Dynamic characteristics of traffic flow with consideration of pedestrians’ road-crossing behavior

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Research Highlights

- We propose some rules for modeling the interaction of vehicles and pedestrians.
- A modified car-following model is introduced with consideration of waiting pedestrian effect.
- Capacity and delay are significantly affected by pedestrians’ crossing behavior.
- Dynamic characteristics of traffic flow are investigated with different pedestrian parameters.
Dynamic Characteristics of Traffic Flow with Consideration of the Pedestrians’ Road-Crossing Behavior

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Abstract:
Pedestrians’ road-crossing behavior can often interrupt traffic flow and cause vehicles queueing. In this paper, we propose some moving rules for modeling the interaction of vehicles and pedestrians. The modified visual angle car-following model is presented for the movement of vehicles with consideration of the lateral effect of waiting pedestrians. The pedestrians’ behavior is summarized as consisting of three steps: pedestrian arrival, gap acceptance, and pedestrian crossing. Some characteristic parameters of pedestrians are introduced to characterize pedestrians’ behavior. Simulation results show that the interaction of vehicles and pedestrians lowers the traffic capacity and increases delays to both vehicles and pedestrians.

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1. Introduction
The car-following model is one of the most celebrated models for describing the evolution of traffic dynamics at the microscopic level, and is the foundation of traffic flow analysis, simulation, and control. Many car-following models have been proposed to describe traffic flow in a single lane over the last 60 years [1, 2]. However, in a real-world environment, there are many external disturbances to traffic flow, such as traffic signals, slowdown areas, lane changing, lateral vehicles, and pedestrians crossing roads. Much research has focused on their influences on traffic flow, using car-following models to analyze the characteristics of traffic flow [3-5]. Sasaki and Nagatani [6] first studied traffic flow controlled by traffic lights on a single-lane roadway by using the optimal velocity car-following model. They used three different strategies

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of traffic light control—simple synchronized, green wave, and random switching—to analyze the
dynamic characteristics of traffic flow. Jiang and Wu [7] found the first- and second-order phase
transitions from free flow to synchronized flow, under speed-limit conditions, using a full
velocity difference car-following model, while Hanaura et al [8] introduced traffic jams
appearing on a single-lane highway with a few slowdown sections. Naito and Nagatani [9]
studied the safety–collision transition induced by lane changing in traffic flow. Jin et al [10]
modified the car-following model to consider lateral vehicles in neighboring lanes, and analyzed
traffic stability.

To our knowledge, pedestrians’ road-crossing behavior is a significant external disturbance,
affecting the safety, capacity, stability, and oscillation of traffic flow. Jiang et al [11], using a
car-following model, have shown that the capacity of the road decreases in the presence of
pedestrians. Helbing et al [12] and Jiang et al [13] also analyzed the interaction between vehicles
and pedestrians, and found some interesting phenomena. None of these studies, however,
considered the characteristics of pedestrians in analyzing the impacts of their crossing behavior
on vehicle traffic flow. The pedestrian model they use is very simple, and does not capture the
actual characteristics of pedestrians.

In this paper, we attempt to fill this gap and develop a modified car-following model and
pedestrian crossing rules so as to analyze the interaction of vehicle traffic and pedestrian flow.
The rest of the paper is organized as follows. In Section 2, we introduce a rule for pedestrians’
crossing behavior, a modified car-following model, and an interacting rule for vehicles and
pedestrians. In Section 3, we analyze the traffic capacity and delays to vehicles and pedestrians,
and assess the impacts of the model parameters. Finally, some implications and extensions of this
study are discussed.

2. Model

Pedestrians’ road-crossing behavior affecting traffic is a very common phenomenon, and the
nature of the behavior in developing countries that affects traffic particularly strongly. This
behavior not only greatly reduces the road capacity but can also have a serious effect on traffic
safety. In this section, we give a typical single-lane situation in which we will analyze the
interaction between vehicle traffic and pedestrian flow. We assume that there is a crosswalk in a
single lane, and pedestrians must cross the road just at this crosswalk. Fig. 1(a) shows the
specific simulation situation. A pedestrian arrives on one side of the street (area A); he/she needs to observe the traffic conditions, estimate whether it is safe to cross, and either decide to cross or wait for a safer gap to emerge at area A. Meanwhile, the driver of vehicle $j$, the nearest vehicle to the stop line, should observe whether there are pedestrians crossing the road or waiting to cross, and maneuver his/her vehicle accordingly so as to avoid a collision. Therefore, in this situation, vehicles and pedestrians are interacting and the moving rules of vehicles and pedestrians are affected by each other, for safety reasons. In order to describe the interacting behavior specifically, we present the following moving rules for vehicles and pedestrians.

