A Re-Examination of the Unbiased Forward Rate Hypothesis in the Presence of Multiple Unknown Structural Breaks

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Abstract

We test the unbiased forward rate (UFR) hypothesis using new tests for cointegration developed by Hatemi-J (2008) that allows for multiple unknown structural breaks. We analyse the AUD, EUR, GBP and JPY (versus the USD) spot rates and forward rates relationship during the period January 7, 1999 to December 28, 2006. We find that the UFR does hold when the effects of the unknown structural breaks are taken into account. The parameters that we obtained were close to unity; hence, taking into account transactions cost and the existence of a risk premium, earning arbitrage profits may still not be possible. Thus, the markets for these currencies may still be considered as efficient.

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1. Introduction

The unbiased forward rate (UFR) hypothesis states that the forward rate is an unbiased and efficient predictor of the future spot rates since it fully reflects all available information concerning agents’ expectations of the future spot rate. This may be expressed as follows:

\[ s_{t+1} = f_t + u_{t+1} \]  

where \( s_{t+1} \) is the spot rate at time \( t+1 \), \( f_t \) is the forward rate at time \( t \), and \( u_{t+1} \) the error term which is a random component with an expected value of zero and serially uncorrelated with the information set available to agents at time \( t \).

The UFR hypothesis is derived based on two assumptions that reflect market efficiency (Hakkio and Rush, 1989; Zacharatos and Sutcliffe, 2002):

(a) Agents are risk neutral, so that the risk premium is zero, which therefore ensures that through arbitrage this equality, is achieved.

\[ E(s_{t+1}) = f_t \]  

Here, \( E(s_{t+1}) \) is the market’s expectation of the spot rate at time \( t + 1 \), which is based on the information set available at that time.

(b) Agents use all available information rationally, so that the expected returns to speculators are zero.

\[ s_{t+1} = E(s_{t+1}) + u_{t+1} \]  

Combining these two conditions therefore produces the unbiased forward hypothesis expressed in Equation 2:

\[ s_{t+1} = E(s_{t+1}) + u_{t+1} \]

\[ = f_t + u_{t+1} \]  

Given the assumptions behind the UFR, if it holds, then it is an indication that the foreign exchange market is efficient.

For purposes of testing the unbiased forward rate hypothesis, Equation 1 is re-formulated into the following equations:
\[ s_{t+1} = a + bf_t + e_t \] 

(5)

\[ (s_{t+1} - s_t) = a + b(f_t - s_t) + v_t \] 

(6)

If UFR is to hold in strong form, this requires that \( a = 0 \) and \( b = 1 \).

Before the development of the cointegration methodology, the second formulation was preferred in order to avoid the issue of non-stationarity. However, this has been discredited by Lin and Maddala (1992) by showing that simultaneous equation bias exists with this specification. This therefore left researchers with a dilemma. With the advent of cointegration, the focus of the testing has shifted to the first formulation.

The bulk of empirical evidence shows that this postulated relationship does not hold and hence, is labeled as a “paradox”. Forward premiums have been found to be negatively, rather than positively related with spot rate changes (Froot and Thaler, 1990; Bekaert, 1996; Bekaert et. al., 1997). There are a number of studies, which have utilised the cointegration approach within a vector autoregression context following the Johansen procedure. Most researchers found cointegration between the two variables but the cointegration vector has not necessarily been \((-1.0 \text{ and } 1.0)\) (see, for instance, Villanueva, 2007; McMillan, 2005 and Jung et al, 1998).

Several economic explanations have been put forward for this “paradox” ranging from the presence of a risk premium (Taylor, 1995 and Froot and Frankel, 1989) to rational systematic errors, i.e. peso problems (Hodrick, 1987 and Engel, 1996). It has also been claimed that this paradox could be simply a statistical phenomenon arising from such problems as autocorrelation in the forward premium, as shown by Baillie and Bollerslev (2000) or simultaneous equation bias, as demonstrated in Barnhart, et. al. (1999). It is not clear from these studies whether the condition of the error term being a white noise has been met since cointegration requires that the error term be stationary.

In this paper, we examine the unbiased forward hypothesis with the use of new cointegration tests suggested by Hatemi-J (2008), which can be used for testing cointegration in the presence of two potential structural breaks, which are unknown and determined by the data. We cover the period from January 7, 1999 to December 28, 2006 using weekly Australian dollar (AUD), Euro (EUR), Japanese yen (JPY), and British pound (GBP) (versus the US dollar or USD) spot rates and 3-month forward rates. We obtained the data from Datastream.
Our study extends the literature on the spot-forward relationship in terms of the application of new approaches to the issue. It sheds further light on the claim by a number of previous researches that the paradox is simply a statistical phenomenon. This group of research used other approaches, and as stated earlier, it is possible that their results could have been affected by statistical inference bias due to the existence of structural breaks in the data.\footnote{It should be pointed out that Choi and Zivot (2007) provide evidence for multiple structural changes in the mean of the G7 countries’ forward rates.}

