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The Use of AR-IDEA Approach for Supplier Selection Problems

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Abstract: To select the best suppliers in the presence of both cardinal and ordinal data and considering weight restrictions, this paper proposes a method, which is based on Assurance Region-Imprecise Data Envelopment Analysis (AR-IDEA). A numerical example demonstrates the application of the proposed method.

Key words: Imprecise data envelopment analysis, Supplier selection, Assurance region, Cardinal and ordinal data

INTRODUCTION

Supplier selection is strategic purchasing decision that commits significant resources and impacts the firm's performance. Selecting suppliers from a large number of possible suppliers with various levels of capabilities and potential is a difficult and time-consuming task that is often driven by multiple factors (Dahel, 2003). In initial models of Data Envelopment Analysis (DEA), the weights are allocated to inputs and outputs freely. This freedom of choice shows the Decision Making Unit (DMU) in the best possible light, and is equivalent to assuming that no input or output has preference to others.

However, an efficient DMU may assign a zero weight to the inputs and/or outputs on which its performance is worst. This might not be acceptable. Because, after spending many times in a careful selection of inputs and outputs, some of them are being ignored by DMUs. In practical problems, such as supplier selection, there may be preferences that should be considered. These preferences can reflect known information about how the inputs and outputs used by the DMUs behave. For example, in supplier selection problem in general, one input (material price) usually overwhelms all other inputs, and ignoring this aspect may lead to biased efficiency results.

On the other hand, supplier selection models are based on cardinal (quantitative) data with less emphasis on ordinal (qualitative) data. However, with respect to the widespread use of manufacturing philosophies such as Just-In-Time (JIT), emphasis has shifted to the simultaneous consideration of cardinal and ordinal data in supplier selection problems.

The objective of this paper is to propose a method for selecting the best suppliers in the presence of weight restrictions and both cardinal and ordinal data. The chief advantage of the proposed model is that it does not need exact weights from the Decision Maker (DM).

This paper proceeds as follows. In Section 2, literature review is presented. Section 3 introduces the method which selects the suppliers. Numerical example and concluding remarks are discussed in Sections 4 and 5, respectively.

2. Literature Review:

Some mathematical programming approaches have been used for supplier selection in the past. Ghodsypour and O'Brien (2001) developed a mixed integer nonlinear programming model to solve the multiple sourcing problems, which takes into account the total cost of logistics, including net price, storage, transportation and ordering costs. The model should be run 2^n times for n suppliers that is burdensome. Dahel (2003) presented a multiobjective mixed integer programming approach to simultaneously determine the number of vendors to employ and the order quantities to allocate to these vendors in a multiple-product, multiple-supplier competitive sourcing environment. Talluri and Baker (2002) presented a multi-phase mathematical programming approach

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for effective supply chain design. More specifically, they developed and applied a combination of multi-criteria efficiency models, based on game theory concepts, and linear and integer programming methods. Ip *et al.* (2004) described the sub-contractor selection problem by a 0-1 integer programming with non-analytical objective function.

Talluri and Narasimhan (2003) proposed a max-min productivity based approach that derives variability measures of vendor performance, which are then utilized in a nonparametric statistical technique in identifying vendor groups for effective selection. To solve the vendor selection problem with multiple objectives, Kumar *et al.* (2004) applied fuzzy goal programming approach. To incorporate the imprecise aspiration levels of the goals, they formulated a vendor selection problem as a fuzzy mixed integer goal programming that includes three primary goals: minimizing the net cost, minimizing the net rejections, and minimizing the net late deliveries subject to realistic constraints regarding buyer's demand, vendor's capacity, vendor's quota flexibility, purchasing value of items, budget allocation to individual vendor, etc.

Hajidimitriou and Georgiou (2002) presented a quantitative model, based on the Goal Programming (GP) technique, which uses appropriate criteria to evaluate potential candidates and leads to the selection of the optimal partner (supplier). Cebi and Bayraktar (2003) proposed an integrated model for supplier selection.

In their model, supplier selection problem has been structured as an integrated Lexicographic Goal Programming (LGP) and Analytic Hierarchy Process (AHP) model including both quantitative and qualitative conflicting factors. Arunkumar *et al.* (2006) proposed a GP model for supplier selection with quantity discounts. They converted the piecewise linear problem into an easier linear problem, thereby decreasing the complexity of the problem. Karpak *et al.* (2001) presented one of the "user-friendly" multiple criteria decision support systems-Visual Interactive Goal programming (VIG). VIG facilitates the introduction of a decision support vehicle that helps improve the supplier selection decisions.

Kameshwaran *et al.* (2007) provided a multiattribute e-procurement system for procuring large volume of a single item. Their system is formulated as a mixed linear integer multiple criteria optimization problem and GP is used as the solution technique. To take into account both cardinal and ordinal data in supplier selection, Wang *et al.* (2004) developed an integrated AHP and Preemptive Goal Programming (PGP) based methodology.

