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Author

Saen, Reza Farzipoor

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Reza Farzipoor Saen

Department of Industrial Management, Faculty of Management and Accounting, Islamic Azad University - Karaj Branch, Karaj, Iran, P. O. Box: 31485-313

Tel: 0098 (261) 4418144-6
Fax: 0098 (261) 4418156
E-mail: farzipour@yahoo.com

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Abstract

Purchasing is one of the most crucial and vital activities of business, as it has a significant impact on finance, operations and competitiveness of the organization. Many organizations are now allocating more resources for outsourcing activities to increase their competitive position. Selecting an appropriate supplier for outsourcing is now one of the most important decisions of the purchasing department. This decision generally depends on a number of different criteria. On the other hand, to encourage the buyers to order more, suppliers usually offer volume discounts. To rank suppliers in volume discount environments in the presence of both cardinal and ordinal data, this paper proposes an innovative algorithm, which is based on Minimax Regret-based Approach (MRA). A numerical example demonstrates the application of the proposed method.

Keywords: Imprecise data envelopment analysis, Supplier ranking, Minimax regret-based approach, Volume discount, Supply chain

1. Introduction

Today, manufacturing companies are facing intense global competition and consequently an incredible pressure to reduce the cost and development time of a new product. It is well known that a substantial proportion of the cost of a typical engineering product is accounted for in raw material, components and other supplies; on average, manufacturers' purchases of goods and services amounts to 55% of revenue (Akarte et al. 2001). Purchasing is thus one of the most crucial and vital activities of business, as it has a significant impact on finance, operations and competitiveness of the organization. In this context, outsourcing is rapidly gaining importance due to a number of reasons related to cost, core competence and managerial complexities of organization and activity specialization. Therefore, many organizations are now allocating more resources for outsourcing activities to increase their competitive position. This is mainly achieved by a judicious combination of in-house manufacturing and outsourcing while preserving the core competencies of the organization. Supplier ranking for outsourcing is now one of the most important decisions of the purchasing department, as it has to fulfill the strategic goals apart from operational requirements of the organization.

Traditional Data Envelopment Analysis (DEA) models require crisp input/output data. However, in real-world problems, especially in supplier selection problems, the inputs and outputs are often imprecise.

Suppliers use discounts as a marketing strategy to secure more purchase from their customers or as a response to competition. The supplier tries to influence the buyer's ordering behavior by offering a lower unit price to encourage larger orders. Moreover, large orders reduce the per-unit order processing and manufacturing set-up costs for the supplier. In business dealing, the buyer controls when to replenish stock and how much quantity to replenish to minimize total relevant cost. When the buyer places a large order taking the advantage of price discount, per-unit ordering cost is also reduced and he/she pays less price per unit item (Das et al. 2004).

The objective of this paper is to propose an innovative algorithm for ranking suppliers in volume discount environments in the presence of both cardinal and ordinal data, which is based on Imprecise Data Envelopment Analysis (IDEA).

This paper proceeds as follows. In Section 2, literature review is presented. In Section 3, the algorithm which ranks the suppliers is introduced. Numerical example and concluding remarks are discussed in Sections 4 and 5, respectively.

2. Literature review

Various studies on the supplier selection are briefly summarized in the following subsections.

2.1 Integer Programming

Zeng et al. (2006) considered a simplified partner selection problem which takes into account only the bid cost and the bid completion time of subprojects, and the due date the project. They modeled the problem as a nonlinear integer programming problem and proved that the decision problem of the partner selection problem is NP-complete. Then they analyzed some properties of the partner selection problem and construct a branch and bound algorithm. Ghodsypour and O'Brien (2001) developed a mixed integer nonlinear programming model to solve the multiple sourcing problems, which takes into account the total cost of logistics, including net price, storage, transportation and ordering costs. The model should be run 2^n times for n suppliers that is burdensome. Talluri and Baker (2002) presented a multi-phase mathematical programming approach for effective supply chain design. More specifically, they developed and applied a combination of multi-criteria efficiency models, based on game theory concepts, and linear and integer programming methods. Talluri and Narasimhan (2003) proposed a max-min productivity based approach that derives variability measures of vendor performance, which are then utilized in a nonparametric statistical technique in identifying vendor groups for effective selection.

2.2 Goal programming

To solve the vendor selection problem with multiple objectives, Kumar et al. (2004) applied fuzzy goal programming approach. To incorporate the imprecise aspiration levels of the goals, they formulated a vendor selection problem as a fuzzy mixed integer goal programming that includes three primary goals: minimizing the net cost, minimizing the net rejections, and minimizing the net late deliveries subject to realistic constraints regarding buyer's demand, vendor's capacity, vendor's quota flexibility, purchasing value of items, budget allocation to individual vendor, etc. Çebi and Bayraktar (2003) proposed an integrated model for supplier selection. In their model, supplier selection problem has been structured as an integrated Lexicographic Goal Programming (LGP) and Analytic Hierarchy Process (AHP) model including both quantitative and qualitative conflicting factors. Cakravastia and Takahashi (2004) proposed a multi-objective model to support the process of supplier selection and negotiation that considers the effect of these decisions on the manufacturing plan. The model also takes into account several theoretical concepts in the negotiation process: concession force, resistance force and effective alternatives. Karpak et al. (2001) presented one of the "user-friendly" multiple criteria decision support systems-Visual Interactive Goal programming (VIG). VIG facilitates the introduction of a decision support vehicle that helps improve the supplier selection decisions. Kameshwaran et al. (2007) provided a multiattribute e-procurement system for procuring large volume of a single item. Their system is formulated as a mixed linear integer multiple criteria optimization problem and Goal Programming (GP) is used as the solution technique.