Fig. 1 Simulation of the interaction between vehicles and pedestrians: (a) the basic simulation situation, where vehicle $j$ is the nearest upstream vehicle from the stop line, (b) when there are any pedestrians crossing, it is assumed that there is a virtual stopped vehicle on the crosswalk, and vehicle $j$ must therefore follow this virtual stopped vehicle and stop behind the stop line, (c) the driver of vehicle $j$ pays attention to the waiting pedestrians, and the visual angle of the waiting pedestrians is used to model lateral discomfort.

2.1 Pedestrians’ Crossing Rules

Pedestrians’ crossing behavior is affected not only by the movement of vehicles on the road, but also by the number of pedestrians arriving at the crossing point, the characteristics of the pedestrians, the pedestrian waiting time, and so on. We now give some pedestrian parameters, to depict their crossing behavior.

Firstly, we assume that each pedestrian’s arrival is independent, and that the probability of a pedestrian arriving during a simulation time interval $\Delta t$, measured in seconds, is $p_{rv}$. Thus, the
pedestrian volume per hour equals $3600p_{ro}/\Delta t$. Next, two important parameters of crossing pedestrians’ personal characteristics are proposed. One is the critical gap. The pedestrian’s critical gap is defined as the minimum time, expressed in seconds, between the nearest upstream vehicle and the stop line, in which the pedestrian believes he/she could cross the road safely without being hit by the nearest vehicle. The critical gap is dependent on the pedestrian’s personality: aggressive pedestrians will have a shorter critical gap and unaggressive pedestrians a longer one. It is commonly assumed that the critical gaps of all pedestrians obey the normal distribution $t_c(i)\sim N(\mu_{cg}, \sigma_{cg}^2)$, where $t_c(i)$ is the $i^{th}$ pedestrian’s critical gap, $\mu_{cg}$ is the average critical gap across all pedestrians, and $\sigma_{cg}$ is the standard deviation of the critical gap. The critical gap is an inherent characteristic parameter of pedestrians. Different critical gap values mean that different pedestrians will make different decisions over whether or not to cross under the same traffic conditions. The other pedestrian parameter is road-crossing speed, which can also be assumed to obey a normal distribution $v_p(i)\sim N(\mu_{cs}, \sigma_{cs}^2)$, where $v_p(i)$ is the $i^{th}$ pedestrian’s road-crossing speed, $\mu_{cs}$ is the average speed across all pedestrians, and $\sigma_{cs}$ is the standard deviation of pedestrian speed. Finally, we introduce “herd mentality”, an important pedestrians’ characteristic that is almost ignored in previous research. Herd mentality describes how people are influenced by their peers to adopt certain behaviors, follow trends, and/or purchase items. In this context, it means that, if an aggressive pedestrian begins to cross, the other waiting pedestrians will follow him/her, and thus all of the waiting pedestrians will begin to cross the road. Thus, the vehicles on the road have to decelerate and stop, and traffic flow will be interrupted. Based on this assumption, the minimum critical gap value of the waiting pedestrians can be used, instead of the distribution of critical gaps. Therefore, the criterion for waiting pedestrians to begin to cross is that the time needed for the nearest vehicle upstream of the stop line to reach the stop line is longer than the minimum critical gap of the waiting pedestrians.

Here, we denote the time it will take the nearest vehicle to reach the stop line as the time-to-collision (TTC) between the vehicle and the crossing pedestrians. The TTC can be defined as follows: the time before the nearest vehicle $j$ will collide with the pedestrians at the stop line, if vehicle $j$ continues at the same speed and on the same path. Thus, TTC can be defined as the distance between the nearest vehicle and the stop line divided by its velocity. In equation form, this is:

$$TTC_j(t) = d_j(t)/v_j(t)$$  
(1)
where vehicle $j$ is the nearest vehicle upstream of the stop line, $TTC_j(t)$ is the time-to-collision of vehicle $j$ at time $t$, $d_j(t)$ is the distance between vehicle $j$ and the stop line at time $t$, and $v_j(t)$ is velocity of vehicle $j$ at time $t$.

