Full Length Research Paper

Revisiting the risk-return relation in the South African stock market

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We investigate the risk-return relation in the South African stock market using data covering the period from 1973 to 2011. Prior research for several countries reveals high sensitivity of the results to data details and models used. Therefore, our analysis of the risk-return nexus in South Africa are based on three different data frequencies (weekly, monthly and quarterly) and are derived from three different generalized autoregressive conditional heteroskedasticity (GARCH) models in addition to a plain vanilla time-series approach. Similar to the findings of Glosten et al. in 1993 and Harvey in 2001, our results fail to support a significantly positive risk-return relationship in South Africa across various data frequencies and model specifications, and this conclusion survives further robustness checks using different sub-periods and index data. Our results further suggest that the recent global financial crisis may have altered market dynamics and distorted the risk-return relation in the South African stock market.

Key words: Risk-return relationship, volatility models, conditional variance, South African stock market.

INTRODUCTION

The trade-off between risk and return in stock markets is an important subject in finance theory. In the seminal paper of Merton (1980), he argues that the conditional expected stock return is positively related to the conditional variance as in:

\[ E_t[R_{t+1}] = \mu + \gamma \text{Var}_t[R_{t+1}] \],

where \( \gamma \) should be positive since it measures the effect of the conditional variance on returns (corresponding to the coefficient of relative risk aversion of a representative investor), and \( \mu \) is a constant and should reduce to zero. However, the empirical research is indecisive on the positive risk/return nexus expected by the underlying theory. In particular, while French et al. (1987), Campbell and Hentschel (1992), and Ghysels et al. (2005) report positive relationships; yet such relations prove statistically weak. In fact, Campbell (1987) and Nelson (1991) support significantly negative relationships. As Glosten et al. (1993) and Harvey (2001) argue the empirical results appear to be highly sensitive to the choice of models and estimation methods.

Most previous empirical studies on the risk/return relation focus on the U.S. and European markets. In this paper, we investigate the risk/return relationship in the South African stock market as different markets display diverse patterns of return and volatility. The Johannesburg Stock Exchange (JSE) began trading in 1887 and it is the largest stock exchange in Africa. The JSE has about 472 listed companies with a market capitalization of US $855.7 billion as of 2011 according to Standard and Poor’s Global Stock Markets Factbook, making the JSE among the top 20 largest stock exchange worldwide¹.

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¹South Africa is also ranked the first out of 139 countries for its regulation of securities exchanges (World Economic Forum Competitiveness Report, 2010 to 2011).
Only a few studies examine the risk/return link in South Africa and, similar to research on other markets, these studies report mixed results. Raputsoane (2009) find a positive relation for the majority of industry indexes but this conclusion is refuted by Mandimika and Chinzara (2010) at the industry and market levels.

This paper examines the risk/return relation in the South African stock market over the period from January 1, 1973 to December 30, 2011. In light of the known sensitivity of the available evidence on the risk/return relation to model specifications and data details, we derive our results from various models and three different data frequencies (weekly, monthly and quarterly)\(^3\). Two main findings are worth highlighting. First, consistent with et al. (1993) and Harvey (2001), our results do not uniformly support a significant positive risk/return relation in South Africa across models and data frequencies. Second, there is evidence suggesting that the recent global financial crisis has significantly impacted the nature of the risk/return relation in the South African market.

The rest of the paper is organized as follows. Section 2 describes the data. Section 3 reports the results from the generalized autoregressive conditional heteroskedasticity (GARCH)-M model, while Section 4 does the same under a plain-vanilla time-series model. Section 5 discusses robustness checks, and Section 6 concludes.

DATA

Our data, culled from ‘Data Stream’, are for the total stock return index in South Africa (Code: TOTMKSA) covering January 1, 1973 to December 30, 2011 (10,205 daily observations). We compute the weekly returns by using every Wednesday price, monthly returns by using the price of the last trading day of that month, and we compute quarterly returns by using the price at the end of each quarter. Table 1 assembles the summary statistics for the market returns at the three time frequencies. The statistics include the mean, median, minimum, maximum, standard deviation, skewness, kurtosis, and autocorrelations. As the table suggests, the returns are somewhat negatively skewed and the monthly returns appear to be normally distributed. The table also shows that the first-order autocorrelations are not very large (less than 0.10 for all the frequencies). The Ljung-Box Q statistics are significant for high frequency data (weekly) but not for low frequency data (monthly and quarterly) (Table 1).