However, one of the GP problems arises from a specific technical requirement. After the purchasing managers specify the goals for each selected criterion (e.g., amount of price, quality level, etc), they must decide on a preemptive priority order of these goals, i.e., determining in which order the goals will be attained. Frequently such a priori input might not produce an acceptable solution and the priority structure may be altered to resolve the problem once more. In this fashion, it may be possible to generate a solution iteratively that finally satisfies the Decision Maker (DM). Unfortunately, the number of potential priority reorderings may be very large. A supplier selection problem with five factors has up to 120 priority reorderings. Going through such a laborious process would be costly and inefficient.

2.3 Analytic hierarchy process

Noorul Haq and Kannan (2006) developed the integrated qualitative decision making of the supplier selection model using fuzzy AHP with that of the quantitative mathematical model for the distribution inventory supply chain using a Genetic Algorithm (GA) to the built-to-order environment. Sha and Che (2006) presented a multi-phased mathematical approach called the

Hybrid Multi-phased-based Genetic Algorithm (HMGA) for network design of supply chain. From the point of network design, the important issues are to find suitable and quality companies, and to decide upon an appropriate production/distribution strategy. It is based on various methodologies that embrace GAs, AHP, and the Multi-Attribute Utility Theory (MAUT) to simultaneously satisfy the preferences of suppliers and customers at each level of the supply chain network. Dulmin and Mininno (2003) presented a proposal for applying a decision model to the final vendor-rating phase of a process of supplier selection. Their model uses a Multiple Criteria Decision Aid (MCDA) technique (PROMETHEE 1 and 2), with a high-dimensional sensitivity analysis approach. They tried to explain how an outranking method and PROMETHEE/GAIA techniques, provides powerful tools to rank alternatives and analyzed the relations between criteria or between DMs. Bhutta and Huq (2002) illustrated Total Cost of Ownership (TCO) and AHP approaches and provided a comparison. They concluded that TCO is better suited to those situations where cost is of high priority and detailed cost data are available to make comparisons. In the case of AHP, it is better suited to solve and decide between suppliers when several conflicting goals exist and, though cost may be an important factor, it is not the overriding one. To decide the total ranking of the suppliers, Liu and Hai (2005) compared the weighted sum of the selection number of rank vote, after determining the weights in a selected rank. They presented a novel weighting procedure in place of pairwise comparison of AHP for selecting suppliers. They provided a simpler method than AHP that is called voting analytic hierarchy process, but which do not lose the systematic approach of deriving the weights to be used and for scoring the performance of suppliers.

However, AHP has two main weaknesses. First subjectivity of AHP is a weakness. Second AHP could not include interrelationship within the criteria in the model.

2.4 Fuzzy mathematical programming

Ohdar and Ray (2004) evaluated the supplier's performance by adopting an evolutionary fuzzy system. One of the key considerations in designing the proposed system is the generation of fuzzy rules. A genetic algorithm-based methodology is developed to evolve the optimal set of fuzzy rule base, and a fuzzy inference system of the MATLAB fuzzy logic toolbox is used to assess the supplier's performance. Chen et al. (2006) presented a fuzzy decision making approach to deal with the supplier selection problem in supply chain system. They used linguistic values to assess the ratings and weights for the criteria. These linguistic ratings can be expressed in trapezoidal or triangular fuzzy numbers. Then, a hierarchy Multiple Criteria Decision Making (MCDM) model based on fuzzy sets theory is proposed to deal with the supplier selection problems in the supply chain system. According to the concept of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), a closeness coefficient is defined to determine the ranking order of all suppliers by calculating the distances to the both Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) simultaneously. Chang et al. (2006) proposed a Fuzzy Multiple Attribute Decision Making (FMADM) method based on the fuzzy linguistic quantifier. However, their proposed method suffers from two main limitations. First, the proposed method does not consider the inputs. Second, the paper does not discuss whether a DM exerts any influence on mental cognition and experiential characteristics when rating the linguistic intervals scale. Li et al. (2007) presented a grey-based approach to deal with the problem of selecting suppliers under an uncertain environment.