Based on the above, we can state the pedestrians’ crossing rules as follows.

(1) Pedestrian arrival

For simplification, we assume that the pedestrians arrive at area A only at discrete time steps, i.e., $t=0, 1, 2, \ldots$. Based on the assumption that the probability of a pedestrian arriving in any given simulation time step is $p_{ro}$, a uniformly distributed random number between 0 and 1, $p_x$, is generated at each time step $t$. If $p_x > p_{ro}$, no pedestrian arrives at area A at time step $t$. If $p_x \leq p_{ro}$, a pedestrian does arrives and is waiting to cross at time step $t$. Each pedestrian $i$ is given two random personal parameters: $t_c(i)$ and $v_p(i)$, as defined above.

(2) Gap acceptance

Pedestrians’ crossing behavior, especially in developing countries, tends to accord with the gap acceptance rule, which refers to the process a pedestrian uses to decide whether or not to accept an available time gap and cross the road. The available gap can also be denoted as the TTC of the nearest vehicle. Therefore, given the above arguments, the pedestrians’ crossing criterion is whether the minimum critical gap of the waiting pedestrians is smaller than the TTC value of the nearest vehicle $j$ at time step $t$. If the crossing criterion is satisfied at the time step $t$ ($TTC_j(t) > \min[t_c(i)]$), the waiting pedestrians will begin to cross the road in a group. In this case, vehicles will stop before the stop line or just decelerate, depending on the gap and the last time of pedestrians crossing. If the crossing criterion is not satisfied, all of the pedestrians will wait at area A for a larger time gap, and the vehicles will continue to move based on the conventional car-following rule.

(3) Pedestrian crossing

When the crossing criterion is satisfied, the pedestrians begin to cross. The most important parameter here is the total crossing time, which is associated with the length and width of the crosswalk, the crossing speed, the number of pedestrian crossing, and so on. The *Highway Capacity Manual* [14] provides the following expression for the total crossing time:
where $T_{ct}$ is the total crossing time (in secs), $t_s$ is the pedestrians start-up time (in secs), $L_c$ is the crosswalk length (in meters) (equal to the lane width), $s_p$ is the average speed of the waiting pedestrians (m/s), $N_{ped}$ is the number of pedestrians crossing during an interval (in persons), $W_c$ is the crosswalk width (m), and $p_1$ and $p_2$ are two calibration parameters. For a single-lane crosswalk, we use $t_s = 1.0$ s, $p_1 = 0.81$, and $p_2 = 0.27$ throughout this paper. Eq. (2) shows that the total crossing time increases linearly with an increasing number of waiting pedestrians.

During the crossing time, in order to increasing the TTC, vehicles must decelerate or stop for allowing the pedestrians to cross safely, and traffic flow is interrupted. While these pedestrians are still crossing, other pedestrians may arrive at area A. To simplify, we assume that the arriving pedestrians do not wait but also cross, behind those who have already started crossing. Therefore, the total crossing time increases. When all of the pedestrians have crossed the road, the stopped vehicles begin moving again, based on the car-following model.

Using the three rules above, we can describe the pedestrians’ crossing behavior precisely, including the effect of the herd mentality. The movement rules of the vehicles, taking into consideration the pedestrians’ interference, will be introduced in the next section.

