Mathematics Education: Is there an App for that?

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Applications (apps) for hand-held devices, such as IPads and smart phones, are in great supply. Many of these focus on mathematics. A recent search revealed more than 4,000 apps for mathematics education. The ease of access and the fact that they are generally low cost, often free, means that they are readily available to the general population but this raises questions as to their quality and what is being learned through the use of these apps. Using two quantitative measures and one qualitative measure, this article evaluates 142 apps which met initial search criteria, and recommends 34 mathematics apps for further evaluation and trial with primary school teachers and students.

Introduction:

This article synthesises the research literature concerning the nexus of mathematics education and iPad apps and then outlines the methodology used to evaluate 142 apps. These apps, having met initial sorting criteria, were assessed using two quantitative and one qualitative measure. Shuler, Levine, and Ree (2012) reported that in mid-2012 there were approximately 500,000 apps available on the Apple iTunes Appstore. General early learning was the most popular subject (47%) and mathematics the second most popular subject (13%) in the education category. In their discussion of mathematics apps, Pelton and Pelton (2012) noted that "while some are commendable, almost all of the rest are simple flashcards, numeric procedures, or mobile textbooks. Very few currently available apps have engaged best practices by integrating visual models to support sense-making" (p. 4426).

Despite the rapid expansion of the use of apps in the educational domain, there is a lack of empirical studies as to their effectiveness in supporting learning, particularly in relation to mathematics. Some recent research in the use of mathematics apps on iPods (Kissane, 2011) and iPhones (Yuan, Chae, Nantwi, & Garg, 2010) has some relevance to this work as the constraints and affordances of iPods and iPhones are similar to those found with iPad apps. Attard and Northcote (2011) presented a brief review of categories of apps available for iPhone and iPad. The fact that only a few brief review of apps exist suggests that is important to conduct substantive research into the use of iPad mathematics apps.

Determining the quality of an app is difficult not only because of the lack of current research, but also because the information available at the Appstore is supplied by the app developer and largely serves as an infomercial. Some developers reference their app content to the U.S. Common Core State Standards (Mathematics). This is helpful; however the references are not always accurate, and these standards are different in any case from those used in the Australian Curriculum Mathematics (Australian Curriculum and Reporting Authority [ACARA], 2012). Nor is it particularly useful to rely on reviews from other internet sources such as, for example, Educators Technology Top 20 Maths Apps (http://www.educatorstechnology.com/2012/10/math-apps-for-ipad.html) as they generally provide product grabs already available from the Appstore. Because of the minimal amount of information available, it is likely that mathematics teachers are unaware of the availability of high-quality apps. This lack of knowledge is exacerbated by the existence of significant time demands on teachers which have, in recent years, been cited as a significant obstacle in the carrying out of novel instructional approaches in the mathematics classroom (Leong & Chick, 2011). The key research question under investigation is whether mathematics apps, categorised as primary education, are appropriate in terms of Australian content and also in terms of the types of mathematics knowledge they promote.

Literature Review:

The efficacy of information and communication technology (ICT) in supporting mathematical learning, if used in developmentally appropriate ways (Haugland & Ruiz, 2002), is supported by a range of research (See Clements & Sarama, 2007; McManis & Gunnewig, 2012; Özgün-Koca, Meagher, & Edwards, 2009/2010; Pelton & Pelton, 2011; Polly, 2011). However, as this article is concerned primarily with apps, the focus of the literature review will be on the use of developmentally appropriate software with students. Kadijevich (cited in Haapasalo, ND) stresses the following two requirements for effective ICT use in mathematics education:

> (1) when utilize (sic) mathematics, don't forget available tool(s); when make (sic) use of tool, don't forget the underlying mathematics; and (2) to solve the assigned task, use, whenever possible, a process approach as well as an object approach, working with different representations (algebraic and graphical, for example) (p.2).

Whilst many iOS apps (iOS is the Apple mobile operating system) appear to have the potential to enhance learning opportunities for young users at school and at home (Pelton & Pelton, 2011), Haugland (1999) sounds a word of caution in relation to a trend in software design, applicable to the burgeoning apps market, in suggesting that the implicit message of many designers and marketers of software is that if children use the software, their learning will be accelerated. The content of this "accelerated learning" software (largely drill-and-practice activities) is often at the skill level of children two or three years older than the target audience and this strategy can result in negative consequences as

> children become frustrated and do not use the software; or children use the software and only rote learning will occur. Their retention of the concepts is poor as well as their ability to apply the concepts to off computer activities" (Haugland, 1999, p.245).