THE RISK/RETURN RELATIONSHIP UNDER GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (GARCH) SPECIFICATIONS

Model specifications

Research on volatility models pioneered by Engle (1982) and Bollerslev (1986) has popularized the use of generalized autoregressive conditional heteroskedasticity (GARCH) models for analyzing the risk/return relation. Similar to our use of three different data frequencies to enhance the robustness of the results, we also employ three different variants of GARCH \((p,q)\) models to ensure robust findings on the nature of the risk/return relationship in South Africa. We select the optimal lags \(p, q\) in all models based on Akaike information criterion (AIC). For illustrative purpose, we discuss the following case for \(p=1\) and \(q=1\).

The mean equation to be estimated for all the four GARCH models is,

\[
R_{t+1} = \mu + \gamma \text{Var}_t[R_{t+1}] + \varepsilon_{t+1}.
\]

(2)

The variance process of the corresponding GARCH (Model 1) is specified as:

\[
\text{Var}_t^{garch} = \omega + \alpha \varepsilon_t^2 + \beta \text{Var}_t^{garch},
\]

(3)

where \(\varepsilon_t = R_t - \mu - \gamma \text{Var}_t^{garch}\). To ensure efficient estimations, we employ the maximum likelihood method to estimate the parameters, \(\mu, \gamma, \omega, \alpha, \beta\) in all models.

Since positive and negative residuals may have different impacts on future volatilities, the exponential GARCH (EGARCH) of Nelson (1991) allows the asymmetric effect of good and bad news on conditional variances. The conditional variance of the EGARCH \((1,1)\) model (Model 2) can be written as:

\[
\ln \text{Var}_t^{egarch} = \omega + \beta \ln \text{Var}_{t-1}^{egarch} + c \frac{\varepsilon_t}{\sqrt{\text{Var}_{t-1}^{egarch}}} + \alpha \left[ \frac{|\varepsilon_t|}{\sqrt{\text{Var}_{t-1}^{egarch}}} - \frac{1}{\sqrt{\pi}} \right]
\]

(4)

Where \(c\) is a parameter that captures the effects that asymmetric positive and negative shocks, \(\varepsilon_t\), have on conditional variance, and \(\varepsilon_t / \sqrt{\text{Var}_{t-1}^{egarch}} \sim N(0,1)\).

Another well-known asymmetric model is the GJR model proposed by Glosten et al. (1993). The GJR model is a simple extension of GARCH with an additional term added to account for possible asymmetries. In the GJR \((1,1)\), the conditional variance (Model 3) takes the form:

\[
\text{Var}_t^{gjr} = \omega + \beta \text{Var}_{t-1}^{gjr} + \alpha \varepsilon_t^2 + c S_{t-1} \varepsilon_t^2,
\]

(5)

where the squared residual is multiplied by \(\alpha + c\) when the return is below its conditional expectation \((S_{t-1} = 1)\), and by \(\alpha\) when the return is above or equal to the expected value \((S_{t-1} = 0)\). If there is leverage effect, we would expect \(c > 0\).

EMPIRICAL RESULTS

We assemble our results from the three GARCH models\(^3\) and three data frequencies in Table 2. The table gives the coefficient estimates and the corresponding

\(^3\)Most prior empirical research on the risk/return nexus employs monthly data. For robustness, besides monthly observations, we also use shorter (weekly) and longer (quarterly) data frequencies.

\(^3\)We assume normally distributed errors. The estimates would remain consistent even without this assumption provided the mean and variance equations are correctly specified (Bollerslev and Wooldridge, 1992).
Table 1. Summary statistics of stock market returns in South Africa

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\rho_3$</th>
<th>Q(12)</th>
<th>Q-Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly</td>
<td>0.0034</td>
<td>0.0193</td>
<td>0.07</td>
<td>-0.39</td>
<td>0.18</td>
<td>-0.95</td>
<td>3.55</td>
<td>0.03</td>
<td>0.00</td>
<td>0.05</td>
<td>10.60</td>
<td>0.56</td>
</tr>
<tr>
<td>Monthly</td>
<td>0.0146</td>
<td>0.0463</td>
<td>0.12</td>
<td>-0.42</td>
<td>0.45</td>
<td>-0.44</td>
<td>1.88</td>
<td>0.02</td>
<td>-0.17</td>
<td>0.05</td>
<td>13.71</td>
<td>0.32</td>
</tr>
</tbody>
</table>

This table provides the mean, median and standard deviation, minimum, maximum and skewness and kurtosis of stock market returns in South Africa in local currency. The table also shows the coefficients of autocorrelation and the Ljung-Box Q statistics for 12 lags. The sample starts from January 2, 1973 to December 30, 2011.

