Intervention: A Process for Assisting Students to Develop Their Own Mathematical Ideas

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A process for assisting students to construct their own mathematical ideas

How does *intervention* differ from *remediation*? Let GEORGE BOOKER explain in the following article.

Mathematics provides powerful tools for thinking that are becoming ever more important in an information driven society. The ways of thinking that will be needed to deal with this new, technological world will be different to those of the more certain past when the formal skills and techniques of arithmetic, elementary geometry and simple measurement were sufficient. A range of estimation, approximation and mental reasoning processes will be essential as well as means of examining and exploring change and uncertainty using the methods of chance and data analysis.

Adapting to these new practices requires understanding over skill. This calls for a rich conceptual understanding and detailed knowledge of the mathematical modes of thinking that need to be built up (Booker, Bond, Briggs, & Davey 1997). At the same time, teaching needs to move from instruction on how to perform and apply techniques to focus on building the meaning and understanding that will enable new situations to be examined and new processes to be applied. A focus on assisting learning rather than telling students what to learn will be required, recognising that students necessarily construct their own ways of knowing. In particular, a means of intervening in a learning cycle at any time is needed in order for students to come to terms with mathematical ideas and maximise the manner in which ways of knowing are formed in order for students to learn to reason with them and communicate about and through mathematics (AEC, 1991).

Intervention rather than remediation

The term *intervention* has evolved to reflect this ongoing involvement in learning at all stages. It applies to understanding and knowledge at a very high level as well as learning at a beginning level. While it can refer to re-teaching of concepts or processes that have been only partly or inappropriately built up, it also encompasses creating new learning and the application of prior learning to new tasks or ideas. Thus, although it is sometimes taken as a replacement for the more familiar term, *remediation*, it is more than just a change of name for a word that has become tired over time. It also reflects concern over the restricted notions that remediation seems to suggest.

Remediation has been concerned with remedying faults found in students identified as having incomplete or flawed development at the conclusion of a particular learning sequence. This often involved someone being withdrawn from a particular teaching situation to participate in a separate program parallel to the class program. By then particular ways of viewing the mathematical situation and carrying out the processes involved were likely to have become ingrained which is hardly appropriate if understanding and sense making are to be the essence of mathematics learning.

In contrast, *intervention* is more suggestive of activities that take place within the regular learning place and program. It requires a cycle of activities from observations which lead to insights about the status of an individual's knowledge that in turn need to be probed so as to be verified or amended before being built on to provide a means to new and deeper understanding (Booker, 1997). This allows mathematical understandings, errors and difficulties to be identified, their sources determined, and suggests how learners might be led to see the need for change. Only then can a process of reconstruction be engaged in to build up full and appropriate ways of thinking and acting that will allow for the development of abilities to use this knowledge to solve problems and form further mathematical generalisations.

With these steps, intervention can be a pivotal part of everyday classroom life, taking place within the regular learning program as an integral component of ongoing teaching so that difficulties can be attended to as soon as they arise and learning can proceed at a time and pace that is suited to each individual.

An example of intervention: difficulties with rounding

One instance of the understanding that has come to the fore with the changing needs for mathematics is the ability to round numbers in order to estimate and make approximate calculations. Many students when asked to round numbers give results such as

Round 57 to the nearest ten:	**60**
Round 328 to the nearest ten:	**300**
Round 5671 to the nearest hundred:	**6000**

It seems natural to immediately try to determine what it is that the students are doing to get some part of the answer wrong and then work on the part of the process that seems in error. While this will matter in due course, if diagnosis is to take place and inform any subsequent intervention, a first step would better be to ask what *understanding* the responses reveal. These students seems to have some rounding procedure; in each case, the largest place has been selected and the number has been adjusted to this place. The idea of what rounding means seems to be present, but the manner of carrying it out seems to be flawed.

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On the other hand, only one of the answers given is actually correct; 328 should have been rounded to 330 and 5671 should have been rounded to 5700. Rather than round to the value asked for, the students have simply treated each example in the same manner; a two-digit number is rounded to the nearest ten, a three-digit number is rounded to the nearest hundred and a four-digit number is rounded to the nearest thousand. The *difficulty* that the students have is to focus solely on the leading digit.

Now that both the students' strengths and weaknesses with these examples have been described in mathematical terms, it makes sense to seek out the *cause* for the errors. In order to continue with the intervention it is necessary to provide further examples, observe what is happening, and make assumptions on the basis of that observation rather than by analysing a completed result which holds no definitive clue as to what occurred. Indeed, it is only after the responses have been given that an observer will realise that something incomplete or inappropriate is occurring. This creation of similar examples to see how a response is given is a crucial part of intervention. Each answer should be accepted at face value and the students encouraged to complete sufficient examples until the procedure being used is clear. This way the same thinking is likely to be present each time, whereas if a result were questioned, it is just as likely that the answer would simply be changed in order to get to what the teacher might be wanting.