2.2 Vehicles’ Movement Rules

The conventional vehicle-moving rule is the car-following model, which has been studied by many researchers. In this paper, when the effect of the pedestrians’ crossing behavior is not taken into account, the vehicle traffic flow is modeled using the visual angle model (VAM) [15]. The VAM takes the drivers’ characteristics into consideration, and can present some complex traffic phenomena, such as density waves, shrinking hysteresis, asymmetry and wide scattering. The motion of vehicle $n$ that follows vehicle $n-1$ is given by,

$$a_n(t) = \alpha \left\{ V \left[ \theta_n(t) - v_n(t) \right] - \lambda \frac{d}{dt} \theta_n(t) \right\}$$

where $a_n(t)$ is the acceleration of vehicle $n$ at time $t$, $\theta_n(t)$ is the visual angle which is observed by the driver of the $n^{th}$ vehicle at time $t$, $V$ is the optimal velocity preferred by a driver based on the
visual angle of the follower, \( v_n(t) \) is the velocity of vehicle \( n \) at time \( t \), \( \alpha \) is the sensitivity coefficient of a driver to the difference between the optimal and current velocities, and \( \lambda \) is the sensitivity coefficient of the stimulus \( d\theta_n(t)/dt \).

The visual angle of the \( n \)th driver can be approximated as:

\[
\theta_n(t) = \frac{w_n}{x_{n-1}(t) - x_n(t) - l_{n-1}} \tag{4}
\]

where \( x_n(t) \) and \( x_{n-1}(t) \) are the positions of the \( n \)th vehicle and \((n-1)\)th vehicles, respectively, and \( l_{n-1} \) and \( w_n \) are the length and width of the \((n-1)\)th vehicle (the leading vehicle), respectively.

The optimal velocity function can be written as follows:

\[
V[\theta_n(t)] = V_1 + V_2 \tanh \left\{ C_1 \left[ \frac{w_n}{x_{n-1}(t) - x_n(t) - l_{n-1}} \right] - C_2 \right\} \tag{5}
\]

in which \( V_1 = 6.75 \text{ m/s}, \ V_2 = 7.91 \text{ m/s}, \ C_1 = 0.13 \text{ m}^{-1}, \ C_2 = 1.57 \) [15, 16].

While the pedestrians are crossing, vehicles cannot move over the crosswalk, and must stop until all of the pedestrians have finished crossing. In order to simplify the movement rule of the vehicles, while the pedestrians are crossing, we assume that there is a virtual stopped vehicle on the crosswalk (see Fig. 1(b)). The position of the rear of the stopped vehicle is on the stop line. Thus, the visual angle of the nearest upstream vehicle \( j \) is as follows:

\[
\theta_j(t) = \frac{w_0}{x_{sl}(t) - x_j(t)} \tag{6}
\]

where \( \theta_j(t) \) is the visual angle of the nearest upstream vehicle \( j \) at time \( t \), \( w_0 \) is the average vehicle width, \( x_{sl}(t) \) is the position of the stop line, and \( x_j(t) \) is the position of the nearest upstream vehicle \( j \) at time \( t \). Substituting Eq. (6) into Eq. (3), the movement of the nearest upstream vehicle \( j \) can be described accurately.

The key assumption of basic car-following is that the follower follows the leader in the middle of the lane. Recently, some car-following models have been proposed that consider the effects of lateral separation [17, 18]. In the pedestrian crossing situation, waiting pedestrians could also affect the moving vehicles. Drivers near the crosswalk should not only pay attention to the leading vehicle, but also to the behavior of the waiting pedestrians. In order to modeling the effect of waiting pedestrians, we assumed the waiting pedestrians would have the same effect on following vehicles as the lateral vehicle [10]. Meanwhile, the different numbers of waiting pedestrians would produce different interferences on the following vehicle. Therefore, we
assume that the waiting pedestrians can seem as a stopped vehicle on the adjacent lane. Introducing the visual angle of waiting pedestrians into VAM (see Fig. 1(c)), we present an improved visual angle car-following model to describe the movement rule for vehicles, taking into account the lateral discomfort effect of waiting pedestrians. The improved visual angle model (IVAM) is given by the following:

\[ a_j(t) = \alpha \left\{ V \left[ \theta_j(t) \right] - v_j(t) \right\} - \lambda \left[ \frac{d}{dt} \theta_j(t) + \frac{d}{dt} \phi_j(t) \right] \]  (7)

where \( \phi_n(t) \) is the visual angle of the waiting pedestrians at time \( t \), which can be calculated approximately as follows [17, 18]:

\[ \phi_j(t) = \frac{W_p}{x_{sl}(t) - x_j(t)} \]  (8)

