Lack of numeration and multiplication conceptual knowledge in middle school students: a barrier to the development of insight into school mathematics?

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Published

2005

Conference Title

Stimulating the "Action" as Participants in Participatory Research

Lack of Numeration and Multiplication Conceptual Knowledge in Middle School Students: A Barrier to the Development of High School Mathematics?

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A general assumption about teaching and learning numeration and multiplication is that these basic, fundamental concepts and processes are learned in primary school. Thus, many high school curricula tend to proceed straight to higher mathematics learning and abstract reasoning based on these ideas irrespective of students' readiness. Consequently, a lack of multiplicative reasoning, which requires a clear conceptual understanding and full knowledge of mathematical processes and the relationships among them, appears to be a major cause of difficulties with further mathematics (Siemon, 2004). In a previous study conducted in a Brisbane state high school situated in a socio-economically disadvantaged area Seah (2004) found that many students demonstrated very limited understanding of the multiplication concepts, with their knowledge restricted to procedural rather than conceptual understanding. This follow-up study indicates that a majority of beginning Year 8 students are yet to achieve Level 3 Mathematics Outcomes (Queensland Studies Authority, 2004), two years below the expected level. Their knowledge is insufficient to develop further mathematics nor are they able to use it to solve everyday problems.

Introduction

The need to think mathematically has become essential for students (Booker, 1998) in a new millennium "awash in numbers" and "drenched with data" (Steen, 2001). Today's society is inundated with quantitative information ranging from statistical reports on the daily newspaper headlines to numbers used as part of the marketing strategy to attract consumers, not forgetting the quantitative thinking required in workplaces. For students, the influence of globalisation and internationalisation, interest in popular music and fashion, attitudes to sex, drugs and alcohol, unemployment and international unrest often take precedence over school work. At the same time, the break down of families forces many children to deal with complex social issues beyond what was once expected. With this continual escalation of economic and political instability, violence and poverty, mathematics education too must transform to meet the needs of today's young people. Yet, schools have been criticised for failing to prepare students for the workforce, and many middle school students are disengaged from school in general and mathematics in particular.

Middle schooling refers to the educational provision for students between 10 to 14 years old who are by nature intensely curious, egocentric, social and active. They need a curriculum that challenges them to think, discuss, and solve problems related to their lives (Pitton, 2001). In reality, the Queensland School Reform Longitudinal Study [QSRLS] (Education Queensland, 2001) found that most middle years of schooling in Australia remain subject-centric, with low levels of productive pedagogy and curriculum integration and with low-stakes assessment regimes. Further, students with continuing difficulties in literacy and numeracy found the transition from primary to secondary schooling especially problematic (Education Queensland, 2003). Although a lack of multiplicative thinking/reasoning ability has been found to be a cause of students' difficulties with higher mathematics (Booker, 1998, 2003; Siemon, 2004), further study is needed to determine other factors that influence students' learning. Behind this investigation is an assumption that the focus on sense making in mathematics has been lost in school classrooms and that a failure to use students' mathematical knowledge as a starting point for making curriculum and instruction decision contributes to students' difficulties in mathematics.

Conceptual knowledge of numeration and multiplication

Number is an abstract entity that describes quantity through the use of symbols. The arbitrary nature of these symbols means that each has to be constructed anew by individual learners (Russell, 1999). The study of numeration requires an understanding of number concepts and notation in order to name, write, read, interpret and process numbers, in particular a secure knowledge of place value and renaming. Place value refers to the written form of a number where the value of any digit in the whole number (or decimal fraction) is determined by its position in that number. Renaming refers to naming a set of numbers according to its value. For example, 356 can be read as 356 ones, 35 tens and 6 ones or 3 hundreds and fifty-six ones, etc. Renaming is fundamental to students' understanding of processes for larger numbers and decimal fractions as they work through various operations. It is critical that these concepts and processes that give meaning to numbers be well established before further mathematical concepts are developed.

