Predictor-Based Neural Dynamic Surface Control of a Nontriangular System With Unknown Disturbances

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***Abstract*—For a class of nontriangular nonlinear systems in presence of unknown disturbances, we propose a predictor-based neural dynamic surface control (PNDSC) strategy in this paper. This nontriangular system is transformed via the mean value theorem, and a predictor is then constructed. To avoid an algebraic loop problem, partial state vectors are employed as input signals of neural networks (NNs) for approximating unknown dynamics, and compensation items are designed to compensate for approximation errors from NNs. Different from the traditional NDSC, the PNDSC in this paper utilizes prediction errors to update learning parameters for improving NNs' learning behaviors with overlarge adaptive gains. On the basis of improved NNs' approximation behaviors, a predictor-based NNs disturbance observer (PNNDO) is constructed for compensation for external disturbances and approximation errors from NNs. Furthermore, with predictors, a normalization method of weights is developed to reduce the number of online learning parameters. On the basis of the aforementioned result, measurement noises are taken into account in our predictor-based neural control strategy. We employ predictor states, rather than measurement information paralyzed by noises, in design of our control strategy. This reduces high-frequency oscillations in control input. A Lyapunov-based stability analysis shows that all signals are ultimately bounded in the closed-loop system. Finally, the effectiveness of the proposed control strategy is verified by a numerical example and a permanent magnet brushless DC motor system.**

***Index Terms*—Adaptive control, neural networks, backstepping, predictor-based neural dynamic surface control, nontriangular system.**

I. Introduction

Nontriangular nonlinear systems [1]–[4] have attracted significant attentions in the control community due to their wide applications, such as electromechanical system [1], Chuas circuit system [2], stirred tank reactor [3] and Brusselator model [4]. The growing interest toward this class of systems is not only owing to practical applications but also research potentials. For example, most of existing control techniques [5]–[13] are infeasible for such nonlinear systems in a direct manner.

There exist challenges in the control methodology design for nontriangular systems. One of remarkable features in triangular systems, such as pure-feedback systems, is that unknown dynamics $f_i(\cdot)$ of the ith subsystem contain states in the previous subsystems $\bar{x}_{i+1} = [x_1, x_2, \cdots, x_{i+1}]^T$. In nontriangular systems, unknown dynamic in the ith subsystem contains not only partial state variables $\bar{x}_{i+1} = [x_1, x_2, \cdots, x_{i+1}]^T$ but all state variables $\bar{x}_n = [x_1, x_2, \cdots, x_n]^T$. In the framework of backstepping control schemes, a state variable x_{i+1} is regarded as a control input signal to the ith subsystem, and a virtual control law α_{i+1} is then constructed. For the existence of a virtual control law, α_{i+1} only contains the states in the previous subsystems in a triangular system. On the contrary, for a nontriangular nonlinear system, a virtual control law α_{i+1} will be a function with all state variables if we use a traditional backstepping scheme. The traditional scheme is not feasible and will cause an algebraic loop problem.

To address the control issue, adaptive control strategies with approximation, such as fuzzy logic systems (FLSs) and neural networks (NNs), have been developed for nontriangular nonlinear systems. Chen introduces a bounded funciton with monotonically increasing property to propose variable separation methods and develops adaptive FLSs and NNs control methods for a nontriangular SISO nonlinear system [14][15]. Subsequently, Tong et al. [16] eliminate this restriction and present a novel adaptive fuzzy backstepping design algorithm. On the basis of the aforementioned results, further researches have reported complex issues for nontriangular SISO nonlinear systems, such as full state constraints [17], high-order nonlinear [18], unmeasurable states [19][20], finite time [21][22] and fault-tolerant control [23]. From SISO nontriangular systems to MIMO ones, an extension method is developed via adaptive FLSs and NNs control methods [24][25][26].

Many efforts are taken to improve transient performance of NNs' learning behavior in neural dynamic surface control (NDSC) approaches. In general, increasing the adaptive gains can reduce the magnitude of tracking errors, and this might cause undesired high-frequency oscillations and have an impact on system stability [27]. For addressing this concern, Peng et al. in [5] construct prediction errors between

This work was supported in part by National Natural Science Foundation of China under Grant 61873130 and under Grant 62103200, in part by Natural Science Foundation of Jiangsu Province under Grant BK20191377 and under Grant BK20191376, in part by the 1311 Talent Project of Nanjing University of Posts and Telecommunications, in part by Natural Science Foundation of Nanjing University of Posts and Telecommunications under Grant NY220194 and under Grant NY221082, in part by the Postgraduate Research and Practice Innovation Program of Jiangsu Province under Grant SJCX21_0292, in part by Key projects of Natural Science Foundation of Hebei Province under Grant E2020203139, in part by the foundation from Key Laboratory of Industrial Internet of Things and Networked Control of the Ministry of Education of China under Grant 2021FF01. (Corresponding authors: Yang Yang; Wenbin Yue.)

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predictors and system states to update NNs' weights. On the basis of the traditional surface errors signal updating method [7], a predictor-based NDSC method provides an additional adjustable parameter and improves transient behaviors of NNs' approximation resulting in smooth control signals. Wang et al. construct a predictor based on surface errors and use prediction errors to update neural adaptation laws for approximating ideal vectors [28]. Xu proposes a composite learning control scheme with tracking errors and prediction errors for updating NNs' weight parameters adaption [29]. The above-mentioned control methods [25]–[29] indeed improve system performance, but they are not able to deal with external disturbances.

Anti-external disturbances is another key topic in nontriangular systems. A nonlinear disturbance observer (NDO) is employed to deal with their influences on nonlinear systems for improving robustness of a closed-loop system [30]–[38]. The principle of an NDO is to estimate unknown disturbances and introduce compensation [32]. An NDO is designed by the difference between a state and an intermediate variable, and the issue of mismatched disturbances in [33]–[34] is addressed. Authors in [35] combine sliding mode NDO with backstepping techniques and propose a robust constrained control scheme for MIMO systems. Shao et al. in [36]–[38] employ approximation property of NNs and construct a neural network disturbance observer (NNDO) for estimating external disturbances. These NNDOs not only fail to consider transient performance of NNs' learning behavior but also require a large number of parameters to be tuned online.

Although many achievements have been made for nontriangular systems and anti-disturbances in the existing references, there still exist technical gaps. Most results are for a nontriangular nonlinear system without external disturbances, and few efforts are made to control such systems against disturbances. To avoid an algebraic loop problem, parts of control strategies are often based on strict assumptions [14][15], such as bounded unknown dynamics with monotonically increasing properties. Also, many existing references mainly focus on the tracking problem of nontriangular systems, and they do not take transient performance of NNs' approximation into account. In fact, scaling up adaptive gains indeed realizes fast behavior for NNs' learning, but overlarge gains may lead to undesirable high-frequency oscillations. Furthermore, few NNDOs are reported for nontriangular systems, and still lack of improvement for disturbance compensation as well as for reducing the number of online parameters.

Motivated by the above-mentioned results and gaps, we propose a predictor-based NDSC strategy for a nontriangular nonlinear systems in the presence of unknown disturbances. A predictor-based NN disturbance observer (PNNDO) is constructed to compensate both external disturbances and NNs' approximation errors, and a normalization method of weights is employed to reduce the number of online parameters. The contributions made in this paper are as follows.

1) For a nontriangular system in the presence of external disturbances, a predictor-based neural dynamic surface control (PNDSC) strategy is proposed. Different from nontriangular systems only in the presence of unknown dynamics [16], the plant in our paper is more complex due to external disturbances. This kind of system is transformed via the mean value theorem, and a predictor in this paper is then constructed. Different from a predictor in a strict-feedback system [5], an improved predictor in this paper introduces a compensation item and a disturbance estimation. The purpose of this improvement is to avoid the algebraic loop problem in a nontriangular system without strict assumptions. Instead of using surface errors, the proposed strategy employs prediction errors from a predictor to drive NNs' learning parameters. This improvement avoids high-frequency oscillations in NNs' learning behaviors caused by overlarge adaptive gains, compared with using surface errors to update signals for a nontriangular system [1].

2) A predictor-based NNDO (PNNDO) is proposed with a normalization method of weights. Compared with the traditional NNDO in [37] in a strict-feedback system, the PNNDO in this paper is designed on the basis of predictor-based NNs. This PNNDO does not only compensate for generalized disturbance, including external disturbances and approximation errors in a nontriangular system, but also improves approximation behaviors. Further, with predictors and a normalization method of weights, we also reduce the number of NNs' learning parameters tuned online compared with that in the traditional NNDO [34].

3) Our control strategy further takes measurement noises into account. To against the influence from measurement noises in sensors, predictor states are employed to replace measurement signals in our control strategy. This replacement improves robustness of the system and the performance of control inputs with fewer oscillations due to the existence of measurement noises.

The remaining part of this paper is organized as follows. Section II presents necessary preliminaries and problem formulation. In Section III, we propose an adaptive control strategy as well as stability analysis. Further, our control strategy takes measurement noises into account in Section IV. Three simulations are illustrated in Section V, and the conclusions are drawn in Section VI with future work.

II. Problem Formulation and Preliminaries

A. Problem Formulation

Consider a nontriangular nonlinear system with external unknown disturbances

$$\begin{cases} \dot{x}_1 = f_1(\bar{x}_n) + d_1(t), \\ \dot{x}_i = f_i(\bar{x}_n) + d_i(t), \\ \dot{x}_n = f_n(\bar{x}_n, u) + d_n(t), \\ y = x_1, \end{cases} \tag{1}$$

where $i = 2, \cdots, n-1$, $\bar{x}_n = [x_1, x_2, \cdots, x_n]^T \in \mathbb{R}^n$ is a state vector, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are control input and output signals, respectively, $d_i(t)$ is external unknown disturbance, and $f_i(\cdot)$ is an unknown smooth function, $i = 1, \cdots, n$.

Assumption 1: ([38]) The external unknown disturbance and its derivative are bounded and satisfy $|d_i| \leq d_i^*$ and $|\dot{d}_i(t)| \leq \breve{d}_i^*$, where d_i^* and $\breve{d}_i^*$ are positive and unknown constants.

Assumption 2: The signs of $\kappa_i(x_1, x_2, \cdots, x_n)$, $i = 1, \cdots, n-1$, and $\kappa_n(x_1, x_2, \cdots, x_n, u)$, are known, and there

exist constants $\kappa_{\max}$ and $\kappa_{\min}$ such that $0 < \kappa_{\min} \leq |\kappa_i(\cdot)| \leq \kappa_{\max}$, where $\kappa_i(x_1, x_2, \cdots, x_n) = \frac{\partial f_i(x_1, x_2, \cdots, x_n)}{\partial x_{i+1}}$, $i = 1, \cdots, n-1$, and $\kappa_n(x_1, x_2, \cdots, x_n, u) = \frac{\partial f_n(x_1, x_2, \cdots, x_n, u)}{\partial u}$.

Remark 1: Since $\kappa_i(\cdot)$ is a continuous function, Assumption 2 indicates that $\kappa_i(\cdot)$ is either strictly positive or strictly negative. This assumption guarantees controllability of the system (1). Without loss of generality, we mainly consider $\kappa_i(\cdot) > 0$, and the other case $\kappa_i(\cdot) < 0$ is in a simple mathematical transformation and can be treated in the same way. Assumption 2 is common in the existing references such as in [1].

