Image Enhancement by Wavelet Multi-scale Edge Statistics

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Abstract

The distribution of wavelet modulus maxima across wavelet scales can be used to characterize edges in an image. In this paper, we present a novel algorithm that performs image enhancement by mapping the distribution of the wavelet modulus maxima of the blurred image to that of a generic sharp image. Experimental results confirm that the proposed algorithm is able to perform image enhancement without introducing unpleasant visual artifacts.

1. Introduction

Edges are important features that affect the perceptual quality of an image. Blurred images usually have edges that appeared overly smooth. One way to improve the perceptual quality of an image is to improve the sharpness of edges in the image. Unsharp masking is one of the main techniques for image enhancement [1-5]. The basic idea of unsharp masking is to add to the blurred image a high-pass filtered version of the image to emphasize high frequency regions such as edges in an image.

Wavelet allows a multi-resolution decomposition of a signal. Recently, an unsharp masking algorithm based on wavelet transform has been proposed for image sharpening [6]. The algorithm makes use of the correlation between different wavelet scales to retain wavelet coefficients related to edges but remove noise-related coefficients. The retained edge-related coefficients are then used to obtain the correction image which is then added back to the original image to enhance the perceptual quality.

In this paper, we introduce a novel image sharpening algorithm based on wavelet multi-scale edge decomposition. Our algorithm is based on the fact that the multi-scale edges extracted by a particular non-decimated wavelet scheme follow a specific distribution according to the degree of regularity of the edges in the image. The novelty of our approach lies in that we exploit the statistical relationship between edges at multiple wavelet scales of the blurred and sharp image to enhance the blurred image. This is done by applying a non-linear mapping to the wavelet coefficients related to multi-scale edges in the blurred image.

2. Wavelet Multi-Scale Edge Analysis

Points of sharp variations are usually one of the most important features for analyzing the properties of signals or images. In images, local sharp structures such as edges contain rich information and are important for many applications such as object segmentation and scene analysis. However, edges in an image could exist in different scales and we need a range of scales to describe these structures effectively.

In [7], Mallat et al. proved that the wavelet modulus maxima of a certain dyadic wavelet transform can detect the location of the singular structures in a signal. Consider a wavelet constructed in such a way that it is the first order derivative of a smoothing function, the
local modulus maxima of the dyadic WT by such a wavelet then characterize sharp variations or edges in a signal at multiples of dyadic scale.

Let $\theta(t)$ be the smoothing function that is used to smooth a signal at various scales. Let $\psi(t)$ be the first derivative of $\theta(t)$, i.e., $\psi(t) = d\theta(t)/dt$, and let the wavelet function and the corresponding smoothing function be defined by

$$\psi_j(t) = 2^{-j/2} \psi(2^{-j} t)$$
$$\theta_j(t) = 2^{-j/2} \theta(2^{-j} t)$$ (1)

The WT of a signal $f(t)$ is then given by the convolution of $f(t)$ with $\psi_j(t)$

$$W_j f(t) = f \ast \psi_j(t) = f \ast (2^j \theta_j(t))$$ (2)

For 2D image $f(x,y)$, the dyadic WT is given by the 2D convolution of $f(x,y)$ with the 2D scaling function $\phi$ and the wavelets $\psi$:

$$W^1_j f(x,y) = f \ast \psi^1_j(x,y)$$
$$W^2_j f(x,y) = f \ast \psi^2_j(x,y)$$
$$S^2_j f(x,y) = f \ast \phi^2_j(x,y)$$ (3)

where the wavelets are partial derivative of a 2-D cubic spline smoothing function. The 2-D WT of an image gives the gradient of the image at multiples of dyadic scales, i.e., $W^1_j f(x,y)$ and $W^2_j f(x,y)$, which give the vertical and horizontal gradient at scale $j$, respectively.

To detect the wavelet modulus maxima at each wavelet scale $k$, we first compute the gradient magnitude image and the gradient normal image at scale $k$. The wavelet modulus maxima at scale $k$ are then obtained by searching along the gradient normal direction such that the gradient magnitude reaches a local maximum. The locations of these local maxima correspond to the multi-scale decomposition of edges in the image.

In Fig.1, we show the distribution of the wavelet modulus maxima across three wavelet scales for a blurred image and a sharp image. The blurred image is obtained by convolving the original 512x512 image with a Gaussian filter of $\sigma = 3$. Row 2 of Fig.1 shows the distribution of the wavelet modulus maxima at scale 1 (y-axis) vs scale 2 (x-axis). Row 3 shows the distribution of the wavelet modulus maxima at scale 3 (y-axis) vs scale 2 (x-axis). We can see from column 1 that when the image is sharp, the magnitudes of the wavelet modulus maxima remain fairly constant across all 3 scales. In contrast, when the image is blurred, the magnitudes of the wavelet modulus maxima decrease as one goes from coarser scale, i.e. scale 3, to finer scale, i.e. scale 1.

