Is maximizing resilience compatible with established ecological goal functions?

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Summary

Cropp & Gabric (2002) used a simple phytoplankton-zooplankton-nutrient model and a genetic algorithm to determine the parameter values that would maximize the value of certain goal functions. These goal functions were maximize biomass, maximize flux, maximize flux to biomass ratio, and maximize resilience. It was found that maximizing goal functions maximized resilience. The objective of this study was to investigate whether the Cropp & Gabric (2002) result was indicative of a general ecosystem principle, or peculiar to the model and parameter ranges used. This study successfully replicated the Cropp & Gabric (2002) experiment for a number of different model types, however a different interpretation of the results is made. A new metric, concordance, was devised to describe the agreement between goal functions. It was found that resilience has the highest concordance of all goal functions trialled, for most model types. This implies that resilience offers a compromise between the established ecological goal functions. The parameter value range used is found to affect the parameter verses goal function relationships. Local maxima and minima affected the relationship between parameters and goal functions, and between goal functions.

Key-words: concordance, emergent properties, resilience, thermodynamic goal function.

Introduction

The search for the fundamental principles that shape ecosystem evolution has occupied ecologists since Lotka (1922) hypothesized that natural selection would operate to preserve organisms that increased the energy flux through a system, subject to the constraints operating on the system. Odum (1983) extended and amplified Lotka's (1922) hypothesis, arguing that natural systems tend to maximize the flow of useful energy - the maximum power principle (Hall 1995), and that theories and corollaries derived from this principle could explain much about the structure and processes of these systems, drawing analogies between ecosystem behavior and the laws of thermodynamics. Although some authors have challenged the rationale for the maximum power principle (e.g. Månsson & McGlade 1993), the search for the laws or goal functions that underly ecosystem organization has continued.

Common to many hypothesized ecosystem goal functions are underlying thermodynamic considerations about how ecosystems utilize the energy that flows through them. Ecosystems exist within the constraints of thermodynamic laws that prescribe the transfer of energy, and may be considered as thermodynamic non-equilibrium dissipative structures, that, in common with their physical counterparts such as hurricanes and Bénard cells, utilize energy fluxes from external sources to maintain organization (Prigogine & Stengers 1984, Toussaint & Schneider 1998).

Several authors have pursued a variety of thermodynamically based principles: maximum entropy formation or dissipation (Aoki 1988), minimum excess entropy (Mauersberger 1995), minimum dissipation (Johnson 1995), maximum exergy storage (Mejer & Jørgensen 1979, Jørgensen & Mejer 1981, Jørgensen 1982), maximum exergy degradation or destruction (Kay & Schneider 1992, Kay & Schneider 1994), and maximum ascendancy (Ulanowicz 1980). Recently, Fath et al. (2001) have suggested that some of these goal functions may indeed be complementary and not mutually exclusive, thus making the identification of a single primary goal function quite difficult.

Recently, Cropp & Gabric (2002) used a genetic algorithm (GA) to simulate the adaptation of the biota in a three-compartment aquatic food web consisting of a limiting nutrient, an autotroph, and a single heterotrophic grazer. The GA searched the model parameter space to optimize each of four goal functions. Three goal functions were formulated by considering the thermodynamic imperatives of exergy, entropy production or dissipation, and ascendancy, together with their ecological analogues: sustainable biomass, productivity per unit biomass and primary productivity. The fourth goal function was resilience, *sensu* DeAngelis (1992). Although there is little evidence to suggest that resilience is a goal function, ecological networks that develop stabilizing feedbacks are considered to be more likely to remain extant than those that do not (Lenton 1998). Indeed, Odum (1969) noted that ecological succession led to mature-stage ecosystems with good resistance to external perturbation.

Interestingly, the Cropp & Gabric (2002) simulations suggested that, within the constraints of the external environment and the genetic potential of their constituent biota, ecosystems will evolve to the state most resilient to perturbation. This maximum resilience hypothesis has recently gained some further support. Laws et al. (2000) applied the assumption of maximum resiliency to a more complex pelagic food web. All but two parameters in the model were assigned values based on deterministic equations. Two independent adaptive characteristics of the model, the relative growth rates of the large phytoplankton and the biomass of filter feeders, were assigned values that maximized the resilience of the steady state system to perturbations. The predictions of the model with respect to the behavior of the export ratio, phytoplankton biomass, and heterotrophic bacterial biomass were in remarkable agreement with field observations encompassing a broad range of environmental conditions.

In the present work we investigate the sensitivity of the maximum resilience hypothesis to the model assumptions used by Cropp & Gabric (2002). In particular we have tested a variety of standard forms for autotroph-nutrient and autotroph-grazer interactions and also examine the relationship between the various goal functions.

Methods

Model Structure

The Cropp & Gabric (2002) model (hereafter the CG model) was described by three differential equations:

$$\frac{dP}{dt} = \mu_P P \left(\frac{N}{N + k_P} \right) - e_Z P Z, \tag{1}$$

$$\frac{dZ}{dt} = e_Z (1 - \eta_Z) P Z - d_Z Z, \tag{2}$$

$$\frac{dN}{dt} = d_Z Z + e_Z \eta_Z P Z - \mu_P P \left(\frac{N}{N + k_P} \right), \tag{3}$$

where P, Z and N are the nutrient concentrations in each compartment, e_Z is the consumption per day of phytoplankton per capita of zooplankton, d_Z is the zooplankton mortality, η_Z is the efficiency of zooplankton conversion of nutrient into biomass, k_P is the nutrient half-saturation concentration for phytoplankton, and μ_P is the maximum phytoplankton nutrient uptake rate. The system is closed with respect to the input and output of nutrients, hence the total sum of nutrients, N_o, is a constant. The CG Model describes a Lotka-Volterra interaction between zooplankton and phytoplankton, and a Holling Type II interaction between the nutrient compartment and phytoplankton (DeAngelis 1992).

The CG Model has been generalized to include a variety of phytoplankton-zooplankton (P-Z) and nutrient-phytoplankton (N-P) interactions, as shown in Table 1:

- All possible combinations of Lotka-Volterra and Holling Type II interactions for $N-P$ (column 2) and $P-Z$ (column 3) were tested (denoted LL, LH, HL and HH);
- An efficiency term for the assimilation of phytoplankton biomass by zooplankton was included or excluded (column 4);
- Where applicable, the path of inefficiently assimilated phytoplankton grazed by zooplankton was passed directly to the nutrient compartment (e.g. DeAngelis 1992) or passed through the zooplankton compartment before the nutrient compartment (e.g. CG Model) (column 5);
- A death term for phytoplankton was either excluded (e.g. Cropp & Gabric 2002) or included (e.g. Druon & Le Fèvre 1999) (column 6).

A full list of differential equations governing all of the models may be found in the *Digital Appendices*.

Parameter Space

Parameter ranges were chosen such that all models possessed a feasible (P and Z values greater than 0 at equilibrium) and stable (negative maximum eigenvalue, or positive resilience) equilibrium. The base parameter values are shown in Table 2. The parameter value range is ±50% of the base values. This is the same as the methodology used by Cropp & Gabric (2002).

HL4 is the rerun of the CG Model. The parameter values for Model HL4 differ from those used by Cropp & Gabric (2002), in that the μ_P median value of 3 (Table 2) is greater than the Cropp & Gabric (2002) median of 0.9. This was necessary for consistency with the other models used, subject to the stability and feasibility constraints.

The Goal Functions

The six goal functions employed were based upon thermodynamic principles thought to underlie the development of ecosystems. These goal functions reflect the attributes of the climax of a successional sequence, the mature ecosystem (Odum 1969). It is assumed that maturity, and hence its heuristically derived attributes (Wilhelm & Brüggemann 2000), are maximized.

1. Maximize phytoplankton biomass at equilibrium, P_{eq}; and

2. Maximize zooplankton biomass at equilibrium, Z_{eq};

   Primary succession necessarily increases biomass from zero. From a thermodynamic perspective, biomass may be thought of as a measure of (genetic) information content (Margalef 1968, Jørgensen 1999), which corresponds to the distance from thermodynamic equilibrium.

3. Maximize flux of nutrients through the system at equilibrium, F_{eq};

   The flux of nutrients through the system is an indicator of production, which is observed to increase to a maximum and then fall slightly during succession (Cooke 1967, Odum 1969, Cooper 1981). Nutrient flux may also be considered a proxy for energy flux (Lotka 1922) and power (Odum 1983).

4. Maximize flux to biomass ratio at equilibrium, $(F/B)_{eq}$;

   The maximization of F/B is an extension of the order-through-fluctuation principle, which encompasses all principles recognizing the tendency of energy gradients to break down through self-organized dissipative structures (Prigogine & Stengers 1984, Choi, Mazumder & Hansell 1999). It can be considered a result of the Second Law of Thermodynamics, where F is a measure of entropy (Johnson 1988).

5. Minimize flux to biomass ratio at equilibrium, $(F/B)_{eq}$;

Mature ecosystems tend to minimize F/B during succession (Margalef 1968, Odum 1969) thereby maximizing their efficiency. The apparent conflict between this goal function and 4 may be considered an artifact of the time frame over which the goal function is said to act. Four may be considered a long-term goal function, and 5 a short-term goal function (Johnson 1988).

6. Maximize resilience at equilibrium, R_{eq}.

   Resilience is defined the negative real part of the eigenvalue closest to zero. Resilience can be thought of as the inverse of the return time to equilibrium after a small perturbation (DeAngelis 1992). The maximization of resilience is not an established goal function, however the maximization of stability (which resilience could be considered one aspect of) is implicit in the Exergy goal function (Jørgensen & Mejer 1977), and has been used to successfully predict ecosystem behavior (Laws et al. 2000).

Concordance - The Sum of Normalized Fitness

A new metric, concordance, was used to quantify the agreement between goal functions. Concordance measures the extent to which maximizing a given goal function also optimizes the value of the other goal functions.

Let the value of each goal function be Γ_i, where [1] $i = 1-6$. Let the parameter set (see Table 2) be denoted by a parameter vector, $\boldsymbol{\alpha}$. Each goal function is a function of the parameter set, $\Gamma_i(\boldsymbol{\alpha})$. Let the parameter set that maximizes goal function j be denoted $\boldsymbol{\alpha}_{max\,j}$, and the parameter set that minimizes goal function j be denoted $\boldsymbol{\alpha}_{min\,j}$.

The normalized values of goal function i using the parameter values that maximize goal

[1] In this study, $i = 1$ is P_{eq}, 2 is Z_{eq}, 3 is F_{eq}, 4 is $(F/B)_{eq}$, 5 is $-(F/B)_{eq}$, and 6 is R_{eq}. Note that Γ_4 and Γ_5 are mutually exclusive.

function j, $\hat{\Gamma}_i(\boldsymbol{\alpha}_{max\ j})$, is

$$\hat{\Gamma}_i(\boldsymbol{\alpha}_{max\ j}) = \frac{\Gamma_i(\boldsymbol{\alpha}_{max\ j}) - \Gamma_i(\boldsymbol{\alpha}_{min\ i})}{\Gamma_i(\boldsymbol{\alpha}_{max\ i}) - \Gamma_i(\boldsymbol{\alpha}_{min\ i})}, \quad 0 \leq \hat{\Gamma}_i(\boldsymbol{\alpha}_{max\ j}) \leq 1 \quad \forall i. \tag{4}$$

The concordance of goal function j, C_j, is the sum of these normalized values[2].

$$C_j = \sum_{\substack{i=1 \\ i \neq j \\ i \neq 4 \text{ or } i \neq 5}}^{i=6} \hat{\Gamma}_i(\boldsymbol{\alpha}_{max\ j}), \quad 0 \leq C_j \leq 4, \tag{5}$$

where C_j is the concordance of goal function j, and n is the total number of goal functions.

Post-processing of the Genetic Algorithm Results

Some goal functions were not a function of every parameter in the model (e.g. in Model LH4, P_{eq} did not depend upon the parameter μ_P). The genetic algorithm would vary these parameter values, misrepresenting the effect of maximizing each goal function on the selection of the parameter. These parameters were set to their base values before being used in calculations, however an investigation of this attribute was made for the CG Model, HL4.

