Progressive Collapse Resistance Demand of Reinforced Concrete Frames under Catenary Mechanism

by Yi Li, Xinzheng Lu, Hong Guan, and Lieping Ye

Progressive collapses are resisted by the catenary mechanism in reinforced concrete (RC) frame structures undergoing large deformations. Research to date has mainly focused on the nonlinear dynamic progressive collapse resistance demand of this type of structures under the beam mechanism (that is, for small deformations), and that the catenary mechanism is lacking. As a first attempt, this study establishes a dynamic amplification factor for evaluating the resistance demands of RC frames under the catenary mechanism. To achieve this, an energy-based, theoretical framework is proposed for calculating the aforementioned demands. Based on this framework, the analytical solution for the collapse resistance demands of regular RC frames under the catenary mechanism is readily obtained. Numerical validation indicates that the proposed equations can accurately describe the progressive collapse demand of RC frames undergoing large deformations.

Keywords: catenary mechanism; energy conservation principle; progressive collapse; reinforced concrete frame; resistance demand.

INTRODUCTION

Progressive collapse of building structures is defined as local damage caused by an accidental event propagating throughout the entire structural system. In the past few decades, progressive collapse events triggered by gas explosions, bombing attacks, fire, and vehicular collisions have been continuously reported. Extensive research has been conducted to investigate and improve progressive collapse resistance of building structures.

Existing experimental and numerical investigations have demonstrated that progressive collapses of reinforced concrete (RC) frame structures are resisted by end moments of the beams for small deformations (namely, the beam mechanism) and by axial tensile forces in the beams for large deformations (namely, the catenary mechanism). The latter is a one-way load-transfer mechanism where the external load is resisted by the beams in one direction. When the frame structures undergo large deformations, cast-in-place RC floor slabs exhibit a two-way membrane tension effect in which the load transfer in each direction is equivalent to a catenary mechanism. The catenary mechanism is the prototype model of the tie force (TF) method adopted in design codes, by which the axial force demand of beams (in one direction) and floor slabs (in two directions) is calculated. Progressive collapse of an RC frame structure can be effectively prevented through catenary action, which enhances the structural integrity after a part of the structure has become a mechanism. Note in the existing TF method that only the nonlinear static (NS) demand under the catenary mechanism is considered. The dynamic effect of the actual progressive collapse process is, however, neglected, and this may lead to an unsafe design.

The nonlinear dynamic (ND) demand is a more realistic representation of the progressive collapse resistance demand, which can be calculated using dynamic equilibrium equations or energy conservation equations. However, to solve dynamic equilibrium equations, nonlinear dynamic finite element analysis is commonly performed, which is time-consuming. Hence, dynamic analyses are generally used for buildings with high security risks. To solve energy conservation equations based on the conventional methods, both energy dissipation and work done by the unbalanced gravity load must be calculated for every single structural member, which is also a tedious task for engineering design.

In view of the aforementioned, more engineer-friendly methods, such as linear static or nonlinear static (NS) methods, have been widely adopted in the existing progressive collapse design code. This is followed by the correction of the aforementioned NS demand to conveniently approximate the ND demand using a DAF. Based on this concept, the progressive collapse resistance demand under the beam mechanism has been studied in the past, using the dynamic method and the energy method. However, little research has been devoted to the demand evaluation under the catenary mechanism for RC frame structures.

In addition to the aforementioned research gap, another critical problem associated with the existing energy-based approaches is the energy distribution in different structural components. In these approaches, the single-degree-of-freedom (SOF) model is widely adopted to represent the mechanical characteristics, including collapse resistance and deformation, of the collapse-resisting substructures, as shown in Fig. 1. It should be noted that the resistance of the SOF model refers to the collapse-resisting capacity of the overall structure, namely the structural resistance. Note that the structural resistance is provided by the structural elements in the substructure, but is not a simple summation of their individual elemental resistance due to complicated load-redistribution characteristics in the structural system. Accordingly, the DAF for the structural resistance is different from that for the elemental resistance. The DAF for the beam mechanism obtained based on the SOF model is the structural-level DAF, $D_A F_s$. However, in engineering designs the elemental-level DAF, $D_A F_e$, is required to determine the resistance demand of the structural elements.
Further, the relationships between the DAFs and the DAFe describe the distribution of the total energy dissipation in resisting progressive collapse throughout the structural system. However, the values of DAFs and DAFe have not been identified in the existing studies and, more specifically, the elemental-level DAFe has not yet been studied before.