where \( W_p \) is the effective total width of the waiting pedestrians. It is clear that \( W_p \) is associated with the number of waiting pedestrians. Thus, \( W_p \) is calculated as:

\[ W_p = W_{p0} \left[ 1 + p_3(N_{ped} - 1) \right] \]  (9)

where \( W_{p0} \) is the effective width of one waiting pedestrian, and \( p_3 \) is the calibration parameter. For simplicity, we assume \( p_3 = 0.3 \) throughout this paper. Eq. (9) shows that the number of waiting pedestrians has a linear effect on the visual angle of the waiting pedestrians. Using Eq. (7), we can describe simply how the waiting pedestrians affect the vehicles’ movement.

In summary, there are three cases determining vehicles’ behavior. Firstly, if there is no pedestrian crossing or waiting to cross, the movements of all vehicles obey Eqs. (3) and (4). Secondly, if there are any pedestrians crossing the road, the movement of the nearest upstream vehicle \( j \) obeys Eqs. (3) and (6), and the other vehicles obey Eqs. (3) and (4). Finally, if there are no pedestrians crossing, but there are some pedestrians waiting to cross, the movement of the nearest upstream vehicle \( j \) obeys Eqs. (7) - (9), and the other vehicles obey Eqs. (3) and (4).

3. Simulation results and discussion

Based on the above assumptions, we can carry out a simulation of the interactions between vehicles and pedestrians. Suppose that there are \( N \) normal vehicles, distributed on the single lane under a periodic boundary condition, and the length and width of all vehicles are the same. The length of the lane is \( L = 1500 \) m. The crosswalk is at position 0.5\( L \), which means that the
The distance between the start of the test lane and the stop line of the crosswalk is $0.5L$, and the width and length of the crosswalk are $W_c$ and $L_c$, respectively. The initial conditions of the vehicles are chosen to be as follows:

$$x_n(t = 0) = (n-1)L/N$$ and $$v_n(t = 0) = V \left[ \frac{w_o}{(L/N-l_o)} \right]$$ for $n = 1, 2, \ldots, N$. The acceleration rates of each vehicle can be calculated from Eqs. (3)-(9) based on different pedestrian crossing conditions, while the position of each vehicle is updated according to

$$v_n(t + \Delta t) = v_n(t) + a_n(t)\Delta t$$ \hspace{1cm} (10)

$$x_n(t + \Delta t) = x_n(t) + \frac{1}{2} \left[ v_n(t) + v_n(t + \Delta t) \right] \Delta t$$ \hspace{1cm} (11)

The pedestrians’ crossing behavior is simulated based on the three rules described above: pedestrian arrival, gap acceptance, and pedestrian crossing. In order to consider the effect of the random arrival of pedestrians, every simulation scenario is simulated ten times. The other parameters for the vehicles and pedestrians that are used throughout this paper are shown in Table 1.

**TABLE 1 Model parameters of the simulation used throughout this paper.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated or typical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity coefficient $\alpha$</td>
<td>0.41</td>
</tr>
<tr>
<td>Sensitivity coefficient of the rate of change of the visual angle $\lambda$</td>
<td>30</td>
</tr>
<tr>
<td>Time step used in the simulation $\Delta t$</td>
<td>0.1 s</td>
</tr>
<tr>
<td>Normal vehicle length $l_0$</td>
<td>5.0 m</td>
</tr>
<tr>
<td>Normal vehicle width $w_0$</td>
<td>1.8 m</td>
</tr>
<tr>
<td>Crosswalk width $W_c$</td>
<td>3.0 m</td>
</tr>
<tr>
<td>Crosswalk length $L_c$</td>
<td>3.5 m</td>
</tr>
<tr>
<td>Effective width of one waiting pedestrian $W_{p0}$</td>
<td>0.6 m</td>
</tr>
<tr>
<td>Average critical gap $\mu_{cg}$</td>
<td>2.0 s</td>
</tr>
<tr>
<td>Standard deviation of critical gap $\sigma_{cg}$</td>
<td>0.2 s</td>
</tr>
<tr>
<td>Average crossing speed of crossing pedestrians $\mu_{cs}$</td>
<td>1.2 m/s</td>
</tr>
<tr>
<td>Standard deviation of crossing speed $\sigma_{cs}$</td>
<td>0.15 m/s</td>
</tr>
</tbody>
</table>

Recent studies [11, 13] that have simulated vehicles interacting with pedestrians’ crossing behavior have shown some interesting phenomena. In the following sections, we will analyze and discuss the dynamic characteristics of vehicle and pedestrian traffic flow.