[2] $i \neq 4$ or $i \neq 5$ is because these goal functions (F/B and $-F/B$) are mutually exclusive.

Results and Discussion

Rerun of the CG Model

The results of Model HL4, the rerun of the CG Model experiment, are shown in Table 3. Using a slightly different parameter range to Cropp & Gabric (2002), each goal function is maximized at the same vertex in parameter space as found by Cropp & Gabric (2002).

Figure 1 shows the normalized goal function values found using the results in Table 3. With the exception of the goal functions 'maximize zooplankton biomass' and 'maximize flux to biomass ratio' [3], the parameter set that maximizes the goal functions optimizes resilience. This supports Cropp & Gabric's (2002) result.

The total height of each bar in Figure 1 is the concordance. It can be seen from Figure 1 that the goal functions 'maximize flux' and 'maximize resilience' have the equal highest concordance. Table 3 indicates that the goal functions 'maximize flux' and 'maximize resilience' are maximized by the same parameter set.

Cropp & Gabric (2002) stated that "the biotic attributes that optimize the thermodynamic goal functions also maximize resilience". This study finds that the concordance of the goal function 'maximize resilience' (and 'maximize flux') is highest. The difference between these two interpretations is that, while Cropp & Gabric (2002) imply that resilience is optimized when other goal functions are maximized, this result suggests that other goal functions are optimized when resilience is maximized.

These two interpretations may not be very different. The directionality of the concordance is an artifact of the way it is formulated, and the different sensitivities of each goal function to

[3] The goal function 'maximize phytoplankton biomass' is not a function of μ_P and k_P, however these parameters can be set such that resilience is optimized with maximum phytoplankton biomass.

the parameters. Concordance necessarily treats each goal function independently, allowing an unambiguous investigation of the effects of one goal function upon the others. However, this means that concordance cannot investigate the effects of maximizing two or more goal functions simultaneously. Indeed, it would be impractical to run a genetic algorithm on every possible combination of goal functions. Interestingly, Figure 1 shows that resilience and flux are one such combination, offering a compromise between the other goal functions.

Does the Resilience Hypothesis Hold for Other Model Types?

Concordance values for each goal function are shown in Table 4 and Table 5 [4]. Table 4 shows the results when $(F/B)_{eq}$ is maximized and Table 5 for when the $(F/B)_{eq}$ is minimized. It can be seen that, for most models, resilience has the highest (or equal-highest) concordance, and hence the maximization of resilience leads to the optimization of the other goal functions. It should be noted that inspection of the raw data revealed that the the converse is not true, that is, the maximization of the other goal functions does not lead to the optimization of resilience.

For minimizing $(F/B)_{eq}$, all models give resilience with the highest or equal-highest (with flux) concordance. This suggests that the resilience hypothesis is robust, and not specific to the CG Model type. It also supports the previously observed (DeAngels et al. 1978) positive relationship between flux and resilience, although not for the flux formulation that DeAngelis (1992) uses.

For maximizing $(F/B)_{eq}$, most models give resilience with the highest or equal-highest (with flux) concordance. Before seeking to explain the exceptions to this, a discussion of the effects of parameter range is required.

[4] The values of the normalized goal functions may be found in the *Digital Appendices*.

Does the Parameter Range Affect Concordance?

Resilience is the negative of the real part of the eigenvalue closest to zero (DeAngelis 1992). The eigenvalues can be real or complex conjugate. Real eigenvalues indicate that the steady state is a simple sink, whereas complex conjugate eigenvalues indicate that there is an oscillatory return to the steady state.

The parameter range chosen was such that all models with a Holling Type II interaction between the phytoplankton and zooplankton compartments (prefixed LH or HH) had the potential to possess either real or complex eigenvalues, whereas those with a Lotka-Volterra interaction (prefixed LL or HL) only possessed complex conjugate eigenvalues.

For all models with potential for both types of eigenvalues, resilience was maximized at the discontinuity between real and complex conjugate eigenvalues. This interior point represents a local maximum in resilience. An example of this is shown in Figure 2. Resilience is plotted against against e_Z for the simplest model, Model LL1. All other parameters are held constant.

Figure 2 shows a typical relationship between parameter value and resilience. When the eigenvalues are complex conjugate, they both have the same real part, which is equal to resilience. In this case, resilience has a positive relationship with the parameter. When the eigenvalues are real, the eigenvalue closest to zero dominates the system, and is used to quantify resilience. In this case, resilience has a negative relationship with the parameter.

From inspection of the raw data, it can be seen that critical points also exist for the goal functions 'maximize zooplankton biomass', and 'maximize flux'.

Where the parameter range is located relative to the interior points changes where the goal function is maximized, and hence, the concordance. It is expected that the best measure of concordance can be made when a parameter range includes its interior point.

When does the resilience hypothesis fail?

Three model formulations (LH4, HH4, HH6) out of 24 do not have resilience with the highest concordance (Table 4). For these models, flux has the highest concordance. These exceptions are all models with a zooplankton grazing inefficiency term where all inefficiently grazed phytoplankton biomass passes through the zooplankton compartment (a Cropp & Gabric (2002) flux formulation), and a Holling Type II interaction exists between zooplankton and phytoplankton.

For HH and LH models, the goal functions 'maximize resilience', 'maximize flux', and 'maximize zooplankton biomass' are maximized at an interior point. This has a general tendency to decrease the concordance of each of the goal functions, compared with their LL and HL counterparts, which are maximized at a vertex. The decrease in concordance is higher for resilience than flux because of the advantage 'maximize flux' has in optimizing 'maximize flux to biomass ratio'. Reversing this goal function to 'minimize flux to biomass ratio' gives resilience the advantage, as can be seen by comparing Table 4 and Table 5.

Concluding Remarks

It was found that resilience had the highest concordance for most model types, and that the result was not specific to the CG model. This implies that resilience offers a robust compromise between the established goal functions, and may provide an additional complimentary goal function to those identified by Fath et al. (2001).

Prior to the Cropp & Gabric (2002) findings, there was little to suggest that resilience is a legitimate goal function. However, it should be noted that stability (which resilience could be considered one aspect of) is an observed attribute of mature (e.g. Odum 1969, Odum 1983, Margalef 1968) and complex (e.g. Naeem & Li 1997, Tilman 1996, Goodman 1975) systems, and forms an underlying assumption for one of the more successful goal functions, exergy (Jørgensen & Mejer 1977), in the form of a buffer capacity. In addition to this, success in using the resilience hypothesis to predict the behavior of real ecosystems (Laws et al. 2000) suggests that a theoretical framework for the resilience hypothesis is worth pursuing.

The observation that resilience offers a compromise between established goal functions does not explain the mechanisms by which ecosystems would structure themselves in order to maximize resilience. If it had been found, for instance, that resilience was maximized as an artifact of the maximization of other goal functions, it may be hypothesized that ecosystems maximize resilience, however, no evidence for this was found. Rather, it has been found that resilience offers one possible way to simultaneously optimize other goal functions. This leaves the question as to why ecosystems would 'choose' resilience over any other compromise between goal functions.

It was observed that the maximally resilient system promoted low grazing efficiency, and that "the herbivore attributes result in organisms that are less fit to compete for limiting resources at the individual level" (Cropp & Gabric 2002). This is reminiscent of DeAngelis's

(1975) observation that the probability of a stable system can be increased by decreasing the assimilation efficiency of species. Together, these observations suggest that the maximization of resilience is not consistent with the individualist 'law' of natural selection. This further emphasizes the need to describe a mechanism for the maximization of resilience independent of the other goal functions.

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Tables and Figures

Table 1: A summary of attributes of models ran.

Model Name	**N-P Interaction**	**P-Z Interaction**	**Z Efficiency Term?**	**All P Pass Through Z?**	**P Mortality Term?**
LL1	Lotka-Volterra	Lotka-Volterra	No	N.A.	No
LL2	Lotka-Volterra	Lotka-Volterra	No	N.A.	Yes
LL3	Lotka-Volterra	Lotka-Volterra	Yes	No	No
LL4	Lotka-Volterra	Lotka-Volterra	Yes	Yes	No
LL5	Lotka-Volterra	Lotka-Volterra	Yes	No	Yes
LL6	Lotka-Volterra	Lotka-Volterra	Yes	Yes	Yes
LH1	Lotka-Volterra	Holling II	No	N.A.	No
LH2	Lotka-Volterra	Holling II	No	N.A.	Yes
LH3	Lotka-Volterra	Holling II	Yes	No	No
LH4	Lotka-Volterra	Holling II	Yes	Yes	No
LH5	Lotka-Volterra	Holling II	Yes	No	Yes
LH6	Lotka-Volterra	Holling II	Yes	Yes	Yes
HL1	Holling II	Lotka-Volterra	No	N.A.	No
HL2	Holling II	Lotka-Volterra	No	N.A.	Yes
HL3	Holling II	Lotka-Volterra	Yes	No	No
HL4	Holling II	Lotka-Volterra	Yes	Yes	No
HL5	Holling II	Lotka-Volterra	Yes	No	Yes
HL6	Holling II	Lotka-Volterra	Yes	Yes	Yes
HH1	Holling II	Holling II	No	N.A.	No
HH2	Holling II	Holling II	No	N.A.	Yes
HH3	Holling II	Holling II	Yes	No	No
HH4	Holling II	Holling II	Yes	Yes	No
HH5	Holling II	Holling II	Yes	No	Yes
HH6	Holling II	Holling II	Yes	Yes	Yes

Table 2: Base parameter values. Range used was ±50%. N_o was taken to be a constant 500 mg/m^2.

Model Name	e_P $\left(\frac{m^2}{mgN\ d}\right)$	e_Z $\left(\frac{m^2}{mgN\ d}\right)$	μ_P $\left(\frac{1}{d}\right)$	μ_Z $\left(\frac{1}{d}\right)$	k_P $\left(\frac{mgN}{m^2}\right)$	k_Z $\left(\frac{mgN}{m^2}\right)$	d_P $\left(\frac{1}{d}\right)$	d_Z $\left(\frac{1}{d}\right)$	η_Z
LL1	0.006	0.006	N.A.	N.A.	N.A.	N.A.	N.A.	0.05	N.A.
LL2	0.006	0.006	N.A.	N.A.	N.A.	N.A.	0.005	0.05	N.A.
LL3 & LL4	0.006	0.006	N.A.	N.A.	N.A.	N.A.	N.A.	0.05	0.4
LL5 & LL6	0.006	0.006	N.A.	N.A.	N.A.	N.A.	0.005	0.05	0.4
LH1	0.006	N.A.	N.A.	3	N.A.	2000	N.A.	0.05	N.A.
LH2	0.006	N.A.	N.A.	3	N.A.	2000	0.005	0.05	N.A.
LH3 & LH4	0.006	N.A.	N.A.	3	N.A.	2000	N.A.	0.05	0.4
LH5 & LH6	0.006	N.A.	N.A.	3	N.A.	2000	0.005	0.05	0.4
HL1	N.A.	0.006	3	N.A.	277	N.A.	N.A.	0.05	N.A.
HL2	N.A.	0.006	3	N.A.	277	N.A.	0.005	0.05	N.A.
HL3 & HL4	N.A.	0.006	3	N.A.	277	N.A.	N.A.	0.05	0.4
HL5 & HL6	N.A.	0.006	3	N.A.	277	N.A.	0.005	0.05	0.4
HH1	N.A.	N.A.	3	3	277	2000	N.A.	0.05	N.A.
HH2	N.A.	N.A.	3	3	277	2000	0.005	0.05	N.A.
HH3 & HH4	N.A.	N.A.	3	3	277	2000	N.A.	0.05	0.4
HH5 & HH6	N.A.	N.A.	3	3	277	2000	0.005	0.05	0.4

Note:
e_P is the consumption per day of nutrient per capita phytoplankton
μ_Z is the maximum zooplankton nutrient uptake rate
k_Z is the nutrient half saturation concentration for phytoplankton
d_P is the phytoplankton mortality

Table 3: Model HL4 results. Parameters that the goal functions were independent of are marked with an *.