However, one of the GP problems arises from a specific technical requirement. After the purchasing managers specify the goals for each selected criterion (e.g., amount of price, quality level, etc), they must decide on a preemptive priority order of these goals, i.e., determining in which order the goals will be attained. Frequently such a priori input might not produce an acceptable solution and the priority structure may be altered to resolve the problem once more. In this fashion, it may be possible to generate a solution iteratively that finally satisfies the DM. Unfortunately, the number of potential priority reorderings may be very large. A supplier selection problem with five factors has up to 120 priority reorderings. Going through such a laborious process would be costly and inefficient.

Pi and Low (2006) provided a more accurate and easier method for quantifying the supplier's attributes to quality-loss using a Taguchi loss function, and these quality losses are also transferred into a variable for decision making by AHP. Noorul Haq and Kannan (2006) developed the integrated qualitative decision making of the supplier selection model using fuzzy AHP with that of the quantitative mathematical model for the distribution inventory supply chain using a Genetic Algorithm (GA) to the built-to-order environment. Wang *et al.* (2005) presented a decision-based methodology for supply chain design that a plant manager can use to select suppliers. The methodology utilizes the techniques of AHP and PGP.

Sha and Che (2006) presented a multi-phased mathematical approach called the Hybrid Multi-phased-based Genetic Algorithm (HMGA) for network design of supply chain. From the point of network design, the important issues are to find suitable and quality companies, and to decide upon an appropriate production/distribution strategy. It is based on various methodologies that embrace GAs, AHP, and the Multi-Attribute Utility Theory (MAUT) to simultaneously satisfy the preferences of suppliers and customers at each level of the supply chain network. Min (1994) suggested MAUT that can effectively deal with both qualitative and quantitative factors in multiple criteria and uncertain decision environments.

Xia and Wu (2007) proposed an integrated approach of AHP improved by rough sets theory and multi-objective mixed integer programming to simultaneously determine the number of suppliers to employ and the order quantity allocated to these suppliers in the case of multiple sourcing, multiple products, with multiple criteria and with supplier's capacity constraints. Bayazit (2006) provided a good insight into the use of Analytic Network Process (ANP) that is a multiple criteria decision making methodology in evaluating supplier selection

problems.

Dulmin and Mininno (2003) presented a proposal for applying a decision model to the final vendor-rating phase of a process of supplier selection. Their model uses a Multiple Criteria Decision Aid (MCDA) technique (PROMETHEE 1 and 2), with a high-dimensional sensitivity analysis approach. They tried to explain how an outranking method and PROMETHEE/GAIA techniques, provides powerful tools to rank alternatives and analyzed the relations between criteria or between DMs. To decide the total ranking of the suppliers, Liu and Hai (2005) compared the weighted sum of the selection number of rank vote, after determining the weights in a selected rank. They presented a novel weighting procedure in place of pairwise comparison of AHP for selecting suppliers. They provided a simpler method than AHP that is called voting analytic hierarchy process, but which do not lose the systematic approach of deriving the weights to be used and for scoring the performance of suppliers. To solve the problems associated with the dynamic nature of supply chain management, Chan (2003) developed a method called Chain Of Interaction (COI). Focusing on the goodwill of AHP, an interactive selection model is suggested. However, AHP has two main weaknesses. First subjectivity of AHP is a weakness. Second AHP could not include interrelationship within the criteria in the model.

Lin and Chen (2004) presented a fuzzy decision making framework for selecting the most favorable strategic supply chain alliance under limited evaluation resources. Ohdar and Ray (2004) evaluated the supplier's performance by adopting an evolutionary fuzzy system. One of the key considerations in designing the proposed system is the generation of fuzzy rules. A genetic algorithm-based methodology is developed to evolve the optimal set of fuzzy rule base, and a fuzzy inference system of the MATLAB fuzzy logic toolbox is used to assess the supplier's performance. Chen *et al.* (2006) presented a fuzzy decision making approach to deal with the supplier selection problem in supply chain system. They used linguistic values to assess the ratings and weights for the criteria. These linguistic ratings can be expressed in trapezoidal or triangular fuzzy numbers. Then, a hierarchy Multiple Criteria Decision Making (MCDM) model based on fuzzy sets theory is proposed to deal with the supplier selection problems in the supply chain system. According to the concept of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), a closeness coefficient is defined to determine the ranking order of all suppliers by calculating the distances to the both Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) simultaneously.

Chang *et al.* (2006) proposed a Fuzzy Multiple Attribute Decision Making (FMADM) method based on the fuzzy linguistic quantifier. However, their proposed method suffers from two main limitations. First, the proposed method does not consider the inputs. Second, the paper does not discuss whether a DM exerts any influence on mental cognition and experiential characteristics when rating the linguistic intervals scale.

