Moving from Diagnosis to Intervention in Numeracy - turning mathematical dreams into reality

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George Booker
Griffith University
Brisbane, Australia
g.booker@griffith.edu.au

Abstract

When students experiences difficulties in mathematics, they require assistance to overcome the misconceptions or inappropriate ways of thinking they have developed. Such difficulties should be identified early and assistance provided to address the needs revealed by a *Diagnosis* of their ways of acting and thinking. *Intervention* can then focus on providing support within a teaching sequence as soon as a difficulty is noted or anticipated as an essential element of ongoing learning. In this way, students be can helped to develop the conceptual understanding and fluent processes needed to acquire and use mathematics. This paper will provide an overview to the intervention process, from determining underlying causes of any difficulties, leading a student to see inadequacies in their ways of proceeding and thus appreciate a need to change, to implementing means of building appropriate ways of thinking, generalising and applying mathematical ideas. A series of numeracy screening tests (Booker 2011) will be presented.

Introduction

> Children should have a robust sense of number … this includes an understanding of place value, meaning for the basic operations, computational facility and a knowledge of how to apply this to problem solving. A thorough understanding of fractions includes being able to locate them on a number line, represent and compare fractions, decimals and per cents, estimate their size and carry out operations confidently and efficiently.
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> Final Report of the National Mathematics Advisory Board, 2008 pp 17 & 18

In a society "awash in numbers" and "drenched in data" (Orrill 2001), numeracy must be considered an essential goal of education for all (National Numeracy Review p.xi, 2008). It is no longer enough to simply study mathematics; mathematical knowledge needs to be able to be used in an ever-widening range of activities. Indeed, those who lack an ability to think mathematically will be disadvantaged, unable to participate in high-level work and at the mercy of other peoples' interpretation and manipulation of numbers and data. As Steen (1997) predicted "an innumerate citizen today is as vulnerable as the illiterate peasant of Gutenberg's time"; both numeracy and literacy are critical components for living full lives in the 3rd millennium. An ability to solve problems, communicate the results and methods used to obtain solutions, to interpret and use the results of mathematical processes and making sense are all essential to being numerate. However, many students fail to achieve even minimal standards of numeracy (DETYA, 2000; National Numeracy Review, 2008; OECD, 2004) and even those who do frequently say that they are 'no good at maths', feel inadequate and are unable to use the elementary mathematics that they have 'acquired'.

Diagnostic assessment

Assessment is integral to teaching and learning. While it can be used to grade students or measure them against National or International benchmarks, more importantly, well-focused assessment can reveal how students think and provide guidance to plan ongoing teaching. It also supplies information about how students are dealing with the mathematical tasks they are exposed to and feedback on particular programs or learning activities, whether they are suited to the students and content in question or whether they need to be modified to produce the expected learning. At the same time, assessment can provide information to the student, parents or caregiver and other teachers about the student's mathematical capabilities and potential. It may also highlight different outcomes across different groups of students, across different classes and schools when the results are discussed with other teachers.

While it is important to know *what* students know, of even more importance is *how* they know – is their knowledge simply memorised routines, or is there a deep understanding based on well-understood concepts and fluent, meaningful processes applied in appropriate ways? Hence the need for *Diagnostic assessment* that is designed to reveal not only what a student knows but also how they know, not to see what they do not know but to reveal what they need to know. Most importantly, it may reveal gaps in a student's mathematical knowledge – critical ideas may not have become part of a student's way of thinking or may not have been included in the steps used to build up a topic. For example, there are many programs where the aspects of renaming needed for computation and other number processes are not developed as an extension of place value or to provide a complete understanding of larger numbers.