In order to obtain the affine external state variable x_{i+1} and control input u, we make use of the mean value theorem [1], and the nonlinear smooth function $f_i(\cdot)$ yields

$$f_i(x_1, \cdots, x_i, x_{i+1}, \cdots, x_n) = f_i(x_1, \cdots, x_i, x_{i+1}^0, \cdots, x_n) + \frac{\partial f_i(\cdot)}{\partial x_{i+1}}\bigg|_{x_{i+1}=x_{i+1}^{\hbar}} (x_{i+1} - x_{i+1}^0), i = 1, \cdots, n-1, \tag{2}$$

where the smooth function $f_i(\cdot)$ is clearly analyzed between $f_i(x_1, \cdots, x_i, x_{i+1}, \ldots, x_n)$ and $f_i(x_1, \cdots, x_i, x_{i+1}^0, \cdots, x_n)$, $x_{i+1}^{\hbar}$ is a point between x_{i+1} and x_{i+1}^0, and $\hbar$ is a constant satisfying $0 < \hbar < 1$, $1 \leq i \leq n-1$.

Specifically, we choose $x_{i+1}^0 = 0$, and (2) yields

$$f_i(x_1, \cdots, x_i, x_{i+1}, \cdots, x_n) = f_i(x_1, \cdots, x_i, x_{i+1}^0, \cdots, x_n) + \frac{\partial f_i(\cdot)}{\partial x_{i+1}}\bigg|_{x_{i+1}=x_{i+1}^{\hbar}} (x_{i+1} - 0), \ i = 1, \cdots, n-1. \tag{3}$$

Similarly, we get

$$f_n(x_1, \cdots, x_n, u) = f_n(x_1, \cdots, x_n, u^0) + \frac{\partial f_i(\cdot)}{\partial u}\bigg|_{u=u^{\hbar}} (u - 0). \tag{4}$$

where $u^{\hbar}$ is a point between u and u^0.

Substituting (3) and (4) into (1) yields

$$\begin{cases} \dot{x}_1 = \bar{f}_1(\bar{x}_n) + \bar{\kappa}_1(x_1, \cdots, x_n)x_2 + d_1(t), \\ \dot{x}_m = \bar{f}_m(\bar{x}_n) + \bar{\kappa}_m(x_1, \cdots, x_n)x_{m+1} + d_m(t), \\ \dot{x}_n = \bar{f}_n(\bar{x}_n, u) + \bar{\kappa}_n(x_1, \cdots, x_n, u)u + d_n(t), \\ y = x_1, \end{cases} \tag{5}$$

where $\bar{f}_m(\bar{x}_n) = f_m(x_1, \cdots, x_m, 0, \cdots, x_n)$, $m = 2, \cdots, n-1$, $\bar{f}_n(\bar{x}_n, u) = f_n(x_1, \cdots, x_n, 0)$, $\bar{\kappa}_j(x_1, \cdots, x_n) = \kappa_j(x_1, \cdots, x_j, x_{j+1}^{\hbar}, \cdots, x_n)$, $j = 1, \cdots, n-1$, and $\bar{\kappa}_n(x_1, \cdots, x_n, u) = \kappa_n(x_1, \cdots, x_n, u^{\hbar})$.

In order to facilitate the design of control strategy, the model (5) is further transformed into

$$\begin{cases} \dot{x}_j = F_j(\bar{x}_n) + x_{j+1} + d_j(t), \quad j = 1, \cdots, n-1, \\ \dot{x}_n = F_n(\bar{x}_n, u) + u + d_n(t), \\ y = x_1, \end{cases} \tag{6}$$

where $F_j(\bar{x}_n) = \bar{f}_j(\bar{x}_n) + \bar{\kappa}_j(x_1, \cdots, x_n)x_{j+1} - x_{j+1}$, and $F_n(\bar{x}_n, u) = \bar{f}_n(\bar{x}_n) + \bar{\kappa}_n(x_1, \cdots, x_n, u)u - u$.

Remark 2: $\bar{\kappa}_j(x_1, x_2, \cdots, x_n)$, as a gain function of affine variable x_{j+1}, is unknown. For designing virtual control laws, additional methods need to be adopted, such as introducing a Nussbaum function [10]. In order to facilitate the design process, the model is further transformed. With the transformation, unknown gains for state variable x_{j+1} and control input u in (5) are removed, and they are known or identity. Only approximating $F_j(\bar{x}_n)$ by NNs is required.

B. Radial Basis Function (RBF) NNs

To deal with uncertainties of the system, RBF NNs are employed to approximate the unknown dynamics. As for a smooth nonlinear function $f : \mathbb{R}^m \to \mathbb{R}$, there exists a RBF basis function vector $\varphi(x) = [\varphi_1(x), \varphi_1(x), \cdots, \varphi_l(x)]^T \in \mathbb{R}^l$, and an ideal weight vector $W^* = [w_1^*, w_2^*, \cdots, w_l^*]^T \in \mathbb{R}^l$ such that $f(x) = W^{*T}\varphi(x) + \varepsilon$, where $x \in \Omega \subset \mathbb{R}^m$ is an input vector, Ω is a compact set, l is the node number of the hidden layer, and ε is the approximation error. ε and its derivative $\dot{\varepsilon}$ satisfy $\varepsilon < \varepsilon^*$ and $|\dot{\varepsilon}| < \breve{\varepsilon}^*$, where ε^* and $\breve{\varepsilon}^*$ are unknown positive constants. The basis function selected in this paper is a Gaussian function $\varphi_i(x) = \exp[-\frac{(x-\zeta_i)^T(x-\zeta_i)}{2\xi_i^2}]$, where $\zeta_i = \left[\zeta_{i,1}, \cdots, \zeta_{i,m}\right]^T$ and ξ_i are the center and width of the activation function, respectively.

Assumption 3 ([7]): The reference signal y_r is a smooth and bounded function, and there exists a positive constant p_0 such that a compact set $\Xi_0 := \left\{(y_r, \dot{y}_r, \ddot{y}_r) : y_r^2 + \dot{y}_r^2 + \ddot{y}_r^2 \leq p_0\right\}$ holds.

The control objective for (1) is to design an adaptive control strategy such that all signals in the closed-loop system are bounded and that the tracking error $y - y_r$ is arbitrarily small.

Remark 3: Due to the fact that unknown dynamics in nontriangular systems contain all state variables, the algebraic loop is an inevitable problem in designing a recursive control strategy for such systems. To address this issue, we use partial state variables as NNs input vectors rather than whole state variables, similar to a method in [1]. However, this method will impact the approximation performance of system unknown dynamics via NNs. Traditional methods do not take approximation performance and the number of learning parameters into account, and it is challenging to improve system performance as well as reduce the number of NNs parameters tuned online.

III. PNDSC via a Disturbance Observer with a Normalization Method of Weights

A. Control Strategy Design

In this section, we propose a predictor-based NDSC strategy for a class of nontriangular nonlinear systems with unknown disturbances. The predictor is combined with the NDSC control design, and NNs parameters are updated by the prediction errors. For the problem of external disturbances in the system, a predictor-based NNs disturbance observer is constructed for compensation of external disturbances and approximation errors of NNs. We combine a normalization method of weights

with PNDSC to reduce the number of parameters tuned online, thus reducing the complexity of PNDSC.

Step 1: Define the first dynamic surface error $s_1 = x_1 - y_r$, which is also called tracking error due to the fact $y = x_1$ in (1). Its time derivative along (6) is given by

$$\begin{aligned}\dot{s}_1 &= \dot{x}_1 - \dot{y}_r \\ &= F_1(\bar{x}_n) + x_2 - \dot{y}_r + d_1(t).\end{aligned} \tag{7}$$

Using NNs for the approximation of $F_1(\bar{x}_n)$, we have $F_1(\bar{x}_n) = W_1^{*T}\varphi_1(\bar{x}_n) + \varepsilon_1(\bar{x}_n)$, and $\varepsilon_1(\bar{x}_n)$ is the approximation error. (7) becomes

$$\dot{s}_1 = W_1^{*T}\varphi_1(\bar{x}_n) + x_2 - \dot{y}_r + \varepsilon_1(\bar{x}_n) + d_1(t), \tag{8}$$

where $W_1^{*T}\varphi_1(\bar{x}_n)$ is a function of all state variables $\bar{x}_n = [x_1, x_2, \ldots, x_n]^T$, and it will be used to design virtual control law α_2, which will be designed later. In the framework of Backstepping, the state variables $[x_2, \cdots, x_n]^T$ can not directly appear in the update law of α_2. Otherwise it will lead to the situation of designing control input u with itself, that is, algebraic loop problem. To address this issue, we introduce $W_1^{*T}\varphi_1(x_1)$, where $\varphi_1(x_1)$ is a function about the variable x_1. Due to the fact that not all state variables are employed as input signals of NNs, there exists an transformation error $|W_1^{*T}(\varphi_1(x_1) - \varphi_1(\bar{x}_n))|$. We have to introduce a compensation term in our control strategy for this transformation error. Further, for reduction of the number of learning parameters, we take a norm calculation and denote $\lambda_1^* = -\|W_1^*\|^2$. The adaptive law for its estimate $\hat{\lambda}_1$ will be designed later.

Define $D_1 = \varepsilon_1(\bar{x}_n) + d_1(t)$, and D_1 is called generalized disturbance, which includes external disturbance $d_1(t)$ and NNs approximation error $\varepsilon_1(\bar{x}_n)$. In order to eliminate the influence of generalized disturbances on the system, the PNNDO is designed as

$$\begin{cases}\hat{D}_1 = c_{d1}(\hat{x}_1 - \hat{d}_1), \\ \dot{\hat{d}}_1 = \hat{\lambda}_1\Phi_1(x_1)\tilde{x}_1 + x_2 + c_{D1}\hat{D}_1 + \bar{c}_{d1}\tilde{x}_1 + \nu_1\hat{\lambda}_1\tilde{x}_1,\end{cases} \tag{9}$$

where $\hat{F}_1 = \hat{\lambda}_1\Phi_1(x_1)\tilde{x}_1$ is an improved architecture of NNs with fewer learning parameters, $\bar{c}_{d1} = c_{d1}^{-1} - (c_1 + c_{p1})$, c_{d1}, c_{D1}, c_{p1} and c_1 are positive constants, $\nu_1\hat{\lambda}_1\tilde{x}_1$ is a compensation item, $\tilde{x}_1 = \hat{x}_1 - x_1$, $\hat{x}_1$ is from the following predictor

$$\dot{\hat{x}}_1 = \hat{\lambda}_1\Phi_1(x_1)\tilde{x}_1 + x_2 - (c_1 + c_{p1})\tilde{x}_1 + \nu_1\hat{\lambda}_1\tilde{x}_1 + \hat{D}_1, \tag{10}$$

ν_1 is a positive constant, and $\hat{\lambda}_1$ is an estimate of $\lambda_1^* = -\|W_1^*\|^2$.

The update law for $\hat{\lambda}_1$ is designed as

$$\dot{\hat{\lambda}}_1 = \Gamma_1[-\Phi_1(x_1)\tilde{x}_1^2 - \nu_1\tilde{x}_1^2 - \zeta_1\hat{\lambda}_1], \tag{11}$$

where $\Phi_1(x_1) = \varphi_1^T(x_1)\varphi_1(x_1)/(4\gamma_1^2)$, and Γ_1, ζ_1 and γ_1 are positive constants. To stabilize s_1, a virtual control law α_2 is proposed

$$\alpha_2 = -\hat{\lambda}_1\Phi_1(x_1)\tilde{x}_1 - \nu_1\hat{\lambda}_1\tilde{x}_1 - c_1 s_1 + \dot{y}_r - \hat{D}_1. \tag{12}$$

Let α_2 pass through a first-order filter to obtain x_{2d} for the estimation of α_2

$$\eta_2\dot{x}_{2d} = \alpha_2 - x_{2d} \tag{13}$$

with $x_{2d}(0) = \alpha_2(0)$, where η_2 is a positive constant.