This paper thus aims to present an energy-based, theoretical framework for calculating the structural and elemental demand relationships between the NS and ND demands of RC frame structures under the catenary mechanism. Based on this framework, the DAFs and DAFe are derived for regular RC frames undergoing large deformations. The two DAFs are validated through a series of numerical examples with varying parameters.

**RESEARCH SIGNIFICANCE**

This study investigates for the first time the dynamic amplification factor (DAF) for RC frames under catenary mechanism. The method proposed is useful for the effective assessment of the progressive collapse resistance demand of RC frames undergoing large deformations, which is a simpler alternative to the direct nonlinear dynamic approach. Unlike existing work, the proposed demand relationship (DAF) is classified into a structural DAF and an elemental DAF, which can be evaluated using an energy-based, theoretical framework developed in this study. This framework will facilitate further studies on the nonlinear dynamic effect of other types of structural systems.

**FUNDAMENTAL CONCEPTS**

The aforementioned resistance demands can be classified by the analysis targets (structural or elemental demand) and the analysis techniques (NS or ND method) as shown in Table 1, which are discussed as follows.

<table>
<thead>
<tr>
<th>Analysis targets</th>
<th>Structural-level</th>
<th>Elemental-level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>Nonlinear static structural demand (NSSD, RcNS)</td>
<td>Nonlinear static elemental demand (NSED, FNS)</td>
</tr>
<tr>
<td>Dynamic</td>
<td>Nonlinear dynamic structural demand (NDSD, RcND)</td>
<td>Nonlinear dynamic elemental demand (NDED, FND)</td>
</tr>
</tbody>
</table>

method. The corresponding outcomes of these methods are, respectively, referred to as the NS and ND demands.

**Detailing requirement**

Under large deformations, the catenary mechanism of a RC frame is substantiated by the detailing requirement to provide effective axial forces through the beams. For example, reinforcing steel bars designed to provide the progressive collapse resistance demand are required to be continuous from one edge to the other and be effectively embedded or anchored.18 Note that this study focuses on the progressive collapse resistance demand under the catenary mechanism. Therefore, RC frame structures with adequate ductility and continuous reinforcement arrangement are considered.

**THEORETICAL FRAMEWORK FOR DEMAND ANALYSIS**

The demand relationships can be categorized into three types (Fig. 2(a)). Type 1 is the relationship between the ND and the NS demands. In this study, these demand relationships at the structural and elemental levels for the catenary mechanism are designated as DAFs and DAFes, respectively. Type 2 is the relationship between the structural and elemental demands, which can be described by the structural-to-elemental demand ratio (SER). In this study, SERNS and SERND are used to describe the NS and ND structural-to-elemental demand ratios, respectively. Type 3 is the relationship between the DAFs and DAFes, which is referred to as the SERDAF.

Figure 2(b) presents the proposed theoretical framework of the energy-based progressive collapse demand analysis which includes the following four major steps:
Take the progressive collapse-resisting substructures as the study object (idealized as a SDOF system, refer to Fig. 1) and establish the expression of the DAF, based on the energy conservation principle;

2. Analyze the mechanical mode of substructures under the catenary mechanism and establish the SER-based DAF, based on which DAF can be converted into DAF;

3. Determine the NSED using the NS analysis; and

4. Correct the NSED to approximate the NDED using the DAF obtained in Step 2.

The framework shown in Fig. 2(b) provides a feasible energy-based method of calculating the DAF in which the calculation of the energy dissipation of every member in the substructure is not required.

