Three-dimensional Analysis of Elastic Marine Cable during Laying

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ABSTRACT

In this paper, we proposed a three-dimensional model for the tension analysis of submarine power cables during laying operations. Both flexural rigidity and axial elasticity were taken into consideration in the form of classical Euler-Bernoulli beam element theory. The governing equations were numerically solved by fourth-order Runge-Kutta methods and the derivatives of the curvature with respect to the cable element arc length can be approximately expressed by the finite difference method. Numerical examples were illustrated to study the influence of some key parameters on the cables and some significant conclusions are drawn.

KEY WORDS: Submarine power cables, tension analysis, configuration curve

INTRODUCTION

Design of submarine power cables laying systems has been intensively studied recently due to the exploitation of oceanic natural resources demands. With the significant commercial benefits, how to ensure the safety of operations during laying is quite a crucial problem. Once the problem is solved, it can avoid many detrimental impair on submarine power cables. Thus, the tension analysis of marine cables is one of hot topics in ocean engineering field in the past few decades.

The pioneering work was done by Zajac (1957), who firstly proposed a stationary model to solve the tension problem during laying or recovery operations. In the analysis, the cable configuration is considered as a straight line and the elevation is constant, which is not practical in the engineering application. Based on Zajac’s framework, Yoshizawa and Yabuta (1983) focused on clarifying the tension distribution of marine cable without tangential drag forces taken into considerations. In their model, the tension force at the touch-down point on the seabed surface and the cable slack can be obtained when the value of elevation angle is given. While Junget et al. (2001) adjusted the pay-out rate to get a desired cable slack and cable tension for various seabed conditions. In Chucheepsakul et al. (2003), a three-dimensional model is presented to simulate the tension distribution of extensible cable with the top tension specified, in which the virtual work-energy functional is formulated to build the governing equations. Additionally, the finite element approach is adopted to get the numerical results. While Wanget al. (2008) focused on the tension analysis without tangential drag forces considered and used fourth-order Runge-Kutta method to improve the computational efficiency and avoid divergence in the iteration process. Comparison to the precious models, the results overall agree well. In this model, self-weight of submarine cables, buoyancy forces and normal drag forces are taken into consideration. Zhang and Hu (2009) simplified the cable configuration curve to a catenary line to analyze the tension force during different stages of laying operations. Meanwhile, the paper discussed the fictional forces between the submarine cables and seabed surface.

The aforementioned researches have fully clarified the laid or towed submarine cable’s behavior under complex sea conditions, the transient problem of marine cable and the effect on the tension distribution has not been given enough attention. Vaz and Patel (1995) proposed a 2D numerical model to solve the problem when the cable laying vessel suddenly accelerated or decelerated from one constant velocity to another. Generally, the top tension is the most crucial parameter during laying and the model can be used to calculate its value. In this paper, they extended a 2D model to study the laid cables and also marine cables towed. During the process, the cable configuration, top tension and the desired pay-out rate are all regarded as the functions of time and the cable’s dynamic characteristics can be derived through the kinematics. However, Vazet al. (1997) mainly solved the three-dimensional transient motion of cables when the laying vessel changed its speeds or sailing directions. Then the partial differential equations are solved by spatial and temporal integration methods. In Vaz and Patel (2000), they developed a 3D model as an extension of their former work. The effect of arbitrary shear currents is included. Furthermore, the model is also practical to the segmented elastic mooring lines and flexible riser problems. Pripić-Oršić and Nabergoj (2005) predicted the motion and tension distribution of marine cables in rough sea, which considered the effect of regular and irregular waves. Nagatomi et al. (2002) proposed the lumped mass method to study the tension value of the cables. The discrete cable elements are interconnected by linear-springs, which has no weight and the spring constant equals to the axial stiffness of submarine cables. The external forces are considered to be concentrated on each mass element. Comparison between the numerical simulations and field experiments are also illustrated to verify the practice of the model.

