A Distributed Inter-Phase Coordination Algorithm for Voltage Control with Unbalanced PV Integration in LV Systems

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Abstract—Most traditional Var compensation-based voltage regulation methods are developed following the single-phase Volt-Var response rule. These methods typically have competent voltage regulation performance with balanced photovoltaic (PV) integration. However, certain randomness of single-phase rooftop PV installation may lead to significant PV power imbalance across three phases, especially in low voltage (LV) distribution systems. In such unbalanced situations, unintended inter-phase Volt-Var response which is ignored in the single-phase Volt-Var response rule will become significant and greatly challenge the effectiveness of the traditional methods on voltage regulation. This can further cause inverter saturation and consequently makes distribution systems vulnerable to overvoltage problems. In this paper, the mathematical equations of unbalanced three-phase Volt-Var response are first derived and analyzed to identify the strong MVE (mutual Var compensation effect) and the weak MVE. This analysis provides the theoretical foundation for the development of the proposed inter-phase coordinated consensus algorithm, which can successfully overcome PV imbalance-induced voltage regulation challenges (e.g. inverter saturation and network overvoltage), while does not need exact system parameters. The effectiveness of this method has been validated by time-series simulations with a real LV distribution system and recorded data.

Index Terms—distributed control, inter-phase coordination, photovoltaic (PV), Var compensation, low voltage distribution systems, voltage regulation.

I. INTRODUCTION

ROOFTOP distributed photovoltaic (PV) generators are rapidly growing due to their environment-friendly nature and drop in cost. According to the Australian PV Institute [1], PV penetration of the Queensland state has exceeded 35.7% so far, and small size PV systems have a predominating role in the total installation capacity.

A. Literature Review

In order to mitigate the system voltage rise caused by PV power integration, PV inverters are requested to actively participate in the system voltage regulation through providing reactive power compensation. Most existing Var compensation methods such as fixed power factor schemes, power factor droop strategies [2, 3] and other model-free methods (including but not limited to [4, 5]) are developed according to the single-phase Volt-Var response rule. Namely, inverters are only designed to absorb reactive power for local overvoltage mitigation, while the interaction of reactive power flow from different phases on voltage regulation is not considered. These methods are capable of system voltage regulation when the PV power connection are roughly balanced.

However, the condition of balanced PV power integration is not always satisfied. According to the industry practice in Australia, PV systems with capacities less than 5kW are single-phase generators. Most of these small-size PV systems are installed on rooftops of residential customers as behind-meter generators, and the connection of a rooftop PV system is entirely dependent on the electricity price of the residential house. Therefore, with such randomness, unbalanced PV power integration across three phases is inevitable in distribution systems [6, 7]. Low voltage (LV) power networks in Australia are extensive three-phase systems, which generally provide electricity supply for several tens to more than one hundred residential customers. Such three-phase LV systems can be seen in [8] and [6]. With a small number of customers and limited diversity in LV systems, arbitrary connection of single-phase rooftop PV generators is very likely to induce significant PV power imbalance across three phases. In such unbalanced situations, traditional Var compensation methods which do not have proper inter-phase coordination design may become ineffective in overvoltage mitigation due to unintended voltage response of reactive power compensation across unbalanced three phases. It is worth noting, the distribution system structure is different in North America, where medium voltage (MV) systems are extensive and complex, but LV systems are short and single-phase. Generally, 4-5 houses share one single-phase distribution transformer, through which LV residential loads are connected to MV systems [9]. With such a system structure, imbalance within LV systems would not be an issue, and significant PV imbalance may occur in MV systems in North America.

Optimization is a widely used mathematic tool for coordination design. Some optimal dispatch algorithms for distributed generators such as in [10] are formulated as centralized optimization problems, targeting at directly calculating optimized operation points for all distributed PV inverters at each time step. However, centralized schemes are not robust to single point failures, and the required communication is generally complicated. Therefore, centralized optimization problems are decomposed and solved in a distributed way in [11, 12], which demonstrate a great advantage for dealing with a large number of distributed generators. Further, the intermittent...
generation of renewable energy resources is another challenge for system voltage regulation [13]. Therefore, to balance the optimal control performance and control response speed, various multi-layer control structures are proposed in recent publications [14-17]. Typically, the higher layer is designed for system level optimal coordination which needs global information; while the lower layer is responsible for real-time voltage regulation requiring only local measurements. In addition, optimal control schemes with uncertainty of renewable energy resources can be formulated as chance constrained optimization problems such as in [18] in order to achieve a robust dispatch. Finally in [19], optimization problems with different scenarios are solved in parallel in advance, and only one strategy will be used according to the real generation of the renewable energy resources.

While, most of optimization-based schemes need exact system parameters such as line impedance and customer load information in real time, which are not always available, especially in LV distribution systems. To reduce the dependence on system parameters, while remain certain coordination among distributed inverters, many rule-based [20, 21] and model-free [4, 5] schemes are proposed for voltage regulation in distribution systems with high PV penetration. The consensus algorithm is one of the model-free algorithms, which has been used to proportionally redistribute reactive power contribution [22, 23] or PV power curtailment [24] among all distributed PV inverters for overvoltage mitigation. So far, most of these methods are designed following the single-phase Volt-Var response rule, and typically only highlight the upstream-downstream coordination, while ignore the inter-phase coordination. Consequently, their voltage regulation performance can be significantly crippled due to unintended Volt-Var response across asymmetrical LV phases under PV imbalance scenarios.