	Parameter					**Value of Goal Functions**				
	e_Z	d_Z	η_Z	μ_P	k_P	P	Z	F	F/B	R
$\boldsymbol{\alpha}_{max}$										
Maximize P	0.003	0.075	0.6	3*	277*	62.5	311.9	58.5	0.16	0.16
Maximize Z	0.003	0.025	0.2	4.5	138.5	10.4	433.3	13.5	0.031	0.09
Maximize F	0.003	0.075	0.6	4.5	138.5	62.5	389.0	73.0	0.16	0.56
Maximize $\frac{F}{B}$	0.009	0.075	0.6	4.5	138.5	20.8	289.2	54.2	0.17	0.06
Maximize R	0.003	0.075	0.6	4.5	138.5	62.5	389.0	73.0	0.062	0.56
$\boldsymbol{\alpha}_{min}$										
Minimize P	0.009	0.025	0.2	3*	277*	3.5	178.2	4.5	0.025	0.0041
Minimize Z	0.009	0.075	0.6	1.5	415.5	20.8	81.5	6.1	0.06	0.010
Minimize F	0.009	0.025	0.2	1.5	415.5	3.5	83.1	2.6	0.03	0.0016
Minimize $\frac{F}{B}$	0.003	0.025	0.2	1.5	415.5	10.4	203.8	6.4	0.030	0.0066
Minimize R	0.009	0.025	0.2	1.5	138.5	3.5	121.7	3.0	0.024	0.0013

Table 4: Concordance (Equation 5) for each goal function in each model. F/B is maximized.

Model Name	Maximize P_{eq}	Maximize Z_{eq}	Maximize F_{eq}	Maximize F/B_{eq}	Maximize R_{eq}
LL1	3.25	0.82	3.90	1.71	3.90
LL2	3.25	0.82	3.90	1.71	3.90
LL3	2.95	0.56	2.84	1.43	3.58
LL4	3.11	0.40	3.74	1.81	3.74
LL5	2.95	0.56	2.84	1.43	3.58
LL6	3.11	0.40	3.74	1.81	3.74
LH1	2.25	0.75	2.53	1.56	3.04
LH2	2.27	0.76	2.54	1.55	3.03
LH3	0.04	0.89	2.44	1.73	2.68
LH4	0.14	0.57	2.57	2.03	2.35
LH5	0.05	0.89	2.45	1.73	2.62
LH6	0.15	0.58	2.58	2.01	2.61
HL1	2.72	0.88	3.92	1.63	3.92
HL2	2.72	0.88	3.92	1.63	3.92
HL3	2.47	0.61	2.84	1.45	3.65
HL4	2.61	0.43	3.78	1.72	3.78
HL5	2.47	0.61	2.84	1.45	3.65
HL6	2.60	0.43	3.78	1.72	3.78
HH1	2.08	0.67	2.42	1.88	3.08
HH2	2.07	0.55	2.50	1.88	3.07
HH3	0.09	0.91	2.39	1.93	2.70
HH4	0.16	0.56	2.60	2.16	1.85
HH5	0.09	1.16	2.58	1.98	2.83
HH6	0.17	0.84	2.26	2.21	2.07
Average	1.824	0.689	2.996	1.756	3.211
CV	0.70	0.29	0.21	0.13	0.20

Table 5: Concordance (Equation 5) for each goal function in each model. F/B is minimized.

Model Name	Maximize P_{eq}	Maximize Z_{eq}	Maximize F_{eq}	Minimize F/B_{eq}	Maximize R_{eq}
LL1	2.38	1.81	3.00	0.96	3.00
LL2	2.38	1.81	3.00	0.95	3.00
LL3	2.37	1.50	2.00	1.00	2.95
LL4	2.36	1.34	2.95	0.73	2.95
LL5	2.37	1.51	1.96	0.99	2.95
LL6	2.36	1.40	2.95	0.72	2.95
LH1	2.12	1.71	1.77	1.44	2.61
LH2	2.13	1.71	1.78	1.44	2.61
LH3	1.02	1.45	1.63	1.01	2.14
LH4	1.12	1.51	1.78	1.11	2.18
LH5	1.03	1.45	1.65	1.01	2.10
LH6	1.12	1.50	1.80	1.10	2.16
HL1	1.84	1.86	3.00	0.72	3.00
HL2	1.84	1.86	3.00	0.72	3.00
HL3	1.87	1.53	1.95	0.76	2.96
HL4	1.86	1.42	2.97	0.53	2.97
HL5	1.87	1.53	1.95	0.75	2.96
HL6	1.86	1.42	2.97	0.52	2.97
HH1	1.93	1.59	1.52	1.35	2.40
HH2	1.93	1.47	1.62	1.35	2.41
HH3	1.06	2.45	1.47	2.05	2.12
HH4	1.13	1.51	1.70	1.12	2.15
HH5	1.07	1.72	1.85	1.04	2.15
HH6	1.14	1.76	1.55	1.12	2.17
Average	1.757	1.576	2.159	0.979	2.619
CV	0.30	0.10	0.28	0.27	0.15

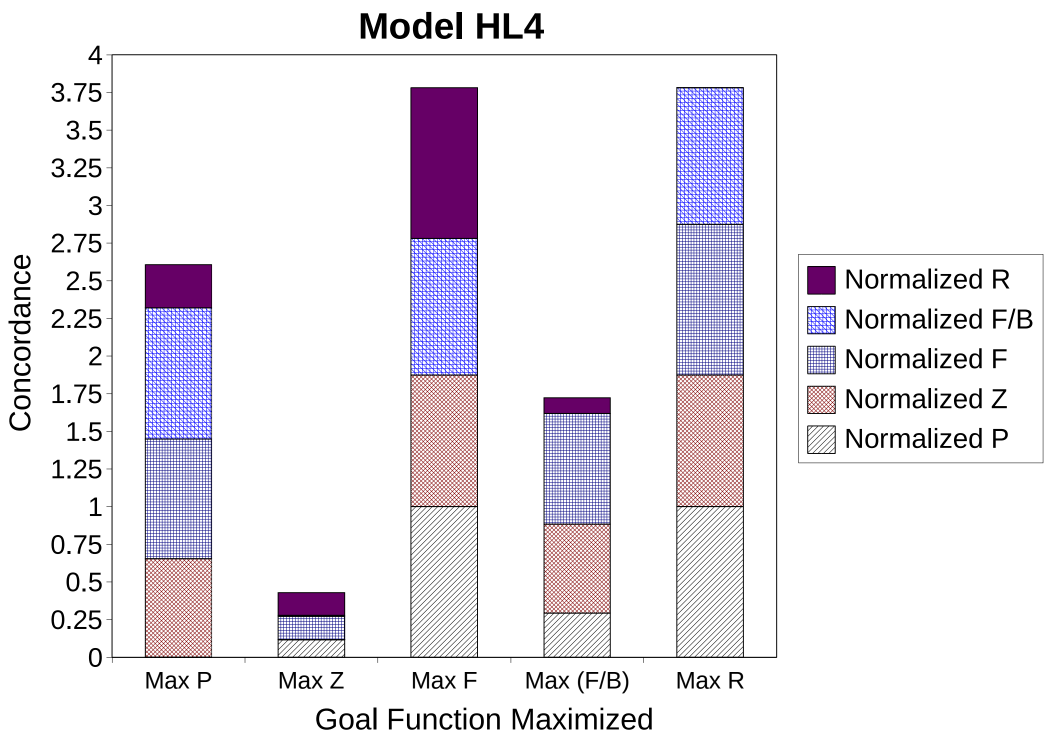

Fig. 1: The normalized effect of maximizing each goal function upon the other goal functions in Model HL4.

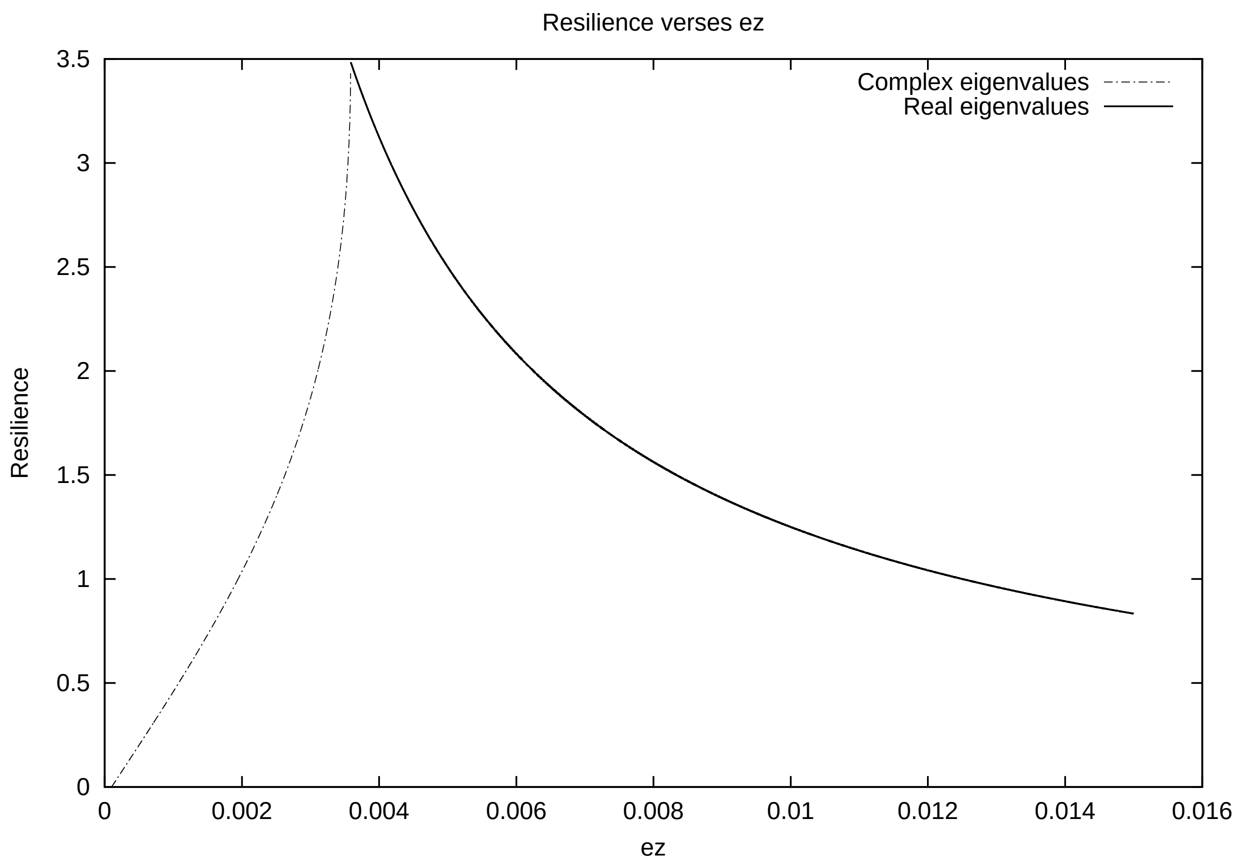

Fig. 2: Resilience verses e_Z for Model LL1. All parameters except e_Z are held constant. Note the maxima of resilience is at the transition from real to complex conjugate eigenvalues.

