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Author

Awrangjeb, Mohammad, Lu, Guojun

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Techniques for Efficient and Effective Transformed Image Identification

Mohammad Awrangjeb^{*,a}, Guojun Lu^b

^a*Cooperative Research Center for Spatial Information, The University of Melbourne
723 Swanston St, Carlton Vic 3053, Australia*

Phone: +61 3 8344 9182, Fax: +61 3 9349 5185

^b*Gippsland School of IT, Monash University, Churchill Vic 3842, Australia*

Phone: +61 3 5122 6857, Fax: +61 3 5122 6879

Abstract

In many applications, one common problem is to identify images which may have undergone unknown transformations. We define this problem as *transformed image identification* (TII), where the goal is to identify geometrically transformed and signal processed images for a given test image. The TII consists of three main stages – *feature detection*, *feature representation*, and *feature matching*. The TII approach by Lowe [1] is one of the most promising techniques. However, both of its feature detection and matching stages are expensive, because a large number of feature-points are detected in the image scale-space and each feature-point is described using a high dimensional vector. In this paper, we explore the use of different techniques in each of the three TII stages and propose a number of promising TII approaches by combining different techniques of the three stages. Our experimental results reveal that the proposed approaches not only improve the computational efficiency and decrease the storage requirement significantly, but also increase the transformed image identification accuracy (robustness).

Key words:

Feature detection, Feature representation, Feature matching, Transformed image identification

*Corresponding author

Email addresses: mawr@unimelb.edu.au (Mohammad Awrangjeb),
Guojun.Lu@infotech.monash.edu.au (Guojun Lu)

1. Introduction

In many applications, such as image copyright protection [2], face recognition [3] and object recognition [1], a common problem is to identify images which may have undergone unknown transformations. We define this common problem as the *transformed image identification* (TII), where the goal is to identify the geometric transformed and the signal processed images for a given image. Therefore, the TII is different from conventional *content-based image retrieval* (CBIR) [4], where all images having the same or similar features, e.g., similar colors, are considered relevant to each other. Li et al. [5] proposed a user feedback-based CBIR technique using learning machines for classification [6].

The TII is also called *near-duplicate image identification* in the literature [7, 3, 8, 9, 10] and consists of three main stages [11]: *feature detection*, *feature representation*, and *feature matching*. In the *feature detection* stage, a set of features, e.g., corners, blobs, T-junctions, are detected. The most valuable property of a feature detector is *repeatability*, i.e., whether it reliably finds the same feature points after the image has undergone different transformations. In the *feature representation* stage, each detected feature point is represented by a feature vector calculated possibly from its neighborhood. In the *feature matching* stage, the feature vectors of the test image and the stored images are compared to identify transformed images for the test image. The matching is often based on a distance, e.g., the Euclidean distance [11], between the feature vectors.

Low's approach [1] has demonstrated its superior performance in identifying transformed images over many other approaches [12, 13, 14]. However, both of its detection and matching stages are expensive, because a large number of keypoints are detected in the scale-space using the difference-of-Gaussian (DoG) filter and each keypoint is described using a 128-dimensional vector known as SIFT (scale invariant feature transform) descriptor. Further research has been carried out in the literature for the dimensionality reduction of the SIFT descriptor, but with the expense of loss of robustness [11, 13, 12]. In this paper, we explore the use of different techniques in each of the TII stages. In stage one, we present two possible solutions for feature-point reduction. First is to down scale the image before the DoG keypoint detection and second is to use corners (instead of DoG keypoints) which are visually significant, more robust, and much smaller in number than the DoG keypoints. In stage two, we explore the use of corner curvature as well as to

38 SIFT descriptors. In stage three, feature point matching techniques based
39 on the geometric point matching technique [15] is used in addition to the
40 nearest-neighbor-distance-ratio based matching technique [1]. Consequently,
41 we propose a number of TII approaches in this paper by combining differ-
42 ent techniques at different stages and will discuss them in Section 4. The
43 two feature-point reduction solutions combined with the SIFT descriptors
44 and our previously proposed feature-point matching technique [15] not only
45 improve the computational efficiency and decrease the storage requirement
46 significantly, but also increase the transformed image identification accuracy
47 (robustness).

48 The organization of this paper is as follows. In Section 2, we briefly re-
49 view the existing feature detection, representation, and matching techniques
50 and highlight key contributions of the paper. Section 3 compares three ex-
51 isting feature detectors in terms of robustness and detection time. In Section
52 4, we present the proposed TII approaches. In Section 5, we discuss the
53 performance study and finally, in Section 6 we conclude the paper.

54 **2. Related Work and Contributions of This Paper**

55 In this section, we first briefly present some relevant existing work in three
56 stages of the TII (feature point detection, feature representation and feature
57 matching). Then we briefly discuss existing promising TII approaches and
58 present the contribution of this paper.

59 *2.1. Feature Detection*

60 A large number of corner and interest-point detectors have been proposed
61 in the literature [1, 16, 14, 17, 18, 19, 20, 21, 22, 23]. While corner detectors
62 detect image spatial locations where edge segments make significant angles,
63 interest-point detectors not only detect corners, but also image locations that
64 have large gradients in all directions at a predetermined scale [1].

65 All corner and interest-point detectors can be broadly classified into two
66 groups: single-scale detectors [22, 21, 23] and multi-scale detectors [1, 16,
67 14, 17, 18, 19, 20]. Single-scale detectors work well if the image has similar
68 size features, but ineffective otherwise; because either fine or coarse scale
69 feature is poorly detected, but images may contain both kinds of features.
70 To improve the effectiveness of the detection stage, multi-scale detectors have
71 been proposed.

72 Corner and interest-point detectors can also be categorized into three
73 groups: intensity-based [1, 16, 14, 22], contour-based [17, 18, 19, 20, 21], and
74 model or template-based [23] methods. Intensity-based methods estimate
75 a measure which is intended to indicate the presence of an interest-point
76 directly from the image pixel values. Contour-based methods first obtain
77 planar curves using some edge detector and then search for the curvature
78 maxima along those curves. Model or template-based methods find corners
79 by fitting the image signal into a predefined model.

80 The main drawback with the model-based detectors is that the corner in
81 natural images cannot be approximated by a model of a perfect corner, as
82 it can take any form of the bidirectional signal change [14]. Moreover, they
83 are computationally too expensive [23] and are not used for general purpose
84 [24]. This paper will focus on the intensity and contour-based detectors only.

85 *2.1.1. Intensity-based Detectors*

86 Probably the most widely used detector is the Harris interest-point de-
87 tector [22] which is based on the eigen values of the second-moment matrix.
88 However, Harris points are not scale-invariant [11]. Lindeberg [16] intro-
89 duced the concept of automatic scale selection which allows detecting inter-
90 est points in an image, each with their own characteristic scale. Mikolajczyk
91 and Schmid [14] refined this technique by creating robust and scale invariant
92 features with high repeatability. They used a scale-adapted Harris measure
93 or the determinant of the Hessian matrix to select the location, and the
94 Laplacian to select the scale. Lowe’s [1] approximation of the Laplacian of
95 Gaussian using the DoG filter speeded up the feature detection stage signif-
96 icantly. The recently proposed fast-Hessian detector in the SURF (speeded
97 up robust features) detector-descriptor scheme [11] used a basic approxima-
98 tion of the Hessian matrix and relied on the integral images to reduce the
99 computational cost.

