Model predictions are in good agreement with measurements. The effect of moment of inertia, bearing torque, viscous resistance and suction and discharge pressures on the torque and compressor output power were investigated using the model. Rotational mass moment of inertia was found to have only a small effect on the compressor torque and power output. Suction and discharge pressures were found to dominate the loading on the motor due to the compressor during startup. Thus, if the startup algorithm
for the motor is appropriately designed (limiting the angular acceleration) and suction and
discharge pressures are equalized before starting the motor, this study shows it is sufficient to
decide the size of the electric motor based on steady operating conditions for the swashplate
compressor. The findings will contribute to optimal dynamic system design and control
strategy to achieve efficient working condition for the electric swashplate compressor.
Bibliography

Chapter 6

Cooling Jacket Design

Statement of contribution to co-authored unpublished paper

This chapter includes a co-authored paper. The bibliographic details of the co-authored paper, including all authors, are:


My contribution to the paper involved: literature review, numerical modelling, experiment, writing and editing manuscript.

Signed: ------- Date: 10/08/2021

PhD Candidate (corresponding author of paper): Mohammad Arqam

Countersigned: ------- Date: 10/08/2021

Principal Supervisor (co-author): Dr. Peter Woodfield

Countersigned: ------- Date: 10 August 2021

Co-supervisor (co-author): Dr. Dzung Viet Dao
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Chapter 7

Stator Windings

Statement of contribution to co-authored unpublished paper

This chapter includes a co-authored paper. The bibliographic details of the co-authored paper, including all authors, are:


My contribution to the paper involved: literature review, numerical modelling, experiment, writing and editing manuscript.

Signed: ------- Date: 10/08/2021

PhD Candidate (corresponding author of paper): Mohammad Arqam

Countersigned: ------- Date: 10/08/2021

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Chapter 8

Thermal Potting Resin

Statement of contribution to co-authored unpublished paper

This chapter includes a co-authored paper. The bibliographic details of the co-authored paper, including all authors, are:


My contribution to the paper involved: literature review, experiment, writing and editing manuscript.

Signed: ------ Date: 10/08/2021

PhD Candidate (corresponding author of paper): Mohammad Arqam

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Principal Supervisor (co-author): Dr. Peter Woodfield

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Chapter 9

Conclusions and Future Recommendations

9.1 Conclusions

This thesis presents an extensive study on the thermomechanical analysis of a compact, high-performance electric swashplate compressor for mobile refrigeration and air-conditioning applications on commercial and heavy vehicles including industrial machinery. Compact electric compressors are the way of the future as the world moves towards a future dominated by electric vehicles. Considering the importance of this particular type of compressor in the mobile refrigeration and air-conditioning industry, it is worth developing analytical and numerical models that can capture the essential physics and thermal performance without being overly complex.

In this thesis, analytical and computational models of varying complexity were considered for design of electric swashplate compressors. Concerning the thermodynamics: ideal gas and real gas approaches were investigated. Concerning the kinematics and dynamics: the swashplate mechanism with and without inertia was considered. Concerning reed valve operation: an ideal model was compared with a minor-loss coefficient approach. Concerning thermal management, cooling jacket design, heat transfer from windings to stator and the use of thermal potting resin were considered.

The simplest analytical model neglected the inertia of components and assumed the refrigerant behaved as an ideal gas. The mathematical model includes kinematics, and the compression process of the swashplate mechanism to predict the performance of the compressor in terms of shaft torque and compressor mechanical power. Some experimental validation comparing predicted and measured drive torque was done to verify this mathematical model. It is concluded that the ideal gas model can be used to give a reasonable prediction of shaft torque for mechanical component design. The ideal gas model also shows that the main contribution to the torque is due to compression of the gas. Differences between the ideal gas model and experimental values can be explained largely through the neglect of the friction between the piston and the cylinder.
To explore the effects of real gas properties on the compressor thermomechanical performance, a practical real gas model was formulated. Real gas properties of R134a and 1234yf were evaluated using the NIST standard reference database. A minor loss discharge coefficient approach was used to model the refrigerant flow rates through the reed valves. It was shown that by using a combination of real-gas properties for the refrigerant and a relatively simple flow model for the valves, it is possible to achieve reasonably good volumetric efficiency predictions as a function of rotational speed for a swashplate compressor. The results showed that the heat transfer from the refrigerant to the cylinder and the pressure drop across discharge valve are found to be of lesser importance than the other parameters considered. The most important consideration was the pressure drop across suction valve in relation to volumetric efficiency prediction. The volumetric efficiency was found to be quite sensitive to the suction valve flow coefficient because any flow resistance for the suction valve influences how much gas enters the cylinder with less time to enter at higher rpm.