Multiplication begins with the need to 'group' together various numbers of objects using repeated addition while bearing in mind possible misconceptions as it is qualitatively different from additive thinking. From a mathematical point of view, multiplication of integers and rational numbers may be considered relatively simple. Nevertheless, it can be modelled by many distinguishable classes of situations and its product is not always a larger number (Greer, 1992). Thus, a fundamental conceptual restructuring is necessary when multiplication is extended beyond the domain of positive integers. Essentially, multiplication underpins the larger context of a multiplicative conceptual field (Vergnaud, 1994) — situations and concepts that involve multiplication, division, fractions, ratio and proportion. An ability to engage in multiplicative thinking requires a clear conceptual understanding and full knowledge of mathematical processes and the relationships among them. Thinking mathematically involves an ability to internalise number concepts through the process of providing problem situations and linking materials with appropriate language and symbols that represent a concept.

High school curricula tend to proceed straight to higher mathematics learning and abstract reasoning based on these ideas, irrespective of students' readiness. Yet Seah (2004) found that many high school students demonstrated limited understanding of multiplication concepts, restricted to procedural [i.e. solely rule-based] rather than conceptual understanding. Such a focus on procedural rather than conceptual understanding has been found to stifle further mathematics learning (Ma, 1999), so it was not surprising that many of the students studied were unable to develop their mathematics education further nor were they able to use the mathematics they had learned to solve everyday problems. This study extends the 2004 investigations to explore the depth and breadth of mathematical knowledge students have as they enter high school. In particular, it looks at students' knowledge of numeration and multiplication and the relationship of mastering both concepts to problem solving.

Method

This study was conducted in a state high school in Brisbane Logan City, traditionally considered socio-economically disadvantaged. The students in this school come from very diverse cultural backgrounds including White Australian, Aboriginal and Torres Strait Islanders, Pacific Islanders, African, Central Asian as well as Cambodians. Historically, the school's overall literacy and numeracy performance is well below the State mean. About one quarter of students in each year level have been officially identified with some forms of disabilities or learning difficulties. Four Year 8 classes took part in this assessment in the second week of term 1. Class A had ten students with disabilities and fourteen learning difficulties students, Class I had fifteen students with learning difficulties. The other two classes had two or three students with learning difficulties. Two diagnostic assessments, the Booker Screening Group Test — Numeration and Multiplication were administered to all students in these four classes. These tests are designed for diagnostic purposes to assess students' numeracy competency at Mathematics Outcomes level 3 and 4 (Queensland Studies Authority, 2004) and are modified from Booker's Profiles in Numeration and Computation (Booker, 1995).

The numeration test consists of twenty questions assessing eight concepts ranging across matching material to symbols, language to symbols, comparing numbers, sequencing numbers, counting, rounding, place value, and renaming. The 15 questions multiplication test is a part oral-and-written and part paper-and-pencil test that addresses knowledge of concepts, basic facts, algorithms and their uses in problem solving. Students were given up to 40 minutes of class time to complete each test, although many completed the test in the suggested time of 25 minutes. Assistance was given to students having difficulties in reading the questions, taking care not to simply lead them to the correct answer. The mathematics teachers for each class marked the test, moderated by the researcher. Qualitative and quantitative data were gathered, compared and analysed.

Results

A total of 143 students from four Year 8 classes participated in the numeration test and 130 students in the multiplication test. Nine disability students in Class A did not take part in the multiplication test as it was decided that they would not have the cognitive functioning needed for this concept.

A. Numeration

Students' performance in the numeration test was consistent across four classes. Figure 1 shows that a majority of the students have no difficulties matching materials to the correct symbols (question 1, 5 and 6). Most also have no difficulties writing two digit number words in symbols (question 2), sequencing two digit numbers (question 4) and comparing a set of numbers. However, the entire cohort showed great difficulties in dealing with larger numbers and numbers that contained internal zeros.

As can be seen in the *language to symbols* section, students able to write larger number words in symbols (question 17) declined by 30 to 60 percent compared to nearly 100% achievement in the same category (question 2). Question 17 asked students to write the number fourteen thousand and twenty-one, to which many of them wrote '1421'. The same error occurred when they were asked to write down the number that is ten less than 7003 in question 13 where many answered '6003'.