Although marine cables were commonly treated as ideal
flexible structures in the conventional methods, the bending stiffness sometimes significantly affects the cable’s configuration curve and tension distribution in low tension or shallow water region. Sun and Leonard (1998) proposed a general set of 3D dynamic field equations and boundary conditions with flexural, torsion and inertial effects, which can eliminate the potential singularity when the tension becomes zero. Park et al. (2003) adopted a numerical investigation into the dynamic response of a towed low tension cable. Then the bending stiffness is considered to cope with the low tension problem and the set of nonlinear differential equations are solved by New-Raphson iteration. Dreyer et al. (1999) obtained the cable curve through both continuous and discrete models, in which the cable is inflexible. The continuous model yielded more accurate results while more costly in computation aspect. In Pinto (1995), they considered the flexural rigidity to overcome the singularities in the geometric stiffness matrix. A weak Galerkin formulation and Newmark methods are used for integration in space and time, respectively. However, the initial and boundary conditions are not easy to assess for the analysis.

THEORETICAL FORMULATION

To obtain the expressions of submarine power cables’ kinematic quantities, a reference system consisting of one Lagrangian coordinates and one frame of reference is adopted, as shown in Fig. 2. The Lagrangian coordinates is stretched arc length \( p \), and another is the inertial frame of reference(I,J,K). In the inertial OXYZ coordinates system, the plane OXY is in the seabed with OZ oriented vertically upwards and the origin is exactly placed at the touch-down point on the seabed surface. The relationship between the two coordinate systems can be expressed as:

\[
\begin{bmatrix}
\cos \theta \cos \psi & \cos \theta \sin \psi & \sin \theta \\
-\sin \theta \cos \psi & -\sin \theta \sin \psi & \cos \theta \\
\sin \psi & \cos \psi & 0
\end{bmatrix}
\]

Or given in the matrix formation:

\[
\begin{bmatrix}
t \\
n \\
b
\end{bmatrix} = C
\begin{bmatrix}
1 \\
J \\
K
\end{bmatrix}
\]

where \( \theta, \psi \) are the elevation and azimuth angles, respectively.

Kinematics of Submarine Power Cables

In the present analysis, we assumed that the pay-out rate and the cable laying vessel velocity are invariable in the negative \( t \) and positive \( J \) directions, respectively. Then, the relative velocity and acceleration vectors between the seawater and the submarine power cables can be given in the inertial coordinate system:

\[
\begin{bmatrix}
v_x \\
v_y \\
v_z
\end{bmatrix} = (-V_p \cos \theta \cos \psi + V_p \cos \phi) + (V_p \sin \psi \sin \theta + V_p \sin \phi)J - V_p \sin \phi K
\]

where \( \phi \) is the angle between the ocean current direction and
positive Y axis;
According to the kinematics properties, the acceleration vectors can be expressed as:

\[
A = \frac{V^2}{1 + \varepsilon} \frac{\partial \theta}{\partial s} \cdot n - \frac{V^2}{1 + \varepsilon} \cos \theta \frac{\partial \psi}{\partial s} \cdot b
\]  

(5)

As shown in Fig. 3, the forces acting on the marine cable elements are external forces, tensional forces and shear forces. Therefore, the dynamic balance forces can be written in the following vector form:

\[
\delta \mathbf{T} + \delta \mathbf{Q} + \mathbf{F}_{\text{ext}} \delta \rho + \mathbf{F}_m \delta \rho = \mathbf{0}
\]  

(6)

where \( \mathbf{T} \) is the tensional forces, \( \mathbf{Q} \) is the shear forces, \( \mathbf{F}_{\text{ext}} \) is the external forces and \( \mathbf{F}_m \) is the physical added-mass forces.

