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An Entangled Web of Crime: Bell's Theorem as a Short Story

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Non-locality of the type first elucidated by Bell in 1964 is a difficult concept to explain to non-specialists and undergraduates. Here we attempt this by showing how such non-locality can be used to solve a problem in which someone might find themselves as the result of a collection of normal, even if somewhat unlikely, events. Our story is told in the style of a Sherlock Holmes mystery, and is based on Mermin's formulation of the "paradoxical" illustration of quantum non-locality discovered by Greenberger, Horne and Zeilinger.

Preamble

With the discovery of Bell's theorem in 1964,[1] and the experiments it prompted over the next two decades,[2] an astonishing fact about the nature of the universe was revealed: it is non-local. That is, that certain things which happen in the universe can only be explained if there is instantaneous action-at-a-distance, although this cannot be used to communication instantaneously. The exact nature of the non-locality is thus very subtle and not as easy to explain to a general readership as other key insights about the physical universe such as the invariance of the speed of light, or even Heisenberg's uncertainty principle.

In this article we illustrate the weird non-locality of quantum mechanics, which is the import of Bell's theorem, using the literary device of a detective story. While a few previous articles have been written with a similar purpose — that is, to explain quantum non-locality using everyday settings[3,4,5,6,7,8] — in none of these was the use of the non-locality required to solve a problem with which someone might be faced as the result of everyday, if rather coincidental, events. This article was motivated by a desire to construct an example of such a situation. We feel that our story should be of use in engaging students, and with this in mind we have included a number of exercises throughout the story, the answers to which are given at the end of the article.

Like Ref.[6] we base our story around the example of nonlocality, described in this form by Mermin, which uses three-party GHZ entanglement.[9,10,11,12] Reference[13] gives a comprehensive review of nonlocality of this sort, which the authors call "quantum pseudo-telepathy". In this the authors show that the GHZ example is the simplest possible in the sense of requiring the smallest Hilbert-space dimension. Other examples from Ref.[13] may however be quicker to explain, in particular one by Aravind[14]. We have been unable to construct a compelling story around this example, but we encourage the reader to try. A much broader review of "strange correlations, paradoxes and theorems" in quantum mechanics may be found in Ref.[15].

We were prompted to write this article as a reaction to the (throwaway) statement by Mermin that "the action at a distance [in Bell's theorem] is entirely useless."[16] As the story shows, it is certainly not useless. Bell-type non-locality does not break Einstein's no-signaling condition, but that does not make it any less real. Of course quantum non-locality is known to be potentially useful for practical tasks such as scheduling with a minimum of classical communication,[17] but a) the protocols for these tasks are far more complicated, and b) the effect appears less dramatic than for the one we describe.

There *are* simple tasks, such as quantum teleportation, or dense coding, which rely upon quantum entanglement and are usually understood to involve quantum non-locality. However, for a non-specialist to appreciate any of the weirdness in these examples he or she must first understand a substantial amount of quantum mechanics.[18] Moreover, recent studies[19,20] show that these (and many other) tasks in quantum information can be simulated in a quantum-like theory which is completely local. It seems that Bell's theorem is still the best way of illustrating the non-locality of the world.

The story below is told in the style of a Sherlock Holmes mystery. The narrator is Mr. Doyle, and the protagonist is Dr. Bell. As is now well-known, the chief inspiration for Arthur Conan Doyle's most famous literary creation, Sherlock Holmes, was a Dr. Bell who lectured Doyle at Edinburgh University Medical School. A fictionalized version of their relationship has been told in a number of recent novels,[21,22] in which Mr. Doyle plays Watson to Dr. Bell's Holmes, and we model our story loosely on that pattern.

The Case of the Two-Colour Gang

It was a late afternoon early in October when first I found myself outside the door of number 8 Hilbert Place, a rather nondescript two-storey house in a small street just south of the city center. While it was mid autumn, the sky was clear, and the afternoon warm as the sun's rays lingered on the trees and grounds of number 8. The bright weather was in some contrast to my mood however, as I was weighed down with a problem which had been occupying my mind for some days. I rang the doorbell, and as I did so my thoughts wandered back over the events of the last few weeks.

I made my living as a barrister, and my private legal practice was doing very well. I enjoyed my work, and had been attracting cases both of increasing interest and importance. A few months before I had been lucky enough to land a defense case which was very much in the public eye — certainly it was the highest profile case of any with which I had then been associated. As events would turn out, it would also be one of my greatest triumphs, for (despite the evidence against them) the case against my clients was dropped. Now, many years later, the details of the case can be told for the first time.