Although a number of generic evaluative criteria for developmentally appropriate software have been proposed ((Ntuli & Kyei-Blankson, 2011; Potter, Johanson, & Hutinger, ND), criteria guiding the evaluation of iOS devices and apps are limited. Emergent research into their use appears to indicate that: "young children learn to use the devices quickly, independently, and confidently and explore freely" (McManis & Gunnewig, 2012, p. 15); these newer devices "seem to be ideally positioned to present mathematical models and manipulatives to support mathematical play, exploration and sense-making both in the classroom and at home" (Pelton & Pelton, 2011, p. 2200); and iOS devices have lower costs (thus increasing the likelihood of uptake in schools). Pelton and Pelton (2011, 2012), in designing a range of iPhone-only apps, were guided by five design principles - keep it simple, provide meaningful models and/or manipulatives, support individual needs, support parents and teachers, and track student progress. Although these five principles are useful for designing apps, they do not translate easily into a scoring rubric for evaluating existing apps (Ntuli & Kyei-Blankson, 2011). What is required is a consideration as to how educators can determine the usefulness of the apps for mathematical learning, given that there is a continual and rapid expansion in the availability of new apps and, inversely, there is limited information available in terms of these apps based on independent evaluations of their appropriateness.

Due to the limitations of the criteria noted above (See Potter et al., ND), the Haugland Software Development Scale (Haugland, 1999) and the Productive Pedagogies Dimensions (Education Queensland, 2001), were modified for use in this research. Both measures have been used in previous research (See Haugland & Ruiz, 2002; Zevenbergen & Lerman, 2007) and thus provide a mechanism for later comparison of findings in the literature with findings from this research. As the Haugland scale emphasises the user and the technical features of the software, and the Productive Pedagogies Dimensions emphasise the potential learning afforded by the technology, the findings generated by their combined use provide a balanced appraisal of the apps in terms of technical features, their ease of use for the end user, and their use as a tool for learning.

From this brief synopsis of the literature, it appears to be the case that, despite the increased availability of mathematics apps, their use has not as yet been rigorously researched. The need for research in the mathematics domain is critical, as many apps are predominantly flash card-based, drill-and-practice-style apps which may not provide the representations that young users require to support sense making (Pelton & Pelton, 2011); or are game-based apps which may be motivational to users but not encourage authentic learning (Yuan et al., 2010).

Methodology:

Data collection commenced with a targeted search for mathematics apps at the Appstore. Two searches were conducted in 2012. The data presented here is from the November 6, 2012 search. Given that the initial search term “mathematics education” returned 3,740 apps the search was narrowed. A search using the term “elementary mathematics” returned 202 apps; “primary mathematics” returned 107 apps, “junior mathematics” returned 20 apps; and “infant mathematics” returned 15 apps. Many of the same apps appeared in two or more of the searches. A final sort of the available +/- 200 apps was conducted and apps were excluded from the final review according to the following criteria.

- If both a free version and a paid version (these present as two different apps) were available, both versions were reviewed only if this were necessary to evaluate the app accurately
- Where there were a number of apps in a series only 1 app was reviewed
- Non-English apps, apps designed for a particular curriculum and bi-lingual apps were included in the review
- Apps that were categorised by iTunes as Games, Entertainment or Lifestyle were excluded. Apps where mathematics was part of a bigger package of reading, writing, and spelling skills were excluded.
- Apps that were calculators or data bases of mathematics terms were excluded.
- Apps that required additional costs for access or further online registration of students or teachers were excluded

At the conclusion of this process, 142 applications remained that were then subjected to a full review. The full review involved three processes: an Australian Curriculum content assessment; a software review using the Haugland Scale (1999); and a pedagogical review using a modified version of the Productive Pedagogies Dimensions (Education Queensland, 2004). Due to space constraints, this article reports full findings from the first review process. However, total scores for the Haugland Scale and the Productive Pedagogies Dimensions were used to generate the list of 34 quality apps for further evaluation.

Findings and Discussion:

As indicated earlier, the categorisation of the apps at the iTunes store is at the discretion of the app designer who may not have a strong mathematics education background or expertise (Kissane, 2011) and any curricula matching is against the Common Core Standards for Mathematics in US schools. The qualitative review process utilised here maps the app against the Australian Curriculum – Mathematics (See Table 1 for two review samples). It provides the app name and cost, relevant strand and sub-strand, specific content descriptors (including descriptor code), and year level appropriateness. An assessment of whether the app primarily presented conceptual, procedural, or declarative knowledge (Miller & Hudson, 2007) was conducted and a short review of the app, useful for teachers to read prior to purchase, was written.

Table 1
Sample of two of the 142 app reviews

App Name	Price	Strand	Sub Strand	Year	Code	Knowledge
Abacus counting frame for preschool	$ 0.99	Number & Algebra	Number & Place Value	2	ACMNA028 ACMNA029	Conceptual

Reviewer Comments: Although aimed at pre-school users, abacus use is Year Two ACARA content. App consists of a four-rung abacus with five balls on each rung. Users can move around any of the balls independently of what has been moved previously. There are no instructions, no tasks to be completed, no sound and the information icon merely provides a link to other products by the app creator.