To further enhance the model through consideration of inertia of pistons and swashplate and the motor start-up torque, a transient compressor model was also developed and validated against experimental data. The model includes essential physics of the swashplate refrigeration compressor, viscous resistance to the piston motion and bearing torque. High speed measurement of phase currents was carried out and used to obtain direct measurements of instantaneous rotational speed and instantaneous current draw and power during the first few seconds of start-up. The effect of moment of inertia, bearing torque, viscous resistance and suction and discharge pressures on the torque and compressor output power were investigated using the model. The results showed that the moment of inertia has only a small effect on compressor torque and power output whereas suction and discharge pressures have the greatest effect on the motor torque loading. The analysis revealed that with appropriate start-up algorithm design and equalised suction and discharge pressures, it is sufficient to decide the size of the electric motor to drive the compressor based on steady operating conditions. The models developed are useful for optimal dynamic system design and for testing control strategies to achieve efficient working conditions for electric swashplate compressors.

Cooling performance of the electric motor is a further key consideration while designing a compact high-performance electric swashplate compressor. A numerical analysis of an air-cooled brushless Direct Current (BLDC) motor was performed using different fin arrangements on the motor housing. It was found that an improvement to the coefficient of performance of around 4% may be expected through using air cooling rather than refrigerant cooling for the
electric drive. The findings also demonstrate that useful estimations of the performance of the cooling jacket can be obtained from a 2D simulation at the expense of losing axial variations. For the practical flow velocities considered, greater enhancement was achieved by adding an extra fin in the cooling flow passage rather than through the inclusion of grooved walls.

Overheating stator windings is one of the major concerns limiting the life and output performance of electric motors. An experimental investigation followed by a numerical validation was carried out to check thermal resistance between windings and the stator core. The measurements were found to be in good agreement with predictions. The model suggests encapsulation materials with higher thermal conductivity could be added into the stator slot to overcome heat transmission issues in air-cooled electric drives. To confirm this, an experiment on 2.5 kW and 4 kW motors driving electric swashplate compressors was conducted to see how effective these encapsulants are if added in the stator winding slots. The results showed that the potting material can reduce the temperature of the windings by 10 °C to 20 °C for electrical power inputs of 2.4 kW to 3.8 kW. Moreover, the winding arrangement also affects the winding temperature significantly. The external case temperature of the motor did not vary much for all three cases showing that the most thermal resistance occurred between the copper windings and the stator electrical steel.

### 9.2 Future recommendations

As proved by this research, experimentally validated analytical and numerical models play an important role to assess the overall compressor performance at the early stages of design and development and may help compressor manufacturers to select the optimum parameters while designing electric swashplate compressors for mobile refrigeration and air-conditioning applications on commercial and heavy vehicles including industrial machinery. However, the work could be extended further in future studies. Therefore, some recommendations are as follows:

1. The models developed in this thesis can be extended to apply to other types of reciprocating compressors with different geometries such as linear compressors (which are currently attracting considerable research interest).

2. Different refrigerants can be tested to see the effect of thermophysical properties on the compressor thermomechanical performance. This is important particularly for exploring the effects of using newly proposed low global-warming potential refrigerants.
3. A more rigorous electric motor model could be included in future studies to identify the effects of different thermal loss mechanisms (viz. iron losses, copper losses and friction losses) and to make the model more complete and comparable with electrical measurements.