The investigation of inter-phase coupled voltage response is mathematically based on the three-phase unbalanced distribution system model [25]. Due to unbalanced line coupling, a counterintuitive voltage response is demonstrated in [26], namely load increase does not necessarily cause voltage magnitude decrease. Further, unbalanced three-phase voltage drops caused by load imbalance and line configurations were qualitatively investigated through comprehensive simulation results in [27]. However, no mathematical analysis is provided. In [28], useful advices are provided for future distribution system planning in order to mitigate the adverse effect of PV imbalance induced voltage regulation challenges. However, [28] did not definitely identify the direction of the inter-phase Volt-Var interaction, and conclusions obtained in [28] mainly comes from specific case studies without rigorous mathematical derivations.

B. Contributions

The contributions of this paper mainly lie in two aspects:

1) The equations governing unbalanced three-phase Volt-Var response are mathematically derived, where voltage response is divided in an innovative way to self and mutual Var compensation components. Detailed derivation is given in Appendix. Such inter-phase coupled Volt-Var interaction is further verified by simulation results as in Fig. 1. This research essentially reveals the physical mechanism that cripples the effectiveness of traditional Volt-var methods in scenarios with unbalanced PV integration across three phases.

2) On the basis of derived three-phase Volt-Var response, the strong (the most influential) mutual Var compensation terms are identified for all phases, which later are utilized as a foundation for an innovative inter-phase coordination design to overcome the inverter saturation issue and hence the overvoltage problem caused by unbalanced PV integration. This design is further embedded into a dynamic consensus algorithm, which does not require the exact system parameters. Simulation results demonstrate the crippled voltage regulation performance of two traditional methods following the single-phase Volt-Var response rule in design, and the effectiveness of the proposed method with inter-phase coordination in PV imbalance scenarios.

II. SINGLE-PHASE VOLT-VAR RESPONSE RULE

In single-phase systems, voltage deviation $\Delta V_{ij}$ between Bus $i$ and Bus $j$ along a feeder can be approximately estimated as [in per unit (p.u.)]

$$\Delta V_{ij} \approx (r P_{ij} + x Q_{ij})/V_0 \approx r P_{ij} + x Q_{ij}$$ (1)

where $r$ and $x$ represent the line resistance and reactance respectively; $P_{ij}$ and $Q_{ij}$ are the active and reactive power flowing from Bus $i$ to Bus $j$ respectively; $V_0 (\approx 1p.u.)$ denotes the voltage magnitude at the beginning of a feeder. Therefore, for single-phase (or balanced three-phase) systems, reverse active (PV) power flow will cause voltage rise at ends of feeders; and Var compensation methods are designed to absorb reactive power from the grid for overvoltage mitigation. This is the single-phase Volt-Var response rule, which forms the fundamental of various Var compensation-based voltage regulation methods nowadays.

Following the single-phase Volt-Var response rule, once local overvoltage is detected by any PV inverter, the inverter is requested to absorb a certain amount of reactive power from the grid in order to eliminate this overvoltage.

$$\Delta Q_{local,i}(t) = \alpha [V_i(t) - V_{max}], \quad V_i(t) > V_{max} \quad (2a)$$

$$Q_{PV,i}(t + \Delta t) = Q_{PV,i}(t) + \Delta Q_{local,i}(t) \quad (2b)$$

where $\Delta Q_{local,i}(t)$ and $V_i(t)$ represent the variation of local reactive power absorption and local detected voltage respectively at time instant $t$ on Bus $i$; $V_{max}$ denotes the upper limit of system voltage; $\Delta t$ is a small time interval for local voltage regulation, which is typically in a time scale of several tens of milliseconds; $\alpha$ is its gain [22, 29].

III. DYNAMIC CONSENSUS ALGORITHM IN VOLTAGE REGULATION

Bidirectional power flows occur as the PV penetration becomes higher and higher, which exposes distribution systems to the risk of overvoltage. Such overvoltage issues are most likely to occur at PV connection points, and PV inverters are requested to provide Var compensation if local overvoltage risk is detected. However, the Var compensation burden of PV inverters are typically very uneven in radial distribution feeders. Some inverters may already become saturated, while at the same time, others may not take part in the system voltage regulation at all due to their relatively healthy local voltage.

In order to share the voltage regulation burden among all PV inverters, consensus algorithms have been used for Var generation redistribution. Traditional static consensus algorithms such as in [22, 23] assumes constant PV power (Var...
compensation) before the converge of the algorithm. While, a dynamic consensus algorithm originally proposed in [30] is adopted in this paper, which is able to track the average of a set of time-varying signals (i.e. reactive power utilization ratios of inverters in this paper) in a multi-inverter network, namely

\[ u^{ave}(k) = \frac{1}{N} \sum_{i=1}^{N} u_i(k) \quad (3) \]

where \( u_i(k) \) is the reference signal of inverters on Bus \( i \) at time step \( k \); \( u^{ave}(k) \) represents the average of all signals at time step \( k \); \( N \) denotes the number of inverters. The adopted dynamic consensus is presented as below [30]:

\[ z_i(k + 1) = z_i(k) - \delta_1 \delta_2 z_i(k) - \delta_1 \delta_3 \sum_{j=1}^{N} L_{ij} \left( z_j(k) + u_j(k) \right) - \delta_1 v_i(k) \quad (4a) \]