There was an equally significant drop in ability when students were asked to sequence and round larger numbers (question 15 and 16). Students were unsure which number to round to, continuing to round the number to the nearest tens despite being asked to round to the nearest hundred. Students did worst in the area of place value of larger numbers and renaming (questions 19, 8, 14 and 20). The most telling example is found in question 20 where students were asked 'how many ten thousands are there in 308 621?' Many wrote none, 3 or 8, demonstrating a severe lack of understanding of the number system. This lack of understanding was not restricted to the students as two teachers found marking the place value and renaming categories challenging. Answers such as 'no tens in 603', 'no hundreds in 7095' were marked correct while '30 ten thousands in 308 621' were marked wrong. Moreover, all four teachers debated whether 75 should round to 70 or 80.

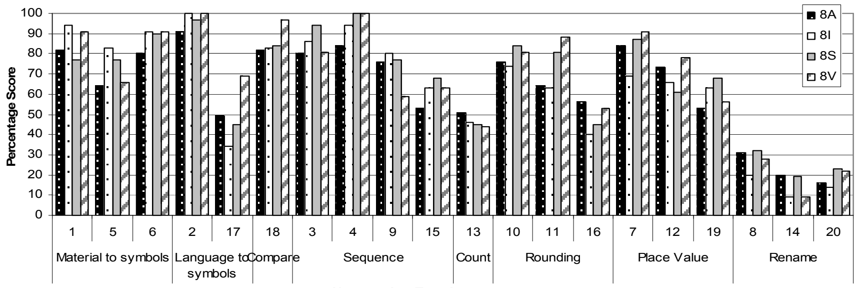


Figure 1.

Percentage score for numeration test.

On the whole, students' knowledge of numeration was well below the expected year level irrespective of their primary school background. That is, the area of students' difficulties was consistent regardless of which primary school they came from (see Figure 2). For example, in the place value and renaming categories, only three out of thirty-one students from school A were able to answer all questions correctly, compared to two from other Queensland schools and school C and one from school D while none in school B is able to answer these questions.

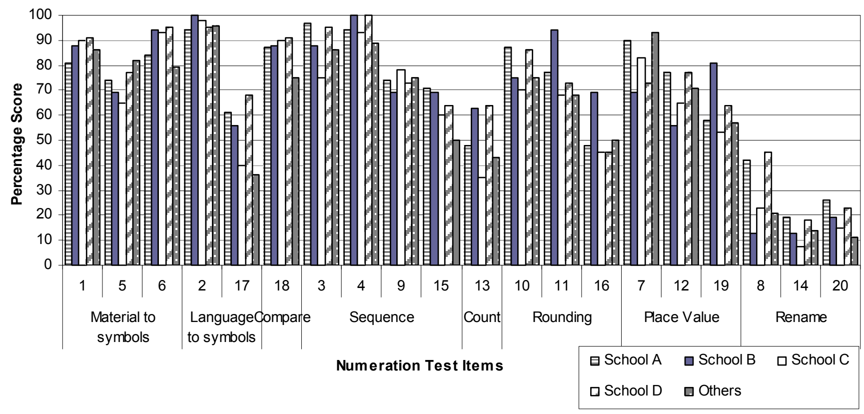


Figure 2.

Breakdown of multiplication test results according to primary school background.

B. Multiplication

As expected, students' performance on the multiplication test was lower then the numeration test. Figure 3 reported the cohort's performance across the four basic multiplication components, concept, word problems, basic facts and algorithms.