RESISTANCE CURVE FOR RC FRAMES UNDER CATENARY MECHANISM

For the regular substructure shown in Fig. 1, when the progressive collapse resistance demands of the beams are satisfied at every story, the demands of the substructures are also satisfied. In this study, the two framed beams are isolated from the substructure to evaluate the demands of the beam elements under the catenary mechanism. This is presented in Fig. 3. A progressive collapse in the framed beams can be resisted by the structural resistance $R$ and the elemental resistance; that is, the axial tensile forces ($F_1$ and $F_2$) in the beams. The framed beams may deform into two different shapes, depending on the type of load and the horizontal stiffness of the support constraints. Under a uniformly distributed load $q$, the beams deform into a curve-type catenary, as shown in Fig. 3(a). This is the calculation model for the code specified TF method, as described in Appendix B of UFC. On the other hand, when subjected to a concentrated load $P$, the beams deform into a straight-type catenary, as shown in Fig. 3(b). The latter has been observed in the published laboratory test results. In practice, when a disproportionate collapse occurs in the intermediate floors of a multi-story building due to the local failure of a vertical element, a large concentrated load from the upper story columns is expected on top of the missing element. This results in a straight-type catenary mechanism. For the top floor, on the other hand, the existence of the uniformly distributed load would lead to a curve type catenary mechanism.

Curve-type catenary mechanism

For the two beams in Fig. 3(a), the maximum vertical displacement $\Delta$ occurs at the midspan—$(L_1+L_2)/2$—of the two beams. For both small and large deformations, $R$, $F_1$, and $F_2$ at the location of maximum deformation must satisfy the following equations, which are obtained through moment equilibrium of the left or the right symmetrical free-body with respect to the support point

$$R' = \frac{8\Delta}{(L_1+L_2)} F_1$$

$$F_1 = F_2$$

For RC beams, the axial forces are provided by the longitudinal reinforcing bars embedded in the beams. Before yielding of the steel bars, the beam ends undergo primarily the plastic hinge rotations, with a small amount of axial deformation. At this stage, the axial force $F_1$ can be calculated from the following equation, according to the deformation mode of the beams

$$F_1 = E_i A_i \frac{\alpha r^2}{2} \left( \frac{L_1 + L_2}{2} \right)$$

where $E_i$ and $A_i$ are the modulus of elasticity and the cross-sectional area of the longitudinal reinforcing bars, respectively. As shown in Fig. 3(a), $r$ is the radius of the catenary arc and $\alpha$ is the subtended angle of the half arc. Note that $\alpha$ can be expressed as $\arcsin\left[\frac{(L_1+L_2)/2}{r}\right]$ and the first two terms of the Taylor’s polynomials of $\alpha$ are given in Eq. (2).
Substituting Eq. (2) into Eq. (1) gives the structural resistance under the catenary mechanism $R_c$ before the beams yield in tension.

$$R_{EA} = \frac{64E_A}{3(L_1 + L_2)} \Delta^3$$

(Corresponding to the expression in Eq. (3a), the curve OAB in Fig. 4 represents the behavior of the catenary action before the beams yield. After yielding of the longitudinal reinforcing bars in the beams, assuming that Beam 1 has a relatively smaller tensile yield strength than Beam 2, the axial force $F_1$ remains the same as the yield force $F_{vy}$ of Beam 1. Based on Eq. (1), the structural resistance under the catenary mechanism $R_{cN}$ after yielding of the beams becomes the following

$$R_{cN} = \frac{8F_{vy}}{(L_1 + L_2)} \Delta$$

The expression in Eq. (3b) refers to the straight line OC in Fig. 4, which represents the catenary action following tension yielding in the beams.

**Straight-type catenary mechanism**

According to the straight-type catenary mechanism shown in Fig. 3(b), $R^c$, $F_1$, and $F_2$ satisfy the following equations

$$R^c = \frac{(L_1 + L_2)}{L_1 L_2} \Delta F_1$$

$$F_1 \approx F_2 \text{ when } \Delta \leq 0.2 \times \text{min}(L_1, L_2)$$

Note that the discrepancy in the axial forces in two beams is less than 2% if the beam length ratio becomes as large as 10, which is rarely practical.