\[ v_i(k + 1) = v_i(k) + \delta_1 \delta_3 \sum_{j=1}^{N} L_{ij} \left( z_j(k) + u_j(k) \right) - \delta_1 \bar{\rho}_i(k) \quad (4b) \]

where \( z_i \) and \( v_i \) are auxiliary variables of Bus \( i \); \( \delta_1, \delta_2, \delta_3 \) are step sizes; \( L_{ij} \) represents the element of Laplacian Matrix \( \mathcal{L} \) in row \( i \) column \( j \); \( \bar{\rho}_i(k) \) denotes the consensus state of Bus \( i \) tracking \( \frac{1}{N} \sum_{k=1}^{N} u_k(k) \). In this paper, the time-varying reference signal \( u_i(k) \) is set to be the reactive power utilization ratio \( \rho_i \) at time step \( k \), namely [31]

\[ \rho_i(k) = \frac{Q_{PV,i}(k)}{\rho_{ave,i}} \quad (5) \]

For the time step \( k + 1 \), the Var generation \( Q_{PV,i} \) of PV inverters on Bus \( i \) is given as

\[ Q_{PV,i}(k + 1) = \bar{\rho}_i(k + 1) C_{PV,i} + \Delta Q_{local,i} \quad (6) \]

where the first term \( \bar{\rho}_i(k + 1) C_{PV,i} \) on the right side represents the component of upstream-downstream coordination by the dynamic consensus algorithm; while the second term \( \Delta Q_{local,i} \) on the right side represents the variation of Var generation for local voltage regulation [following equation (2)] during the interval of time step \( k \) and \( k + 1 \). It is worth noting, it is infeasible for distributed PV inverters to have a zero-error perfect tracking performance with time-varying stochastic signals since it takes time for information to propagate throughout the network [32] in real life applications. The steady-state error of the adopted dynamic consensus algorithm is provided by the Theorem below.

**Theorem [30]:** For strongly connected and weight-balanced\(^2\) graph \( \mathcal{G} \), if the variations of the time-varying reference signal matrix \( \Delta u \) satisfies \( \| \Pi_N \Delta u \|_{ess} = \gamma < \infty, \) for any \( \delta_3 > 0 \) and \( \delta_1 \in \left( 0, \min \left\{ \delta_2^{-1}, \delta_3^{-1} (d_{\text{out}}^{\text{max}})^{-1} \right\} \right) \), the above dynamic consensus algorithm initialized with \( z_i(0), v_i(0) \in \mathbb{R} \) and \( \sum_{j=1}^{N} u_j(0) = 0 \) has an upper-bounded steady-state error:

\[ \lim_{k \to \infty} \left[ \rho_i(k) - \frac{1}{N} \sum_{j=1}^{N} u_j(k) \right] \leq \left( \delta_1 \delta_3 \lambda_2 \right)^{-1} \gamma, \quad i \in \{1, \ldots, N\} \]

where \( \Pi_N \) is defined as \( \Pi_N = I_N - \frac{1}{N} 1_N 1_N^T \) with \( I_N \) a \( N \times N \) identity matrix and \( 1_N \) a vector of \( N \) ones; \( \| u \|_{ess} \) denotes the

\(^{1}\) A weighted directed graph can be represented as \( \mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathbf{A}) \), where \( \mathcal{V} = \{1, \ldots, N\} \) is the set of nodes and \( \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \) is the set of edges. \( \mathbf{A} \in \mathbb{R}^{N \times N} \) is a weighted adjacency matrix, with \( a_{ij} > 0 \) if \( (i,j) \in \mathcal{E} \) and \( a_{ij} = 0 \), otherwise. The out-degree matrix \( \mathbf{D}^{\text{out}} \) is a diagonal matrix with \( d_{\text{out}}^{\text{max}}(i) = \sum_j a_{ij} \) for all \( i \in \mathcal{V} \). The (out) Laplacian Matrix \( \mathbf{L} \) is defined as \( \mathbf{L} = \mathbf{D}^{\text{out}} - \mathbf{A} \).

\(^{2}\) A graph is weight balanced, if at each node \( i \in \mathcal{V} \), the weighted out-degree \( d_{\text{out}}^{\text{off}}(i) = \sum_j a_{ij} \) and the weighted in-degree \( d_{\text{in}}^{\text{off}}(i) = \sum_j a_{ji} \) coincide.

### IV. INTER-PHASE COORDINATION DESIGN

Australian LV distribution systems are unbalanced in nature. Further, arbitrary phase connection of rooftop PV generators makes system imbalance even more significant. Due to unbalanced line coupling, Var compensation will not only influence its local voltage variation, but also have a significant impact on the voltage regulation of other phases (inter-phase impact). Such inter-phase Volt-Var response is roughly offset by each other in balanced situations, but it becomes non-negligible with unbalanced PV integration. Var compensation from different phases may adversely affect each other on voltage regulation if inter-phase coordination is not properly designed. As a result, the effectiveness of Var compensation schemes can be significantly crippled on voltage regulation.