The importance of the relationship between the spot rate and forward rate in the foreign exchange market is well-recognised as evidenced by the continuing studies conducted on this issue\footnote{In addition to the literature cited earlier, see also Sercu, et al (2008); Diamandis, et al (2008); Chen (2007); Sekioua (2006), Wang (2005); Apergis and Eleftheriou (1997) as well as Ahmed and Ansari (1997).}. In addition to its implication on market efficiency, theoretically, this relationship is an important input into foreign exchange models. It is also highly important to policymakers in terms of setting appropriate foreign exchange policies and to investors in the foreign exchange markets in terms of their search for profitable opportunities. Another importance of knowing this relationship is in terms of hedging as forward contracts are used to hedge spot investments.

The remaining parts of the paper are organised in this manner. Section 2 discusses the methodology while Section 3 presents the empirical results. Section 4 provides a conclusion to the study.

2. Methodology

As mentioned earlier, we utilise cointegration tests developed by Hatemi-J (2008) that allow for multiple structural breaks. In order to apply these tests, first, we reformulate equation (5), as follows, in order to take into account the impact of two structural breaks:

\[ S_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \beta_0 F_t + \beta_1 D_{1t} F_t + \beta_2 D_{2t} F_t + \mu_t, \quad (7) \]

where \( D_{1t} \) and \( D_{2t} \) are binary indicator variables that are defined in the following form:

\[
D_{1t} = \begin{cases} 
0 & \text{if } t \leq [n \tau_1] \\
1 & \text{if } t > [n \tau_1] 
\end{cases}
\]

\[
D_{2t} = \begin{cases} 
0 & \text{if } t \leq [n \tau_2] \\
1 & \text{if } t > [n \tau_2] 
\end{cases}
\]
and

\[ D_{2t} = \begin{cases} 0 & \text{if } t \leq \lfloor n \tau_2 \rfloor \\ 1 & \text{if } t > \lfloor n \tau_2 \rfloor \end{cases} \]

The unknown parameters \( \tau_1 \in (0, 1) \) and \( \tau_2 \in (0, 1) \) measure the relative timing of the regime shift points. The bracket denotes the integer part since \( n \) is the number of observations.

We utilise three tests for cointegration in the presence of two unknown structural breaks, which are denoted by \( ADF, Z_\alpha \) and \( Z_t \) respectively. We calculate the \( ADF \) test based on the \( t \)-test for the slope of \( \bar{u}_{t-1} \) in a regression of \( \Delta \bar{u}_t, \Delta \bar{u}_{t-1}, \ldots, \Delta \bar{u}_{t-k} \), where \( \bar{u}_t \) is the residuals from equation (7). We determine the lag order \( k \) by minimizing an information criterion suggested by Hatemi-J (2003, 2008b).\(^3\) The \( Z_\alpha \) and \( Z_t \) test statistics are estimated based on the following bias-corrected first-order serial correlation coefficient:\(^4\)

\[
\hat{\rho}^* = \frac{\sum_{t=1}^{n-1} \bar{u}_t \bar{u}_{t+1} - \sum_{j=1}^{B} w(j/B) \hat{\gamma}(j)}{\sum_{t=1}^{n-1} \bar{u}_t^2},
\]

where \( w(\cdot) \) is a function that determines the kernel weights fulfilling the standard conditions for spectral density estimators and \( B \) represents the bandwidth. The autocovariance function is calculated as

\[
\hat{\gamma}(j) = \frac{1}{n} \sum_{t=j+1}^n (\bar{u}_{t-j} - \hat{\rho} \bar{u}_{t-j-1})(\bar{u}_t - \hat{\rho} \bar{u}_{t-1}),
\]

where \( \hat{\rho} \) is the ordinary least squares estimate of the effect (without intercept) of \( \bar{u}_{t-1} \) on \( \bar{u}_t \).

The \( Z_\alpha \) and \( Z_t \) test statistics are then obtained as

\[
Z_\alpha = n(\hat{\rho}^* - 1),
\]

and

\[
Z_t = \frac{(\hat{\rho}^* - 1)}{(\hat{\gamma}(0) + 2 \sum_{j=1}^{B} w(j/B) \hat{\gamma}(j)) / \sum_{t=1}^{n-1} \bar{u}_t^2},
\]

\(^3\) According to the simulation experiments conducted by Hatemi-J (2008b) this information criterion performs well in situations where unit roots or ARCH effects prevail.