Step $i (i = 2, \cdots, n-1)$: Define the ith surface error $s_i = x_i - x_{id}$, and its time derivative along (6) is

$$\begin{aligned}\dot{s}_i &= \dot{x}_i - \dot{x}_{id} \\ &= F_i(\bar{x}_n) + x_{i+1} - \dot{x}_{id} + d_i(t).\end{aligned} \tag{14}$$

From RBFNNs, we have $F_i(\bar{x}_n) = W_i^{*T}\varphi_i(\bar{x}_n) + \varepsilon_i(\bar{x}_n)$, where $\varepsilon_i(\bar{x}_n)$ is the approximation error. Then, we get

$$\dot{s}_i = W_i^{*T}\varphi_i(\bar{x}_n) + x_{i+1} - \dot{x}_{id} + \varepsilon_i(\bar{x}_n) + d_i(t). \tag{15}$$

According to the analysis in the Step 1, $W_i^{*T}\varphi_i(\bar{x}_n)$ can not be used in the control design in the Step i. We will design the control strategy in a similar way to the Step 1. Define $D_i = \varepsilon_i(\bar{x}_n) + d_i(t)$, and D_i is called generalized disturbance in this paper, including external disturbance $d_i(t)$ and NNs approximation error $\varepsilon_i(\bar{x}_n)$. To eliminate the influence of D_i on the system, we design a PNNDO as

$$\begin{cases}\hat{D}_i = c_{di}(\hat{x}_i - \hat{d}_i), \\ \dot{\hat{d}}_i = \hat{\lambda}_i\Phi_i(\bar{x}_i)\tilde{x}_i + x_{i+1} + c_{Di}\hat{D}_i + \bar{c}_{di}\tilde{x}_i + \nu_i\hat{\lambda}_i\tilde{x}_i,\end{cases} \tag{16}$$

where $\hat{F}_i = \hat{\lambda}_i\Phi_i(\bar{x}_i)\tilde{x}_i$ is the improved architecture of an NN for fewer learning parameters, $\bar{c}_{di} = c_{di}^{-1} - (c_i + c_{pi})$, c_{di}, c_{Di}, c_{pi} and c_i are positive constants, $\tilde{x}_i = \hat{x}_i - x_i$,

$$\dot{\hat{x}}_i = \hat{\lambda}_i\Phi_i(\bar{x}_i)\tilde{x}_i + x_{i+1} - (c_i + c_{pi})\tilde{x}_i + \nu_i\hat{\lambda}_i\tilde{x}_i + \hat{D}_i, \tag{17}$$

ν_i is a positive constant, and $\hat{\lambda}_i$ is an estimate of $\lambda_i^* = -\|W_i^*\|^2$.

The virtual control law with an adaptive law is

$$\alpha_{i+1} = -\hat{\lambda}_i\Phi_i(\bar{x}_i)\tilde{x}_i - \nu_i\hat{\lambda}_i\tilde{x}_i - c_i s_i + \dot{x}_{id} - \hat{D}_i, \tag{18}$$

$$\dot{\hat{\lambda}}_i = \Gamma_i[-\Phi_i(\bar{x}_i)\tilde{x}_i^2 - \nu_i\tilde{x}_i^2 - \zeta_i\hat{\lambda}_i], \tag{19}$$

where $\Phi_i(\bar{x}_i) = \varphi_i^T(\bar{x}_i)\varphi_i(\bar{x}_i)/(4\gamma_i^2)$, and Γ_i, ζ_i and γ_i are positive constants.

Let α_{i+1} pass through a first-order filter to obtain an estimation of α_{i+1} denoted by $x_{(i+1)d}$

$$\eta_{i+1}\dot{x}_{(i+1)d} = \alpha_{i+1} - x_{(i+1)d}, \quad x_{(i+1)d}(0) = \alpha_{i+1}(0), \tag{20}$$

where η_{i+1} is a positive constant.

Remark 4: In the framework of designing an NDO in [34], our PNNDO includes two improvements. One is replacement of driven signals for constructing NNs, and the other is introduction of a normalization method of weights. In [34], authors construct a nonlinear disturbance observer by an error value between the intermediate variable $\hat{d}_i$ and the system state x_i. Here we employ a predictor $\hat{x}_i$, instead of the system state x_i, to calculate an error value with $\hat{d}_i$, and this error is taken as a driven signal for estimation of disturbances. This replacement yields smooth estimation behaviors. Further, by introducing a normalization method of weights, we reduce the number of learning parameters tuned online in a PNNDO. Fortunately, this normalization method provides a feasible solution for stability analysis with predictor signals.

Step n: Define the nth surface error $s_n = x_n - x_{nd}$, and the time derivative of s_n along (6) is

$$\begin{aligned}\dot{s}_n &= \dot{x}_n - \dot{x}_{nd} \\ &= F_n(\bar{x}_n, u) + u - \dot{x}_{nd} + d_n(t). \end{aligned} \tag{21}$$

From (6), (21) becomes

$$\dot{s}_n = W_n^{*T}\varphi_n(\bar{x}_n, u) + u - \dot{x}_{nd} + \varepsilon_n(\bar{x}_n, u) + d_n(t). \tag{22}$$

Define $D_n = \varepsilon_n(\bar{x}_n, u) + d_n(t)$, and D_n is called generalized disturbance, which includes external disturbance $d_n(t)$ and NNs' approximation error ε_n.

In order to eliminate the influence of D_n, the PNNDO is designed as

$$\begin{cases} \hat{D}_n = c_{dn}(\hat{x}_n - \hat{d}_n), \\ \dot{\hat{d}}_n = \hat{\lambda}_n\Phi_n(\bar{x}_n)\tilde{x}_n + u + c_{Dn}\hat{D}_n + \bar{c}_{dn}\tilde{x}_n + \nu_n\hat{\lambda}_n\tilde{x}_n, \end{cases} \tag{23}$$

where $\hat{F}_n = \hat{\lambda}_n\Phi_n(\bar{x}_n)\tilde{x}_n$, $\bar{c}_{dn} = c_{dn}^{-1} - (c_n + c_{pn})$, c_{dn}, c_{Dn}, c_{pn} and c_n are positive constants, $\tilde{x}_n = \hat{x}_n - x_n$, $\hat{x}_n$ is from the state predictor

$$\dot{\hat{x}}_n = \hat{\lambda}_n\Phi_n(\bar{x}_n)\tilde{x}_n + u - (c_n + c_{pn})\tilde{x}_n + \nu_n\hat{\lambda}_n\tilde{x}_n + \hat{D}_n, \tag{24}$$

ν_n is a positive constant, and $\hat{\lambda}_n$ is an estimate of $\lambda_n^* = -\|W_n^*\|^2$.

Choose the control input u

$$u = -\hat{\lambda}_n\Phi_n(\bar{x}_n)\tilde{x}_n - \nu_n\hat{\lambda}_n\tilde{x}_n - c_n s_n + \dot{x}_{nd} - \hat{D}_n \tag{25}$$

and the update law for $\hat{\lambda}_n$

$$\dot{\hat{\lambda}}_n = \Gamma_n[-\Phi_n(\bar{x}_n)\tilde{x}_n^2 - \nu_n\tilde{x}_n^2 - \zeta_n\hat{\lambda}_n], \tag{26}$$

where $\Phi_n(\bar{x}_n) = \varphi_n^T(\bar{x}_n)\varphi_n(\bar{x}_n)/(4\gamma_n^2)$, and Γ_n, ζ_n and γ_n are positive constants.

Remark 5: We design $\hat{\lambda}_i$ to reduce the number of NNs parameters tuned online. From $-W_i^{*T}\phi_i(\bar{x}_i)\tilde{x}_i \le -\lambda_i^*\Phi_i\tilde{x}_i^2 + \gamma_i^2$, we transform $W_i^{*T}\phi_1(\bar{x}_n)\tilde{x}_1$ into $\lambda_1^*\Phi_1\tilde{x}_1$. In the hidden layer, we normalize activation functions and stack all these functions into $\Phi_i(\bar{x}_i)$. With this technique, the number of dynamic learning parameters is reduced from $\sum_{i=1}^{n} l_i$ to n, where l_i is the number of NNs nodes used in Step i. It is worth mentioning that, although the normalization method of weights reduces the number tuned online, it may degrade approximation behaviors of NNs. For this end, we introduce a PNNDO to compensate for approximation errors from NNs.

Remark 6: NNs' learning parameters are usually updated by surface errors in dynamic surface control for a nontriangular system. At the initial stage, this surface error is often not zero, and with an overlarge adaptive gain, system overshoot may be too large and high-frequency oscillations may occur. In this paper, NNs' weight parameters are updated by prediction errors generated by the predictor instead of surface errors, and the dynamic learning ability is separated from the tracking performance. Thus, the NNs in this paper improve the transient performance even in the presence of overlarge adaptive gains.

Remark 7: In this paper, we utilize $\tilde{x}_i$ to update the NNs learning parameters instead of surfaces errors, and this replacement reduces generation of high-frequency oscillations and improves transient performance. The improvement is attributed to the following two aspects. On one hand, it is possible, by selecting the suitable parameter c_{pi}, to steer prediction error, rather than surface error in a faster manner. On the other hand, by setting $\hat{x}_i(0) = x_i(0)$, initial predictor errors are zero resulting in peaceful approximation behaviors even with an overlarge adaptive gain Γ_i during initial period. Accordingly, approximation performance remains smooth regardless of an overlarge adaptive gain.

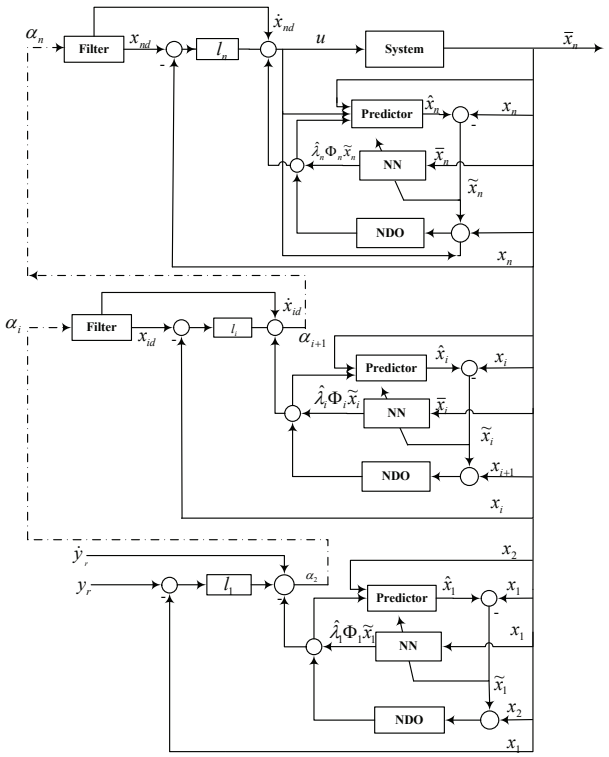


Fig. 1: The block diagram of the control strategy.

Fig. 1 shows the block diagram of the adaptive control strategy. In the n steps, unknown nonlinearities in the system are approximated by RBF NNs, the prediction errors are constructed based on the design predictor, and the adaptive parameters are updated by prediction errors. The adaptive parameters learn the norm of NNs' weights to estimate the unknown dynamics of the system, which are applied to the PNNDO to compensate for generalized disturbances, and finally the control input is obtained.