**A. Mathematical Analysis of Three-Phase Volt-Var Response**

In unbalanced situations, the widely used single-phase Volt-Var response rule as described in (1) becomes insufficient for Var compensation design. In order to consider the impact of inter-phase Volt-Var response induced by unbalanced line coupling on voltage regulation, generalized three-phase Volt-Var response equations are mathematically derived as in (7) ~ (9), where \( r^{h,g} \) and \( x^{h,g} \) \((h,g \in \{a,b,c\})\) represent the real and imaginary part of the mutual impedance among Phases \( h \) and \( g \); \( P_i^h \) and \( Q_i^h \) are active and reactive power of Phase \( h \) flowing from upstream Bus \( i \) to downstream Bus \( j \) respectively [positive values represent forward power flow (absorption)]; \( \Delta V_i^h \) denotes the voltage magnitude deviation between Bus \( i \) and Bus \( j \) on Phase \( h \) (positive values represent voltage drop). This derived three-phase Volt-Var response equations will be the foundation of the inter-phase coordination design in this paper. Please refer to the Appendix Part A for the detailed derivation.

Since active power (PV generation) is regarded as uncontrollable in Var compensation design, only the voltage response of reactive power is considered in this section. The terms \( \chi^{h,g} Q_i^h (h, g \in \{a, b, c\}) \) on the right side of the three-phase Volt-Var response equations [(7) ~ (9)] have a similar form to its counterpart in the single-phase Volt-Var response rule (1). These terms represent the voltage response of Var compensation from its own phase. Therefore, the same as in single-phase (balanced three-phase) situations, inverters absorbing (injecting) reactive power can reduce (increase) their local voltage. Such Var compensation effect is called self Var compensation effect (SVE) in this paper. While, due to line coupling, the system voltage on one phase can also be influenced by reactive power flows from the other two phases, as shown in the terms with square brackets in (7) ~ (9). Such interaction cannot be ignored, especially in LV distribution systems with unbalanced PV installation and substantial line coupling across phases. In this paper, the effect of Var compensation that contributes to the voltage variation on other phases is called mutual Var compensation effect (MVE). Therefore, the total voltage response of Var compensation is the combined effect of both SVE and MVE.
Different from SVEs which always have positive signs, the signs of MVEs in the three-phase Volt-Var response rule are dependent on the phase sequence. For example, in the voltage response of Phase A as in (7), inductive MVE (absorption) from Phase B will induce a voltage rise on Phase A [i.e. \((\frac{-\sqrt{3} r_{ab}}{2} - \frac{x_{ab}}{2}) Q_{ij}^{ab} \) in (7)], since \(x_{ab}\) and \(r_{ab}\) are always positive (this applies to all overhead lines of real distribution systems). Therefore, if a voltage rise on Phase A needs to be mitigated, except for having inductive SVE (absorption) on its own phase (Phase A), an alternative way is making capacitive MVE (injection) on Phase B. Whilst, the impact of MVE from Phase C on the voltage response of Phase A can be described as \((\frac{-\sqrt{3} r_{ac}}{2} - \frac{x_{ac}}{2}) Q_{ij}^{ac} \) in (7). If \(\frac{\sqrt{3} r_{ac}}{2} > \frac{x_{ac}}{2}\), inductive MVE from Phase C will reduce the voltage of Phase A; otherwise, the inductive MVE will induce a reverse voltage response (increase) on Phase A. Therefore, the sign of MVE from Phase C to Phase A depends on specific system parameters. It is worth noting, for the same amount of Var compensation, MVE from Phase C has much weaker impact on the voltage regulation of Phase A than that of MVE from Phase B. This is because the absolute value of \((\frac{-\sqrt{3} r_{ac}}{2} - \frac{x_{ac}}{2})\) is much smaller than that of \((\frac{-\sqrt{3} r_{ab}}{2} - \frac{x_{ab}}{2})\).

Further, Phase B and Phase C have similar voltage response structures as that on Phase A, namely one SVE, one strong MVE and one weak MVE, as shown in (8) and (9) respectively. Finally, the three-phase Volt-Var response rule is summarized as in Table I, where an upside arrow shows an increase and a downside arrow means a decrease; sign “\(\cdots\)” represents parameter-dependent Volt-Var response (i.e. weak MVE), which will not be used in the inter-phase coordination design.

**Fig. 1** demonstrates the three-phase Volt-Var responses of a real LV distribution system obtained from numerical simulations, which shows the same voltage variation tendencies as described in (7) ~ (9). For example in Fig. 1 (a), inductive \(Q_{ij}^{ab}\) can significantly reduce the voltage of Phase A \([x_{ab} Q_{ij}^{ab} \text{ (SVE)}]\) and increase the voltage of Phase C \([-(\frac{\sqrt{3} r_{ca}}{2} - \frac{x_{ca}}{2}) Q_{ij}^{ac}]\) (strong MVE). However, its voltage response on Phase B is trivial \([(\frac{\sqrt{3} r_{ba}}{2} - \frac{x_{ba}}{2}) Q_{ij}^{ab}]\) (weak MVE).

