Answering Tree Pattern Queries Using Views: a Revisit

Junhu Wang  
Griffith University, Gold Coast  
Campus, Australia  
J.Wang@griffith.edu.au

Jiang Li  
Griffith University, Gold Coast  
Campus, Australia  
Jiang.Li@griffithuni.edu.au

Jeffrey Xu Yu  
Chinese University of Hong Kong  
Kong, China  
yu@se.cuhk.edu.hk

ABSTRACT
We revisit the problem of answering tree pattern queries using views. We first show that, for queries and views that do not have nodes labeled with the wildcard * , there is an alternative to the approach of query rewriting which does not require us to find any rewritings explicitly yet which produces the same answers as the maximal contained rewriting. Then, using the new approach, we give a simple criterion and a corresponding algorithm for identifying redundant view answers, which are view answers that can be ignored when evaluating the maximal contained rewriting. Finally, for queries and views that do have nodes labeled *, we provide a method to find the maximal contained rewriting and show how to answer the query using views without explicitly finding the rewritings.

Categories and Subject Descriptors
H.2.4 [Systems]: Query processing

General Terms
Theory, Algorithms

Keywords
XML, XPath, tree pattern, view, containment, rewriting

1. INTRODUCTION
Answering queries using views has been studied extensively for relational databases and found applications in fields such as query optimization, data integration, and query answering when the original data is inaccessible, e.g., in a network environment. Motivated by similar potential applications, the problem of answering tree pattern queries using views has recently attracted attention from many researchers.

A main approach for answering tree pattern queries using views exploits the technique of query rewriting, which transforms a query $Q$ into a set of new queries, and then evaluates these new queries over the materialized answers to the view $V$. In the literature, there are two types of rewritings, equivalent rewritings and contained rewritings. An equivalent rewriting will produce all answers to the original query, and a contained rewriting may produce only part of the answers. This work is about closely related to contained rewritings. Previously [8] gave a method to find the maximal contained rewriting (MCR) (i.e., the union of all contained rewritings) when both the view and query are in $P(\text{/**}/d\text{//})$ (i.e., they do not have nodes labeled with the wildcard *), and [11] gave a method for finding the maximal contained rewriting for views in $P(\text{/**}/*\text{/**})$ and queries that have no *-nodes connected to a //node and no leaf node $v$ such that $v$ and the parent of $v$ are both labeled *.

To actually find the answers to a new query $Q$, we need to evaluate the rewritings over the materialized view answers. A question arises as to whether it is possible to answer a new query without finding the rewritings. Furthermore, all previous works require us to materialize all answers to the view and evaluate the MCR over all such answers. Since an answer to a tree pattern is a sub-root of the original data tree, and some answers may be sub-trees of other answers, it is likely that we do not need to evaluate the MCR over all of the view answers. In other words some view answers may be redundant in that any answers that can be found by evaluating the MCR over them can also be found by evaluating the MCR over other view answers. In our experiments we found that on average 36.91% of view answers for the dataset XMark are redundant, 73.67% percent of view answers for the BIOML dataset are redundant, and up to 69.48% of view answers can be redundant for the dataset Treebank (see Section 4.3). Identifying view answers which do not contribute to the answering of new queries will help us minimize view maintenance and speed-up query evaluation. In this paper, we study these issues and make the following contributions:

1. For query $Q$ and view $V$ in $P(\text{/**}/d\text{//})$ we show there is an alternative approach for answering $Q$ using $V$ which does not require us to find the MCR of $Q$ using $V$ explicitly, yet which produces the same set of answers to $Q$ as can be found by the MCR.

2. For $Q$ and $V$ in $P(\text{/**}/d\text{//})$, we provide a simple criteria for identifying redundant view answers which are view answers that can be ignored when evaluating the MCR. We also provide two procedures to find the redundant view answers, one is fast but incomplete, the other is less efficient but complete. We conducted experiments about redundant view answers which in-
We show how the above results can be extended to cases where \( V, Q \) are in \( P(\cdot, \cdot, \cdot) \), but \( Q \) does not have \(-\)-edges, or \( Q \) does not have \(*\)-nodes connected to \(-\)-edges or leaf \(*\)-nodes.

For the more general case where both \( Q \) and \( V \) can be any pattern in \( P(\cdot, \cdot, \cdot) \), we present a method to find the MCR of \( Q \) using \( V \), and show how to answer \( Q \) using answers to \( V \) without finding explicit rewritings.

The rest of the paper is organized as follows. Section 2 gives the preliminaries. Section 3 presents the new approach for answering tree patterns queries using views when the view and query are both in \( P(\cdot, \cdot, \cdot) \). Based on the new approach, Section 4 provides the condition and an algorithm for finding redundant view answers. In Section 5 we study queries and views in \( P(\cdot, \cdot, \cdot) \). We first give a method for finding the MCR, then show how to answer \( Q \) using \( V \) without rewriting, and then discuss removing redundant view answers with respect to other classes of patterns than \( P(\cdot, \cdot, \cdot) \). Section 6 compares our work with some closely related work. We conclude the paper in Section 7.

## 2. PRELIMINARIES

### 2.1 XML data tree and tree patterns

Let \( \Sigma \) be an infinite set of tags which does not contain the symbol \(*\). An XML data tree, or simply a data tree, is a tree in which every node is labeled with a tag in \( \Sigma \). A tree pattern (TP), or simply a pattern, in \( P(\cdot, \cdot, \cdot) \) is a tree in which every node is labeled with a symbol in \( \Sigma \cup \{\ast\} \), every edge is labeled with either \(-\) or \(\//\)-edges, and there is a unique selection node. The path from the root to the selection node is called the selection path. Figure 1 shows some examples TPs, where single and double lines are used to represent \(-\)-edges and \(\//\)-edges respectively, and a circle is used to indicate the selection node. A TP corresponds to an XPath expression. The TPs in Figure 1 (a) and (b) correspond to the XPath expressions \( a[\langle c\rangle b]\[\langle d\rangle \] \) and \( a[\langle c\rangle b][\langle x\rangle y] \) respectively.

Let \( P \) be a TP. We will use \( \text{sn}(P) \) and \( \text{sp}(P) \) to denote the selection node and the selection path of \( P \) respectively. For any tree \( T \), we will use \( N(T) \), \( E(T) \) and \( rt(T) \) to denote the node set, the edge set, and the root of \( T \) respectively. We will also use \( \text{label}(v) \) to denote the label of node \( v \), and call a node labeled \( x \) an \( x \)-node. In addition, if \( (u, v) \) is a \(-\)-edge (resp. \(\//\)-edge), we say \( v \) is a \(-\)-child (resp. \(\//\)-child) of \( u \).

A matching of \( TP \) in a data tree \( t \) is a mapping \( \delta \) from \( N(P) \) to \( N(t) \) that is (1) root-preserving: \( \delta(rt(P)) = rt(t) \), (2) label-preserving: \( \forall v \in N(P), \text{label}(v) = \delta(\text{label}(v)) \) or \( \text{label}(v) = \ast \), and (3) structure-preserving: for every \(-\)-edge \( (x, y) \) in \( P \), \( \delta(y) \) is a child of \( \delta(x) \); for \(\//\)-edge \( (x, y) \), \( \delta(y) \) is a descendant of \( \delta(x) \). Each matching \( \delta \) produces a subtree of \( t \) rooted at \( \delta(\text{sn}(P)) \), which is known as an answer to \( TP \).

We use \( P(t) \) to denote the set of all answers of \( P \) over \( t \). For a set \( S \) of data trees, we define \( P(S) = \cup_{t \in S} P(t) \).

### 2.3 Contained rewritings of tree pattern queries using views

A view is an existing TP. Let \( V \) be a view and \( Q \) be a TP. In this paper we will implicitly assume that \( \text{label}(rt(Q)) = \text{label}(rt(V)) \neq \ast \). A contained rewriting (CR) of \( Q \) using \( V \) is a TP \( Q' \) such that (1) for any data tree \( t \), \( Q'(V(t)) \subseteq Q(t) \); (2) there exists a data tree \( t \) such that \( Q'(V(t)) \neq \emptyset \); Figure 1 shows a view \( V \), a TP \( Q \), and two CRs \( Q' \) and \( Q'' \), of \( Q \) using \( V \). The maximal contained rewriting (MCR) of \( Q \) using \( V \), denoted MCR\((Q, V)\), is defined to be the union of all CRs of \( Q \) using \( V \).