Similar derivations as presented previously are adopted herein. Under the straight-type catenary mechanism, the structural resistances $R_{c1}$ and $R_{cN}$ before and after yielding of the longitudinal reinforcing bars are given by Eq. (5a) and (5b), respectively

$$R_{c1} = \frac{E_A(L_1 + L_2)}{2L_1L_2} \Delta^3$$

$$R_{cN} = \frac{(L_1 + L_2)}{L_1 L_2} F_i \Delta$$

Equations (5a) and (5b) indicate that RC frames under the straight-type catenary mechanism exhibit similar mechanical behavior, as illustrated in Fig. 4.

**Effect of beam mechanism**

It is well accepted that RC-framed beams behave in the form of a beam mechanism before exhibiting the catenary mechanism. Therefore, as Fig. 5 shows, the entire resistance curve of an RC frame structure is defined by the polyline ODEFA under the beam mechanism and the straight line AC under the catenary mechanism. The polyline connects such key points as the reference point O (0, 0), the yield point D ($\Delta_{by}$, $R_{by}$), the peak point E ($\Delta_{bp}$, $R_{bp}$), the ultimate point F ($\Delta_{bu}$, $R_{bu}$), and the failure point G ($\Delta_{bf}$, 0) of the resistance curve of the RC frame structures under the beam mechanism. In the figure, $\Delta_{cf}$ and $R_{cf}$ are the ultimate displacement and the ultimate structural resistance, respectively, of an RC frame under the catenary mechanism. In this paper, the structural and elemental resistance demands of the RC frames exhibiting the catenary mechanism are studied first. This is followed by further analysis of the effect of the beam mechanism on the demands under the catenary mechanism.

**STRUCTURAL DEMAND RELATIONSHIP OF RC FRAMES UNDER CATENARY MECHANISM**

The analytical expression of the DAF under the curve-type catenary mechanism, $DAF_{c}$, is derived in this section. The DAF, under the straight-type mechanism will be discussed at the end of this section. The NSSD and NDS are notionally illustrated in Fig. 6. The NSSD, $R^c_{NSS}$, is represented by Point B, which satisfies the static equilibrium condition under the unbalanced gravity load $G$ and can be evaluated by the preliminary NS analysis. The NDSD, $R^c_{NDS}$, is represented by Point E, which satisfies an additional demand induced by the nonlinear and dynamic effects during the
collapse process, and it can be determined by correcting the analysis through the DAF.

In the existing codes of practice, the ultimate displacement $\Delta_c f$ of the RC-framed beams under the catenary mechanism is specified as $0.2L$, where $L$ is the span length of the beams. Therefore, the ND and NS displacements, $\Delta_{ND}$ and $\Delta_{NS}$, are identical, as shown in Fig. 6. To simplify the theoretical discussion, it is assumed that the yield displacements $\Delta_{y1}$ and $\Delta_{y2}$, corresponding to the yield resistances $R_{cy1}$ and $R_{cy2}$, respectively, are equal, as indicated in Fig. 6. Note that the energy dissipated before reaching $\Delta_{y1}$ and $\Delta_{y2}$ only consumes a very small proportion of the total energy. For example, the framed beams discussed later in Fig. 7 only dissipate 3.32% of the total energy before reaching $\Delta_{y1}$ or $\Delta_{y2}$. Therefore, under the aforementioned assumption, slight discrepancies are expected in the solution.

In accordance with the energy conservation condition, in which the work done by the unbalanced gravity load (the area OFBC) and is equal to the energy dissipation by the structure (the area ODEC), the energy equation of the RC frames under the curve-type catenary mechanism is as follows

$$G\Delta_{ND} = \int_0^{\Delta_{c}} \frac{64E_A A_{ND}}{3(L_1 + L_2)} \Delta'_c d\Delta_c + \frac{1}{2}(R_{cy2} + R_{ND}) (\Delta_{c} - \Delta_{y2})$$

where $A_{ND}$ is the cross-sectional area of the longitudinal reinforcing bars in the framed beams for the ND demand. For $R_{cy2}$ and $\Delta_{y2}$ to satisfy Eq. (3a), we have the following

$$R_{cy2} = \frac{64E_A A_{ND}}{3(L_1 + L_2)} (\Delta_{y2}^2)$$

The yield factor $\beta^c$ of the substructures is defined as

$$\beta^c = \frac{R_{cy1}^c}{R_{NS}}$$

Considering the aforementioned discussion on $\Delta_{ND}$, $\Delta_{y1}$, and $\Delta_{y2}$, Eq. (3b) and (8a) produce the following

$$\beta^c = \frac{R_{cy2}^c}{R_{ND}}$$

Fig. 6—Resistance demands of RC frame structures under catenary mechanism.