2.5 Data envelopment analysis

Because of the complexity of the decision making process involved in supplier selection, all the aforementioned literature relied on some form of procedures that assigns weights to various performance measures. The primary problem associated with arbitrary weights is that they are subjective, and it is often a difficult task for the DM to accurately assign numbers to preferences. It is a daunting task for the DM to assess weighting information as the number of performance criteria increased. Therefore, a more robust mathematical technique that does not demand too much and too precise information, i.e., ordinal preferences instead of cardinal weights, from the DM can strengthen the supplier evaluation process. To this end, Weber (1996) demonstrated how DEA can be used to evaluate vendors on multiple criteria and identified benchmark values which can then be used for this purpose. Weber et al. (2000) presented an approach for evaluating the number of vendors to employ in a procurement situation using Multi-Objective Programming (MOP) and DEA. The approach advocates developing vendor-order quantity solutions (referred to as supervendors) using MOP and then evaluating the efficiency of these supervendors on multiple criteria using DEA. To evaluate the aggregate performances of suppliers, Liu et al. (2000) proposed to employ DEA. This extends Weber's (1996) research in using DEA in supplier evaluation for an individual product. Forker and Mendez (2001) proposed an analytical method for benchmarking using DEA that can help companies identify their most efficient suppliers, the suppliers among the most efficient with the most widely applicable Total Quality Management (TQM) programs, and those suppliers who are not on the efficient frontier but who could move toward it by emulating the practices of their “best peer” supplier(s). Talluri and Sarkis (2002) focused upon the supplier performance evaluation and monitoring process, which assist in maintaining effective customer-supplier linkages. They tried to improve the discriminatory power of BCC model proposed by Banker et al. (1984).

Recently, to select the best suppliers in the presence of both cardinal and ordinal data, Farzipoor Saen (2007) proposed a method, which is based on IDEA. However, he did not introduce any method for ranking the best suppliers in the presence of ties among efficient suppliers. As well, Farzipoor Saen and Zohrehbandian (2008) presented a super-efficiency model for ranking the suppliers in the presence of volume discount. However, the proposed model suffers from infeasibility problem. Meanwhile, their model did not consider both cardinal and ordinal data. Again, Farzipoor Saen (2009) introduced a model which selects the best suppliers in the presence of cardinal and ordinal data in the conditions that they offer volume discounts. But, he did not propose any solution for ranking the best suppliers.

Nevertheless, all of the abovementioned references do not deal with supplier ranking in volume discount environments in the presence of both cardinal and ordinal data.

While extensive research on economic order quantities with quantity discounts exists, only a few methods address the problem from the perspective of supplier selection and ranking. Dahel (2003) presented a multiobjective mixed integer programming approach to simultaneously determine the number of vendors to employ and the order quantities to allocate to these vendors in a multiple-product, multiple-supplier competitive sourcing environment. Arunkumar et al. (2006) proposed a GP model for supplier selection with quantity discounts. They converted the piecewise linear problem into an easier linear problem, thereby decreasing the complexity of the problem. Xia and Wu (2007) proposed an integrated approach of AHP improved by rough sets theory and multi-objective mixed integer programming to simultaneously determine the number of suppliers to employ and the order quantity allocated to these suppliers in the case of multiple sourcing, multiple products, with multiple criteria and with supplier's capacity constraints.

To the best of author's knowledge, there isn't any reference that deals with supplier ranking in volume discount environments in the presence of both cardinal and ordinal data.

In a nutshell, the presented approach in this paper has some distinctive contributions:

- The introduced approach does not need exact weights from the DM.
- Cardinal and ordinal data, and volume discounts assumption are considered concurrently. This aids DMs to select suppliers using a thorough approach that goes beyond just purchase prices.
- In the case of ties among efficient suppliers, a ranking procedure is introduced to break the ties.

3. Proposed algorithm for ranking suppliers

Imprecise data implies that some data are known only to the extent that the true values lie within prescribed bounds while other data are known only in terms of ordinal relations. Cook et al. (1993, 1996) showed how DEA could be extended to treat ordinal data. To deal with all aspects of imprecise data in DEA, Cooper et al. (1999) proposed a body of concepts and methods that go by the name of IDEA. There have since been a number of refinements, extensions, and applications (Cooper et al. 2001a,b; Kim et al., 1999).

Outcome of DEA models is an efficiency score equal to one to efficient DMUs and less than one to inefficient DMUs. Therefore, for inefficient DMUs a ranking is given but efficient DMUs can not be ranked. One problem that has been discussed frequently in the DMUs ranking literature, has been the lack of discrimination in DEA applications, in particular when there are insufficient DMUs or the number of inputs and outputs is too high relative to the number of DMUs. This paper proposes to use a formulation called "Minimax Regret-based Approach" (MRA) to rank the suppliers. There have been many researches on ranking efficient DMUs that some of them are as follow:

Jahanshahloo et al. (2005a) using Monte Carlo method, developed a method which is able to rank all efficient (extreme and non-extreme) DMUs. Jahanshahloo et al. (2005b) introduced a method for ranking of DMUs using Common Set of Weights (CSW). Jahanshahloo and Afzalinejad (2006) suggested a ranking method which basically differs from previous methods. In this ranking method, DMUs are compared against a full-inefficient frontier. This method can be used to rank all DMUs to get analytic information about the system, and also to rank only efficient DMUs to discriminate between them. Amirteimoori et al. (2005) described a new DEA ranking approach that uses l_2-norm. Jahanshahloo et al. (2006) showed that the technique used for rendering model of Mehrabian et al. (1999) unit-invariant causes the ranking to change when some inputs of some inefficient DMUs change, without causing any change in the new Production Possibility Set (PPS). They modified model of Mehrabian et al. (1999) so that this problem will not occur. Saati et al. (2001) suggested a modification for model of Mehrabian et al. (1999) and proved that the modified version is always feasible and the ranking lies in (0, 1]. Unlike the previous models, model of Saati et al. (2001) is both input and output oriented, simultaneously. However, all the aforementioned references do not provide a ranking model of DMUs in the presence of both ordinal and cardinal data. In this section, the model which can rank the suppliers in the presence of both ordinal and cardinal data in volume discount environment is presented.