Let \( Q' \) be a CR of \( Q \) using \( V \). It is clear that \( \text{label}(rt(Q')) = \text{label}(\text{sn}(V)) \) or at least one of \( \text{label}(rt(Q')) \) and \( \text{label}(\text{sn}(V)) \) will be \( \ast \). We use \( Q \odot V \) to represent the TP obtained by merging \( rt(Q') \) and \( \text{sn}(V) \). The merged node will be labeled with \( \text{label}(rt(Q')) \) if \( \text{label}(rt(Q')) \neq \ast \), otherwise it will be labeled with \( \text{label}(\text{sn}(V)) \). The selection node of \( Q' \odot V \) is that of \( Q' \). Note that condition (1) in the definition of CR is equivalent to \( Q' \odot V \subseteq Q \).

Given TP \( Q \) and view \( V \) in \( P(\cdot, \cdot, \cdot) \), [8] shows that the existence of a CR of \( Q \) using \( V \) can be characterized by the...
existence of a useful embedding from \( Q \) to \( V \). In brief, a useful embedding from \( Q \) to \( V \) is a partial mapping \( f \) from \( N(Q) \) to \( N(V) \) that is root-preserving, label-preserving, structure-preserving as defined in a containment mapping (except that they are required only for the nodes of \( Q \) on which the function is defined), and that satisfies the following conditions. (1) If \( f(x) \) is defined, then \( f(x) \) is defined on parent\( (x) \) (the parent of \( x \)). (2) For every node \( x \) on the selection path \( \text{sp}(Q) \), if \( f(x) \) is defined, then \( f(x) \in \text{sp}(V) \), and if \( f(\text{sn}(Q)) \) is defined, then \( f(\text{sn}(Q)) = \text{sn}(V) \). (3) For every path \( p \in Q \), either \( p \) is fully embedded, i.e., \( f \) is defined on every node in \( p \), or if \( x \) is the last node in \( p \) such that \( f(x) \) is defined and \( f(x) \in \text{sp}(P) \), and \( y \) the child of \( x \) on \( p \) (call \( y \) the anchor node), then either \( f(x) = \text{sn}(V) \), or the edge \((x, y)\) is a \(/\)-edge.

Given a useful embedding \( f \) from \( Q \) to \( V \), a Clip-Away Tree (\( \text{CAT} \)), denoted \( \text{CAT}_f \), can be constructed as follows. (i) Construct the root of \( \text{CAT}_f \) and label it with label\( (\text{sn}(V)) \). (ii) For each path \( p \in Q \) that is not fully embedded, find the anchor node \( y \) and attach the subtree of \( Q \) rooted at \( y \) (denoted \( Q_y \)) under rt\( (\text{CAT}_f) \) by connecting rt\( (Q_y) \) and rt\( (\text{CAT}_f) \) with the same type of edge as that between \( y \) and its parent. The selection node of \( \text{CAT}_f \) is the node corresponding to \( \text{sn}(Q) \). For example, in Figure 1 the query \( Q \) has two useful embeddings to \( V \), both of them map the \( a \)-node and \( c \)-node in \( Q \) to the \( a \)-node and \( c \)-node in \( V \), and one also maps the \( b \)-node in \( Q \) to the \( b \)-node in \( V \), but the other does not. The corresponding CATs are shown in Figure 1 (c) and (d) respectively. Each CAT is a CR of \( Q \) using \( V \), and every CR of \( Q \) using \( V \) is contained in one of these CATs. Thus if \( h_1, \ldots, h_k \) are all the useful embeddings from \( Q \) to \( V \), then \( \text{MCR}(Q, V) = \bigcup_{i \in [1, k]} \text{CAT}_{h_i} \).

3. EVALUATING TPS OVER ANNOTATED VIEW ANSWERS - AN ALTERNATIVE TO QUERY REWRITING

Given view \( V \), new query \( Q \), the rewriting approach finds a set of new queries (i.e., the rewritings) and evaluates the rewritings over the materialized view answers in order to find answers to \( Q \). In this section, we present an alternative approach for answering \( Q \) using only the answers to \( V \). The new approach does not require us to find any explicit rewritings, instead, it evaluates \( Q \) over a set of annotated view answers. We will show that the new approach produces the same set of answers as evaluating the MCR over the original view answers in \( V(t) \). We focus on queries and views in \( P^{(\approx, \approx)} \) in this section and will deal with queries and views in \( P^{(\approx, \approx)} \) in Section 5.

Definition 3.1: (annotated view answer) Let \( V \in P^{(\approx, \approx)} \) be a view, \( t \) be a data tree, and \( t_i \) be a view answer in \( V(t) \). We construct a new tree from \( V(t) \) and \( t_i \), denoted \( t \cdot V \), by merging the root of \( t_i \) with the selection node of \( V \). The merged node will be labeled with label\( (rt(t_i)) \). We call this tree an annotated answer of \( V \) over \( t \) or an annotated view answer when \( V \) and \( t \) are clear from context.

Figure 2 (a) shows a data tree \( t \). There are two answers, \( t_1 \) and \( t_2 \), to the view \( V \) shown in Figure 1 (a), as indicated by the surrounding boxes. The annotated view answers are shown in Figure 2 (b) and (c).

An annotated view answer will be treated like a data tree and used to find answers to a query over the original data tree \( t \), hence it has no selection node. The evaluation of a query over an annotated view answer is similar to that over a normal data tree. Formally, for any query \( Q \), we evaluate \( Q \) over \( t \cdot V \) by finding matchings of \( Q \) in \( t \cdot V \). A matching of \( Q \) in \( t \cdot V \) is a mapping from \( N(Q) \) to \( N(t \cdot V) \) which is root-preserving, label-preserving, structure-preserving and which maps the selection node of \( Q \) into a node in \( t \). If \( h \) is such a matching, then the subtree of \( t \) rooted at \( h(\text{sn}(Q)) \) is called an answer of \( Q \) over \( t \cdot V \). We use \( Q(t \cdot V) \) to denote the set of all answers of \( Q \) over \( t \cdot V \).

The annotated view answers have the following properties.

**Lemma 3.1:** Let \( V \in P^{(\approx, \approx)} \) be a view, \( t \) be a data tree, and \( t_i \in V(t) \). Then \( t_i \in V(t \cdot V) \). □

**Proposition 3.1:** Let \( V \in P^{(\approx, \approx)} \) be a view, \( t \) be a data tree, and \( V(t) = \{t_1, \ldots, t_n\} \). Let \( Q \in P^{(\approx, \approx)} \) be a query. If \( Q \) has no CR using \( V \), then \( Q(t_i \cdot V) = \emptyset \) for all \( i \in [1, n] \). □

The proof is straightforward because any matching of \( Q \) in \( t_i \cdot V \) implies a useful embedding from \( V \) to \( Q \), if we restrict the matching to those nodes of \( Q \) whose image is in the \( V \)-part of \( t_i \cdot V \), the matching becomes a useful embedding from \( Q \) to \( V \).

**Theorem 3.1:** Let \( V \in P^{(\approx, \approx)} \) be a view, \( t \) be a data tree, and \( V(t) = \{t_1, \ldots, t_n\} \). Let \( Q \in P^{(\approx, \approx)} \) be a query, \( h_1, \ldots, h_k (k \geq 1) \) be all of the useful embeddings from \( Q \) to \( V \). Then

\[
(1) \quad Q(t_i \cdot V) = \text{CAT}_{h_j}(t_i) \cup \cdots \cup \text{CAT}_{h_k}(t_i), \forall i \in [1, n].
\]

\[
(2) \quad \text{CAT}_{h_j}(V(t)) \cup \cdots \cup \text{CAT}_{h_k}(V(t)) = Q(t_1 \cdot V) \cup \cdots \cup Q(t_n \cdot V).
\]

**Proof.** Denote \( \text{CAT}_{h_j} \) by \( Q_j \) for \( j = 1, \ldots, k \). (1) We show \( Q(t_i) \cup \cdots \cup Q_h(t_i) \subseteq Q(t_i \cdot V) \) and \( Q(t_i \cdot V) \subseteq Q(t_i) \cup \cdots \cup Q(t_i \cdot V) \). □

![Figure 2: Data tree t and the annotated view answers for the view V in Figure 1](image-url)
... \cup Q_k(t_i)$. First, for every matching $h$ of $Q_j$ in $t_i$, we can construct a matching $h'$ of $Q$ in $t_i \cdot V$ as follows: for every node $u \in Q$, if $u$ is mapped into the $V$-part of $t_i \cdot V$ by $h_j$, then let $h'(u) = h_j(u)$. If $u$ is not mapped into $V$ by $h_j$, then it will be a node in $\text{CAT}_{h_j} (= Q_j)$, and we let $h'(u) = h(u)$. Clearly $h'$ is a matching of $Q$ in $t_i \cdot V$, and $h'(\text{sn}(Q))$ and $h'(Q(t_i))$ are the same node in $t_i$. Therefore, $Q_i(t_i) \subseteq Q(t_i \cdot V)$. Thus $Q_i(t_i) \cup \cdots \cup Q_k(t_i) \subseteq Q(t_i \cdot V)$.