B. Stability Analysis

To facilitate stability analysis, we define the following error signals

$$\begin{cases} \tilde{\lambda}_i = \hat{\lambda}_i - \lambda_i^*, \\ \tilde{D}_i = \hat{D}_i - D_i, \\ q_{i+1} = x_{(i+1)d} - \alpha_{i+1}, \\ \tilde{x}_i = \hat{x}_i - x_i, \\ \hat{s}_i = \hat{x}_i - x_{id}, \end{cases} \tag{27}$$

and then, the error dynamics in terms of $\hat{s}_i$, $\tilde{\lambda}_i$, $\tilde{x}_i$, q_{i+1} and $\tilde{D}_i$ yield

$$\begin{cases}\dot{\hat{s}}_i = \hat{s}_{i+1} - \tilde{x}_{i+1} + q_{i+1} - c_i\hat{s}_i - c_{pi}\tilde{x}_i,\\ \dot{\hat{s}}_n = -c_n\hat{s}_n - c_{pn}\tilde{x}_n,\end{cases} \tag{28}$$

$$\dot{\tilde{\lambda}}_i = \Gamma_i[-\Phi_i(\bar{x}_i)\tilde{x}_i^2 - \zeta_i\hat{\lambda}_i - \nu_i\tilde{x}_i^2], \tag{29}$$

$$\begin{aligned}\dot{\tilde{x}}_i =&\dot{\hat{x}}_i - \dot{x}_i\\ =&\hat{\lambda}_i\Phi_i(\bar{x}_i)\tilde{x}_i - (c_i + c_{pi})\tilde{x}_i + \tilde{D}_i - W_i^{*T}\varphi_i(\bar{x}_i)\\ &+ W_i^{*T}\varphi_i(\bar{x}_i) - W_i^{*T}\varphi_i(\bar{x}) + \nu_i\hat{\lambda}_i\tilde{x}_i,\end{aligned} \tag{30}$$

$$\dot{q}_{i+1} = -\frac{q_{i+1}}{\eta_{i+1}} - B_{i+1}(\cdot), \tag{31}$$

and

$$\begin{aligned}\dot{\tilde{D}}_i &= \dot{\hat{D}}_i - \dot{D}_i\\ &= c_{di}(1 - c_{Di})\hat{D}_i - \tilde{x}_i - \dot{D}_i,\end{aligned} \tag{32}$$

where $B_{i+1}(\cdot) = \dot{\alpha}_{i+1}(\hat{s}_1, \cdots, \hat{s}_i, \tilde{x}_1, \cdots, \tilde{x}_i, \hat{\lambda}_1, \cdots, \hat{\lambda}_i, q_2, \cdots, q_{i+1}, \hat{D}_1, \cdots, \hat{D}_i, y_r, \dot{y}_r)$.

In what follows, we present the stability and analysis results.

Theorem 1: Consider a nontriangular nonlinear system suffering from external unknown disturbances defined by (28)-(32) with Assumption 1–Assumption 3. For bounded initial condition, satisfying $\sum_{i=1}^{n}(\hat{s}_i^2(0)+\frac{1}{\Gamma_i}\tilde{\lambda}_i^2(0)+\tilde{x}_i^2(0)+\tilde{D}_i^2(0))+\sum_{j=1}^{n-1} q_{i+1}^2(0) \le 2p$, where p is a positive constant, all error signals in the closed-loop system, i.e., $\hat{s}_i$, $\tilde{\lambda}_i$, $\tilde{x}_i$, $\tilde{D}_i$ and q_{i+1} are bounded, and the tracking error converges to a small neighborhood around the origin.

Proof : Consider the Lyapunov function candidate

$$V = \sum_{i=1}^{n}(\frac{1}{2}\hat{s}_i^2 + \frac{1}{2\Gamma_i}\tilde{\lambda}_i^2 + \frac{1}{2}\tilde{x}_i^2 + \frac{1}{2}\tilde{D}_i^2) + \sum_{i=1}^{n-1}\frac{1}{2}q_{i+1}^2. \tag{33}$$

The time derivative of V is

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{n-1}\hat{s}_i\Big(\hat{s}_{i+1} - \tilde{x}_{i+1} + q_{i+1} - c_i\hat{s}_i - c_{pi}\tilde{x}_i\Big)\\ &+ \hat{s}_n\Big(-c_n\hat{s}_n - c_{pn}\tilde{x}_n\Big) + \sum_{i=1}^{n}\tilde{x}_i\Big(\hat{\lambda}_i\Phi_i(\bar{x}_i)\tilde{x}_i\\ &- (c_i + c_{pi})\tilde{x}_i + \tilde{D}_i - W_i^{*T}\varphi_i(\bar{x}_i) + W_i^{*T}\varphi_i(\bar{x}_i)\\ &- W_i^{*T}\varphi_i(\bar{x}_n) + \nu_i\hat{\lambda}_i\tilde{x}_i\Big) + \sum_{i=1}^{n}\tilde{D}_i\Big(c_{di}(1 - c_{Di})\hat{D}_i\\ &- \tilde{x}_i - \dot{D}_i\Big) + \sum_{i=1}^{n}\tilde{\lambda}_i\Big[-\Phi_i(\bar{x}_i)\tilde{x}_i^2 - \zeta_i\hat{\lambda}_i - \nu_i\tilde{x}_i^2\Big]\\ &+ \sum_{i=1}^{n-1}q_{i+1}\Big(-\frac{q_{i+1}}{\eta_{i+1}} - B_{i+1}(\cdot)\Big).\end{aligned} \tag{34}$$

Define the sets $\Xi_0 = \{y_r^2 + \dot{y}_r^2 + \ddot{y}_r^2 \le p_0\}$ and $\Xi_i = \sum_{j=1}^{i}\{(\hat{s}_j^2 + \frac{1}{\Gamma_i}\tilde{\lambda}_i^2 + \tilde{x}_i^2 + \tilde{D}_i^2) + \sum_{i=1}^{n-1} q_{i-1}^2 \le 2p\}$, where p_0 and p are positive constant. Due to that Ξ_0 and Ξ_1 are compact sets in $\mathbb{R}^3$ and $\mathbb{R}^{5i-1}$, it comes the result that the set $\Xi_0 \times \Xi_1 \in \mathbb{R}^{5i+2}$ is also compact. Thus, $B_{i+1}(\cdot)$ has a upper bound M_{i+1} over $\Xi_0 \times \Xi_1$. According to Assumption 1 and boundedness of NNs approximation error, we denote D_i^* and $\breve{D}_i^*$ as upper bounds of D_i and $\dot{D}_i$, respectively. Using Young's inequality, the following inequalities $|\hat{s}_i\hat{s}_{i+1}| \le \frac{1}{2}\hat{s}_i^2 + \frac{1}{2}\hat{s}_{i+1}^2$, $|\hat{s}_i q_{i+1}| \le \frac{1}{2}\hat{s}_i^2 + \frac{1}{2}q_{i+1}^2$, $c_{pi}\hat{s}_i\tilde{x}_{i+1} \le \frac{1}{2}c_{pi}^2\hat{s}_i^2 + \frac{1}{2}\tilde{x}_{i+1}^2$, $|\tilde{D}_i\dot{D}_i| \le \frac{1}{2}\tilde{D}_i^2 + \frac{1}{2}\breve{D}_i^{*2}$, $-\tilde{\lambda}_i\hat{\lambda}_i \le -\frac{1}{2}\tilde{\lambda}_i^2 + \frac{1}{2}\lambda_i^{*2}$, $|q_{i+1}B_{i+1}| \le \frac{1}{2}q_{i+1}^2 + \frac{1}{2}M_{i+1}^2$, $-\tilde{D}_i\hat{D}_i \le -\frac{1}{2}\tilde{D}_i^2 + \frac{1}{2}D_i^{*2}$, $-W_i^{*T}\varphi(\bar{x}_i)\tilde{x}_i \le \lambda_i\Phi_i\tilde{x}_i^2 - \hat{\lambda}_i\Phi_i\tilde{x}_i^2 + \gamma_i^2$, $\tilde{x}_i(W_i^{*T}\varphi_i(\bar{x}_i) - W_i^{*T}\varphi_i(\bar{x}_n)) \le -\nu_i\tilde{x}_i^2\lambda_i^* + \frac{l_i}{\nu_i}$ hold. It follows that

$$\begin{aligned}\dot{V} \le& -(c_1 - \frac{3}{2} - \frac{c_{p1}^2}{2})\hat{s}_1^2 - \sum_{i=2}^{n}(c_i - 2 - \frac{c_{pi}^2}{2})\hat{s}_i^2\\ &- (c_1 + c_{p1} - \frac{1}{2})\tilde{x}_1^2 - \sum_{i=2}^{n-1}(c_i + c_{pi} - 1)\tilde{x}_i^2\\ &- (c_n + c_{pn} - \frac{1}{2})\tilde{x}_n^2 - \sum_{i=1}^{n}(\frac{c_{di}(c_{Di} - 1)}{2} - \frac{1}{2})\tilde{D}_i^2\\ &- \sum_{i=1}^{n}\frac{\zeta_i}{2\Gamma_i^{-1}}\Gamma_i^{-1}\tilde{\lambda}_i^2 - \sum_{i=1}^{n-1}(\frac{1}{\eta_{i+1}} - 1)q_{i+1}^2 + \Pi,\end{aligned} \tag{35}$$

where $\Pi = \frac{1}{2}\sum_{i=1}^{n}\breve{D}_i^{*2} + \frac{1}{2}\sum_{i=1}^{n}\lambda_i^{*2} + \frac{1}{2}\sum_{i=1}^{n}D_i^{*2} + \sum_{i=1}^{n}\gamma_i^2 + \sum_{i=1}^{n}\frac{l_i}{\nu_i} + \frac{1}{2}\sum_{i=1}^{n}M_{i+1}^2$.

Let $\chi_{\hat{s}} = \min_{i=1,\ldots,n}\{c_1 - \frac{3}{2} - \frac{c_{p1}^2}{2}, c_i - 2 - \frac{c_{pi}^2}{2}, c_n - \frac{c_{pn}^2}{2}\}$, $\chi_{\tilde{\lambda}} = \min_{i=1,\ldots,n}\{\frac{\zeta_i}{2\Gamma_i^{-1}}\}$, $\chi_{\tilde{x}} = \min_{i=1,\ldots,n}\{c_1 + c_{p1} - \frac{1}{2}, c_i + c_{pi} - 1, c_n + c_{pn} - \frac{1}{2}\}$, $\chi_q = \min_{i=1,\ldots,n-1}\{\frac{1}{\eta_{i+1}} - 1\}$ and $\chi_{\tilde{D}} = \min_{i=1,\ldots,n}\{\frac{c_{di}(c_{Di}-1)}{2} - \frac{1}{2}\}$. It follows that

$$\dot{V} \le -\chi V + \Pi, \tag{36}$$

where $\chi = \min\{2\chi_{\hat{s}}, 2\chi_{\tilde{\lambda}}, 2\chi_{\tilde{x}}, 2\chi_q, 2\chi_{\tilde{D}}\}$. Solving the inequality (36) gives

$$V(t) \le \frac{\Pi}{\chi} + (V(0) - \frac{\Pi}{\chi})\mathrm{e}^{-\chi t}. \tag{37}$$

From (37), we obtain the bound of the Lyapunov function V and all signals in the closed-loop system. From (33), one has $|\hat{s}_1| \le \sqrt{2\frac{\Pi}{\chi}}$ and $|\tilde{x}_1| \le \sqrt{2\frac{\Pi}{\chi}}$. Due to the definition $s_1 = \hat{s}_1 - \tilde{x}_1$, we get $|s_1| \le 2\sqrt{2\frac{\Pi}{\chi}}$. This completes this proof.

IV. PNDSC with Measurement Noises

In the previous section, with assumption that measured states are noise-free, we propose a predictor-based NDSC strategy for a nontriangular system with unknown disturbances. In fact, this scenario in Section III is ideal, and there inevitably exist noises over the measurement channel. In what follows, we will consider measurement noises and further improve control strategy in the previous section.

