ABSTRACT

In this study, a numerical model is developed using the Finite Difference Method to determine the response of sandy seabed under different loading conditions. The quasi-dynamic (u-w-p) model is presented detailed in this paper. In u-w-p model, acceleration, velocity, and displacement terms are considered different for both solid and fluid phases. u-w-p model has considered sophisticated and general condition of seabed motion. The governing equations of the model are deduced from constitutive law and conservation law under certain assumptions. In the model, numerical solutions are developed by using the finite difference method (FDM) both in 1-D with finite seabed depth and 2-D with infinite seabed depth. Three major factors (pore water pressure, effective stress and shear stress) are assessed from the proposed models. The results show that both the effective stress and the pore water pressure vary according to the depth. When the excess pore pressures are increasing, the surface settlement of seabed can be observed. Hydraulic flow is downward at the crest and upward below the trough of the sea wave. Thus, liquefaction depths below the crest are obviously higher than the liquefaction depth below the trough of wave. However, the liquefaction occurs between phase angles of π/2 and 3π/2, instead of just under the wave crest, which indicates the existence of phase lags under wave actions. Unlike the variation of pore water pressure, the change of shear stress is relatively linear to the wave propagation. The proposed model will provide a better understanding of the mechanism of soil-wave interaction.

KEYWORDS: Wave-seabed-structure; seabed instability; numerical model

INTRODUCTION

Coastal structures have been highly developed in last 30 years, no matter in a developing country or developed country, such as marine pipelines, anchors, drilling platform and different types of breakwaters. These offshore facilities are playing different roles in different locations, but all of them are always faced with the same kind of threat—the threat from waves. Wave mainly can damage offshore structures in two ways, one is to scour the structure directly, another is to disturb the seabed soil and damage structure from their fundament. With the wide application of high strength material, the scour damage has already been efficiently reduced (Henkel, 1970), however, there is not convinced method to reduce damage caused by wave-induced seabed instability. Therefore, considerable efforts have been dedicated to the phenomenon of the wave-sea-bed structure interaction (Yamamoto, 1977;
Madsen, 1978; Hsu and Jeng, 1994; Jeng 1996 and Jeng 2012). From previous studies, there are two kinds of seabed failure mode. One is liquefaction, and another is shear failure. When the pore pressure become excessive with accompanying decrease in effective stresses, a sedimentary bed may be moved in either horizontal (liquefaction) or vertical directions (shear failure), then lead to an instability of the seabed (Jeng, 1996).

According to linear wave theory, ocean waves could be simplified as sinusoidal waves. These waves will induce pressure variation at the seabed surface, and the changes in pressure will influence the stresses and pore water pressure of seabed soils. Furthermore, it will affect the stability of the bottom sediments and offshore structures. Previous researchers have considered seabed stability to be a major problem, but there has been little effort to examine the effects of cyclic bed flow on the effective stress state within the bed and the consequences of bed flow with regards to soil stability. They are concentrated in some specific method of analysis for a specific soil and no comparative study has been carried out for the different modeling of soil to calculate the transient response of seabed under wave loading.

Herein, a u-w-p model (including vertical displacements, horizontal displacements and pore water pressure, denote as “u-w-p”), are selected to analyze the transient response of the seabed under wave action. Also the finite different method (FDM) is applied to obtain the results. Analyses are carried to the different types of soils with changing stiffness, permeability and degree of saturation parametrically for some materials.

**u-w-p model:** In this model, acceleration, velocity, and displacement terms are considered different for both solid and fluid phases. This is the most complex model, and simplified models could be derived based on it.

In this paper, the general behaviors of sandy seabed under wave action are discussed. Numerical models (under three different conditions) have been developed based on conservation law and constitutive law and linear wave theory. The models are expected to simulate the dynamic movement of real seabed under different wave conditions. The variation of pore water pressure, liquefaction potential and shear stress will be mainly discussed and performed. With a better understanding of these factors, the seabed failure process could be reproduced, and the critical point during the failure process also could be clarified clearly. This will enhance the knowledge of seabed stability and benefit the coastal embankment design, and reduce the damage caused by wave on coastal structures in future.

**LITERATURE REVIEW**

Breakwaters are the most common structure which was adopted to protect and enhance the utility of coastlines. At the early stage of breakwater design, engineers will try to strengthen breakwater by increasing the armor, while this was proved to be not effective later, because the scouring damage is not the key criteria of breakwater failure (Smith et. al., 1984). In the next several years, research of breakwater failure became much more comprehensive. Oumeraci (1994) had made a review and conclusion of vertical breakwater failure by analyzing 22 typical breakwater failure cases. Further, Takahashi et al. (1994) made a failure report of different types of breakwater. According to their researches, seabed instability had been deemed to play an important role during the breakwater failure process.

In seabed instability analysis, there are two major failure modes. One is shear failure, and another is seabed liquefaction. (Jeng, 1996) The shear failure of seabed caused by waves was first forwarded by Henkel (1970). Mitchell and Hull (1974) followed Henkel’s path and substantiated Henkel’s findings. Later Wright and Dunham (1972) first tried to use the finite element method to derive the relationship between wave-induced stresses and seabed displacement. This approach had been proved to be effective and applied by many other researchers later.
Raham (1991) made a comparatively overall summary of mechanisms of wave-induced seabed instability, especially the instability caused by the shear force. In wave-induced seabed instability research, shear failure draw more attention in early stage and shear failure was regarded as the major factor of seabed instability before 1980s, but the wave-induced liquefaction has arrested more attention since 1980s. Much evidence has been reported in the literature to show that some failures of breakwaters are attributed to foundation failure instead of structural causes (Slivester and Hsu, 1989).