Table 2. The Risk-return relation under various GARCH specifications.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>GARCH (p,q)</th>
<th>EGARCH (p,q)</th>
<th>GJR-GARCH (p,q)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$</td>
<td>$\gamma$</td>
<td>$p$ $q$ LLR</td>
</tr>
<tr>
<td>Weekly</td>
<td>0.002</td>
<td>1.886</td>
<td>1 4215</td>
</tr>
<tr>
<td></td>
<td>(1.15)</td>
<td>(0.99)</td>
<td>1</td>
</tr>
<tr>
<td>Monthly</td>
<td>0.013**</td>
<td>0.707</td>
<td>1 612</td>
</tr>
<tr>
<td></td>
<td>(1.98)</td>
<td>(0.54)</td>
<td>1</td>
</tr>
<tr>
<td>Quarterly</td>
<td>0.030</td>
<td>1.375</td>
<td>2 116</td>
</tr>
<tr>
<td></td>
<td>(1.58)</td>
<td>(1.28)</td>
<td>2</td>
</tr>
</tbody>
</table>

This table shows estimates of the risk-return relation, $E_t[R_{t+1}] = \mu + \gamma \text{Var}[R_{t+1}]$ with the GARCH, EGARCH, and GJR. Estimators of the conditional variance are given by equations (3) to (5). The coefficients and the corresponding Bollerslev and Wooldridge’s (1992) robust $t$-statistics (in parentheses) are shown. LLR denotes the log likelihood ratio. $p$, $q$ are the optimal lag numbers chosen based on AIC, with the maximum lags for both $p$ and $q$ being three lags. ** and * denote significance at the five and ten percent levels, respectively.

Bollerslev and Wooldridge’s (1992) robust $t$-statistics. The results suggest that lags $p=1$ and $q=1$ generate the best models. Across the three data frequencies, the estimated risk aversion coefficients $\gamma$ are not statistically significant in the GARCH and GJR model. In contrast, the risk aversion coefficients proved positive and significant in the EGARCH model but only for the monthly data. However, the EGARCH results from the weekly and quarterly data, although significant, are perversely negative implying high risk is associated with low return.

In sum, the results reported in Table 2 from different models and data frequencies do not consistently support a significantly positive relation between risk and return in South Africa. The results appear highly sensitive to the models and data frequencies used.

THE RISK-RETURN RELATIONSHIP UNDER A PLAIN VANILLA TIME-SERIES MODEL

Literature on the risk/return nexus reveals some interest in the relation between stock returns and the past realized variance of stock returns. Following et al. (2005) and Bali et al. (2009), we use a plain vanilla time-series model to investigate the relation between returns and their conditional variance. That is:

$$R_{t+1} = \mu + \gamma E_t[\sigma^2_{t+1}] + \epsilon_{t+1},$$  \hspace{1cm} (6)

where $E_t[\sigma^2_{t+1}]$ is a conditional variance of the market portfolio as represented by the one-period lagged realized variance obtained from daily market returns, and $\epsilon_{t+1}$ is an error term. We compute the conditional variance using daily returns as follows:

$$\sigma^2_{t} = \sum_{s=1}^{S} r_s^2,$$  \hspace{1cm} (7)

where $\sigma^2_{t}$ is the realized variance of stock market return, $S$ is the number of trading days in the period and $r_s$ is the market return on day $s$.

Table 3 reports the time-series regression estimates from Equation (6) for the weekly, monthly and quarterly frequencies. The dependent variable is one period ahead

...
and the independent variables are a constant and the realized variance within the period. The Newey-West adjusted $t$-statistics with four lags are in parentheses below the parameter estimates.

### FURTHER ROBUSTNESS TESTS

Our evidence thus far suggests the absence of a positive and significant relation between risk and return in the South African market. This lack of evidence seems consistent across several models and various data frequencies. In this section, we investigate if other factors may have contaminated our results and led to biased conclusions.

Given the lengthy time span of our sample (almost 40 years), a structural break is conceivable which could render the results suspect. We split our sample period in two different manners. First, the recent global financial crisis began in 2007 may have impacted market dynamics worldwide, including South Africa. Therefore, we deleted the potentially turbulent post-2007 crisis sub-period and re-estimated our models over the pre-crisis sub-period. We report the results in Tables 4 and 5. Under the GARCH and GJR models, the results from the pre-crisis data are similar to those derived earlier from the full period. However, the results from the EGARCH model support a significant and positive risk/return relationship. Thus, according to the EGARCH apparatus, the global financial crisis may have weakened the risk/return relation in the South African market.

Of course the global financial crisis is not the only important event in the past four decades that could have contaminated our results. Thus, we follow Farley et al. (1975) and divide our sample at the midpoint to examine the risk/return relation in the two sub-periods. The results, also shown in Tables 4 and 5, are similar in the first sub-period to those in the pre-crisis period. However, in the second sub-period, the EGARCH model no longer indicates a positive risk/return relation. In fact, this relationship became significantly negative in the second half of the period. It is further possible that the results may be suspect due to our use of the total return index rather than the price index (the latter excludes dividends). To check this possibility, we re-estimate our models on the basis of the price index and report the results in Tables 4 and 5. Again, the results persist in rejecting a significantly positive risk/return relation.