\(^4\) The \( Z_\alpha \) and \( Z_t \) tests were originally introduced by Philips (1987). It should be mentioned that Gabriel et al (2002) suggested applying a Markov switching approach to test for cointegration in the presence of breaks.
where \( \hat{\gamma}(0) + 2 \sum_{j=1}^{B} w(j/B) \hat{\gamma}(j) \) is the long-run variance measure of the residuals of a regression of \( \hat{a}_t \) on \( \hat{a}_{t-1} \). The appropriate test statistics are the minimum values of these three tests across all possible values for \( \tau_1 \) and \( \tau_2 \), with \( \tau_1 \in T_1 = (0.15, 0.70) \) and \( \tau_2 \in T_2 = (0.15 + \tau_1, 0.85) \). We choose the minimum value of each test because the smallest value provides the empirical evidence in favor of the cointegration relationship. The minimum values of these test statistics are mathematically defined as

\[
ADF^* = \inf_{(\tau_1, \tau_2) \in T} ADF(\tau_1, \tau_2),
\]

\[
Z_t^* = \inf_{(\tau_1, \tau_2) \in T} Z_t(\tau_1, \tau_2),
\]

\[
Z_a^* = \inf_{(\tau_1, \tau_2) \in T} Z_a(\tau_1, \tau_2),
\]

where \( T = (0.15n, 0.85n) \). We truncate the data by 15% on each side, following Gregory and Hansen (1996), who developed tests for cointegration for one unknown structural break. Using the same reasoning, we also allow the distance between the two regime shifts to be at least 15%. The calculations are conducted by utilizing a statistical software component in Gauss produced by Hatemi-J (2009). This statistical software is available on line.

3. Empirical Results

Prior to testing for cointegration, we conducted unit roots tests. The results, not reported but available on request, showed that each variable is integrated of the first order even in the presence of two breaks in their deterministic components. The graphical illustration of these variables depicted in Figure 1 also confirms this conclusion. This implies that we have to pay attention to the integration properties of the data in order to avoid spurious and false inference.

[INSERT FIGURE 1 HERE]
The results of the tests for cointegration in the presence of two unknown structural breaks are presented in Table 1. As can be seen from these results, the null hypothesis of no cointegration is strongly rejected by all three tests for each currency. The timing of each break as determined by the method used in this study is shown as follows:


[INSERT TABLE 1 HERE]

The 2001 breaks are a result of the September 11 attack on the US while the 2003 ones, could be due to the effect of the Iraqi invasion which had significantly disrupted oil supply resulting in an abrupt change in oil prices which then affected exchange rates.

[INSERT TABLE 2 HERE]

As can be seen in Table 2, the slope value for the pre-break period is very close to unity for each exchange rate (see the estimated value for $\beta_0$) and it is strongly significant in each case. There are two significant breaks in the slope for the AUD (see the estimated values for $\beta_1$ and $\beta_2$). First, there is an increase in the parameter value and then a decrease in the parameter value. Despite being statistically significant, these changes are marginal. The first change is 0.015 and the second change is -0.054. The same story is true regarding the JPY with the difference that both changes are positive. The first change is 0.028 and the second is 0.045 in the JPY. In the case of the GBP, there is only one significant change in the slope in the second period, which is equal to -0.02. Both changes in the slope parameter are non-significant in the case of EUR. The risk premium is statistically significant in all cases except EUR (see the estimated values for $\alpha_0$). There are two significant changes in the case of the AUD and the JPY (see the estimated values for $\alpha_1$ and $\alpha_2$). In the case of the GBP, there is only one significant change in the premium, which happens in the second break period. No significant changes in the premium were found in the case of EUR. Based on this estimates
we can conclude that the unbiased forward rate hypothesis holds most strongly in the case of the EUR.

4. Conclusion

The unbiased forward rate hypothesis stipulates that the forward rate is an unbiased predictor of future spot rates. Empirically, if this is to hold within a regression framework, there must be cointegration within the variables with point-for-point response. This paper is the first attempt to investigate these issues in the presence of two unknown structural breaks by applying a new test method recently developed in the literature. The timing of each break is selected by the method. In some cases the selected structural breaks coincide with the September 11 terrorist attack on the US and the 2003 Iraqi invasion.

Our results show that the UFR does hold and the parameters that we derived are close to unity. There are significant changes in the parameters for all exchange rates except for the EUR. However, the changes in the parameters are marginal. These results indicate that the market for these currencies are still efficient as these values may not allow the existence of arbitrage profits after taking into account transaction costs and risk premium.
References


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<th>Currency</th>
<th>Test Statistic</th>
<th>Estimated Test Value</th>
<th>1% Critical Value</th>
<th>5% Critical Value</th>
<th>10% Critical Value</th>
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Notes: The critical values are collected from Hatemi-J (2008).
Table 2. The Estimated Values of the Parameters

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Notes: The t-values are presented in the parentheses. The Newey-West heteroskedasticity and autocorrelation consistent standard errors are used.
Figure 1: Forward and Spot rates for Australian dollar (AUD), Japanese yen (JPY), British pound (GBP) and Euro.

Note that F after each currency indicates forward rate and S indicates the spot rate.