Research of soil liquefaction was first developed in earthquake analysis. In the late 1970s, some scientists recognized that the seafloor may have similar phenomena as earthquake liquefaction when strong wave propagated along the seafloor. (Dalrymple, 1979). Although the phenomenon looks similar and both of them could be classified into cyclic liquefaction, yet the mechanism of wave-induced liquefaction is quite different from the earthquake. Robertson and Wride (1998) sort liquefaction into two different types: one is due to cyclic loadings, where the effective stress can reach zero or even smaller than zero under certain circumstance, this one is called cyclic liquefaction; another one is due to strain softening with a loss of cumulative shear strength, this one is called flow liquefaction or static liquefaction. Flow liquefaction can be observed during earthquake triggered by static and cyclic loading, however, as to seabed only the cyclic loading can result in liquefaction (Robertson, 2010). So seabed liquefaction is a kind of cyclic liquefaction.

Fung (1965) had given the solutions for the amplitudes of the harmonic stresses in a semi-infinite elastic medium, with harmonic wave loading on the surface. Demars (1980) plotted these stress distributions in dimensionless. In further study of liquefaction, scientists mainly focus on two factors of liquefaction “when’ and “where”. One is “When the liquefaction may happen?” another is “where or what depth the liquefaction may occur?” As to the first question “when”, liquefaction occurs at wave trough phase, at this stage the pore water pressure becomes larger and the effective stress decreases and comes to zero (Sakai et al., 1992). In loose seabed with relative density around 45%, the soil liquefied to the flume bottom. In the dense seabed with relative density around 60%, the soil liquefied to an intermedia depth. This was also observed by Tzang (1998) in silt and Sassa and Sekiguchi (1999) in sandy seabed. Another research focus in liquefaction is about how liquefaction acts in different soil type is carried by Wit and Kranenburg (1997).

Seabed liquefaction is a major topic in wave-induced soil analysis. Many different approaches have been conducted to find the result, like using different assumptions (seabed thickness is finite or infinite, seabed is saturated or unsaturated and etc.) or applying different basic theory (Biot consolidation theory or conservation law or even some newly developed method).

Wave-induced Seabed Analysis

Jeng (2003) has made a summary and evaluation of researches on wave-induced soil analysis and wave-induced offshore structure analysis. He concluded that there were two major research directions in this area, one was the experimental analysis, and another one was theoretical analysis, which included analytical analysis and numerical analysis.

The analytical analysis was first developed based on the Biot’s Three Dimensional Consolidation formula (1956). In wave-induced soil analysis, analytical analysis was first developed by Yamamoto (1977) and Madsen (1978), they derived the wave-induced pore pressure, soil displacement and effective stress from mathematics approaches. These analytical solutions were widely used in engineering practice at that time, and it was called Yamamoto-Madsen Solutions (1978).

In addition, Okusa (1985) simplified Yamamoto and Madsen method (1978) with conditional elastic theory. He also pointed out that there are two different circumstances may
happen to pore water pressure and stresses under the wave action. If the seabed was fully saturated, isotropic and infinite depth, the seabed soil behavior will only be affected by the wave property, no relationship with seabed soil property. However, the pore water pressure and soil stresses will be influenced by both soil property and wave property under all other conditions. He implied that the simplified assumption of seabed (fully saturated, isotropic and infinite depth) was not comprehensive enough. Hsu and Jeng (1994) developed another important analytical solution. This analytical solution provided a 3-D solution for seabed with infinite depth, and 2-D solution for both finite and infinite seabed thickness. This was also the first creditable solution for finite thick seabed. The applied governing equation of this solution was a combination of Three Dimension Biot Consolidation Theory (1941) and Storage equation of Verruijt (1969).

Hsu and Jeng’s (1994) solutions had two parts: 3-D solution for finite thickness and 2-D solution for infinite thickness. The 3-D solution still had a room for further refinement, but the 2-D solution for infinite thickness was convinced and practicable (Jeng, 2001). Jeng (1996) made further analysis based on the solution developed by Hsu and Jeng (1994), and he also made a further study by using numerical analysis. Jeng proved that their solution in 1994 was more coincide with experimental result than Okusa (1985), Zen and Yamazaki (1993) and Tsai (1965) in liquefaction analysis. Seymour, Hsu and Jeng (1996) also deduced another analytical solution based on the VS equation developed by Varley and Seymour (1988). Later Jeng and Seymour (1997) extend their analytical solution to different kinds of seabed of both infinite and finite thickness. When they compared the result with other analytical solution which assumed the seabed has the same permeability all through the deep, they found the seabed surface layer stress could have a 23% difference. They also implied that the result was not convinced if the whole seabed was assumed to have the same permeability. Further, Jeng (1997) assumed that the seabed was cross-anisotropic: the permeability in the horizontal direction was different from the vertical direction. He concluded if the seabed was assumed to be isotropic, the result had underestimated the effective stresses and overestimated the pore water pressure. He also pointed the importance of permeability in wave-induced seabed analysis again.