Choy *et al.* (2002) presented an Intelligent Supplier Management Tool (ISMT) using the Case-Based Reasoning (CBR) and Neural Network (NN) techniques to select and benchmark suppliers. Choy and Lee (2003) suggested an intelligent Generic Supplier Management Tool (GSMT) using the CBR technique for outsourcing to suppliers and automating the decision making process when selecting them. Choy *et al.* (2004) discussed an Intelligent Supplier Relationship Management System (ISRMS) integrating a company's Customer Relationship Management (CRM) system, Supplier Rating System (SRS) and Product Coding System (PCS) by the CBR technique to select preferred suppliers during the New Product Development (NPD) process. In order to develop a flexible data access framework, and to support the partner selection activity, the combination of OnLine Analytical Processing (OLAP), and CBR was proposed by Lau *et al.* (2005).

Vokurka *et al.* (1996) proposed to incorporate expert system technology into a decision-support framework. Their expert system integrates the judgment and expertise of purchasing professionals with the formal approaches of earlier works. Humphreys *et al.* (2003) introduced a framework for integrating environmental factors into the supplier selection process. They developed a decision support tool that helps companies to integrate environmental criteria into their supplier selection process. Subsequently, a framework of the supplier selection process that incorporates environmental performance is developed. In their framework, the user should give weightings to the environmental categories in order to represent its importance in the analysis.

Kreng and Wang (2005) constructed an optimal expected total cost model, which can offer a guideline to assess and select appropriate suppliers to support multiple products manufacturing. However, in their proposed model, total cost is the only criterion for supplier selection. Chen and Chen (2006) developed an evaluation model that assesses the quality performance of suppliers. Their developed model applies the process incapability index. Lasch and Janker (2005) described a multivariate analysis tool for managing a pool of engaged or future suppliers. A constructed ideal supplier serves as a reference to compare all suppliers by means of factor analysis method. Lee *et al.* (2003) proposed a High-Quality-Supplier Selection (HQSS) model to deal with supplier selection problems in supply chain management. In selecting a supplier, quality

management factors are considered first, and then price, delivery, etc.

Linn *et al.* (2006) proposed a new approach to supplier selection using Capability index and Price Comparison (CPC) chart. The CPC chart integrates the process capability and price information of multiple suppliers and presents them in a single chart for the management to make supplier selection decisions. Pi and Low (2005) proposed an evaluation and selection system of suppliers using Taguchi loss functions. Hong *et al.* (2005) proposed a supplier selection method to maintain a continuous supply-relationship with suppliers. They suggested a mathematical programming model that considers the change in supplier's supply capabilities and customer needs over a period in time. Azoulay-Schwartz *et al.* (2004) used Gittins indices to optimally select a supplier. Chandra *et al.* (2005) presented a model for selecting suppliers with geographical location as a critical factor using a Dual-Matrix approach.

Because of the complexity of the decision making process involved in supplier selection, all the aforementioned literature relied on some form of procedures that assigns weights to various performance measures. The primary problem associated with arbitrary weights is that they are subjective, and it is often a difficult task for the DM to accurately assign numbers to preferences. It is a daunting task for the DM to assess weighting information as the number of performance criteria increased. Therefore, a more robust mathematical technique that does not demand too much and too precise information, i.e., ordinal preferences instead of cardinal weights, from the DM can strengthen the supplier evaluation process.

To this end, Weber (1996) illustrated whether DEA can be used to evaluate suppliers on multiple criteria and identified benchmark values which can then be used for this purpose. Weber *et al.* (2000) presented an approach for evaluating the number of vendors to employ in a procurement situation using Multi-Objective Programming (MOP) and DEA. The approach advocates developing vendor-order quantity solutions (referred to as supervendors) using MOP and then evaluating the efficiency of these supervendors on multiple criteria using DEA.

Braglia and Petroni (2000) described a MAUT based on the use of DEA, aimed at helping purchasing managers to formulate viable sourcing strategies in the changing market place. To evaluate the aggregate performances of suppliers, Liu *et al.* (2000) proposed to employ DEA. This extends Weber's (1996) research in using DEA in supplier evaluation for an individual product. Forker and Mendez (2001) proposed an analytical method for benchmarking using DEA that can help companies identify their most efficient suppliers, the suppliers among the most efficient with the most widely applicable Total Quality Management (TQM) programs, and those suppliers who are not on the efficient frontier but who could move toward it by emulating the practices of their "best peer" supplier(s).

Talluri and Sarkis (2002) focused on the supplier performance evaluation and monitoring process, which assist in maintaining effective customer-supplier linkages. They tried to improve the discriminatory power of BCC model proposed by Banker *et al.* (1984). To select appropriate suppliers, Talluri *et al.* (2006) suggested a Chance-Constrained Data Envelopment Analysis (CCDEA) approach in the presence of multiple performance measures that are uncertain.

Recently, to select the best suppliers in the presence of both cardinal and ordinal data, Farzipoor Saen (2007) proposed an innovative method, which is based on Imprecise DEA (IDEA). However, he did not consider the weights restrictions. Again, Farzipoor Saen (2008) demonstrated a new pair of Assurance Region-Imprecise Data Envelopment Analysis (AR-IDEA) model for selecting the best suppliers in the presence of both weight restrictions and imprecise data. However, the main limitation of the proposed model is that it does not consider anchoring value used in IDEA.