DIGITAL APPENDICES

Differential Equations Governing Models and Goal Functions

Model LL

Differential Equations:

$$\frac{dP}{dt} = e_P NP - e_Z PZ - d_P P, \tag{App 1a}$$

$$\frac{dZ}{dt} = e_Z(1-\eta_Z)PZ - d_Z Z, \tag{App 1b}$$

$$\frac{dN}{dt} = d_Z Z + d_P P + e_Z \eta_Z PZ - e_P NP. \tag{App 1c}$$

Goal Functions:

Phytoplankton biomass at equilibrium:

$$P_{eq} = \frac{d_Z}{e_Z(1-\eta_Z)}. \tag{App 2a}$$

Zooplankton biomass at equilibrium:

$$Z_{eq} = \frac{e_P N_{eq} - d_P}{e_Z}, \tag{App 2b}$$

where nutrient at equilibrium is

$$N_{eq} = \frac{d_Z(N_o - P_{eq}) + e_Z \eta_Z P_{eq}(N_o - P_{eq}) + d_P P_{eq}}{d_Z + e_Z \eta_Z P_{eq} + e_P P_{eq}}.$$

Flux at equilibrium for the DeAngelis (1992) flux formulation:

$$F_{eq} = (1 - \eta_Z) e_Z P_{eq} Z_{eq} = d_Z Z_{eq} = e_P N_{eq} P_{eq} - \eta_Z e_Z P_{eq} Z_{eq} - d_P P_{eq}. \quad \text{(App 2c)}$$

Flux at equilibrium for the Cropp & Gabric (2002) flux formulation:

$$F_{eq} = e_Z P_{eq} Z_{eq} = d_Z Z_{eq} + \eta_Z e_Z P_{eq} Z_{eq} = e_P N_{eq} P_{eq} - d_P P_{eq}. \quad \text{(App 2d)}$$

Flux to biomass ratio at equilibrium:

$$\left(\frac{F}{B}\right)_{eq} = \frac{F_{eq}}{P_{eq} + Z_{eq}}. \quad \text{(App 2e)}$$

Resilience at equilibrium is found from (refer to *Digital Appendix: Finding Resilience* for details):

$$\mathbf{A} = e_P(N_{eq} - P_{eq}) - e_Z Z_{eq} - d_P, \quad \text{(App 2f)}$$

$$\mathbf{B} = -P_{eq}(e_P + e_Z), \quad \text{(App 2g)}$$

$$\mathbf{C} = e_Z(1 - \eta_Z) Z_{eq}, \quad \text{(App 2h)}$$

$$\mathbf{D} = 0. \quad \text{(App 2i)}$$

Model LH

Differential Equations:

$$\frac{dP}{dt} = e_P NP - \frac{\mu_Z PZ}{k_Z + P} - d_P P, \tag{App 3a}$$

$$\frac{dZ}{dt} = \frac{\mu_Z (1 - \eta_Z) PZ}{k_Z + P} - d_Z Z, \tag{App 3b}$$

$$\frac{dN}{dt} = d_Z Z + d_P P + \frac{\mu_Z \eta_Z PZ}{k_Z + P} - e_P NP. \tag{App 3c}$$

Goal Functions:

Phytoplankton biomass at equilibrium:

$$P_{eq} = \frac{d_Z k_Z}{\mu_Z (1 - \eta_Z) - d_Z}. \tag{App 4a}$$

Zooplankton biomass at equilibrium:

$$Z_{eq} = \frac{(e_P N_{eq} - d_P)(k_Z + P_{eq})}{\mu_Z}, \tag{App 4b}$$

where nutrient at equilibrium is

$$N_{eq} = \frac{d_Z (k_Z + P_{eq})(N_o - P_{eq}) + \mu_Z \eta_Z P_{eq} (N_o - P_{eq}) + d_P P_{eq} (k_Z + P_{eq})}{d_Z (k_Z + P_{eq}) + \mu_Z \eta_Z P_{eq} + e_P P_{eq} (k_Z + P_{eq})}.$$

Flux at equilibrium for the DeAngelis (1992) flux formulation:

$$F_{eq} = \frac{\mu_Z (1 - \eta_Z) P_{eq} Z_{eq}}{k_Z + P_{eq}} = d_Z Z_{eq} = e_P N_{eq} P_{eq} - \frac{\mu_Z \eta_Z P_{eq} Z_{eq}}{k_Z + P_{eq}} - d_P P_{eq}. \tag{App 4c}$$

Flux at equilibrium for the Cropp & Gabric (2002) flux formulation:

$$F_{eq} = \frac{\mu_Z P_{eq} Z_{eq}}{k_Z + P_{eq}} = d_Z Z_{eq} + \frac{\mu_Z \eta_Z P_{eq} Z_{eq}}{k_Z + P_{eq}} = e_P N_{eq} P_{eq} - d_P P_{eq} \tag{App 4d}$$

Flux to biomass ratio at equilibrium:

$$\left(\frac{F}{B}\right)_{eq} = \frac{F_{eq}}{P_{eq} + Z_{eq}}. \quad \text{(App 4e)}$$

Resilience at equilibrium is found from (refer to *Digital Appendix: Finding Resilience* for details):

$$\mathbf{A} = e_P(N_{eq} - P_{eq}) - \frac{\mu_Z k_Z Z_{eq}}{k_Z + P_{eq}} - d_P, \quad \text{(App 4f)}$$

$$\mathbf{B} = -P_{eq}\left(e_P + \frac{\mu_Z}{k_Z + P_{eq}}\right), \quad \text{(App 4g)}$$

$$\mathbf{C} = \frac{\mu_Z k_Z (1 - \eta_Z) Z_{eq}}{(k_Z + P_{eq})^2}, \quad \text{(App 4h)}$$

$$\mathbf{D} = 0. \quad \text{(App 4i)}$$

Model HL

Differential Equations:

$$\frac{dP}{dt} = \frac{\mu_P N P}{k_P + N} - e_Z P Z - d_P P, \tag{App 5a}$$

$$\frac{dZ}{dt} = e_Z(1-\eta_Z) P Z - d_Z Z, \tag{App 5b}$$

$$\frac{dN}{dt} = d_Z Z + d_P P + e_Z \eta_Z P Z - \frac{\mu_P N P}{k_P + N}. \tag{App 5c}$$

Goal Functions:

Phytoplankton biomass at equilibrium:

$$P_{eq} = \frac{d_Z}{e_Z(1-\eta_Z)}. \tag{App 6a}$$

Zooplankton biomass at equilibrium:

$$Z_{eq} = \left(\frac{1}{e_Z}\right)\left(\frac{\mu_P N_{eq}}{k_P + N_{eq}} - d_P\right), \tag{App 6b}$$

where nutrient at equilibrium is

$$N_{eq} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where:

$$a = e_Z, \tag{App 6c}$$

$$b = -\mu_P + d_P + e_Z(N_o - P_{eq} - k_P), \tag{App 6d}$$

$$c = k_P e_Z (N_o - P_{eq}) + d_P k_P. \tag{App 6e}$$

Flux at equilibrium for the DeAngelis (1992) flux formulation:

$$F_{eq} = e_Z(1-\eta_Z)P_{eq}Z_{eq} = d_Z Z_{eq} = \frac{\mu_P N_{eq} P_{eq}}{k_P + N_{eq}} - e_Z \eta_Z P_{eq} Z_{eq} - d_P P_{eq}. \tag{App 6f}$$

Flux at equilibrium for the Cropp & Gabric (2002) flux formulation:

$$F_{eq} = e_Z P_{eq} Z_{eq} = e_Z \eta_Z P_{eq} Z_{eq} + d_Z Z_{eq} = \frac{\mu_P N_{eq} P_{eq}}{k_P + N_{eq}} - d_P P_{eq}. \tag{App 6g}$$

Flux to biomass ratio at equilibrium:

$$\left(\frac{F}{B}\right)_{eq} = \frac{F_{eq}}{P_{eq} + Z_{eq}}. \tag{App 6h}$$

Resilience at equilibrium is found from (refer to *Digital Appendix: Finding Resilience* for details):

$$\mathbf{A} = \frac{\mu_P(N_{eq} - P_{eq})(k_P + N_{eq}) + \mu_P P_{eq} N_{eq}}{(k_P + N_{eq})^2} - e_Z Z_{eq} - d_P, \tag{App 6i}$$

$$\mathbf{B} = \frac{-\mu_P P_{eq} k_P}{(k_P + N_{eq})^2} - e_Z P_{eq}, \tag{App 6j}$$

$$\mathbf{C} = e_Z(1 - \eta_z) Z_{eq}, \tag{App 6k}$$

$$\mathbf{D} = 0. \tag{App 6l}$$

Model HH

Differential Equations:

$$\frac{dP}{dt} = \mu_P P \left(\frac{N}{N + k_P} \right) - \mu_Z Z \left(\frac{P}{P + k_Z} \right), \tag{App 7a}$$

$$\frac{dZ}{dt} = \mu_Z Z \left(\frac{(1 - \eta_Z)P}{k_Z + P} \right) - d_Z Z, \tag{App 7b}$$

$$\frac{dN}{dt} = d_Z Z + \mu_Z Z \left(\frac{\eta_Z P}{k_Z + P} \right) - \mu_P P \left(\frac{N}{N + k_P} \right). \tag{App 7c}$$

Goal Functions:

Phytoplankton biomass at equilibrium:

$$P_{eq} = \frac{d_Z k_Z}{(1 - \eta_Z)\mu_Z - d_Z}. \tag{App 8a}$$

Zooplankton biomass at equilibrium:

$$Z_{eq} = \left(\frac{k_Z + P}{\mu_Z} \right) \left(\frac{\mu_P N}{k_P + N} - d_P \right), \tag{App 8b}$$

where nutrient at equilibrium is

$$N_{eq} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{App 8c}$$

where:

$$a = -\mu_Z, \tag{App 8d}$$

$$b = \mu_P(k_Z + P_{eq}) + \mu_Z(k_P - N_o + P_{eq}) - d_P(k_Z + P), \tag{App 8e}$$

$$c = -\mu_Z k_P(N_o - P_{eq}) - d_P(k_Z + P). \tag{App 8f}$$

Flux at equilibrium for the DeAngelis (1992) flux formulation:

$$F_{eq} = \mu_Z Z_{eq} \left(\frac{(1-\eta_Z)P_{eq}}{k_Z + P_{eq}} \right) = \mu_P P_{eq} \left(\frac{N_{eq}}{k_P + N_{eq}} \right) - \mu_Z Z_{eq} \left(\frac{\eta_Z P_{eq}}{k_Z + P_{eq}} \right) = d_Z Z_{eq}. \quad \text{(App 8g)}$$

Flux at equilibrium for the Cropp & Gabric (2002) flux formulation:

$$F_{eq} = \mu_Z Z_{eq} \left(\frac{P_{eq}}{k_Z + P_{eq}} \right) = \mu_P P_{eq} \left(\frac{N_{eq}}{k_P + N_{eq}} \right) = d_Z Z_{eq} + \mu_Z Z_{eq} \left(\frac{\eta_Z P_{eq}}{k_Z + P_{eq}} \right). \quad \text{(App 8h)}$$

Flux to biomass ratio at equilibrium:

$$\left(\frac{F}{B} \right)_{eq} = \frac{F_{eq}}{P_{eq} + Z_{eq}}. \quad \text{(App 8i)}$$

Resilience at equilibrium is found from (refer to for details):

$$\mathbf{A} = \frac{\mu_P (N_{eq} - P_{eq})(k_P + N_{eq}) + \mu_P P_{eq} N_{eq}}{(k_P + N_{eq})^2} - \frac{\mu_Z Z_{eq} k_Z}{(k_Z + P_{eq})^2} - d_P, \quad \text{(App 8j)}$$

$$\mathbf{B} = \frac{-\mu_P P_{eq} k_P}{(k_P + N_{eq})^2} - \frac{\mu_Z P_{eq}}{k_Z + P_{eq}}, \quad \text{(App 8k)}$$

$$\mathbf{C} = \frac{\mu_Z Z_{eq} k_Z (1 - \eta_Z)}{(k_Z + P_{eq})^2}, \quad \text{(App 8l)}$$

$$\mathbf{D} = 0. \quad \text{(App 8m)}$$

Finding Resilience.