In this subsection, the system model (1) is redefined

$$\begin{cases}\dot{x}_{jp} = F_j(\bar{x}_{np}) + x_{(j+1)p} + d_j(t), \quad j = 1, \cdots, n-1,\\ \dot{x}_{np} = F_n(\bar{x}_{np}, u) + u + d_n(t),\\ y = x_{1p},\\ x_i(t) = x_{ip}(t) + \rho_i(t), \quad i = 1, \cdots, n,\end{cases} \tag{38}$$

where $\bar{x}_{np} = [x_{1p}, x_{2p}, \cdots, x_{np}]^T \in \mathbb{R}^n$ is the system state vector, $x_i(t)$ is measured state, $\rho_i(t)$ represents the measurement noise. We assume $\rho_i(t)$ is bounded such that $|\rho_i(t)| \leq \rho_i^*$, where ρ_i^* is a positive constant.

The control objective for (38) is to design an adaptive control strategy with measurement noise such that all signals in the closed-loop system are bounded and that the tracking error $y - y_r$ is arbitrarily small.

A. Control Strategy Design

Due to the existence of measurement noises, only measured state x_i appears in the control strategy in traditional method. Fortunately, in our control strategy, we can use predictor states $\hat{x}_{ip}$ to complete the design, where $\hat{x}_{ip}$ is the estimate of system state x_i. Define estimate error $\tilde{x}_i = \hat{x}_{ip} - x_i$. Then, the predictors, NDOs, virtual control laws and the control input can be redesigned with (38).

First, predictors are designed using measured states x_i as

$$\dot{\hat{x}}_{ip} = \hat{\lambda}_i \Phi_i(\hat{\tilde{x}}_{ip})\tilde{x}_i + x_{i+1} - (c_i + c_{pi})\tilde{x}_i + \nu_i \hat{\lambda}_i \tilde{x}_i + \hat{D}_i, \quad i = 1, \cdots, n_i - 1 \tag{39}$$

$$\dot{\hat{x}}_{np} = \hat{\lambda}_n \Phi_n(\hat{\tilde{x}}_{np})\tilde{x}_n + u - (c_n + c_{pn})\tilde{x}_n + \nu_n \hat{\lambda}_n \tilde{x}_n + \hat{D}_n. \tag{40}$$

Then, PNNDOs are designed for generalized disturbance D_i as

$$\begin{cases} \hat{D}_i = c_{di}(\hat{x}_{ip} - \hat{d}_i), \ i = 1, \cdots, n_i - 1 \\ \dot{\hat{d}}_i = \hat{\lambda}_i \Phi_i(\hat{\tilde{x}}_{ip})\tilde{x}_i + x_{i+1} + c_{Di}\hat{D}_i + \bar{c}_{di}\tilde{x}_i + \nu_i \hat{\lambda}_i \tilde{x}_i, \end{cases}$$

$$\begin{cases} \hat{D}_n = c_{dn}(\hat{x}_{np} - \hat{d}_n), \\ \dot{\hat{d}}_n = \hat{\lambda}_n \Phi_n(\hat{\tilde{x}}_{np})\tilde{x}_n + u + c_{Dn}\hat{D}_n + \bar{c}_{dn}\tilde{x}_n + \nu_n \hat{\lambda}_n \tilde{x}_n \end{cases} \tag{41}$$

with adaptive laws

$$\dot{\hat{\lambda}}_i = \Gamma_i[-\Phi_i(\hat{\tilde{x}}_{ip})\tilde{x}_i^2 - \nu_i \tilde{x}_i^2 - \zeta_i \hat{\lambda}_i], \ i = 1, \cdots, n_i. \tag{42}$$

Now, virtual control laws α_{i+1} and the control input u are chosen with predictor states $\hat{x}_i$ as

$$\alpha_{i+1} = -\hat{\lambda}_i \Phi_i(\hat{\tilde{x}}_{ip})\tilde{x}_i - \nu_i \hat{\lambda}_i \tilde{x}_i - c_i \hat{s}_i + \dot{x}_{id} - \hat{D}_i, \quad i = 1, \cdots, n_i - 1, \tag{43}$$

and

$$u = -\hat{\lambda}_n \Phi_n(\hat{\tilde{x}}_{np})\tilde{x}_n - \nu_n \hat{\lambda}_n \tilde{x}_n - c_n \hat{s}_n + \dot{x}_{nd} - \hat{D}_n, \tag{44}$$

respectively, where $\hat{s}_i = \hat{x}_{ip} - x_{id}$, $i = 1, \cdots, n_i$.

Remark 8: In design process of our control strategy in this section, we employ a predictor state $\hat{x}_{ip}$ and an auxiliary variable $\hat{s}_i$ to replace a measured state x_i and a dynamic surface error s_i, respectively. Also, we utilize predictor states instead of the measured states with noises as input vectors of NNs to approximate an unknown function. This displacement realizes the design of the control strategy indirectly using the measurement state and avoids the measurement noise directly affecting control performance. This is because the design of the predictor states has the characteristic of filter. It filters the noise in the measurement states and alleviates influence from measurement noises on the control input directly and reduces oscillations.

Algorithm 1: PNDSC with measurement noises.

Initialization: Parameters c_i, c_{pi}, ν_i, c_{di}, c_{Di}, Γ_i, and ζ_i, $i = 1, \cdots, n$;
Initial conditions $\hat{x}_{ip}(0)$, $\hat{D}_i(0)$, and $\hat{\lambda}_i(0)$, $i = 1, \cdots, n$;
Time step=0;

```
while time step < simulation time step limit do
    for each i ∈ {1, ··· , n} do
        Surface error: calculate s_i;
        PNNDO: construct D̂_i from Eq. (41);
        Predictor: construct state x̂_ip from Eq. (39);
        Prediction error: calculate x̃_i;
        for each j ∈ {1, ··· , l_i} do
            RBF NN: update φ_i(x̂̃_ip) and calculate Φ_i(x̂̃_ip);
        end
        Calculate λ̂_i Φ_i(x̂̃_ip) x̃_i;
        if i < n then
            Virtual control variable: calculate α_{i+1} from Eq. (43);
            First-order filter: let α_{i+1} pass through the filter to obtain x_{(i+1)d} and ẋ_{(i+1)d};
        else
            Control law: calculate u from Eq. (44);
        end
    end
    Increasement time step;
end
```

B. Stability Analysis

To facilitate stability analysis, we define the following error signals

$$\begin{cases} \tilde{\lambda}_i = \hat{\lambda}_i - \lambda_i^*, \\ \tilde{D}_i = \hat{D}_i - D_i, \\ q_{i+1} = x_{(i+1)d} - \alpha_{i+1}, \\ \tilde{x}_i = \hat{x}_{ip} - x_i, \\ \hat{s}_i = \hat{x}_{ip} - x_{id}, \end{cases} \tag{45}$$

and then, the error dynamics in terms of $\hat{s}_i$, $\tilde{\lambda}_i$, $\tilde{x}_i$, q_{i+1} and $\tilde{D}_i$ yield

$$\begin{cases} \dot{\hat{s}}_i = \hat{s}_{i+1} - \tilde{x}_{i+1} + q_{i+1} - c_i \hat{s}_i - (c_i + c_{pi})\tilde{x}_i, \\ \dot{\hat{s}}_n = -c_n \hat{s}_n - c_{pn}\tilde{x}_n, \end{cases} \tag{46}$$

$$\dot{\tilde{\lambda}}_i = \Gamma_i[-\Phi_i(\hat{\tilde{x}}_{ip})\tilde{x}_i^2 - \zeta_i \hat{\lambda}_i - \nu_i \tilde{x}_i^2], \tag{47}$$

$$\begin{aligned}\dot{\tilde{x}}_i =&\dot{\hat{x}}_{ip} - \dot{x}_i\\ =&\hat{\lambda}_i\Phi_i(\hat{\bar{x}}_{ip})\tilde{x}_i - (c_i + c_{pi})\tilde{x}_i + \tilde{D}_i - W_i^{*T}\varphi_i(\hat{\bar{x}}_{ip})\\ &+ W_i^{*T}\varphi_i(\hat{\bar{x}}_{ip}) - W_i^{*T}\varphi_i(\bar{x}_{np}) + \nu_i\hat{\lambda}_i\tilde{x}_i + \rho_i(t), \quad (48)\end{aligned}$$

$$\dot{q}_{i+1} = -\frac{q_{i+1}}{\eta_{i+1}} - B_{i+1}(\cdot), \quad (49)$$

and

$$\begin{aligned}\dot{\tilde{D}}_i &= \dot{\hat{D}}_i - \dot{D}_i\\ &= c_{di}(1 - c_{Di})\hat{D}_i - \tilde{x}_i - \dot{D}_i, \quad (50)\end{aligned}$$

where $B_{i+1}(\cdot) = \dot{\alpha}_{i+1}(\hat{s}_1, \cdots, \hat{s}_i, \tilde{x}_1, \cdots, \tilde{x}_i, \hat{\lambda}_1, \cdots, \hat{\lambda}_i, q_2, \cdots, q_{i+1}, \hat{D}_1, \cdots, \hat{D}_i, y_r, \dot{y}_r)$.

In what follows, we present the stability and analysis results.

Theorem 2: Consider a nontriangular nonlinear system with measurement noises defined by (46)-(50) with Assumption 1–Assumption 3. For bounded initial condition, satisfying $\sum_{i=1}^{n}(\hat{s}_i^2(0)+\frac{1}{\Gamma_i}\tilde{\lambda}_i^2(0)+\tilde{x}_i^2(0)+\tilde{D}_i^2(0))+\sum_{j=1}^{n-1} q_{i+1}^2(0) \le 2p$, where p is a positive constant, all error signals in the closed-loop system, i.e., $\hat{s}_i$, $\tilde{\lambda}_i$, $\tilde{x}_i$, $\tilde{D}_i$ and q_{i+1} are bounded, and the tracking error converges to a small neighborhood around the origin.