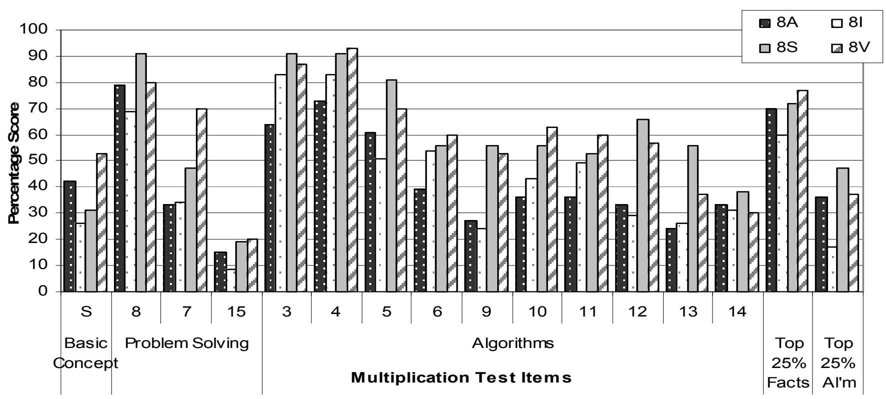
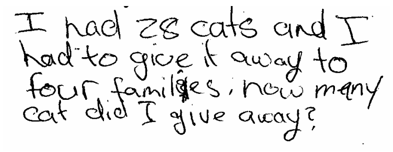


Figure 3.

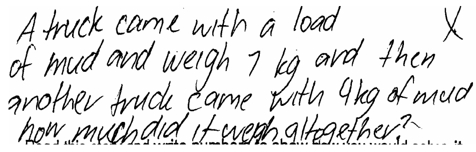
Percentage score for multiplication test.

Most students have no difficulties answering simple word problems as well as doing single digit multiplication. The success rate for those able to write a reasonable story for the multiplication question presented dropped by 35% to 50%. Although some students were able to write an appropriate story, they story were based solely on the concept of repeated addition and lack the depth that is needed for developing multiplicative thinking. A large proportion of the cohort wrote inappropriate stories that involve other operations such as:

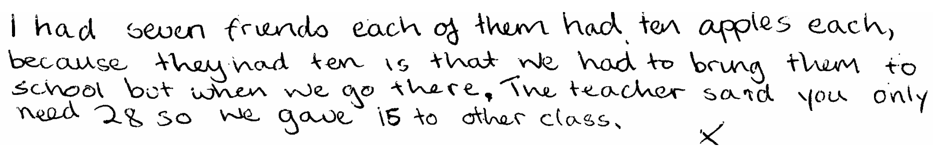
Student A wrote:



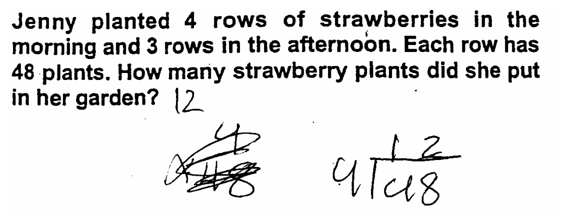
Student B wrote:



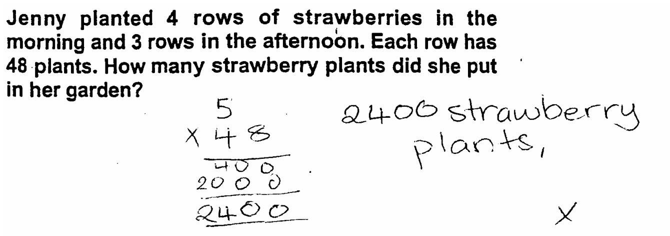
Others knew that the answer is 28 and attempted to write a story to match the answer.



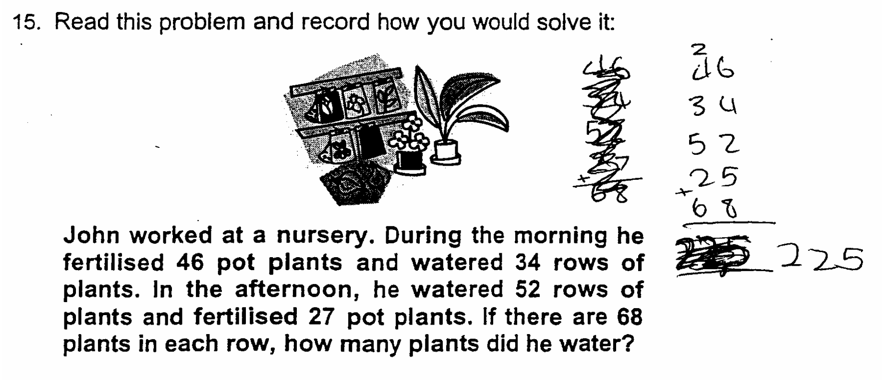
An inability to comprehend the basic concept of multiplication of whole number may have contributed to students' poor performance in word problems and algorithm. While 80% of the students were able to answer simple word problems (question 8), the rate of success dropped to 16% when students were confronted with problems that involve multiplying two sets of data. On average, only 15 students per class or 46% were able to perform two-steps word problems (question 14). When word problems involved too much information, the rate was reduced to an average of five per class or 15.6%. A large number of students were unable to come up with an appropriate strategy for the problem. Some chose a wrong strategy for the problem.