Table 1 lists the nomenclatures used to formulate the problem under consideration.

Table 1. The nomenclatures

Problem parameters

$j = 1,\ldots,n$ collection of suppliers (DMUs)
j_o , DMU_o = the DMU under evaluation

c_{jk} = unit price quoted by supplier j in discount interval k

d_j = discount interval offered by supplier j

$r = 1,\ldots,s$ the index of outputs
$i = 1,\ldots,m$ the index of inputs
y_{rj} = the rth output of jth DMU
x_{ij} = the ith input of jth DMU
y_{rj}^L, x_{ij}^L = lower bound
y_{rj}^U, x_{ij}^U = upper bound
θ_{jo}^U = the best possible relative efficiency achieved by DMU$_o$
θ_{jo}^L = the lower bound of the best possible relative efficiency of DMU$_o$
χ_r and η_i = the parameters on the degree of preference intensity provided by decision maker
σ_r = small positive number reflecting the ratio of the possible minimum of $\{y_{rj} | \, j=1,\ldots, n\}$ to its possible maximum
$\hat{y}_{rj}$ = the rth output of jth DMU after scale transformation

Decision variables
u_r = weight of the rth output
v_i = weight of the ith input

Suppose that there are n suppliers (DMUs) to be evaluated which produce multiple outputs y_{rj} (r = 1, 2, …, s), by utilizing multiple inputs x_{ij} (i = 1, 2, …, m). Without loss of generality, it is assumed that all the input and output data x_{ij} and y_{rj} (i=1, …, m; r=1, …, s; j=1, …, n) cannot be exactly obtained due to the existence of uncertainty. They are only known to lie within the upper and lower bounds represented by the intervals $\left[x_{ij}^L, x_{ij}^U\right]$ and $\left[y_{rj}^L, y_{rj}^U\right]$, where $x_{ij}^L > 0$ and $y_{rj}^L > 0$.

In order to deal with such an uncertain situation, the following pair of linear programming models has been developed to generate the upper and lower bounds of interval efficiency for each DMU (Wang et al. 2005).

$$
\begin{aligned}
& Max\theta_{jo}^{U} = \sum_{r=1}^{s} u_r y_{rj_o}^{U} \\
& s.t. \\
& \qquad \sum_{i=1}^{m} v_i x_{ij_o}^{L} = 1, \\
& \qquad \sum_{r=1}^{s} u_r y_{rj}^{U} - \sum_{i=1}^{m} v_i x_{ij}^{L} \le 0, \qquad j = 1, \cdots, n, \\
& \qquad u_r, v_i \ge \varepsilon \qquad \forall r, i.
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
& Max\theta_{jo}^{L} = \sum_{r=1}^{s} u_r y_{rj_o}^{L} \\
& s.t. \\
& \qquad \sum_{i=1}^{m} v_i x_{ij_o}^{U} = 1, \\
& \qquad \sum_{r=1}^{s} u_r y_{rj}^{U} - \sum_{i=1}^{m} v_i x_{ij}^{L} \le 0, \qquad j = 1, \cdots, n, \\
& \qquad u_r, v_i \ge \varepsilon \qquad \forall r, i.
\end{aligned} \tag{2}
$$

where j_o is the DMU under evaluation (usually denoted by DMU$_o$); u_r and v_i are the weights assigned to the outputs and inputs; θ_{jo}^{U} stands for the best possible relative efficiency achieved by DMU$_o$ when all the DMUs are in the state of best production activity, while θ_{jo}^{L} stands for the lower bound of the best possible relative efficiency of DMU$_o$. They constitute a possible best relative efficiency interval $\left[\theta_{jo}^{L}, \theta_{jo}^{U}\right]$. ε is the non-Archimedean infinitesimal.

For more details on proof and verification of the Models (1) and (2), as well definitions 1 & 2, please see Wang et al. (2005). As well, the mathematical background of interval arithmetic has been given in Appendix A.

In order to judge whether a DMU is DEA efficient or not, the following definition is given.

Definition 1. A DMU, DMU$_o$, is said to be DEA efficient if its best possible upper bound efficiency $\theta_{jo}^{U*} = 1$; otherwise, it is said to be DEA inefficient if $\theta_{jo}^{U*} < 1$.

Now, the method of transforming ordinal preference information into interval data is discussed, so that the interval DEA models presented in this paper can still work properly even in these situations (Wang et al. 2005).

Suppose some input and/or output data for DMUs are given in the form of ordinal preference information. Usually, there may exist three types of ordinal preference information: (1) strong ordinal preference information such as $y_{rj} > y_{rk}$ or $x_{ij} > x_{ik}$, which can be further expressed as

$y_{rj} \geq \chi_r y_{rk}$ and $x_{ij} \geq \eta_i x_{ik}$, where $\chi_r > 1$ and $\eta_i > 1$ are the parameters on the degree of preference intensity provided by decision maker; (2) weak ordinal preference information such as $y_{rp} \geq y_{rq}$ or $x_{ip} \geq x_{iq}$; (3) indifference relationship such as $y_{rl} = y_{rt}$ or $x_{il} = x_{it}$. Since DEA model has the property of unit-invariance, the use of scale transformation to ordinal preference information does not change the original ordinal relationships and has no effect on the efficiencies of DMUs. Therefore, it is possible to conduct a scale transformation to every ordinal input and output index so that its best ordinal datum is less than or equal to unity and then give an interval estimate for each ordinal datum.