Conversely, for every matching $\delta$ of $Q$ in $t_i \cdot V$, there is a useful embedding $h_i$ from $Q$ to $V$, which is obtained by restricting $\delta$ to those nodes of $Q$ that are mapped into the $V$-part of $t_i \cdot V$ by $\delta$. The corresponding CAT $\text{CAT}_{h_i} (= Q_i)$ has a matching in $t_i$, which is obtained by restricting $\delta$ to those nodes of $Q$ that are mapped into the $t_i$-part of $t_i \cdot V$. This matching maps $\text{sn}(Q_i)$ to the same node as $\delta(\text{sn}(Q))$. Therefore, $Q(t_i \cdot V) \subseteq Q_i(t_i) \cup \cdots \cup Q_k(t_i)$.

\begin{align*}
Q_i(V(t_i)) & \cup \cdots \cup Q_k(V(t_i)) \\
= (Q_i(t_1) \cup \cdots \cup Q_i(t_n)) & \cup \cdots \cup (Q_i(t_1) \cup \cdots \cup Q_k(t_n)) \\
= (Q_i(t_1) & \cup \cdots \cup Q_k(t_1)) \cup \cdots \cup (Q_i(t_n) \cup \cdots \cup Q_k(t_n)) \\
= Q(t_i \cdot V) & \cup \cdots \cup Q(t_n \cdot V).
\end{align*}

\hfill \Box

Proposition 3.1 and Theorem 3.1 imply that, if $Q$ and $V$ are both in $P^{(~32)}$ (i.e., $\mathcal{H}^{(~32)}$), then evaluating $Q$ directly over the annotated view answers (hereafter referred to as the annotated view answer approach) will produce the same answers that can be found by evaluating $\text{MCR}(Q,V)$ over the original view answers, thus proving our claim at the beginning of this section.

\section*{Variation of annotated view answer}

Let us use $V^{U_V}$ to denote the pattern obtained from view $V$ by removing all nodes under $\text{sn}(V)$, and $V^{C}$ to denote the pattern consisting of $\text{sn}(V)$ and all nodes and edges under $\text{sn}(V)$. It is not difficult to see that Proposition 3.1 and Theorem 3.1 still hold if we replace $t_i \cup V$ with $t_i \cdot V^C$ (for $i = 1, \ldots, n$). This is because $t_i$ is a tree in $V(t_i)$, thus there is a matching of $V^C$ in $t_i$.

\section*{Discussion on evaluation and performance}

Assuming naive algorithms for query evaluation for both the rewriting approach and the annotated view answer approach, there are extreme cases where one approach is faster than the other. For instance, when the number of non-redundant CATs is small and the number of view answers is large, the rewriting approach appears faster. In particular, if there are no useful embeddings from $Q$ to $V$, then the rewriting approach knows immediately that no answers to $Q$ can be obtained using the view answers (although it has the cost to find out that there are no useful embeddings), but the annotated view answer approach still has to evaluate $Q$ over each annotated view answer. On the other hand, in cases where the number of non-redundant CRs is large and the number of view answers is small, the annotated view answer approach appears faster. We use the following example to illustrate this point.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{example.png}
\caption{$V$, $Q$, $t$ and $t_1 \cdot V$ for Example 3.1}
\end{figure}

\begin{example}
Consider the query $Q$ and view $V$ in Figure 3. There are a total of 32 (=2^5) non-redundant CATs of $Q$ using $V$. Some example CATs are listed in Figure 3 (c) to (l). Consider the data tree $t$ shown in Figure 3 (c). It is easy to verify that $V(t)$ has three view answers $t_1$, $t_2$ and $t_3$, rooted at nodes $c_1$, $c_2$ and $c_3$ respectively (we use subscripts to indicate nodes with the same label), and $t_2$ and $t_3$ are redundant (see next section for redundant view answers). If we evaluate $Q$ directly over the annotated view answer $t_1 \cdot V$ (Figure 3 (d)), we will quickly obtain two answers, one rooted at $c_1$ and the other rooted at $c_2$, and the evaluation will take less time than evaluating all 32 CATs over $t_1$.
\end{example}

In the above example, the better efficiency of the new approach comes from the fact that when evaluating $Q$ directly over $t_1 \cdot V$ we only need to verify whether $\text{sn}(Q)$ can be matched to each of the $c$-nodes by some matching, and in doing so, we can avoid many repeated computations that are done when we evaluate all CATs over $t_1$. Take $c_3$ for example, if we evaluate $Q$ over $t_1 \cdot V$, we find that if we map $\text{sn}(Q)$ to $c_3$, then the $d$-node under $\text{sn}(Q)$ cannot be mapped to anywhere. Thus we know immediately that $c_3$ is not in $Q(t_1 \cdot V)$. On the other hand, if we evaluate all of the CATs over $t_1$, we in effect have to repeat the same comparison 32 times. Similarly with $c_2$, if we evaluate $Q$ over $t_1 \cdot V$, once we find a matching that maps $\text{sn}(Q)$ to $c_2$, we do not need to consider other matchings that also map $\text{sn}(Q)$ to $c_2$. However, if we evaluate all of the CATs over $t_1$, we in effect have
to find all matchings that produce $c_2$.

A more careful comparison of performance requires us to design better algorithms for both approaches and there are obvious ways to do so. For instance, for the rewriting approach, for each view answer $t$, we can first find a set of potential nodes which are the nodes in the same label as $\text{an}(Q)$, and filter out those potential nodes $u$ such that the subtree of $Q$ rooted at $\text{an}(Q)$ has no matching in the subtree rooted at $u$. Then we check the remaining potential nodes, and if one is found to be an answer to $Q$ using one of the CATs, then there is no need to check it using other CATs.

A more detailed discussion on the valuation of MCRs can be found in [16]. For the annotated view approach, once we have found the matching $\delta$ of $Q$ in $t_1 \cdot V$, we can extract the part $\delta'$ of $\delta$ which maps the nodes in $Q$ to the $V$-part, and reuse it to find other matchings or when we evaluate $Q$ over $t_2 \cdot V$. Since every matching of $Q$ in an annotated view answer implies a useful embedding, algorithms can be designed for the annotated view answer approach that takes, at most, the same amount of time as for finding the CATs and evaluating the CATs over the view answers in $V(t)$.

One application of the new approach is that it enables us to find a simple condition to identify redundant view answers, as will be discussed in the next section.

4. REDUNDANT VIEW ANSWERS

The previous section shows that to answer a new query $Q$ using answers to $V$, we can either evaluate the rewritings over the original view answers in $V(t)$, or evaluate $Q$ over the annotated view answers. In practice, $V(t)$ may be a large set and many view answers may be subtrees of other view answers. Thus a natural question to ask is whether we need to evaluate the rewritings over all of the view answers.

We will address this problem in this section.

Definition 4.1: View answer subsumption and redundancy. Let $V$ be a view, and $P$ be a class of TPs. Suppose $t_1, \ldots, t_{m-1} \in V(t)$ are view answers. If for every TP $Q \in P$, $\text{MCR}(Q, V)(t_{m-1}) \subseteq \text{MCR}(Q, V)(t_1) \cup \cdots \cup \text{MCR}(Q, V)(t_{m-2})$, then we say $t_m$ is subsumed by $t_1, \ldots, t_{m-1}$ wrt $P$. A view answer is said to be redundant wrt $P$ if it is subsumed by some other view answers wrt $P$.

Intuitively, all answers that can be obtained by evaluating $\text{MCR}(Q, V)$ for all $Q \in P$ over the redundant view answers can be obtained by evaluating $\text{MCR}(Q, V)$ over some other view answers.