The preceding summer had seen a number of break-ins at the Museum of Semi-Classical Art. This had been the cause of considerable concern as the museum was due to host the exhibition *Local Realism*, a collection of very valuable works by artists of the world-renowned realist school that arose in our city. In view of this the curator had increased security by placing four guards in the newly built Isosceles wing, which was to house the collection. This precaution was indeed prudent, for a mere three days after the exhibition opened a very daring robbery was attempted. Three robbers somehow defeated the perimeter alarms and broke into the museum at midnight. They split up and dashed through the corridors of the Isosceles wing, each aiming to grab a particularly valuable piece of art. Their plans were foiled by the guards, however, who spotted them and raised the alarm. While this prevented the robbers from carrying off any of the art, the guards did not manage to catch them.

Fortunately for the police, the descriptions given by the guards fitted three well known criminals, and the next day they made a dawn raid on their home. There the police found further evidence linking the three to the attempted robbery, and they were subsequently arrested and charged. I had myself only just put down the evening paper, where I learnt of the arrest, when quite out of the blue I received a call from the three men in custody. Having no other matter of importance on hand at the time, I agreed to represent them, but certainly had no idea what a curious turn the case would take.

My reverie on the doorstep of number 8 Hilbert Place was broken by footsteps in the hall. The door opened to reveal a young woman with a pleasant face and a bright smile. She was dressed in casual clothes, with a loose-fitting pullover and blue jeans. While one would not immediately associate such attire with that of a consultant to a prestigious legal firm, it was only when I noticed the fluffy Bugs Bunny slippers that I wondered for an instant if I had indeed knocked at the right door.

'You must be Mr. Doyle', said the young woman. 'I'm Alice Bell. Do come in.'

'Thank you', I said. 'It was good of you to see me at such short notice'

'Not at all Mr. Doyle. As you probably know I do most of my consulting for the legal firm Greenberger-Horne-Zeilinger, but they have offered few cases of late which exhibit those singular features so necessary if the problem is to provide any real interest for me. I assure you the debt will be more than repaid if your case is of sufficient curiosity'.

'It certainly seems so to me, I must admit', I replied.

'Excellent', said Dr. Bell, 'Cup of tea?'

I realized that I was indeed thirsty, and as she handed me a steaming cup and took a plate of cookies from the sideboard, I realized that I was also quite hungry, having not eaten since breakfast. She then led me up some stairs to a pleasantly furnished office — along with the mandatory desk and laptop it housed two comfortable and somewhat weatherbeaten leather chairs and a small coffee table. Shelves lined two adjacent walls, and while many were filled with books, others contained jars of various shapes and sizes which I assumed at first to contain chemicals, although later inspection revealed a much greater and more unusual variety of contents. The third wall was covered by a large and detailed map of the world, with the final wall devoted almost completely to a huge window, affording a good view of the city center, including a hint of the harbor and northern hills beyond. To the side of the desk was what seemed to be some kind of electronic apparatus, but apart from the brand name 'Cryptolightning' which was written on the side there was no indication as to its function.

Placing the cookies on the coffee table she sat down and motioned me to take the other chair. I did so, and as I helped myself to a cookie she slid a little further into her chair, and placing the tips of her fingers together said with a slight smile and an unmistakable air of anticipation 'So Mr. Doyle, what is it that brings you here?'

'You have heard of the break-in at the Museum of Semi-Classical Art?', I asked.

'It is hardly possible not to have', she said 'You may safely assume that I am familiar with all that has been in the papers, but no more'.

'Then I shall begin straight in with the details' I said. 'As you know from the papers, my clients were discovered with a number of body suits of the type worn by cat burglars. While some of these were a single color, being red or green, the others were more unusual in that they were green on the front and red on the back, or vice versa. Obviously the prosecution wanted to form as strong a link as possible between these suits and those that the robbers were wearing, so naturally I cross-examined the guards very carefully on this point.'

'Naturally', murmured Dr. Bell as I paused to take another bite.