### 3.1 Vehicle capacity

The most important effect of pedestrians crossing a road is that they can interrupt the vehicle traffic flow and decrease the road’s vehicle capacity. Fig. 2 shows the number of vehicles...
passing the crosswalk in one hour, with different pedestrian arrival probabilities $p_p$ and different traffic flow densities. In the density-volume phase space, the plots can be classified into three categories. In the lower and higher density regions, the vehicle numbers passing the crosswalk almost accord with the theoretical curve, and the capacity remains unaltered whether there are pedestrians or not. It is easy to explain this phenomenon. When the traffic density is low, the time headway or TTC between two continuous vehicles is significantly larger than the pedestrians’ critical gap and crossing time. Thus, the pedestrians have enough of a time gap to cross safely and have little influence on vehicle traffic. Similarly, when the traffic density is high, the speed of the traffic is very slow and the time headway is again larger than the pedestrians’ critical gap, so that the crossing pedestrians do not interrupt the traffic flow. Only when the density is in the intermediate region are the maximum passing vehicle numbers for different densities less than the theoretical capacity. This means that the presence of the crossing pedestrians has a large influence on the vehicle traffic flow. Another feature that can be seen from Fig. 2 is that the capacity quickly drops with an increasing number of pedestrians. When $p_p=0.04$, and the pedestrian flow is 1,440 persons per hour, the capacity drops from around 1,800 vehicles per hour to between 400 and 600 vehicles per hour. Therefore, the pedestrians’ crossing behavior has a significant influence on the vehicle traffic flow, especially near the intermediate density region.
Fig. 2 Numbers of vehicles passing the crosswalk under different pedestrian arrival probabilities, based on the simulation, where (a) $p_p=0.01$ (b) $p_p=0.02$ (c) $p_p=0.03$ (d) $p_p=0.04$. The gray solid line represents the theoretical capacity curve when no pedestrian crossing behavior is modeled.

3.2 Delays to vehicles and pedestrians

The pedestrians’ crossing behavior not only reduces the road’s vehicle traffic capacity, but also increases the delay for vehicles. The interactions between vehicles and pedestrians also cause pedestrians to have to wait for a time gap and thus increase delays for them too. The delays to vehicles and pedestrians respectively can be calculated as

$$D_v = \frac{1}{k} \sum_k \left[ t_{OD}(k) - L_{OD}/v_{opt} \right]$$

$$D_p = \frac{1}{m} \sum_m \left[ T_a(m) - T_l(m) \right]$$

where $D_v$ is the average delay for the vehicles, $D_p$ is the average delay for the pedestrians, $t_{OD}(k)$ is the time it takes for the $k^{th}$ vehicle to pass from point O to point D (see Fig. 1(c)), $L_{OD}$ is the distance between those points, $v_{opt}$ is the optimal speed for the current traffic density level, $T_a(m)$ is the arrival time of the $m^{th}$ pedestrian, and $T_l(m)$ is the time it takes for the $m^{th}$ pedestrian to begin crossing the road.

Fig. 3 shows the delays for vehicles and pedestrians under different pedestrian flow levels and for $N=60$. It can be seen in Fig. 3(a) that, with an increase in pedestrian flow, the delays to vehicles increase significantly. This is due to the fact that increasing numbers of pedestrians interrupt the vehicle traffic flow since crossing takes longer. In contrast, the delays to pedestrians decrease with an increasing flow of pedestrians, for two reasons: the herd mentality effect and lateral discomfort. The herd mentality means that most waiting pedestrians will follow an
aggressive pedestrian, who has less critical gap and easily meets the crossing condition $TTC_j > t_c$, to cross. Therefore, with an increase of numbers of pedestrians, the arrival probability of pedestrians with a shorter critical gap is larger, which shortens the waiting time and delays to pedestrians. Lateral discomfort means that when there are more waiting pedestrians, there is a deceleration effect on the moving vehicles, and the crossing condition $TTC_j$ increases, which leading to easier crossing and the decrease of pedestrian delays.