Now, consider the transformation of ordinal preference information about the output y_{rj} ($j=1,\ldots, n$) for example. The ordinal preference information about input and other output data can be converted in the same way.

For weak ordinal preference information $y_{r1} \geq y_{r2} \geq \cdots \geq y_{rn}$, we have the following ordinal relationships after scale transformation:

$$1 \geq \hat{y}_{r1} \geq \hat{y}_{r2} \geq \cdots \geq \hat{y}_{rn} \geq \sigma_r,$$

where σ_r is a small positive number reflecting the ratio of the possible minimum of $\{y_{rj} | j=1,\ldots, n\}$ to its possible maximum. It can be approximately estimated by the decision maker. It is referred as the ratio parameter for convenience. The resultant permissible interval for each $\hat{y}_{rj}$ is given by

$$\hat{y}_{rj} \in [\sigma_r, 1], \qquad j = 1, \cdots, n.$$

For strong ordinal preference information $y_{r1} > y_{r2} > \cdots > y_{rn}$, there is the following ordinal relationships after scale transformation:

$$1 \geq \hat{y}_{r1}, \qquad \hat{y}_{rj} \geq \chi_r \hat{y}_{r,j+1} \quad (j = 1, \cdots, n-1) \qquad \text{and} \qquad \hat{y}_{rn} \geq \sigma_r,$$

where χ_r is a preference intensity parameter satisfying χ_r>1 provided by the decision maker and σ_r is the ratio parameter also provided by the decision maker. The resultant permissible interval for each $\hat{y}_{rj}$ can be derived as follows:

$$\hat{y}_{rj} \in \left[\sigma_r \chi_r^{n-j}, \chi_r^{1-j}\right], \qquad j = 1, \cdots, n \quad \text{with} \quad \sigma_r \leq \chi_r^{1-n}.$$

Finally, for indifference relationship, the permissible intervals are the same as those obtained for weak ordinal preference information.

Through the scale transformation above and the estimation of permissible intervals, all the ordinal preference information is converted into interval data and can thus be incorporated into interval DEA models.

In interval efficiency assessment, since the final efficiency score for each DMU is characterized by an interval, a simple yet practical ranking approach is thus needed for ranking the efficiencies of different DMUs. Here the MRA developed by Wang et al. (2005) is introduced. The approach is summarized as follows:

Let $A_i = [a_i^L, a_i^U] = \langle m(A_i), w(A_i) \rangle$ $(i = 1, \cdots, n)$ be the efficiency intervals of n DMUs, where $m(A_i) = \frac{1}{2}(a_i^U + a_i^L)$ and $w(A_i) = \frac{1}{2}(a_i^U - a_i^L)$ are their midpoints (centers) and widths. Without loss of generality, suppose $A_i = [a_i^L, a_i^U]$ is chosen as the best efficiency interval. Let $b = \max_{j \neq i} \{a_j^U\}$. Obviously, if $a_i^L < b$, the decision maker might suffer the loss of efficiency (also called the loss of opportunity or regret) and feel regret. The maximum loss of efficiency he/she might suffer is given by

$$\max(r_i) = b - a_i^L = \max_{j \neq i} \{a_j^U\} - a_i^L.$$

If $a_i^L \geq b$, the decision maker will definitely suffer no loss of efficiency and feel no regret. In this situation, his/her regret is defined to be zero, i.e. $r_i = 0$. Combining the above two situations, there is

$$\max(r_i) = \max\left[\max_{j \neq i}(a_j^U) - a_i^L, 0 \right].$$

Thus, the minimax regret criterion will choose the efficiency interval satisfying the following condition as the best (most desirable) efficiency interval:

$$\min_i \{\max(r_i)\} = \min_i \left\{ \max\left[\max_{j \neq i}(a_j^U) - a_i^L, 0 \right] \right\}.$$

Based on the analysis above, the following definition for ranking efficiency intervals is given.

Definition 2. Let $A_i = [a_i^L, a_i^U] = \langle m(A_i), w(A_i) \rangle$ $(i = 1, \cdots, n)$ be a set of efficiency intervals. The maximum loss of efficiency (also called maximum regret) of each efficiency interval A_i is defined as

$$R(A_i) = \max\left[\max_{j \neq i}(a_j^U) - a_i^L, 0 \right] = \max\left[\max_{j \neq i} \{m(A_j) + w(A_j)\} - (m(A_i) - w(A_i)), 0 \right], \qquad i = 1, \cdots, n.$$

It is evident that the efficiency interval with the smallest maximum loss of efficiency is the most desirable efficiency interval.

To be able to generate a ranking for a set of efficiency intervals using the maximum losses of efficiency, the following eliminating steps are suggested:

Algorithm A:

Step 1: Calculate the maximum loss of efficiency of each efficiency interval and choose a most desirable efficiency interval that has the smallest maximum loss of efficiency (regret). Suppose A_{i_1} is selected, where $1 \le i_1 \le n$.