4. The compressor transient model can further be extended to capture the overall refrigeration circuit start-up behaviour. This would involve transient models for the condenser, evaporator and finite capacity of the heat exchanger tubes and supply lines. This would enable direct prediction of transient pressures in the circuit.

5. As shown, flow through suction valves is critical for the volumetric efficiency. Therefore a complete thermomechanical model of the thermal expansion valve (TXV) can be developed to more thoroughly investigate restrictions in the flow through the suction line and achieve higher performances in relation to volumetric efficiency.

6. The transient start-up model could further be used to identify optimal dynamic system designs and control strategies to achieve efficient working conditions for the electric swashplate.

7. Future studies may also investigate the effects of controller housing/heat sink design on the power-electronics components and the overall cooling performance of the electric drive.

8. A more complete electric compressor drive model could be developed in future studies to consider the effect of suction line pressure drop, leakage through reed valves, and the heat transfer between suction refrigerant and the motor to investigate their influence on the compressor volumetric efficiency. Moreover, a detailed analysis should be performed when compressor is operating under the heat pump cycle at low ambient temperature.
Appendix A

Analytical models

A.1. Working principal of swashplate compressor

The subject of the current study is a fixed displacement swashplate compressor with five double headed piston. The pistons are mounted along the periphery of the swashplate in such a way that the compression in one cylinder causes expansion in the symmetrically opposite cylinder. The compression process (Fig. A-1 (a)) starts when the piston makes displacements (infinitesimal) towards Top Dead Centre (TDC) by compressing the refrigerant inside the cylinder, and as a result the in-cylinder pressure and temperature increase. The discharging process (Fig. A-1 (b)) starts when in-cylinder pressure exceeds the discharge pressure. If the in-cylinder pressure exceeds the discharge pressure, the discharge valve opens but the flow is restricted through a minor loss coefficient. After the piston changes direction, a certain amount of refrigerant is present in the clearance volume at high pressure and expands as the piston moves away from TDC as shown in (Fig. A-1 (c)). The suction process (Fig. A-1 (d)) starts when the pressure in the cylinder falls below the suction pressure as the piston goes down to the Bottom Dead Centre (BDC).
A.2. Model validation for real gas

Fig. A-2 compares the model predictions with experimental data for discharge temperature, volumetric efficiency and mass flow rate. Clearly the real-gas, real-valve model is in better agreement with experiment than the ideal model. For the experiment, the inlet conditions and hence the pressure ratio changed with rpm. As a result, the ideal gas predictions in Fig. A-2 do not follow straight lines. To highlight that this is due to changes in experimental conditions rather than rpm, the ideal-gas, ideal-valve case with a constant pressure ratio is also plotted in Fig. A-2 as a dashed red line. As expected from Eqs. (2.1) and (2.2), ideal predictions are straight lines if the only condition varied is the rotational speed.
Figure A-2 Comparison with experiment for 150 cc compressor. The solid lines use the experimentally measured pressure ratio for each data point while the dashed line assumes a fixed pressure ratio.

When the detailed behavior for one cylinder is added to that of all ten cylinders, the net inputs and outputs of the compressor can be determined as shown in Fig. A-3. The red lines correspond to the ideal-gas, ideal-valve model while the black lines represent the more realistic model. The ten pulses correspond to the ten cylinders. The outlet mass flow rate (dashed line in Fig. A-3 (a) and (b) shows much greater variation than the inlet. This is due to the shorter period of discharge compared to suction as shown in Fig. 4-4 (a). A longer period of suction means greater periods of overlap between the various cylinders and hence smaller suction pulses. In terms of mass flow rate, the ideal gas and real gas calculations give surprisingly similar results (~0.06 kg/s) in Fig. A-3 (a) and (b) despite the reduced compressibility as shown in Fig. 4-4 (c) for the real gas case. This can be explained by the observation that the valve opens a bit earlier for the ideal gas as shown in Fig. 4-4 (a).
Figure A- 3 Cyclic variation of mass flow rate and torque (200 cc compressor at 1800 rpm)