General Jacobian Matrix for PZN System

The Jacobian Matrix is used to find the resilience of the system. The Jacobian Matrix is described by:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial}{\partial P}\left(\frac{dP}{dt}\right) & \frac{\partial}{\partial Z}\left(\frac{dP}{dt}\right) \\ \frac{\partial}{\partial P}\left(\frac{dZ}{dt}\right) & \frac{\partial}{\partial Z}\left(\frac{dZ}{dt}\right) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}. \tag{App 9}$$

From Equation App 9 it can be said that the eigenvalue, λ, is found by

$$|\mathbf{J} - \lambda I| = 0 = \begin{vmatrix} \frac{\partial}{\partial P}\left(\frac{dP}{dt}\right) - \lambda & \frac{\partial}{\partial Z}\left(\frac{dP}{dt}\right) \\ \frac{\partial}{\partial P}\left(\frac{dZ}{dt}\right) & \frac{\partial}{\partial Z}\left(\frac{dZ}{dt}\right) - \lambda \end{vmatrix}. \tag{App 10}$$

The quadratic equation to be solved from Equation App 10 is

$$0 = \lambda^2 - (\mathbf{A} + \mathbf{D})\lambda + (\mathbf{AD} - \mathbf{BC}), \tag{App 11}$$

for equilibrium values of P and Z.

Note that for complex eigenvalues Equation App 12 must be satisfied

$$(\mathbf{A} + \mathbf{D})^2 - 4(\mathbf{AD} - \mathbf{BC}) < 0, \tag{App 12}$$

for which the resilience, R, is the negative of the real part of the eigenvalues

$$R = -\frac{\mathbf{A} + \mathbf{D}}{2}. \tag{App 13}$$

For real eigenvalues, the larger (most positive) of the two eigenvalues found from the characteristic equation (Equation App 10) is used to find resilience.

Raw Data: Parameter Values

Table 1: Model LL1 and LL2

	Parameter Values									**Goal Function Values**					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model LL1															
Max P_{eq}	0.006	0.003						0.075		25.0	316.7	23.8	0.070	0.0750	No
Max Z_{eq}	0.009	0.003						0.025		8.3	368.8	9.2	0.024	0.0375	No
Max F_{eq}	0.009	0.003						0.075		25.0	356.3	26.7	0.070	0.1125	No
Max $(F/B)_{eq}$	0.009	0.009						0.075		8.3	245.8	18.4	0.073	0.0375	No
Max R_{eq}	0.009	0.003						0.075		25.0	356.3	26.7	0.070	0.1125	No
Min P_{eq}	0.006	0.009						0.025		2.8	198.9	5.0	0.025	0.0083	No
Min Z_{eq}	0.003	0.009						0.075		8.3	122.9	9.2	0.070	0.0125	No
Min F_{eq}	0.003	0.009						0.025		2.8	124.3	3.1	0.024	0.0042	No
Min $(F/B)_{eq}$	0.003	0.003						0.025		8.3	245.8	6.1	0.024	0.0125	No
Min R_{eq}	0.003	0.009						0.025		2.8	124.3	3.1	0.024	0.0042	No
Model LL2															
Max P_{eq}	0.006	0.003					0.0050	0.075		25.0	316.1	23.7	0.070	0.0750	No
Max Z_{eq}	0.009	0.003					0.0025	0.025		8.3	368.5	9.2	0.024	0.0375	No
Max F_{eq}	0.009	0.003					0.0025	0.075		25.0	356.0	26.7	0.070	0.1125	No
Max $(F/B)_{eq}$	0.009	0.009					0.0025	0.075		8.3	245.7	18.4	0.073	0.0375	No
Max R_{eq}	0.009	0.003					0.0050	0.075		25.0	355.8	26.7	0.070	0.1125	No
Min P_{eq}	0.006	0.009					0.005	0.025		2.8	198.6	5.0	0.025	0.0083	No
Min Z_{eq}	0.003	0.009					0.0075	0.075		8.3	122.3	9.2	0.070	0.0125	No
Min F_{eq}	0.003	0.009					0.0075	0.025		2.8	123.7	3.1	0.024	0.0042	No
Min $(F/B)_{eq}$	0.003	0.003					0.0075	0.025		8.3	244.6	6.1	0.024	0.0125	No
Min R_{eq}	0.003	0.009					0.005	0.025		2.8	123.9	3.1	0.024	0.0042	No

Table 2: Model LL3 and LL4

	Parameter Values									**Goal Function Values**					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model LL3															
Max P_{eq}	0.006	0.003						0.075	0.6	62.5	291.7	21.9	0.062	0.1875	No
Max Z_{eq}	0.009	0.003						0.025	0.2	10.4	367.2	9.2	0.024	0.0469	No
Max F_{eq}	0.009	0.003						0.075	0.2	31.3	351.6	26.4	0.069	0.1406	No
Max $(F/B)_{eq}$	0.009	0.009						0.075	0.2	10.4	244.8	18.4	0.072	0.0469	No
Max R_{eq}	0.009	0.003						0.075	0.6	62.5	328.1	24.6	0.063	0.2813	No
Min P_{eq}	0.006	0.009						0.025	0.2	3.5	198.6	5.0	0.025	0.0104	No
Min Z_{eq}	0.003	0.009						0.075	0.6	20.8	119.8	9.0	0.064	0.0313	No
Min F_{eq}	0.003	0.009						0.025	0.6	6.9	123.3	3.1	0.024	0.0104	No
Min $(F/B)_{eq}$	0.003	0.003						0.025	0.6	20.8	239.6	6.0	0.023	0.0313	No
Min R_{eq}	0.003	0.009						0.025	0.2	3.5	124.1	3.1	0.024	0.0052	No
Model LL4															
Max P_{eq}	0.006	0.003						0.075	0.6	62.5	291.7	54.7	0.154	0.1875	No
Max Z_{eq}	0.009	0.003						0.025	0.2	10.4	367.2	11.5	0.030	0.0469	No
Max F_{eq}	0.009	0.003						0.075	0.6	62.5	328.1	61.5	0.158	0.2813	No
Max $(F/B)_{eq}$	0.009	0.009						0.075	0.6	20.8	239.6	44.9	0.173	0.0938	No
Max R_{eq}	0.009	0.003						0.075	0.6	62.5	328.1	61.5	0.158	0.2813	No
Min P_{eq}	0.006	0.009						0.025	0.2	3.5	198.6	6.2	0.031	0.0104	No
Min Z_{eq}	0.003	0.009						0.075	0.6	20.8	119.8	22.5	0.160	0.0313	No
Min F_{eq}	0.003	0.009						0.025	0.2	3.5	124.1	3.9	0.030	0.0052	No
Min $(F/B)_{eq}$	0.003	0.003						0.025	0.2	10.4	244.8	7.6	0.030	0.0156	No
Min R_{eq}	0.003	0.009						0.025	0.2	3.5	124.1	3.9	0.030	0.0052	No

Table 3: Model LL5 and LL6

	Parameter Values									**Goal Function Values**					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model LL5															
Max P_{eq}	0.006	0.003					0.005	0.075	0.6	62.5	291.1	21.8	0.062	0.1875	No
Max Z_{eq}	0.009	0.003					0.0025	0.025	0.2	10.4	367.0	9.2	0.024	0.0469	No
Max F_{eq}	0.009	0.003					0.0025	0.075	0.2	31.3	351.4	26.4	0.069	0.1406	No
Max $(F/B)_{eq}$	0.009	0.009					0.0025	0.075	0.2	10.4	244.7	18.3	0.072	0.0469	No
Max R_{eq}	0.009	0.003					0.005	0.075	0.6	62.5	327.7	24.6	0.063	0.2813	No
Min P_{eq}	0.006	0.009					0.005	0.025	0.2	3.5	198.3	5.0	0.025	0.0104	No
Min Z_{eq}	0.003	0.009					0.0075	0.075	0.6	20.8	119.2	8.9	0.064	0.0313	No
Min F_{eq}	0.003	0.009					0.0075	0.025	0.6	6.9	122.6	3.1	0.024	0.0104	No
Min $(F/B)_{eq}$	0.003	0.003					0.0075	0.025	0.6	20.8	238.3	6.0	0.023	0.0313	No
Min R_{eq}	0.003	0.009					0.005	0.025	0.2	3.5	123.7	3.1	0.024	0.0052	No
Model LL6															
Max P_{eq}	0.006	0.003					0.005	0.075	0.6	62.5	291.1	54.6	0.154	0.1875	No
Max Z_{eq}	0.009	0.003					0.0025	0.025	0.2	10.4	367.0	11.5	0.030	0.0469	No
Max F_{eq}	0.009	0.003					0.0025	0.075	0.6	62.5	327.9	61.5	0.157	0.2813	No
Max $(F/B)_{eq}$	0.009	0.009					0.0025	0.075	0.6	20.8	239.4	44.9	0.172	0.0938	No
Max R_{eq}	0.009	0.003					0.005	0.075	0.6	62.5	327.7	61.4	0.157	0.2813	No
Min P_{eq}	0.006	0.009					0.005	0.025	0.2	3.5	198.3	6.2	0.031	0.0104	No
Min Z_{eq}	0.003	0.009					0.0075	0.075	0.6	20.8	119.2	22.3	0.160	0.0313	No
Min F_{eq}	0.003	0.009					0.0075	0.025	0.2	3.5	123.5	3.9	0.030	0.0052	No
Min $(F/B)_{eq}$	0.003	0.003					0.0075	0.025	0.2	10.4	243.5	7.6	0.030	0.0156	No
Min R_{eq}	0.003	0.009					0.005	0.025	0.2	3.5	123.7	3.9	0.030	0.0052	No

Table 4: Model LH1 and LH2

	Parameter Values									Goal Function Values					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model LH1															
Max P_{eq}	0.006			1.5		3000		0.075		157.9	317.0	23.8	0.050	0.1968	Yes
Max Z_{eq}	0.009			1.5		2040		0.025		34.6	430.8	10.8	0.023	0.1530	No
Max F_{eq}	0.009			3.3		2460		0.075		57.2	386.5	29.0	0.065	0.2517	No
Max $(F/B)_{eq}$	0.009			4.4238		1000		0.075		17.2	325.5	24.4	0.071	0.0656	No
Max R_{eq}	0.009			1.78		2558.2		0.075		112.8	360.6	27.0	0.057	0.5026	Yes
Min P_{eq}	0.006			4.5		1000		0.025		5.6	283.2	7.1	0.025	0.0132	No
Min Z_{eq}	0.003			4.5		1000		0.075		16.9	195.2	14.6	0.069	0.0182	No
Min F_{eq}	0.003			4.5		1000		0.025		5.6	198.4	5.0	0.024	0.0059	No
Min $(F/B)_{eq}$	0.003			1.5		3000		0.025		50.8	385.9	9.6	0.022	0.0747	No
Min R_{eq}	0.003			4.5		1000		0.025		5.6	198.4	5.0	0.024	0.0059	No
Model LH2															
Max P_{eq}	0.006			1.5		3000	0.005	0.075		157.9	316.2	23.7	0.050	0.1962	Yes
Max Z_{eq}	0.009			1.5		2002.9	0.0042	0.025		33.9	430.4	10.8	0.023	0.1501	No
Max F_{eq}	0.009			4.0601		3000	0.0028	0.075		56.5	386.2	29.0	0.065	0.2493	No
Max $(F/B)_{eq}$	0.009			4.5		1000	0.005	0.075		16.9	323.5	24.3	0.071	0.0643	No
Max R_{eq}	0.009			1.93		2792.8	0.005	0.075		112.36	260.4	27.0	0.057	0.4800	Yes
Min P_{eq}	0.006			4.5		1000	0.005	0.025		5.6	282.7	7.1	0.025	0.0132	No
Min Z_{eq}	0.003			4.5		1000	0.0075	0.075		16.9	194.2	14.6	0.069	0.0183	No
Min F_{eq}	0.003			4.5		1000	0.0075	0.025		5.6	197.4	4.9	0.024	0.0059	No
Min $(F/B)_{eq}$	0.003			1.5		3000	0.0075	0.025		50.8	383.8	9.6	0.022	0.0747	No
Min R_{eq}	0.003			4.5		1000	0.0025	0.025		5.6	198.1	5.0	0.024	0.0059	No