'Now, one must understand' I continued, 'that the lighting in the Gallery was rather odd, changing in color and intensity from place to place in accordance with the artwork. As a result the guards were not able to discern completely the colors of the robbers' clothes. However, the first three asserted that they had had a clear view of one of the robbers, and that he was wearing a red suit, but none of them could remember which robber it was. Thus it may even have been a different robber in each case. In addition, they were sure that the other two robbers were wearing the same color, but they could not be sure what color it was under the lighting conditions. The

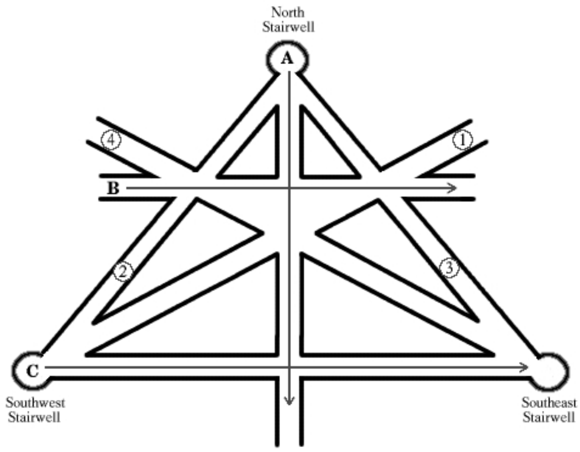

FIG. 1: Here is shown Mr. Doyle's map of the Isosceles wing of the Museum for Semi-Classical art, with the positions of the four guards (marked by the numbered circles), and the passage of the robbers A, B and C through the gallery as recorded by the infra-red security cameras.

testimony of the fourth and final guard was only a little different. He asserted that one of the robbers was wearing a green suit, and that the other two were wearing the same (although again unknown) color.'

'So can we sum up the guards' statements by saying that the first three guards saw an odd number of robbers wearing red, and the fourth saw an even number wearing red?', asked Dr. Bell.

'Most artfully put,' I said. 'Now while it was not possible to conclude much from these statements alone, further evidence was provided by the infra-red security cameras, which showed clearly the paths taken by the robbers as they ran through the gallery. In addition, the guards made definite statements as to their respective locations when they saw the robbers. In the light of this I was able to find an inconsistency in the guards' testimony.'

'I have here a map of the Gallery on which I have indicated the positions of the guards and the paths taken by the robbers.' I drew the map from my briefcase, and handing it to Dr. Bell, added 'I have labeled the guards with the numbers 1 to 4, and the robbers with the letters A B and C' (Note: Mr. Doyle's map is reproduced in Fig. 1)

I paused for a few moments, allowing Dr. Bell the chance to take in the map.

'So each of the Guards saw only the back or the front of each robber, but not both?', she asked, looking up from the map.

'Yes, indeed', I replied, most impressed by her perspicacity. 'Thus the testimony of each guard only refers to either the back or the front of each of the robbers. From the map we know, for example, that the first guard saw the back of robber A but the fronts of robbers B and C. The statements of the guards therefore refer not to three,

	Robber A	**Robber B**	**Robber C**
Guard 1	Back	Front	Front
Guard 2	Front	Back	Front
Guard 3	Front	Front	Back
Guard 4	Back	Back	Back

TABLE I: Each of the guards saw either the back or the front of each of the robbers. Mr. Doyle's table reproduced here shows which of the two it was for each guard and each robber.

but to six different things, being the two sides of each of the three robbers. I found it convenient to summarize which sides were seen by the various guards in a table.', and fishing the table out of my briefcase I handed it to Dr. Bell. (Note: Mr. Doyle's table is reproduced here in Table I)[23]

'From this table, and the statements of the guards,' I went on, 'I was able to show that while any three of the guards' claims are consistent, all four are not — one of them must be lying, or at the least mistaken.' In response to the Doctor's raised eyebrows I proceeded to explain my reasoning, with which I must say I was rather pleased.

Exercise 1: Reproduce Mr. Doyle's argument.

'After I had presented the argument in court,' I continued, 'the prosecution asked for a private word. It turned out that the police suspected that one of the guards was working for the gang that organized the break-in, but didn't know which one it was. If this was the case, then the guard in question was almost certainly away from his post turning off the perimeter alarms when the robbers broke in, and this would mean that he would have to have fabricated his evidence. If the police could find out which guard was lying, it would give them a new lead to the mastermind behind the robberies.'