When the cable element length tends to be relative small, the above equation can be rewritten in the partial differential forms as:

\[
\frac{\partial \mathbf{T}}{\partial \rho} + \frac{\partial \mathbf{T}}{\partial \rho} + \mathbf{F}_{\text{ext}} - m \mathbf{A} = \mathbf{0}
\]  

(7)

where \( m \) and \( \mathbf{A} \) are the physical mass of marine cable elements and acceleration vector, respectively.

During laying operations, the tension distribution of submarine power cables is mainly up to its self-weight per unit length in water. Thus, the shear force and tensional force vectors are approximately parallel to the normal and tangential directions, respectively. Hence:

\[
(T + \Delta T) (t + \Delta t) - \mathbf{T} dt = \mathbf{T} dt + \Delta \mathbf{T} (8)
\]

\[
(Q + \Delta Q) (n + \Delta n) - \mathbf{A} \Delta \mathbf{n} = \mathbf{A} \Delta \mathbf{n} + \Delta \mathbf{A} \quad (9)
\]

where the second order differentials have been neglected.

The derivatives of the unit vectors \( \mathbf{t}, \mathbf{n}, \mathbf{b} \) with respect to the stretched arc length \( \rho \) can be written in the following matrix form:

\[
\frac{\partial}{\partial \rho} \left\{ \begin{array}{c} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{array} \right\} = \frac{\partial \mathbf{C}}{\partial \rho} \left\{ \begin{array}{c} \mathbf{I} \\ \mathbf{J} \\ \mathbf{K} \end{array} \right\} = \frac{\partial \mathbf{C}}{\partial \rho} \mathbf{C} \left\{ \begin{array}{c} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{array} \right\}
\]  

(10)

Because \( \mathbf{C} \) is an orthogonal transformation matrix, thus,

\[
\frac{\partial}{\partial \rho} \left\{ \begin{array}{c} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{array} \right\} = \frac{\partial \mathbf{C}}{\partial \rho} \mathbf{C} \left\{ \begin{array}{c} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{array} \right\}
\]  

(11)

In this analysis, the flexural rigidity is included in the governing equations, which is usually neglected in the conventional models. The differential governing equations can be expressed by the five-grid difference format. The derivatives of the curvature with respect to the cable element length are approximately expressed by the five-grid difference format. In the governing equations, four unknown quantities are to be solved: three coordinates \( x, y, z \) and the tensional force \( T \). To get the differential equations solved, three boundary conditions \( (x_0, y_0, z_0) \) and two other initial conditions \( \theta_0, \psi_0 \) at the touch-down point should be specified. Additionally, the tension force at TDP should also be given. To determine the cable configuration, the nodal coordinates \( (x_i, y_i, z_i) \) are necessary to be obtained. Due to the geometric compatibility, the coordinates \( (x_i, y_i, z_i) \) are given by:

\[
x_{i+1} = x_i + (1 + \varepsilon) \cos \theta \cos \psi \cdot ds
\]  

(12)

\[
y_{i+1} = y_i + (1 + \varepsilon) \cos \theta \sin \psi \cdot ds
\]  

(13)

\[
z_{i+1} = z_i + (1 + \varepsilon) \sin \theta \cdot ds
\]  

(14)

\[
\kappa_i = \frac{\theta_{i+1} - \theta_i}{\Delta s}
\]  

(15)

\[
\varepsilon = T / EA
\]  

(16)

where \( (x_0, y_0, z_0), (x_{i+1}, y_{i+1}, z_{i+1}) \) are the coordinate values at the i-th node and i+1-th node, \( \varepsilon \) is the axial strain, \( E \) is the Young modulus of submarine power cable and \( A \) is the cross section area. \( \theta \), \( \psi \) are the elevation and azimuth angles at the i-th node, respectively.

It’s noted that the length of submarine power cable elements should not be too small. Otherwise, the integration process will not converge effectively.