Table 5: Model LH3 and LH4

	Parameter Values									Goal Function Values					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model LH3															
Max P_{eq}	0.006			1.5		3000		0.075	0.6	428.6	66.6	5.0	0.010	0.0110	Yes
Max Z_{eq}	0.009			1.5		1819.1593		0.025	0.2	38.7	423.3	10.6	0.023	0.1706	No
Max F_{eq}	0.009			4.5		2841.6422		0.075	0.2	60.5	374.9	28.1	0.065	0.2660	No
Max $(F/B)_{eq}$	0.009			4.5		1000		0.075	0.2	21.3	321.4	24.1	0.070	0.0810	No
Max R_{eq}	0.009			2.8196		2137.8299		0.075	0.5	113.1	339.7	25.5	0.056	0.4883	Yes
Min P_{eq}	0.006			4.5		1000		0.025	0.2	7.0	282.6	7.1	0.024	0.0166	No
Min Z_{eq}	0.003			1.5		3000		0.075	0.6	428.6	62.3	4.7	0.010	0.0111	Yes
Min F_{eq}	0.003			1.5		3000		0.075	0.6	428.6	62.3	4.7	0.010	0.0111	Yes
Min $(F/B)_{eq}$	0.003			1.5		3000		0.075	0.6	428.6	62.3	4.7	0.010	0.0111	Yes
Min R_{eq}	0.003			4.5		1000		0.025	0.2	7.0	198.0	5.0	0.024	0.0074	No
Model LH4															
Max P_{eq}	0.006			1.5		3000		0.075	0.6	428.6	66.6	5.0	0.010	0.0110	Yes
Max Z_{eq}	0.009			1.5		1819.1593		0.025	0.2	38.7	423.3	13.2	0.029	0.1706	No
Max F_{eq}	0.009			3.4765		1486.8035		0.075	0.6	84.8	333.3	62.5	0.149	0.3615	No
Max $(F/B)_{eq}$	0.009			4.5		1000		0.075	0.6	43.5	308.6	57.9	0.164	0.1679	No
Max R_{eq}	0.009			2.8196		2137.8299		0.075	0.5	113.1	339.7	25.5	0.056	0.4883	Yes
Min P_{eq}	0.006			4.5		1000		0.025	0.2	7.0	282.6	8.8	0.030	0.0166	No
Min Z_{eq}	0.003			1.5		3000		0.075	0.6	428.6	62.3	11.7	0.024	0.0111	Yes
Min F_{eq}	0.003			4.5		1000		0.025	0.2	7.0	198.0	6.2	0.030	0.0074	No
Min $(F/B)_{eq}$	0.003			1.5		3000		0.075	0.6	428.6	62.3	11.7	0.024	0.0111	Yes
Min R_{eq}	0.003			4.5		1000		0.025	0.2	7.0	198.0	6.2	0.030	0.0074	No

Table 6: Model LH5 and LH6

	Parameter Values									**Goal Function Values**					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model LH5															
Max P_{eq}	0.006			1.5		3000	0.005	0.075	0.6	428.6	65.8	4.9	0.010	0.0109	Yes
Max Z_{eq}	0.009			1.544		1897.3607	0.0025	0.025	0.2	39.2	423.0	10.6	0.023	0.1730	No
Max F_{eq}	0.009			3.9751		2626.5885	0.0025	0.075	0.2	63.4	374.7	28.1	0.064	0.2790	No
Max $(F/B)_{eq}$	0.009			4.5		1000	0.0025	0.075	0.2	21.3	321.2	24.1	0.070	0.0810	No
Max R_{eq}	0.009			4.0337		2589.4428	0.005	0.075	0.56	113.3	331.2	24.8	0.056	0.4987	Yes
Min P_{eq}	0.006			4.5		1000	0.005	0.025	0.2	7.0	282.1	7.1	0.024	0.0166	No
Min Z_{eq}	0.003			1.5		3000	0.0075	0.075	0.6	428.6	60.2	4.5	0.009	0.0107	Yes
Min F_{eq}	0.003			1.5		3000	0.0075	0.075	0.6	428.6	60.2	4.5	0.009	0.0107	Yes
Min $(F/B)_{eq}$	0.003			1.5		3000	0.0075	0.075	0.6	428.6	60.2	4.5	0.009	0.0107	Yes
Min R_{eq}	0.003			4.5		1000	0.0025	0.025	0.2	7.0	197.7	4.9	0.024	0.0074	No
Model LH6															
Max P_{eq}	0.006			1.5		3000	0.005	0.075	0.6	428.6	65.8	12.3	0.025	2.1855	Yes
Max Z_{eq}	0.009			1.544		1897.3607	0.0025	0.025	0.2	39.2	423.0	13.2	0.029	0.1730	No
Max F_{eq}	0.009			4.5		2000	0.0025	0.075	0.6	87.0	333.0	62.4	0.149	0.3763	No
Max $(F/B)_{eq}$	0.009			4.5		1000	0.0025	0.075	0.6	43.5	308.4	57.8	0.164	0.1679	No
Max R_{eq}	0.009			4.0337		2589.4428	0.005	0.075	0.56	113.3	331.2	24.8	0.056	0.4987	Yes
Min P_{eq}	0.006			4.5		1000	0.005	0.025	0.2	7.0	282.1	8.8	0.030	0.0166	No
Min Z_{eq}	0.003			1.5		3000	0.0075	0.075	0.6	428.6	60.2	11.3	0.023	0.0107	Yes
Min F_{eq}	0.003			4.5		1000	0.0075	0.025	0.2	7.0	197.0	6.2	0.030	0.0074	No
Min $(F/B)_{eq}$	0.003			1.5		3000	0.0075	0.075	0.6	428.6	60.2	11.3	0.023	0.0107	Yes
Min R_{eq}	0.003			4.5		1000	0.0025	0.025	0.2	7.0	197.7	6.2	0.030	0.0074	No

Table 7: Model HL1 and HL2

	Parameter Values									**Goal Function Values**					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model HL1															
Max P_{eq}		0.003	3		277			0.075		25.0	335.3	25.1	0.070	0.0598	No
Max Z_{eq}		0.003	4.5		138.5			0.025		8.3	435.1	10.9	0.025	0.0682	No
Max F_{eq}		0.003	4.5		138.5			0.075		25.0	421.0	31.6	0.071	0.2102	No
Max $(F/B)_{eq}$		0.009	4.5		138.5			0.075		8.3	294.0	22.1	0.073	0.0230	No
Max R_{eq}		0.003	4.5		138.5			0.075		25.0	421.0	31.6	0.071	0.2102	No
Min P_{eq}		0.009	3		277			0.025		2.8	178.4	4.5	0.025	0.0033	No
Min Z_{eq}		0.009	1.5		415.5			0.075		8.3	82.7	6.2	0.068	0.0038	No
Min F_{eq}		0.009	1.5		415.5			0.025		2.8	83.2	2.1	0.024	0.0013	No
Min $(F/B)_{eq}$		0.003	1.5		415.5			0.025		8.3	204.4	5.1	0.024	0.0053	No
Min R_{eq}		0.009	1.5		138.5			0.025		2.8	121.8	3.0	0.024	0.0011	No
Model HL2															
Max P_{eq}		0.003	3		277		0.005	0.075		25.0	334.6	25.1	0.070	0.0596	No
Max Z_{eq}		0.003	4.5		138.5		0.0025	0.025		8.3	435.0	10.9	0.025	0.0681	No
Max F_{eq}		0.003	4.5		138.5		0.0025	0.075		25.0	420.8	31.6	0.071	0.2099	No
Max $(F/B)_{eq}$		0.009	4.5		138.5		0.0025	0.075		8.3	293.8	22.0	0.073	0.0230	No
Max R_{eq}		0.003	4.5		138.5		0.0025	0.075		25.0	420.8	31.6	0.071	0.2099	No
Min P_{eq}		0.009	3		277		0.005	0.025		2.8	177.9	4.4	0.025	0.0032	No
Min Z_{eq}		0.009	1.5		415.5		0.0075	0.075		8.3	81.9	6.1	0.068	0.0038	No
Min F_{eq}		0.009	1.5		415.5		0.0075	0.025		2.8	82.4	2.1	0.024	0.0013	No
Min $(F/B)_{eq}$		0.003	1.5		415.5		0.0075	0.025		8.3	202.6	5.1	0.024	0.0052	No
Min R_{eq}		0.009	1.5		138.5		0.0075	0.025		2.8	121.0	3.0	0.024	0.0011	No

Table 8: Model HL3 and HL4

	Parameter Values									Goal Function Values					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model HL3															
Max P_{eq}		0.003	3		277			0.075	0.6	62.5	311.9	23.4	0.062	0.1602	No
Max Z_{eq}		0.003	4.5		138.5			0.025	0.2	10.4	433.3	10.8	0.024	0.0856	No
Max F_{eq}		0.003	4.5		138.5			0.075	0.2	31.3	415.7	31.2	0.070	0.2653	No
Max $(F/B)_{eq}$		0.009	4.5		138.5			0.075	0.2	10.4	293.2	22.0	0.072	0.0289	No
Max R_{eq}		0.003	4.5		138.5			0.075	0.6	62.5	389.0	29.2	0.065	0.5570	No
Min P_{eq}		0.009	3		277			0.025	0.2	3.5	178.2	4.5	0.025	0.0041	No
Min Z_{eq}		0.009	1.5		415.5			0.075	0.6	20.8	81.5	6.1	0.060	0.0098	No
Min F_{eq}		0.009	1.5		415.5			0.025	0.6	6.9	82.8	2.1	0.023	0.0032	No
Min $(F/B)_{eq}$		0.003	1.5		415.5			0.025	0.6	20.8	200.7	5.0	0.023	0.0135	No
Min R_{eq}		0.009	1.5		138.5			0.025	0.2	3.5	121.7	3.0	0.024	0.0014	No
Model HL4															
Max P_{eq}		0.003	3		277			0.075	0.6	62.5	311.9	58.5	0.156	0.1602	No
Max Z_{eq}		0.003	4.5		138.5			0.025	0.2	10.4	433.3	13.5	0.031	0.0856	No
Max F_{eq}		0.003	4.5		138.5			0.075	0.6	62.5	389.0	72.9	0.162	0.5570	No
Max $(F/B)_{eq}$		0.009	4.5		138.5			0.075	0.6	20.8	289.2	54.2	0.175	0.0602	No
Max R_{eq}		0.003	4.5		138.5			0.075	0.6	62.5	389.0	72.9	0.162	0.5570	No
Min P_{eq}		0.009	3		277			0.025	0.2	3.5	178.2	5.6	0.031	0.0041	No
Min Z_{eq}		0.009	1.5		415.5			0.075	0.6	20.8	81.5	15.3	0.149	0.0098	No
Min F_{eq}		0.009	1.5		415.5			0.025	0.2	3.5	83.1	2.6	0.030	0.0016	No
Min $(F/B)_{eq}$		0.003	1.5		415.5			0.025	0.2	10.4	203.8	6.4	0.030	0.0066	No
Min R_{eq}		0.009	1.5		138.5			0.025	0.2	3.5	121.7	3.8	0.030	0.0014	No