3. Multi-scale Edge Enhancement

Based on the observation of the relationship between image sharpness and wavelet modulus maxima evolution across scales, we proposed to perform image sharpening by remapping the distribution of the wavelet modulus maxima of a blurred image $M_1$ to that of a sharper image $M_2$. We consider a WT of 3 scales. Let $X$, $Y$, $Z$ denote the random variables that correspond to the wavelet modulus maxima at scales 1, 2 and 3, respectively, and let $\rho_1(x,y,z)$ and $\rho_2(x,y,z)$ be the joint probability density function (pdf) for the wavelet modulus maxima of the blurred image and the sharper image, respectively. We need to find a transformation function $T$ such that the pdf of $\tilde{M}_1 = T(M_1)$ resembles that of $M_2$, i.e., we require $\{u,v,w\} = T(x,y,z)$ such that $\rho_1(u,v,w) = \rho_2(x,y,z)$. 

![Fig. 1 Wavelet modulus maxima across scales for original (left column) and blurred “Lena” (right column). Row 1 is the images, row 2 is the wavelet modulus maxima distribution at scale 1 (y-axis) vs scale 2 (x-axis). Row 3 is the wavelet modulus maxima distribution at scale 3 (y-axis) vs scale 2 (x-axis).]
We realize the transformation $T$ by three separate transformations, $T_x, T_y, T_z$, operating on $X, Y, Z$, respectively. Let $\Phi$ be the mapping defined by

$$\Phi(\bullet) = C_2^{-1} \cdot C_1(\bullet)$$  (4)

where the subscripts 1 and 2 in (4) denote the blurred image and the sharp image, respectively. The cumulative distribution function $C_i(t)$ is given by

$$C_i(t) = \int_{-\infty}^{t} \rho_i(\tau) d\tau$$  (5)

and the quantile function $C_i^{-1}(q)$ is given by

$$C_i^{-1}(q) = \inf_{t \in \mathbb{R}} \{ C_i(t) \geq q \}, \quad q = C_i(t)$$  (6)

The transformations $T_x, T_y, T_z$ are then given by

$$T_x = \Phi(x)$$  
$$T_y = \Phi(y)$$  
$$T_z = \Phi(z)$$  (7)

Equation (7) effectively performs a remapping of the wavelet modulus maxima of the source image at scale 1, 2, and 3 to that of the target image by histogram matching on individual scale.

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As the evolution of wavelet modulus maxima across scale is related to the regularity of the edges, for images with similar degree of sharpness, the distributions of their wavelet modulus maxima across scale would be similar. This has been observed to be the case. Hence, to sharpen an image, we remap the distributions of the wavelet modulus maxima to that derived from an ensemble of sharp images.

Fig. 2 shows the distribution of the wavelet modulus maxima across scales before and after the remapping using (7) for the blurred “Lena” Fig. 3 shows the transformations $T_x, T_y, T_z$ that are applied to get the results in Fig. 2.

Once the remapping is done, the enhanced image can be obtained by doing an inverse wavelet transform to the modified wavelet coefficients. However, to ensure that the remapped distributions are preserved in the final reconstruction, the projection onto convex set (POCS) algorithm is used to iterate to the final solution.

In POCS-based image reconstruction [8-11], every known property of the original image $f$ is formulated as a corresponding convex set in a Hilbert space $H$. The original image is then assumed to lie in the intersection of these convex sets,

$$f \in C_0 = \bigcap_{i=1}^{k} C_i$$  (8)

Let $Q = P_1 P_2 \ldots P_k$, where $P_i$ are the projection operator onto $C_i$, then $x_n = Q^n x$ converges to an element in $C_0$ as $n \to \infty$, for any starting point $x \not\in C_0$. In our reconstruction algorithm, $Q = P_2 P_1$. The projection operator $P_1$ is realized by setting the value of $W_1^2 f(x, y)$ and $W_2^2 f(x, y)$ at the modulus maxima positions to that of the remapped values. The projection operators $P_2$ is defined by

$$P_2 = W \cdot R \cdot W^{-1}$$  (9)

where $R$ is the truncation operator given by

$$R_x = \begin{cases} 
0 & x < 0 \\
255 & x > 255 \\
x & otherwise
\end{cases}$$  (10)

Only 2 to 3 iterations are needed to obtain the final solution.
4. Results

In Fig. 4, we show the result of our algorithm on the blurred “Lena”. The reference distributions are obtained from an ensemble of sharp images. In Fig. 5, we show the zoom-in view of our result and the result of cubic unsharp masking. The cubic unsharp masking algorithm [1] performs adaptive non-linear sharpening of an image by modulating the linear unsharp filter using a quadratic term that depends on the local gradient in the image. It has been shown to perform significantly better than the linear unsharp masking algorithm [1, 3]. We see that sharp edges are recovered in our result but not in cubic unsharp masking.

Fig. 4 Left: Blurred “Lena” image. Right: Enhanced image.

Fig. 5 Zoom-in view of: (left) blurred image, (middle) enhanced using our algorithm, (right) enhanced using cubic unsharp masking.

5. Conclusion

In this paper we present a novel image enhancement method based on multi-scale analysis of the distribution of edge magnitude in the wavelet space. By remapping the distribution of the wavelet modulus maxima of the blurred image to that of a generic sharp image and reconstructing the image using POCS, an enhanced image that has crisp edges can be obtained. Our algorithm achieves significantly better result compared to a state-of-the-art adaptive unsharp masking technique.

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References