Others failed to consider the relevance of the strategy used and check if the answer is reasonable:

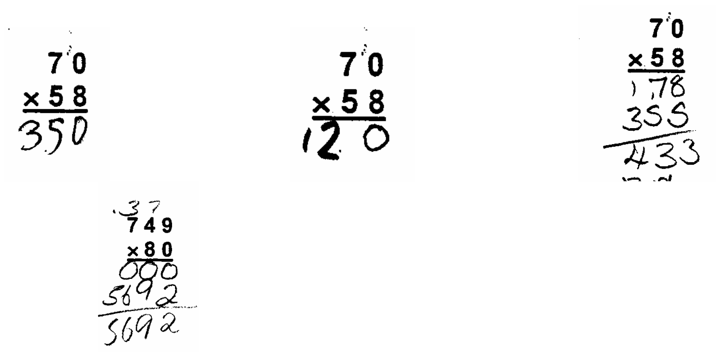


Many simply added all the number symbols appeared in the word problems:



Contrary to common belief, this data shows that there is no correlation between mastery of basic facts and the ability to complete algorithms correctly. On average, about 70% of the cohort was able to recall most of the basic facts, whereas only 34% could compute algorithms correctly. With the algorithms, about 83% of the students were able to compute 1 digit by 2 digit multiplication (question 3 and 4). When the more difficult tasks of multiplying 1 digit by 3 digit numbers were given, the rate of success dropped to 65% for question 5 and then to 51% for question 6. A majority of students had difficulties multiplying 2 digit by 2 digit numbers with only 34% of the cohort able to solve question 14 without error. Some errors found in the algorithms were due to incorrect basic facts, but most arose from confusion between multiplication procedures and other computation. Some students simply applied previously learnt addition procedures to the multiplication algorithm, multiplying the ones and then the tens without cross multiplying all the necessary data:

Zero posed a significant problem for many students, especially those who did not have an understanding of place value, or who had restricted multiplication concept development:



Students' knowledge of multiplication was well below the expected year level irrespective of their primary school background (see Figure 4), parallelling the results found for numeration. The fact that none of the students from school B were able to solve word problems involving multi-digit multiplication (question 15), compared to six correct answer from school A, two from school C, seven from school D, and three from other Queensland schools is an issue worth further investigation. The fact that these students also failed to do well in the place value and renaming aspects of numeration that underpin multiplication points to one likely reason. However, further discussion with their teachers is needed to establish the cause of these difficulties to provide guidance for appropriate intervention.