Proof 1: Consider the Lyapunov function candidate

$$V = \sum_{i=1}^{n}(\frac{1}{2}\hat{s}_i^2 + \frac{1}{2\Gamma_i}\tilde{\lambda}_i^2 + \frac{1}{2}\tilde{x}_i^2 + \frac{1}{2}\tilde{D}_i^2) + \sum_{i=1}^{n-1}\frac{1}{2}q_{i+1}^2. \quad (51)$$

The time derivative of V is

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{n-1}\hat{s}_i\Big(\hat{s}_{i+1} - \tilde{x}_{i+1} + q_{i+1} - c_i\hat{s}_i - (c_i + c_{pi})\tilde{x}_i\Big)\\ &+ \hat{s}_n\Big(-c_n\hat{s}_n - (c_n + c_{pn})\tilde{x}_n\Big) + \sum_{i=1}^{n}\tilde{x}_i\Big(\hat{\lambda}_i\Phi_i(\hat{\bar{x}}_{ip})\tilde{x}_i\\ &- (c_i + c_{pi})\tilde{x}_i + \tilde{D}_i - W_i^{*T}\varphi_i(\hat{\bar{x}}_{ip}) + W_i^{*T}\varphi_i(\hat{\bar{x}}_{ip}) + \rho_i(t)\\ &- W_i^{*T}\varphi_i(\bar{x}_n) + \nu_i\hat{\lambda}_i\tilde{x}_i\Big) + \sum_{i=1}^{n}\tilde{D}_i\Big(c_{di}(1 - c_{Di})\hat{D}_i\\ &- \tilde{x}_i - \dot{D}_i\Big) + \sum_{i=1}^{n}\tilde{\lambda}_i\Big[-\Phi_i(\hat{\bar{x}}_{ip})\tilde{x}_i^2 - \zeta_i\hat{\lambda}_i - \nu_i\tilde{x}_i^2\Big]\\ &+ \sum_{i=1}^{n-1}q_{i+1}\Big(-\frac{q_{i+1}}{\eta_{i+1}} - B_{i+1}(\cdot)\Big). \quad (52)\end{aligned}$$

Define the sets $\Xi_0 = \{y_r^2 + \dot{y}_r^2 + \ddot{y}_r^2 \le p_0\}$ and $\Xi_i = \sum_{j=1}^{i}\{(\hat{s}_j^2 + \frac{1}{\Gamma_i}\tilde{\lambda}_i^2 + \tilde{x}_i^2 + \tilde{D}_i^2) + \sum_{i=1}^{n-1} q_{i-1}^2 \le 2p\}$, where p_0 and p are positive constant. Due to that Ξ_0 and Ξ_1 are compact sets in $\mathbb{R}^3$ and $\mathbb{R}^{5i-1}$, it comes the result that the set $\Xi_0 \times \Xi_1 \in \mathbb{R}^{5i+2}$ is also compact. Thus, $B_{i+1}(\cdot)$ has a upper bound M_{i+1} over $\Xi_0 \times \Xi_1$. According to Assumption 1 and boundedness of NNs approximation error, we denote D_i^* and $\breve{D}_i^*$ as upper bounds of D_i and $\dot{D}_i$, respectively. Using Young's inequality, the following inequalities $|\hat{s}_i\hat{s}_{i+1}| \le \frac{1}{2}\hat{s}_i^2 + \frac{1}{2}\hat{s}_{i+1}^2$, $|\hat{s}_i q_{i+1}| \le \frac{1}{2}\hat{s}_i^2 + \frac{1}{2}q_{i+1}^2$, $(c_i + c_{pi})\hat{s}_i\tilde{x}_{i+1} \le \frac{c_i + c_{pi}}{2}\hat{s}_i^2 + \frac{c_i + c_{pi}}{2}\tilde{x}_{i+1}^2$, $|\tilde{D}_i\dot{D}_i| \le \frac{1}{2}\tilde{D}_i^2 + \frac{1}{2}\breve{D}_i^{*2}$, $-\tilde{\lambda}_i\hat{\lambda}_i \le -\frac{1}{2}\tilde{\lambda}_i^2 + \frac{1}{2}\lambda_i^{*2}$, $|q_{i+1}B_{i+1}| \le \frac{1}{2}q_{i+1}^2 + \frac{1}{2}M_{i+1}^2$, $-\tilde{D}_i\hat{D}_i \le -\frac{1}{2}\tilde{D}_i^2 + \frac{1}{2}D_i^{*2}$, $-W_i^{*T}\varphi(\hat{\bar{x}}_{ip})\tilde{x}_i \le \tilde{\lambda}_i\Phi_i\tilde{x}_i^2 - \hat{\lambda}_i\Phi_i\tilde{x}_i^2 + \gamma_i^2$, $\tilde{x}_i(W_i^{*T}\varphi_i(\hat{\bar{x}}_{ip}) - W_i^{*T}\varphi_i(\bar{x}_{np})) \le -\nu_i\tilde{x}_i^2\lambda_i^* + \frac{l}{\nu_i}$ hold. It follows that

$$\begin{aligned}\dot{V} \le& -(\frac{c_1}{2} - \frac{3}{2} - \frac{c_{p1}}{2})\hat{s}_1^2 - \sum_{i=2}^{n}(\frac{c_i}{2} - 2 - \frac{c_{pi}}{2})\hat{s}_i^2\\ &- (\frac{c_1 + c_{p1}}{2} - 1)\tilde{x}_1^2 - \sum_{i=2}^{n-1}(\frac{c_i + c_{pi}}{2} - \frac{3}{2})\tilde{x}_i^2\\ &- (\frac{c_n + c_{pn}}{2} - 1)\tilde{x}_n^2 - \sum_{i=1}^{n}(\frac{c_{di}(c_{Di} - 1)}{2} - \frac{1}{2})\tilde{D}_i^2\\ &- \sum_{i=1}^{n}\frac{\zeta_i}{2\Gamma_i^{-1}}\Gamma_i^{-1}\tilde{\lambda}_i^2 - \sum_{i=1}^{n-1}(\frac{1}{\eta_{i+1}} - 1)q_{i+1}^2 + \Pi, \quad (53)\end{aligned}$$

where $\Pi = \frac{1}{2}\sum_{i=1}^{n}\breve{D}_i^{*2} + \frac{1}{2}\sum_{i=1}^{n}\lambda_i^{*2} + \frac{1}{2}\sum_{i=1}^{n}D_i^{*2} + \sum_{i=1}^{n}\gamma_i^2 + \sum_{i=1}^{n}\frac{l}{\nu_i} + \frac{1}{2}\sum_{i=1}^{n}M_{i+1}^2 + \frac{1}{2}\sum_{i=1}^{n}\rho_i^{*2}$.

Let $\chi_{\hat{s}} = \min_{i=1,\ldots,n}\{\frac{c_1}{2} - \frac{3}{2} - \frac{c_{p1}}{2}, \frac{c_i}{2} - 2 - \frac{c_{pi}}{2}\}$, $\chi_{\tilde{\lambda}} = \min_{i=1,\ldots,n}\{\frac{\zeta}{2\Gamma_i^{-1}}\}$, $\chi_{\tilde{x}} = \min_{i=1,\ldots,n}\{\frac{c_1 + c_{p1}}{2} - 1, \frac{c_i + c_{pi}}{2} - \frac{3}{2}, \frac{c_n + c_{pn}}{2} - 1\}$, $\chi_q = \min_{i=1,\ldots,n-1}\{\frac{1}{\eta_{i+1}} - 1\}$ and $\chi_{\tilde{D}} = \min_{i=1,\ldots,n}\{\frac{c_{di}(c_{Di} - 1)}{2} - \frac{1}{2}\}$. It follows that

$$\dot{V} \le -\chi V + \Pi, \quad (54)$$

where $\chi = \min\{2\chi_{\hat{s}}, 2\chi_{\tilde{\lambda}}, 2\chi_{\tilde{x}}, 2\chi_q, 2\chi_{\tilde{D}}\}$. Solving the inequality (36) gives

$$V(t) \le \frac{\Pi}{\chi} + (V(0) - \frac{\Pi}{\chi})\mathrm{e}^{-\chi t}. \quad (55)$$

From (37), we obtain the bound of the Lyapunov function V and all signals in the closed-loop system. From(33), one has $|\hat{s}_1| \le \sqrt{2\frac{\Pi}{\chi}}$ and $|\tilde{x}_1| \le \sqrt{2\frac{\Pi}{\chi}}$. Due to the definition $s_1 = \hat{s}_1 - \tilde{x}_1 - \rho_1(t)$, we get $|s_1| \le 2\sqrt{2\frac{\Pi}{\chi}} + \rho_1^*$. This completes this proof.

Remark 9: According to the stability analysis, the magnitude of tracking error mainly depends on the values of χ and Π. We can improve tracking accuracy by increasing χ or decreasing Π. Specifically, χ can be increased by scaling design parameters up appropriately, such as such c_i, c_{pi}, c_{di}, c_{Di} and Γ_i, and, from the notation of Π, its value is generally insensitive to design parameters. In fact, due to existence of disturbances in Π, we cannot eliminate the impact from unknown disturbances completely and only alleviate disturbances impact as much as possible.

Remark 10: For the problem of measurement noises in practical systems, one of intuitive solutions is introducing an observer-based control strategy. However, due to the inapplicability of the separation principle [5], it is a tough task to apply such observer in adaptive control. Fortunately, our control strategy utilizing predictor states, instead of direct measurement states, can not only reduce the impact of measurement noises on control but also simplify stability analysis. The proposed strategy provides a feasible solution for the treatment of measurement noises.

V. Simulation Studies

In this section, we are going to illustrate the effectiveness of the proposed control strategy by a numerical example and a permanent magnet brushless DC motor system.

Example 1: We consider a three-order nonlinear system

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, x_3) + d_1(t), \\ \dot{x}_2 = f_2(x_1, x_2, x_3) + d_2(t), \\ \dot{x}_3 = f_3(x_1, x_2, x_3, u) + d_3(t), \\ y = x_1, \end{cases} \tag{56}$$

where $f_1(x_1, x_2, x_3) = 1 - \cos(x_1 + x_3) + x_3 + 2.5x_2 + 0.5\sin(x_1)\cos(x_3)x_2$, $f_2(x_1, x_2, x_3) = 0.1x_1x_2e^{x_2} + (2 + \sin(x_1x_2))x_3$, $f_3(x_1, x_2, x_3, u) = x_1x_2e^{x_2} + x_3\cos(x_1x_2) + 2u + 0.1\sin(x_1x_2x_3)u$, $d_1(t) = 1 + 2\sin(0.8t) + 0.4\cos(2t)$, $d_2(t) = 1 + 0.8\cos(0.6t) + 0.2\cos(0.4t)$ and $d_3(t) = 1.6 + \sin(2t) + 2\cos(0.1t)$. It is worth mentioning that the functions $\frac{\partial f_1}{\partial x_2} = 2.5 + 0.5\sin(x_1)\cos(x_3) > 0$, $\frac{\partial f_2}{\partial x_3} = 2 + \sin(x_1x_2) > 0$ and $\frac{\partial f_3}{\partial u} = 2 + 0.1\sin(x_1x_2x_3) > 0$ satisfy Assumption 2. The reference signal is $y_r = 0.5(\sin(t) + \sin(0.5t))$. The designed parameters are set as $\nu_i = 1$, $\Gamma_i = 10$, and $\sigma_i = 0.01$, $i = 1, 2, 3$. The initial conditions are $x_1(0) = \hat{x}_1(0) = 0.1$, $x_2(0) = \hat{x}_2(0) = 0$, and $x_3(0) = \hat{x}_3(0) = 0.2$.

The simulation results are shown in Fig. 2–Fig. 4. It is seen from Fig. 2 that the output signal y follows the reference signal y_r well. Fig. 3 shows the values of control input. The approximation performances of NNs are depicted in Fig. 4. Traditionally, in order to reduce the tracking errors, we often increase the adaptive gains, and overlarge adaptive gain may result in undesired high-frequency oscillations in NNs' learning behavior and control signals. To demonstrate the improvement of avoiding this phenomenon, a performance comparison has been made for our strategy and the traditional NN with a disturbance observer [34]. As shown in Fig. 5, the two systems have the condition that an overlarge adaptive gain is $\Gamma_i = 500$ and other parameters and initial conditions are the same. It can be seen that undesired high-frequency oscillations appear in the traditional NNs and control signals [34], and the control strategy including PNNDO proposed in this paper holds smooth approximation performance. This is because the prediction error updating NNs is used to separate the relationship between the convergence of the surface error and the unknown dynamic estimation of the system. Also, by selecting the initial value of the predictor $\hat{x}_i(0) = x_i(0)$ and adjusting the gain of the predictor κ_i, the prediction error holds faster convergence speed and thus improves the transient performance.

