# Tidal fluctuations in a leaky confined aquifer: Dynamic effects of an overlying phreatic aquifer.

## Author

Li, L, Barry, DA, Jeng, DS

## Published

2001

## Journal Title

Water Resources Research

## DOI

10.1029/2000WR900402

WATER RESOURCES RESEARCH, VOL. 37, NO. 4, PAGES 1095–1098, APRIL 2001

# Tidal fluctuations in a leaky confined aquifer: Dynamic effects of an overlying phreatic aquifer

L. Li and D. A. Barry

School of Civil and Environmental Engineering, University of Edinburgh, Edinburgh, Scotland, United Kingdom
Contaminated Land Assessment and Remediation Research Centre, University of Edinburgh, Edinburgh Scotland, United Kingdom

D.-S. Jeng

School of Engineering, Griffith University, Gold Coast, Australia

**Abstract.** Tidal fluctuations in a leaky confined coastal aquifer are damped significantly due to leakage into an overlying phreatic aquifer. *Jiao and Tang* [1999] presented an analytical solution to a simple model describing this phenomenon. Their solution assumes that the tidal fluctuations in the overlying phreatic aquifer are negligible (i.e., a static phreatic aquifer). Here we examine dynamic effects of the overlying aquifer based on a new approximate analytical solution. The numerical results indicate that the dynamic effects can be significant for a relatively large leakage and a high transmissivity of the phreatic aquifer.

## 1. Problem Set Up

As shown in Figure 1, we consider tidal fluctuations in both the confined and the phreatic aquifer. These two aquifers interact with each other through a thin leaky layer. The governing equations of the head fluctuations in both aquifers are [e.g., *Bear and Verruijt*, 1987]

$$s_1 \frac{\partial h_1}{\partial t} = T_1 \frac{\partial^2 h_1}{\partial x^2} + L(h_2 - h_1), \tag{1a}$$

$$s_2 \frac{\partial h_2}{\partial t} = T_2 \frac{\partial^2 h_2}{\partial x^2} + L(h_1 - h_2), \tag{1b}$$

where $h_1$ and $h_2$ are the heads in the confined and the phreatic aquifers, respectively; $T_1$ and $T_2$ are the transmissivities of these two aquifers, respectively; $s_1$ is the specific yield of the phreatic aquifer and $s_2$ is the storativity of the confined aquifer; and $L$ is the specific leakage of the semipermeable layer. Note that linearization has been applied for the governing equation of the phreatic aquifer, (1a), under the assumption that the tidal amplitude is small relative to the aquifer's thickness [*Parlange et al.*, 1984]. *Jiao and Tang* [1999] assumed $h_1$ to be constant. Here we shall consider $h_1$ variable, reflecting tidal water table fluctuations in the phreatic aquifer. The boundary conditions for these semi-infinite aquifers are

$$h_1(0, t) = h_2(0, t) = h_{\mathrm{MSL}} + A \cos(\omega t), \tag{2a}$$

$$\left.\frac{\partial h_1}{\partial x}\right|_{x=\infty} = \left.\frac{\partial h_2}{\partial x}\right|_{x=\infty} = 0, \tag{2b}$$

where $h_{\mathrm{MSL}}$ is the averaged mean sea level and $A$ and $\omega$ are the tidal amplitude and frequency, respectively. Physically, (2a) and (2b) describe a periodic boundary condition at the origin in an aquifer without a regional flow component. Only one tidal constituent is considered here. Since the problem is linear, solutions for multiple-constituent tides can be obtained easily from the solution given below using superposition. In (2a) we also have assumed a vertical ocean-land interface and negligible seepage face for the phreatic aquifer. The effects of beach slope and seepage face formation on tidal propagation in a phreatic aquifer have been discussed elsewhere [*Li et al.*, 2000, 2001].