Fig. 7—Schematic diagram of validation method for progressive collapse resistance demand under catenary mechanism.

Parameters of demonstrated example: $q = 31.5 \text{ kN/m}$, $L = 8 \text{ m}$, dimensions $= 600 x 300 \text{ mm}$, uniformly distributed load, no seismic design. (Note: 1 kN.m–1 = 0.0685 kip.ft–1; 1 m = 3.281 ft; 1 mm = 0.0394 in.; 1 kN = 0.225 kip.)
and
\[
\Delta_{y1}^c = \Delta_{y2}^c = \beta' \Delta_{yS}^c = \beta' \Delta_{yD}^c
\]  
(9)

Substituting Eq. (7) to (9) into Eq. (6) yields the NDSD, \(R_{ND}^c\), under the curve-type catenary mechanism.

\[
R_{ND}^c = \frac{4G}{2 - (\beta')^2}
\]  
(10)

Considering the DAF\(_c\), as defined previously and Eq. (10), the structural demand relationship DAF\(_c\) of an RC frame structure under the curve-type catenary mechanism can be expressed as follows

\[
DAF_{NS}^c = \frac{R_{NS}^c}{R_{ND}^c} = \frac{4}{2 - (\beta')^2}
\]  
(11)

It should be noted that the initial energy can be dissipated by the beam mechanism. Thus, the resistance demand under the catenary mechanism is decreased. Assuming that the energy dissipation \(U_b\) by the beam mechanism is as follows

\[
U_b = \psi G \Delta_{yD}^c
\]  
(12)

where \(\psi\) is the energy dissipation factor representing the ratio of the partial (by the beam mechanism) to the total energy dissipation, we then have the DAF\(_s\), considering the energy dissipation of the beam mechanism, as follows

\[
DAF_s^c = \frac{4(1 - \psi)}{2 - (\beta')^2}
\]  
(13)

Equation (13) describes the relationship between NDSD and NSSD of an RC frame structure under the curve-type catenary mechanism.

The expressions for DAF\(_s\), for the straight-type catenary mechanism can be derived following the same procedure as described previously, except that the expressions associated with the straight-type catenary mechanism should be adopted in Eq. (6) and (7). This indicates that the relationships between the ND and NS demands for the two mechanisms are identical.

**NUMERICAL VALIDATION**

**Validation method**

The numerical validation is accomplished using the nonlinear finite element program THUFIBER.\(^{13}\) Published literatures show that RC frames exhibiting a flexural and axial behavior can be well simulated using THUFIBER.\(^{12,23}\) The independent NS and ND analysis are conducted to obtain the NS and ND demands respectively. Figure 7(a) shows a theoretical model of one-story two-span continuous framed beams used to validate the proposed demand relationships. Details of the model are given in the next section, and the validation procedure is summarized below in five stages.

**Stage 1:** Calculate the actual NSED and NSSD—An NS pushdown analysis is conducted herein. The amount of longitudinal reinforcing steel is iteratively adjusted until the pushdown load equals the unbalanced gravity load \(G\) when the joint displacement reaches the target displacement—that is, \(0.2L\).\(^{10,12}\) The obtained frame satisfies the NS demand—the static equilibrium condition under the unbalanced gravity load \(G\). The corresponding axial force in the beams is therefore taken as the actual NSED, \((F_{yS})_{NS}\). Note that the actual NSSD equals to the gravity load \(G\).