App Name	Price	Strand	Sub Strand	Year	Code	Knowledge
Butterfly Brunch	$ 0.99	Measurement & Geometry	Location & Transformation	6	ACMMG143	Procedural

Reviewer Comments: Users have option to choose one quadrant (effectively +x and +y) or four quadrants (includes –x and –y). They then are given a target co-ordinate (e.g. -3, 4) and are to guide a butterfly to the co-ordinates using a series of clicks on directional arrows. Once they reach the target they press feed. If they are correct a caterpillar climbs up a stalk; if incorrect the caterpillar does not move and a buzzer sounds. There are no instructions as to how Cartesian co-ordinates work. Although Cartesian co-ordinates are Year 6 ACARA content, the design of the app is more appropriate for 6-7 year olds.

These short reviews were guided by the five design principles of Pelton and Pelton (2011) and by consideration of what is recommended as best practice for learning in mathematics, including the use of concrete-pictorial models and a focus on processes such as problem solving, reasoning, representation, exploration, and sense making (Pelton & Pelton, 2011, 2012). The full review of 142 apps can be made available on request. It is anticipated that the review data will be hosted at an online site thus facilitating further reviewing of apps by me, and other mathematics teachers.

The high usage of apps in non-school environments possibly contributes to the number of apps dedicated to drill-and-practice-type activities. On numerous occasions the description of the app provided by the Appstore did not match the actual experience of using the app. The major mismatches were incorrectly labelled app names; inaccurate descriptions concerning content; age level suggestions that did not match content outcomes or misleading descriptions which indicated conceptual knowledge development but provided only drill-and-practice-type activities. It is clearly the case that Appstore information is not sufficient for teachers to make a valid judgement on whether to purchase the app or not. In addition, the price charged for the apps is not necessarily an indicator of quality as a number of the top apps were free and many of the weaker apps were relatively costly.

Analysis of the content of the 142 apps against the three mathematical strands in the Australian Curriculum revealed that two apps included content from all three strands and seven apps included content from the number and algebra and measurement and geometry strands. The remaining apps involved only one strand. Table 3 provides a further breakdown of apps mapped to the Australian Curriculum sub-strands.

Table 2
Number of apps with content from Australian Curriculum Sub-Strands

Strand	Sub-Strand	No. of apps
Number and Algebra	Number and place value	105*
	Fractions and decimals	10
	Patterns and algebra	7
	Money and financial mathematics	3
	Linear and non-linear relationships	2
	Real numbers	1
Measurement and Geometry	Using units of measurement	15
	Shape	12
	Geometric reasoning	4
	Location and transformation	3
	Pythagoras and trigonometry	0#
Statistics and Probability	Data representation and interpretation	4
	Chance	3

***NB**: Total app count exceeds 142 as a number of apps include content from more than one sub-strand and are therefore counted more than once. # Pythagoras and trigonometry is only introduced in Australian secondary schools and so was beyond the scope of this review.

It is clear from the data presented in Table 2 that the number and algebra strand, and more specifically the number and place value sub-strand, are dominant in terms of content. Given previous research into mathematics apps for iPods (Kissane, 2011) and iPhones (Yuan et al., 2010) it is perhaps unsurprising that approximately 82.9% of the number and place value apps were dedicated to the four operations, with multiplication being the most popular. The two dominant sub-strands in measurement and geometry were using units of measurement (60% of these apps dealt with time) and shape (largely early childhood apps focusing on identifying 2D shapes and 3D objects). There was a scarcity of apps incorporating content from the statistics and probability strand.

The apps were also evaluated according to the type(s) of mathematical knowledge they aimed to develop. The definition of these three types of knowledge, as outlined by Miller and Hudson (2007), is used here. Table 3 provides a breakdown of the relative occurrence of apps that support the development of conceptual, procedural, or declarative knowledge. Conceptual knowledge is defined as "a connected web of information in which the linking relationships are as important as the pieces of discrete information that are linked" (Goldman et al., in Miller & Hudson, 2007, p. 49) and involves a deep understanding related to the meaning of mathematics. Procedural knowledge is the ability to follow a set of sequential steps to solve a mathematical task. Declarative knowledge is information that students retrieve from memory without hesitation and information that students know at a glance (Miller & Hudson, 2007).