**B. Inter-Phase Coordination Design**

Following the three-phase Volt-Var response rule as in (7) ~ (9) and Table I, the inter-phase coordinated Var compensation scheme is developed in this section, and both SVE and strong MVE will be used for voltage regulation. For example, if an overvoltage issue on Phase A cannot be eliminated by its own SVE anymore due to local Var generation saturation, a request of Var retreat (RVR) signal will be sent to inverters on Phase B (at the same bus) as shown in Fig. 2. According to the three-phase Volt-Var response rule, reducing Var absorption (or increasing Var injection) on Phase B can mitigate the voltage rise on Phase A [strong MVE \((\frac{\sqrt{3} r_{ab}}{2} - \frac{x_{ab}}{2}) Q_{ij}^{ab} \) in (7)]. Similarly, following the derived response structure, if overvoltage issues cannot be eliminated locally on Phase B and Phase C, inverters on these two phases will also seek help from Phase C and Phase A respectively, as shown in Fig. 2.

![Fig. 2 Request of Var retreat (RVR) signal flow.](image-url)

**Fig. 1** Volt-Var responses in a three-phase unbalanced system (a) only reactive power absorption from Phase A \((Q_{ij}^{ab})\), (b) only reactive power absorption from Phase B \((Q_{ij}^{ab})\), (c) only reactive power absorption from Phase C \((Q_{ij}^{ac})\).
This inter-phase coordination design is integrated with the real-time local voltage regulation, with a flow chart shown in Fig. 3. Once local overvoltage ($V_{\text{local}} > V_{\text{max}}$) is detected, the SVE will be immediately triggered, and inverters start to absorb reactive power for local overvoltage mitigation. Namely, SVE will be given the priority for voltage regulation. Once Var generation become saturated while local overvoltage problems still exist, inverters will emit RVR signals following the phase sequence as specified in Fig. 2. In such situations, inverters will ignore any received RVR signals from other phases, until their local overvoltage is eliminated.

Once received RVR signals, inverters will execute the MVE service as long as its local voltage is healthy ($V_{\text{local}} < V_{\text{thr}} \leq V_{\text{max}}$) and its Var generation is not saturated. According to the three-phase Volt-Var response rule, the MVE service can mitigate the voltage rise of the targeted phase while increase the local voltage at the same time. Therefore, inverters should avoid local overvoltage during the period of providing MVE service. The Var generation for MVE service is designed as below

$$\Delta Q_{\text{int},i}(t) = \beta (V_{\text{thr}} - V_{\text{local},i}(t)) C_{PV,i}^h \quad (10a)$$
$$Q_{PV,i}(t + \Delta t) = Q_{PV,i}(t) + \Delta Q_{\text{int},i}(t) \quad (10b)$$

where $\Delta Q_{\text{int},i}(t)$ is the Var variation for MVE service on Bus $i$ Phase $h$ at time instant $t$; $C_{PV,i}^h$ means the inverters capacity on Bus $i$ Phase $h$; $\beta$ represents the step size which is set to be 3 in this paper with $V_{\text{thr}}$ and $V_{\text{local},i}^h$ in p.u. and $C_{PV,i}^h$ in kW. While, if inverters that received RVR signals have local voltage higher than voltage threshold $V_{\text{thr}} (V_{\text{thr}} \leq V_{\text{max}})$, the received RVR signals will not be executed, but passed to inverters on the next phase according to the phase sequence in Fig. 2. An example is provided in case studies for further explanation. To avoid duplication, the control logic as in Fig. 3 is fully demonstrated only on Phase A (with similar control logic on both Phases B and C).

![Fig. 3 Flow chart of inter-phase coordination design.](image)

### C. Inter-Phase Coordinated Consensus Algorithm

The proposed inter-phase coordinated consensus algorithm in this paper is a distributed voltage control scheme, which does not need exact system parameters. Specifically, the local Var compensation (i.e. SVE service) is responsible for real-time local voltage regulation; the inter-phase coordination (i.e. MVE service) is designed to adjust Var generation in order to avoid adverse Volt-Var response across phases; while the dynamic consensus algorithm is for rapid propagation of the Var generation through the whole network. Finally, the Var generation of PV inverters on Bus $i$ Phase $h$ is given as below,

$$Q_{PV,i}(k + 1) = \rho_i(k + 1)C_{PV,i}^h + \Delta Q_{\text{int},i}^h + \Delta Q_{\text{local},i}^h \quad (11)$$

where $\Delta Q_{\text{int},i}^h$ and $\Delta Q_{\text{local},i}^h$ represent the variation of Var generation for MVE service and local voltage regulation during the interval of time step $k$ and $k + 1$.

Different from traditional consensus algorithms that only aim at upstream-downstream coordination, the proposed method is designed to achieve both upstream-downstream coordination and inter-phase coordination in a distributed manner. As shown in Fig. 4, the RVR signals are sent/received among inverters which are installed on the same bus but in different phases for the inter-phase coordination. While, the consensus signals are exchanged between inverters installed on the same phase but different buses for the upstream-downstream coordination. The consensus signal will bypass houses that do not have rooftop PV generators, and exchange the information with the nearest neighbors on the same phase.

![Fig. 4 RVR signals and consensus signals in the proposed method.](image)

### D. Discussion

In the current practice, rooftop PV systems are installed behind electricity meters. Therefore, the rooftop PV system and the residential load are connected to the same phase for one house. If the utility tries to mitigate the PV imbalance through changing the phase of PV connection, individual electricity meters are needed for PV systems, which requires extra investment. In addition, it may be complicated to change the connection of PV and load together in order to save one electricity meter, as this may change the load balance in the night-time when PV generation is zero.

The proposed method follows the three-phase Volt-Var response in design, which makes this method competent in all possible PV imbalance scenarios. In modern systems, smart meters typically have the function of communication. Therefore, only a minor revision is needed for smart meters in order to execute the proposed algorithm.