Step 2: Eliminate A_{i_1} from the consideration, recalculate the maximum loss of efficiency of every efficiency interval and determine a most desirable efficiency interval from the remaining (*n*-1) efficiency intervals. Suppose A_{i_2} is chosen, where $1 \le i_2 \le n$ but $i_2 \ne i_1$.

Step 3: Eliminate A_{i_2} from the further consideration, re-compute the maximum loss of efficiency of every efficiency interval and determine a most desirable efficiency interval A_{i_3} from the remaining (*n*-2) efficiency intervals.

Step 4: Repeat the above eliminating process until only one efficiency interval A_{i_n} is left. The final ranking is $A_{i_1} \succ A_{i_2} \succ \cdots \succ A_{i_n}$, where the symbol $"\succ"$ means “is superior to”.

The above ranking approach is referred to as the MRA. Now, consider a procurement situation that suppliers provide different levels of price, product quality, delivery performance, etc. Also, depending on the buyer's purchase quantity, supplier *j* offers a volume discount having d_j discount intervals according to business volume.

At this point, the algorithm that deals with supplier ranking in volume discount environments in the presence of both cardinal and ordinal data is presented (Farzipoor Saen 2008, Farzipoor Saen and Zohrehbandian 2008).

Algorithm B

Step 1. Determine quantity of demand: in this step the buyer determines his demand quantity of material.

Step 2. Determine intersections of price breaks of all suppliers: in this step, for each supplier, piecewise linear function of material price is partitioned so that the material price of each supplier in the related interval becomes a constant parameter, i.e., intersections of material price breaks of all suppliers are computed. Assume that there are three suppliers. Fig. 1 shows this step graphically.

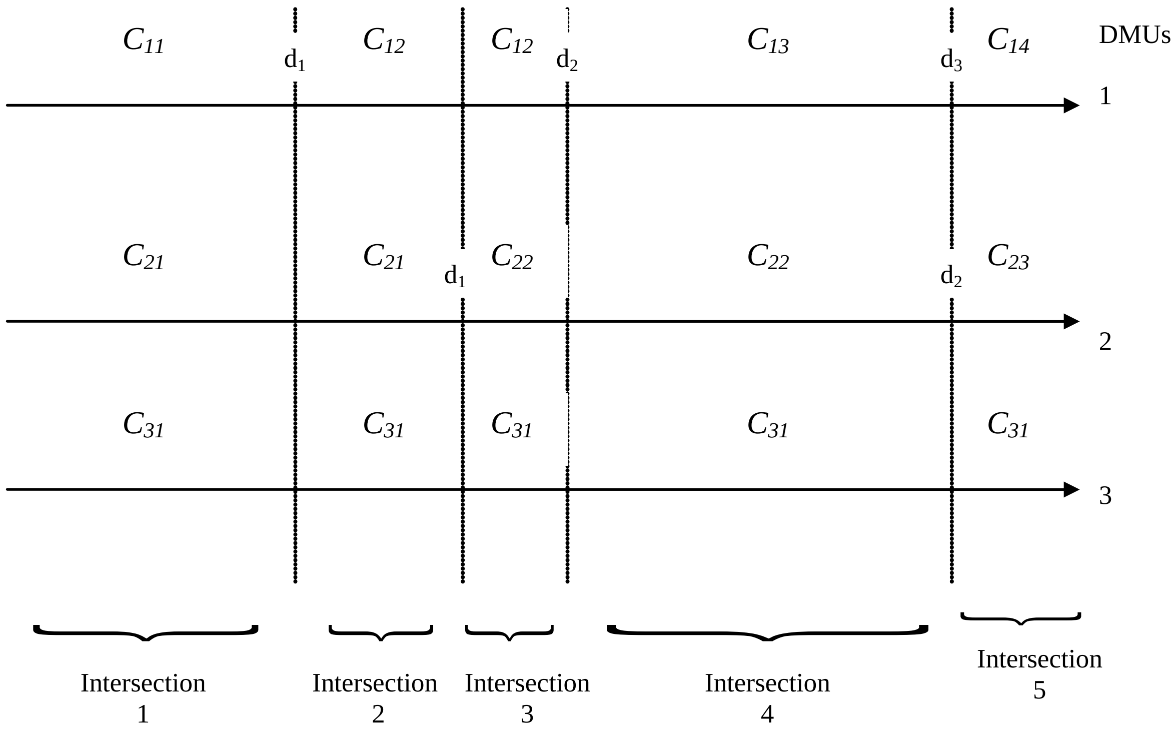

Fig. 1: Intersections of price breaks of three suppliers

Step 3. For each intersection, conduct a scale transformation to every ordinal input and output data, then apply models (1) and (2) and finally use algorithm A: in this step with respect to material price in each intersection (including *n* suppliers) and other criteria such as quality, delivery performance, etc; firstly ordinal data are transformed to interval data, secondly models (1) and (2) are applied and lastly algorithm A is used.

Step 4. Interpret the results: in this step, the results are interpreted according to previous steps, i.e., with regard to step 1, the desired intersection of price break is chosen. Then with regard to step 3, suppliers are ranked.