Table 9: Model HL5 and HL6

	Parameter Values									Goal Function Values					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model HL5															
Max P_{eq}		0.003	3		277		0.005	0.075	0.6	62.5	311.3	23.3	0.062	0.1597	No
Max Z_{eq}		0.003	4.5		138.5		0.0025	0.025	0.2	10.4	433.2	10.8	0.024	0.0855	No
Max F_{eq}		0.003	4.5		138.5		0.0025	0.075	0.2	31.3	415.5	31.2	0.070	0.2650	No
Max $(F/B)_{eq}$		0.009	4.5		138.5		0.0025	0.075	0.2	10.4	293.0	22.0	0.072	0.0289	No
Max R_{eq}		0.003	4.5		138.5		0.0025	0.075	0.6	62.5	388.9	29.2	0.065	0.5563	No
Min P_{eq}		0.009	3		277		0.005	0.025	0.2	3.5	177.8	4.4	0.025	0.0041	No
Min Z_{eq}		0.009	1.5		415.5		0.0075	0.075	0.6	20.8	80.8	6.1	0.060	0.0098	No
Min F_{eq}		0.009	1.5		415.5		0.0075	0.025	0.6	6.9	82.0	2.1	0.023	0.0032	No
Min $(F/B)_{eq}$		0.003	1.5		415.5		0.0075	0.025	0.6	20.8	198.9	5.0	0.023	0.0134	No
Min R_{eq}		0.009	1.5		138.5		0.0075	0.025	0.2	3.5	120.9	3.0	0.024	0.0014	No
Model HL6															
Max P_{eq}		0.003	3		277		0.005	0.075	0.6	62.5	311.3	58.4	0.156	0.1597	No
Max Z_{eq}		0.003	4.5		138.5		0.0025	0.025	0.2	10.4	433.2	13.5	0.031	0.0855	No
Max F_{eq}		0.003	4.5		138.5		0.0025	0.075	0.6	62.5	388.9	72.9	0.162	0.5563	No
Max $(F/B)_{eq}$		0.009	4.5		138.5		0.0025	0.075	0.6	20.8	289.0	54.2	0.175	0.0601	No
Max R_{eq}		0.003	4.5		138.5		0.0025	0.075	0.6	62.5	388.9	72.9	0.162	0.5563	No
Min P_{eq}		0.009	3		277		0.005	0.025	0.2	3.5	177.8	5.6	0.031	0.0041	No
Min Z_{eq}		0.009	1.5		415.5		0.0075	0.075	0.6	20.8	80.8	15.1	0.149	0.0098	No
Min F_{eq}		0.009	1.5		415.5		0.0075	0.025	0.2	3.5	82.4	2.6	0.030	0.0016	No
Min $(F/B)_{eq}$		0.003	1.5		415.5		0.0075	0.025	0.2	10.4	202.0	6.3	0.030	0.0066	No
Min R_{eq}		0.009	1.5		138.5		0.0075	0.025	0.2	3.5	120.9	3.8	0.030	0.0014	No

Table 10: Model HH1 and HH2

	Parameter Values									**Goal Function Values**					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model HH1															
Max P_{eq}			3	1.5	277	3000		0.075		157.9	327.0	24.5	0.051	0.1757	Yes
Max Z_{eq}			4.5	1.5	138.5	1142.71		0.025		19.4	459.6	11.5	0.024	0.23	No
Max F_{eq}			4.5	3.2625	138.5	1437.92		0.075		33.8	429.0	32.2	0.070	0.33	No
Max $(F/B)_{eq}$			4.5	4.5	138.5	1000		0.075		16.9	395.1	29.6	0.072	0.08	No
Max R_{eq}			4.5	2.9164	144.2	2689.15		0.075		70.98	413.5	31.0	0.064	0.855	Yes
Min P_{eq}			3	4.5	277	1000		0.025		5.6	287.0	7.2	0.025	0.0063	No
Min Z_{eq}			1.5	4.5	415.5	1000		0.075		16.9	150.7	11.3	0.067	0.0039	No
Min F_{eq}			1.5	4.5	415.5	1000		0.025		5.6	151.5	3.8	0.024	0.0011	No
Min $(F/B)_{eq}$			1.5	1.5	415.5	3000		0.025		50.8	388.5	9.7	0.022	0.068	No
Min R_{eq}			1.5	4.5	138.5	1000		0.025		5.6	222.2	5.6	0.024	0.0007	No
Model HH2															
Max P_{eq}			3	1.5	277	3000	0.005	0.075		157.9	326.5	24.5	0.051	0.1755	Yes
Max Z_{eq}			4.5	4.5	138.5	2638.31	0.0025	0.025		14.7	456.4	11.4	0.024	0.16	No
Max F_{eq}			4.5	4.5	138.5	2200.39	0.0025	0.075		37.3	429.7	32.2	0.069	0.38	No
Max $(F/B)_{eq}$			4.5	4.5	138.5	1000	0.0025	0.075		16.9	394.9	29.6	0.072	0.0883	No
Max R_{eq}			4.5	2.2	147.7	2040.08	0.005	0.075		71.7	412.4	30.8	0.064	0.8820	Yes
Min P_{eq}			3	4.5	277	1000	0.005	0.025		5.6	286.4	7.2	0.025	0.0063	No
Min Z_{eq}			1.5	4.5	415.5	1000	0.0075	0.075		16.9	149.3	11.2	0.067	0.0039	No
Min F_{eq}			1.5	4.5	415.5	1000	0.0075	0.025		5.6	150.2	3.8	0.024	0.0011	No
Min $(F/B)_{eq}$			1.5	1.5	415.5	3000	0.0075	0.025		50.8	386.2	9.7	0.022	0.067	No
Min R_{eq}			1.5	4.5	138.5	1000	0.0075	0.025		5.6	220.9	5.5	0.024	0.0007	No

Table 11: Model HH3 and HH4

	Parameter Values									Goal Function Values					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model HH3															
Max P_{eq}			3	1.5	277	3000		0.075	0.6	428.6	68.6	5.1	0.010	0.0110	Yes
Max Z_{eq}			4.5	2.9282	138.5	2317.6931		0.025	0.2	25.0	455.0	11.4	0.024	0.3070	No
Max F_{eq}			4.5	3.6554	138.5	1480.9384		0.075	0.209	39.4	420.4	31.5	0.069	0.3721	No
Max $(F/B)_{eq}$			4.5	4.5	138.5	1000		0.075	0.2	21.3	392.3	29.4	0.071	0.1131	No
Max R_{eq}			4.5	3.7791	138.5	2874.8878		0.0678	0.227	68.28	413.0	28.0	0.058	0.8409	Yes
Min P_{eq}			3	4.5	277	1000		0.025	0.2	7.0	286.6	7.2	0.024	0.0080	No
Min Z_{eq}			1.5	1.5	415.5	3000		0.075	0.6	428.6	63.6	4.8	0.010	0.0110	Yes
Min F_{eq}			1.5	4.5	415.5	1000		0.025	0.6	14.1	150.9	3.8	0.023	0.0031	No
Min $(F/B)_{eq}$			1.5	1.5	415.5	3000		0.075	0.6	428.6	63.6	4.8	0.010	0.0110	Yes
Min R_{eq}			1.5	4.5	138.5	1000		0.025	0.2	7.0	222.1	5.6	0.024	0.0009	No
Model HH4															
Max P_{eq}			3	1.5	277	3000		0.075	0.6	428.6	68.6	12.9	0.026	0.0110	Yes
Max Z_{eq}			4.5	2.9282	138.5	2317.6931		0.025	0.2	25.0	455.0	14.2	0.030	0.3070	No
Max F_{eq}			4.5	4.1422	138.5	1375.3666		0.075	0.6	65.2	389.0	72.9	0.161	0.5729	No
Max $(F/B)_{eq}$			4.5	4.5	138.5	1000		0.075	0.6	43.5	377.9	70.9	0.168	0.2534	No
Max R_{eq}			4.5	3.7791	138.5	2874.8878		0.0678	0.227	68.28	413.0	28.0	0.058	0.8409	Yes
Min P_{eq}			3	4.5	277	1000		0.025	0.2	7.0	286.6	9.0	0.031	0.0080	No
Min Z_{eq}			1.5	1.5	415.5	3000		0.075	0.6	428.6	63.6	11.9	0.024	0.0111	Yes
Min F_{eq}			1.5	4.5	415.5	1000		0.025	0.2	7.0	151.4	4.7	0.030	0.0015	No
Min $(F/B)_{eq}$			1.5	1.5	415.5	3000		0.075	0.6	428.6	63.6	11.9	0.024	0.0111	Yes
Min R_{eq}			1.5	4.5	138.5	1000		0.025	0.2	7.0	222.1	6.9	0.030	0.0009	No

Table 12: Model HH5 and HH6

	Parameter Values									Goal Function Values					
Goal	e_P	e_Z	μ_P	μ_Z	k_P	k_Z	d_P	d_Z	η_Z	P_{eq}	Z_{eq}	F_{eq}	$(F/B)_{eq}$	R_{eq}	$R_{eq}\mathbb{R}$?
Model HH5															
Max P_{eq}			3	1.5	277	3000	0.005	0.075	0.6	428.6	68.2	5.1	0.010	0.0109	Yes
Max Z_{eq}			4.5	2.02	138.5	2364.61	0.0025	0.025	0.2	37.0	450.1	11.3	0.023	0.50	No
Max F_{eq}			4.5	2.7522	138.5	2186.70	0.0025	0.075	0.2	77.1	405.7	30.4	0.063	0.5647	Yes
Max $(F/B)_{eq}$			4.5	4.5	138.5	1000	0.0075	0.075	0.2	21.3	391.9	29.4	0.071	0.11	No
Max R_{eq}			4.5	3.7522	138.5	2491.69	0.0075	0.072	0.310	71.0	407.5	29.2	0.061	0.8522	Yes
Min P_{eq}			3	4.5	277	1000	0.005	0.025	0.2	7.0	286.0	7.2	0.024	0.0080	No
Min Z_{eq}			1.5	1.5	415.5	3000	0.0075	0.075	0.6	428.6	61.7	4.6	0.009	0.0107	Yes
Min F_{eq}			1.5	4.5	415.5	1000	0.0075	0.025	0.6	14.1	149.5	3.7	0.023	0.0032	No
Min $(F/B)_{eq}$			1.5	1.5	415.5	3000	0.0075	0.075	0.6	428.6	61.7	4.6	0.009	0.0107	Yes
Min R_{eq}			1.5	4.5	138.5	1000	0.0025	0.025	0.2	7.0	221.7	5.5	0.024	0.0009	No
Model HH6															
Max P_{eq}			3	1.5	277	3000	0.005	0.075	0.6	428.6	68.2	12.8	0.026	4.51	Yes
Max Z_{eq}			4.5	2.02	138.5	2364.61	0.0025	0.025	0.2	37.0	450.1	14.1	0.029	0.50	No
Max F_{eq}			4.5	3.71	138.5	1938.41	0.0025	0.075	0.6	103.1	372.3	69.8	0.147	2.05	No
Max $(F/B)_{eq}$			4.5	4.5	138.5	1000	0.0025	0.075	0.6	43.5	377.7	70.8	0.168	0.25	No
Max R_{eq}			4.5	3.7522	138.5	2491.69	0.0075	0.072	0.310	71.0	407.5	29.2	0.061	0.8522	Yes
Min P_{eq}			3	4.5	277	1000	0.005	0.025	0.309	8.1	285.7	10.3	0.035	0.009	No
Min Z_{eq}			1.5	1.5	415.5	3000	0.0075	0.075	0.6	428.6	61.7	11.6	0.024	1.46	Yes
Min F_{eq}			1.5	4.5	415.5	1000	0.0075	0.025	0.2	7.0	150.1	4.7	0.030	0.0015	No
Min $(F/B)_{eq}$			1.5	1.5	415.5	3000	0.0075	0.075	0.6	428.6	61.7	11.6	0.024	1.46	Yes
Min R_{eq}			1.5	4.5	138.5	1000	0.0025	0.025	0.2	7.0	221.7	6.9	0.030	0.0009	No