In wave-induced seabed study, three major types of numerical analysis methods are applied. They are finite differential analysis, boundary element method, and finite element method. Zen and Yamazaki (1990) assumed that seabed thickness was much smaller than wave length, and then they simplify the problem from 2-dimensional boundary problem into 1-dimensional problem. Cheng et al. (2001) also applied an analytical solution to establish their 1-dimensional finite differential model of excess pore water pressure under the wave action and pointed out that the pore water pressure was highly affected by the shear in shallow water. Jeng (1996) has also conducted a series of numerical analysis about wave-induced seabed instability. Jeng (1996) has proved the validity of Hsu and Jeng’s analytical analysis, and also pointed out the three domain soil parameters in wave-induced seabed analysis. They were permeability, degree of saturation and seabed thickness. The maximum liquefaction depth increased when the seabed thickness and permeability decreased, but the maximum failed depth decreased. In addition, the degree of saturation increased, the seabed was easier to be instable. This implied that the liquefaction and shear failure were not consistent under wave-action, not like the wave factors.

Later Jeng and Lin (1997) extended their numerical model to non-uniform seabed and non-linear waves. The result from this finite element model was consistent with the analytical solutions and experimental solutions. Lin and Jeng (2000) made further extension of the numerical model. The model was able to simulate conditions of anisotropic and non-uniform seabed under the wave action.

As the time goes on, some new governing equations have been widely applied. For example, the Volume-Averaged Reynolds-Averaged Navier-stokes (VARANS) equation was used as governing equation in Zhang et al. (2011) and Jeng (2013). In Zhang et al. (2011), it
was summarized that longer wave period and larger wave height will induce larger pore water pressure at the leading edge of a breakwater, and reaffirm the importance of soil permeability and degree of saturation in dynamic seabed analysis.

On the whole, the development of wave-induced seabed analysis has three major stages. The core theoretical framework at the first stage is Yamamoto and Madsen’s method (1978), this method is simple to apply and suitable to estimate the dynamic stress in industry. Most of the relative researches adopt their research method to conduct further analysis at that time, like Oksa (1985) and Raham (1991). However, Yamamoto and Madsen’s method (1978) is based on several simplified assumptions, like the seabed is assumed to be infinite thickness, saturated, isotropic and homogeneous. As Jeng (1997) proved in 1997, these simplified assumptions could affect the final result up to 22%.

Then the research about wave-induced seabed instability comes to the second stage. The most significant finding in this stage is Hsu and Jeng’s analytical solution (1994). Jeng (1996, 1997, 1999 and 2001) has made a lot of numerical studies based on this analytical solution, and extend the results to more complicated seabed conditions, like the seabed could be finite thickness, unsaturated, inhomogeneous and anisotropic. In addition, Jeng also presents a reasonable research methodology. His research begins from analytical solutions, and gets further development through numerical analysis. This is a very clear and logical track. However, Hsu and Jeng’s method (1994) still has some limitations. It is not so convenient to obtain the seabed soil parameters (elastic modulus, shear modulus, Poisson ratio, permeability and so on) from this method. In general, Hsu and Jeng’s method (1994) is convinced in describing the seabed behavior under wave actions, but it’s not suitable in seabed soil parameter analyses.

In the third stage of wave-induced seabed analysis, the most common work is to develop more accurate numerical models to estimate the result. Most of the research is about the governing equation, like the Volume-Averaged Reynolds-Averaged Navier-Stokes (VARANS) equation applied by Jeng et al. (2013). Due to the progress in computer application, some complicated governing equations are able to be analyzed now. This may provide better numerical models and highly increase the accuracy of research. This may also be the major research trends in this decade.

**GOVERNING EQUATION**

Following assumptions are made in this study:

- Seabed is homogeneous, isotropic and flat surface.
- Linear elastic theory is assumed in all the constitutive models for stress vs. strain relationship.
- Flow behavior is also assumed linear with employing Darcy’s Law between hydraulic gradient and pore water flow velocity.
- Sea wave and associated water pressure on seabed surface are given with: Linear wave theory

Constitutive law for poro-elastic material

The constitutive laws for solid phase are expressed based on the linear elastic theory. (For solid phase)

\[
\Delta \sigma_y = F \Delta \varepsilon_{ik} \delta_{ik} + 2G \Delta \varepsilon_y, \quad \text{or} \quad \Delta \varepsilon_y = -F \Delta \sigma_{ik} \delta_{ik} + \frac{1}{2G} \Delta \sigma_y
\] (1)
(For fluid phase)

\[(1 - n) \Delta \varepsilon_{\|} + n \Delta \varepsilon_{\perp} = -(\Delta u_{i,j} + \Delta w_{i,j}) = \left(1 - n \right) \frac{1}{K_s} + n \frac{1}{K_f} \Delta p = \frac{1}{B_f} \Delta \bar{p}; \quad (2)\]

\[B_f = \frac{K_f}{n}, \quad \frac{1}{K_f} = \frac{1}{K_j} S_r + \frac{1}{K_s} (1 - S_r), \quad K_s >> K_f\]

where: parameters \(K_s, K_f, K_j\) and \(K_g\) are the bulk module of solid particles, fluid phase, liquid phase and gas phase, respectively. The bulk modulus of fluid phase \(K_f\) is a function of degree of saturation \(S_r\). The parameters with suffix ‘f’ are for those of fluid phase. The newly introduced tensor \(w_i\) is the relative displacement of fluid phase with respect to the solid phase; the absolute displacement of fluid is, then, given by

\[(\text{Absolute displacement of fluid phase}) = u_i + \frac{\Delta w_i}{n}\]