Further probing often reveals that these students only have access to a 'rule' that tells them to consider the first digit, check the value of the next digit, and then 'round up' when that digit is greater than 5 or 'round down' when the digit is less than 5. When that digit is 5, uncertainty reigns and many students will answer that they need to 'round up' at first, others that they 'round down', but most will oscillate between rounding 'up' and 'down' each time the question is repeated.

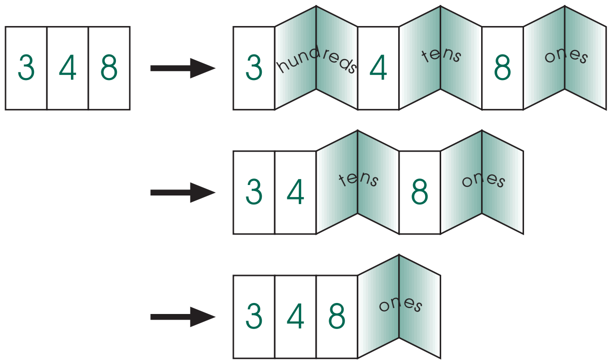
At this point, the mathematical difficulty has been described and the source of that difficulty is probably clear. The issue is now to bring about *change* in the student's way of thinking and acting. But change is neither easy to implement nor to accept. Before people can change their behaviour or way of thinking, it will be necessary that they accept that there is a need for change and that changing will benefit them. What is needed is a means of challenging their way of thinking that can lead them to see the need to change and in turn allow them to construct a meaningful process that can be used confidently and unambiguously.

The two-digit number board is helpful in providing such a means:

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

When those numbers about which there are no confusion are examined, some, such as 41, 42, 43, 44 round to the tens that they have, 4 tens or 40. This means that the five numbers 40, 41, 42, 43 and 44, all round to 4 tens or 40. Other numbers such as 49, 48, 47 and 46 round to the next ten, 5 tens or 50. The symmetry of the number system shows that for each multiple of ten, five numbers round to the tens that they have and five numbers round to the next ten. Not only does this examination of place value show that there is no need for confusion regarding a number with 5 in the ones place, it also shows a method for readily determining what each number rounds to.

By considering the number of tens, hundreds, or whatever that a number has, this thinking is readily extended to cover all instances of rounding. Thus, 328 has 32 tens. It can round to 32 tens or 33 tens. 33 tens is closer so 328 rounds to 330. Similarly, 5671 has 56 hundreds. It can round to 56 hundreds or 57 hundreds. 57 hundreds is closer so 5671 rounds to 5700. If the intention had been to round 328 to the nearest hundred, the thinking would have been that 328 has 3 hundreds so that it can round to 3 hundreds or 4 hundreds. Since 328 is closer to 3 hundreds, it rounds to 300.



328 is 32 tens 8 ones or 3 h 2t 8 ones

By integrating the process for the two-digit numbers with the understanding of how many tens, hundreds, thousands or whatever a number has as shown by a number expander, a rounding procedure is readily *constructed* to account for any number to whatever place is required.

Of course there would need to be sufficient practice to consolidate this understanding, not only to ensure rounding to the nearest ten or hundred, but also to allow for the *generalisation* of this understanding to build up rounding for larger numbers that follow. Indeed, a common difficulty seen among older students (Booker, 1995) is for them to consider 6 digit and larger numbers as if only the first portion needs to be rounded to the nearest hundred thousand, ten thousand or thousand, and leave the rest of the number with their answer:

Round 683 478 to the nearest hundred thousand: **700 478**

Round 845 329 to the nearest ten thousand: **850 329**

Round 534 217 to the nearest hundred thousand: **500 217**

The larger numbers have been broken into two 3-digit segments and the procedure only applied to the part representing the thousands. A more appropriate 'rule' has been adopted, but still not one that takes on an understanding of the full meaning of the rounding process. Expecting and building an ability to describe the thinking underlying the process of rounding or whatever is crucial in this phase if students are not to remain 'answer-focussed' and be led to view understanding of processes and concepts as the real core of doing mathematics.