To the best of author's knowledge, there is not any reference that deals with supplier selection in the conditions that weight restrictions in the presence of both cardinal and ordinal data are present, while there is anchoring value.

3. Proposed Method for Suppliers Selection:

DEA proposed by Charnes *et al.* (1978) (CCR model) and developed by Banker *et al.* (1984) (BCC model) is an approach for evaluating the efficiencies of DMUs. In many real world applications (especially supplier selection problems), it is essential to take into account the existence of ordinal (qualitative) factors. Very often, it is the case that for a factor such as supplier reputation, one can, at most, provide a ranking of the DMUs from best to worst relative to this attribute. The capability of providing a more precise, quantitative measure reflecting such a factor is generally beyond the realm of reality. In some situations, such factors can be legitimately quantified, but very often; such quantification may be superficially forced as a modeling convenience. In situations such as that described, the data for certain influence factors (inputs and outputs) might better be represented as rank positions in an ordinal, rather than numerical sense. Refer again to the

supplier reputation example. In certain circumstances, the information available may permit one to provide a complete rank ordering of the DMUs on such a factor. Therefore, the data may be imprecise.

On the other hand, one serious drawback of DEA applications in supplier selection has been the absence of DM judgment, allowing total freedom when allocating weights to input and output data of supplier under analysis. This allows suppliers to achieve artificially high efficiency scores by indulging in inappropriate input and output weights. The most widespread method for considering judgments in DEA models is, perhaps, the weight restrictions inclusion. Weight restrictions allow for the integration of managerial preferences in terms of relative importance levels of various inputs and outputs.

The idea of conditioning the DEA calculations to allow for the presence of additional information arose first in the context of bounds on factor weights in DEA's multiplier side problem. This led to the development of the cone-ratio (Charnes *et al.* (1989)) and assurance region models (Thompson *et al.* (1990)). Both methods constrain the domain of feasible solutions in the space of the virtual multipliers. Weights restrictions may be applied directly to the DEA weights or to the product of these weights with the respective input or output level, referred to as *virtual input* or *virtual output*. Restrictions on virtual weights were proposed first by Wong and Beasley (1990). Sarrico and Dyson (2004) suggest, in line with Thompson *et al.* (1990), the use of virtual assurance regions, concluding that they can overcome problems of infeasibility as well as interpretation of target and efficiency scores, whilst retaining the benefit of the natural representation of preference structures. Cooper *et al.* (2001) developed the AR-IDEA model to deal not only with imprecise data and cardinal data but also with weight restrictions. In this paper, this model is used to restrict the flexibility of the suppliers (DMUs) in selecting the weights while ordinal and cardinal data are present.

The following model, used to initiate the discussion, extends an ordinary DEA model in a manner that is described next.

$$\begin{aligned}
 \max z_o &= \sum_{r \in R} y_{ro} \mu_r, \\
 \text{s.t.} & \left. \begin{aligned}
 \sum_{r \in R} y_{rj} \mu_r - \sum_{i \in I} x_{ij} \omega_i &\leq 0 \\
 \sum_{i \in I} x_{io} \omega_i &= 1
 \end{aligned} \right\} \forall j, \tag{1}
 \end{aligned}$$

$$\left. \begin{aligned}
 \mathbf{x}_i &= (x_{ij}) \in D_i^- \\
 \mathbf{y}_r &= (y_{rj}) \in D_r^+
 \end{aligned} \right\} \forall i, r, \tag{2}$$

$$\omega_i, \mu_r \geq \varepsilon \quad \forall i, r \tag{3}$$

Here the y_{rj} and x_{ij} represent outputs $r \in R$ and inputs $i \in I$, respectively, for $DMU_j, j=1, \dots, n$, and the y_{ro} and x_{io} are the outputs and inputs for DMU_o , the DMU that is to be evaluated. The μ_r, ω_i are, respectively, output and input "multiplier" variables, and $\varepsilon > 0$ is a non-Archimedean infinitesimal, which restricts the variable values to be positive.

The constraints in Relations (1) and (3) are in the standard form of "multiplier" DEA models that assume all data are known exactly. Relations (1) and (2), however, are assumed that subsets (or all) the data take imprecise forms such as $\underline{y}_{rj} \leq y_{rj} \leq \bar{y}_{rj}$, $\underline{x}_{ij} \leq x_{ij} \leq \bar{x}_{ij}$, so the true values of these data are known

only to lie within the upper and lower limits indicated by $\underline{y}_{rj}, \overline{y}_{rj}$ and $\underline{x}_{ij}, \overline{x}_{ij}$. These imprecise data form the sets D_r^+, D_i^- for the vectors y_r, x_i with components y_{rj} and x_{ij} , respectively. These sets can also include the data that are known only to satisfy order relations such as $y_{rj} \leq y_{rk}, x_{ij} \leq x_{ik} \quad \forall j \neq k$, for some r and i , as well as cardinal data.