The relative velocity of pore fluid, or flow of pore fluid, can be described as

\[\Delta \dot{w}_i = -k_f \Delta h_{f,i} = -k_f \left( \frac{\Delta p_{f,i}}{\rho_w g} - \rho_f \Delta \psi_i \right) = -\frac{k_f}{\rho_w g} \left( \Delta p_{f,i} - \rho_f \Delta \psi_i \right) \]

\[r_{ij} \Delta \dot{w}_i + \Delta p_{f,i} = \rho_f \Delta \psi_i ; \quad r_{ik} \frac{k_f}{\rho_w g} = \delta_{ij}\]

where variable parameter \(h\) is potential hydraulic head, and \(k_f\) is a tensor for Darcy’s permeability law, and the tensor \(r_{ij}\) is the inversed form of \(k_f\). The first term and second term in the right part of the equation correspond to pressure head and position head, respectively. For an isotropic permeability, the tensors for permeability become diagonal tensors with uniform components.

\[k_{ij} = k \delta_{ij}, \quad r_{ij} = \frac{\rho_w g}{k} \delta_{ij}\]

where \(k\) is a permeability coefficient for ordinary use, based on Darcy’s law. Conservation law for poro-elastic material

The equilibrium condition for overall material including solid phase and fluid phase is derived from Equation (1);

\[\rho_i \Delta \dot{u}_i + \rho_f \Delta \dot{w}_i + \Delta \sigma_{f,i,j} + \Delta p_{f,i} = \rho_t \psi_i \]

\[\rho_i \Delta \dot{u}_i + \rho_f \Delta \dot{w}_i - F \Delta u_{f,j} - G(\Delta u_{f,i,j} + \Delta u_{f,j,i}) + \Delta p_{f,i} = \rho_t \psi_i\]

where total density \(\rho_t\) is

\[\rho_t = (1 - n) \rho_i + n \rho_f\]
The equilibrium condition for the fluid phase is given as the modification of Darcy’s permeability law (Equation 4).

\[ \rho_f (\Delta u_i + \frac{\Delta \bar{u}_j}{n}) + r_g \Delta \bar{w}_j + \Delta p_j = \rho_f \Delta \psi_i \]  

(8)

Governing equations for poro-elastic material

The governing equation is expressed as simultaneous partial differential equations. In the following four types of constitutive models are presented for dynamic and quasi-dynamic and static analyses. In dynamic analysis, acceleration terms of the solid phase and the fluid phase are fully taken into consideration; however, the acceleration terms are neglected in quasi-dynamic analysis, and only displacement term of the solid phase and velocity term of the fluid phase is considered in static analysis.

The first, the [u-w-p] model can be presented as follows:

[u-w-p] model; dynamic analysis

\[ \rho_s \Delta u_i + \rho_f \Delta \bar{u}_j \bar{u}_j + G(\Delta u_{ij} + \Delta u_{ji}) + \Delta p_j = \rho_f \Delta \psi_i \]

[u-w-p] model; quasi-dynamic analysis

\[ \rho_f (\Delta u_i + \frac{\Delta \bar{u}_j}{n}) + r_g \Delta \bar{w}_j + \Delta p_j = \rho_f \Delta \psi_i \]

\[ B_f (\Delta \bar{u}_i + \Delta \bar{w}_j) + \Delta \bar{p} = 0 \]  

(9)

[u-w-p] model; static analysis

\[ -F \Delta u_{ij} - G(\Delta u_{ij} + \Delta u_{ji}) + \Delta p_j = \rho_f \Delta \psi_i \]

\[ r_g \Delta \bar{w}_j + \Delta p_j = \rho_f \Delta \psi_i \]

\[ B_f (\Delta \bar{u}_i + \Delta \bar{w}_j) + \Delta \bar{p} = 0 \]  

(10)

These governing equations were deduced from Constitutive law and Conservation Law. This model combined in the governing equation is named as [u-w-p] model. Sometimes the model is simplified according to the appropriate assumptions concerned in the problems. For example [u-p] model, where the relative acceleration of the fluid phase is neglected; that is, the acceleration of fluid is taken to be equal to that of the solid phase. If the effect of pore water flow is negligible as in the problems where the clay ground is concerned under high frequency region or in short term, the seepage flow can be eliminated from the governing equation and the undrained condition can be assumed. In this case the model is called [u]-model; the relative velocity of fluid phase as well as the relative acceleration are neglected. If only the pore water pressure and the pore water flow are concerned, and the deformation of the solid phase is out of the scope of the analysis, the model can be simplified and the solid phase is assumed rigid. This model named [w-p] model is effective for the material with high permeability, such as coarse sand and gravel, and for the static...
WAVE AND SOIL PARAMETERS

The numerical solution of the finite difference method is presented. All the four constitutive models are employed: \( u \cdot w \cdot p \) model, \( u \cdot p \) model, \( u \) model, \( w \cdot p \) model. Use of parameters in different conditions is shown in table 1; wave parameter and soils properties as in table 2 and table 3. In the problem, the variable parameters, such as displacements, velocity and pressure, are represented by a function

\[
\text{Displacement} = a(z) e^{(i(\omega t + k x))}
\]

(12)

Table 1: Uses of Parameters in Different Conditions

<table>
<thead>
<tr>
<th>Condition</th>
<th>( \Delta \dot{u}_i )</th>
<th>( \Delta \dot{w}_i )</th>
<th>( \Delta u_i )</th>
<th>( \Delta \dot{w}_i )</th>
<th>( \Delta \dot{p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Quasi- dynamic</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Static</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