Contrary to our findings from the total return index data (Table 3), the results from the price index under the EGARCH model indicate the presence of a significantly negative risk/return relationship in South Africa. Such a perverse outcome reveals some caution in using the price index since it ignores dividends that are an important component of returns in addition to capital gains.

Finally, results reported in Table 4 from the plain-vanilla time-series model over different sample periods and from the price index do not fare any better and they too fail to uncover a significantly positive relation between risk and return.

In sum, our results from a multitude of models, data frequencies, index data, and sample periods consistently suggest the absence of a credible positive risk/return relation in the South African stock market.

### Conclusion

This paper examines the risk/return relation in the South African stock market using data from 1973 to 2011 at three different frequencies (weekly, monthly and quarterly) and using various GARCH models as well as the plain vanilla time-series approach. We do not find a significantly positive risk/return relation across data frequencies and models used. This conclusion generally

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5To conserve space, we confine our test of structural break to the common monthly data frequency.

6One virtue of splitting the sample at the midpoint when testing for structural instability is to maximize the test efficiency by having sufficient degrees of freedom in both sub-periods (Farley et al., 1975).

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| Table 3. The Risk-Return Relation under the Plain-Vanilla Specification |
|-----------------|-----------------|-----------------|
| **Frequency**   | **Plain-vanilla model** |
|                 | $\mu$          | $\gamma$       | $R^2$       |
| Weekly          | 0.003** (2.91) | 0.044 (0.36)   | 0.00%       |
| Monthly         | 0.014** (3.60) | 0.287 (0.38)   | 0.04%       |
| Quarterly       | 0.041** (3.22) | 0.267 (0.34)   | 0.05%       |

This table shows estimates from $R_{it} = \mu + \gamma E(t_{it}) + \varepsilon_{it}$, where $E(t_{it})$ is a conditional variance of the market portfolio as approximated by the one-period lagged realized variance obtained from daily market returns and $\varepsilon_{it}$ is an error term. $R^2$ is the $R$ squared statistic. The Newey-West adjusted $t$-statistics with four lags are in parentheses below the parameter estimates.
Table 4. Robustness Tests (GARCH specifications).

<table>
<thead>
<tr>
<th>Sample</th>
<th>GARCH (p,q)</th>
<th>EGARCH (p,q)</th>
<th>GJR-GARCH (p,q)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$</td>
<td>$\gamma$</td>
<td>$p$</td>
</tr>
<tr>
<td>Excluding GFC</td>
<td>0.013</td>
<td>0.822</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(1.40)</td>
<td>(0.44)</td>
<td>1</td>
</tr>
<tr>
<td>1973:1-1992:12</td>
<td>0.008</td>
<td>1.513</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.27)</td>
<td>1</td>
</tr>
<tr>
<td>1993:1-2011:12</td>
<td>0.014**</td>
<td>0.674</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(2.69)</td>
<td>(0.61)</td>
<td>1</td>
</tr>
<tr>
<td>Using price index</td>
<td>0.011*</td>
<td>0.443</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(1.68)</td>
<td>(0.34)</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5. Robustness tests (plain-vanilla model).

<table>
<thead>
<tr>
<th>Sample</th>
<th>Plain-Vanilla Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$</td>
</tr>
<tr>
<td>Excluding GFC</td>
<td>0.014**</td>
</tr>
<tr>
<td></td>
<td>(3.29)</td>
</tr>
<tr>
<td>1973:1-1992:12</td>
<td>0.013**</td>
</tr>
<tr>
<td></td>
<td>(2.01)</td>
</tr>
<tr>
<td>1993:1-2011:12</td>
<td>0.015**</td>
</tr>
<tr>
<td></td>
<td>(3.32)</td>
</tr>
<tr>
<td>Using price index</td>
<td>0.011**</td>
</tr>
<tr>
<td></td>
<td>(2.75)</td>
</tr>
</tbody>
</table>

GFC denotes the recent global financial crisis. Using price index" means the returns are calculated based on the price index rather than the total return index.

persists despite the use of various sub-periods and different index data. That finding is perhaps not that surprising since many studies for other countries have also concluded that the risk/return relation is tenuous at best (Glosten et al., 1993; Harvey, 2001). We have further presented some evidence that the recent global financial crisis may have altered market dynamics and distorted the risk/return relation. This is because our results from the EGARCH model estimated over the pre-crisis period support the presence of a significantly positive risk/return relation in South Africa.

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