Fig. A-3 (c) shows that the torque required for the refrigerant with real-gas properties is lower than that required for an ideal gas. The torque fluctuation is similar for both cases. The lower average torque for the real gas case is expected from a consideration of Fig. 4-4 (a). The area enclosed by the cycle in Fig. 4-4 (a) represents the net work out per cycle. At higher rpm, the area enclosed by the real gas is smaller than the ideal gas case and hence the torque is lower (see also Fig. 4-6 (a)). Note here that the model does not account for viscous friction in bearings and between the piston and cylinder wall. Unlike the work of compression, the contribution of viscous friction to the torque is expected to increase as velocities increase and the total torque may increase.
Appendix B

Transient start-up

B.1. Raw power data for hot start-up

Figure B-1 Raw power data for hot start
B.2. Raw power data for cold start-up:
Figure B-4 Smoothed data for cold start
Appendix C
Thermal management

C.1. Transient heat-up of BLDC (bench-top motor)

Transient heating calculations were done with ANSYS fluent for the BLDC bench-top motor. The mesh used for the calculation is shown in Fig. C-1. Calculations were done for half of the domain to achieve for grid independent results.

![Computational Mesh for 3D simulation](image)

Fig. C-1 Computational Mesh for 3D simulation

Fig. C-2 shows the calculated steady state temperature distributions for 9m/s of air velocity without extended surfaces. Testing with a fine mesh lead to higher computational cost so calculations were done on half of the domain using a bit coarser grid.

![Steady-state temperature distribution at 9m/s](image)

Figure C-2 Steady-state temperature distribution at 9m/s
Transient (unsteady-state) heating calculation was done at (25°C) ambient temperature to see how long the motor should take to reach steady state temperature from room temperature. The velocity of air was assumed to be 9m/s and the initial temperature of the motor was same as that of the surrounding temperature. Fig. C-3 shows the variation of temperature with respect to time until the steady-state temperature achieved. The time increment was set to be 50 sec and the total time was 9000 sec.

![Transient Heating (Bench-top motor)](image)

Figure C-3 Transient heating at 9m/s

The calculation results suggest that in order to get the steady state temperature from room temperature in the motor at 9m/s of air velocity, we should wait for around 136 min.

**C.2 Transient heat-up of BLDC at different velocities**

Transient heating calculations were also done for the BLDC bench-top motor with 3m/s, 9m/s air flows and 400, 700 W heat generation respectively. Calculations were done for half of the domain to achieve for grid independent results. Fig. C-4 shows the calculated steady state temperature distributions for 3m/s, 9m/s of air velocities without extended surfaces. Testing with a fine mesh led to higher computational cost so calculations were done on half of the domain using a bit coarser grid.
Transient (unsteady-state) heating calculations were done at (25, 26°C) room temperatures to see how long the motor should take to reach steady state temperature from the room temperature. The velocities of air were assumed to be 3m/s, 9m/s and the initial temperature of the motor was same as that of surrounding temperature. Fig. C-5 shows the variation of temperature with respect to time until the steady-state temperature reached. The time increment was set to be 50 sec and the total time was 300 min.

The calculation results suggest that in order to get the steady state temperature from room temperature in the motor at 3m/s, 9m/s of air velocities, we should wait for around 136, 192 min respectively.
Appendix D
Heat up on suction side

It was found experimentally that the suction line inlet temperature is lower than the measured inlet temperature on the PT sensor for electric compressor drive. Based on simulations, the corrected temperature for when the compressor is at 1800 rpm can be estimated using:

\[ T_{suction} = T_{meas\ in} - 0.133 \times (T_{metal} - T_{meas\ in}) \]

where \( T_{meas\ in} \) is the PT suction side (inlet) sensor reading and \( T_{metal} \) is the temperature of the compressor housing. The correction becomes larger at lower rpm.

