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1. Introduction

[1] We thank *Hilberts and Troch* [2006] for their comment on our paper [*Cartwright et al.*, 2005]. Before proceeding with our specific replies to the comments we would first like to clarify the definitions and meanings of equations (1)–(3) as presented by *Hilberts and Troch* [2006]. First, equation (1) is the fundamental definition of the (complex) effective porosity as derived by *Nielsen and Perrochet* [2000]. Equations (2) and (3), however, represent the linear frequency response function of the water table in the sand column responding to simple harmonic forcing. This function, which was validated by *Nielsen and Perrochet* [2000], provides an alternative method for estimating the complex effective porosity from the experimental sand column data in the absence of direct measurements of h_{tot} (which are required if equation (1) is to be used).

2. Low-Frequency Response: Effects of Proximity of the Sand Surface

[2] First, we would like to clarify that our conclusion that at lower frequencies the complex effective porosity $n_\omega \to n$ was based on the experimental data from three different soil types which can be described by the empirical equation [*Cartwright et al.*, 2005, equation (5)],

$$n_\omega = \frac{n}{1 + 2.5\left(i\frac{n\omega H_\psi}{K}\right)^{2/3}} \tag{1}$$

where ω is the angular frequency, H_ψ is the equivalent saturated height of the capillary fringe and K is the saturated hydraulic conductivity. The functional form of the equation is based on that which is derived based on the *Green and Ampt* [1911] approximation [cf. *Cartwright et al.*, 2005, equation (7)]. The *Green and Ampt* [1911] version predicts $|n_\omega|$ to have an asymptotic slope of -1 as opposed to the $-2/3$ seen in the data.

[3] Inspection of (1) reveals that as $\omega \to 0$ then $n_\omega \to n$. We will now address the comment of *Hilberts and Troch* [2006] regarding the influence of proximity to the sand surface on the low-frequency limit of the complex effective porosity (n_ω).

[4] We agree with *Hilberts and Troch* [2006] that the proximity of the sand surface can significantly influence water table and moisture dynamics. However, on the basis of the experimental data the proximity of the sand surface played no measurable role on the response of the water table. This is evidenced as follows.

[5] We draw the reader's attention to section 4.2 of *Cartwright et al.* [2005]. In this section, the results of some of our earlier experimental work [*Cartwright et al.*, 2004] are used to eliminate the influence of the sand surface proximity on the 2005 experimental data. That is, the 2004 data indicate that the sand surface begins to influence the behavior of the water table when $z_s \leq h_{\max} + 0.5H_\psi$. Where z_s is the sand surface elevation and $h_{\max}$ is the maximum water table elevation ($=d + |\eta|$).

[6] For the specific case of the glass bead tests, the corresponding values (calculated as the average for all the glass bead tests, see tests 1–21 in Table 2 of *Cartwright et al.* [2005]) were $h_{\max} \approx 0.67$ m and $H_\psi = 1.5$ m leading to $h_{\max} + 0.5H_\psi \approx 1.42$ m which is significantly less than $z_s = 1.8$ m. Therefore we conclude that, based on the experimental evidence, the sand surface had no influence on the behavior of the water table. Even if the "worse case" is considered where $h_{\max} = 0.735$ m (test 10), this yields $h_{\max} + 0.5H_\psi \approx 1.49$ m. We agree with *Hilberts and Troch* [2006] that the moisture dynamics near the sand surface may be affected by the sand surface however, based on the experimental evidence any such affect does not appear to influence the water table dynamics for the range of parameters we tested.

[7] In further support of this, the experimentally determined complex effective porosity for all three materials follows the same trend as described by equation (1) above [cf. *Cartwright et al.*, 2005, Figure 4], and so the water table is not critically close to the sand surface.

3. High-Frequency Decay of $|n_\omega|$

[8] In section 2.3 of *Hilberts and Troch* [2006] a quantitative analysis is presented to better explain the

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relationship we observed between the van Genuchten parameters and the results of a nonhysteretic Richards equation model. However, this type of (nonhysteretic) model is unable to reproduce the experimental data [cf. *Cartwright et al.*, 2005, Figure 7]. The modeling of *Hilberts and Troch* [2006] also confirms this (see their Figure 2 where the asymptotic slope of $|n_\omega/n|$ goes as ≈ -0.9 as opposed to the observed $-2/3$).

[9] Figure 7 of *Cartwright et al.* [2005] also illustrates the inability of the nonhysteretic model to predict the high-frequency limit of $Arg\{n_\omega\}$ (the phase lag between oscillations in the equivalent height of total moisture, h_{tot}, relative to the water table, h). That is the model predicts $Arg\{n_\omega\} \rightarrow \pi/2$ as opposed to the $\pi/3$ seen in the data. These limitations were shown to be overcome with the application of a hysteretic model (see section 6 based on the results of *Werner and Lockington* [2003]).

[10] The "curious relationship" we discuss in section 7 is the fact that, in order to reproduce the experimental data (i.e., the asymptotic slope of $-2/3$ for $|n_\omega/n|$ and $\pi/3$ for $-Arg\{n_\omega\}$) with a nonhysteretic model, a van Genuchten β value of 3 is required. Whether there is anything physical in this is still open for debate.

References

Cartwright, N., P. Nielsen, and L. Li (2004), Experimental observations of watertable waves in an unconfined aquifer with a sloping boundary, *Adv. Water Resour.*, *27*, 991–1004.

Cartwright, N., P. Nielsen, and P. Perrochet (2005), Influence of capillarity on a simple harmonic oscillating water table: Sand column experiments and modeling, *Water Resour. Res.*, *41*(8), W08416, doi:10.1029/2005WR004023.

Green, W. H., and G. A. Ampt (1911), Studies on soil physics 1. The flow of air and water through soils, *J. Agric. Sci.*, *4*(1), 1–24.

Hilberts, A. G. J., and P. A. Troch (2006), Comment on "Influence of capillarity on a simple harmonic oscillating water table: Sand column experiments and modeling" by Nick Cartwright et al., *Water Resour. Res.*, *42*, W11601, doi:10.1029/2006WR005042.

Nielsen, P., and P. Perrochet (2000), Watertable dynamics under capillary fringes: Experiments and modelling, *Adv. Water Resour.*, *23*(1), 503–515. (Errata, *Adv. Water Resour.*, *23*, 907–908, 2000.)

Werner, A. D., and D. A. Lockington (2003), Influence of hysteresis on tidal capillary fringe dynamics in a well-sorted sand, *Adv. Water Resour.*, *26*(11), 1199–1204.

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