A Numerical Comparison between EFIE/MoM and CCD Methods for EM Scattering in Two Dimensions

Author

Seagar, A, Espinosa, H

Published

2016

Conference Title

PROCEEDINGS OF THE 2016 INTERNATIONAL CONFERENCE ON ELECTROMAGNETICS IN ADVANCED APPLICATIONS (ICEAA)

Version

Accepted Manuscript (AM)

DOI

10.1109/ICEAA.2016.7731389

A Numerical Comparison between EFIE/MoM and CCD Methods for EM Scattering in Two Dimensions

A. Seagar* H. Espinosa†

Abstract — **Two methods for calculating the electromagnetic radiation scattered by a perfectly conducting object: the Electric Field Integral Equation formulated within the Method of Moments, and the Clifford-Cauchy-Dirac technique; are compared numerically for a particular test case. Both methods involve the calculation of fields over the surface of the scatterer from integral equations; one integral with a kernel of Green's functions and the other with a Cauchy kernel. Although both start with Maxwell's equations the two methods differ fundamentally in several ways.**

1 INTRODUCTION

It is common to calculate electromagnetic scattering from perfect and imperfect conductors numerically using the well established Method of Moments (MoM) [1, 2] based on one or other electric (EFIE), magnetic (MFIE) or combined (CFIE) field integral formulation of Maxwell's equations with Green's functions in the algebraic framework of vector calculus. A newer and less well established technique, known as the Clifford-Cauchy-Dirac (CCD) technique, has been reported recently [3, 4] to offer an alternative numerical method based on an integral formulation using the Cauchy integral in the framework of Clifford algebra.

The Cauchy integral possesses the property of a reflection operator in that it returns the original field after application twice in succession. As a consequence it is straightforward to construct from the Cauchy (reflection) integral operator, projection operators which lead to a direct although iterative solution without any formal specification of system matrix or matrix inversion. The EFIE, MFIE and CFIE methods lack the reflection property and the solution cannot be formulated in the same way.

It therefore follows at least in principle that the two kinds of solutions are fundamentally different. The CCD technique is fundamentally geometric in an abstract multidimensional Banach (functional) space, whereas the conventional methods are fundamentally algebraic with no obvious support from a geometric structure at the outer level.

Numerical tests in one dimension [3] have already shown that the CCD method recovers the scattered field numerically for all types of material, conductor, dielectric and in-between, without exception from a single Cauchy integral formulation. However one-dimensional scattering is of limited use, and for that reason this article addresses two dimensions with a numerical comparison of the field scattered from a simple test object for both the EFIE/MoM and CCD methods.

2 TEST CASE

The test case chosen involves illuminating a cylindrical scatterer, being of circular cross section in two dimensions and having infinite extent in the third dimension, with a TEM plane wave linearly polarised in the direction of the axis of the cylinder. The scatterer is taken to conduct electrical current perfectly in order to simplify implementation of the EFIE equation for the Method of Moments. The Cauchy integral solution is implemented for all kinds of materials, including of course the perfect conductor as a particular setting of the values of certain parameters.

The diameter of the conductor is taken $\sqrt{2} = 1.4142$ metres in length, and the frequency of the incident wave is $f = 15c/2\pi = 715.7\,\text{MHz}$, so that the free-space wavelength is $\lambda = 2\pi/15 = 0.419\,\text{m}$. Under these conditions the reflected field creates an interference pattern which varies in both angular and radial directions.

3 CALCULATIONS

The numerical tests involve calculating the scattered field in the vicinity of the scatterer, and plotting $\Re(\mathbf{E})$, the real part of the phasor of the electric field.

For both techniques the surface of the scatterer was divided into 500 equally sized linear elements, with pulse basis functions being adopted, and with integration over the singularity being treated so as to deliver a proper result. For the MoM point matching was used, but for the CCD there is no such step.

Both techniques were solved iteratively. The

*School of Engineering, Griffith University, Gold Coast, QLD, Australia, e-mail: dr_andrew_seagar@ieee.org, tel.: +61 07 55529175, fax: +61 07 55528065.