Paper number 2000WR900402.
0043-1397/01/2000WR900402$09.00

## 2. Perturbation Solution

*Walker* [1987] described a general approach for solving (1) exactly. The solution, however, is very complicated, containing double integrals involving Bessel functions. Here we will adopt a perturbation approach to obtain simple solutions that reveal more directly the physical behavior of the leaky confined aquifer system. We seek solutions in the following forms:

$$h_1 = h_{10} + \varepsilon h_{11} + O(\varepsilon^2), \tag{3a}$$

$$h_2 = h_{20} + \varepsilon h_{21} + O(\varepsilon^2), \tag{3b}$$

where the perturbation variable ε is chosen to be $L/\omega$ based on dimensional analysis. This choice of ε, physically representing the importance of leakage flows in relation to tidal fluctuations in the aquifers, allows us to investigate *Jiao and Tang*'s [1999] assumption of a static phreatic aquifer; that is, the tidal forcing will propagate in this aquifer and be transmitted to the lower layer through leakage. Substituting (3) to (1) and (2) results in $O(\varepsilon^0)$:

$$s_1 \frac{\partial h_{10}}{\partial t} - T_1 \frac{\partial^2 h_{10}}{\partial x^2} = 0, \tag{4a}$$

$$s_2 \frac{\partial h_{20}}{\partial t} - T_2 \frac{\partial^2 h_{20}}{\partial x^2} = 0, \tag{4b}$$

$$h_{10}(0, t) = h_{20}(0, t) = h_{\mathrm{MSL}} + A \cos(\omega t), \tag{4c}$$

$$\left.\frac{\partial h_{10}}{\partial x}\right|_{x=\infty} = \left.\frac{\partial h_{20}}{\partial x}\right|_{x=\infty} = 0, \tag{4d}$$

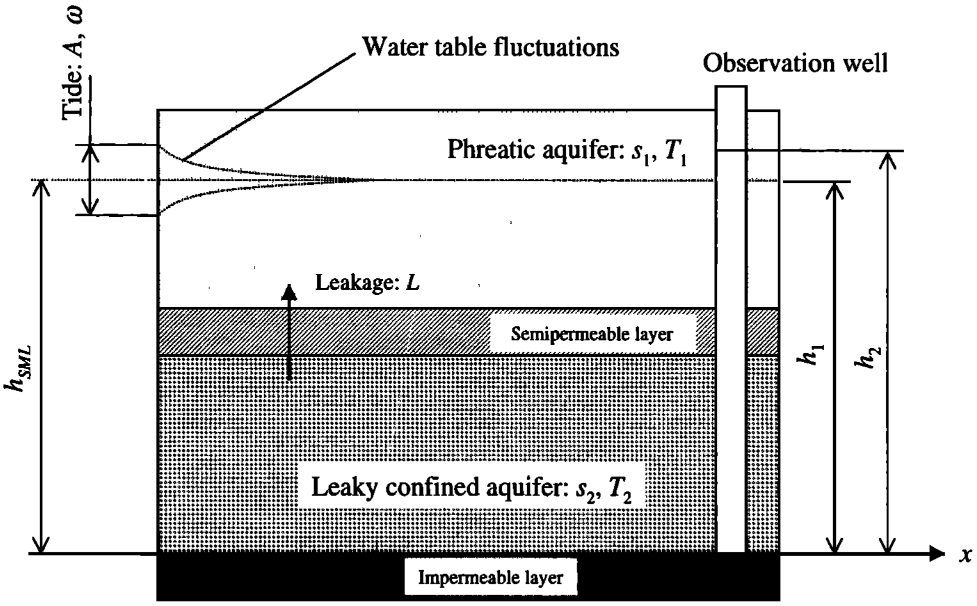


**Figure 1.** Schematic diagram of a leaky confined aquifer with an overlying phreatic aquifer.

and $O(\varepsilon^1)$:

$$s_1 \frac{\partial h_{11}}{\partial t} - T_1 \frac{\partial^2 h_{11}}{\partial x^2} = \omega(h_{20} - h_{10}), \tag{5a}$$

$$s_2 \frac{\partial h_{21}}{\partial t} - T_2 \frac{\partial^2 h_{21}}{\partial x^2} = \omega(h_{10} - h_{20}), \tag{5b}$$

$$h_{11}(0, t) = h_{21}(0, t) = 0, \tag{5c}$$

$$\left.\frac{\partial h_{11}}{\partial x}\right|_{x=\infty} = \left.\frac{\partial h_{21}}{\partial x}\right|_{x=\infty} = 0. \tag{5d}$$