'So the prosecution offered me a deal. They said that if my clients would tell them which guard was lying, they would drop the case against them. Now this deal is indeed attractive, because there is considerable evidence against my clients, and I certainly cannot guarantee a victory. However, my clients deny any involvement in the attempted robbery, and if they are telling the truth then they do not have the information the police want. And if (heaven forbid) they *are* members of the crime ring, it would be unwise for them to aid the police; if the crime boss discovered that they led the police to him then my clients would be better off in jail.'

'I thought at first that we might be able simply to make up a story as to what the robbers were wearing, so as to accept the offer and satisfy the prosecution. However, this fails for two reasons. The first is that the police informed me that one of the guards is an undercover officer who's testimony is beyond doubt. Thus, if our story conflicts with his testimony they will know we are lying. The second reason is that, if by chance our story incriminates the guard working for the crime ring, my clients could be in worse trouble.'

'I had virtually decided that I would have no option but to reject the offer. However, I mentioned the problem last night to a colleague of mine who works for GHZ, and she was adamant that before giving up I should come and see you — so there you have it.' Having finished my exposition I sat back and drained my cup.

Dr. Bell was silent for a few minutes, apparently lost in thought. At last she said, 'Your situation is indeed an interesting one, Mr. Doyle. Let us consider what would happen if we let the police ask just one question of each of your clients, being *either* what color his suit was on the front, *or* what color it was on the back.'

I picked up my table again, which Dr. Bell had placed on her coffee table, and examined it as she continued.

'I think you will find that in this case the police will only be able to test one of the guards' statements, rather than all four, but they will be able to test any one of the statements by choosing which question they ask each of your clients'.

'Yes, that seems to be right', I said, after studying the table. However, I don't see how this would help us. It is true that if my clients were to know which question each was to be asked, they would know which guard's statement was being tested, and thus what to answer so as to confirm that statement. That way they could be sure neither to contradict the undercover policeman, nor to unwittingly finger the crooked guard. However, the police will surely demand to question each of my clients separately. Moreover, in a case as important as this they will no doubt place each of them in a sealed room, in different buildings, so as to prevent absolutely any form of communication between them. Thus none of them will know which questions the others are being asked. Without that information they won't know what to answer.'

Exercise 2: (a) Reproduce Dr. Bell's reasoning that the police can test any one of the guards' statements by questioning in the manner she suggests. (b) Reproduce Mr. Doyle's reasoning that it is not possible for his clients to know which statement is being tested unless they communicate.

'Indeed', said Dr. Bell. 'If the police accept the offer of asking a single question of each of your clients they will wish to make sure that communication between them is impossible precisely to ensure that your clients cannot know which guard's statement is being tested.' However, I think that there may yet be a way to solve this problem. The theory which describes the behavior of elementary particles, called quantum mechanics, has a very strange property referred to as *non-locality*. While it does not allow instantaneous communication, it may nevertheless be sufficient to solve our problem. I must investigate the question further. How long do we have?'

'Two or three days at the outside, I would say', I replied.

'Excellent!', said Dr. Bell. 'Then call me mid morning tomorrow, and we shall see if I do not have something for you.'

Well I must say that I was highly sceptical. This "quantum mechanical non-locality" to which the Doctor referred sounded to me more like the ravings of an eccentric than hard science. Perhaps the good Doctor's recent boredom had sent her a little over the edge? However, I agreed to call the next morning, and thanking her for the advice and the cookies, I left for home.

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Calling Dr. Bell the next morning, as I had promised, I found her in excellent spirits.

'I have good news for you Mr. Doyle', she said, 'Quantum mechanical non-locality is indeed sufficient to solve your problem, so long as the police will agree to ask each guard a single question. Moreover, I have contacted some colleagues of mine at a laboratory which specializes in quantum information, and they are able to construct the devices which you will require. I should have the gadgets in my possession by tomorrow afternoon.'

This was superlative news indeed! I lost no time in making Dr. Bell's suggested counter-offer to the prosecution. To my gratification they accepted it later that day, and the following afternoon I was back in Dr. Bell's office, sitting in one of her comfortable chairs and tucking into another batch of freshly baked cookies.

'We are lucky, Mr. Doyle, that quantum technology is now at the point where we can manufacture these little gismos.' Dr. Bell was holding a small object, the size and shape of an electronic car key, and there were two more like it on the coffee table between us. On each were two buttons, labelled "lock" and "unlock".