In order to verify the effectiveness of the proposed disturbance observer, we add an extra disturbance $d^e(t)$. As shown in Fig. 6, when $t \leq 10$s, the system is disturbance-free, $d^e(t) = 0$, and the approximation performance of NNs with/without NDO are all well. We add an external disturbance $d^e(t) = 3 + \sin(t + 2)$ during the period $10s < t \leq 20s$. It can be seen that NNs without NDO can not approximate the unknown dynamics of the system well, and approximation performance of PNNDO remains satisfied. When $t > 20s$, the disturbance will disappear with $d^e(t) = 0$ and NNs with/without NDO can approximate the unknown dynamics

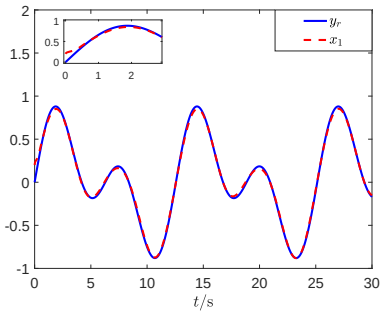


Fig. 2: Output signal y in Example 1.

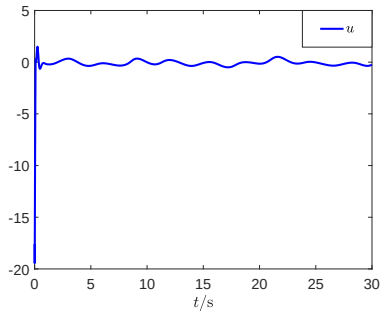


Fig. 3: Control input u in Example 1.

well. The comparison with/without our disturbance observer is shown in Fig. 6, where a gray area represents an interval that the system is subjected to additional disturbances. It can be found that if there is no disturbance observer, only NNs are not able to approximate uncertain dynamics well as the system is suddenly subjected to external disturbances. Fig. 7 shows the tracking errors with/without disturbance observers, and we can see that the tracking behavior is better with our PNNDO.

Example 2: A permanent magnet brush DC motor system [1] is conducted in this subsection. Its model is

$$\begin{cases} M\ddot{q} + B\dot{q} + N\sin(q) = I, \\ L\dot{I} = V_0 - RI - K_B\dot{q}, \end{cases} \tag{57}$$

where $N = mL_0G/(2K_\tau) + M_0L_0G/K_\tau, M = J/K_\tau + mL_0^2/(3K_{tau}) + M_0L_0^2/K_\tau + 2M_0R_0^2/(5K_\tau), B = B_0/K_\tau$,

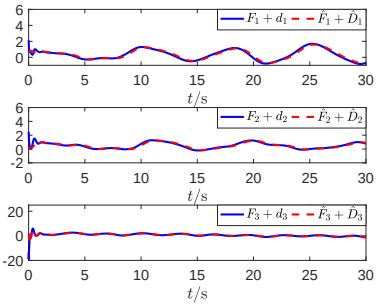
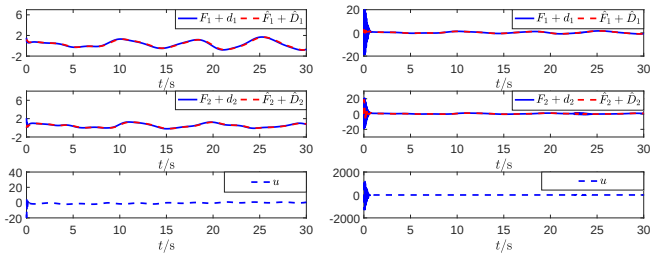
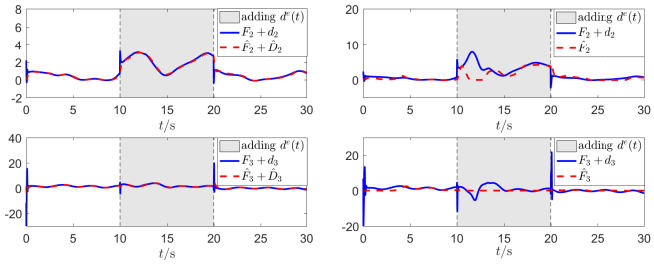


Fig. 4: Approximation performance of NNs and NDO in Example 1.



(a) with predictor (b) without predictor [34]

Fig. 5: Dynamic approximation effect of system with/without predictor in Example 1.



(a) with NDO (b) without NDO

Fig. 6: Approximation behaviors with/without NDO in Example 1.

M_0 is the load mass, J is the rotor inertia, m is the link mass, L_0 is the link length, B_0 is the coefficient of viscous friction at the joint, K_τ is the coefficient which characterizes the electromechanical conversion of armature current to torque, R_0 is the radius of the load, $q(t)$ is the angular motor position (and hence the position of the load), $I(t)$ is the motor armature current, V_0 is input control voltage, K_B is the back-emf coefficient, G is the gravity coefficient, R is the armature resistance, and L is the armature inductance. The detailed model parameters are found in [1]. Denote $x_2 = \dot{q}$, $x_3 = I$ and $u = V_0$, and we rewrite (57) as

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, x_3), \\ \dot{x}_2 = f_2(x_1, x_2, x_3), \\ \dot{x}_3 = f_3(x_1, x_2, x_3, u), \\ y = x_1, \end{cases} \tag{58}$$

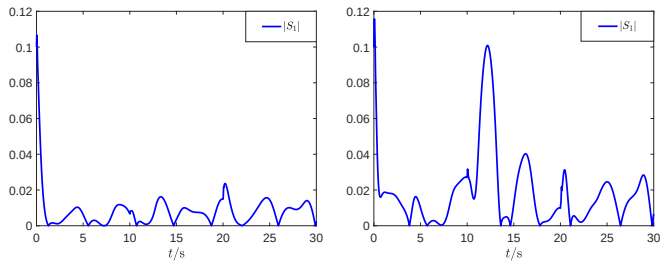
where $f_1(x_1, x_2, x_3) = x_2$, $f_2(x_1, x_2, x_3) = \frac{x_3}{M} - \frac{N}{M}\sin(x_1) - \frac{B}{M}x_2$, $f_3(x_1, x_2, x_3, u) = \frac{u}{L} - \frac{K_B}{L}x_2 - \frac{R}{L}x_3$. The reference signal is $y_r = \sin(t)$. Assume there exist uncertain dynamics in (57), where $\Delta f_1(x_1, x_2, x_3) = 0.2\sin(x_1 x_3)$, $\Delta f_2(x_1, x_2, x_3) = 0.2\exp(-x_1^4 x_3^2) + \frac{B}{M}\cos x_2 \sin x_3$, and $\Delta f_3(x_1, x_2, x_3, u) = \sin\frac{0.1\exp(-x_3^2)}{1+u^2}$. The System (58) is then rewritten as

$$\begin{cases} \dot{x}_1 = \bar{f}_1(x_1, x_2, x_3), \\ \dot{x}_2 = \bar{f}_2(x_1, x_2, x_3), \\ \dot{x}_3 = \bar{f}_3(x_1, x_2, x_3, u), \\ y = x_1, \end{cases} \tag{59}$$

where $\bar{f}_1(x_1, x_2, x_3) = f_1(x_1, x_2, x_3) + \Delta f_1(x_1, x_2, x_3)$, $\bar{f}_2(x_1, x_2, x_3) = f_2(x_1, x_2, x_3) + \Delta f_2(x_1, x_2, x_3)$, $\bar{f}_3(x_1, x_2, x_3, u) = f_3(x_1, x_2, x_3, u) + \Delta f_3(x_1, x_2, x_3, u)$. Also, the following functions $|\frac{\partial \bar{f}_1}{\partial x_2}| = 1 > 0$, $|\frac{\partial \bar{f}_2}{\partial x_3}| = 15 - 0.4x_1^4 x_3 \exp(-x_1^4 x_3^2) + 0.3\cos x_2 \cos x_3 > 0$, $\frac{\partial \bar{f}_3}{\partial u} = 40 - \frac{0.2u\exp(-x_3^2)}{(1+u^2)^2}\cos(\frac{0.1\exp(-x_3^2)}{1+u^2}) > 0$ satisfy Assumption 2.

The simulation results are shown in Fig. 8–Fig. 9, which reveal that the trajectories of tracking performance and control inputs (current and voltage). To show that updating NNs' learning parameters by the predictor can avoid high-frequency oscillation caused by selecting excessive adaptive gains, we compare the effects of traditional NNs method and control strategy we proposed with large adaptive gains $\Gamma_i = 2000$ after increasing the amplitude of disturbance after $t = 10$s, and the results are shown in Fig. 10. It can be found that under the condition of large adaptive gains, the traditional NNs method leads to high frequency oscillation after the system is suddenly disturbed, and the approximation performance of NNs and PNNDO based on the prediction error designed in this paper still remains satisfactory.

The numbers of NNs' learning parameters are shown in Tab. I. It shows that with the increase of the number of hidden layer nodes, the number of NNs' learning parameters tuned online in NNDO in [34] also increases, but the number of NNs' learning parameters in PNNDO in our control strategy remains unchanged. To show the advantage in calculation time, we compare the simulation time for our control strategy with the time for the traditional scheme in [34]. The hidden layer nodes of the NNs are all selected as 100, and the parameters of the two methods are the same. The results are listed in Tab. II. It can be seen that the control strategy in this paper needs about 0.464s for completing the task, while the time of the traditional NNs method is about 0.693s. The whole simulation running time is reduced by about 22.9%. This is because the normalization method of weights in this paper reduces the number of learning parameters tuned online.



(a) with NDO (b) without NDO

Fig. 7: Tracking performance with/without NDO in Example 1.

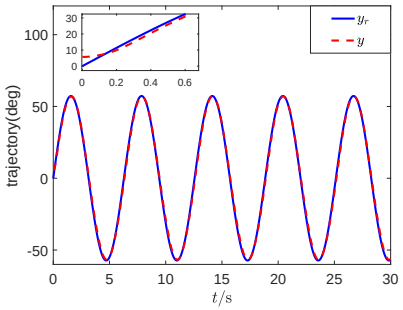


Fig. 8: Output response of y in Example 2.

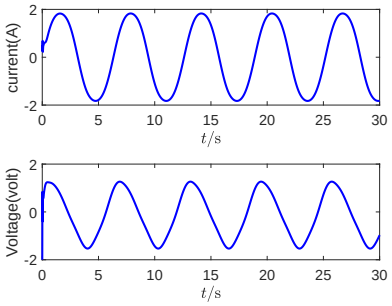


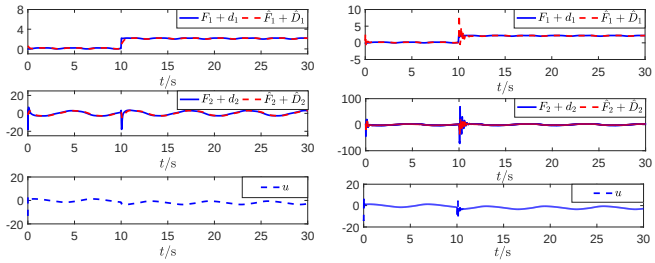
Fig. 9: Input current $I(t)$ and control voltage $V(t)$ of the motor in Example 2.

To illustrate the effectiveness of the proposed disturbance observer, we present approximation performance in the system with/without observers. The curves are depicted in Fig. 11, where the gray areas represent intervals that the system is subjected to additional external disturbances at higher magnitudes. Add an external disturbance $d^e(t) = 1 + 0.1\cos(t)$ when $10s < t \leq 20s$. When $t > 20s$, the external disturbance $d^e(t) = 3 + 2\sin(t)$. It can be seen that the approximation performance of the system with NNDO maintains well all the time, but the approximation performance of the system without NNDO will deteriorate when it is suddenly subjected to external disturbances. We conclude that a PNNDO approximates unknown dynamics more accurately even if the system is suddenly subjected to external disturbances.

We further take measurement noises in this example. The noises are generated by a 'Gaussian Noise Generator' block in Simulink. It can be seen, from Fig. 12(a), that the approximation behaviors by the traditional NDSC appear oscillations and that our control strategy mitigates oscillations effectively. This is because signals from predictors, rather than paralyzed information, are employed in NNs' input vector. Fig. 12(b) reveals that the amplitude of our control input is smaller than that of the traditional NDSC method. The comparison results indicate that, desipte of the existence of measurement noises, the activity of our input signal is reduced obviously due to improvement of approximation behaviors.