### V. CASE STUDIES

A real 415V LV distribution system in Australia (shown in Fig. 5) with recorded data [PV power, load demand and upstream voltage as shown in Fig. 6 (a)–(c) respectively] are used for time-
series simulations in this section. The red triangle in Fig. 5 represents the MV/LV transformer with delta-grounded wye connection. The line impedance matrix (\(\Omega/Km\)) is provided in Appendix Part B. There are 41 customers have rooftop PV generators according to the record. PV generators from 3 customers are connected to the LV grid through three-phase inverters, and other 38 customers are equipped with signal-phase PV inverters. In total, the PV installation capacities are 48kWp, 34kWp and 78kWp in Phase A, Phase B and Phase C respectively. However, due to privacy issues, the utility cannot release more detailed data of each residential customers. In simulations of this paper, PV capacities are assumed to be equally installed on the rooftop of each residential house connected to the same phase. In addition, three-phase load profiles as in Fig. 6 (b) are also aggregated data, which are recorded from the secondary side of the MV/LV distribution transformer, and the upstream voltage as in Fig. 6 (c) is assumed to be roughly balanced. The voltage regulation performance on Bus 16 is selected for the comparison of different Var compensation methods, since Bus 16 is the most remote bus and it is therefore vulnerable to overvoltage problems. According to the Australia Standard AS60038 [33], the upper limit of line-to-neutral voltage is 253V (=1.0542pu) for 240/415V LV systems.

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fig. 5 A real 415V LV distribution system with 101 customers in Australia.
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The time interval of each consensus iteration as in (4a) to (4c) is provided in Fig. 5 is 4 seconds in this paper. Since each iteration of the consensus algorithm only needs simple algebraic operations and limited communication, 4-second is more than enough for all possible computation and communication time delay in real life applications.

### A. Power Factor Droop Curve

Power factor droop curves are widely used Var compensation methods for voltage regulation suggested by standards [2, 3]. Fig. 7 demonstrates a typical power factor droop curve (\(V_{\text{nom}}=0.95\text{pu}, V_{\text{high}}=1.05\text{pu}\)), through which an inverter can adaptively adjust its power factor according to the detected local voltage. Specifically, inverters will operate with lagging power factors (absorb reactive power) if their local voltage is high; while inverters are required to inject reactive power when local voltage is low. In addition, the power factor of inverters should not be lower than 0.9 [2].

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fig. 7 Power factor droop curve.
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Fig. 8 demonstrates the three-phase voltage profiles as well as the corresponding power factor profiles if the power factor droop curve as shown in Fig. 7 is applied on all PV inverters for voltage regulation. Following the power factor droop curve, all inverters operate with lagging power factors (negative power factors in this figure) during the noon period. As shown in Fig. 8 (a), the voltage of Phase B frequently violates the upper limit (253V), which exposes PV inverters on Phase B to the risk of disconnection due to high voltage. For example, the voltage on Phase B exceeds its upper limit at 10:40 and keeps increasing afterwards. However, the power factor of inverters on Phase B has already reached its lowest allowable value (-0.9) during this period [Fig. 8 (b)]. Consequently, voltage rise on Phase B cannot be sufficiently compensated by its local Var absorption anymore, which makes Phase B vulnerable to overvoltage problems.

### B. Traditional Consensus Algorithm

Crippled voltage regulation performance can also be observed with the consensus method without inter-phase coordination design as shown in Fig. 9 (a). Following the single-phase Volt-Var response rule as in (1), inverters are only expected to absorb...
reactive power (operating with lagging power factors) for overvoltage mitigation. Please refer to Section II for the detailed scheme. Since the consensus algorithm aims to redistribute Var generation among all inverters proportional to their installation capacities, power factor profiles as shown in Fig. 9 (b) are overlapped at the most of time. Compared with Fig. 8, the traditional consensus method tends to drive inverters down to lower power factors, however, this does not provide a better voltage regulation performance.

The crippled performance of Var compensation-based methods in voltage regulation as in Fig. 8 (a) and Fig. 9 (a) is caused by unintended inter-phase Volt-Var responses according to the established mathematical model in (7) ~ (9). However, such inter-phase Volt-Var responses is ignored in the traditional single-phase Volt-Var response rule. As in this unbalanced case, the extremely high voltage on Phase B as in Fig. 8 (a) and Fig. 9 (a) comes not only from the active PV power injection to its own phase, but also from the improper Var compensation on other phases. Specifically, large amount of PV inverters (78kWp) operating with lagging power factor (absorb Q) on Phase C can cause a significant voltage rise on Phase B [refer to the strong MVE \( \frac{-\sqrt{3}r_{bc}}{2} \frac{x_{bc}^2}{2} Q_{ij}^c \) in (8)]. While, the PV installation capacity on Phase B is relatively low (34kWp), which cannot provide sufficient Var support to compensate local voltage rise. As a result, overvoltage problems on Phase B become inevitable in such situations. Therefore, inter-phase coordinated Var compensation is indispensable for voltage regulation with unbalanced PV integration across three phases.