'These devices contain elementary particles — in this case electrons — in a joint quantum state which is described as being *entangled*. Because of this the results of measurements on individual particles will be correlated. You must slip these to your clients when you next meet, and explain to them what to do. Each of your clients is to take one of them with him in his pocket when he is questioned. If he is asked about the color of the back of his suit, then he should press the "lock" button. The device will then vibrate for a few seconds. If it vibrates constantly, then he should answer "green". If it vibrates in pulses he should answer "red". Alternatively, if he is asked about the color of the front of his suit, then he should press the "unlock" button, and answer depending on the vibration in the same way. This will guarantee that no matter which guard's testimony is being tested by the police, it will be confirmed by your clients answers.'

'But without the device's communicating to each other which questions the police have asked each of my clients, surely that is impossible!', I replied.

'Not, impossible, Mr. Doyle, just very strange', Dr. Bell assured me. 'Although there is no physically detectable signal between the devices, the quantum particles do influence each other, both at a distance and apparently instantaneously, '

Having done a physics-for-poets course in my law degree, I was not put off this easily. 'That can't be right. An instantaneous action at a distance would violate Einstein's theory of relativity which forbids faster-than-light communication'

Dr. Bell smiled enigmatically. 'One might think so', she said 'but it turns out that this nonlocal influence cannot under any circumstances be used to communicate information. You will note that when your clients use the devices, none of them will learn what questions the other two have been asked, or what their answers are, so no information is communicated between them. Einstein's theory survives, although only by the skin of its teeth. This is one of the reasons Einstein was never comfortable with quantum mechanics.'

'That is truly remarkable', I said, pocketing the devices and handing over a well-earned check. 'Well, I cannot thank you enough for your help. You have indeed solved a problem which I thought to be impossible.'

'Really the pleasure is mine Mr. Doyle. It is delightful to find a real-life use for something as curious and apparently arcane as quantum non-locality.'

Exercise 3: Explain in detail how Dr. Bell's gadgets worked.

Answers to the Exercises

Exercise 1

There is, in fact, an elegant way to see that the four statements cannot all be true by using the properties of multiplication.[10] Note first that the guards' statements concern 6 different things, these being the two sides (the back and front) of each of the three robbers, and that each guard saw three of these sides. As Dr. Bell saw, the statements of the first three guards are equivalent to each of them claiming that "of the three sides of the robbers that I saw, an *even* number were green", and the statement of the fourth guard amounts to "of the three sides of the robbers that I saw, an *odd* number were green". Now see what happens if we associate a number with the front and back of each robber (giving 6 real numbers), making the value 1 if the color is red, and -1 if the color is green. Now, since the first three guards saw an even number of green sides, the product of their three numbers is plus one, while the product of the three numbers for the forth guard is minus one. Therefore, the four statements together imply that the product of all of the guards' numbers (i.e. twelve numbers) is minus one. However, using the rules of multiplication it is easy to see that this is not possible. If we examine (using Table I), the twelve various sides that the guards saw, we see that each of the six different sides appears exactly twice in this set of twelve. Thus, the product of the associated set of twelve numbers is actually the product of the squares of the six numbers associated with each side. Since squares are always positive, this product must be positive. Thus all four statements cannot be true simultaneously.

We do not know of a similarly elegant procedure which demonstrates that any three of the statements alone are consistent, but it is enough to find four situations which satisfy each of the four subsets of three statements, and this is not difficult to do by inspection of Table I. In fact, since the statements of the first three guards are symmetric under an interchange of two of the robbers, we need only find two situations, one which satisfies the first three statements, and one which satisfies the last statement along with two of the first three; interchanging the identities of the robbers will then provide the others. If all the robbers have red backs and green fronts, then the statements of the first three guards are true. If robber A is green on the back and front, and B and C are green on the front and red on the back, then the statements of guards 2, 3 and 4 are true. Thus any three of the guards could be telling the truth, but at least one is either mistaken or lying.

Exercise 2

(a) With only three yes/no questions, the prosecution can only find out the color of three of the robber's sides. Now, from the discussion in the answer to Exercise 1, above, we know that each guard's statement concerns only whether there are an even or odd number of a given color among the three sides that he saw. As a consequence, to verify any one of the statements the prosecution must know the colors of *all* of the sides which the statement in question concerns. Thus, since each of the guard's statements concerns a different set of three sides, the prosecution can only determine the truth or falsity of one of the statements.