100 *2.1.2. Contour-based Detectors*

101 The CSS (curvature scale-space) detector in [19] is one of the earlier
102 contour-based multi-scale detectors. It detected corners at a high scale and
103 tracked them through multiple lower scales in order to improve localization.
104 Since different curves require different smoothing-scales and there may be
105 different sizes of corners on the same curve, this detector is highly sensitive
106 to the use of a single corner detection scale and a single fixed curvature-
107 threshold. He & Yung [21] improved this detector by using the adaptive

108 curvature-threshold and the dynamic region-of-support on both sides of each
109 curvature extremum point. Zhang et al. [20] further improved it by intro-
110 ducing the idea of curvature-product. In terms of the curvature product,
111 the strong corners become more distinguishable from the weak and false cor-
112 ners. Awrangjeb et al. [17] proposed another improvement (known as affine
113 resilient CSS - ARCSS - detector [25]) by selecting three corner detection
114 scales based on the curve’s affine-length.

115 Recently Awrangjeb and Lu [18] pointed out the two main problems of the
116 above CSS-based detectors. First, the CSS curvature estimation technique
117 is highly sensitive to the local variation and noise on the curve. Second, the
118 CSS corner detection technique requires appropriate Gaussian smoothing-
119 scale selection which is a difficult task. To overcome these two problems,
120 they proposed a new corner detector [18] based on the *chord-to-point distance*
121 *accumulation* (CPDA) for the discrete curvature estimation [26], which is less
122 sensitive to the local variation and noise on the curve and does not require
123 appropriate Gaussian smoothing-scale selection.

124 2.2. Feature Representation

125 In order to facilitate feature-point matching in any subsequent applica-
126 tion, each feature-point must be represented with some of its associated in-
127 formation. The more the representation is distinctive, the less the number of
128 false candidate matches will be obtained in the matching stage. The feature
129 representation is also known as the feature descriptor in the literature.

130 There are two different types of representations found in the literature:
131 geometric descriptors and local descriptors.

132 2.2.1. Geometric Descriptors

133 This type of representation [27, 28] is purely geometric, where each corner
134 or feature-point is represented using its curvature, angle, and distances from
135 neighbor corner-points. Zhou et al. [27] used the angles of Delaunay triangles
136 which are formed among the Harris interest-points [22]. These angles are
137 invariant to image translation, rotation, and uniform scaling. Huttenlocher
138 used [28] distance ratios defined by the quadruples of the feature-points. The
139 distance ratios are invariant to affine transformations.

140 Awrangjeb and Lu [15] used the curvature descriptors for their proposed
141 *geometric point matching* (GPM) technique. In fact, their proposed ARCSS
142 [25] and CPDA [18] detectors provide various information available for later
143 use. For each corner, these detectors provide its position, absolute curvature

144 value, angle with its two neighboring corners or with the endpoints when the
145 necessary number of neighbor corners are not found and the affine-lengths
146 between neighboring corners on the same curve. All of the above information
147 can be used collectively as the ‘curvature descriptor’ to represent the corner.

148 Though the geometric representation is easy to design and requires quite
149 small amount of storage per feature-point, the representation is not unique.
150 As a consequence, either the true correspondences are missed or a huge
151 number of false correspondences are obtained between two images and the
152 matching procedure becomes prohibitively expensive. This is why, the use of
153 geometric representation is quite limited in the literature [29].

154 2.2.2. Local Descriptors

155 This type of representation [1, 13, 11, 12] is based on the pixel values in a
156 specific neighborhood around each feature-point. They are harder to design,
157 but more distinctive than the geometric representations discussed above.

158 The SIFT-based descriptors, originally proposed by Lowe in [1], have been
159 described as the best among the different types of feature descriptors [12].
160 The original SIFT descriptor [1] captures a substantial amount of informa-
161 tion about the spatial intensity patterns around a feature point. For each
162 descriptor it computes a 3-D histogram of gradient location and orientation,
163 where the location is quantized into a 4×4 location grid and for each loca-
164 tion grid (total 16) the gradient angle is quantized into 8 orientations. The
165 resulting descriptor is a 128-dimensional vector (8 orientation bins for each of
166 4×4 location bins). This vector is robust to small deformations or localiza-
167 tion errors. To obtain illumination invariance, the descriptor is normalized
168 by the square root of the sum of squared components.

169 Further research has been carried out in the literature either to reduce
170 the descriptor’s dimension [11, 13] or to make it more distinctive [13, 12].
171 The PCA-SIFT (principal component analysis-SIFT) [13] descriptor encodes
172 the salient aspects of the image gradient in the feature point’s neighborhood;
173 however, instead of using SIFT’s smoothed weighted histograms, it applies
174 PCA to the normalized gradient patch to reduce the descriptor’s dimen-
175 sion. The SURF descriptor, on the other hand, describes a distribution of
176 Haar-wavelet responses within the interest point neighborhood and uses the
177 integral images to reduce the computational time. Though the lower dimen-
178 sional PCA-SIFT [13] and SURF [11] are helpful for fast feature matching,
179 PCA-SIFT was proved to be less robust than SIFT under affine transfor-
180 mations [12] and SURF was designed to handle rotation and scale attacks

181 only as a compromise between feature complexity and robustness [11]. The
182 GLOH (gradient location-oriented histogram) [12] is another variant of SIFT.
183 It considers more spatial regions for the histograms and reduces the higher
184 dimensionality of the descriptor using PCA. Though it was proved to be more
185 distinctive and robust than the original SIFT [12], it is computationally more
186 expensive [11]. Consequently, in spite of the above refinement versions, the
187 original SIFT is still the most popular.

188 The above local descriptors are invariant to image rotation, scaling, and
189 translations. The affine invariant local descriptors which are based on the it-
190 erative affine-region detection around the feature-points are computationally
191 very expensive [14].

192 2.3. Feature Matching

193 Mikolajczyk and Schmid [12] evaluated three feature matching techniques.
194 In *threshold*-based matching, two features are matched if the distance be-
195 tween their descriptors is below a predefined threshold. A feature may have
196 several matches in this strategy. In *nearest-neighbor*-based matching, two
197 features F_1 and F_2 with descriptors D_1 and D_2 respectively are matched if
198 the descriptor D_2 is the nearest neighbor of D_1 and their distance is below a
199 threshold. With this approach a feature should have at most one match. In
200 *nearest-neighbor-distance-ratio*-based matching (NNDR matching), which is
201 similar to nearest-neighbor-based matching, the threshold is applied to the
202 distance ratio between the first and second nearest neighbor matches, i.e.,
203 two features F_1 and F_2 with descriptors D_1 and D_2 respectively are matched
204 if $|D_1 - D_2|/|D_1 - D_3| < t$, where D_3 is the descriptor of the second nearest
205 neighbor match F_3 of F_1 . In this approach, a feature has also at most one
206 match.