**Stage 2:** Calculate the actual NDED and DAF\(_s\)—For the framed beams established in Stage 1, a gravity load \(G\) is initially considered to be applied to the beams, and a static analysis is conducted until the static equilibrium state is reached. This state corresponds to 1s in Fig. 7(b), which is followed by a sudden removal of the middle support. A
transient ND analysis is subsequently conducted to simulate the dynamic response of the framed beams, as shown in Fig. 7(b). During the ND analysis, the amount of longitudinal reinforcing steel is iteratively adjusted until the catenary mechanism takes place and the corresponding displacement reaches the target displacement specified in the existing codes, 0.2L. The obtained frame satisfies the ND demand—the energy equilibrium condition under the unbalanced gravity load $G$. The corresponding maximum axial force in the beams is recorded as the actual NDED, $(F_{yD})_{ND}$. Thus, the actual value of $DAF_c$ can be expressed as follows

$$
(DAF_c)_{act} = \frac{(F_{yD})_{ND}}{(F_{yD})_{NS}}
$$

(17)

Stage 3: Calculate the actual NDSD and $DAF_c$—An NS pushdown analysis is conducted again for the framed beams obtained in Stage 2, satisfying the ND demand, to calculate the actual NDSD, as shown in Fig. 7(c). The target displacement, 0.2L, and the load pattern in the pushdown analysis are identical to those in the transient ND analysis (Stage 2). The pushdown load at the target displacement 0.2L is taken as the actual NDSD, $R'_{ND}$. Thus, the actual value of $DAF_c$, $(DAF_c)_{act}$, is given by the following

$$
(DAF_c)_{act} = \frac{R'_{SD}}{R_{NS}} = \frac{R_{SD}}{G}
$$

(18)

Stage 4: Calculate and validate the predicted $DAF_c$ and compare the structural and elemental $DAF_c$—For the numerical model being analyzed, the actual structural $DAF_c$, $(DAF_c)_{act}$ obtained in Stage 3, is compared with the predicted $DAF_c$, $(DAF_c)_{pre}$. Note that $(DAF_c)_{pre}$ is calculated using Eq. (13), in which the energy dissipation factor $\psi$ under the beam mechanism is determined by the ratio of the area below ODEFA to that below ODEFAC of the pushdown curve calculated in Stage 2, as illustrated in Fig. 7(c). The yield factor $\beta$ in Eq. (13) is given by Eq. (8), in which the yield displacement $\Delta y$ of the catenary mechanism is determined by the transformation point from the beam mechanism to the catenary mechanism (Point A in Fig. 7(c)). The actual structural and DAFs, $(DAF_c)_{act}$ and $(DAF_c)_{act}$, are also compared in the following section to validate Eq. (15), describing their theoretical relationship.

Stage 5: Calculate the predicted NDED and validate the framework in Fig. 2(b)—Following the proposed framework in Fig. 2(b), the predicted NDED, $NDED_{pre}$, is calculated by correcting the actual NSED obtained in Stage 1 using the predicted $(DAF_c)_{pre}$ obtained in Stage 4. Then, the $NDED_{pre}$ is compared with the actual NDED, $NDED_{act}$, obtained by the direct ND analysis in Stage 2.

Model parameters

To ensure the representativeness of the validation, different parameters, including the load type and magnitude, the span length, and the cross-sectional dimension of the beams, are considered, as summarized in Table 2. This results in 180 models in total. A uniformly distributed load and a concentrated load are applied to investigate the resistance demand under the curve- and straight-type catenary mechanisms, respectively.

To investigate the effect of the beam mechanism, two different reinforcing scenarios are considered, as shown in Fig. 8. In the first scenario, the structure is reinforced with two steel bars with diameters of 22 mm (0.866 in.) at the top of the beam ends. This corresponds to a 0.22 to 0.33% reinforcement ratio, which is greater than the minimum ratio of 0.2% specified in the Chinese Design Code of Concrete Structures. This minimum ratio is widely adopted in frames designed without consideration for seismic effects. In the second scenario, the structure is reinforced with six steel bars with diameters of 22 mm (0.866 in.) at the top of the beam ends, which corresponds to a 1.3 to 2.1% reinforcement ratio. This ratio is smaller than the code-specified maximum ratio of 2.5% and is commonly used in frames designed with consideration for seismic effects. The aforementioned two scenarios cover the upper and lower limits of the energy dissipation effects of the beam mechanism. For all the models, two additional bars with diameters of 22 mm (0.866 in.) are also placed at the top sections at the midspan of the beams. The bottom bars are positioned along the entire length of the beams. The cross-sectional areas of the bottom bars are iteratively adjusted until the target displacement of 0.2L is reached for the catenary mechanism.