The process of intervention

Consideration of this example has drawn attention to the full cycle of steps that need to be followed when intervening in mathematics. The approach assists individuals to come to know mathematics by using assessment in an ongoing way to inform a process that helps learners to construct mathematical ways of thinking and knowing for themselves. In particular, the cycle of observations, assumptions and further probing leading to intervention may be summarised as:

Further analysis of the steps in this intervention process will bring out its value and indicate how it can become a central feature of everyday teaching:

1. Identify *and describe the mathematical understanding as well as any error or difficulty*

This demands a familiarity with the basic concepts and processes of numeration, computation, geometry, measurement and so on at a conceptual level rather than a procedural one. Notions such as place value, renaming, and the meaning for the various operations and processes need to be to the fore and the understanding or difficulty needs to be described in mathematical terms.

observations → assumptions → probes → intervention

- identify
- source
- lead to see need for change
- re-construct
- practice/generalisation

A result obtained from a particular form of probing leads to an observation which in turn provokes an assumption and leads to further probing to confirm or validate the meaning that is taken. This cycle of diagnostic assessment of what a student understands and is able to use can then be something which drives a teacher's ability to plan for individual students, to review a proposed sequence of work or activities, and to determine whether to move on to new ideas or applications or to step back to where difficulties are arising for a student or group of students. It can be summarised as:

The process of intervention

1. Identify and describe the mathematical understanding, error or difficulty
2. Determine the source of this understanding or difficulty
3. Lead the learner to see the need for change (the inadequacy of this thinking)
4. Assist the learner to (re)construct an appropriate way of thinking
5. Provide sufficient motivating practice to consolidate the new thinking and to provide for the generalisation of the strategy to more complex situations

2. Determine the source of the difficulty

In other words, identify the thinking that is being used and why. Often this will require a cycling back through the assessment phase to ask a series of questions related to similar examples as an awareness that something was amiss would only occur after a response had been given. It may be necessary to systematically work through several hypothesised sources until the particular way of thinking or operating is identified. This will also build on an understanding of the ways in which students approach new topics and will require an anthropological stance as the observer tries to see the events from the learner's perspective.

3. Lead the learner to see the need for change by examining the inappropriateness or inadequacy of this thinking

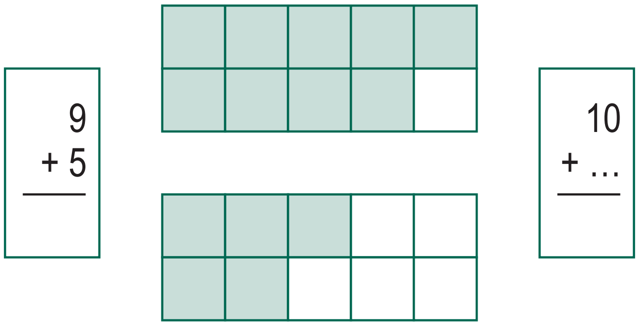
When the source of the difficulty has been identified, means to bring the learner to see that his or her ways of thinking do not match what is needed must be found and put to the learner to see if some form of re-conceptualising the situation can be provoked. Different aspects of mathematical learning will require distinct approaches:

- When a concept is incomplete or confused, story situations which were used to build up the initial concept can be revisited to provide a focus for discussing meanings.
- When an inappropriate or inefficient method is being used, such as counting on fingers, rulers or dots placed on a page, situations which embody more efficient means can be put before the student. For instance, using two dice for games or activities, one with words or digits, the other with 1, 2, or 3 dots, rather than a single dice with 1 – 6 dots make it unlikely that counting all will continue to be used.

5, 6, 7, 8 rather than 1, 2, 3, 4, 5, 6, 7, 8

When counting backwards by tens in the hundreds, questions about how many tens in 356, 346, 336, 326, 316 can lead to a realisation that these numbers are counted ***35***6, ***34***6, ***33***6, ***32***6, ***31***6 because the number of tens are decreasing one at a time. The numbers that follow are then ***30***6, ***29***6, ***28***6, ...

Basic facts learnt by rote can also be challenged in ways that lead to a consideration of the strategies that generate them more efficiently. Faced with a situation where the thinking behind a fact is emphasised more than a quick answer, a student can be lead to re-establish the manner in which a result is given. Thus a child who counts on to obtain answers for all of the basic addition facts could be challenged on some simple make to ten facts such as '9 and 5' where it is so much easier to simply move one to make ten and then see that the result must be 14:



When an incorrect procedure is being used such as with a computational algorithm, some form of cognitive conflict needs to be provided whereby what the learner thinks should occur is opposed to what they 'see' occurring. The use of materials is the most obvious way for this to be brought about, and readily applies to most of the errors that occur with addition, subtraction or division.