(b) The reason that Mr. Doyle's clients cannot know which statement is being tested against their answers is as follows. At the time of questioning each suspect will know only whether he is being asked about the color of his front or his back. An inspection of Table I shows that for each side of each robber, there are two of the guards' statements which apply to it. Thus each suspect will know only that one of these two possible statements is being tested. Now, since any two of the statements are mutually consistent each suspect can choose his answers to agree with those two statements. However, further examination shows that for each statement that the prosecution might test against, each suspect will be trying to satisfy a *different* pair of questions. For example, from Table I, if the prosecution decides to test the statement of the first guard, then suspect A will know that he is being tested against either statement 1 or 4, suspect B that it is statement 1 or 3, and suspect C that it is statement 1 or 2. Thus, together, Mr. Doyle's three clients will be trying to satisfy *all* four statements. But since the four statements are inconsistent, they cannot decide beforehand on a set of answers which will do this.

Exercise 3

In order for Mr. Doyle's clients to answer the questions put to them in such a way that they could guarantee their answers are consistent with the statement which the prosecution is testing, they would have to determine their answers in a coordinated fashion. In a universe which obeyed the rules of classical physics, this would be impossible, because they are prevented from communicating. However, they are able to achieve this task by using the following remarkable non-local property which quantum systems possess: It is possible to prepare two or more quantum systems in a joint state, such that when the systems are separated (so that communication between them is impossible), the relationship between the results of measurements made on the separated systems depends upon *what* measurements were made on the distant systems. It is *as if* the quantum systems had been able to communicate about what measurements were being made on them, and used this information to arrange the relationship between the measurement outcomes. This effect cannot be used for communication by three people in possession of the quantum systems, however, because each person cannot influence *which* outcome the others receive, merely the relationship between all their outcomes.

Dr. Bell's plan was to use the non-locality of quantum mechanics by preparing three quantum systems in a joint state, giving one to each suspect, and then at the time of questioning, having each suspect make one of two possible measurements on his own system depending upon which of the two questions they were asked. They would then answer their respective questions by using the result each obtains from his measurement. The joint state that gives precisely the right answers is the Greenberg-Horne-Zeilinger, or GHZ, state of three spin-half particles.[9] If you are not familiar with the mathematical formalism which is used to describe states of, and measurements upon, quantum systems, then unfortunately at this point you will just have to take our word for it that the GHZ state, along with suitable measurements, allows the suspects to answer the questions so as to cheat the prosecution. However, if you are familiar with elementary quantum mechanics, then the details of the scheme may be explained quite simply.

If we denote the two spin-half eigenstates of the operator for spin in the z direction as $|\uparrow\rangle$ for "spin up" and $|\downarrow\rangle$ for "spin down", then the GHZ state is

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_{\mathrm{A}}\,|\uparrow\rangle_{\mathrm{B}}\,|\uparrow\rangle_{\mathrm{C}} - |\downarrow\rangle_{\mathrm{A}}\,|\downarrow\rangle_{\mathrm{B}}\,|\downarrow\rangle_{\mathrm{C}}). \tag{1}$$

Here the subscripts indicate which system belongs to which suspect (A, B, or C). Dr. Bell's gadgets work as follows: If a suspect is asked what color his suit was on the front, then he presses the "lock" button. This triggers a measurement which projects his system onto one of the basis states $\{|\otimes\rangle, |\odot\rangle\}$, where these states are given by

$$|\otimes\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + i\,|\downarrow\rangle), \tag{2}$$

$$|\odot\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - i\,|\downarrow\rangle). \tag{3}$$

This corresponds to a measurement of the spin of the particle in the y-direction. If the gadget gets the result corresponding to $|\otimes\rangle$ then it vibrates in a way that tells him to answer that the color was red. Similarly, if the result is $|\odot\rangle$ then he will know to answer that the color was green. Alternatively, if a suspect is asked what color his suit was on the back, he presses the "unlock" button and this makes a measurement which projects the system onto one of the states $\{|{\rightarrow}\rangle, |{\leftarrow}\rangle\}$, where

$$|{\rightarrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle), \tag{4}$$

$$|{\leftarrow}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle). \tag{5}$$