The solutions to (4) are

$$h_{10} = h_{\mathrm{MSL}} + Re\lfloor Ae^{i(\kappa_{10}x - \omega t)}\rfloor, \tag{6a}$$

$$h_{20} = h_{\mathrm{MSL}} + Re\lfloor Ae^{i(\kappa_{20}x - \omega t)}\rfloor, \tag{6b}$$

where $\kappa_{10} = (1 + i)(s_1\omega/2T_1)^{1/2}$ and $\kappa_{20} = (1 + i)(s_2\omega/2T_2)^{1/2}$. Substituting (6) into (5) gives

$$s_1 \frac{\partial h_{11}}{\partial t} - T_1 \frac{\partial^2 h_{11}}{\partial x^2} = Re[\omega Ae^{i(\kappa_{20}x - \omega t)} - \omega Ae^{i(\kappa_{10}x - \omega t)}], \tag{7a}$$

$$s_2 \frac{\partial h_{21}}{\partial t} - T_2 \frac{\partial^2 h_{21}}{\partial x^2} = Re[\omega Ae^{i(\kappa_{10}x - \omega t)} - \omega Ae^{i(\kappa_{20}x - \omega t)}]. \tag{7b}$$

The second forcing term (on the right-hand side) in (7a) is a secular term that resonates with the operators on the left-hand side. The particular solution for this term is proportional to $xe^{i(\kappa_{10}x - \omega t)}$, which is unbounded in $x$. The same problem exists with (7b). To eliminate these secular terms, we expand the wave numbers following *Liu and Wen* [1997], i.e.,

$$\kappa_1 = \kappa_{10} + \varepsilon\kappa_{11} + O(\varepsilon^2), \tag{8a}$$

$$\kappa_2 = \kappa_{20} + \varepsilon\kappa_{21} + O(\varepsilon^2), \tag{8b}$$

and take

$$h_{10} = h_{\mathrm{MSL}} + Re\lfloor Ae^{i(\kappa_1 x - \omega t)}\rfloor, \tag{8c}$$

$$h_{20} = h_{\mathrm{MSL}} + Re\lfloor Ae^{i(\kappa_2 x - \omega t)}\rfloor. \tag{8d}$$

Then, the leading-order equations, (4a) and (4b), remain the same, while the $O(\varepsilon)$ equations, (7a) and (7b), become

$$s_1 \frac{\partial h_{11}}{\partial t} - T_1 \frac{\partial^2 h_{11}}{\partial x^2} = Re[\omega Ae^{i(\kappa_2 x - \omega t)} - (2\kappa_{10}\kappa_{11}T_1 + \omega)Ae^{i(\kappa_1 x - \omega t)}], \tag{9a}$$

$$s_2 \frac{\partial h_{21}}{\partial t} - T_2 \frac{\partial^2 h_{21}}{\partial x^2} = Re[\omega Ae^{i(\kappa_1 x - \omega t)} - (2\kappa_{20}\kappa_{21}T_2 + \omega)Ae^{i(\kappa_2 x - \omega t)}]. \tag{9b}$$

To eliminate the secular terms, we must have

$$\kappa_{11} = -\frac{\omega}{2\kappa_{10}T_1}, \tag{10a}$$

$$\kappa_{21} = -\frac{\omega}{2\kappa_{20}T_2}. \tag{10b}$$

The solutions to (9) subject to boundary conditions (5c) and (5d) are

$$h_{11} = Re\left\{\frac{A\omega}{\kappa_2^2 T_1 - is_1\omega}[e^{i(\kappa_2 x - \omega t)} - e^{i(\kappa_{10}x - \omega t)}]\right\}, \tag{11a}$$

$$h_{21} = Re\left\{\frac{A\omega}{\kappa_1^2 T_2 - is_2\omega}[e^{i(\kappa_1 x - \omega t)} - e^{i(\kappa_{20}x - \omega t)}]\right\}. \tag{11b}$$

The final solution is thus

$$h_1 = h_{\mathrm{MSL}} + Re\left\{Ae^{i(\kappa_1 x - \omega t)} + \frac{AL}{\kappa_2^2 T_1 - is_1\omega}[e^{i(\kappa_2 x - \omega t)} - e^{i(\kappa_{10}x - \omega t)}]\right\}, \tag{12a}$$