207 The worst case (as well as the average case and the best case) running
208 time of these algorithms is $O(mn)$, where m and n are the numbers of features
209 in two images. Note that using these matching techniques a repeated feature
210 may be missed or there may be some false positive matches.

211 The GPM (Geometric Point Matching) technique by Awrangjeb and Lu
212 [15, 25] is shown in Fig. 1. Let S_1 and S_2 be two sets of feature-points
213 (corners) from two given images where each feature-point is described with a
214 descriptor. The initial candidate matches are obtained using the *threshold*-
215 based matching discussed above. Then for each combination of three can-
216 didate matches, it estimates the affine transformation parameters between
217 the images using an iterative procedure. In each iteration, it transforms all

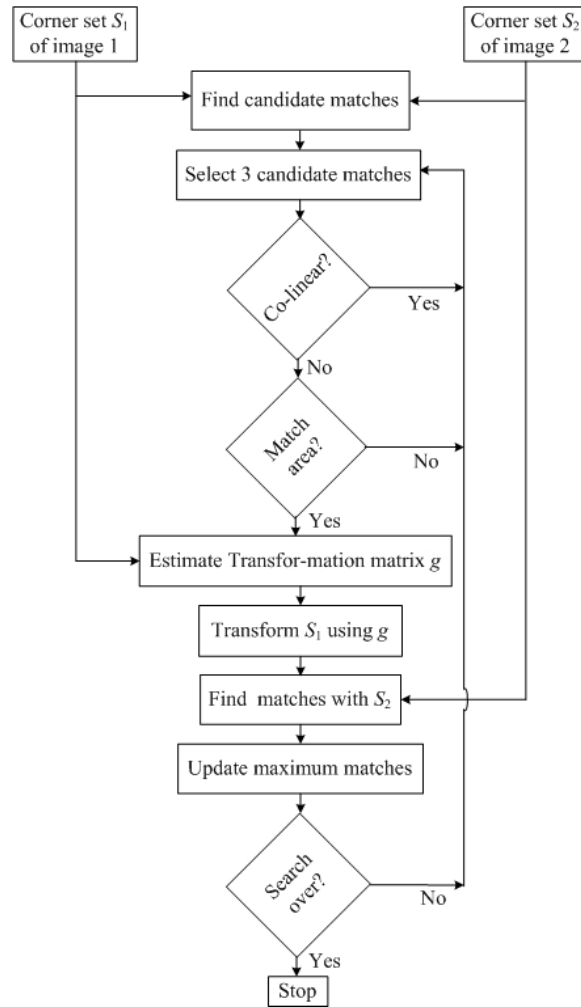


Figure 1: Block diagram for the geometric point matching technique.

218 the feature-points in one image using the estimated parameters and finds
 219 matches in the other image. The algorithm tracks the set of parameters that
 220 offer the highest number of matches over all the iterations. Consequently,
 221 the GPM technique works in following three steps:

- 222 • *Step 1*: Find the candidate corner matches by matching the descriptors
 223 in S_1 and S_2 .
- 224 • *Step 2*: If three candidate matching corners are non-collinear on each
 225 image and the ratio of the areas of the corresponding triangles in both

226 of the images is within a specific range, estimate the transformation
227 matrix g between these triangles.

- 228 • *Step 3*: Transform all the corners in S_1 using g and determine the
229 number of corner matches with S_2 allowing a maximum localization
230 error of 3-pixels.

231 This above algorithm, though offers better matching performance, is more
232 expensive than any of the above three matching techniques. However, in
233 practice, if the feature representation is very distinctive, it costs close to the
234 above techniques, i.e., $O(mn)$ [25, 29].

235 Note that though the idea of the GPM technique [15] is similar to that
236 of the RANSAC (Random Sample Consensus) [30], the former differs from
237 the later in the following aspects. Firstly, the GPM obtains the candidate
238 matches using the feature descriptors. Since the standard RANSAC does
239 not use such local matching process, it performs poorly when the propor-
240 tion of the outliers increases above 50% [1]. Secondly, the RANSAC sets a
241 termination condition, e.g., terminate the algorithm once a specific number
242 of iterations are executed. We conjecture that if the feature descriptors are
243 distinctive, then the number of false candidate matches will be low and we
244 do not need to set any termination condition. Finally, the RANSAC requires
245 a careful setup of a number of algorithmic parameters to ensure its good
246 performance. The GPM requires only a few parameter setups based on the
247 range of geometric transformations we want to consider.

248 2.4. Promising TII Approaches

249 In this section we briefly present promising TII approaches and their main
250 problem.

251 Zhang and Chang [9] formulated a stochastic attributed relational graph
252 matching for TII. Corners were considered as vertices and their positions,
253 corresponding pixel values and wavelet features were used to represent (at-
254 tribute) the vertices. However, the graph matching method involves a com-
255 plex process of stochastic belief propagation and thus identification speed is
256 slow [8]. Lowe [1] used DoG keypoints, SIFT descriptors and NNDR match-
257 ing technique. Ke et al. [10] and Zhao et al. [8] used PCA-SIFT descriptors
258 to represent DoG keypoints. While Ke et al. [10] used the RANSAC al-
259 gorithm to match them; Zhao et al. [8] used a bipartite graph matching
260 algorithm. Xu et al. [7] used SIFT descriptors around DoG keypoints to
261 apply a multi-level (block-by-block) image matching method.

262 However, because of the large number of interest points in images (pos-
263 sibly exceeding 1000), matching based on interest points is extremely time
264 consuming and inappropriate for many applications.

265 Since Lowe’s TII approach has demonstrated its superior performance
266 [12], we compare it with our proposed TII approaches.

267 *2.5. Contributions of This Paper*

268 All three stages of a TII system are important for its performance. In
269 Lowe’s approach [1], DoG keypoints, SIFT feature vector and NNDR are used
270 in the three stages respectively. We propose to use alternative techniques in
271 each of the three stages.

272 In stage one, since the number of DoG keypoints is high, we propose
273 two solutions to reduce the number of keypoints. In the first solution, the
274 detection stage becomes faster and the number of detected DoG keypoints is
275 reduced largely, which also speeds up the later stages. In the second solution,
276 we propose using corners instead of DoG keypoints. In general, corners offer
277 the following advantages over keypoints:

- 278 • Corners are visually distinguishable and more robust than their key-
279 points counterparts.
- 280 • In an image, the number of corners is much lower than the number of
281 keypoints.
- 282 • Corners can be ranked based on their strength like the curvature value
283 or the number of corners can be controlled by changing the detection
284 thresholds. Therefore, a particular number of strong corners can be
285 selected based on the application and it gives further reduction in com-
286 putational cost during matching. In contrast, it is very hard to rank
287 the DoG keypoints.
- 288 • Corner detection requires less time than keypoint detection in the scale-
289 space.

290 In stage two, corners can be represented with the curvature descriptors
291 as well as with the SIFT feature vectors.

