CONSIDERING STUDENT EXPERIENCE AND KNOWLEDGE OF CONTEXT IN PLANNING MATHEMATICS LEARNING

Peter Sullivan — Monash University
Peter Grootenboer — Griffith University
Robyn Jorgensen — Griffith University

Abstract: In this study, 42 Australian Indigenous children were posed a series of related questions on addition and subtraction. The questions were posed both in the context of money and without any context. The data revealed an alignment between the students' capacity to work with money and to work with numbers. The money context did not increase the cognitive load or task complexity suggesting that relevant contexts can be used by teachers to support mathematics learning.

THE CONTEXT OF THE RESEARCH

The following is a discussion of some data collected at two Indigenous Australian Community Schools in a remote region of Western Australia, as part of the *Maths in the Kimberley* research project[1]. The project investigated ways of supporting the teaching of mathematics in small community-run schools, motivated by general concerns about the achievement of students in such schools.

All recent reports that synthesise results from large scale assessments of learning in Australian schools include analysis and discussion of the extent to which Indigenous Australian students are outperformed by their non Indigenous peers. Thompson, de Bortoli, Nicholas, Hillman, and Buckley (2010), for example, reported that Indigenous students were almost two full years of schooling behind non-Indigenous students in mathematical literacy. The disparity in achievement is a direct result of mismatches between the goals and processes of schools, and the experience and aspirations of Indigenous students (see Jorgensen & Sullivan, 2010).

While recognising that there are competing view on this, we agree with Hughes (2010) that the pathway to modernisation for Indigenous communities and the creation of opportunities for Indigenous students is through the learning of the conventional mathematics content, rather than through modified curricula, and we explored issues associated with the teaching of that mathematics. The project design recognised the complexity of the educational challenges in such small communities, acknowledged those who have addressed these issues previously, and emphasised collaboration with the respective communities at each stage.

Building on the experience of Indigenous Australian students

The project recognised that one of the key challenges for teachers in remote schools is connecting the mathematical concepts they are teaching to the experience of the

[1] The Maths in the Kimberley project was funded by the Australian Research Council, led by Robyn Jorgensen, and conducted on the invitation of the Association of Independent Schools of Western Australia.

students. Of course, connection to experience is a consistent theme in advice to all teachers. Hattie and Timperley (2007), for example, reviewed a large range of studies on the characteristics of effective classrooms. They found that feedback was one of the main influences on student achievement, and the key elements were "where am I going?", "how am I going?", and "where am I going to next?". To provide this sort of feedback, it is clear that teachers need to have an understanding of what mathematics the students know and can do. Similarly, Tzur (2008) argued that instruction should begin with what the students already know and are confident with, and then move to content that is unfamiliar, rather than what he claims is the common approach of starting with unfamiliar content. The clear implication is that it is best if instructional decisions are informed by, and grounded in, what the students already know and experiences with which they are familiar.

This connection with experience also forms a consistent theme in recommendations for teaching Indigenous students. Stanton (1994), for example, argued that the curriculum and pedagogies for Indigenous students could incorporate "both ways" or "common ground" approaches, and that this applies to school policy, management, structure, curriculum and pedagogy. In the case of mathematics, he suggested that the curriculum should: be negotiated; build on aspects of traditional culture; incorporate technology; recognize the interfaces with language; and utilize contexts. Similarly, Frigo, Corrigan, Adams, Hughes, Stephens, and Woods (2003), in reporting a study of schools with high proportions of Indigenous students, listed among key elements of effective numeracy teaching as presenting skills in real life contexts, and building on what the students know. The theme of understanding the students' mathematical knowledge, capabilities and identities, especially in contexts where the teachers and students are from different cultures, is a prominent and recurring one.

It seems that both traditionally, as reflected in the language, and currently, in terms of the limited use of quantities in their everyday lives, number activities tend to be remote from Indigenous students' experience. To overcome this, we sought to explore ways of building connections between money, with which the students have some familiarity and more abstract number ideas. This generally involves making connections to realistic contexts with which the students are familiar.

This creates its own challenges. Bransford, Brown, and Cocking (1999) argued that real-life contexts can be confusing and increase cognitive load. Cooper and Dunne (1998) found that contextualising mathematics tasks created particular difficulties for low socio-economic status (SES) students. Likewise, Lubienski (2000) found that pupils who preferred the contextualised trial materials and found them easier all had high SES backgrounds, while most pupils who preferred closed, context free tasks were low SES. In fact, many of the low SES pupils claimed to be worse off with contextualised problems, and none found the contextualised materials easier.