4.1 Conditions for subsumption

Next we investigate conditions under which $t_m$ is subsumed by $t_1, \ldots, t_{m-1}$. We limit our discussion to TPs in $P^{(\neq \beta)}$, i.e., $V \in P^{(\neq \beta)}$ and $P = P^{(\neq \beta)}$. In this case, due to results in Section 3, $t_m$ is subsumed by $t_1, \ldots, t_{m-1}$ if and only if for any $Q \in P^{(\neq \beta)}$, $Q(t_m, V) \subseteq Q(t_1, V) \cup \cdots \cup Q(t_{m-1}, V)$.

We start with the case where $m = 2$. Obviously, if $t_2$ is not a subtree of $t_1$, then it cannot be subsumed by $t_1$. Therefore, when looking for subsumed view answers we only need to examine view answers which are subtrees of other view answers. In what follows, we will use $t_2 < t_1$ to denote $t_2$ is a subtree of $t_1$. The following example shows that, in general, $t_2 < t_1$ does not imply $t_2$ is subsumed by $t_1$.

Example 4.1: Consider the data tree $t$ and view $V$ shown in Figure 4 (a) and (b) respectively. For convenience we use subscripts to indicate different nodes labeled with the same label. $V(t)$ contains two subtrees of $t$, one rooted at $b_1$ (denoted $t_1$) and the other rooted at $b_4$ (denoted $t_2$). $t_2$ is a subtree of $t_1$. Let $Q$ be the same as $V$, then $Q(t_1 \cdot V)$ contains the subtree of $t$ rooted at $b_4$, but $Q(t_1 \cdot V)$ does not.

We now prove the following theorem, which provides a necessary and sufficient condition for $t_2$ to be subsumed by $t_1$.

Theorem 4.1: Let $t_1, t_2 \in V(t)$ and $t_2$ be a subtree of $t_1$. The following three conditions are equivalent:

1. $V(t_2 \cdot V) \subseteq V(t_1 \cdot V)$
2. $t_2 \in V(t_1 \cdot V)$
3. $\forall Q. Q(t_2 \cdot V) \subseteq Q(t_1 \cdot V)$

Proof. (1) $\rightarrow$ (2): By Lemma 3.1 $t_2 \in V(t_2 \cdot V)$. Thus if $V(t_2 \cdot V) \subseteq V(t_1 \cdot V)$, then $t_2 \in V(t_1 \cdot V)$.

(2) $\rightarrow$ (3): Suppose $t_2 \in V(t_1 \cdot V)$. That is, there is a matching $h$ of $V$ in $t_1 \cdot V$ such that $h(\text{an}(V)) = rt(t_2)$. Let us extend $h$ to a mapping from $N(t_2 \cdot V)$ to $N(t_1 \cdot V)$ by mapping every node in the $t_2$-part of $t_2 \cdot V$ to the corresponding node in the $t_2$-part of $t_1 \cdot V$. Now for any query $Q$, suppose $f$ is a matching of $Q$ in $t_2 \cdot V$, then $hf$ is a matching of $Q$ in $t_1 \cdot V$. Furthermore, $f(\text{an}(Q))$ and $hf(\text{an}(Q))$ correspond to the same node in $t_2$. Therefore, $Q(t_2 \cdot V) \subseteq Q(t_1 \cdot V)$.

(3) $\rightarrow$ (1): If $\forall Q. Q(t_2 \cdot V) \subseteq Q(t_1 \cdot V)$, then we can take $Q$ as $V$ and obtain $V(t_2 \cdot V) \subseteq V(t_1 \cdot V)$.

The above theorem indicates that to test whether one view answer $t_2$ is subsumed by another view answer $t_1$, we only need to test whether $t_2$ is in $V(t_1 \cdot V)$. For example, for the
For easy identification of the nodes, we use subscripts to
view $V$ and tree $t$ in Figure 3, it is easy to verify that both
t2 and t3 are in $V(t_1 \cdot V)$, hence they are subsumed by t1.

Using the above theorem, we can show that $t_m$ is subsumed by
t1, . . . , $t_{m−1}$ if and only if it is subsumed by one of the
them.

**Corollary 4.1:** Suppose $t_1, . . . , t_m \in V(t)$, $t_m < t_i$ for $i = 1, . . . , m − 1$. If $∀Q, Q(t_m \cdot V) \subseteq Q(t_1 \cdot V) \cup ⋯ \cup Q(t_{m−1} \cdot V)$, then there exists $i \in [1, m − 1]$ such that $∀Q, Q(t_m \cdot V) \subseteq Q(t_i \cdot V)$.

**Proof.** Suppose $∀Q, Q(t_m \cdot V) \subseteq Q(t_1 \cdot V) \cup ⋯ \cup Q(t_{m−1} \cdot V)$, then $V(t_m \cdot V) \subseteq V(t_1 \cdot V) \cup ⋯ \cup V(t_{m−1} \cdot V)$, hence $t_m \in V(t_1 \cdot V) \cup ⋯ \cup V(t_{m−1} \cdot V)$. Therefore there exists $i \in [1, m − 1]$ such that $t_m \in V(t_i \cdot V)$. By Theorem 4.1, $∀Q, Q(t_m \cdot V) \subseteq Q(t_i \cdot V)$.

Now suppose $t_1, t_2, t_3 \in V(t)$, and $t_3 < t_2 < t_1$. If $t_3 \in V(t_2 \cdot V)$ and $t_2 \in V(t_1 \cdot V)$, then by Theorem 4.1, $V(t_2 \cdot V) \subseteq V(t_1 \cdot V)$, hence $t_3 \in V(t_1 \cdot V)$. But the following examples demonstrate that $t_3 \in V(t_2 \cdot V)$ does not imply $t_3 \in V(t_1 \cdot V)$, and $t_2 \in V(t_1 \cdot V)$ does not imply $t_3 \in V(t_1 \cdot V)$. In other
words, $t_3$ is subsumed by $t_2$ or $t_2$ is subsumed by $t_1$ does not
imply $t_3$ is subsumed by $t_1$. This observation is useful when
we design the algorithm to find redundant view answers.

**Example 4.2:** Consider the example $t$ and $V$ in Figure 5.
For easy identification of the nodes, we use subscripts to
indicate distinct nodes with the same label, i.e., the nodes
$b_1, . . . , b_{10}$ are all labeled with $b$. It is easy to verify that
there are three subtrees of $t$ in $V(t)$, which are the sub-
trees rooted at $b_6$, $b_7$ and $b_{10}$ respectively. We denote these
subtrees by $t_1, t_2$ and $t_3$. Although $t_3 < t_2 < t_1$ and $t_3 \in V(t_1 \cdot V)$, $t_3$ is not in $V(t_2 \cdot V)$.

**Example 4.3:** Consider the example $t$ and $V$ in Figure 6.
In the figure the nodes $b_1, . . . , b_{10}$ are all labeled with $b$. It is
easy to verify that there are three subtrees of $t$ in $V(t)$, which
are the subtrees rooted at $b_6$, $b_7$ and $b_{10}$ respectively. We denote these subtrees by $t_1, t_2$ and $t_3$. Although $t_3 < t_2 < t_1$, and $t_2 \in V(t_1 \cdot V)$, $t_3$ is not in $V(t_1 \cdot V)$.

Using Theorem 4.1 and the above observations, we can de-
sign an algorithm for finding all redundant view answers in
$V(t)$: to see whether $t_2 \in V(t)$ is redundant, we only need to
test whether there exists $t_1 \in V(t)$ such that $t_2 < t_1$ and $t_2 \in V(t_1 \cdot V)$, and we may have to test $t_2$ against all view answers of which $t_2$ is a subtree. However, testing $t_2 \in V(t_1 \cdot V)$ can sometimes be costly. Next we will provide a more efficient (although incomplete) method for identifying redundant view answers.

Suppose $t_1, t_2 \in V$ and $t_2$ is a subtree of $t_1$. The corollary
below gives a sufficient condition under which $t_2$ is subsumed
by $t_1$.

**Proposition 4.1:** Let $t_1, t_2$ be in $V(t)$, and $t_2$ be a subtree
of $t_1$. Let $(x, y)$ be the last //edge on $sp(V)$ (counting from
$rt(V)$). If the length of the path from $y$ to $nV$ is less or
equal to the length of the path from $rt(t_1)$ to $rt(t_2)$, then
$t_2 \in V(t_1 \cdot V)$.