Validation results

The validation for the structural demand relationship, $DAF_c$, is illustrated in Fig. 9. It is evident that the predicted values of $(DAF_c)_{pre}$ are very close to the actual values of $(DAF_c)_{act}$, with the majority of the errors within ±10% and the mean absolute percentage errors are 2.61% and 2.44% for the curve-type and straight-type catenary mechanisms, respectively. This indicates that the proposed Eq. (13) is able to accurately predict the relationship of the structural
resistance demands of RC frame structures under the catenary mechanism. It is also observed that the DAF\textsubscript{c} under the catenary mechanism ranges from approximately 1.5 to 2.0. This value is larger than the DAF\textsubscript{b} under the beam mechanism, which is below 1.34 for regular RC frame structures with a ductility ratio greater than 1.0 in the NS analysis.\textsuperscript{10}

In the existing design codes, the NS elemental demand is directly adopted for the TF design. The validation results reveal that the actual ND elemental demand is approximately 1.5 to 2.0 times the NS elemental demand. This indicates that the demand calculated by the existing TF method is insufficient for collapse prevention; therefore, the method should be further modified to ensure safe design for the catenary mechanism. It is also noticed that only the DAF for the beam mechanism is regulated in the existing design codes, which is much lower than that for the catenary mechanism. If the currently recommended DAF is used to correct the demand under the catenary mechanism, the solution would be underestimated.

To improve the demand calculation method for the catenary mechanism, the proposed equations (Eq. (13) and (15)) can be used to correct the NS demand in the existing TF design. The effect of the beam mechanism on the structural demand under the catenary mechanism is illustrated in Fig. 10. The square dots denote the relationship between the energy dissipation factor $\psi$ and (DAF\textsubscript{c})\textsubscript{act} for all 180 numerical models. Substituting the average value of $\beta$ of the validation models into Eq. (13) gives the nominal average value of (DAF\textsubscript{c})\textsubscript{act} as a function of $\psi$, which is represented by the solid line in Fig. 10. It is apparent that when the energy dissipation by the beam mechanism increases, the structural demand under the catenary mechanism decreases. It is well-known that the beams in RC frame structures constructed in earthquake zones are designed to have higher bending moment capacities than those constructed in non-earthquake zones. As a result, RC frame structures with seismic design considerations have higher energy-dissipating capacities under the beam mechanism in a progressive collapse process. Consequently, the collapse resistance demand of these structures under the catenary mechanism decreases.

In Fig. 11, the actual values of the elemental (DAF\textsubscript{e})\textsubscript{act} are compared to those of the structural (DAF\textsubscript{c})\textsubscript{act} for all 180 models. Again, the mean ratios of (DAF\textsubscript{c})\textsubscript{act} and (DAF\textsubscript{e})\textsubscript{act} are well within ±10% and the mean absolute percentage errors are 2.49% and 2.64% for the curve-type and straight-type catenary mechanisms, respectively. This further validates the proposed relationship (Eq. (15)) under the catenary mechanism.

The NDED calculated based on the framework proposed in Fig. 2(b) is validated by the results presented in Fig. 12. The majority of the mean ratios of NDED\textsubscript{act} and NDED\textsubscript{pre} are within ±10% and the mean absolute percentage errors are 3.39% and 1.79% for the curve-type and straight-type catenary mechanisms, respectively. That demonstrates that the NDED of the numerical examples can be well assessed by the proposed framework.

**CONCLUSIONS**

Under large deformations, RC frame structures resist progressive collapse through the catenary mechanism. An energy-based framework is proposed for calculating the progressive collapse resistance demands under the catenary mechanism at both the structural and elemental levels. The following conclusions can be drawn.

1. Two types of catenary mechanisms, the curved type and the straight type, are similar mechanisms and are identical in their DAF values.