Table 3
Number of apps developing differing forms of mathematics knowledge

Type of Knowledge	Number of Apps (*n*=142)	Percentage* *(to nearest 0.1)*
Declarative	63	44.4
Procedural	42	29.6
Conceptual	12	9.9
Both conceptual and procedural	14	8.5
Both conceptual and declarative	2	1.4
Both procedural and declarative	7	4.9
All three knowledge types	2	1.4

Of the 142 apps, 44.4% developed only declarative knowledge and 52.1% developed a combination of declarative and other types of knowledge. In terms of the sub-strand of number and place value the percentage of apps including a declarative knowledge component rises to 63%. These percentages reflect findings reported in the literature in relation to iPod and iPhone math apps. Pelton and Pelton (2011) found that "the majority of existing iOS apps for learning in mathematics are drill oriented (primarily flash card), do not provide resources to support sense-making, and fail to support active learning" (p. 2201). Kissane (2011) reports, in relation to iPod apps, "that despite enthusiastic claims to the contrary by the designers, many of the apps examined seem to offer not much more than heaps of practice, often timed" (p. 937).

A stated outcome of this research was the generation of a list of quality mathematics apps to be used in future research with primary school teachers and students. Although not fully reported here, due to space constraints, two quantitative measures, the Haugland Scale (Haugland, 1999) and the Productive Pedagogies Dimensions (Education Queensland, 2004) were also used to evaluate the apps. Each measure generated a rank order list. A full list of the scores of all 142 apps, and analysis of sub-themes that emerged from data analysis of both quantitative tools, is available on request. Table 4 lists the 34 apps, ranked in tiers, which were rated in the top 40 using each quantitative measure. What is apparent from the data is the high degree of congruence in terms of the top 40 apps. Comparison of the two "top 40" lists shows that 7 apps appeared in the top 10 of each measure; a further 10 apps appeared in the top 20 of each measure; a further 7 apps appeared in the top 30 of each measure; and a further ten apps appeared in the top 40 of each measure. It would appear from the high occurrence of the same apps in both lists (70% similarity of top 10; 85% similarity of top 20; 80% similarity of top 30, and 85% similarity of the top 40) that the two scales complement each other in terms of evaluating apps. In addition, Cronbach's Alpha for each scale indicated a high degree of internal consistency (Haugland Scale, $\alpha = 0.77$; Productive Pedagogies Dimensions, $\alpha = 0.94$). It is therefore considered valid to continue to use these two measures to evaluate new mathematics apps as they become available and to also trial the two measures with classroom teachers in future research.

Table 4
Top apps according to Haugland Scale and Productive Pedagogies Dimensions

Apps are listed as to their rank in the Top 10, 20, 30, or 40 of both lists.

Tier One (Top 10) - Area of Rectangles | Common Core Number and Operations in Base Ten (K-2) | Early Numbers: Maths Wizard Counting | Find and count | I See! Math 1| Mathemagica - Kids Math | Miracle Learning for Calculation

Tier Two (T20) - 123 Counting Fun Lite | Adding Beads | Friends of Ten | Hands-On Equations | Hands-On Maths Number Sense | Learn Math 1 (Mondiso) - Add | Subtract | Learn Numbers: Learn2Count | Marble Math Junior | Math Dream | Maths Skill Builders

Tier Three (T30) - Fact Families - + and - | Fun Count App | Hands-on Maths Attribute Blocks | Math Galaxy Fractions Fun | Math Model | Time Math Free | Visual Math 1

Tier Four (T40) - Astromat Lite | Base Ten Number Blocks | Column + - * / | Kindergarten Math | Math Grade One | Middle School Math HD | Patterns, Colors and Shapes | Statistics!!! | Telling Time HD | Toddler Counting 123 Lite

Limitations and a way forward:

As identified in similar research on iPods (Kissane, 2011) and iPhones (Yuan et al., 2010), it is necessary, when researching Appstore apps, to outline the difficulties associated with delineating a clear data set. The first limitation is the initial difficulty in finding and reviewing all the relevant apps. The problem of the sheer number of apps is compounded by a further limitation, that is, the poorly structured iTunes Appstore user interface. Search results are presented graphically as icons; not sorted in alphabetical order; often labelled inaccurately; and in a state of flux as new apps are added, renamed, upgraded, or deleted. The initial location of apps is therefore both a time consuming and potentially inaccurate process and it is possible that some appropriate mathematics apps were not located and therefore not reviewed. In summary, this article reported the results of an evaluation of 142 apps as a subset of a larger number of apps initially selected according to a range of search criteria. It identified key themes from the literature and outlined the research methodology used to evaluate the apps. Finally, the article reported the key findings of the qualitative review of mathematics apps in relation to the Australian Curriculum and generated a list of 34 quality mathematics apps to be evaluated further, using Gee's (2003) learning principles for video games and software evaluation tools proposed by Howard (2008) and Klopfer (2008), for use with primary school teachers and students.

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