The challenge for teachers is to find ways to build on students' experience without increasing the complexity of the formulation of tasks, and the cognitive load that multi-step problems create. As highlighted previously, this means using contexts that are familiar and inviting to the Indigenous learners. This study was an attempt to examine what this might mean in project classrooms, and to explore the potential of using money to establish a the basis for important generalisable number ideas.

The context of the data collection

The focus of the data collection reported below was prompted by classroom observations of lessons in which the teacher was seeking to develop generalisable number concepts such as partitioning and additive thinking. The teacher had earlier noticed that the students were adept with using money at the school fete, more so than seemed evident in their classroom mathematics. In the course of developing some learning experiences associated with partitioning, some classroom activities were presented that required the students to perceptually recognise amounts of money (see Sullivan, Youdale, & Jorgensen (2009) for a detailed report of the lesson observations). *Inter alia*, the observer noted students who were able to recognise immediately and accurately amounts made up of coins in various denominations, apparently using what Sousa (2008) described as perceptual recognition. This created the impetus for the data collection reported here.

The focus of the instruction was on ways of partitioning amounts such as $1 (e.g., 70c + 30c, 60c + 40c), and other amounts and how this could be used for operations such as combining particular money amounts (e.g., 80c + 50c is the same as $1 + 30c, etc).

In order to explore this further, the research questions were:

- Are the students fluent with recognising money amounts, and if so, what amounts can they perceptually recognise?
- What is the relationship between fluency with recognising money amounts and items involving equivalent addition tasks posed using only numbers?
- What is the relationship between the coins and addition items, and equivalent subtraction items both involving coins and numbers?

The framework used for the creation of the particular items was that used by the *Early Numeracy Research Project* (ENRP) (Clarke et al., 2001) and consists of sets of growth points that describe sequential development of concepts, with four number domains. We drew on the addition and subtraction section of the framework, the relevant growth points of which were:

2. Counts on from one number to find the total of two collections.
3. Given a subtraction situation, chooses appropriately from strategies including count back, count down to and count up from
4. Given an addition or subtraction problem, strategies such as doubles, commutativity, adding 10, tens facts, and other known facts are evident
5. Given an addition or subtraction problem, strategies such as near doubles, adding 9, build to next ten, fact families and intuitive strategies are evident

This hierarchical framework has been shown to be robust and to represent growth of learning in students. Students in our project schools had been earlier interviewed based on this framework, and most students in the middle primary years were able to answers questions such as: 'I have 8 biscuits, and I eat 3. How many do I have left?' and; 'What is 10 – 7?' Nearly all of the students interviewed were able to: count by 10s past 100; count by 5s to 90; calculate 9 + 4, where the 9 objects were covered, requiring counting; state the answer to 2 + 19. At the same time, few students were about the answer the question "I have 12 strawberries, and eat 9. How many do I have left?"

We used these results as the basis of a task based interview, and sought to extend the complexity of the tasks from there. We considered an interview as the most appropriate way to gain insights into what the students could do. This was partly so a student could answer without concern for what other class members might say, and partly so that the researcher could monitor the progress of the student and adapt the protocol accordingly. The interviews were conducted in a quiet place separate from the rest of the class (e.g., in a separate room) so the student could focus on the tasks and questions at hand. The students were familiar with the researchers because we had spent some time in the classes before the interviews were undertaken, and during the interviews the researchers endeavoured to make the students feel comfortable, valued and relaxed so they could answer freely and honestly.

An interview protocol was developed that posed 10 sets of items, with four items in each set (see Figure 1). For example, the first set of items was as follows:

> *Recognition of the value of a collection of coins:* Two 20c pieces were covered then shown for 2 seconds ("and 1 and 2") and then covered. The students were asked "how much money is under my hand?"
>
> *Number addition:* Students were shown a card on which was written "20 + 20" and the question "what is 20 plus 20?" was posed.
>
> *Calculation of difference using money:* The researcher put out two 20c pieces on the table without the students seeing them, and uncovered one 20c piece. The question was posed "There is 40c on the table altogether. How much is still covered by my hand?"
>
> *Calculation of difference using number:* Students were shown a card on which was written "40 – ? = 20" and the question "what number do I take from 40 to get 20?" was posed. If this was not clear the first time, the question was asked another way.

Depending on the level of response, these items were addressing the growth point levels 2, 3 and 4 described above. In posing the items, the first item in each set was presented first. In other words, all the coins recognition items were posed in sequence, and then the researcher went back to pose the number addition items in sequence, and so on.