**Proof Sketch.** Let $f$ be the matching of $V$ in $t$ that
generates $t_2$. Since the length of the path from $y$ to $nV$ is
less or equal to the length of the path from $rt(t_1)$ to $rt(t_2)$,
and there are only //edges on the path from $y$ to $nV$, all
nodes from $y$ to $nV$ will be mapped into $t_1$, and the sub-
tree of $V$ rooted at $y$, $V_y$, will be mapped into $t_1$ by $f$. Therefore,
we can construct a mapping $h$ from $V$ to $t_1 \cdot V$ such that
for every node $u$ in $V_y$, $h(u) = f(u)$, and for every other
node $u' \in N(V) − N(V_y)$, $h$ maps $u'$ to the corresponding
node in the $V$ part of $t_1 \cdot V$. Since $(x, y)$ is a //edge, $h$ is a
matching of $V$ in $t_1 \cdot V$, and $h(nV) = rt(t_2)$. Therefore
$t_2 \in V(t_1 \cdot V)$.

Let us call the path from $y$ to $nV$ in $Proposition 4.1$ the
last //segment of $V$. Using the proposition and Theorem 4.1,
if the length of the last /-segment of $V$ is not longer than the path from $rt(t_1)$ to $rt(t_2)$, than $t_2$ is subsumed by $t_1$. For example, for the view and data tree in Figure 3, the last /-segment (i.e., the path from the second a-node to the selection node) of $V$ is shorter than the path from $rt(t_1)$ to $rt(t_2)$. Therefore $t_2$ is subsumed by $t_1$. Similarly $t_3$ is subsumed by $t_1$.

A special case of Proposition 4.1 is when the selection node $n(V)$ is connected to its parent with a //edge. In this case, all view answers in $V(t)$ that are subtrees of other view answers are redundant.

4.2 The algorithm

Based on Theorem 4.1 and Proposition 4.1 we design an algorithm to remove redundant view answers from $V(t)$. Let us use $L_V$ to denote the length of the last /-segment on $\uparrow p(V)$, and $d(t_1, t_2)$ to denote the length of the path from $rt(t_1)$ to $rt(t_2)$, if $t_2$ is a subtree of $t_1$. The removal of redundant view answers is done by two scans. The first, called $FastScan$, scans the view answers using Proposition 4.1 to remove some redundant view answers in $V(t)$ quickly, and the second, called $CompleteScan$, uses Theorem 4.1 to remove the remaining redundant view answers. For easy exposition, we assume the view answers in $V(t)$ have been arranged into lists where each list consists of the view answers $t_1, \ldots, t_n$ such that $t_n \prec t_{n-1} \prec \cdots \prec t_1$. We associate each $t_i$ with a Boolean variable $t\_del$ which indicates whether $t_i$ has been deleted. Initially $t\_del = \text{FALSE}$ for all $i \in [1, n]$. The input to each scan is such a list, and the output is the same list with possibly modified $t\_del$ values. The two scans are listed in Algorithm 1.

**Implementation and Complexity:** To implement Algorithm 1, we organize the view answers into disjoint subtrees of $t$, two view answers are in the same subtree if and only if one is a subtree of another or they are both subtrees of another view answer. We use an attribute $u\_ans$ for each node $u$ to indicate whether $u$ is the root of some view answer in $V(t)$. To apply $FastScan$, we check the nodes which are roots of some view answers level by level and top-down. If one view answer is found redundant, then all view answers below it will also be redundant. In the worst case, every view answer (except those which are not subtrees of others) will be checked once. Thus $FastScan$ is linear in the number of view answers in $V(t)$, that is, it takes $O(|V(t)|)$. Checking whether $t_2$ is in $V(t \cdot V)$ can be done in $|V| + |V|$ using a variation of Algorithm 3 of [10], and $CompleteScan$ needs to make $(m-1)n/2$ such tests. In the worst case the roots of all view answers in $V(t)$ are on the same path and $n = |V(t)|$, and $FastScan$ finds no redundant view answers (thus $m = |V(t)|$) and we have to resort to $CompleteScan$. Therefore the worst-case complexity of Algorithm 1 is $O(|V(t)| + |V||rt(t)|)$ where $t_1$ is the largest view answer (i.e., one with the largest number of nodes) in $V(t)$.

4.3 Experiments

The purpose of our experiments is to get some insight on the percentage of view answers which are redundant, and the percentage of redundant view answers that can be found using $FastScan$, as well as the time required to find the redundancies, for some real datasets. We implemented $FastScan$ and $CompleteScan$ with C++, and all our experiments were carried out on a Dell Latitude D430 laptop with Intel(R) 1.20GHz CPU and 2G RAM running Windows XP.

**Datasets and view sets** Obviously for there to be redundant view answers in $V(t)$ the data tree $t$ must have recursion (i.e., there must be root-to-leave paths which contain multiple occurrences of the same label), and the view $V$ must have //edges. The depth of $t$, the number of occurrences of $\text{label}(n(V))$ in $t$, and the size of $V$ will all impact the number of redundancies. In our experiments we used three datasets, i.e., the benchmark dataset XMark [1], a real dataset TreeBank [2], and a dataset generated from the BIOML DTD [3] using IBM XML generator¹. For XMark, the dataset is around 111MB, contains more than 1.6 million elements and has limited recursion. For TreeBank, the data tree is deep and has a large number of recursive elements. The maximal depth is 36 and there are more than 2.4 million elements. For BIOML, the generated data tree is around 344MB with a depth of 19 and contains more than 2.46 million elements. For TreeBank we randomly generated 40 views of depth between 2 and 10. For XMark we randomly generated 30 views of depth between 2 and 5 involving the recursive elements $\text{parlist}$ and/or $\text{listitem}$ on the selection path, and for BIOML we generated 40 views of depth between 3 and 7. A few example views are listed in Table 1 (three views for each dataset TreeBank, XMark and BIOML).

**Experimental result** The result of our experiments is summarized in Table 2. For TreeBank, on average 18.13% of the view answers were found redundant, and 81.87% of the redundancies were found by $FastScan$. For some views the percentage of redundant view answers can be as high as 69.48%. For XMark and BIOML, on average 36.91% and 73.67% respectively, of the view answers were found to be redundant, and all of the redundancies were found by $FastScan$. We also recorded the time taken by the two scans for the three data sets. Table 2 show the total time spent on each set of queries by $FastScan$, $CompleteScan$ after $FastScan$, and $CompleteScan$ alone. Our experiment shows that using $FastScan$ before $CompleteScan$ we can find the redundancies in less than 100 milliseconds per view on average, and


```
Algorithm 1 Removing redundant view answers

INPUT: $t_n, \ldots, t_1 \in V(t)$ s. t. $t_n \prec t_{n-1} \prec \cdots \prec t_1$

/* FastScan */
1: $m \leftarrow n$
2: for $i = 2$ to $n$ do
3: if $L_V \leq d(t_1, t_i)$ then
4: $t_i\_del, t_{i+1}\_del, \ldots, t_n\_del \leftarrow \text{TRUE}$
5: $m \leftarrow i - 1$
6: go to $CompleteScan$

/* CompleteScan */
7: for $i = 1$ to $m - 1$ do
8: if $t_i\_del = \text{FALSE}$ then
9: for $j = i + 1$ to $m$ do
10: if $t_j\_del = \text{FALSE}$ and $t_j \in V(t, \cdot V)$ then
11: $t_j\_del \leftarrow \text{TRUE}$
```
it only takes a tiny fraction of the time required by CompleteScan alone for all three sets of data and views.

5. QUERIES AND VIEWS IN \( P(\cdot,\cdot,\cdot,\cdot,\cdot) \)

In this section, we consider answering TP queries using views when the view \( V \) and the query \( Q \) are in \( P(\cdot,\cdot,\cdot,\cdot,\cdot) \). We first present a method for finding the MCR, then we present the annotated view answer approach and show it produces the same answers as the MCR. Finally, we discuss extension of the process for removing redundant view answers to some special cases.

5.1 Finding the MCR

We make a slight modification to the definition of contained rewriting: in addition to the conditions (1) and (2) in Section 2.3, we add another condition: (3) If \( |\text{label}(\text{an}(V))| \neq * \), then \( \text{label}(\text{rt}(Q')) \neq * \). This modification does not change the essence of MCR because, if \( Q' \) is a CR of \( Q \) using \( V \) and \( \text{rt}(Q') \) is labeled \(*\), then we can change the label of \( \text{rt}(Q') \) to \( \text{label}(\text{an}(V)) \) to obtain another rewriting \( Q'' \), and \( Q'(V(t)) = Q''(V(t)) \). The only purpose of this modification is to make our presentation easier.