In the next section, a numerical example is presented.

4. Numerical example

For illustration purposes, the problem of supplier ranking is introduced. The data set for this example is taken from Farzipoor Saen (2009). The data set for this example contains specifications on 12 suppliers. The cardinal input considered is price. Supplier Reputation (SR) is included as a qualitative input while Number of Bills received from the supplier without errors (NB) will serve as the bounded data output. SR is an intangible factor which is not usually explicitly included in evaluation model for supplier. This qualitative variable is measured on an ordinal scale. The price breaks for quantity ordered are given in Table 2, according to the discount range of each supplier. Note that, the inputs and outputs selected in this paper are not exhaustive by any means, but are some general measures that can be utilized to evaluate suppliers. In an actual application of this methodology, decision makers must carefully identify appropriate inputs and outputs measures to be used in the decision making process. Table 3 depicts other supplier attributes.

Table 2. Price breaks

Supplier No. (DMU)	Ranges (Tons)	Price ($)
1	[0,10]	10
	(10,15]	9
	(15,+∞)	7
2	[0,5]	8
	(5, +∞)	5
3	[0,6]	20
	(6,10]	18
	(10,15]	15
	(15,+∞)	13
4	[0,+∞)	12
5	[0,8]	10
	(8,+∞)	9
6	[0,11]	15
	(11,+∞)	12
7	[0,+∞)	13
8	[0,4]	11
	(4,7]	10
	(7,+∞)	8
9	[0,12]	8
	(12,+∞)	7
10	[0,15]	14
	(15,20]	12
	(20,+∞)	9
11	[0,+∞)	11
12	[0,+∞)	14

Table 3. Other attributes for 12 suppliers

Supplier No. (DMU)	Input SR* x_{2j}	Output NB y_{1j}
1	5	[50, 65]
2	10	[60, 70]
3	12	[40, 50]
4	6	[100, 160]
5	4	[45, 55]
6	2	[85, 115]
7	8	[70, 95]
8	1	[100, 180]
9	9	[90, 120]
10	7	[50, 80]
11	11	[250, 300]
12	3	[100, 150]

* Ranking such that $12 \equiv$ highest rank,…, $1 \equiv$ lowest rank ($x_{2,3} > x_{2,11} \ldots > x_{2,8}$)

First, assume that the buyer's demand is 9 tons. Next, intersections of price breaks for all suppliers are determined. Table 4 shows the related price vector of 9 tons.

Table 4. The related price vector of 9 tons

Supplier No. (DMU)	Price ($) x_{1j}
1	10
2	5
3	18
4	12
5	9
6	15
7	13
8	8
9	8
10	14
11	11
12	14

Suppose the preference intensity parameter and the ratio parameter about the strong ordinal preference information are given (or estimated) as $\eta_2 = 1.12$ and $\sigma_2 = 0.01$, respectively. Using the transformation technique described in algorithm B, an interval estimate for SR of each supplier can be derived, which is shown in the Table 5.

Therefore, all the input and output data are now transformed into interval numbers and can be evaluated using interval DEA models. Table 6 reports the results of efficiency assessments for the 12 suppliers obtained by using interval DEA models (1) and (2). The non-Archimedean infinitesimal was set to be $\varepsilon = 0.0001$.

Table 5. Interval estimate for the 12 suppliers after the transformation of ordinal preference information

Supplier No. (DMU)	SR
1	[.0157352, .4523492]
2	[.0277308, .7971939]
3	[.0347855, 1]
4	[.0176234, .5066311]
5	[.0140493, .4038832]
6	[.0112, .3219732]
7	[.0221068, .6355181]
8	[.01, .2874761]
9	[.0247596, .7117802]
10	[.0197382, .5674269]
11	[.0310585, .8928571]
12	[.012544, .36061]

Table 6. The efficiency interval for the 12 suppliers

Supplier No. (DMU)	Efficiency Interval
1	[.183326, .276816]
2	[.439966, .513333]
3	[.0814737, .113055]
4	[.305541, .575628]
5	[.183326, .260675]
6	[.207772, .570092]
7	[.197424, .30619]
8	[.458321, 1]
9	[.412472, .55]
10	[.130945, .24861]
11	[.833262, 1]
12	[.261896, .664065]

Based on the definition 1, suppliers 8 and 11 both have the possibility to be DEA efficient. If they are able to use the minimum inputs to produce the maximum outputs, they are DEA efficient (efficient in scale); otherwise, they are not DEA efficient. Although suppliers 8 and 11 both have the possibility to be DEA efficient, due to the differences in the lower bound efficiencies, their performances are in fact different.

In order to rank the efficiencies of the 12 suppliers (DMUs), the MRA is employed to compute the maximum loss of efficiency for each supplier (see Appendix B). As computations show, supplier_{11} is selected as the best supplier.

5. Concluding remarks

The decision of selecting the best supplier from a wide supplier base is an unstructured, complicated and time-consuming problem. The process involves evaluation of different alternatives based on various criteria, some of which have to be maximized (outputs) and others minimized (inputs).

In the meantime, appearance of a new discount pricing schedule entitled "volume discount" becomes a major obstacle for procurement managers in finding the best purchasing strategy. In the context of volume discount, a supplier offers discounts on total amount of volume purchased from the supplier. This paper introduces an algorithm for ranking the suppliers in the presence of both cardinal and ordinal data in the conditions that they offer volume discounts.