The particular number and money amounts from the respective sets of items are shown in Figure 1:

Item	Perceptual recognition of collection of coins	Number Addition	Difference using money	Number Difference
1	20c, 20c	20+20	40c - 20c	20 + ? = 40
2	20c, 20c, 10c	20+20+10	50c - 10c	60 + ? = 50
3	50c, 10c	50+10	60c - 10c	70 + ? = 60
4	50c, 20c, 10c	50+20+10	80c - 20c	60 + ? = 80
5	50c, 20c, 20c, 10c	50+20+20+10	$1 - 20c	80 + ? = 100
6	50c, 20c, 10c, 10c, 5c, 5c	50+20+10+10+5+5	$1 - 10c	90 + ? = 100
7	50c, 20c, 20c, 20c	50+20+20+20	$1.10 - 20c	90 + ? = 110
8	$2, $2, $2	2+2+2	$6 - $2	4 + ? = 6
9	$2, $2, $2, $1	2+2+2+1	$7 - $1	6 + ? = 7
10	$2, $2, $2, $2, $1, $1	2+2+2+2+1+1	$10 - $2	8 + ? = 10

Figure 1: The ten sets of four related items.

The items with cents became progressively more difficult, as did the items using dollar amounts. Once a student experienced difficulty with a sequence of items, we jumped to the start of the next sequence (i.e., item 8). For example, when a student was having difficulty with an item on the value of coins involving cents we would skip forward to the items involving $1 and $2 coins.

Results

The following is a selection from the responses to the interview questions. The number of students who correctly recognised the value of the coins, when the coins were shown for 2 seconds then covered, is shown in Table 1 below. Note that once a student experienced difficulty with a sequence of items, we jumped to the start of the next sequence.

Amount in coins	Number of students correct
20c, 20c	31
20c, 20c, 10c	28
50c, 10c,	31
50c, 20c, 10c	25
50c, 20c, 20c, 10c	25
50c, 20c, 10c, 10c, 5c, 5c	11
50c, 20c, 20c, 20c	5
$2, $2, $2	31
$2, $2, $2, $1	32
$2, $2, $2, $2, $1, $1	27

Table 1: Number of students who correctly recognised amounts of coins (n = 42)

Around three quarters of the students could do the easier items, and most of them could also do the items of medium difficulty as well. Noting that the coins were only shown briefly, this suggested that many of the students were able to recognise the money readily. In other words, many were not forming a mental image and then adding the value of the coins, but seemed to be using some other process. There were students, including some of the more successful students, who did form such mental images nevertheless.

Overall, the range of results was broad with some young students getting none of the items correct, and a few students (from grades 4 to 6) getting the majority of them correct. There was about one quarter of the students who could not say the money amounts even with the easier items. The diversity in the responses is discussed below.

To explore the nature of the responses and to allow comparison across the sets of items, the responses were compared by grade level for the equivalent items for item 4 in Figure 1 involving 50c, 20c, 10c (see Table 2 below). This was the fourth item in each set and is illustrative of the other results since the profile of responses is similar.

Grade	n	Coin recognition	Number addition	Money subtraction	Number subtraction
3	18	7	7	6	6
4	9	6	7	6	2
5	13	10	10	9	5
6	2	2	2	1	1
Total	42	25	26	22	14

Table 2: Correct responses to the items involving 50c, 20c, 10c

It appears that the overall facilities of the coin recognition, number addition, and money subtraction items were similar. Given that they were only shown the coin recognition task briefly it confirms that many students have an interesting fluency with recognition of the money amounts. The majority of the grade 4, 5 and 6 students were able to do the coin recognition for this combination, but only a minority of the grade 3s. This suggests that the questions were pitched at the appropriate level to probe these students' capacity at the items. It can be inferred that students who completed all of these tasks would be at growth point 4 or 5 in the framework for addition and subtraction presented earlier.

Most of the students who were not able to recognise the amount of these coins were in grade 3. This suggests that the collective experience of the students influences their capacity. It seems that, whatever experiences are needed for recognising coin amounts, most students had had those experiences by the time they were in grade 4.

Our hypothesis, based on classroom observations, was that the students would be better with recognising coin amounts than with straight calculations, and that we would use this facility in designing instructional programs. This was not the case for equivalent pairs of items involving addition.

To illustrate the extent to which the responses to one are dependent on the response to the related item, the numbers in each of the four possible combinations of responses to the equivalent addition items are presented in Table 3.