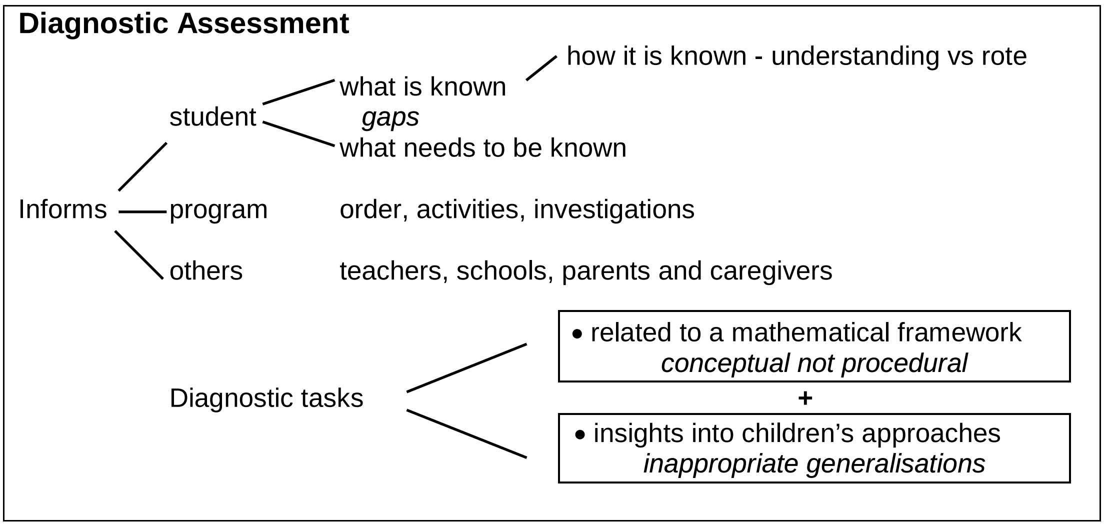


Figure 1: Components of Diagnostic Assessment

In this way, diagnostic assessment highlights the strengths that a leaner brings to a topic and also the weaknesses in their prior knowledge that may cause errors or difficulties.

A diagnostic test is different from other forms of testing. It needs to provide insight into all the steps required to develop facility with a topic and not just measure how well a final outcome or particular points in the development have been attained. However, the sequence of questions posed cannot simply mimic those used in teaching a topic as this may allow a student to refresh their knowledge and mask aspects that have not become central to their thinking. It is often difficulties with steps at the outset of developing facility with a topic that cause deficiencies rather than just an inability to or apply the final process. Misconceptions or errors might also arise from insufficient understanding of mathematical aspects that were developed outside of a particular topic but are essential to the processes. For example, many difficulties with computation are in fact due to underlying aspects of numeration such as zero, place value and renaming.

	87 - 40 40	86 + 42 148	702 - 348 264
Difficulties with	*zero*	*place value*	*renaming* & *zero*

Similarly, difficulties with measurement may be due to insufficient understanding of spatial properties or of the significance of zero or decimal fractions in the way measuring instruments are applied.

Many errors are based in confusion with and among the rules that have been acquired or else in insufficient understanding of the numbers being worked with. A further source of difficulties are *inappropriate generalisations*, where something that worked in one situation is taken to another setting where the conditions that permitted it no longer apply. For example, *additive thinking* is often used within multiplicative situations for computation, fractions or measurement.

	46 x 58 248	46 x 58 408	35 x 47 425 200 625
Multiplying only	ones by ones, tens by tens		*renaming* difficulties as well

Strengths and weaknesses revealed in diagnostic assessment then need to be stated in terms of the mathematical ideas that underlie them, reasons need to be proposed for why they came about and the underlying causes of the difficulties identified so that appropriate teaching strategies to overcome difficulties can be planned, Close observation is critical to provide insight into the ways of thinking being applied. Mostly it will require a task chosen to elicit the ways in which a student is acting then systematically exploring the possible forms this takes. Consequently, any initial attribution of reasons can only be an assumption, usually based on experience with this type of behaviour. Nonetheless it may not be the actual thinking being evinced and must be treated as only a possibility that needs to be investigated further. This will require further probing to determine what is in fact occurring and then analysis of the likely underlying causes.

assumptions

observations probing

↓

causes

Figure 2: Cycle of Diagnostic assessment

Often several possibilities for an error may need to be considered and the process of probing and observing continued in order to first dismiss one or more before the likely reason or reasons can be determined. In this way, a cycle of observations, assumptions and probing will eventually lead to an understanding of the underlying causes and suggest what is needed to overcome them.

From Diagnosis to Intervention

Diagnostic assessment is critical in planning how to teach or re-teach those essential aspects of mathematics that underpin numeracy. When misconceptions, difficulties and gaps in a student's knowledge have been identified, means to intervene in the learning can be planned and implemented in a manner appropriate to both the learner and the way in which concepts and processes are best established and consolidated. Both what is known and needs to be known must be identified and described in terms of the underlying mathematical ways of thinking rather than rules or procedures that may be followed in a less than meaningful way.