“+” = considered, “−” = neglected

Table 2: Wave parameters

<table>
<thead>
<tr>
<th>Types of wave</th>
<th>H (m)</th>
<th>T (sec)</th>
<th>d (m)</th>
<th>L (m)</th>
<th>p (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave</td>
<td>10.0</td>
<td>13.0</td>
<td>20.0</td>
<td>167.6</td>
<td>37.90</td>
</tr>
</tbody>
</table>

and the function \( a(z) \) is solved as a kind of one-dimensional problem by finite difference method. The finite difference solution is derived exclusively for two-dimensional dynamic analysis, however, the solution obtained can be easily modified for the other dimensional condition, and analytical conditions as follows:

\[
\begin{align*}
\left\{ \begin{array}{l}
\kappa \neq 0 \text{ or } L \neq \infty : \text{two-dimensional analysis} \\
\kappa = 0 \text{ or } L = \infty : \text{one-dimensional analysis}
\end{array} \right.
\]

(13)

\[
\left\{ \begin{array}{l}
\rho_s \neq 0, \rho_f \neq 0, \quad \phi = 0 : \text{dynamic analysis} \\
\rho_s = 0, \rho_f = 0, \quad \phi = 0 : \text{quasi-dynamic analysis} \\
\rho_s = 0, \rho_f = 0, \quad \phi = 0 : \text{static analysis}
\end{array} \right.
\]

(14)

The behavior of seabed induced by sea wave is complicated and variously changes as a function of soil types. In this section the influence of soil type, that is mechanical properties of soil,
is introduced and discussed. The calculation result of the exact solution and the numerical solution are also discussed. Therefore five types of soils: loose sand, dense sand, silt, normally consolidated clay, and gravel with the parameter as listed in Table 3 are considered, and however, only u-w-p model in quasi-dynamic, 2-D analysis is concerned.

Table 3: Mechanical properties of some typical soils selected for the demonstration of frequency dependent wave properties.

<table>
<thead>
<tr>
<th>Material Type</th>
<th>Loose Sand</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_t ): bulk density of wet material (kg/m(^3))</td>
<td>1.90×10(^3)</td>
<td></td>
</tr>
<tr>
<td>( \rho_s ): density of solid phase (kg/m(^3))</td>
<td>2.65×10(^3)</td>
<td></td>
</tr>
<tr>
<td>( n ): porosity</td>
<td>0.454</td>
<td>( e \approx n / (1 - n) )</td>
</tr>
<tr>
<td>( s ): shear modulus of solid phase (N/m(^2))</td>
<td>0.4×10(^8)</td>
<td></td>
</tr>
<tr>
<td>( \nu_s ): Poisson’s ratio</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>( B’ ): Skempton’s B-value in 1-D</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>( k ): coefficient of permeability (m/s)</td>
<td>1.0×10–4</td>
<td></td>
</tr>
<tr>
<td>( \lambda_s ): Lamé’s constants (N/m(^2))</td>
<td>0.6×10(^8)</td>
<td>( \lambda_s = 2G\nu_s / (1 - 2\nu_s) )</td>
</tr>
<tr>
<td>( E_s ): Young’s modulus of solid phase (N/m)</td>
<td>1.04×10(^8)</td>
<td>( E_s = 2(1 + \nu_s)G_s )</td>
</tr>
<tr>
<td>( E_{us} ): stiffness in 1-D of solid phase (N/m(^2))</td>
<td>1.40×10(^8)</td>
<td>( E_{us} = 2(1 - \nu_s)G_s / (1 - 2\nu_s) )</td>
</tr>
<tr>
<td>( K_f ): bulk modulus of fluid phase (N/m(^2))</td>
<td>0.424×10(^8)</td>
<td>( K_f = nE_o, B_f (1 - B_f) )</td>
</tr>
<tr>
<td>( B_f ): averaged bulk modulus (N/m(^2))</td>
<td>0.933×10(^8)</td>
<td>( B_f = K_f / n )</td>
</tr>
<tr>
<td>( \rho_f ): bulk density of fluid phase (kg/m(^3))</td>
<td>9.930×10(^2)</td>
<td>( \rho_f = \rho_o (1 + Sr) + \rho_s Sr )</td>
</tr>
<tr>
<td>( \rho_d ): bulk density of dry material (kg/m(^3))</td>
<td>1.45×10(^3)</td>
<td>( \rho_i = (1 - n)\rho_o )</td>
</tr>
<tr>
<td>( \omega_o ): characteristic angular frequency (rad/sec)</td>
<td>4.48×10(^4)</td>
<td>( \omega_o = n\rho_o / k\rho_f )</td>
</tr>
<tr>
<td>Maximum ( B’ ) value</td>
<td>0.974</td>
<td>( B’ = K_f / (K_f + nE_o) )</td>
</tr>
</tbody>
</table>

Bulk modulus of saturated water, \( K_w = 2.31\times10^9 \) (N/m\(^2\))
Bulk modulus of air, \( K_a = 3.03\times10^5 \) (N/m\(^2\))
Bulk density of water, \( \rho_w = 1000.0 \) (kg/m\(^3\)) Bulk density of air, \( \rho_a = 0.0 \) (kg/m\(^3\))