This is a measurement of the spin of the particle in the x-direction. If the suspect gets the result corresponding to $|{\rightarrow}\rangle$ then he answers that the color was red, otherwise he says that it was green. A quantum mechanical analysis of the two measurements shows that every set of possible outcomes of three of these measurements is consistent with all the statements of the guards, and in particular will be consistent with any statement that the police choose to test by asking their three questions. For example, assume the police are checking if the suspects' answers are consistent with the first guard's statements. Then suspect A will be asked about the back of his suit, so he will make an x spin measurement. The result of the measurement is either $|{\rightarrow}\rangle$ or $|{\leftarrow}\rangle$, and let us assume that it is $|{\rightarrow}\rangle$ or red. In this case the action of the measurement is to apply the projection operator $|{\rightarrow}\rangle\langle{\rightarrow}|$ to A's system, and renormalise the state (which in this case involves multiplying by $\sqrt{2}$). The state of the three systems after the measurement is then

$$\begin{aligned}\sqrt{2}(|{\rightarrow}\rangle\langle{\rightarrow}|)_{\mathrm{A}}\,|\mathrm{GHZ}\rangle &= |{\rightarrow}\rangle_{\mathrm{A}}\,(\langle\uparrow| + \langle\downarrow|)_{\mathrm{A}}\,|\mathrm{GHZ}\rangle \\ &= |{\rightarrow}\rangle_{\mathrm{A}}\left(\frac{|\uparrow\rangle_{\mathrm{B}}\,|\uparrow\rangle_{\mathrm{C}} - |\downarrow\rangle_{\mathrm{B}}\,|\downarrow\rangle_{\mathrm{C}}}{\sqrt{2}}\right) \\ &\equiv |{\rightarrow}\rangle_{\mathrm{A}}\,|\mathrm{BC}\rangle \end{aligned} \tag{6}$$

Now suspect B is asked about the front of his suit, so he makes a y spin measurement. The result can be either $|\otimes\rangle$ or $|\odot\rangle$, but let's assume the result is $|\otimes\rangle$ or red. The state in Eq. (6) now becomes

$$\begin{aligned}\sqrt{2}(|\otimes\rangle\langle\otimes|)_{\mathrm{B}}|{\rightarrow}\rangle_{\mathrm{A}}|\mathrm{BC}\rangle &= |\otimes\rangle_{\mathrm{B}}(\langle\uparrow| - i\,\langle\downarrow|)_{\mathrm{B}}|{\rightarrow}\rangle_{\mathrm{A}}|\mathrm{BC}\rangle \\ &= |{\rightarrow}\rangle_{\mathrm{A}}\,|\otimes\rangle_{\mathrm{B}}\left(\frac{|\uparrow\rangle_{\mathrm{C}} + i\,|\downarrow\rangle_{\mathrm{C}}}{\sqrt{2}}\right) \\ &= |{\rightarrow}\rangle_{\mathrm{A}}\,|\otimes\rangle_{\mathrm{B}}\,|\otimes\rangle_{\mathrm{C}}\,, \end{aligned} \tag{7}$$

(Note the minus sign appearing in $\langle\otimes|_{\mathrm{B}}$ because $\langle\otimes|_{\mathrm{B}}$ is the adjoint of $|\otimes\rangle_{\mathrm{B}}$.) Since C's system is now in the state $|\otimes\rangle$, when C is asked about the front of his suit his measurement must give $|\otimes\rangle$ or red. Thus, the police find that there are no green suits among the answers, which is consistent with the first guard's testimony that he saw an even number of green suits. One could try out all the other possibilities and one would find that in each case the suspects' answers will be consistent with the testimony of the guard chosen by the police. Also, notice that the order in which the suspects give their answers does not matter. There is an elegant treatment of this problem in Mermin's paper,[10] where this three-particle GHZ-style proof of Bell's theorem was first presented.

The gadgets provided by Dr. Bell do not yet exist. However, spin-based qubits[24] are one of the contenders for scalable quantum information processing. With the current rapid advances in quantum information technology,[25] there is no reason to assume that devices operating as we describe could not be built within the next few decades.

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We thank Damian Pope for helpful discussions. We note also that the basic structure of Figure 1 was inspired by the diagram illustrating the GHZ result in Ref.[26]

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sides of the robbers seen by the four guards. Finally, the two colours that the detectors can flash correspond to the same two colors of the robbers' suits. Thus, in Mermin's case when he sets the detector settings to 111, for example, and states that an even number of the three detectors flash red, that corresponds in our case to saying that the fourth guard saw an even number of the robbers wearing red.

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