VI. Conclusion

A predictor-based neural dynamic surface control (PNDSC) strategy has been designed in this paper for a class of nontrian-



(a) with predictor in this paper (b) without predictor [34]

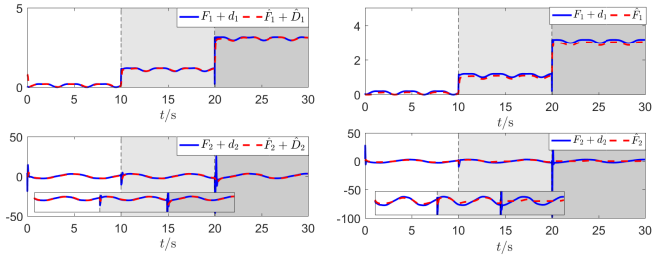
Fig. 10: Dynamic approximation with/without predictors in Example 2.

TABLE I: Comparison of the number of NNs' learning parameters.

Number of hidden layer nodes of the NNs	Number of NNs' learning parameters in NNDO in [34]	Number of NNs' learning parameters in PNNDO in this paper
10	10×3=30	3
50	50×3=150	3
100	100×3=300	3

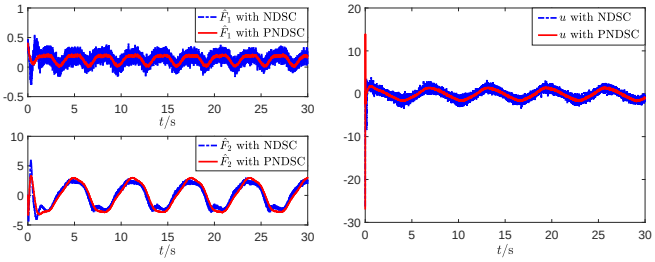
TABLE II: Comparison of simulation time of different strategies

Control strategies	l	t/s
Traditional NNs-based method [34]	100	0.693
Our strategy in this paper	100	0.464
Improved	-	22.9%



(a) with NDO (b) without NDO

Fig. 11: Approximation performances with/without NDO in Example 2.



(a) approximation performances with NDSC/PNDSC (b) control inputs with NDSC/PNDSC

Fig. 12: Approximation performances and control inputs with measurement noises in Example 2.

gular nonlinear systems with unknown disturbances. Predictor-based neural networks have been employed to approximate uncertain dynamics resulting in smooth learning behaviors without incurring high-frequency oscillations as adaptive gains are overlarge. The PNNDO then has been developed for generalized disturbances including unknown external disturbances and NNs approximation errors. This paper reduces the number of NNs parameters tuned online by introducing a normalization method of weights. Furthermore, a PNDSC strategy is discussed for a nontriangular system with measurement noises. The simulation results have verified the effectiveness of our control strategy. Future work will extend our PNDSC technique to secure control of nontriangular systems.

REFERENCES

[1] Y. Li, X. Shao and S. Tong. Adaptive fuzzy prescribed performance control of nontriangular structure nonlinear systems. IEEE Transactions on Fuzzy Systems, 2020, 28(10): 2416-2426.

[2] Y. Xu, J. Sun, G. Wang and Z. -G. Wu. Dynamic triggering mechanisms for distributed adaptive synchronization control and its application to circuit systems. IEEE Transactions on Circuits and Systems I: Regular Papers, 2021, 68(5): 2246-2256.

[3] Y. Li and S. Tong. Adaptive fuzzy output constrained control design for multi-input multioutput stochastic nonstrict-feedback nonlinear systems. IEEE Transactions on Cybernetics, 2017, 47(12): 4086-4095.

[4] B. Chen, K. Liu, X. Liu, P. Shi, C. Lin and H. Zhang. Approximation based adaptive neural control design for a class of nonlinear systems. IEEE Transactions on Cybernetics, 2014, 44(5): 610-619.

[5] Z. Peng, D. Wang and J. Wang. Predictor-based neural dynamic surface control for uncertain nonlinear systems in strict-feedback form. IEEE Transactions on Neural Networks and Learning Systems, 2017, 28(9): 2156-2167.

[6] L. Liu, T. Gao, Y. J. Liu and et al. Time-varying IBLFs-based adaptive control of uncertain nonlinear systems with full state constraints. Automatica, 2021, 129: 109595.

[7] D. Wang, and J. Huang. Neural network-based adaptive dynamic surface control for a class of uncertain nonlinear systems in strict-feedback form. IEEE Transactions on Neural Networks, 2005, 16(1): 195-202.

[8] Y. Li, Y. Liu, S. Tong. Observer-based neuro-adaptive optimized control of strict-feedback nonlinear systems with state constraints. IEEE Transactions on Neural Networks and Learning Systems, 2021.

[9] H. Wang, P. Shi, H. Li and Q. Zhou. Adaptive neural tracking control for a class of nonlinear systems with dynamic uncertainties. IEEE Transactions on Cybernetics, 2017, 47(10): 3075-3087.

[10] J. Ma, Z. Zheng and P. Li. Adaptive dynamic surface control of a class of nonlinear systems with unknown direction control gains and input saturation. IEEE Transactions on Cybernetics, 2015, 45(4): 728-741.

[11] B. Niu, H. Li, T. Qin and H. R. Karimi. Adaptive NN dynamic surface controller design for nonlinear pure-feedback switched systems with time-delays and quantized input. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2018, 48(10): 1676-1688.

[12] Y. Li and S. Tong. Adaptive neural networks prescribed performance control design for switched interconnected uncertain nonlinear systems. IEEE Transactions on Neural Networks and Learning Systems, 2018, 29(7): 3059-3068.

[13] Y. Li, J. Zhang and S. Tong. Fuzzy adaptive optimized leader-following formation control for second-order stochastic multi-agent systems. IEEE Transactions on Industrial Informatics, doi: 10.1109/TII.2021.3133927.

[14] B. Chen, X. Liu and S. Ge. Adaptive fuzzy control of a class of nonlinear systems by fuzzy approximation approach. IEEE Transactions on Fuzzy Systems, 2012, 20(6): 1012-1021.

[15] H. Wang, K. Liu, X. Liu, B. Chen and C. Lin. Neural-based adaptive output-feedback control for a class of nonstrict-feedback stochastic nonlinear systems. IEEE Transactions on Cybernetics, 2015, 45(9): 1977-1987.

[16] S. Tong, Y. Li and S. Sui. Adaptive fuzzy tracking control design for SISO uncertain nonstrict feedback nonlinear systems. IEEE Transactions on Fuzzy Systems, 2016, 24(6): 1441-1454.

[17] K. Sun, S. Mou, J. Qiu and et al. Adaptive fuzzy control for nontriangular structural stochastic switched nonlinear systems with full state constraints. IEEE Transactions on Fuzzy Systems, 2018, 27(8): 1587-1601.

[18] W. Zha, C. Qian and S. Yan. Controller design for a class of nontriangular nonlinear systems with input dependent growth rate. International Journal of Robust and Nonlinear Control, 2019, 29(5): 1325-1338.

[19] T. Li, J. Yang, S. Li and et al. Semiglobal output feedback control for uncertain nontriangular nonlinear systems with sector-bounded unknown measurement. International Journal of Robust and Nonlinear Control, 2020, 30(1): 1-16.

[20] S. Y. Oh and H. L. Choi. Robust observer-based output feedback controller for nonlinear systems with uncertain triangular and nontriangular nonlinearities and diagonal terms. International Journal of Robust and Nonlinear Control, 2019, 29(4): 1182-1200.

[21] L. Zhao, X. Chen, J. Yu and P. Shi. Output feedback-based neural adaptive finite-time containment control of non-Strict feedback nonlinear multi-agent systems. IEEE Transactions on Circuits and Systems I: Regular Papers, doi: 10.1109/TCSI.2021.3124485.

[22] H. Wang, K. Xu and J. Qiu. Event-triggered adaptive fuzzy fixed-time tracking control for a class of nonstrict-feedback nonlinear systems. IEEE Transactions on Circuits and Systems I: Regular Papers, 2021, 68(7): 3058-3068.

[23] Li Y, Ma Z and Tong S. Adaptive fuzzy fault-tolerant control of nontriangular structure nonlinear systems with error constraint. IEEE Transactions on Fuzzy Systems, 2017, 26(4): 2062-2074.

[24] X. Zhao, H. Yang, H. R. Karimi and Y. Zhu. Adaptive neural control of MIMO nonstrict-feedback nonlinear systems with time delay. IEEE Transactions on Cybernetics, 2016, 46(6): 1337-1349.

[25] L. Ma, X. Huo, X. Zhao and G. Zong. Observer-based adaptive neural tracking control for output-constrained switched MIMO nonstrictfeedback nonlinear systems with unknown dead zone. Nonlinear Dynamics, 2020, 99: 1019-1036.

[26] S. Sui, C. L. P. Chen and S. Tong. Fuzzy adaptive finite-time control design for nontriangular stochastic nonlinear systems. IEEE Transactions on Fuzzy Systems, 2018, 27(1): 172-184.

[27] T. Yucelen and W. M. Haddad. Low-frequency learning and fast adaptation in model reference adaptive control. IEEE Transactions on Automatic Control, 2013, 58(4): 1080-1085.

[28] W. Wang and S. Tong. Adaptive fuzzy bounded control for consensus of multiple strict-feedback nonlinear systems. IEEE Transactions on Cybernetics, 2018, 48(2): 522-531.

[29] B. Xu. Composite learning finite-time control with application to quadrotors. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2018, 48(10):1806-1815.

[30] F. Liu, Y. Li and H. Yang. Consensus for leader-follower multi-agent systems with mismatched disturbance and correlated uncertainties. Information and Control, 2018, 47(1): 111-118, 128.

[31] H. Wang, W. Bai, X. Zhao and P. X. Liu. Finite-time-prescribed performance-based adaptive fuzzy control for strict-feedback nonlinear systems with dynamic uncertainty and actuator faults. IEEE Transactions on Cybernetics, doi: 10.1109/TCYB.2020.3046316.

[32] W. Chen, J. Yang, L. Guo and S. Li. Disturbance-observer-based control and related methodsłan overview. IEEE Transactions on Industrial Electronics, 2016, 63(2): 1083-1095.

[33] F. Liu, H. Yang, Y. Yang and et al. Finite-time integral sliding-mode control for multi-agent systems with mismatched disturbances. Acta Automatica Sinica, 2019, 45(4): 749-758.

[34] M. Chen and S. Ge. Adaptive neural output feedback control of uncertain nonlinear systems with unknown hysteresis using disturbance observer. IEEE Transactions on Industrial Electronics, 2015, 62(12): 7706-7716.

[35] M. Chen, P. Shi and C. Lim. Robust constrained control for MIMO nonlinear systems based on disturbance observer. IEEE Transactions on Automatic Control, 2015, 60(12): 3281-3286.

[36] S. Shao, M. Chen and Y. Zhang. Adaptive discrete-time flight control using disturbance observer and neural networks. IEEE Transactions on Neural Networks and Learning Systems, 2019, 30(12): 3708-3721.

[37] B. Xu. Composite learning control of flexible-link manipulator using NN and DOB. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2018, 48(11): 1979-1985.

[38] W. Wang, H. Liang, Y. Pan and T. Li. Prescribed performance adaptive fuzzy containment control for nonlinear multiagent systems using disturbance observer. IEEE Transactions on Cybernetics, 2020, 50(9): 3879-3891.