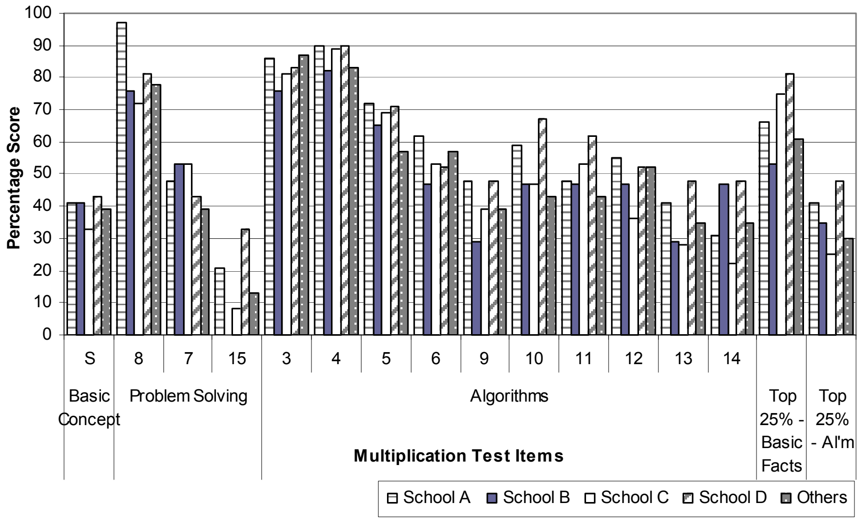


Figure 4.

Breakdown of multiplication test results according to primary school background.

Comparison of incorrect strategies versus no strategy used by students (Figures 5 and 6) confirmed that most students attempted all questions rather than leaving any unanswered. The data also shows that as the tasks became more difficult involving complex word problems or the multiplication of multi-digit numbers, incorrect or inappropriate strategies were used more frequently. Thus, it appears that students' true ability is revealed when conceptual knowledge is required to solve problems.

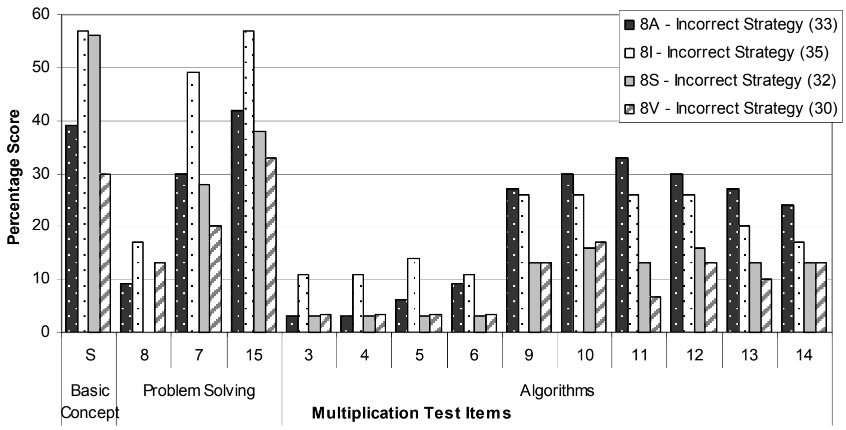


Figure 5.

Comparison of incorrect strategies used in multiplication test.

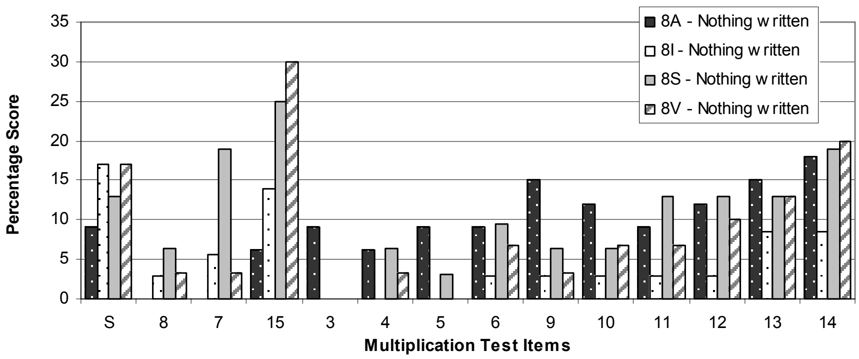


Figure 6.

Comparison of unattempted questions in multiplication test.

Conclusions

Several conclusions can be drawn from this data. In general, students at this high school demonstrated very limited understanding of the concepts for numeration and multiplication. In particular, the majority of students' knowledge is restricted to procedural rules rather than conceptual understanding. Further, the results for this study indicated five points.