Fig. 3 Delays under different levels of pedestrian flow, based on the simulation results, where (a) shows vehicle delay and (b) shows pedestrian delay. The gray solid lines represent the fitted delay curves of the vehicles and pedestrians.

In order to demonstrate the delays to pedestrians more precisely, Fig. 4 shows them under different vehicle density levels, when $p_p=0.03$ (pedestrian flow equals 1,080 persons per hour). Similarly to vehicle capacity, pedestrian delays can also be classified into three categories. In the lower and higher density regions, the delay is less than in the intermediate region because of the larger time headways between vehicles. When the density is in the intermediate region, it is clear from Fig. 4 that the delay points are more sparsely scattered than in the other two regions. The reason is that, in this region, the vehicle flow is approaching capacity and the time headway is very small, so that most of the pedestrians are unable to cross the road and spend the entire simulation period waiting for a sufficient time gap. Therefore, the delay to the pedestrians is infinite and the delay points are missing from the figure.
Fig. 4 Pedestrian delays based on simulation results under different vehicle densities.

3.3 The effect of the parameters of the pedestrians’ characteristics

The parameters of the pedestrians’ characteristics play an important role in determining the pedestrians’ crossing behavior, and have a great impact on traffic flow characteristics. The pedestrians’ parameters, such as the critical gap, crossing speed, and lateral effect, determine whether a group of pedestrians can cross safely without being hit by a vehicle.

Fig. 5 shows the number of vehicles that pass the crosswalk, according to whether or not we consider the lateral effect of the waiting pedestrians, the black points showing the results when we do consider the lateral effect and the red open triangles showing them when we do not. It is obvious that the maximum number of vehicles passing the crosswalk decreases greatly when we consider the waiting pedestrians, something that has been neglected in previous studies. Using the improved visual angle car-following model (IVAM), we can reproduce the capacity drop that is due to the lateral discomfort effect on traffic flow. The modified moving rules for vehicles and pedestrians are very necessary for us to simulate the interaction phenomenon of crossing behavior realistically. The space-over-time plot of the vehicle trajectories also clearly exhibits the characteristics of the lateral effect, as shown in Fig. 6. Compared with Fig. 6(a) (no lateral effect), traffic flow is more frequently interrupted by waiting or crossing pedestrians in Fig. 6(b) (lateral effect considered), while the queue and density wave form quickly. These figures show that the waiting pedestrians have a great degree of influence on vehicles, exacerbating go-and-stop traffic flow, and reducing the road capacity.
A quantitative comparison of the delays and capacities, depending on whether or not the waiting pedestrians’ lateral effect is taken into consideration, is shown in Table 2. The results of the simulation with the lateral effect are more consistent with the field data. It seems that both the delays and the capacities are significantly different when considering the lateral effect than without it. The capacity drops by about 10%-30% for different pedestrian arrival probabilities when we consider the lateral discomfort caused by the waiting pedestrians. More serious is the fact that the delay to vehicles increases significantly, by between 100% and 400%. The results clearly show the interaction between the waiting pedestrians and the vehicles. This means that the waiting pedestrians cause the drivers to pay more attention to them and prevent some aggressive pedestrians from force their way across the crosswalk. The driver of the nearest vehicle must decelerate when he/she notices pedestrians waiting to cross, and the deceleration increases with an increasing number of waiting pedestrians.

![Graphs showing volume and density](image)

**Fig. 5** Numbers of vehicles that pass the crosswalk, based on the simulation results, where (a) $p_p=0.02$ and (b) $p_p=0.03$. The black points represent the results of considering the lateral effect of waiting pedestrians, and the red open triangles represent the results of not considering the lateral effect. The gray solid line represents the theoretical capacity curve with no pedestrian crossing behavior.
Fig. 6 Representative space-over-time plot of vehicle trajectories for $N=60$ and $p_p=0.03$, where in (a) there is no consideration of the lateral discomfort effect of waiting pedestrians, which causes less vehicle queueing, and in (b) the lateral discomfort effect of waiting pedestrians is considered, meaning that vehicle queues form frequently.