†School of Engineering, Griffith University, Nathan, QLD, Australia, e-mail: h.espinosa@griffith.edu.au, tel.: +61 07 37358432, fax: +61 07 37355198.

CCD technique is cast in terms of projection operators, which makes iterated (alternating) projections [5] the most natural approach to use for all cases. For the MoM, direct matrix inversion can also be used provided the matrix is not too large.

The iterative calculations were continued for both methods until the convergence criteria as appropriate for the different techniques were met. For the CCD that resulted in termination after 9 iterations, and for the MoM termination after 20 iterations.

Each technique produces as result a field over the surface of the scatterer. For the EFIE the field is the surface current. For the CCD the field is a combination of transmitted and reflected electromagnetic fields. In the case of a perfect electrical conductor the transmitted field is identically zero so the field over the surface contains the reflected component alone.

For the purpose of comparison, fields away from the surface have been calculated from the surface currents using equation (5.84) from [6], or by using the Cauchy extension formula as described in [4], according to need.

4 RESULTS

Figure 1 shows the scattered field in a region $(-2.5, -3) \leq (x, y) \leq (5.5, 3)$ in the vicinity of the scatterer centred at the origin $(x, y) = (0, 0)$, calculated using the CCD technique, for an incident wave approaching the front of the cylinder from the left hand side.

Figure 2 shows the surface current density field calculated using the EFIE/MoM technique running anticlockwise from the right hand side (back) of the scatterer, and figure 3 shows the electric field calculated from the current.

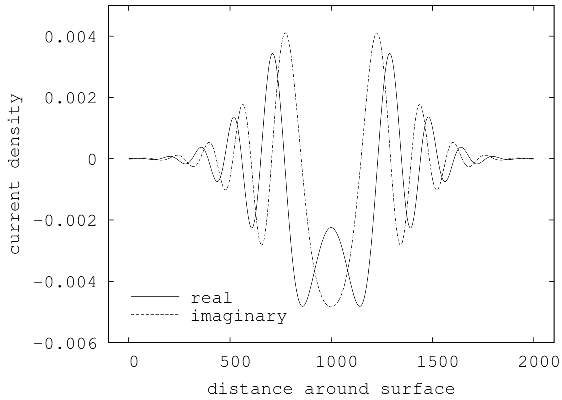

Figure 2: Surface current density calculated using EFIE/MoM technique.

Direct calculation of the current from equation (5.109) of [6] verifies that the MoM as implemented for these tests is producing accurate results.

The fields produced by the two methods are similar in the major features but differ in detail. Figure 4 shows that although the differences are not large, they are significant. This result was not anticipated. In one dimension, the CCD produces perfect results [3, 4].

5 DISCUSSION

The Cauchy integral plays the central role in the calculations for the CCD technique. When the numerical implementation is not sufficiently accurate the results cannot be perfect.

In implementing the Cauchy integral the approach has been to make it, in this first instance, as simple as possible. Accuracy of the implementation can be tested by applying the Cauchy inte-

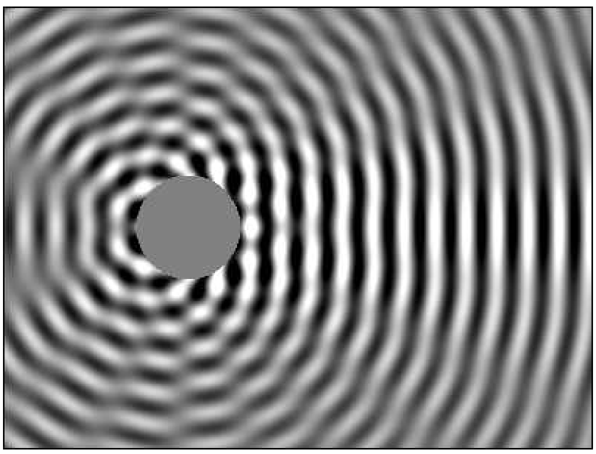
Figure 1: Scattered field calculated using CCD technique.