When addition is worked from left to right regardless of place value or renaming materials will show 4 tens 7 ones and 2 tens 6 ones giving 6 tens 13 ones which can then be seen to be 7 tens 3 ones or 73 and not 613

```
   4 7
 + 2 6
 -----
   6 1 3
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When subtraction is completed so that the smaller number is always subtracted from the larger materials are only used to show 6 tens 5 ones 8 ones need to be taken away, which can only occur if 6 tens 5 ones is renamed as 5 tens 15 ones

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   6 5
 – 2 8
 -----
   4 3
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If division is completed without regard to place value sharing materials onto a place value chart will show that since 5

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   7 4 0 3
8 )5 9 2 3
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thousands cannot be shared among 8, 59 hundreds will need to be shared, resulting in 7 hundreds each. Hence the 7 will need to be recorded in the hundreds place, not the thousands place.

However, the use of materials to see that something is inappropriate with multiplication is not really feasible. Showing 36 groups of 48 would take such an amount of materials and such a long time, that no one experiencing difficulties would be likely to complete the task, let alone see by means of the experience that something was amiss with the computations:

$$\begin{array}{r} 48 \\ \times\ 36 \\ \hline 288 \\ 138 \\ \hline \end{array} \qquad \text{or} \qquad \begin{array}{r} 48 \\ \times\ 36 \\ \hline 1248 \end{array}$$

Rather, the use of a counter example where the same method but with an example chosen that would be more likely to be seen to be incorrect is necessary:

$$\begin{array}{r} 48 \\ \times\ 10 \\ \hline 48\ ? \end{array} \qquad \text{or} \qquad \begin{array}{r} 41 \\ \times\ 36 \\ \hline 126 \end{array}$$

Other possibilities might involve estimation or checking but these are unlikely to be very convincing to the learner as they usually rely on just those understandings that led him or her astray in the first place. Similarly, using a calculator is merely asking someone to accept that an electronic device be an arbiter and is no more successful than a teacher simply saying, 'That's wrong, do it this way.'

4. Assist the learner to (re)construct *an appropriate way of thinking*

There are a number of strategies that can be adopted to assist a learner to put aside an ineffective or inefficient way of thinking and adopt one more suited to the process or application. While these apply as much to initial teaching as to re-teaching, all provide methods which focus on the learner constructing knowledge for him or herself rather than be told or provided with a method by a knowledgeable teacher. Too often if one, finished method is simply put before a student, he or she will simply resort to rote learning to memorise a procedure which ultimately has little meaning and therefore little chance of being used outside of the manner in which it was presented.

- The use of *games* and activities which foster the development of understanding and build up underlying skills is one of the most effective ways of assisting to build more powerful ways of conceiving and using mathematics (Booker, 1996). Not only on account of the fact that it is likely to be quite different to the approaches met in the past, but also because it can allow the manipulation of materials in a way which allows them to realistically enter the student's world of experiences and thus become his or her own representation of the mathematics. In the social actions that accompany playing, actions, thoughts and interpretations will need to be verbalised and justified to others. As well, most games require strategies for winning. When a player focuses on a way of playing to maximise success, the mathematics which underlies the game must be able to be taken for granted, so that the game itself forces an internalisation and understanding of the mathematics in order that play might occur at a more demanding or sophisticated level.
- Explicit *modelling* of the process, strategy or way of thinking using materials and involving the learners at each step in a problem solving manner is often needed. When students have to determine for themselves the ways of proceeding and have to justify their thinking, the mathematics can become something over which they have power and control. At the same time, care can be taken with the way the materials are introduced and the expectations for their use so that they match the final, recorded form of any process.
- A focus on meaningful *language* and recording growing out of the discussion about the problem situations from which new ways of operating or thinking arise is also crucial.
- Perhaps the most important point to bear in mind when introducing new ideas, concepts and processes, is to allow the *connections* between the different processes and topics to be brought out explicitly. Concepts and processes will then not only be differentiated one from the other but also retrieved

more readily when needed. Attached to a meaning that is seen as part of a whole view of a subject, accessing them is a matter of sensing the fundamental meaning and then retrieving that particular aspect of mathematical knowledge rather than floundering through a vast array of individual facts and techniques.

- As in the development of all learning, a teacher must guide the development of thinking from where the learner is towards the final, accepted form. This means that any development starts from where knowledge is secure, rather than the point where new knowledge might come into being, building on these secure understandings to reach toward the new ideas. It is also important to allow time for discussion and reconciliation of uncertainties in this development and to encourage learners to describe the new understandings that they have, the ways in which they have come to know them, and the manner in which they have been found useful in applications and problem solving.

5. Provide sufficient motivating practise to consolidate the new thinking and to provide for the generalisation of the strategy to more complex situations

Practise is often seen as an ends in itself, a way of ensuring that learners commit to memory some procedure or fact. If this is the case, it might well undermine or sidestep any attempts that have been made at constructive, meaningful learning. The practise that is needed is of the form that makes sense of mathematics and creates fundamental ways of thinking that become the learner's own. In this way, the basis is laid for the development of new ideas and the invention of new solutions and processes.

Building connected mathematical ideas

If all new mathematical ideas can be constructed using this sequence of steps and building on existing knowledge, not only will new concepts and strategies be efficiently built out of established ways of thinking and proceeding, but the learning will also be more effective as a connected view of the subject is formed. In turn, this will lead to students who view learning as a problem solving task and who are autonomous in their ability to use, apply and extend mathematics. Mathematics will be viewed as a way of thinking about the world that will enable each school leaver to participate fully in all aspects of their social and working life.

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Note

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