Raw Data: Normalized Goal Function Values

Table 13: Normalized Goal Function Values. Model LL

Goal Maximized	**Goal Normalized**					
	P	Z	F	(F/B)	R	$-(F/B)$
Model LL1						
P_{eq}	1.00	0.79	0.87	0.94	0.65	0.06
Z_{eq}	0.25	1.00	0.26	0.01	0.31	0.99
F_{eq}	1.00	0.95	1.00	0.95	1.00	0.05
$(F/B)_{eq}$	0.25	0.50	0.65	1.00	0.31	0.00
R_{eq}	1.00	0.95	1.00	0.95	1.00	0.05
$-(F/B)_{eq}$	0.25	0.50	0.13	0.00	0.08	1.00
Model LL2						
P_{eq}	1.00	0.79	0.87	0.94	0.65	0.06
Z_{eq}	0.25	1.00	0.26	0.01	0.31	0.99
F_{eq}	1.00	0.95	1.00	0.95	1.00	0.05
$(F/B)_{eq}$	0.25	0.50	0.65	1.00	0.31	0.00
R_{eq}	1.00	0.95	1.00	0.95	1.00	0.05
$-(F/B)_{eq}$	0.25	0.50	0.13	0.00	0.08	1.00
Model LL3						
P_{eq}	1.00	0.69	0.81	0.79	0.66	0.21
Z_{eq}	0.12	1.00	0.26	0.03	0.15	0.97
F_{eq}	0.47	0.94	1.00	0.94	0.49	0.06
$(F/B)_{eq}$	0.12	0.51	0.66	1.00	0.15	0.00
R_{eq}	1.00	0.84	0.92	0.82	1.00	0.18
$-(F/B)_{eq}$	0.29	0.48	0.12	0.00	0.09	1.00
Model LL4						
P_{eq}	1.00	0.69	0.88	0.87	0.66	0.13
Z_{eq}	0.12	1.00	0.13	0.00	0.15	1.00
F_{eq}	1.00	0.84	1.00	0.89	1.00	0.11
$(F/B)_{eq}$	0.29	0.48	0.71	1.00	0.32	0.00
R_{eq}	1.00	0.84	1.00	0.89	1.00	0.11
$-(F/B)_{eq}$	0.12	0.51	0.07	0.00	0.04	1.00
Model LL5						
P_{eq}	1.00	0.69	0.81	0.79	0.66	0.21
Z_{eq}	0.12	1.00	0.26	0.03	0.15	0.97
F_{eq}	0.47	0.94	1.00	0.94	0.49	0.06
$(F/B)_{eq}$	0.12	0.51	0.66	1.00	0.15	0.00
R_{eq}	1.00	0.84	0.92	0.82	1.00	0.18
$-(F/B)_{eq}$	0.29	0.48	0.12	0.00	0.09	1.00
Model LL6						
P_{eq}	1.00	0.69	0.88	0.87	0.66	0.13
Z_{eq}	0.12	1.00	0.13	0.00	0.15	1.00
F_{eq}	1.00	0.84	1.00	0.89	1.00	0.11
$(F/B)_{eq}$	0.29	0.49	0.71	1.00	0.32	0.00
R_{eq}	1.00	0.84	1.00	0.89	1.00	0.11
$-(F/B)_{eq}$	0.12	0.50	0.07	0.00	0.04	1.00

Table 14: Normalized Goal Function Values. Model LH

Goal Maximized	**Goal Normalized**					
	P	Z	F	(F/B)	R	$-(F/B)$
Model LH1						
P_{eq}	1.00	0.52	0.78	0.57	0.38	0.43
Z_{eq}	0.19	1.00	0.24	0.02	0.30	0.98
F_{eq}	0.34	0.81	1.00	0.88	0.49	0.12
$(F/B)_{eq}$	0.08	0.55	0.81	1.00	0.12	0.00
R_{eq}	0.70	0.70	0.92	0.71	1.00	0.29
$-(F/B)_{eq}$	0.30	0.81	0.20	0.00	0.14	1.00
Model LH2						
P_{eq}	1.00	0.52	0.78	0.57	0.40	0.43
Z_{eq}	0.19	1.00	0.24	0.02	0.30	0.98
F_{eq}	0.33	0.81	1.00	0.88	0.51	0.12
$(F/B)_{eq}$	0.07	0.55	0.80	1.00	0.12	0.00
R_{eq}	0.70	0.70	0.92	0.71	1.00	0.29
$-(F/B)_{eq}$	0.30	0.80	0.19	0.00	0.15	1.00
Model LH3						
P_{eq}	1.00	0.01	0.01	0.01	0.01	0.99
Z_{eq}	0.08	1.00	0.25	0.22	0.34	0.78
F_{eq}	0.13	0.87	1.00	0.91	0.54	0.09
$(F/B)_{eq}$	0.03	0.72	0.83	1.00	0.15	0.00
R_{eq}	0.25	0.77	0.89	0.77	1.00	0.23
$-(F/B)_{eq}$	1.00	0.00	0.00	0.00	0.01	1.00
Model LH4						
P_{eq}	1.00	0.01	0.11	0.01	0.01	0.99
Z_{eq}	0.08	1.00	0.13	0.03	0.34	0.97
F_{eq}	0.18	0.75	1.00	0.89	0.74	0.11
$(F/B)_{eq}$	0.09	0.68	0.92	1.00	0.33	0.00
R_{eq}	0.25	0.77	0.74	0.59	1.00	0.41
$-(F/B)_{eq}$	1.00	0.00	0.10	0.00	0.01	1.00
Model LH5						
P_{eq}	1.00	0.02	0.02	0.01	0.01	0.99
Z_{eq}	0.08	1.00	0.26	0.22	0.34	0.78
F_{eq}	0.13	0.87	1.00	0.90	0.55	0.10
$(F/B)_{eq}$	0.03	0.72	0.83	1.00	0.15	0.00
R_{eq}	0.25	0.75	0.86	0.76	1.00	0.24
$-(F/B)_{eq}$	1.00	0.00	0.00	0.00	0.01	1.00
Model LH6						
P_{eq}	1.00	0.02	0.11	0.01	0.01	0.99
Z_{eq}	0.08	1.00	0.13	0.04	0.34	0.96
F_{eq}	0.19	0.75	1.00	0.89	0.75	0.11
$(F/B)_{eq}$	0.09	0.68	0.92	1.00	0.33	0.00
R_{eq}	0.25	0.75	0.89	0.73	1.00	0.27
$-(F/B)_{eq}$	1.00	0.00	0.09	0.00	0.01	1.00

Table 15: Normalized Goal Function Values. Model HL

Goal Maximized	**Goal Normalized**					
	P	Z	F	(F/B)	R	$-(F/B)$
Model HL1						
P_{eq}	1.00	0.72	0.78	0.94	0.28	0.06
Z_{eq}	0.25	1.00	0.30	0.01	0.32	0.99
F_{eq}	1.00	0.96	1.00	0.96	1.00	0.04
$(F/B)_{eq}$	0.25	0.60	0.68	1.00	0.10	0.00
R_{eq}	1.00	0.96	1.00	0.96	1.00	0.04
$-(F/B)_{eq}$	0.25	0.35	0.10	0.00	0.02	1.00
Model HL2						
P_{eq}	1.00	0.72	0.78	0.94	0.28	0.06
Z_{eq}	0.25	1.00	0.30	0.01	0.32	0.99
F_{eq}	1.00	0.96	1.00	0.96	1.00	0.04
$(F/B)_{eq}$	0.25	0.60	0.68	1.00	0.10	0.00
R_{eq}	1.00	0.96	1.00	0.96	1.00	0.04
$-(F/B)_{eq}$	0.25	0.34	0.10	0.00	0.02	1.00
Model HL3						
P_{eq}	1.00	0.65	0.73	0.80	0.29	0.20
Z_{eq}	0.12	1.00	0.30	0.04	0.15	0.96
F_{eq}	0.47	0.95	1.00	0.95	0.48	0.05
$(F/B)_{eq}$	0.12	0.60	0.68	1.00	0.05	0.00
R_{eq}	1.00	0.87	0.93	0.84	1.00	0.16
$-(F/B)_{eq}$	0.29	0.34	0.10	0.00	0.02	1.00
Model HL4						
P_{eq}	1.00	0.65	0.79	0.87	0.29	0.13
Z_{eq}	0.12	1.00	0.16	0.01	0.15	0.99
F_{eq}	1.00	0.87	1.00	0.91	1.00	0.09
$(F/B)_{eq}$	0.29	0.59	0.73	1.00	0.11	0.00
R_{eq}	1.00	0.87	1.00	0.91	1.00	0.09
$-(F/B)_{eq}$	0.12	0.35	0.05	0.00	0.01	1.00
Model HL5						
P_{eq}	1.00	0.65	0.73	0.80	0.29	0.20
Z_{eq}	0.12	1.00	0.30	0.04	0.15	0.96
F_{eq}	0.47	0.95	1.00	0.95	0.47	0.05
$(F/B)_{eq}$	0.12	0.60	0.68	1.00	0.05	0.00
R_{eq}	1.00	0.87	0.93	0.84	1.00	0.16
$-(F/B)_{eq}$	0.29	0.34	0.10	0.00	0.02	1.00
Model HL6						
P_{eq}	1.00	0.65	0.79	0.87	0.29	0.13
Z_{eq}	0.12	1.00	0.16	0.01	0.15	0.99
F_{eq}	1.00	0.87	1.00	0.91	1.00	0.09
$(F/B)_{eq}$	0.29	0.59	0.73	1.00	0.11	0.00
R_{eq}	1.00	0.87	1.00	0.91	1.00	0.09
$-(F/B)_{eq}$	0.12	0.34	0.05	0.00	0.01	1.00

Table 16: Normalized Goal Function Values. Model HH

Goal Maximized	**Goal Normalized**					
	P	Z	F	(F/B)	R	$-(F/B)$
Model HH1						
P_{eq}	1.00	0.57	0.73	0.57	0.20	0.43
Z_{eq}	0.09	1.00	0.27	0.04	0.27	0.96
F_{eq}	0.19	0.90	1.00	0.95	0.39	0.05
$(F/B)_{eq}$	0.07	0.79	0.91	1.00	0.10	0.00
R_{eq}	0.43	0.85	0.96	0.84	1.00	0.16
$-(F/B)_{eq}$	0.30	0.77	0.21	0.00	0.08	1.00
Model HH2						
P_{eq}	1.00	0.58	0.73	0.57	0.20	0.43
Z_{eq}	0.06	1.00	0.27	0.04	0.18	0.96
F_{eq}	0.21	0.91	1.00	0.94	0.44	0.06
$(F/B)_{eq}$	0.07	0.80	0.91	1.00	0.10	0.00
R_{eq}	0.43	0.86	0.95	0.83	1.00	0.17
$-(F/B)_{eq}$	0.30	0.77	0.21	0.00	0.08	1.00
Model HH3						
P_{eq}	1.00	0.01	0.05	0.01	0.01	0.99
Z_{eq}	0.04	1.00	0.27	0.23	0.36	0.77
F_{eq}	0.08	0.91	1.00	0.96	0.44	0.04
$(F/B)_{eq}$	0.03	0.84	0.92	1.00	0.13	0.00
R_{eq}	0.15	0.89	0.87	0.79	1.00	0.21
$-(F/B)_{eq}$	1.00	0.00	0.04	0.00	0.01	1.00
Model HH4						
P_{eq}	1.00	0.01	0.12	0.01	0.01	0.99
Z_{eq}	0.04	1.00	0.14	0.04	0.36	0.96
F_{eq}	0.14	0.83	1.00	0.95	0.68	0.05
$(F/B)_{eq}$	0.09	0.80	0.97	1.00	0.30	0.00
R_{eq}	0.15	0.89	0.46	0.35	1.00	0.65
$-(F/B)_{eq}$	1.00	0.00	0.11	0.00	0.01	1.00
Model HH5						
P_{eq}	1.00	0.02	0.05	0.01	0.01	0.99
Z_{eq}	0.07	1.00	0.28	0.22	0.59	0.78
F_{eq}	0.17	0.89	1.00	0.87	0.66	0.13
$(F/B)_{eq}$	0.03	0.85	0.96	1.00	0.13	0.00
R_{eq}	0.15	0.89	0.95	0.84	1.00	0.16
$-(F/B)_{eq}$	1.00	0.00	0.03	0.00	0.01	1.00
Model HH6						
P_{eq}	1.00	0.02	0.12	0.01	0.01	0.99
Z_{eq}	0.07	1.00	0.14	0.04	0.59	0.96
F_{eq}	0.23	0.80	1.00	0.85	0.38	0.15
$(F/B)_{eq}$	0.08	0.81	1.02	1.00	0.30	0.00
R_{eq}	0.15	0.89	0.58	0.45	1.00	0.55
$-(F/B)_{eq}$	1.00	0.00	0.11	0.00	0.01	1.00