292 In stage three, a matching technique has to be used to obtain all the
293 true matches between images. The NNDR matching technique described
294 in Section 2.3 matches feature descriptors between two images but does not

295 consider the underlining geometric relations between them. Therefore, it may
296 miss some true matches or find some false matches. On the other hand, the
297 GPM technique not only matches feature descriptors between images but
298 also considers their geometric relations. Consequently, the GPM provides
299 better matching performance than the NNDR matching [29].

300 The three main contributions of this paper are as follows:

- 301 1. We compare corners with DoG and scaled-DoG keypoints in Section
302 3 and through experimentation we show that corners perform better
303 than their keypoints counterparts in terms of both repeatability and
304 localization error.
- 305 2. In Section 4, we propose five different TII approaches by combining
306 different techniques in different stages.
- 307 3. Through experimentation and analysis we show that the two feature-
308 point reduction solutions combined with the SIFT descriptors and the
309 GPM technique not only improve the computational efficiency and de-
310 crease the storage requirement, but also improve the TII accuracy (see
311 Section 5).

312 **3. Comparison of Feature Detectors**

313 What feature detectors and what type of feature-points to use in the stage
314 one of TII have a significant effect on both of the efficiency and effectiveness
315 of a TII system. In this section, we briefly describe three feature detectors
316 – namely, ARCSS corner detector [17, 25], CPDA corner detector [18], and
317 DoG keypoint detector [1] and present a comparative performance study
318 of these three detectors. We compare the detectors in terms of robustness
319 (repeatability and localization error) and efficiency (detection time). We will
320 use these three detectors in our proposed TII approaches later presented in
321 Section 4.

322 *3.1. ARCSS Detector*

323 The ARCSS corner detector [17, 25] extracts edges in the gray-scale im-
324 age using the Canny edge detector [31]. It then parameterizes each curve
325 (edge) with the affine-length. In order to eliminate noise, it convolves each
326 parameterized curve using the Gaussian kernel in one of three medium scales
327 based on the curve’s affine-length. Thereafter, it calculates absolute cur-
328 vature value on each point of the smoothed curves and considers curvature

329 maxima points as candidate corners. Both weak and false corners are re-
330 moved using the appropriate thresholds. Finally, corners are tracked down
331 to the finest scale to improve localization.

332 The experimental results in [17, 25] showed that in geometric transforma-
333 tions the ARCSS detector outperformed existing CSS detector [19], which
334 had outperformed many other detectors including the Harris interest-point
335 detector [22].

336 3.2. CPDA Detector

337 The CPDA corner detector [18] first extracts planar curves from the edge
338 image detected by the Canny edge detector [31]. Each curve is then smoothed
339 with a small width Gaussian kernel in order to remove quantization noise and
340 trivial details. In order to make strong and weak corners more distinguish-
341 able, we first use three chords of different lengths to estimate three normal-
342 ized discrete curvature values on each point of the smoothed curve. Then we
343 multiply the normalized curvatures to obtain the curvature product (a single
344 estimated curvature) at each point. The maxima of the absolute curvature
345 products along the smoothed curve are then obtained as candidate corners.
346 Finally, it follows a two-step refinement process that removes weak and false
347 corners using thresholds.

348 The experimental results in [18] showed that the CPDA detector outper-
349 formed many contour-based detectors including CSS [19], multi-scale CSS
350 [20], and adaptive threshold-based CSS [21] detectors.

351 3.3. DoG Keypoint Detector

352 The DoG keypoint detector [1] first blurs the input image with a small
353 width Gaussian filter to prevent aliasing and to increase the stability of the
354 keypoints. It then builds the Gaussian pyramid by repeatedly smoothing
355 the blurred image. The difference-of-gaussian (DoG) pyramid is built by
356 subtracting the adjacent Gaussian smoothed images in the same octave of
357 the Gaussian pyramid. Finally, they obtain the local extrema positions in the
358 DoG pyramid as keypoints. When an extrema is found, two tests are applied
359 before labeling it as a keypoint. First, it must have sufficient contrast and
360 second, it should not be an edge point.

361 In one of our solutions, we scale down the original images before DoG
362 keypoint detection. We name this modified detector as ‘scale-DoG detector’
363 in the rest of the paper.

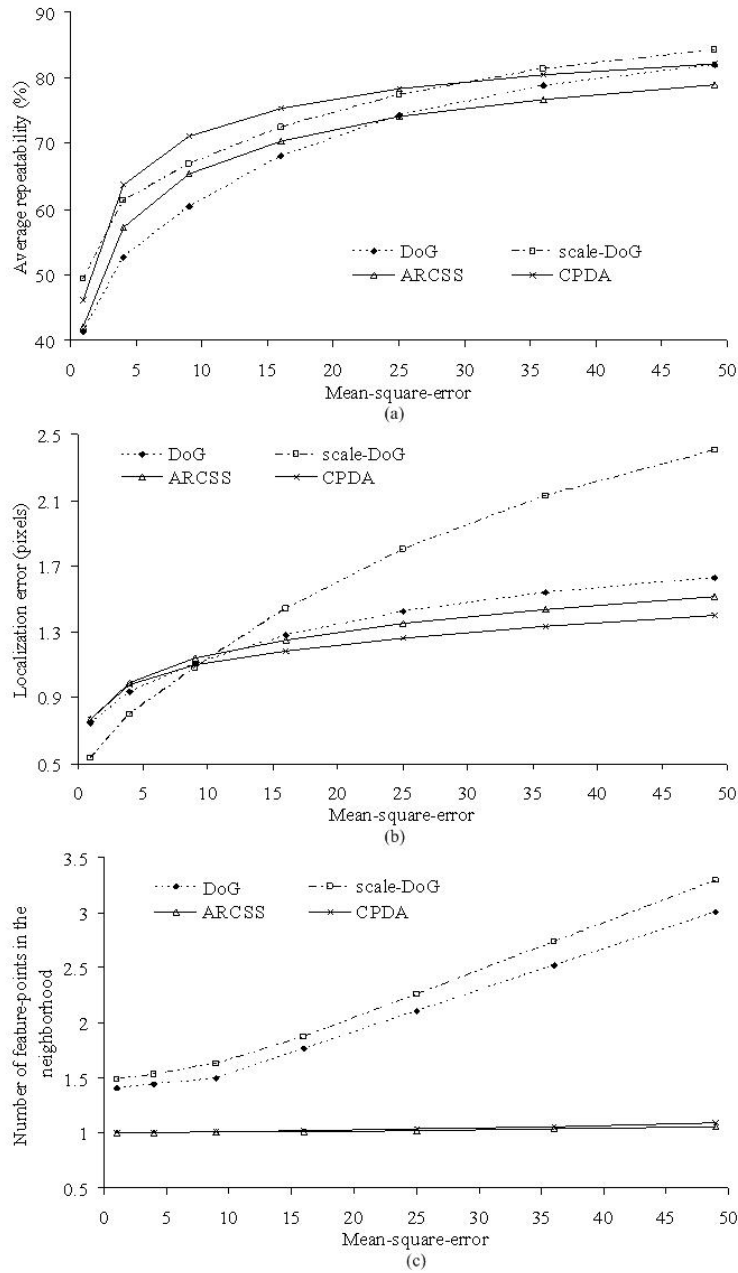


Figure 2: Comparing Detector's Performance: (a) average repeatability, (b) localization error, and (c) number of feature-points in the neighborhood.