The most effective way of constructing new ways of thinking will mostly begin with the use of materials to draw out the patterns on which the ideas are developed, linking to a language that provides meaning and only moving to the symbolic expressions that express what is happening succinctly when the learner has adopted the way of thinking as his or her own. Simply showing a student what to do using a written procedure is rarely successful in replacing procedures that have led to errors. At best they will try to copy and remember a teacher's approach but it may be that the link to what they do know is not apparent in the purely recorded form. Rather, engaging and different practice activities, often in the form of games in which learners willingly participate, are an essential part of learning to bring a concept to the forefront of a learner's mind and enable a process to become fluent.

In this way, intervention can build from the understandings that are essential for the development of further concepts and processes and provide the links needed to extend the ideas to enable applications to new situations, means of solving a range of problems and make possible extensions to further mathematics. This process can be summarised:

The process of intervention

1. the identification of *understandings and errors* and the description of them in terms of the underlying mathematical concepts and processes
2. uncovering *sources of difficulties* – not only inappropriate thinking but also the degree of understanding of why processes and responses are correct
3. revealing *inadequacies in thinking* to a child in order to build an appreciation of a need for change
4. the implementation of means of *constructing or re-constructing* appropriate ways of thinking
5. *practice* that is focused and motivating to allow a way of thinking to become secure and provide a basis for generalisation to more complex problems and applications or to the development of further mathematics

Figure 3: The process of intervention

Case study

A student was observed to have difficulties with decimal fractions involving tenths:

Write the numbers that are 3 tenths more:

8.4 __*11.4*__ **9.7** _*12.7*__ **6** __*9*__ **0.8** _*3.8*_ **2.9** _*5.9*__

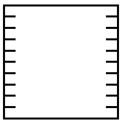
In this case the student has read the question as ‘write the numbers that are 3 more’ either because they have simply focused on the number 3 and used the cue word *more* when reading the question or because *place value* is not secure for decimal fractions. For instance, many students say ‘8 point 4’ for the number 8.4 without any emphasis given to the place values involved. 8 and 4 are simply seen as digits separated by ‘.’ rather than 8 ones 4 tenths which is then read as ‘8 and 4 tenths. In either case, no meaning for the ‘3 tenths’ requested in the question would have been present.

To increase a number by 3 tenths means that the digit in the tenths place is increased by 3 which may also involve renaming 10 tenths as 1 one:

8.4	*8.7*	**9.7**	*10*	**6**	*6.3*	**0.8**	*1.1*	**2.9**	*3.2*

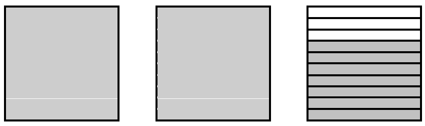
In order to provide intervention on the underlying difficulties the diagnosis has revealed, the fraction concept and place value for decimal fractions need to be built up. This requires teaching to

- revisit the model for fractions using rectangles to link model and names based on ordinal numbers
- use a square with the beginning lines and have the student connect the lines to see there are 10 equal parts - tenths



10 equal parts - tenths

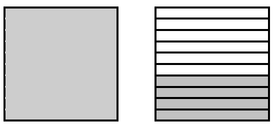
- name fractions with ones and tenths



2 ones and 7 tenths – 2 and 7 tenths

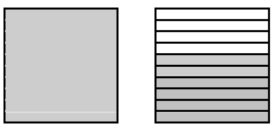
ones	tenths	
2	7	2.7

- shade ones and tenths diagrams to match symbols and read names



1 and 4 tenths 1.4

- shade decimal fractions that are 2 tenths more and name them



1 and 6 tenths 1.6

- write the symbols for the initial fractions and the fractions that are 2 tenths more

2 tenths more than 1.4 is 1.6

Note that the example chosen to (re-)establish the way of thinking is different to those that were completed incorrectly so as to focus on the underlying ideas rather than simply be seen to correct an example that was answered incorrectly. When several examples like these have been examined and the student has developed an understanding of what is needed, then the fractions originally answered can be looked at again to see that the student can not only obtain correct results but is able to see what was done incorrectly when he was first asked these questions.

Conclusion

Building numeracy in all students is a critical aspect of contemporary schooling. Understanding how concepts and processes are constructed and connected provides a basis for overcoming misconceptions and inappropriate ways of thinking that may have developed (Hiebert & Grouws, 2007, p.391). Appropriate intervention programs can then be planned and implemented to build students' competence and confidence with fundamental mathematical ideas. In this way, students will be prepared to engage with further mathematical ideas and be inclined to use their knowledge of mathematics in the many everyday and work contexts where reasoning and sense making will be required.

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