**FDM SOLUTION OF THE MODEL**

This section shows the stress parameters obtained from the governing equation under boundary conditions as shown in Eq. (50-52). The velocities and displacements in both vertical and horizontal directions have been considered. The pore water pressure and seepage velocity have been applied, too.
Governing equation:

\[
\rho_l \frac{\partial^2 \Delta u_x}{\partial t^2} + \rho_f \frac{\partial \Delta w_x}{\partial t} - (F + G) \left( \frac{\partial^2 \Delta u_x}{\partial x^2} + \frac{\partial^2 \Delta u_z}{\partial z^2} \right) - G \left( \frac{\partial^2 \Delta u_x}{\partial x^2} + \frac{\partial^2 \Delta u_z}{\partial z^2} \right) + \frac{\partial \Delta p}{\partial x} = 0
\]  \tag{15}

\[
\rho_l \frac{\partial^2 \Delta u_z}{\partial t^2} + \rho_f \frac{\partial \Delta w_z}{\partial t} - (F + G) \left( \frac{\partial^2 \Delta u_x}{\partial x^2} + \frac{\partial^2 \Delta u_z}{\partial z^2} \right) - G \left( \frac{\partial^2 \Delta u_x}{\partial x^2} + \frac{\partial^2 \Delta u_z}{\partial z^2} \right) + \frac{\partial \Delta p}{\partial z} = 0
\]  \tag{16}

\[
\rho_f \left( \frac{\partial^2 \Delta u_x}{\partial t^2} + \frac{1}{n} \frac{\partial \Delta w_x}{\partial t} \right) + \frac{\rho_f g}{k} \Delta w_x + \frac{\partial \Delta p}{\partial x} = 0
\]  \tag{17}

\[
\rho_f \left( \frac{\partial^2 \Delta u_z}{\partial t^2} + \frac{1}{n} \frac{\partial \Delta w_z}{\partial t} \right) + \frac{\rho_f g}{k} \Delta w_z + \frac{\partial \Delta p}{\partial z} = 0
\]  \tag{18}

\[
B_f \left( \frac{\partial^2 \Delta u_x}{\partial t \partial x} + \frac{\partial^2 \Delta u_z}{\partial t \partial z} + \frac{\partial \Delta w_x}{\partial \partial x} + \frac{\partial \Delta w_z}{\partial \partial z} \right) + \frac{\partial \Delta p}{\partial t} = 0
\]  \tag{19}

Since the governing equation is a linear simultaneous differential equation with constant coefficients, the fundamental form of the solution becomes

\[
\begin{align*}
\Delta u_x &= a_{ux} e^{i\kappa x} e^{i\omega t} \\
\Delta u_z &= a_{uz} e^{i\kappa z} e^{i\omega t} \\
\Delta w_x &= a_{wx} e^{i\kappa x} e^{i\omega t} \\
\Delta w_z &= a_{wz} e^{i\kappa z} e^{i\omega t} \\
\Delta p &= a_p e^{i\kappa x} e^{i\omega t}
\end{align*}
\]  \tag{20}

These equations are interpreted into finite difference notation

\[
\left( -\omega^2 \rho_l + \kappa^2 (F + 2G) \right) a_{ux(k)} - G \frac{a_{ux(k+1)} - 2a_{ux(k)} + a_{ux(k-1)}}{\Delta z^2} - ik(F + G) \frac{a_{ux(k+1)} - a_{ux(k-1)}}{2\Delta z} + i\omega \rho_f a_{wx(k)} + ik a_{p(k)} = 0
\]  \tag{21}

\[
-k(F + G) a_{ux(k-1)} - \left( \omega^2 \rho_l - \kappa^2 G \right) a_{ux(k)}
\]  \tag{22}
For the response of seabed to sea wave, the following boundary conditions must be satisfied.

\[\Delta p(x, 0, t) = p_0 e^{i(\sigma t + \xi x)}, \quad z = 0; \quad B.C.1\]
\[\Delta \sigma_{xz}(x, 0, t) = 0, \quad z = 0; \quad B.C.2\]
\[\Delta \sigma_{zz}(x, 0, t) = 0, \quad z = 0; \quad B.C.3\]
\[\Delta w_z(x, H, t) = -k \frac{\partial \Delta p}{\partial z}(x, H, t) = 0, \quad z = H; \quad B.C.4\]
\[\Delta u_z(x, H, t) = 0, \quad z = H; \quad B.C.5\]
\[\Delta u_x(x, H, t) = 0, \quad z = H; \quad B.C.6\]
Then

\[ \Delta z = \frac{H}{N} \]  

(30)

The boundary conditions at top are interpreted as

\[ \Delta \rho(x, 0, t) = a_p(0) e^{i(\omega t + \kappa x)} = p_o e^{i(\omega t + \kappa x)} ; \quad a_p(0) = p_o \]  

(31)

\[ \Delta \sigma_{zz}(x, 0, t) = \left\{ -i \kappa F a_{uz}(0) - (F + 2G) \frac{a_{uz}(0) - a_{uz}(-1)}{2\Delta z} \right\} e^{ix \xi x} e^{i\omega t} = 0; \]  

(32)

\[ a_{uz}(-1) = a_{uz}(0) + 2i \kappa \Delta z \frac{F}{F + 2G} a_{uz}(0) \]

\[ \Delta \sigma_{zz}(x, 0, t) = \left( -G \frac{a_{uz}(0) - a_{uz}(-1) - i \kappa G a_{uz}(0)}{2\Delta z} \right) e^{ix \xi x} e^{i\omega t} ; \]

(33)

\[ a_{uz}(-1) = a_{uz}(0) + 2i \kappa \Delta z a_{uz}(0) \]