Table 1: Performance summary of different detectors.

Properties	DoG	scale-DoG	ARCSS	CPDA
<i>Feature-points in images</i>	586	82	30	41
<i>Feature detection time (seconds)</i>	2.71	0.19	0.17	2.6
<i>Average repeatability (%)</i>	65.36	70.47	66.40	71.04
<i>Localization error (pixels)</i>	1.24	1.46	1.21	1.15
<i>Feature-points in (transformed) neighborhood</i>	1.93	2.12	1.06	1.06

364 3.4. Performance Comparisons

365 We used a database of 23 original images and their more than 8,500 test
366 (transformed and signal processed, see [25, 29] for details of this database)
367 images to evaluate the performance of the detectors. In the case of the
368 scale-DoG detector, the original (512×512) images were down scaled to
369 (128×128). The test images were then generated from the scaled-down im-
370 ages. In order to find repeated corners between the original and transformed
371 images, we transformed the feature-points detected in the original image us-
372 ing the known transformation matrix and then found their repetitions with
373 the feature-points detected in the test image. In this case, we allowed a
374 *mean-square-error* (MSE) of e square pixels, which means a feature-point in
375 the original image was considered as repeated if at least one feature-point
376 was found in its (transformed) neighborhood in the test image when the
377 maximum distance was e square pixels. The nearest feature-point in the
378 neighborhood was considered as repeated.

379 We evaluated the performance in terms of *average repeatability* and *lo-*
380 *calization error* [25, 29]. *Repeatability* indicates how stable the detected
381 feature-points are under different geometric transformations and signal pro-
382 cessing attacks. The *average repeatability* is defined as the percentage of the
383 total observed feature-points repeated between the original and test images:

$$R_{avg} = \frac{100 \times N_r}{2} \left(\frac{1}{N_0} + \frac{1}{N_t} \right), \quad (1)$$

384 where N_0 and N_t are the number of feature-points in the original and test
385 images, respectively, and N_r is the number of repeated feature-points between
386 them.

387 *Localization error* shows how accurately a detected feature-point is local-
388 ized by the detector. A lower localization error indicates a better accuracy.

389 It is measured using the *root-mean-square-error* (RMSE) of the detected
 390 feature-points:

$$L_e = \sqrt{\frac{1}{N_r} \sum_{i=1}^{N_r} [(x_{oi} - x_{ti})^2 + (y_{oi} - y_{ti})^2]}, \quad (2)$$

391 where (x_{oi}, y_{oi}) and (x_{ti}, y_{ti}) are the positions of the i -th repeated feature-
 392 point in the original and test images respectively.

393 We also calculated the number of detected feature-points in the original
 394 images, the feature detection time, and the number of detected feature-points
 395 in the neighborhood (in the test image) for each repeated feature-point in
 396 the original image [29]. A low number of feature-points is essential for many
 397 applications like image copyright protection [2], fast image registration [15],
 398 and mobile robot vision [32]. A low feature detection time is always im-
 399 portant. A larger number of detected feature-points in the neighborhood
 400 of a repeated feature-point indicates a higher chance of mismatching during
 401 image matching.

402 Fig. 2 shows the detail comparative performance and Table 1 presents
 403 the performance summary (average results). The scale-DoG detector offered
 404 the higher average repeatability than the original DoG detector. However,
 405 the scale-DoG detector offered the higher localization error with the increase
 406 of the neighborhood size. The ARCSS and CPDA detectors showed higher
 407 average repeatability than the DoG and scale-DoG detectors respectively
 408 when the neighborhood size was small. The opposite scenario was observed
 409 when the neighborhood size was large. The reason is, in the case of the DoG
 410 and scale-DoG detectors the detected keypoints are very close to each other.
 411 Fig. 2(c) shows that for these two detectors the number of keypoints in the
 412 transformed image is very high. Consequently, a close but different keypoint
 413 could be chosen as repeated offering high repeatability. Moreover, the DoG
 414 and scale-DoG detectors suffered from high localization error as shown in
 415 Fig. 2(b).

416 From the results in Table 1 we can say that the ARCSS detector was
 417 the fastest and it detected the lowest number of feature-points among the
 418 four. On the other hand, the CPDA detector offered the highest average re-
 419 peatability and the lowest localization error. However, it was slower than the
 420 ARCSS and scale-DoG detectors. In the next section we will use all these four
 421 feature detectors to compare the performance of different TII approaches.

Table 2: Proposed transformed image identification approaches.

Approaches	Detectors	Descriptors	Matching
<i>Approach 1</i>	ARCSS	curvature	GPM
<i>Approach 2</i>	CPDA	curvature	GPM
<i>Approach 3</i>	scale-DoG	SIFT	GPM
<i>Approach 4</i>	ARCSS	SIFT	GPM
<i>Approach 5</i>	CPDA	SIFT	GPM

422 4. Proposed Transformed Image Identification Approaches

423 For identifying the transformed images for a given test image, all three
 424 stages – feature detection, description, and matching – should be efficient
 425 and effective. The original SIFT approach detects a several hundreds to a
 426 few thousands of keypoints from a medium size image. Its detection stage
 427 is slow because of the use of scale-space. Moreover, for many applications,
 428 e.g., image copyright protection, this huge number of keypoints also make
 429 the later stages unnecessarily slow.

430 To overcome this problem, we propose two solutions. The first solution is
 431 to down scale the input image before DoG keypoint detection. So, the detec-
 432 tion stage becomes faster and the number of detected keypoints is reduced
 433 largely, which also speeds up the later stages. The highly distinctive SIFT
 434 descriptors are used to represent the keypoints and the GPM technique is
 435 used for keypoint matching.

436 As a second solution, we propose using corners instead of DoG keypoints.
 437 We use corners detected by our previously proposed two detectors: ARCSS
 438 [17, 25] and CPDA [18] detectors. Either the curvature descriptor or the
 439 highly distinctive SIFT descriptors at corner locations can be used to rep-
 440 resent corners and the GPM is used for corner matching. While finding the
 441 SIFT descriptors at a corner location we considered all the SIFT descriptors
 442 (detected by Low’s approach) within the 3-pixel neighborhood around the
 443 corner in question.

444 Consequently, in this paper we propose five approaches of TII as shown
 445 in Table 2.

446 4.1. Approach 1: ARCSS corners with Curvature Descriptors

447 Approach 1 uses the ARCSS corners as feature-points, the curvature de-
 448 scriptors to represent the corners and the GPM for corner matching. In the

449 first step (see Section 2.3) of the GPM technique, the candidate matches are
450 obtained by matching the curvature descriptors of the query image and a
451 database image. Once the candidate point matching set is obtained, Steps 2
452 and 3 of the GPM technique are applied to identify the transformed images.

453 *4.2. Approach 2: CPDA corners with Curvature Descriptors*

454 Approach 2 uses the CPDA corners as feature-points, the curvature de-
455 scriptors to represent the corners and the GPM for corner matching.