A point to be noted is that some of the constraints in Relation (3) can be regarded as examples of Assurance Regions (ARs). However, AR bounds are identified more realistically, if these can be obtained, which are represented in the following form:

$$\left. \begin{aligned} \mu &= (\mu_r) \in A^+ \\ \omega &= (\omega_i) \in A^- \end{aligned} \right\} \tag{4}$$

The vectors μ and ω contain component variables μ_r and ω_i , which are to be assigned values that are constrained to lie within AR bounds defined by the sets A^+ and A^- , respectively.

To make the point more concrete, the set A^+ is exhibited explicitly as follows:

$$\alpha_r^- \leq \frac{\mu_r}{\mu_{r+1}} \leq \alpha_r^+, \tag{5}$$

where α_r^-, α_r^+ represent fixed lower and upper bounds for the ratio of the values μ_r and μ_{r+1} associated with outputs r and $r+1$ for DMU_{*j*}.

To indicate that such imprecisely known data are to be treated by DEA, the above model is referred as IDEA. It is also worth noting that Relation (4) can be incorporated into Relation (1) in the place of Relation (3). This yields an extension referred to as the AR-IDEA model.

The model represented in Relations (1)-(3) forms a nonlinear (nonconvex) programming problem. However, this problem can be transformed into an ordinary linear programming problem by methods that are begun to sketch below.

First, all data are rescaled by means of the following formulas:

$$\begin{aligned} \hat{x}_{ij} &= \frac{x_{ij}}{\max_j \{x_{ij}\}} & \forall i \\ \hat{y}_{rj} &= \frac{y_{rj}}{\max_j \{y_{rj}\}} & \forall r \end{aligned} \tag{6}$$

Thus, each \hat{x}_{ij} and \hat{y}_{rj} is obtained by normalizing the original data on the maximal value in the column where this variable appears.

The rescaled version of Models (1)-(3) can then be represented as

$$\left. \begin{aligned} \max z_o &= \sum_{r \in R} \hat{y}_{ro} \hat{\mu}_r, \\ s.t. & \\ & \sum_{r \in R} \hat{y}_{rj} \hat{\mu}_r - \sum_{i \in I} \hat{x}_{ij} \hat{\omega}_i \leq 0 \\ & \sum_{i \in I} \hat{x}_{io} \hat{\omega}_i = 1 \end{aligned} \right\} \forall j, \quad (7)$$

$$\left. \begin{aligned} \hat{\mathbf{x}}_i &= (\hat{x}_{ij}) \in \hat{D}_i^- \\ \hat{\mathbf{y}}_r &= (\hat{y}_{rj}) \in \hat{D}_r^+ \end{aligned} \right\} \begin{matrix} \forall i \\ \forall r \end{matrix}, \quad (8)$$

$$\hat{\omega}_i, \hat{\mu}_r \geq \varepsilon \quad \forall i, r. \quad (9)$$

where the notations D_r^+, D_i^- in Models (1)-(3) have been replaced by \hat{D}_r^+, \hat{D}_i^- in Models (7)-(9). The $\hat{\mu}_r, \hat{\omega}_i$ in Models (7)-(9) are used for the variables (=weights or multipliers) instead of μ_r, ω_i in Models (1)-(3) to reflect the following relations:

$$\begin{aligned} \hat{\omega}_i &= \omega_i \cdot \max_j \{x_{ij}\} & \forall i, \\ \hat{\mu}_r &= \mu_r \cdot \max_j \{y_{rj}\} & \forall r. \end{aligned} \quad (10)$$

Now, to obtain an ordinary linear programming problem, new variables X_{ij} and Y_{rj} are defined as follows:

$$X_{ij} = \hat{x}_{ij} \hat{\omega}_i; \quad Y_{rj} = \hat{y}_{rj} \hat{\mu}_r \quad (11)$$

for all i, r and every j . Since $\hat{x}_{ij}, \hat{y}_{rj} \geq 0$ and $\hat{\omega}_i, \hat{\mu}_r \geq 0$, then $X_{ij}, Y_{rj} \geq 0 \quad \forall i, r, j$.
Now, let

$$\hat{x}_{ij}^o = \max_j \{ \hat{x}_{ij} \}; \quad \hat{y}_{rj}^o = \max_j \{ \hat{y}_{rj} \} \quad (12)$$

$$X_{ij}^o = \max_j \{ X_{ij} \}; \quad Y_{rj}^o = \max_j \{ Y_{rj} \}, \quad (13)$$

and further,

$$X_{ij}^o = \hat{x}_{ij}^o \hat{\omega}_i; \quad Y_{rj}^o = \hat{y}_{rj}^o \hat{\mu}_r \quad (13)$$

for each input i and output r .

It follows that $\hat{x}_{ij}^o = \hat{y}_{rj}^o = 1$, because these are the column maxima of each i, r , as noted in Equation (12). Therefore, there is

$$\hat{\omega}_i = X_{ij}^o; \quad \hat{\mu}_r = Y_{rj}^o \quad \forall i, r, \tag{15}$$

and hence,

$$(\hat{\omega}_i, \hat{\mu}_r) \geq \varepsilon \rightarrow (X_{ij}^o, Y_{rj}^o) \geq \varepsilon, \tag{16}$$

which means that the positivity conditions on the weights in Relation (9) can be represented by the same positivity conditions on the new variables corresponding to the column maxima.