As designed, RVR signals are emitted by the inverters on Phase A to the inverters on Phase C for MVE service request. Unfortunately, the local voltage of Phase C is already very close to its upper limit (\( V_{local} > V_{ther} \), refer to Fig. 3) at this moment, and providing MVE service will further increase its local voltage. Therefore, the inverters on Phase C pass RVR signals to the inverters on Phase A. Since the voltage at Phase A is far from its upper limit at 12:17:00, the inverters on Phase A can immediately reduce its Var absorption (or increase Var injection), which will mitigate the voltage rise on Phase C and increase the voltage on Phase A. With the help of MVE from Phase A, the inverters on Phase C can further reduce its local Var absorption (namely providing MVE for Phase B) without inducing overvoltage on its own phase. Meanwhile, the consensus algorithm continuously instructs the Var generation to upstream inverters to achieve the upstream-downstream coordination. Fig. 10 (c) shows the power factor profiles of upstream inverters on Bus 5, which are similar to those of downstream inverters on Bus 16 as in Fig. 10 (b).

As shown in Fig. 6 (a), the PV power keeps increasing from 12:17:00 to 12:17:40 without inducing any voltage violation problem under the control of the proposed method. During this period, the inverters on Phase A significantly reduce power factor from 0.99 leading to 0.9 leading; while the inverters on Phase C slightly increase power factor from 0.95 lagging to around 0.97 lagging as in Fig. 10 (b). This combined effect prevents the overvoltage problem on Phase B, and meanwhile it increases the voltage on Phase A and makes the voltage on Phase C almost unchanged. It is worth noting, PV systems gradually reduce their generation in the period from 3 pm to 6 pm, and at the same time, load demand grows to a higher level approaching to its evening peak. During this period, load flow changes its direction from reverse power flow to forward power flow. The change of power flow direction will not challenge the effectiveness of the proposed method as shown in Fig. 10 (a).

In addition, inverters are assumed to have the capacity for Var compensation, and operating with 0.9 power factor will not lead to the overload issue of inverters. Further, the 0.9 power factor
limit is one kind of widely used Var compensation scheme for voltage regulation suggested by the AS/NZS 4777.2 standard [2] and the IEEE 1547 standard [3]. As shown in Fig. 10 (b) and (c), power factors of all inverters are restricted within the range from 0.9 leading to 0.9 lagging.

Different from traditional methods where inverters are only designed to absorb reactive power for overvoltage mitigation, inverters on Phase A are requested to inject reactive power in this unbalanced scenario for inter-phase coordinated voltage regulation. Such Var compensation design goes beyond the field of the traditional single-phase Volt-Var response rule.

VI. CONCLUSION

With random connection of rooftop photovoltaic (PV) generators, PV penetration tends to be unbalanced across three phases, especially in low voltage (LV) distribution systems. In such situations, the traditional single-phase Volt-Var response rule becomes insufficient for Var compensation design due to ignoring the inter-phase Volt-Var response which is non-negligible in unbalanced scenarios. In this paper, new equations of unbalanced three-phase Volt-Var response are mathematically formulated and innovatively decomposed into self Var compensation effect (SVE), strong mutual Var compensation effect (MVE) and weak MVE components. This has formed the theoretical base for the design of the proposed inter-phase coordinated consensus method, which can effectively solve the inverter saturation and network overvoltage issues caused by unbalanced PV integration in distribution LV systems. In this method, exact system parameters are not required.

Time-series simulations based on a real LV distribution system and recorded data demonstrate the voltage regulation challenges of the traditional schemes and the effectiveness of the proposed method. Due to a lack of inter-phase coordination in the traditional power factor control and consensus schemes, Var compensation from different phases will adversely affect each other through uneven network coupling under PV imbalance scenarios, resulting in inverter saturation and extremely high voltage on one phase (Phase B as shown in Fig. 8 and Fig. 9). Consequently, the effectiveness of the traditional methods on overvoltage mitigation is greatly impaired. While, the proposed method properly utilizes the coupled Volt-Var response across phases (strong MVE services from Phase A and Phase C). Through such inter-phase coordination, inverter saturation in Phase B is effectively reduced, and hence the Phase B voltages are successfully controlled within the allowable range (as shown in Fig. 10). Control schemes of storage systems considering their characteristics are successfully controlled within the allowable range (as shown in Fig. 10). Control schemes of storage systems considering their characteristics are successfully controlled within the allowable range (as shown in Fig. 10).

VII. APPENDIX

A. Derivation of Three-Phase Volt-Var Response Equations

For three-phase LV distribution systems, the line voltage drop between Bus \(i\) and Bus \(j\) can be expressed as [25],

\[
\begin{bmatrix}
V_i^a \\
V_i^b \\
V_i^c
\end{bmatrix} = \begin{bmatrix}
V_j^a \\
V_j^b \\
V_j^c
\end{bmatrix} + \begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ba} & Z_{bb} & Z_{bc} \\
Z_{ca} & Z_{cb} & Z_{cc}
\end{bmatrix} \begin{bmatrix}
I_j^a \\
I_j^b \\
I_j^c
\end{bmatrix}
\]  

(A1)

where \(V_i^h (h \in \{a, b, c\})\) represents the line-to-neutral voltage phasor at Phase \(h\) Bus \(i\); \(V_j^h (h \in \{a, b, c\})\) is the line current phasor of Phase \(h\) flowing from Bus \(i\) to Bus \(j\); \(Z_{hg} (h, g \in \{a, b, c\})\) denotes self or mutual impedances among three phases. For Phase A, (A1) can be rewritten as (A2) for convenience.