Based on Newton’s second law of motion, the governing equations of submarine power cables are obtained and can be given as:

\[
\frac{d^2}{ds^2} \left[ T \right] \mathbf{V} + \frac{d}{ds} \left[ \frac{E I}{C} \mathbf{V} \right] - k \mathbf{V} \mathbf{C} \left[ 1 + \omega \mathbf{V} \mathbf{C} \right] = \mathbf{0}
\]  

(17)

\[
\frac{d}{ds} \left[ \mathbf{V} \mathbf{C} \right] \frac{d}{ds} \left[ \mathbf{V} \mathbf{C} \right] - \omega \mathbf{V} \mathbf{C} \left[ 1 + \omega \mathbf{V} \mathbf{C} \right] = \mathbf{0}
\]  

(18)

\[
\frac{d}{ds} \left[ \mathbf{V} \mathbf{C} \right] \frac{d}{ds} \left[ \mathbf{V} \mathbf{C} \right] - \omega \mathbf{V} \mathbf{C} \left[ 1 + \omega \mathbf{V} \mathbf{C} \right] = \mathbf{0}
\]  

(19)

where \( \rho_a, \rho_b \) are the equivalent physical density per unit length of submarine power cables in normal and binomial directions, respectively.

The expression can be given as:

\[
\rho^*_a = \frac{\rho_a}{g} + \rho_a C_{am} \frac{\pi D^2}{4}, \quad \rho^*_b = \frac{\rho_b}{g} + \rho_b C_{ab} \frac{\pi D^2}{4}
\]  

(20)
Where $V_{rs}$ is the normal component of the relative velocity between the submarine power cables and seawater during laying operations and $V_{rs} = \sqrt{v_{rs}^2 + \kappa v_{rs}}$, $\kappa$ is the curvature of submarine power cables configuration in water.

RESULTS & DISCUSSIONS

In this section, some numerical examples are presented to investigate the influence of crucial parameters on submarine power cables, such as, the bending stiffness, the cable laying vessel velocity, the current profile, the pay-out rate and self-weight etc. The input data for the numerical cases are given in Table 1 or specified otherwise.

Table 1. Marine properties and operations conditions

<table>
<thead>
<tr>
<th>Marine Cables</th>
<th>LA</th>
<th>HA</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Diameter</td>
<td>80</td>
<td>110</td>
<td>mm</td>
</tr>
<tr>
<td>Weight in air</td>
<td>196.1</td>
<td>392.2</td>
<td>N/m</td>
</tr>
<tr>
<td>Weight in seawater</td>
<td>145.5</td>
<td>296.7</td>
<td>N/m</td>
</tr>
<tr>
<td>Seawater density</td>
<td>1025</td>
<td>1025</td>
<td>kg/m3</td>
</tr>
<tr>
<td>Normal drag coefficient</td>
<td>1.50</td>
<td>1.50</td>
<td>—</td>
</tr>
<tr>
<td>Tangential drag coefficient</td>
<td>0.03</td>
<td>0.03</td>
<td>—</td>
</tr>
<tr>
<td>Flexural rigidity</td>
<td>0/500</td>
<td>0/1000</td>
<td>N.m²</td>
</tr>
<tr>
<td>Added-mass coefficient</td>
<td>1.00</td>
<td>1.00</td>
<td>—</td>
</tr>
<tr>
<td>Water depth</td>
<td>100</td>
<td>100</td>
<td>m</td>
</tr>
</tbody>
</table>

**Effects of Flexural Rigidity**

The effects of the flexural rigidity on submarine cables are illustrated in Fig. 4, in which two different types of submarine power cables are analyzed. The properties of the cables are presented in Table 1. The cable laying vessel is assumed to move at a constant speed of 0.5 m/s, which equals to the pay-out rate. Then the current velocity varies from 0.5 m/s at the seabed surface to 1.5 m/s on the seawater surface in the positive X axis. The numerical results clearly indicate that the bending stiffness is insignificant for both configuration and tension distribution during laying operations. However, the influence of self-weight per unit length in water is remarkable and the trend becomes more obvious as the water depth increases. It concludes that the difference between these two types of submarine power cables mainly come down to their self-weight and are almost independent of the flexural rigidity.