456 The differences of Approach 2 with Approach 1 are – (i) instead of ARCSS
457 corners Approach 2 uses the CPDA corners and (ii) instead of affine-length
458 parameterized Euclidean curvature (see [18]) Approach 2 uses the CPDA
459 discrete curvature (see [25]) to represent corners.

460 *4.3. Approach 3: Scale-DoG Keypoints with SIFT Descriptors*

461 Approach 3 uses the DoG keypoints as feature-points, the SIFT descrip-
462 tors to represent the keypoints and the GPM for keypoint matching.

463 As discussed above, in Low’s approach [1], the DoG detector detects sev-
464 eral hundred to a few thousand keypoints from a medium sized image. Ap-
465 plying the GPM technique to such a huge number of keypoints will be com-
466 putationally very expensive. Moreover, since each descriptor of a keypoint
467 is a 128-dimensional vector of floating point numbers, storing the descriptor
468 information of an image would take a few megabytes of memory, which in
469 turn would take several gigabytes of memory for a moderate image database.

470 In order to reduce both of the matching and storage costs, the images are
471 scaled down before feature detection to reduce the number of keypoints. In
472 this case, only the coarse scale features are detected which are more robust
473 than the fine scale features.

474 The candidate keypoint matches are obtained by matching the SIFT de-
475 scriptors of the detected keypoints using the NNDR matching technique de-
476 scribed in Section 2.3. Once the candidate point matching set is obtained,
477 Steps 2 and 3 of the GPM technique are applied to identify the transformed
478 images.

479 *4.4. Approach 4: ARCSS corners with SIFT Descriptors*

480 Approach 4 uses the ARCSS corners as feature-points, the SIFT descrip-
481 tors to represent the corners and the GPM for corner matching.

482 It was observed that the number of false candidate matches is very high
483 with Approach 1 and, therefore, the matching stage becomes expensive. To

484 overcome this problem, Approach 4 uses the highly distinctive SIFT de-
485 scriptor, instead of the curvature descriptor, at each corner position while
486 obtaining the candidate corner matching set. This strategy improves the
487 matching performance significantly in terms of both efficiency and effective-
488 ness, though it increases the storage requirement. Similar to the Approach
489 3 discussed above, the candidate corner matching set is obtained using the
490 NNDR matching and the transformed images are identified using the GPM
491 technique.

492 Once the candidate point matching set is obtained, Steps 2 and 3 of the
493 GPM technique are applied to identify the transformed images.

494 4.5. Approach 5: CPDA corners with SIFT Descriptors

495 Approach 5 uses the CPDA corners as feature-points, the SIFT descrip-
496 tors to represent the corners and the GPM for corner matching.

497 The only difference between Approach 5 and Approach 4 is, instead of
498 using the ARCSS corners Approach 5 uses the CPDA corners.

499 5. Performance Study

500 We implemented the five proposed approaches using MATLAB 7 on a
501 machine with the following configuration: Dual Core AMD Opteron(tm)
502 Processor (265 × 2) with 4GB RAM. This section presents the experimental
503 results on a large database and compares the proposed approaches with the
504 existing most popular Low's approach [1]. It presents the results using the
505 precision-recall graph [4].

506 5.1. Test Database

507 We randomly selected 1,050 images from David Nister's recognition database
508 [33].¹ Each of the images was of size 480 × 640 and converted to gray-scale.
509 Then the following 10 simple transformations were applied to each image:

- 510 (i) rotation-crop: $\theta = 30^\circ$,
- 511 (ii) scale: $s_x = 1.2, s_y = 0.8$,
- 512 (iii) rotation-scale: $\theta = 20^\circ, s_x = 1.2, s_y = 0.8$,
- 513 (iv) shear: $s_{hx} = s_{hy} = 0.012$,
- 514 (v) rotation-scale-shear: $\theta = 10^\circ, s_x = 1.1, s_y = 0.9, s_{hx} = s_{hy} = 0.01$,

¹Available at <http://www.vis.uky.edu/~dnister/>. Permission is provided for research purposes on this web site.

- 515 (vi) jpeg: *qualityfactor* = 20,
- 516 (vii) Gaussian noise: *mean* = 0, *variance* = 0.001,
- 517 (viii) Gaussian blurring: *sigma* = 3, *window* = 3 × 3,
- 518 (ix) rotation-scale-jpeg: $\theta = 20^\circ$, $s_x = 1.2$, $s_y = 0.8$, *quality* = 20 and
- 519 (x) rotation-scale-shear-jpeg: $\theta = 20^\circ$, $s_x = 1.2$, $s_y = 0.8$, $s_{hx} = s_{hy} = 0.01$, *quality* = 20.

520 As a result, there were a total of 1,050 groups of images in the database.
 521 Each group had 10 relevant images. Therefore, there were a total of 10,500
 522 images in the database.

523 5.2. Evaluation Metrics

524 We used *precision* and *recall* [4] collectively to measure the identification
 525 performance. *Recall* measures the system’s capacity to retrieve the relevant
 526 images from the database. It is defined as the ratio between the number of
 527 retrieved relevant images r and the total number of relevant images r_l (group
 528 size) in the database:

$$Recall = \frac{r}{r_l}. \quad (3)$$

529 *Precision* measures the retrieval accuracy. It is defined as the ratio between
 530 r and the number of retrieved images r_t :

$$Precision = \frac{r}{r_t}. \quad (4)$$

531 In practice, the performance of an information retrieval system is pre-
 532 sented using the *precision-recall graph*, where the higher the precision at a
 533 given recall value the better the performance of the retrieval system [4].

534 In order to calculate the *precision* and *recall* for a given query image us-
 535 ing (3) and (4), we first rank all the database images based on the number
 536 of feature-point matches with the query image. We then consider the mini-
 537 mum top r_t images as the retrieved images such that they include all $r_l = 10$
 538 relevant images of the query image.

539 5.3. Approaches to be Compared

540 In the performance study, seven different TII approaches were considered
 541 as shown in Table 3. Approaches 1 to 5 in Table 3 are the proposed ones
 542 discussed in Section 4. Approach 3 scaled down each original image of size
 543 480×640 to 60×80 . Approaches 6 and 7 were implemented to compare
 544 with the proposed approaches. Approach 6 is Low’s approach [1] which used
 545 the DoG keypoints, SIFT descriptors and NNDR matching. Approach 7 is

Table 3: Transformed image identification approaches used in the performance study.

Approaches	Detectors	Descriptors	Matching
<i>Approach 1</i>	ARCSS	curvature	GPM
<i>Approach 2</i>	CPDA	curvature	GPM
<i>Approach 3</i>	scale-DoG	SIFT	GPM
<i>Approach 4</i>	ARCSS	SIFT	GPM
<i>Approach 5</i>	CPDA	SIFT	GPM
<i>Approach 6</i>	DoG	SIFT	NNDR
<i>Approach 7</i>	scale-DoG	SIFT	NNDR

546 similar to Approach 3, but uses the NNDR matching instead of the GPM
 547 technique to show the superior performance of the GPM technique.