Further, from Equations (11) and (15), there is

$$\hat{x}_{ij} = \frac{X_{ij}}{X_{ij}^o}; \quad \hat{y}_{rj} = \frac{Y_{rj}}{Y_{rj}^o} \quad \forall i, r, j. \tag{17}$$

This means that the sets of constraints on the rescaled data values in Model (8), viz., $\hat{\mathbf{x}}_i = (\hat{x}_{ij}) \in \hat{D}_i^-$ and $\hat{\mathbf{y}}_r = (\hat{y}_{rj}) \in \hat{D}_r^+$, can be converted into new variables for use in these same constraints.

These new constraint sets can be expressed in the sets $\mathbf{X}_i = (X_{ij}) \in B_i^-$, $\mathbf{Y}_r = (Y_{rj}) \in B_r^+$, which are represented in the following linear programming version of the model of (1)-(3) via Models (7)-(9):

$$\left. \begin{aligned} \max z_o &= \sum_{r \in R} Y_{ro}, \\ s.t. & \\ & \sum_{r \in R} Y_{rj} - \sum_{i \in I} X_{ij} \leq 0 \\ & \sum_{i \in I} X_{io} = 1 \end{aligned} \right\} \quad \forall j, \tag{18}$$

$$\left. \begin{aligned} \mathbf{X}_i &= (X_{ij}) \in B_i^- & \forall i \\ \mathbf{Y}_r &= (Y_{rj}) \in B_r^+ & \forall r \end{aligned} \right\}, \tag{19}$$

$$X_{ij}^o, Y_{rj}^o \geq \varepsilon \quad \forall i, r, \tag{20}$$

where all variables X_{ij}, Y_{rj} , except for those in (20), are nonnegative. Here, this model is referred as IDEA-CCR to emphasize that it is the CCR version of the DEA model.

Because of the problems of weak orderings, strict orderings as an alternative possibility are considered. For this purpose, relations such as

$$B_i^- = \{ X_{ij} - X_{ij+1} \geq \eta; \quad X_{ij+1} - X_{ij+2} \geq \eta; \quad \dots; X_{in} \geq \eta \} \tag{21}$$

are used.

Here η is a positive scalar used to distinguish rank positions strictly, with larger values of η tending to provide greater discriminations in the resulting efficiency values, and $\eta = 0$, representing the case of weak order contributing little or no discrimination.

Turning from weak to strict order introduces the problem of how to specify the value of η . For this purpose, Cook *et al.* (1996) suggested following model:

$$\begin{aligned} \eta_o^* &= \max \quad \eta_o \\ \text{s.t.} \quad & \sum_{r \in R} Y_{rj} - \sum_{i \in I} X_{ij} \leq 0 \quad \forall j \\ & \sum_{i \in I} X_{io} = 1, \end{aligned} \tag{22}$$

$$\begin{aligned} X_{ij}^o - X_{i,j+1} &\geq \eta; \quad X_{i,j+1} - X_{i,j+2} \geq \eta; \quad \dots; \quad X_{in} \geq \eta && \text{for ordinal input } i, \\ Y_{rj}^o - Y_{r,j+1} &\geq \eta; \quad Y_{r,j+1} - Y_{r,j+2} \geq \eta; \quad \dots; \quad Y_{rn} \geq \eta && \text{for ordinal output } r, \\ \mathbf{X}_i &\in B_i^-; \quad \mathbf{Y}_r \in B_r^+ && \text{for exact and bounded input } i \text{ and output } r. \end{aligned}$$

Here $X_{ij}^o - X_{i,j+1} \geq \eta$ means DMU_{*j*} outranks DMU_{*j+1*} in its use of input *i* by at least η ; and similarly, $X_{i,j+1} - X_{i,j+2} \geq \eta$ means DMU_{*j+1*} outranks DMU_{*j+2*} by at least η , in this same input, and so on, with similar characterizations applying to $Y_{rj}^o - Y_{r,j+1} \geq \eta$, etc.

Following Cook *et al.* (1996), the value of η is obtained from the above model to obtain the maximal discriminating power by choosing $\eta_o^* = \max \eta_o$ subject to the above constraints. Note that this constraint set is allowed to reflect cardinal data in the sets B_i^-, B_r^+ , as well as ordinal relations. In fact, for each DMU to which this problem is applied, a maximum value, *viz.*, $\hat{\eta} = \min_j \{ \eta_j^* \}$ is achieved as the largest feasible value that the ordinal constraints will admit.

Now, to demonstrate how to use information obtained from managerial assessments in the model, AR efficiency analysis is presented. Specifications for the AR are usually effected in an *ex ante* manner based on price/cost data, expert opinion or other considerations.