The problem considered in this study is at initial stage of investigation and much further researches can be done based on the results of this paper. Some of them are as follows:

Similar research can be repeated for dealing with ordinal data and bounded data by fuzzy sets. Other potential extension to the algorithm includes the case that some of the suppliers are slightly non-homogeneous. One of the assumptions of all the classical models of DEA is based on complete homogeneity of DMUs (suppliers), whereas this assumption in many real applications cannot be generalized. In other words, some inputs and/or outputs are not common for all the DMUs occasionally. For instance at the university there are different departments. To determine relative efficiency of departments most of inputs and outputs are common, but there are a few input(s) and/or output(s) for some departments that may not be common to all. The engineering departments are equipped with machineries and laboratories whereas an input of such kind for department of political sciences may be meaningless. It is clear that zero value allocation for this type of input, causes relative efficiency of political sciences department to increase unrealistically. So, there is a need to a model that deals with these conditions.

In this study, the proposed model has been used to a problem related to supplier selection. However, the same model could be applied, with minor modifications, to other problem related to selection of third-party reverse logistics (3PL) providers.

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Appendix A

Interval arithmetic:

If $a=[a_1, a_2]$ and $b=[b_1, b_2]$ are two positive intervals, then

$a+b=[a_1+b_1, a_2+b_2]$;

$a-b=[a_1-b_2, a_2-b_1]$;

$a*b=[a_1*b_1, a_2*b_2]$;

$a/b=[a_1/b_2, a_2/b_1]$.

The above +, -, *, and / represent operation rules.

Appendix B

$R(\text{supplier}_1)$=max[max(.513333, .113055, .575628, .260675, .570092, .30619, 1, .55, .24861, 1, .664065)- .183326, 0]= .816674

$R(\text{supplier}_2)$=max[max(.276816, .113055, .575628, .260675, .570092, .30619, 1, .55, .24861, 1, .664065)- .439966, 0]= .560034

$R(\text{supplier}_3)$=max[max(.276816, .513333, .575628, .260675, .570092, .30619, 1, .55, .24861, 1, .664065)- .0814737, 0]= .9185263

⋮

$R(\text{supplier}_{12})$=max[max(.276816, .513333, .113055, .575628, .260675, .570092, .30619, 1, .55, .24861, 1)- .261896, 0]= .738104

Obviously, supplier_{11} has the smallest maximum loss of efficiency. So, supplier_{11} is rated as the best supplier and eliminated from the further consideration. Therefore for the remaining suppliers, maximum losses of efficiency are recalculated as follows:

$R(\text{supplier}_1)$=max[max(.513333, .113055, .575628, .260675, .570092, .30619, 1, .55, .24861, .664065)- .183326, 0]= .816674

$R(supplier_2)=\max[\max(.276816,\ .113055,\ .575628,\ .260675,\ .570092,\ .30619,\ 1,\ .55,\ .24861,\ .664065)-\ .439966,\ 0]=\ .560034$

$R(supplier_3)=\max[\max(.276816,\ .513333,\ .575628,\ .260675,\ .570092,\ .30619,\ 1,\ .55,\ .24861,\ .664065)-\ .0814737,\ 0]=\ .9185263$

$\vdots$

$R(supplier_{12})=\max[\max(.276816,\ .513333,\ .113055,\ .575628,\ .260675,\ .570092,\ .30619,\ 1,\ .55,\ .24861)-\ .261896,\ 0]=\ .738104$

Among the above regrets, the maximum loss of efficiency of $supplier_8$ is the smallest, so $supplier_8$ is rated as the second best supplier and eliminated from the further consideration. So, for the remaining suppliers, maximum losses of efficiency are recalculated and shown below:

$R(supplier_1)=\max[\max(.513333,\ .113055,\ .575628,\ .260675,\ .570092,\ .30619,\ .55,\ .24861,\ .664065)-\ .183326,\ 0]=\ .480739$

$R(supplier_2)=\max[\max(.276816,\ .113055,\ .575628,\ .260675,\ .570092,\ .30619,\ .55,\ .24861,\ .664065)-\ .439966,\ 0]=\ .224099$

$R(supplier_3)=\max[\max(.276816,\ .513333,\ .575628,\ .260675,\ .570092,\ .30619,\ .55,\ .24861,\ .664065)-\ .0814737,\ 0]=\ .5825913$

$\vdots$

$R(supplier_{12})=\max[\max(.276816,\ .513333,\ .113055,\ .575628,\ .260675,\ .570092,\ .30619,\ .55,\ .24861)-\ .261896,\ 0]=\ .313732$

Since $supplier_2$ has the smallest maximum loss of efficiency, so it is rated as the third best supplier and eliminated from the further consideration. Repeating the above process, the ranking order of 12 suppliers is obtained as follows:

$Supplier_{11} \succ supplier_8 \succ supplier_2 \succ supplier_9 \succ supplier_{12} \succ supplier_4 \succ supplier_6 \succ supplier_7 \succ$
$Supplier_1 \succ supplier_5 \succ supplier_{10} \succ supplier_3$

Therefore, $supplier_{11}$ is selected as the best supplier.

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