\[
V_i^a = V_j^a + Z_{aa}I_j^a + Z_{ab}I_j^b + Z_{ac}I_j^c
\]  

(A2)

Multiplying both sides of the equation (A2) by its own conjugates, (A3) is obtained.

\[
|V_i^a|^2 = |V_j^a|^2 + \Delta_a + (\Delta_a^*) + \Delta_a
\]  

(A3)

where

\[
\Delta_a = V_j^a(Z_{aa}I_j^a + Z_{ab}I_j^b + Z_{ac}I_j^c)^* \\
\Delta_a^* = |Z_{aa}I_j^a + Z_{ab}I_j^b + Z_{ac}I_j^c|^2
\]  

(A4)

In the following, \(\Delta_a\) and \(\Delta_a^*\) in (A3) will be discussed separately. In \(\Delta_a\),

\[
\Delta_a = (r_{aa} - jx_{aa})(P_{ij}^a + jQ_{ij}^a) + (r_{ab} - jx_{ab})(P_{ij}^b + jQ_{ij}^b)
\]

\[
+ (r_{ac} - jx_{ac})(P_{ij}^c + jQ_{ij}^c)
\]  

(A5)

In order to remove the nonlinear terms \(\frac{v_{ij}^a}{V_j^a}\) and \(\frac{v_{ij}^a}{V_j^a}\) in (A8), two assumptions are applied:

1) Due to short line length and small power flow, the difference of voltage angles along an LV distribution feeder is negligible, namely

\[
|V_i^h| \approx |V_j^h|, |V_i^h| \approx |V_j^h| |∠0, |V_i^h| \approx |V_j^h| |∠−120°, |V_i^h| \approx |V_j^h| |∠−120° (A9)
\]

2) For a same instant, magnitude mismatch of line-to-neutral voltages on one bus are typically within 5%, therefore

\[
|V_i^h| \approx |V_j^h| (h, g \in \{a, b, c\})
\]  

(A10)

With these two assumptions, \(\frac{v_{ij}^a}{V_j^a}\) and \(\frac{v_{ij}^a}{V_j^a}\) in (A8) can be approximately expressed as \(\frac{v_{ij}^a}{V_j^a} \approx 1∠120°, \frac{v_{ij}^a}{V_j^a} \approx 1∠−120°\). On this basis, \(\Delta_a\) can be expressed with its real and imaginary parts separately. It is worth noting, the term \(\Delta_a^*\) in (A3) only contains \(\Delta_a^*\), which

\[
\Re(\Delta_a^*) = \Re[(\Delta_a^*)^*] \in (A3) only has its real part, and \Re(\Delta_a^*) = \Re[(\Delta_a^*)^*] \in (A3) only has its real part, and \Re(\Delta_a^*) = \Re[(\Delta_a^*)^*] always holds. Therefore, only the real part of \(\Delta_a^*\) is calculated here.

\[
\Re(\Delta_a^*) = (r_{aa}P_{ij}^a + x_{aa}Q_{ij}^a) + (r_{ab} - \frac{x_{ab}}{2})(P_{ij}^b + jQ_{ij}^b)
\]

\[
+ (r_{ac} - \frac{x_{ac}}{2})(P_{ij}^c + jQ_{ij}^c)
\]  

(A11)

While, for \(\Delta_a^*\) as in (A5), it can be further expressed as

\[
\Delta_a^* = |Z_{aa}I_j^a + Z_{ab}I_j^b + Z_{ac}I_j^c|^2 = |\Delta_{ij}^*|^2 = (\Delta_{ij}^*)^2\]  

(A12)

Therefore, \(\Delta_a^*\) is a higher order quantity of \(\Delta_{ij}^*\) (voltage magnitude deviation between Bus \(i\) and Bus \(j\) on Phase \(a\)), which can be ignored with an acceptable mismatch [35, 36] in order to remove its nonlinearity. Consequently, (A3) can be approximately expressed as

\[
|V_i^a|^2 \approx |V_j^a|^2 + 2\Re(\Delta_a)
\]  

(A13)
Further, considering $|V_i|^2 + |V_j|^2 \approx 2$ in per unit, $|V_i|^2 - |V_j|^2 = ((|V_i|^2 + |V_j|^2)(|V_i|^2 - |V_j|^2)) \approx 2(|V_i|^2 - |V_j|^2)$ Therefore, (A13) can be finally converted to $\Delta V|^2 = |V_i|^2 - |V_j|^2 \approx \Im(\Delta^2)$ (A14) which is exactly the Volt-Var response equation as given in (7) for Phase A. Coupled voltage response on Phase B and Phase C as in (8) and (9) can also be similarly derived. Simulation results as in Fig. 1 further verify the correctness of the derived three-phase Volt-Var response rule. Namely, the derived analytical model and computer simulation results have same voltage variation tendencies.

B. Overhead Line Impedance Matrix

$$
\begin{bmatrix}
0.4405 + j0.6565 & 0.1439 + j0.3442 & 0.1379 + j0.3118 \\
0.1439 + j0.3442 & 0.4168 + j0.6792 & 0.1275 + j0.3717 \\
0.1379 + j0.3118 & 0.1275 + j0.3717 & 0.4065 + j0.6893
\end{bmatrix}
$$

VIII. REFERENCES


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