To incorporate conditions (4) and (5) into the model represented by (18)-(20), (10) and (15) are used to obtain the rescaled constraint sets \hat{A}^-, \hat{A}^+ . To see how the set A^- is transformed into \hat{A}^- , $\omega_i = \hat{\omega}_i / \max_j \{ x_{ij} \}$, is used, as in (10). Similarly, the set A^+ is transformed into \hat{A}^+ by using (10). Using the equations (15), \hat{A}^-, \hat{A}^+ can be replaced by \hat{B}^-, \hat{B}^+ . Therefore, there is final linear inequality form for the AR bounds, which are used to analyze AR efficiency in supplier selection problem.

Substituting the AR constraints in \hat{B}^- , \hat{B}^+ in place of (20), an AR efficiency score for each supplier is obtained. The linear programming problem for the AR-IDEA model is then finally given by

$$\left. \begin{aligned} \max z_o &= \sum_{r \in R} Y_{ro}, \\ \text{s.t.} \quad & \sum_{r \in R} Y_{rj} - \sum_{i \in I} X_{ij} \leq 0 \\ & \sum_{i \in I} X_{io} = 1 \end{aligned} \right\} \quad \forall j, \tag{23}$$

$$\left. \begin{aligned} \mathbf{X}_i &= (X_{ij}) \in B_i^- \quad \forall i \\ \mathbf{Y}_r &= (Y_{rj}) \in B_r^+ \quad \forall r \end{aligned} \right\}, \tag{24}$$

$$\left. \begin{aligned} (X_{ij}^o) &\in \hat{B}^- \\ (Y_{rj}^o) &\in \hat{B}^+ \end{aligned} \right\} \tag{25}$$

$$X_{ij}, Y_{rj} \geq 0 \quad \forall i, r, j \tag{26}$$

Therefore, one unified approach that deals with all aspects of the imprecise data and weight restrictions in a direct manner have been introduced. Fig. 1 exhibits the above mentioned discussions concisely.

In the next section, a numerical example is presented.

4. Numerical Example:

The data set for this example is partially taken from Farzipoor Saen (2007) and contains specifications on 18 suppliers. In particular, this example is used to show how ordinal and bounded data, as well as weight restrictions, can be combined into the one unified approach provided by AR-IDEA. The cardinal inputs considered are Total Cost of shipments (TC) and Price. Supplier Reputation (SR) is included as a qualitative input while Number of Bills received from the supplier without errors (NB) will serve as the bounded data output. NB of supplier 14 was considered to be 350 in accordance with the anchoring used in IDEA. SR is an intangible factor that is not usually explicitly included in evaluation model for supplier. This qualitative variable is measured on an ordinal scale so that, for instance, reputation of supplier 18 is given the highest rank, and supplier 17, the lowest. Note that, the measures selected in this paper are not exhaustive by any means, but are some general measures that can be utilized to evaluate suppliers. In an application of this methodology, DMs must carefully identify appropriate inputs and outputs to be used in the decision making process. Table 1 depicts the supplier's attributes.

According to the decision of DM, the importance of TC, as expressed by the weight ω_1 , must be greater than Price, expressed by the weight ω_2 . Assume that TC is at least twice as important as Price and at most thrice as important as Price. So the following inequality bounds, using the formulation in (5), are constructed:

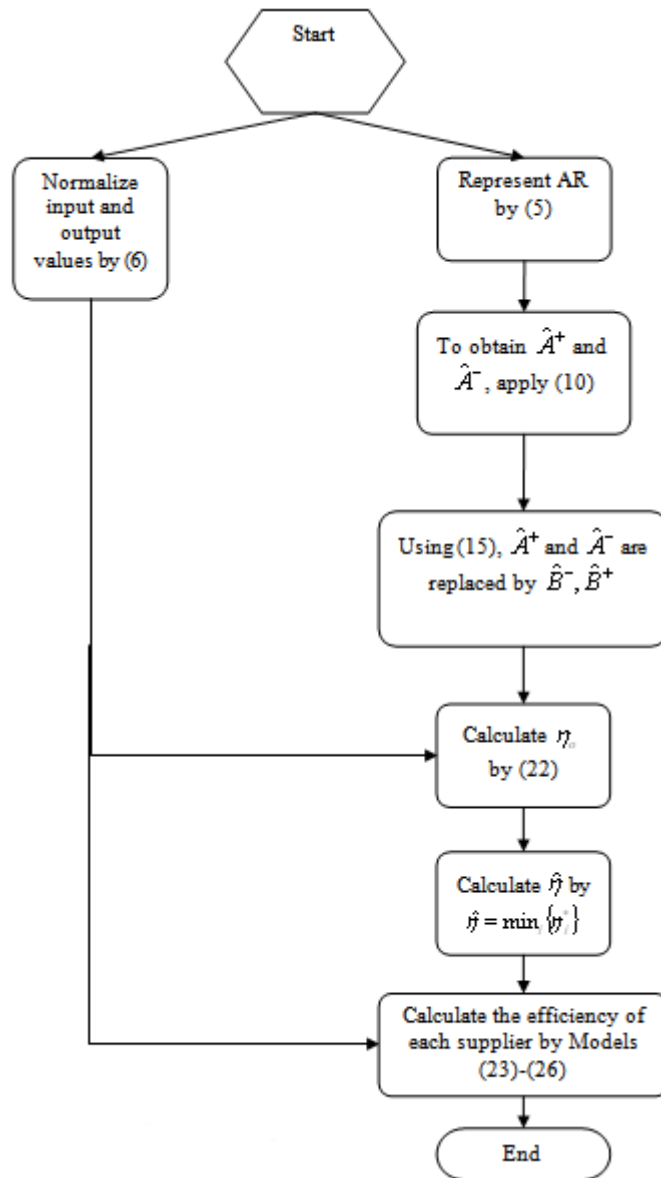


Fig. 1: Summary of discussions

$$A^- = \left\{ 2 \leq \frac{\omega_1}{\omega_2} \leq 3 \right\}$$

where A^- represent the constraint set, as defined in Equation (4), for input multiplier ratio bounds. To incorporate this condition into the model represented by (23)-(26), (10) and (15) are used to obtain the rescaled constraint set \hat{A}^- :

$$\hat{A}^- = \left\{ 0.2672 \leq \frac{\hat{\omega}_1}{\hat{\omega}_2} \leq 0.4008 \right\}$$

Using the equations $\hat{\omega}_i = X_{ij}^o$, as in (15), and observing that $\{X_{ij}^o, i = 1, 2\} = \{X_{1,16}, X_{2,16}\}$ as

utilized in moving from Relation (9) to (20), the \hat{A}^- can be replaced by

$$\hat{B}^- = \{0.2672X_{2,16} \leq X_{1,16} \leq 0.4008X_{2,16}\}$$

Substituting the above AR constraint in place of (20), an AR efficiency score for each supplier is obtained.

Now, to obtain a composite evaluation, joint treatment of AR bounds (\hat{B}^-) and strict ordinal relations in (21) are considered. To this end, first $\hat{\eta}$, the maximum possible value of η , is calculated by (22). These values are presented in the last column of Table 1. The maximum value is $\hat{\eta} = \min_j \{\eta_j^*\} = 0.0588235$.

Using this value for the ordinal relations under X_3 and applying Models (23)-(26), the efficiency scores of suppliers (DMUs) have been presented in Table 2.

Table 1: Related attributes for 18 suppliers

Supplier No. (DMU)	Inputs		Output		η
	TC x_{1j}	Price x_{2j}	SR* x_{3j}	NB y_{ij}	
1	253	2000	5	[50, 65]	.2
2	268	1800	10	[60, 70]	.1
3	259	2100	3	[40, 50]	.333
4	180	2150	6	[100, 160]	.16667
5	257	1900	4	[45, 55]	.25
6	248	2500	2	[85, 115]	.5
7	272	2200	8	[70, 95]	.125
8	330	1900	11	[100, 180]	.0909091
9	327	2040	9	[90, 120]	.11111
10	330	1890	7	[50, 80]	.142857
11	321	2000	16	[250, 300]	.0625
12	329	1950	14	[100, 150]	.0714286
13	281	2350	15	[80, 120]	.066667
14	309	2300	13	350	.0769231
15	291	2000	12	[40, 55]	.083333
16	334	2010	17	[75, 85]	.0588235
17	249	1990	1	[90, 180]	1
18	216	2153	18	[90, 150]	1

* Ranking such that 18 \equiv highest rank, ..., 1 \equiv lowest rank ($x_{2,18} \ x_{2,16} \dots \ x_{2,17}$)

Table 2: Efficiency scores of 18 suppliers

Supplier No. (DMU)	Efficiency
1	.552
2	.389
3	.49
4	1
5	.519
6	1
7	.621
8	.824
9	.706
10	.575
11	.392
12	.392
13	.235
14	1
15	.216
16	.056
17	1
18	.000008

Models (23)-(26) identified suppliers 4, 6, 14, and 17 to be efficient with a relative efficiency score of 1. The remaining 14 suppliers with relative efficiency scores of less than 1 are considered to be inefficient. Therefore, DM can choose one or more of these efficient suppliers.

5. Concluding Remarks:

Increasing attention toward supplier partnership not only raises the importance of supplier selection but also increases the significance of considering ordinal factors in this decision making process. DEA can be very useful in involving several factors. By improving DEA with ordinal data, the qualitative judgment can be quantified.

This paper has provided and summarized the use of AR-IDEA on the supplier selection. The AR-IDEA model offers viable approach to supplier selection with imprecise data and weight restrictions because of the availability of Lingo software package that provides efficient computational tools for solving the model.

The problem considered in this study is at initial stage of investigation and much further researches can be done based on the results of this paper. Some of them are as follows:

Similar research can be repeated for suppliers ranking in the presence of both cardinal and ordinal data and considering weight restrictions. Other potential extension to the methodology includes the case that some of the suppliers are slightly non-homogeneous. One of the assumptions of all the classical models of DEA is based on complete homogeneity of DMUs (suppliers), whereas this assumption in many real applications cannot be generalized. In other words, some inputs and/or outputs are not common for all the DMUs occasionally. Therefore, there is a need to a model that deals with these conditions.

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