548 Note that the GPM technique was found computationally very expensive
 549 when it was applied to match the DoG keypoints due to the large number of
 550 keypoints, where each keypoint is represented using a 128-dimensional SIFT
 551 feature-vector (so it is not reported here). However, the accuracy was found
 552 better than using the NNDR matching (Approach 6).

553 5.4. Experimental Results and Discussions

554 Fig. 3 shows the TII performance of different approaches. Due to high
 555 computational complexity of some of the approaches, the graphs in Fig. 3
 556 were averaged on 177 random queries only. The following sub-sections present
 557 and discuss the experimental results.

558 5.4.1. Scale-DoG vs Usual DoG Keypoint Detectors

559 Approach 7 performed worse than Low’s approach (Approach 6) for the
 560 following two reasons: (i) in Approach 7, though the number of feature-
 561 points is decreased to speed up both of the detection and matching stages,
 562 the number of matching features using the NNDR matching is also decreased;
 563 and (ii) in contrast, the main reason behind the success of Low’s approach
 564 is, it detects a lot of features and a large number of them are found matched
 565 between the relevant images.

566 5.4.2. GPM vs NNDR matching

567 The NNDR matching is based on the individual descriptors (see Section
 568 2.3). In contrast, the GPM matching first estimates the transformation ma-
 569 trix between images using the candidate matches, transforms corners in one

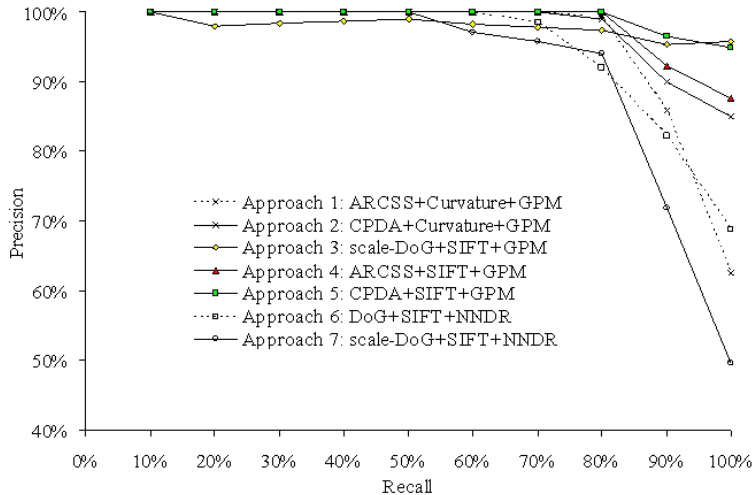


Figure 3: Transformed image identification performance by different approaches.

570 image and then finds matches with the other image. Consequently, the GPM
 571 matching is like a global matching technique and offers a higher number of
 572 matches than the NNDR matching.

573 Approach 7 uses the NNDR matching only. Approach 3 uses the GPM
 574 technique which in addition to obtaining the candidate matches using the
 575 NNDR matching, estimates the transformation matrix before finding the
 576 corner matches. As a result, Approach 3 performed better than Approach
 577 7 as shown in Fig. 3. This shows the superior performance of the GPM
 578 technique compared to the NNDR matching.

579 5.4.3. Corner-Curvature vs DoG Keypoint-SIFT Combinations

580 Approach 1 required lower storage and total computational costs, but
 581 still showed slightly higher identification accuracy than Approach 6. While
 582 the former used the curvature descriptor with the GPM technique, the latter
 583 used the SIFT descriptors with the NNDR matching technique. Approach 2
 584 offered higher identification accuracy than Approach 1, but required a higher
 585 computational cost. This is because the number of CPDA corners was higher
 586 than the number of ARCSS corners and, thus, there may be a larger number
 587 of false candidate matches in Approach 2 than in Approach 1.

588 However, Approach 6 outperformed Approach 1 at higher recall values as
 589 shown in Fig. 3. The reason is, as the average number of detected corners

590 was low using the ARCSS detector (Approach 1), some irrelevant images
 591 (usually with a higher number of detected corners than the relevant images)
 592 may also be retrieved. The performance was increased at higher recall values
 593 by increasing the number of corners, as evident from the performance of Ap-
 594 proach 2 (using CPDA corners). However, this made the matching procedure
 595 expensive.

Table 4: Comparing different transformed image identification approaches of Table 3.

Approaches	Features ^a	Storage	DR time	Mat. time	Total time	Accuracy
<i>Approach 1</i>	46	<i>low</i>	0.17	4.330	4.50	94.8
<i>Approach 2</i>	58	<i>low</i>	2.6	5.36	7.96	97.4
<i>Approach 3</i>	37	<i>moderate</i>	0.54	0.004	0.544	97.9
<i>Approach 4</i>	71	<i>moderate</i>	0.17	0.045	0.215	98.0
<i>Approach 5</i>	90	<i>moderate</i>	2.6	0.17	2.77	99.1
<i>Approach 6</i>	585	<i>high</i>	2.70	3.380	6.08	94.2
<i>Approach 7</i>	37	<i>moderate</i>	0.54	0.002	0.542	90.9

^a*Features* = number of features per image; *DR time* = feature detection and representation time per image; *Mat. time* = time to match a pair of images (a query image and a database image); all times are in seconds; *Accuracy* is the average precision over all the recall values shown in Fig. 3; total number of features and *DR time* were averaged over the whole database discussed in Section 5.1; and *Mat. time* was averaged over 177 random queries.

597 5.4.4. Corner-Curvature vs Corner-SIFT Combinations

598 When the SIFT descriptors were used to represent the corners the match-
 599 ing cost reduced drastically as the number of false candidate matches de-
 600 creased considerably due to the highly distinctive nature of the SIFT de-
 601 scriptor [1] (see Approaches 4 and 5 in Fig. 3).

602 5.4.5. The Best TII Approaches

603 Table 4 summarizes the performance of different approaches. Approach
 604 5 performed the best in terms of identification accuracy, followed by Ap-
 605 proaches 3 and 4. However, Approach 5 was much slower than Approaches 3
 606 and 4 due to its slow corner detector. Approach 6 required very high storage
 607 for descriptors and all of its stages were quite expensive. The identifica-
 608 tion accuracies of Approaches 1 and 2 were slightly better than the SIFT.
 609 Moreover, Approaches 1 and 2 required much lower storage than Approach