2. For regular RC frame structures, the elemental DAF equals the structural DAF. These two values may not be identical for irregular frame structures with varied structural arrangement from story to story. This aspect merits further investigation.

3. For structures with seismic design considerations, the beam mechanism capacity increases, which subsequently reduces the catenary mechanism demand, due to
the noticeable contribution of the energy dissipation of the beam mechanism.

4. Numerical validation using a total of 180 framed beam models demonstrates that the proposed relationships are accurate and that the DAF<sub>cs</sub> under the catenary mechanism is larger than the DAF<sub>b</sub> under the beam mechanism. The proposed DAF<sub>ce</sub> can be used to correct the NS elemental demand to approximate the ND elemental demand required for engineering design.

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**NOTATION**

- A<sub>1</sub>, E<sub>1</sub> = cross-sectional area and elastic modulus of longitudinal reinforcing bars, respectively
- A<sub>ND</sub> = cross-sectional area of longitudinal reinforcing bars for ND demand
- DAF = dynamic amplification factor
- DAF<sub>e</sub>, DAF<sub>s</sub> = structural DAF and elemental DAF, respectively
- DAF<sub>cs</sub>, DAF<sub>ce</sub> = structural and elemental DAFs under catenary mechanism, respectively
- (DAF<sub>cs</sub>)<sub>act</sub>, (DAF<sub>cs</sub>)<sub>pre</sub> = actual and predicted values of DAF<sub>cs</sub>, respectively
- (DAF<sub>ce</sub>)<sub>act</sub>, (DAF<sub>ce</sub>)<sub>pre</sub> = actual and predicted values of DAF<sub>ce</sub>, respectively
- F, F<sub>1</sub>, F<sub>2</sub> = elemental demand under catenary mechanism
- F<sub>1y</sub> = yield force of Beam 1
- F<sub>NS</sub>, (F<sub>1y</sub>)<sub>NS</sub> = nonlinear static elemental demand
- F<sub>ND</sub>, (F<sub>1y</sub>)<sub>ND</sub> = nonlinear dynamic elemental demand
- G = total unbalanced gravity load
- g = balanced, uniformly distributed gravity load
- L, L<sub>1</sub>, L<sub>2</sub> = span length of beam
- P = unbalanced concentrated gravity load
- q = unbalanced, uniformly distributed gravity load
- q<sub>p</sub> = uniformly distributed pushdown load
- R<sup>c</sup>, R<sup>e</sup> = structural demand
- R<sub>c</sub>, R<sub>e</sub> = structural demand under beam mechanism and catenary mechanism, respectively
- R<sup>c</sup> before and after beam yield is in tension
- R<sup>e</sup> before and after beam yield is in tension
- r<sub>1</sub>, r<sub>2</sub> = radius of catenary arc
- R<sub>se</sub>, R<sub>eD</sub> = structural-to-elemental ratio for NS demand, ND demand, and DAF, respectively
- α = subtended angle of the half arc

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**Fig. 10**—Effect of beam mechanism on structural demand under catenary mechanism.

**Fig. 11**—Comparison between actual values of DAF<sub>ce</sub> and DAF<sub>cs</sub>.
APPENDIX—PROOF OF UNIVERSALITY OF DEMAND RELATIONSHIPS

The theoretical derivations presented in the aforementioned sections are based on the simplified model illustrated in Fig. 3, with pre-defined beam dimensions and material parameters. When the geometric dimensions and material parameters of Beam 1 and Beam 2 vary independently, $E$, $A$, $L$, $\Delta$, and $F_i$ in Eq. (1) to (5) would change accordingly, which would in turn lead to a variation of the ratios of the structural to elemental demand ($SER_{\text{S}}$ and $SER_{\text{D}}$) regulated by Eq. (14). It should be noted, however, that the parameters $L$, $\Delta$, and $F_i$ are eliminated in the calculation process as evident in Eq. (15). This indicates that the geometric dimensions and material parameters have no influence on the DAF$_{\text{S}}$ and DAF$_{\text{D}}$. The aforementioned discussion clearly demonstrates that the structural and elemental demand relationships derived from the substructure in Fig. 3 are universal for all regular RC frame structures.

REFERENCES


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