1. Students' knowledge of the number systems is limited to three or four digit numbers, which in turn affected their ability to deal with larger numbers.
2. The greatest difficulties students encountered in numeration are related to place value, renaming and internal zeros.
3. In turn, a lack of conceptual understanding of numeration and other previously learned mathematical ideas, together with little understanding of the interconnections among them, are the source of students' difficulties with multiplication. This can be seen by the way many students mixed addition and subtraction strategies when completing the multiplication algorithm and their inability to use place value knowledge to rename the numbers that resulted from multiplication of the partial products involved in the algorithm.
4. Word problems posed the greatest difficulties, with a majority of students not able to use any appropriate strategy to come to terms with the problems let alone provide an appropriate answer.
5. Areas of difficulties were consistent across the four classes and the primary schools from which they came. A lack of exposure and encouragement in learning how to think mathematically may have contributed to primary school B students' poor performance in place value, renaming, and solving word problems.

According to the new Queensland Mathematics Syllabus (Queensland Studies Authority, 2004), students should be achieving at Level 4 outcomes by the end of Year 7. At Year 8, students are expected to be able to confidently perform multi-digit multiplication and work towards multiplication of fractions. Based on the data collected, many students in this school are yet to achieve Level 3 Outcomes and in no position to move to the Level 5 outcomes expected in early High School.

Implications

Mathematics is a way of seeing the world and thinking about it (Sierpinska, 1998). Traditionally, mathematics education has placed a heavy emphasis on practising the basic functionality of procedural knowledge especially in the middle and senior years of schooling. For many students and teachers, mathematics was primarily concerned with memorising rules and facts which could then be applied to solve routine problems. Consequently, most people perceived mathematics not just as difficult, "but as remote, opaque in its language and symbolism, and lacking a human face" (Anderson, 1999, p. 16). The data collected and analysed in this study shows the consequences of such an approach. A lack of understanding of the conceptual knowledge underpinning mathematics and the interconnectedness among this knowledge causes major difficulties at the initial level of applying elementary ideas to straightforward problems and also fails to provide a basis for further learning.

To determine the forms of mathematical knowledge these students had prior to entering the high school, the researcher interviewed a sample of students who failed to answer the place value and renaming questions in the numeration test. When these students were shown a place value chart, many of them immediately recognised it and were able to correctly name each place value. However, they were unable to transfer their knowledge of this relationship to the symbols alone, particular within the operations. That is to say, many of them see place value as a topic on its own and are unable to comprehend connections between place value and the base 10 system. Nor were they able to use this knowledge to help them make sense of the operations or analyse word problems in order to attempt solutions. Moreover, many students were unable to recall basic facts automatically nor were they able to call on knowledge of the multiplication concept to identify the various situations involved in these word problems.

Several students also stated that they preferred to attempt all questions rather than leaving any blank so as not to be seen as "a loser". This suggests that intervention for students with mathematical difficulties needs to consider their self-concept and motivation just as much as their inadequacies with the underlying mathematical concepts and processes. Further, some of the high school mathematics teachers also displayed a limited understanding of the fundamental mathematical knowledge needed to teach primary mathematics as well as a tendency to focus on mathematics at a formal level beyond the grasp of the students whom they are teaching.

Recommendations for intervention need to focus on four factors, in particular:

(a) To establish a transition program between primary and high schools teachers in the area of mathematics curriculum development to ensure consistency and continuity of learning.

(b) To develop instructional designs that focus on outcome based instructions and concept building. In particular, to link concepts and processes in order to help students understand the interconnectedness among mathematical ideas.
(c) A major focus should be placed on teaching place value and renaming within numeration and extending this understanding to computation involving all four operations.
(d) To provide a professional development package for teaching staff in primary and high school settings that focuses on concept learning and teaches sense making through the use of meaningful activities and instructional games that engage students in situations that are real to them.

The learning of mathematics is a process of each individual's negotiation of "taken-as-shared" meanings within a community of practice (Cobb & Bauersfeld, 1995). The future of mathematics education requires a collaborative team effort among schools and between year levels, focusing on the teaching of conceptual knowledge and linking these concepts across settings. Only by doing this can students be encouraged to construct their own mathematical knowledge to have as a means to solve problems and generalise new mathematical knowledge from that they have previously constructed.

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