TABLE 2 Simulation results of the delays and numbers of vehicles passing the crosswalk under both a lateral effect and no lateral effect. The percentage decreases in capacity and increases in delays are based on comparing the no lateral effect data with the lateral effect data.

<table>
<thead>
<tr>
<th>Number of vehicles (N)</th>
<th>Pedestrian arrival probability ($p_p$)</th>
<th>No lateral effect</th>
<th>Lateral effect</th>
<th>Percentage decrease in capacity</th>
<th>Percentage increase in delay</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Number of vehicles (veh)</td>
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Another very important characteristic parameter of the pedestrians is the critical gap, which is the core of pedestrian gap acceptance theory and determines whether pedestrians begin crossing
the road or wait for a longer time gap. With an increase in the average critical gap, the arriving pedestrians should wait for a longer time gap between the nearest vehicle $j$ and the stop line of the crosswalk, before they begin crossing. Fig. 7 presents the simulation results of the number of vehicles that go through the crosswalk under different pedestrian critical gap values. When the critical gap is in the low range, the pedestrians often choose a smaller time gap in which to cross and the traffic flow is interrupted frequently. Thus, the number of vehicles passing the crosswalk is significantly less than the theoretical capacity. With an increase in the critical gap value, all pedestrians are more careful, and the number of vehicles going through the crosswalk increases simultaneously. When the critical gap exceeds a certain value, for example around 3.2 seconds in Fig. 7, there is no time gap in the traffic flow that the pedestrians will accept and no pedestrian can cross the road until the end of the simulation period. Therefore, as is obvious from the figure, we can see that, in the large critical gap range, the number of vehicles passing the crosswalk is the same for different pedestrian flow levels and always equals the theoretical capacity. This means that the delays for the pedestrians are very large. Therefore, in this situation, pedestrian traffic signal controls must be considered to separate the interaction of vehicles and pedestrians, and improve the operational efficiency of both vehicles and pedestrians at the crosswalk.

Fig. 7 Number of vehicles that pass the crosswalk for different pedestrian critical gap values, $N=60$ and $p_p=0.02, 0.03, \text{ and } 0.04.$
4 Summary

In this paper, we have proposed some rules of pedestrians’ crossing behavior and modified the visual angle car-following model. The pedestrians’ behavior rules can be summarized into three steps: pedestrian arrival, gap acceptance, and pedestrian crossing. Characteristic parameters of pedestrians are introduced, such as the pedestrian arrival probability, the critical gap, the crossing speed, and the herd mentality effect. Using these improved models, we have represented the interaction between vehicles and pedestrians in a simulated single-lane pedestrian crossing situation. The simulation results show that the capacity and delays of vehicles are both affected by crossing pedestrians, and with large numbers of pedestrians the traffic flow is interrupted frequently. Lateral effects of waiting pedestrians on traffic flow are introduced using both qualitative and quantitative analysis. We also analyze how the values of the pedestrian parameters affect the dynamic characteristics of traffic flow. It is hoped that this work will contribute to the description of the dynamic characteristics of traffic flow, taking pedestrians’ crossing behavior into account, and help inform decisions over whether to install pedestrian crossing signals.

Acknowledgments

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References

Title:
Dynamic Characteristics of Traffic Flow with Consideration of the Pedestrians’ Road-Crossing Behavior

Authors
Sheng Jin, Xiaobo Qu, Cheng Xu, Dian-Hai Wang*

Abstract
Pedestrians’ road-crossing behavior can often interrupt traffic flow and cause vehicles queueing. In this paper, we propose some moving rules for modeling the interaction of vehicles and pedestrians. The modified visual angle car-following model is presented for the movement of vehicles with consideration of the lateral effect of waiting pedestrians. The pedestrians’ behavior is summarized as consisting of three steps: pedestrian arrival, gap acceptance, and pedestrian crossing. Some characteristic parameters of pedestrians are introduced to characterize pedestrians’ behavior. Simulation results show that the interaction of vehicles and pedestrians lowers the traffic capacity and increases delays to both vehicles and pedestrians.

Keywords
Pedestrians’ crossing behavior, Critical gap, Herd mentality, Visual angle, Car-following