D.1. Theory

Heat transfer takes place via convection in the flow passage between the suction side inlet and the location of the suction side pressure/temperature transducer. The flow passage for the 200 cc compressor is approximately as shown in Fig. D-1.
Figure D-1 (a) Compressor geometry (b) flow passage for refrigerant

The goal is to estimate $T_{suction}$ shown in Fig. D-1 based on $T_{meas.\_in}$:

Using the basic principle of heat transfer in a duct with isothermal walls, the temperatures can be related using a log-mean temperature difference:

$$
\dot{m}c(T_{meas.\_in} - T_{suction}) = hA \Delta T_{lm}
$$

(D.1)

Where the log-mean temperature difference is:

$$
\Delta T_{lm} = \frac{(T_{metal} - T_{suction}) - (T_{metal} - T_{meas.\_in})}{ln \left( \frac{T_{metal} - T_{suction}}{T_{metal} - T_{meas.\_in}} \right)}
$$

The above equations can be rearranged to give:

$$
T_{suction} = T_{meas.\_in} - (e^{hA/(\dot{m}c)} - 1)(T_{metal} - T_{meas.\_in})
$$

(D. 2)

Where $h$ is the heat transfer coefficient, $A$ is the surface area, $\dot{m}$ is the mass flow rate and $c$ is the specific heat capacity. For turbulent flow, the heat transfer coefficient $h$ can be found using the Dittus Boelter equation:

$$
Nu_D = 0.023Re_D^{0.8}Pr^{0.4}
$$

(D. 3)

Where

$$
h = \frac{k}{D}Nu_D
$$

(D. 4)

Where $k$ is the thermal conductivity, $Pr$ is the Prandtl number and $D$ is the hydraulic diameter.
The mass flow rate of refrigerant along the flow passage shown in Fig. D-1 is related to RPM, volumetric efficiency, $\eta_{vol}$, compressor size, piston geometry and fluid density:

$$\dot{m} = \rho \eta_{vol} \frac{Vol_{comp} \cdot RPM}{60} \quad (D. 5)$$

For the 200 cc compressor, $Vol_{comp} = 0.0002 \text{ m}^3$. This is divided by 6 in the above equation since there are three flow passages to the back of the compressor and half of the refrigerant enters from the front. Reynolds number can be determined as follows:

$$Re_D = \frac{\rho vD}{\mu} = \frac{\dot{m}D}{\mu A_c} = \frac{\rho \eta_{vol}Vol_{comp} \cdot RPM \cdot D}{360 \mu A_c} \quad (D. 6)$$

Making use of Eqs. (3) to (6) we can write:

$$\frac{hA}{mc} = 0.023 \frac{A}{A_c} Pr^{-0.6} \left( \frac{D \rho \eta_{vol}Vol_{comp} \cdot RPM}{360 \mu A_c} \right)^{-0.2} \quad (D. 7)$$

Eqs (D. 2) and (D. 7) can be used to find the suction temperature. For 1800 rpm and a range of different metal temperatures

$$e^{\ln A/\dot{m}c} - 1 \approx 0.133$$

This means that for target operating conditions

$$T_{suction} \approx T_{meas_{in}} - 0.133(T_{metal} - T_{meas_{in}}) \quad (D. 8)$$

If we want to account better for RPM, we could use the following form with Table D. 2 giving the values of $C$

$$T_{suction} \approx T_{meas_{in}} - C(T_{metal} - T_{meas_{in}}) \quad (D. 9)$$

Alternatively, we can account for RPM by using

$$T_{suction} = T_{meas_{in}} - (e^{0.559 \cdot RPM^{-0.2}} - 1)(T_{metal} - T_{meas_{in}}) \quad (D. 10)$$
Figure D- 2 ‘C’ in Eq. D.9 is not sensitive to metal temperature

Figure D- 3 ‘C’ in Eq. D.9 is only slightly sensitive to suction superheat
Table D. 1 Range of parameters considered for Figs. D-2 and D-3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of slot</td>
<td>0.115 m</td>
</tr>
<tr>
<td>Perimeter of slot</td>
<td>0.085 m</td>
</tr>
<tr>
<td>Cross section area</td>
<td>0.00025 m$^2$</td>
</tr>
<tr>
<td>$T_{metal}$</td>
<td>45 °C to 70 °C</td>
</tr>
<tr>
<td>Suction side $T_{sat}$</td>
<td>-1 °C</td>
</tr>
<tr>
<td>Suction superheat</td>
<td>0.1 to 20 °C</td>
</tr>
<tr>
<td>Refrigerant</td>
<td>R134a</td>
</tr>
</tbody>
</table>