We also modify the definition of useful embeddings and CATs in order to deal with the possibility that \( \text{label}(\text{an}(V)) = * \). The modification to useful embeddings is to relax the requirement of label-preservation to weak label-preservation as defined below.

**Definition 5.1:** A mapping \( h \) from \( N(Q) \) to \( N(V) \) is weak label-preserving if (1) for every \( v \in N(Q) \), either \( \text{label}(v) = * \), or \( \text{label}(v) = \text{label}(h(v)) \), or \( h(v) = \text{an}(V) \) and \( \text{label}(\text{an}(V)) = * \), (2) all non-* nodes that are mapped to \( \text{an}(V) \) have the identical label.

Correspondingly, if \( \text{label}(\text{an}(V)) = * \), and \( f \) is a useful embedding from \( Q \) to \( V \) which maps some node \( u \) in \( Q \) to \( \text{an}(V) \), and \( \text{label}(u) \neq * \), then the root of \( \text{CAT}_f \) will be labeled with \( \text{label}(u) \).

Our main results in this subsection are summarized in Theorems 5.1 and 5.2 below.

**Theorem 5.1:** Let \( V, Q \) be TPs in \( P(\cdot,\cdot,\cdot,\cdot,\cdot) \). If \( Q \in P(\cdot,\cdot,\cdot,\cdot,\cdot) \cup P(\cdot,\cdot,\cdot,\cdot,\cdot) \) or \( V \in P(\cdot,\cdot,\cdot,\cdot,\cdot) \), then \( \text{MCR}(Q, V) = \text{CAT}_{h_1} \cup \cdots \cup \text{CAT}_{h_k} \), where \( h_1, \ldots, h_k \) are all of the useful embeddings from \( Q \) to \( V \).

We outline the proof here and leave the formal proof to the full version of this paper. We need to consider two cases.

**Case 1:** \( Q \in P(\cdot,\cdot,\cdot,\cdot,\cdot) \cup P(\cdot,\cdot,\cdot,\cdot,\cdot) \). We note that if \( Q \in P(\cdot,\cdot,\cdot,\cdot,\cdot) \cup P(\cdot,\cdot,\cdot,\cdot,\cdot) \), then for any \( P \in P(\cdot,\cdot,\cdot,\cdot,\cdot) \), \( P \subseteq Q \) iff there is a CM from \( Q \) to \( P \) [14]. It can be shown that the same property holds for \( Q \in P(\cdot,\cdot,\cdot,\cdot,\cdot) \cup P(\cdot,\cdot,\cdot,\cdot,\cdot) \). Therefore, if \( Q \in P(\cdot,\cdot,\cdot,\cdot,\cdot) \cup P(\cdot,\cdot,\cdot,\cdot,\cdot) \), and \( Q' \) is a CR of \( Q \) using \( V \), then there is a CM from \( Q \) to \( Q' \). This CM implies a useful embedding from \( Q \), and the corresponding CAT contains \( Q' \). Therefore, \( \text{MCR}(Q, V) = \text{CAT}_{h_1} \cup \cdots \cup \text{CAT}_{h_k} \).

**Case 2:** \( V \in P(\cdot,\cdot,\cdot,\cdot,\cdot) \). Although in this case \( V \subseteq Q \), generally there may not exist a CM from \( Q \) to \( V \). Therefore, \( \text{MCR}(Q, V) \) may be a pattern in \( P(\cdot,\cdot,\cdot,\cdot,\cdot) \). Thus it is not straightforward to see whether the MCR can be found by finding the useful embeddings and CATs. To prove the theorem for this

Before describing the next result we need more notations.

Let \( x, y \in \Sigma \cup \{\ast\} \) be two labels. Given TP \( Q \in P^{(\ast/\ast;\ast)} \), we define \( Q(x, y) \) to be the set of \(*\)-chains from an \( x \)-node or \(*\)-node to a \( y \)-node or \(*\)-node:

\[
Q(x, y) = \{ u \leadsto v \in Q \mid \text{label}(u) \in \{x, \ast\}, \text{label}(v) \in \{y, \ast\} \}
\]

Define the function \( L_Q(x, y) \) as follows:

\[
L_Q(x, y) = \begin{cases} 
-1 & \text{if } Q(x, y) = \emptyset; \\
\max \{|u \leadsto v| \mid u \leadsto v \in Q(x, y)\} & \text{otherwise}
\end{cases}
\]

For example, for the TP \( Q \) in Figure 7, \( L_Q(a, b) = -1, L_Q(c, d) = 0, \) and \( L_Q(\ast, d) = -1. \)

Let \( V, Q \) be TPs in \( P^{(\ast/\ast;\ast)} \). We will use \( \text{Mod}_Q(V) \) to denote the set of TPs obtained from \( V \) by replacing each \( /\)-edge \( e = (u, v) \) with a \(*\)-chain \( u \leadsto v \) of length \( l_e \), where \( l_e \) is an integer in \([0, L_Q(\text{label}(u), \text{label}(v)) + 1]\).

We can now describe our next main result as follows.

**Theorem 5.2:** Let \( V, Q \) be TPs in \( P^{(\ast/\ast;\ast)} \). Suppose \( \text{Mod}_Q(V) = \{V_1, \ldots, V_n\} \). Then

1. \( Q' \) is a CR of \( Q \) using \( V \) iff it is a CR of \( Q \) using \( V_i \) for all \( i = 1, \ldots, n. \)

2. \( \text{MCR}(Q, V) = \text{MCR}(Q, V_1) \cap \cdots \cap \text{MCR}(Q, V_n). \)

We discuss how to use the above theorem to find \( \text{MCR}(Q, V) \) before outlining its proof.

**Example 5.1:** Consider the view \( V \), query \( Q \) shown in Figure 7, where \( V \) and \( Q \) are modified from the examples in [10]. It is easy to verify that \( x/y \) is a CR of \( V \) using \( Q \), but this rewriting cannot be found by finding all useful embeddings from \( Q \) to \( V \). However, the rewriting can be found by first splitting the view into 2 subviews (the \( /\)-edge \( (c, d) \) may be replaced with a \(*\)-chain of length 0 or 1, and the other \( /\)-edges can be replaced with a \( /\)-edge) with \( /\)-edges only, and then finding the CR \( x/y \) of \( Q \) using each of the subviews, and finally intersecting the two identical CRs.

**Complexity** The above method for finding the MCR of \( Q \) using \( V \) requires us to find the MCRs of \( Q \) using each subview in \( \text{Mod}_Q(V) \). We do not need to actually do the intersection of the MCRs, i.e., to transform \( \text{MCR}(Q, V_1) \cap \cdots \cap \text{MCR}(Q, V_n) \).
$\text{MCR}(Q, V_\ast)$ into a union of tree patterns. Instead, we can take the intersection $\text{MCR}(Q, V_\ast)(V(t)) \cap \cdots \text{MCR}(Q, V_\ast)(V(1))$ when evaluating it over $V(t)$. The cost for finding each of $\text{MCR}(Q, V_\ast)$ has been discussed in [8]. The cost of finding $\text{MCR}(Q, V_\ast)$ would be the total cost of finding all of $\text{MCR}(Q, V_1), \ldots, \text{MCR}(Q, V_n)$ (if we ignore the cost to find $\text{Mod}_Q(V)$ which is negligible).

Since the subviews in $\text{Mod}_Q(V)$ have similar structures, it is highly likely that we can design a more efficient algorithm to find all of $\text{MCR}(Q, V_1), \ldots, \text{MCR}(Q, V_n)$ altogether using the ideas of match sets [10]. We leave it as future work.

Next we outline the proof of Theorem 5.2. The proof uses Lemma 5.3 below and can be done as follows:

**Proof of Theorem 5.2:** (2) is a direct corollary of (1), so we focus on the proof of (1).

Let $Q'$ be a CR of $Q$ using $V$, that is, $Q' \subset V \subseteq Q$. Since $V_i \subseteq V$, we know $Q' \subset V_i \subseteq Q$ (for $i \in [1, n]$). Hence $Q'$ is a CR of $Q$ using $V_i$.