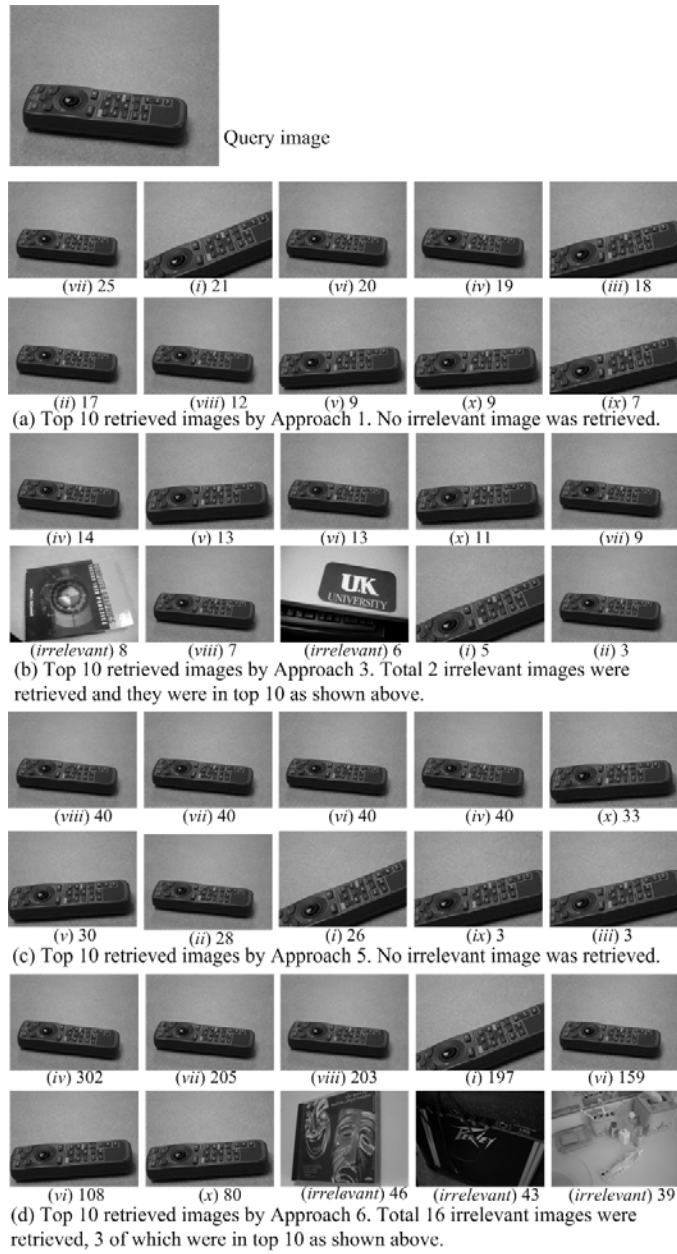


Figure 4: Transformed image retrieval examples by Approaches 1, 3, 5 and 6. Approach 4 performed the same (no irrelevant images were retrieved) as Approach 1 as shown in (a) above. And Approach 2 performed the same (no irrelevant images were retrieved) as Approach 5 as shown in (c) above. The number of detected features in the query image by these Approaches were 19, 37, 66 and 488 respectively. The roman number (inside parentheses) under each image indicates a particular transformation discussed in Section 5.1 and the number after it is the number of feature matches with the query image in (a).

610 6. The storage requirement, total time and the identification accuracy of
611 Approach 6 were significantly improved by the application of the proposed
612 first feature-point reduction solution (see Approach 3). The total time and
613 the identification accuracy of Approaches 4 and 5 (the second solution) were
614 also considerably improved by using the distinctive SIFT descriptors at the
615 corner locations, though the storage requirement was moderately increased.
616 This proves that the highly distinctive SIFT descriptor is more effective than
617 the curvature descriptor.

618 Therefore, by combining the advantages of corners, SIFT descriptors and
619 the GPM technique, Approaches 3, 4 and 5 can be considered the overall
620 best.

621 Note that we used the machine code provided by D. Lowe³ for the fea-
622 ture detection and representation stages of Approaches 3, 6 and 7. The DR
623 time shown in Table 4 is not important when the feature-point detection
624 and representation stages are done offline. However, the DR time for query
625 processing is important.

626 5.5. Transformed Image Identification Examples

627 Fig. 4 presents an example of the transformed image identification by
628 Approaches 1, 3, 5 and 6. In this example, Approach 6 retrieved 16 irrelevant
629 images, Approach 3 retrieved 2 irrelevant images and Approaches 1 and 5
630 retrieved no irrelevant images. The top 10 retrieved images are shown in
631 Fig. 4 for each Approach, where 3 irrelevant retrieved images by Approach
632 6 and 2 irrelevant images by Approach 3 are shown. Note Approaches 2 and
633 4 performed the same (no irrelevant images were retrieved) as Approach 5
634 (Fig. 4(c)) and 1 (Fig. 4(a)) respectively.

635 6. Conclusion and Future Work

636 We have shown that both of the feature detection and matching stages
637 of TII can be significantly speeded up by reducing the number of detected
638 feature-points. We have presented two feature-point reduction solutions.

639 The first solution is to down scale the image before the DoG keypoint
640 detection. We have found that this solution significantly increases the TII
641 performance when the scale-DoG keypoints are matched using the GPM tech-
642 nique, which finds more repeated features than the existing NNDR matching

³Available at <http://www.cs.ubc.ca/~lowe/keypoints/>

643 technique with the expense of little more computational time. However, this
644 additional expense can be considered negligible because of high improvement
645 in the identification accuracy (comparing accuracy of Approach 3 with that
646 of Approaches 6 and 7 in Table 4).

647 The second solution is to use corners instead of DoG keypoints. Either the
648 curvature descriptor or the highly distinctive SIFT descriptors at corner lo-
649 cations can be used to represent corners. The corner-curvature combinations,
650 though require higher matching time, perform better than the keypoint-SIFT
651 combinations (comparing performance of Approaches 1 and 2 with that of
652 Approaches 6 and 7 in Table 4). However, when the corners are represented
653 using the SIFT descriptors (corner-SIFT combinations), not only the match-
654 ing time is significantly reduced, but also the TII accuracy (robustness) is
655 improved (comparing performance of Approaches 4 and 5 with that of Ap-
656 proaches 1 and 2 in Table 4).

657 Therefore, the two feature-point reduction solutions combined with the
658 SIFT descriptors and the GPM technique not only improve the computa-
659 tional efficiency and decrease the storage requirement, but also improve the
660 TII accuracy.

661 However, there are at least two more directions for further improvement.
662 Firstly, the reduction in the number of feature-points in the proposed solu-
663 tions could limit the applicability in some applications like matching images
664 with different lighting conditions and backgrounds where the repeatability
665 rate of the detected feature-points is very low. Increasing the repeatability
666 rate significantly in such cases, while keeping the number feature-points low,
667 could be a future research interest.

668 For example, we could use the tensor-based feature representation (in the
669 form of a high-order matrix [34]), which preserves the distinctive information.
670 As a result, the repeatability rate could be improved despite the presence of
671 a small number of feature-points. Xu et al. [35] successfully applied tensor
672 features for human gait recognition.

673 Secondly, the matching time shown in Table IV is the time to match a
674 pair of images. Consequently, the query time (to get search results after
675 searching the whole database) is very high for a large database. There could
676 be two solutions: 1) Representation of the database as a vocabulary tree
677 [33], where each image is represented as a number of nodes (features) in the
678 tree. 2) Interactive image searching techniques, which are currently being
679 used for user feedback-based CBIR systems [5, 6], could be employed. A set
680 of initial searched images are first shown for user feedbacks. Then based on

681 user feedbacks, the CBIR system iteratively refines the search results.

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