Table D. 2 Values to use with Eq. D.9

<table>
<thead>
<tr>
<th>RPM</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.2148</td>
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<tr>
<td>250</td>
<td>0.2045</td>
</tr>
<tr>
<td>300</td>
<td>0.1964</td>
</tr>
<tr>
<td>350</td>
<td>0.1898</td>
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<tr>
<td>400</td>
<td>0.1843</td>
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<tr>
<td>450</td>
<td>0.1796</td>
</tr>
<tr>
<td>500</td>
<td>0.1755</td>
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<tr>
<td>550</td>
<td>0.1719</td>
</tr>
<tr>
<td>600</td>
<td>0.1687</td>
</tr>
<tr>
<td>650</td>
<td>0.1658</td>
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<tr>
<td>700</td>
<td>0.1631</td>
</tr>
<tr>
<td>750</td>
<td>0.1607</td>
</tr>
<tr>
<td>800</td>
<td>0.1585</td>
</tr>
<tr>
<td>850</td>
<td>0.1564</td>
</tr>
<tr>
<td>900</td>
<td>0.1545</td>
</tr>
<tr>
<td>950</td>
<td>0.1527</td>
</tr>
<tr>
<td>1000</td>
<td>0.1510</td>
</tr>
<tr>
<td>1050</td>
<td>0.1494</td>
</tr>
<tr>
<td>1100</td>
<td>0.1479</td>
</tr>
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<td>1150</td>
<td>0.1465</td>
</tr>
<tr>
<td>1200</td>
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<td>1250</td>
<td>0.1439</td>
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<td>1300</td>
<td>0.1427</td>
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<tr>
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<tr>
<td>1450</td>
<td>0.1394</td>
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<tr>
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<tr>
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</tr>
<tr>
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</tr>
<tr>
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<tr>
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<td>1800</td>
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</tr>
<tr>
<td>1850</td>
<td>0.1323</td>
</tr>
</tbody>
</table>
D.2. Estimating compressor metal temperature

One weakness with Eq. D. 9 is that the compressor metal temperature is not measured with our current hardware arrangement. At a guess, one would expect it to be somewhere between the suction inlet temperature and the discharge temperature. The actual value for $T_{\text{metal}}$ will also be influenced by the motor temperature and the environment temperature. Some experimental data would help to obtain a good estimate based on available measurements.

D.3. Estimating suction line temperature

Thermal tests were done on the 200 cc unit in the large test chamber for temperatures from 25 °C to 40 °C using R134a. It is found that suction and discharge line temperatures can be estimated by extrapolating and interpolating from the on-board measured temperatures by the PT sensors.

The inlet suction temperature estimations are found to be:

$$T_{\text{suction}} = T_1 - 0.71 \times (T_2 - T_1)$$

(D.11)

where $T_1$ is the PT sensor suction temperature and $T_2$ is the PT sensor discharge temperature.

Fig. D-4 shows the key measurements and the proposed interpolation and extrapolation. The interpolation and extrapolation work quite well except in the transient region (e.g. time = 380 min in Fig. 1). In the previous experiment, the readings for PT$_1$ and PT$_2$ were swapped around in the firmware at about 80 min. This swap has been corrected in Fig. D-4.
D.4. Summary

If we have a good estimate of the metal temperature then Eq. (D. 9) together with Table D.2 will be effective for estimating the suction inlet temperature including the effect of RPM. If we simply assume the metal temperature is a constant value or a weighted average of the measured inlet and outlet temperatures from the PT sensors then it may be sufficient to use Eq. D. 8. Since we have used empirical equations with an approximation of the geometry, it is recommended that we compare the calculation with experimental data. The test results shown in Fig. D-4 suggest the suction line temperature can be estimated from simple extrapolation of the on-board PT sensor temperatures.