	Coin recognition correct	Coin recognition incorrect
Number addition correct	24	1
Number addition incorrect	2	5

Table 3: Comparison of student responses of the pairs of addition items connected to 50c, 20c, 10c

This table shows that 29 out of the 32 students that attempted both were either correct or incorrect on both, and only 3 (less than 10%) were correct on only one. For the equivalent items in this case, it seems that the capacity to add 50, 20 and 10, is connected to the ability to perceptually recognise this amount in coins. In fact, the cross tabulation for seven sets of items (items 1, 2, 3, 4, 5, 8 and 9) is similar.

Across the ten items, the facility on the items involving the *difference* between coin amounts was also very close to the coin recognition items, and therefore, to the number addition items.

Conclusion

In terms of the research questions, it seems that many of these Indigenous Australian primary students were able to state the amount of collections of coins quickly and readily. This was compatible with our classroom observations. It also seems that nearly all students who could do this were able to respond to number addition questions using comparable numbers. While it makes sense that this should happen, it seems that the students were more fluent with the number calculation tasks than seemed evident from observations of their classroom responses. It is possible that the interview environment is more conducive to concentration.

An inference overall is that the facility with the straight number tasks is connected directly to the ability to perceptually recognise the money amounts, although it is not clear which skill informs the other.

This fluent recognition of collections of coins seems a strength of these students, and teachers could explore the extent to which such strengths could be used to introduce students to addition tasks, for promoting confidence in solving contextual problems, for challenging those students who are ready, and for allowing students to recognise the advantages of developing number fluency. In other words, the context with which these students are familiar can be used to enrich their learning of mathematics.

Part of the challenge of teaching mathematics to students whose experience with modern mathematical ideas is limited due to culture and remoteness is that it is difficult to identify contexts that are both familiar to the students and which also exemplify key mathematical ideas. Connected to this is that some contexts increase the cognitive load on the students making learning more difficult. This research suggests that thoughtfully selected contexts can be used effectively to engage students in learning mathematics.

References

Bransford, J. B., Brown, A. L., & Cocking, R. R. (Eds.) (1999). *How people learn: Brain, mind, experience, and school*. London: Committee on Developments in the Science of Learning, National Research Council.

Clarke, D., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, M., McDonough, A., Montgomery, P., Roche, A., Sullivan, P., Clarke, B., & Rowley, G. (2002). Early Numeracy Research Project (ENRP): Final report. Retrieved September 11, 2010 from: http://www.sofweb.vic.edu.au/eys/num/enrp.htm.

Cooper, B., & M. Dunne (1998). Anyone for tennis? Social class differences in children's responses to national curriculum mathematics testing. *The Sociological Review*, (Jan), 115–148.

Frigo, T., Corrigan, M., Adams, I., Hughes, C., Stephens, M., & Woods, D. (2003). *Supporting English literacy and numeracy learning for indigenous students in the early years.* ACER Monograph 57: Camberwell.

Hattie, J., & Timperley, H. (2007) The power of feedback. *Review of Educational Research, 77* (1), 81-112.

Hughes, P. (2010). Keynote address at Dare to Lead conference, Yulara NT. April.

Jorgensen, R., & Sullivan, P. (2010). Scholastic heritage and success in school mathematics: Implications for remote Aboriginal learners. In I. Synder & J. Nieuwenhuysen (Eds). *Closing the Gap: Improving outcomes in Southern World Societies.* Clayton, Melbourne; Monash University Press.

Lubienski, S. T. (2000). Problem solving as a means toward mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education, 31*, 454–482.

Sousa, D. (2008). *How the brain learns mathematics.* California: Corwin Press.

Stanton, R. (1994). Mathematics “both ways”: A mathematics curriculum for aboriginal teacher education students. *For the Learning of Mathematics, 14*(3), 15-23.

Sullivan, P., Youdale, R., & Jorgensen, R. L. (2009). The link between planning and teaching mathematics: An exploration in an indigenous community school In C. Hurst, M. Kemp, B. Kissane, L. Sparrow, & T. Spencer (Eds.), *Proceedings of the 22nd Biennial Conference of the Australian Association of Mathematics Teachers Inc.* (pp. 247-255). Adelaide: The Australian Association of Mathematics Teachers.

Thompson, S., de Bortoli, L., Nicholas, M., Hillman, K., & Buckley, S. (2010). Challenges for Australian education: Results from PISA 2009. Melbourne: Australian Council of Educational Research.

Tzur, R. (2008). Profound awareness of the learning paradox. In B. Jaworski & T. Wood (Eds). *The Mathematics Teacher Educator as a developing professional* (pp. 137-156).