From Eq. 21 \( a_{uz(-1)} \) and \( a_{uz(-1)} \) are eliminated

\[ \left( -\omega^2 \rho + \kappa^2 (F + 2G) \right) a_{uz}(0) - \frac{2a_{uz}(0) - 2a_{uz}(0)}{\Delta z^2} + 2i \kappa \Delta z a_{uz}(0) \]

(34)

\[ -\kappa^2 \frac{F(F + G)}{F + 2G} a_{uz}(0) + i \omega \rho_f a_{uz}(0) + i \kappa a_{uz}(0) = 0 \]

From Equation (22) \( a_{uz(-1)} \) and \( a_{uz(-1)} \) are eliminated

\[ -\kappa^2 (F + G) a_{uz}(0) - \left( \omega^2 \rho + \kappa^2 G \right) a_{uz}(0) \]

\[ -(F + 2G) \frac{2a_{uz}(0) - 2a_{uz}(0)}{\Delta z^2} + 2i \kappa \Delta z a_{uz}(0) \]

(35)

\[ + \frac{a_{p(2)} + 4a_{p(1)} - 3a_{p(0)}}{2\Delta z} = 0 \]

From Equation (23)

\[ -\omega^2 \rho_f a_{uz}(0) + \left( i \omega \rho_f + \frac{\rho_a e}{k} \right) a_{uz}(0) + i \kappa a_{uz}(0) = 0 \]  

(36)
From Equation (24)

\[ a_{p(0)} = P_0 \] (38)

\[ \Delta \sigma_{xx} = \left\{-i\kappa (F + 2G) a_{ux(0)} - F - \frac{a_{ux(2)} + 4a_{ux(1)} - 3a_{ux(0)}}{2\Delta z} \right\} e^{j\kappa x} e^{j\omega t} \] (39)

\[ \Delta \sigma_{zz} = \left\{-i\kappa Fa_{ux(0)} - (F + 2G) \right\} \frac{-a_{uz(2)} + 4a_{uz(1)} - 3a_{uz(0)}}{2\Delta z} e^{j\kappa x} e^{j\omega t} \] (40)

\[ \Delta \sigma_{xt} = \left\{-G \frac{-a_{ux(2)} + 4a_{ux(1)} - 3a_{ux(0)}}{2\Delta z} - i\kappa Ga_{uz(0)} \right\} e^{j\kappa x} e^{j\omega t} \] (41)

The boundary conditions at bottom are interpreted as

\[ \Delta u_x(x, H, t) = a_{ux(x)} e^{j\kappa x} e^{j\omega t} = 0; \quad a_{ux(N)} = 0 \] (42)

\[ \Delta u_z(x, H, t) = a_{uz(x)} e^{j\kappa x} e^{j\omega t} = 0; \quad a_{uz(N)} = 0 \] (43)

\[ \Delta w_z(x, H, t) = a_{wz(x)} e^{j\kappa x} e^{j\omega t} = 0; \quad a_{wz(N)} = 0 \] (44)

The boundary conditions:

\[ a_{ux(N)} = 0 \] (45)

\[ a_{uz(N)} = 0 \] (46)

From Equation (23)

\[-\omega^2 \rho_f a_{ux(N)} + \left( \frac{i\omega \rho_f}{\eta} + \frac{\rho_w g}{k} \right) a_{ux(N)} + i\kappa a_{p(N)} = 0 \] (47)

Boundary condition

\[ a_{wz(N)} = 0 \] (48)

From Equation (25)

\[-\kappa \omega B_f a_{wz(N)} + i\omega B_f \frac{3a_{wz(N)} - 4a_{wz(N-1)} + a_{wz(N-2)}}{2\Delta z} + i\kappa B_f a_{wz(N)} \]

\[ + B_f \frac{3a_{wz(N)} - 4a_{wz(N-1)} + a_{wz(N-2)}}{2\Delta z} + i\omega a_{p(N)} = 0 \] (49)
and stress parameters are

\[
\Delta \sigma_{zz} = \sum_{i=1}^{n} \left( -i\kappa (F + 2G)u_{zz}^{(i)} - \frac{3u_{zz}^{(i)} - 4u_{zz}^{(i-1)} + u_{zz}^{(i-2)}}{2\Delta z}, \dot{u}_{zz}^{(i)} \right) \right) e^{i\omega t} e^{i\varphi}
\]

\[
\Delta \sigma_{z\alpha} = \sum_{i=1}^{n} \left( -i\kappa (F + 2G)u_{z\alpha}^{(i)} - \frac{3u_{z\alpha}^{(i)} - 4u_{z\alpha}^{(i-1)} + u_{z\alpha}^{(i-2)}}{2\Delta z}, \dot{u}_{z\alpha}^{(i)} \right) \right) e^{i\omega t} e^{i\varphi}
\]

\[
\Delta \sigma_{\alpha\alpha} = \sum_{i=1}^{n} \left( -G - \frac{3u_{\alpha\alpha}^{(i)} - 4u_{\alpha\alpha}^{(i-1)} + u_{\alpha\alpha}^{(i-2)}}{2\Delta z}, \dot{u}_{\alpha\alpha}^{(i)} \right) \right) e^{i\omega t} e^{i\varphi}
\]

GENERAL BEHAVIOR OF THE SEABED UNDER THE WAVE ACTION

The major objective of the study is to identify the effects of different soil parameters in wave-induced seabed instability. To achieve this, it’s necessary to clarify the general behavior of the seabed under the wave action first. In this section, three main factors are applied to introduce the seabed behavior. They are pore water pressure, effective stress, and shear stress, and the study of soil parameters (like permeability, friction angle, and so on) will be carried on based on these results in further research. A u-w-p model in quasi-dynamic 2-D is employed to introduce the behavior of the seabed consisting of loose sand in detail.