**Effects of the Pay-out Rate and Cable Laying Vessel Velocity**

In this section, the pay-out rate is assumed to be equal to the vessel velocity and the tension distribution. As shown in Fig. 5, the difference of cable configuration for HA is clearly shown with 0.3 m/s (solid line), 0.5 m/s (dotted line), 0.8 m/s (dash-dotted line) and 1.0 m/s (dashed line) results. Above a small portion of seabed, the cable configuration curve is basically identical. While the difference trends to become obvious with the increase of seawater depth. However, the tension distribution is not sensitive to the change of the pay-out rate $V_p$ at all as illustrated in Fig. 5 (c).
Effects of Current Directions

The current profile is a crucial parameter in the tension analysis of submarine power cable. Fig. 6 presents the response of submarine power cable for four typical values of current directions range from 0 to \( \pi / 2 \), under identical magnitude of velocity. As shown in Fig. 6 (a) ~ (b), when the current direction is parallel to the Y axis (\( \phi = \pi / 2 \)), the cable profile keeps in the OYZ plane and can be simplified to a two-dimensional problem. It’s worth noting that when the current velocity \( V_c \) is perpendicular to the sailing direction \( V_s (\phi = 0) \), the cable configuration changes much steeper.

Effects of Current Velocity Value

The cases presented in this section mainly focus on the analysis of the magnitude of current in the X axis. The velocity is 0.5 m/s at the seabed surface and varies linearly to 0.5 m/s, 1.0 m/s, 1.5 m/s and 2.0 m/s on the seawater level. The coordinates at the water surface increase as the
$V$ increases as shown in Fig. 7 (a) ~ (b). For example, the coordinates is $(8.5, 49.0, 100)$ for $V = 0.5$ m/s and reaches $(23.38, 52.12, 100)$ for $V = 2.0$ m/s. While Fig. 7(c) indicates that different current velocities make no difference for the tensional forces of the marine cables, which mainly attributes to the large self-weight of submarine power cables.

Effects of Pre-tension at Touch-down Point

The bottom tension is also considered as other critical parameters during laying operations. As illustrated in Fig.7, the cable curves are almost consistent in the OXZ plane for various pre-tension value. While the influence is the sailing orientation is significantly for $T_o$ specified to be parallel to Y axis. For example, the Y coordinates values for different $T_o$ are 28.40m, 38.97m, 44.90m and 51.30m, which means the lower the pre-tension at the touch-down point, the flatter the cable configuration varies. Additionally, the numerical results clearly show that the tensional forces nearly increase linearly with the water.
The depth and approximately equals to \( T_e + \rho h \), which is quite different from the light and small diameter tele-communication cables.

**CONCLUSIONS**

In this study, a three-dimensional model is proposed to analyze the behaviors of submarine power cables with flexural rigidity and elasticity taken into consideration. The comprehensive effects of flexural rigidity, ocean current profile, CLV velocity, pay-out rates and material properties are investigated in the numerical study. Based on the numerical results, some significant conclusions are drawn:

1. The flexural rigidity has insignificant effect on the tension distribution and configuration curve, which indicates it’s practical to take submarine power cables as ideal flexural structures in the conventional analysis. While the self-weight does remarkably affect the cable’s properties during laying operations and the tension forces increase linearly with the increase of self-weight per unit length in water.

2. For the large diameter and self-weight of submarine power cables, the pay-out rate \( V_p \), cable laying vessel velocity \( V \) and current profile does remarkably affect cable configuration but are insignificant for the tensional force as illustrated in Fig. 5 (c) ~ Fig. 7 (c). In the tension analysis of submarine power cables, we can just consider the self-weight and neglect all the other environmental loadings.

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