Conversely, suppose $Q'$ is a CR of $Q$ using $V_i$ (for all $i = 1, \ldots, n$), then $Q \subset V_i \subseteq Q$. Suppose $Q$ is a pattern in $\text{Mod}_Q(Q')$. Then $Q_i \subseteq Q'$, thus $Q_i \cap V_i \subseteq Q$. It is easy to verify that $\text{Mod}_Q(Q' \cap V) = (Q_i \cap V_i | Q_i \in \text{Mod}_Q(Q'), V_i \in \text{Mod}_Q(V))$. Therefore, by Lemma 5.3, $Q' \subset V \subseteq Q$. That is, $Q'$ is a CR of $Q$ using $V$.

**Lemma 5.3:** Let $P, Q$ be TP in $P^{(i//, s, t)}$. If every pattern in $\text{Mod}_Q(P)$ is contained in $Q$, then $P \subseteq Q$.

To prove Lemma 5.3, we only need to prove Lemma 5.4 below. This is because that the containment of tree patterns in $P^{(i//, s, t)}$ can be reduced to the containment of Boolean patterns [10], which are tree patterns that do not have selection nodes, and which will return either TRUE or FALSE when evaluated over a data tree $t$, depending on whether there is a matching of the pattern in $t$. For Boolean patterns $P$ and $Q$, $P \subseteq Q$ means $\forall t, P(t) \Rightarrow Q(t)$. We use $\text{mod}(P)$ to denote the set of all data trees in which $P$ has a matching, that is, $\text{mod}(P) = \{ t | P(t) \Rightarrow \text{TRUE} \}$.

**Lemma 5.4:** Let $P, Q$ be Boolean patterns. Let $z$ be a label that is not present in $Q$. Let $\text{Mod}_Q(P)$ be the set of data trees obtained from $P$ by first replacing each $//-$edge $e = (u, v)$ with a *-chain $u \leadsto v$ of length $l_e$ where $0 \leq l_e \leq L_Q(\text{label}(u), \text{label}(v)) + 1$, and then replacing *- with $z$. If $\text{Mod}_Q(P) \subseteq \text{mod}(Q)$, then $P \subseteq Q$.

To prove Lemma 5.4, we need Lemma 5.5 and Lemma 5.6 below.

**Lemma 5.5:** [10] Let $e$ be a matching of $P$ in $t$. There exists a unique pattern $P^o$ obtained from $P$ by replacing each $//-$edge with a *-chain such that, there is a unique matching $e'$ of $P^o$ in $t$ such that $\forall v \in N(P)$, $e(v) = e'(v)$.

**Lemma 5.6:** Let $t_1$ be a data tree and $P^o$ be a Boolean pattern such that $P^o(t_1) = \text{FALSE}$. Let $x_0, x_1, \ldots, x_n, x_{n+1}$ be a path in $t_1$ such that $n > L_{P^o}(\text{label}(x_0), \text{label}(x_{n+1})) + 1$ and $x_{i+1}$ is the unique child of $x_i$ for $i = 1, \ldots, n - 1$, and none of the labels $\text{label}(x_1), \ldots, \text{label}(x_n)$ has appeared in $P^o$. Let $t_2$ be the tree obtained from $t$ by deleting the node $x_n$ and transforming all its children into children of $x_{n-1}$. Then $P^o(t_2) = \text{FALSE}$.

**Figure 8:** Illustration of the proof of Lemma 5.6
structure-preserving. For any edge \((z, z')\) in \(P_1\), the number of edges between \(e_1(z)\) and \(e_1(z')\), denoted \(d(e_1(z), e_1(z'))\), is either \(d(e_2(z), e_2(z'))\) or \(d(e_2(z), e_2(z')) + 1\). This is because \(e_1(z')\) is either identical to \(e_2(z')\) or one node below \(e_2(z')\), while if \(e_1(z)\) is one node below \(e_2(z)\), then either \(e_1(z)\) is one node below \(e_2(z')\) as well or \(e_2(z) = x_{n-1}\), \(e_1(z) = x_n\) and \(e_1(z') = e_2(z')\). Therefore, \(e_1(z')\) is always a descendant of \(e_1(z)\). We now only need to show if \((z, z')\) is a \(/\)-edge, then \(e_1(z')\) is a child of \(e_1(z)\). The only possible case where \(e_1(z')\) is not a child of \(e_1(z)\) is when \(e_1(z) = e_2(z) = x_{n-1}\) and \(e_1(z') = e_2(z')\) is a node below \(x_n\). (Refer to Figure 8). This happens only if \(z'\) is not in \(C\), that is, \(z'\) is not below a \//edge whose end node is mapped to \(C\). Consider the path in \(P_1\) from \(z\) to \(rt(P_1)\): \(y_1 = z, y_2, \ldots, y_k = rt(P_1)\). Let \(m\) be the largest integer such that \(e_2(y_{m-1}) \in C\), hence \(e_2(y_{m-1}) \notin C\) (there exists such \(m\) since \(e_2(rt(P_1)) \in C\)). Consider the path \(y_m, y_{m-1}, \ldots, y_1\) in \(P_1\). All the nodes in it are mapped to \(C\) hence they are all labeled *\). Furthermore, all edges \((y_{i-1}, y_i)\) for \(i \in [1, m]\) are \//edges, since otherwise \(z = y_1\) would be in \(C\), while we already know \(z \notin C\). So the path from \(y_{m-1}\) to \(z'\) is a *-chain. Hence its length \(m\) is less or equal to \(L_{P_1}(\text{label}(y_{m-1}), \text{label}(z'))\). On the other hand, since all edges are \//edges, the images of \(y_m, \ldots, y_1\) under \(e_2\) must include all nodes \(x_1, \ldots, x_{n-1}\), thus \(m = n - 1\). This implies \(n - 1 \leq L_{P_1}(\text{label}(y_{m-1}), \text{label}(z'))\), which is a contradiction to the assumption. □

Proof of lemma 5.4 We are now ready to prove Lemma 5.4. The proof is similar to that of Proposition 3 of [10]. It is a proof by contradiction. The idea is to show that if \(P \not\subseteq Q\), then there is \(t_1 \in mod_m(P)\) (note: \(mod_m(P)\) denotes the set of data trees obtained from the patterns in \(mod_m(P)\) by replacing the wildcard * with \(z\)) for some sufficiently large integer \(m\), such that \(Q(t_1) = \text{FALSE}\) (Step 1). From \(t_1\), we can construct \(t' \in Mod_m(P)\) such that \(Q(t') = \text{FALSE}\), contradicting the assumption \(Mod_m(P) \subseteq mod(Q)\) (Step 2).

Step 1 of the proof is similar to that of Proposition 3 of [10]. Step 2 can be completed using Lemma 5.6 as follows. If \(t_1 \in Mod_m(P)\) then the proof is complete. Suppose \(t_1\) is not in \(Mod_m(P)\). That is, there is a \(-/\)-edge \((u, v)\) in \(P\) such that in \(t_1\), the number of nodes between \(u\) and \(v\) is larger than \(L_Q(x, y) + 1\), where \(x = \text{label}(u), y = \text{label}(v)\). Then we can apply Lemma 5.6 repeatedly to reduce the length of the path until the number of nodes between \(u\) and \(v\) is less or equal to \(L_Q(x, y) + 1\). This process can be repeated for every \(-/\)-edge in \(P\) until we find \(t' \in Mod_m(P)\).

5.2 The alternative approach

We show how to answer \(Q\) using \(V\) without explicitly finding the MCR in this section.

Let \(Q, V \in P(//; ++; -\rightarrow)\) be TP's. Let \(t'\) be any data tree such that \(\text{label}(rt(t')) = \text{label}(an(V))\) or \(\text{label}(an(V)) = \ast\). We use \(t' \circ V\) to denote the tree obtained from \(t'\) and \(V\) by merging \(rt(t')\) and \(an(V)\), where the merged node is labeled with \(\text{label}(rt(t'))\). Let \(V' \in Mod(V)\) be a subview, \(t'\) be a data tree, and \(t_1\) be a tree in \(V(t)\). We call \(t_1 \circ V'\) a new answer annotated by \(V'\) (note that \(t_1\) may be outside \(V'(t)\)). Similar to Theorem 3.1, we can prove the following lemma.