Pore water Pressure

The calculation results of pore water pressure from the numerical solution are shown in Figure 1 and Figure 2. Pore water pressure is similar to the moving wave in amplitude and phase angle at the surface of seabed. As shown in Figure 2 with increasing depth, pore water pressure amplitude decreases from 1 to 0.4. This type of damping may be attributed to the frictional effect between the solid and fluid phase in the seabed materials. This frictional effect is modeled with Darcy’s Law and taken into account in the analyses.

Peak of cyclic pore water pressure also changes with depth with respect to the peak of the ocean wave. Waveform tends to shift to the opposite direction of the wave travelling. With an increase of depth, the effect of phase angle (\( \dot{\theta} \)) to normalize pore water pressure in seabed is decreased. As shown in Figure 1, pore water pressures are affected by moving wave. When the excess pore pressures are positive, the seabed tends to decrease in volume. If excess pore pressures are negative so that the soil tends to increase in volume.
Cyclic change in effective stress is caused due to the change of the hydraulic gradient. Upward hydraulic gradient which is associated with upward seepage flow reduces effective stress and downward hydraulic gradient increases the effective stress. If the upward seepage flow is notable and exceeds a certain value, the effective stress may become negative; this condition is recognized as a kind of liquefaction. This type of liquefaction is classified as ‘cyclic liquefaction’. On the other hand, another type of liquefaction may occur which is caused by accumulation of excess residual pore water pressure that is associated with cyclic application of shear stress such as during earthquakes, is classified as ‘cumulative liquefaction’.

As shown in Figure 3, the amplitude of effective stresses increase with depth from 0 to 0.6. Effective stresses as shown in Figure 4 is corresponding to the distribution of pore water pressure associated with the friction between pore water and solid skeleton. With an increase of depth, the effective stress increases in both neutral value and amplitude. And variation of effective stress tends to shift in the same direction as that of wave movement that is the opposite direction of that of pore water pressure. Since the hydraulic flow is downward around the crest of the sea wave and upward below the trough of the sea wave, $z'$ is high (increases) below the crest and low (decrease) below the trough of wave.
Figure 2: Pore water pressure amplitude in u-w-p model in quasi-dynamic 2D for loose sand

Figure 3: Effective stress amplitude in u-w-p model in quasi-dynamic 2D for loose sand
Figure 4: Effective stress in u-w-p model in quasi-dynamic 2D for loose sand

Figure 5: Effective stress VS Phase angle in u-w-p model quasi-dynamic 2D for loose sand
The effective stress falls into negative in a limited part of shallow seabed when the range of phase angle of $2\pi/4$ and $3\pi/4$; as shown in Figure 5. The negative value of effective stress occurs when sea wave is at the crest. This negative effective stress suggests the occurrence of cyclic liquefaction. Liquefaction occurs up to the depth of 2.3 m and near the trough of sea wave and it tends to decrease with an increase of depth. The occurrence and intensity of liquefaction are influenced by many factors, such as wave type and material type. In the following research, the influence of the analytical condition and modeling on the cyclic liquefaction will be discussed in detail.

Shear Stress

As shown in Figure 6, the phase angle reduces effect with increasing depth and almost zero in both the phase angles of $\pi/2$ and $3\pi/2$, which correspond to the crest and trough of sea wave. As shown in Figure 7, the amplitude of shear stress tends to increase with depth. This cyclic change in shear stress would potentially cause the seabed to liquefy and is named cumulative liquefaction. The mechanism of the cumulative liquefaction is similar as that caused by the earthquake. Pore water pressure may be generated due to the cyclic shear deformation induced by sea wave loading and accumulated in a series of sea waves travelling. Especially, this cumulative generation of pore water pressure may be remarkable in the case of loose sand and finally ground reduces effective stress and loses stiffness and strength. The cumulative behavior of pore water pressure is a function of soil type and cyclic stress condition.

Figure 6: Shear stress vs Phase angle in u-w-p model quasi-dynamic 2D for loose sand
CONCLUSION

The numerical method applied in this study, which including four types of models under different dimensional conditions and different analytical conditions, provides the means for evaluating the stability of seabed against sea wave loading. In this paper, the u-w-p model under quasi-dynamic condition is presented. Based on the u-w-p model, the general behaviors of the seabed under wave actions are discussed. The following concluding remarks can be drawn:

• Both the effective stress and the pore water pressure vary according to the depth. This negative effective stress suggests the occurrence of cyclic liquefaction. Liquefaction occurs up to the depth of 2.3 m and near the trough of sea wave.

• Amplitude of effective stresses increase with depth from 0 to 0.6. Effective stress is corresponding to the distribution of pore water pressure associated with the friction between pore water and solid skeleton. With increase in depth, the effective stress increases in both neutral value and amplitude. And cyclic variation of effective stress tends to shift in the same direction as that of wave movement that is the opposite direction of that of pore water pressure.

Figure 7: Shear stress amplitude in u-w-p model in quasi-dynamic 2D for loose sand
• The effective stress falls into negative in a limited part of shallow seabed when the range of phase angle of $2\pi/4$ and $3\pi/4$. The negative value of effective stress occurs when sea wave is at the crest.

REFERENCES