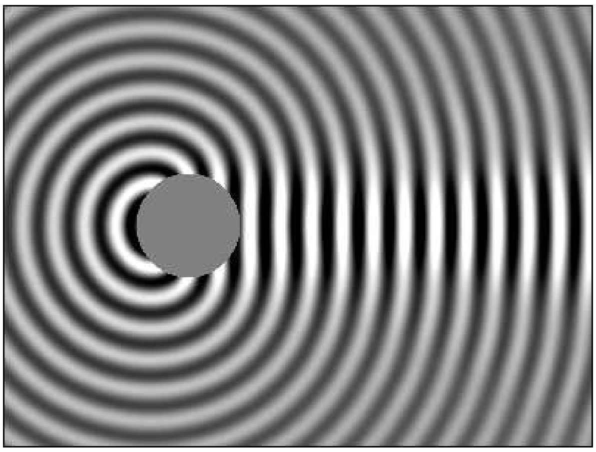
Figure 3: Scattered field calculated using EFIE/MoM technique.

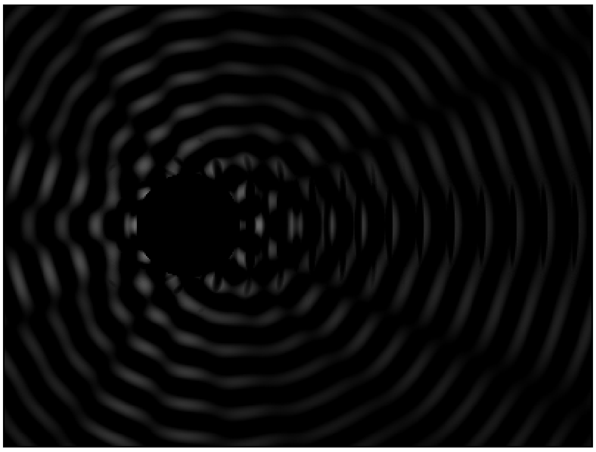

Figure 4: Difference between fields calculated using EFIE/MoM and CCD techniques.

gral to the field on the surface twice in succession. That should in principle return the original field. The implementation here fails to do that with a high enough level of accuracy, indicating that the approach adopted is too simple. Inspection of the Cauchy kernel on elements adjacent to the singularity shows that the function is varying rapidly. For more accurate results it is necessary to take that into account.

For the existing results it is worth noting that the iterative method of solution using Hardy projections as described in [3, 4], when started with an initial estimate of zero, produces a sequence of revised estimates falling successively closer to the known solution. This provides supporting evidence that the method itself not only applies equally well in two dimensions as in one dimension, but is imbued with a certain robustness against errors in the process.

Future work is required to implement the Cauchy integral with sufficient sophistication to produce results for the CCD technique which are indistinguishable from those produced by the EFIE/MoM.

References

[1] Roger F. Harrington. *Field Computation by Moment Methods*. Macmillan, New York, US-NY, 1968. (reprinted John Wiley 1993).

[2] Walton C. Gibson. *The Method of Moments in Electromagnetics*. Chapman and Hall/CRC, Boca Raton, US-FL, 2nd edition, 2015.

[3] Andrew Seagar and Ajalawit Chantaveerod. An EM scattering algorithm for all materials. In *International Conference on Electromagnetics in Advanced Applications (ICEAA'15)*, pages 256–259, Turin, Italy, September 7–11 2015.

[4] Andrew Seagar. *Application of Geometric Algebra to Electromagnetic Scattering – the Clifford-Cauchy-Dirac Technique*. Springer, Singapore, 2016.

[5] C. Badea, S. Grivaux, and V. Müller. The rate of convergence in the method of alternating projections. *St. Petersburg Mathematical Journal*, 23(3):413–434, 2012.

[6] R.F. Harrington. *Time-Harmionic Electromagnetic Waves*. Wiley–IEEE Press, New York, US-NY, 2001.