**Lemma 5.7:** Let \(Q\) and \(V\) be TP's in \(P(//; ++; -\rightarrow)\). Let \(h_1, \ldots, h_k\) be all of the useful embeddings from \(Q\) to \(V\). Then \(Q(t' \circ V) = \text{CAT}_{h_1}(t') \cup \cdots \cup \text{CAT}_{h_k}(t')\).

Using the lemma above, we can prove

**Theorem 5.3:** For any data tree \(t\), suppose \(V(t) = \{t_1, \ldots, t_m\}\), and \(\text{Mod}(V)\) is \(\{V_1, \ldots, V_n\}\). Then

\[
\text{MCR}(Q, V)(V(t)) = \bigcap_{i=1}^{n} (Q(t_1 \circ V_i) \cup \cdots \cup Q(t_m \circ V_i))
\]

**Proof sketch.** By Theorem 5.2 (2),

\[
\text{MCR}(Q, V)(V(t)) = \bigcap_{i=1}^{n} (\text{MCR}(Q, V_i)(V(t)) \\
= \bigcap_{j=1}^{m} (\text{MCR}(Q, V_j)(t_j) \cup \cdots \cup \text{MCR}(Q, V_j)(t_m))
\]

By Theorem 5.1 and Lemma 5.7, \(\text{MCR}(Q, V_j)(t_j) = Q(t_j \circ V_i)\) for \(j \in [1, m], i \in [1, n]\). Therefore, \(\text{MCR}(Q, V)(V(t)) = \bigcap_{i=1}^{n} (Q(t_1 \circ V_i) \cup \cdots \cup Q(t_m \circ V_i))\).

The above theorem indicates that we can answer \(Q\) without explicitly finding \(\text{MCR}(Q, V)\). Instead, we can evaluate \(Q\) over \(t_j \circ V_i\) (for \(i \in [1, n], j \in [1, m]\)) and intersect \(Q(t_1 \circ V_i) \cup \cdots \cup Q(t_m \circ V_i)\) for all subviews \(V_i\) in \(\text{Mod}(V)\).

5.3 Redundant view answers wrt \(P = P(//; ++; -\rightarrow)\)

It is easy to verify that all propositions, theorems and corollaries in Sections 3 and 4 still hold if \(V\) and \(Q\) are changed to any patterns in \(P(//; ++; -\rightarrow)\). Therefore, because of Theorem 5.1, in the special cases \(Q \in P(//; ++; -\rightarrow) \cup P(//; ++; -\rightarrow)\) or \(V \in P(//; ++; -\rightarrow)\), for all data tree \(t\), \(\text{MCR}(Q, V)(V(t)) = \bigcap_{i \in V(t)} Q(t' \circ V)\). That is, evaluating the query \(Q\) directly over the annotated view answers will generate the same answers as the MCR over the original answers. Furthermore, we can use the same criteria and procedures as described in Section 4 to identify/remove redundant view answers with respect to \(P = P(//; ++; -\rightarrow) \cup P(//; ++; -\rightarrow)\), for any view \(V \in P(//; ++; -\rightarrow)\).

6. RELATED WORK

In this section we compare our work with previous ones that are the mostly closely related to ours. As mentioned in Section 1, apparently [8] is the first paper on MCR of tree pattern queries using views, and it proposed the technique of useful embeddings for queries and views in \(P(//; ++; -\rightarrow)\). The paper also proposed an algorithm to find the MCR under a non-recursive and non-disjunctive DTD which can be represented as an acyclic graph. The basic idea is to reduce the original problem to one without DTD by chasing the tree patterns repeatedly using constraints that can be derived from the DTD. Recently, [11] proposed a method for finding the MCR for queries that have no *-nodes connected to a //edge and no leaf node \(u\) such that \(u\) and the parent of \(u\) are both labeled *, based on the concepts of trap embedding (and its induced pattern) and trap relay, where a trap embedding is a mapping from a tree pattern to a tree, and a trap relay is a mapping from a pattern to another pattern.
It can be shown that the induced pattern of a trap embedding from $Q$ to a pattern $V$ in $\text{mod}_{L_{2}+1}(V)$ is the same as a CAT if the attach point (see [11]) of $V$ is $\text{sn}(V_{i})$. Thus Theorem 3.3 of [11] can be derived from Theorem 2.2 (1) of this paper (but not the other way around). Previously, [13] studied maximal contained rewriting using multiple views for queries and views in $\text{P}(\text{P}(-/-,\cdot))$. More recently, [7] and [16] studied the evaluation of MCRs, and [12] gave an algorithm for identifying redundant contained rewritings, which is orthogonal to Section 4 of this work. It should be noted that the redundant view answers discussed in this paper are with respect to the union of all CATs, and they may not be redundant with respect to equivalent rewritings or with respect to a subset of CATs.

There have been works on equivalent rewritings (ERs), where an equivalent rewriting is a special contained rewriting $Q'$ which satisfies the condition $Q' \circ V = Q$. Among them [15] showed that if $Q$ and $V$ are in $\text{P}(\text{P}(-/-,\cdot))$, $\text{P}(\text{P}(-/-,\cdot))$ or they are normalized linear patterns, there is an equivalent rewriting of $Q$ using $V$ if $Q_{k}$ is an equivalent rewriting where $k$ is the position of $\text{sn}(V)$ on the selection path of $V$, and $Q_{k}$ is the subtree of $Q$ rooted at the $k$-th node on the selection path of $Q$. [4] extended the above result to $Q, V \in \text{P}(\text{P}(-/-,\cdot))$ and showed that for many common special cases, there is an equivalent rewriting of $Q$ using $V$ iff $Q_{k}$ or $Q_{k}^{'\text{ii}}$ is an equivalent rewriting, where $Q_{k}^{'\text{ii}}$ is the pattern obtained from $Q_{k}$ by changing all edges connected to the root to //edges. Thus in those special cases, to find an equivalent rewriting we only need to test whether $Q_{k}$ or $Q_{k}^{'\text{ii}}$ is an equivalent rewriting. [9] assumed both $Q$ and $V$ are minimized, and reduced tree pattern matching to string matching, allowing a more efficient algorithm for finding equivalent rewritings. They further proposed a way to organize the materialized views in Cache to enable efficient view selection and cache look-up. [6] investigated equivalent rewritings using a single view or multiple views for queries and views in $\text{P}(\text{P}(-/-,\cdot))$, where the set of views is represented by grammar-like rules called query set specifications. The work can be seen as an extension to an earlier work [5] which investigated the same problem for explicit views.

None of the previous works dealt with the redundant view answers problem or the approach of finding answers to a new TP using annotated view answers. Finally, Section 5 of this work borrows techniques from [10], and some of our results on query containment, namely Lemmas 5.1 and 5.3, are extensions to the results in [10].

7. CONCLUSION

We revisited the problem of answering TP queries using views, and showed that for $Q$ in $\text{P}(\text{P}(-/-,\cdot))$ evaluating $Q$ directly over the annotated view answers will find the same answers as evaluating the MCR of $Q$ using $V$ over the original view answers. We also provided an algorithm for identifying redundant view answers in this case. Our preliminary experiments with TreeBank and XMark showed that there can be a large percentage of redundant view answers and the majority of the them can be found using the linear algorithm FastScan. We showed that the new approach and the techniques for finding redundant view answers can also be applied if we limit $Q$ to $\text{P}(\text{P}(-/-,\cdot,\cdot))$ or $\text{P}(\text{P}(-/-,\cdot,\cdot))$. We showed that if $V$ has no //edges then for any $Q \in \text{P}(\text{P}(-/-,\cdot,\cdot))$ the approach of useful embeddings can be used to find MCR($Q, V$). For $V$ and $Q$ which are general TP's in $\text{P}(\text{P}(-/-,\cdot,\cdot))$, we presented a method for finding MCR($Q, V$) by dividing $V$ into a finite set of subviews and finding the MCRs of $Q$ using each of the subviews. We discussed how to answer $Q$ using $V$ without explicit rewriting in the general case.

As future work we plan to do experiments to compare the actual performance of the rewriting approach and the annotated view answer approach, design more efficient algorithms for finding the MCR and investigate the redundant view answer problem for the general case where $Q, V \in \text{P}(\text{P}(-/-,\cdot,\cdot))$.

Acknowledgement: This work is supported by the Australian Research Council Discovery Grant DP1093404 and by grant of the Research Grants Council of the Hong Kong SAR, China No. 419008. We thank Professor Wenfei Fan for helpful comments on an earlier draft of this paper.

8. REFERENCES