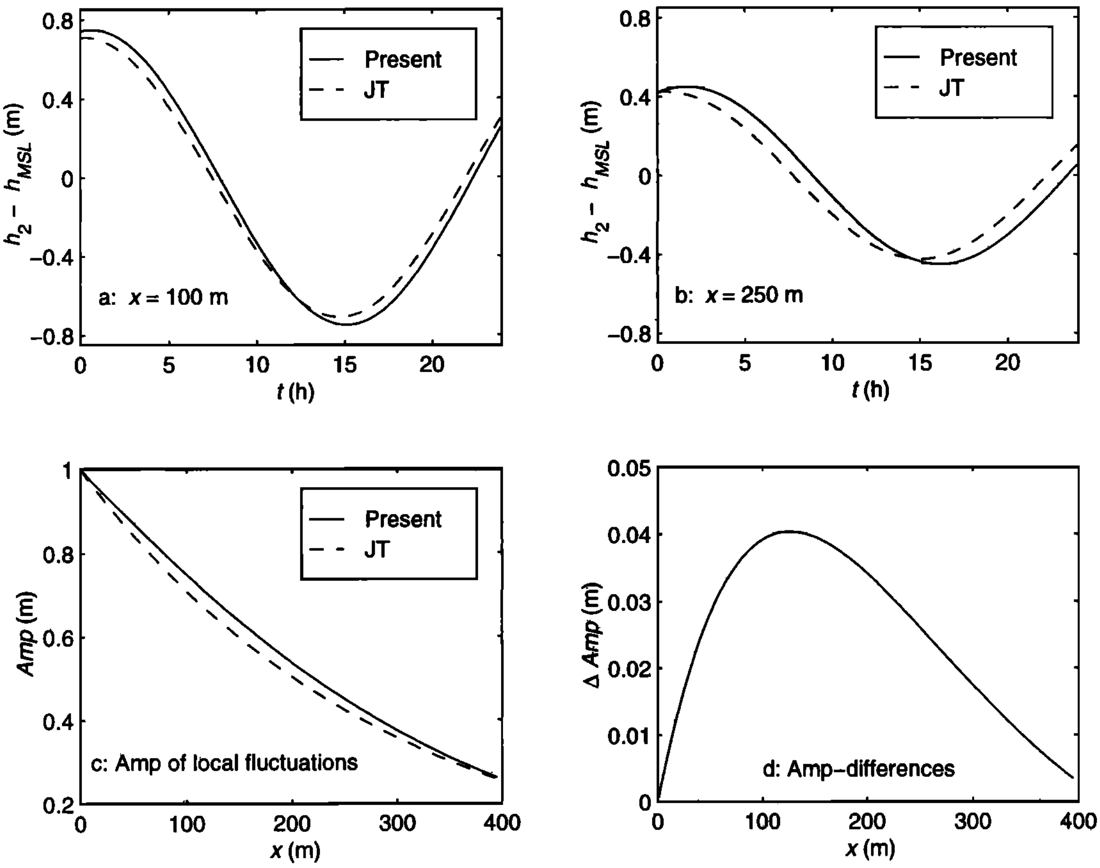


**Figure 2.** Predicted head fluctuations by the present solution (solid lines) and that of *Jiao and Tang* [1999] (dashes) for (a) $x$ = 100 and (b) 250 m. (c) Comparison of the local fluctuation amplitudes predicted by the present solution and that of *Jiao and Tang* [1999]. (d) Differences between the two solutions of the fluctuation amplitudes, i.e., the dynamic effects.

$$h_2 = h_{\mathrm{MSL}} + Re\left\{Ae^{i(\kappa_2 x-\omega t)} + \frac{AL}{\kappa_1^2 T_2 - is_2\omega}\left[e^{i(\kappa_1 x-\omega t)} - e^{i(\kappa_{20} x-\omega t)}\right]\right\}. \tag{12b}$$

Here we obtain the solution to $O(\varepsilon)$ only. The approach can be extended to obtain solutions of higher orders. These solutions are complicated, and hence we shall focus on the first-order solution.

## 3. Dynamic Effects of the Phreatic Aquifer on Tidal Fluctuations in the Leaky Confined Aquifer

Equation (12b) describes the tidal head fluctuations in the leaky confined aquifer. The first fluctuating term, $Re\lfloor Ae^{i(\kappa_2 x-\omega t)}\rfloor$, is a truncated solution of *Jiao and Tang*'s [1999], $h_{jt} = Re\lfloor Ae^{i(\kappa_{jt} x-\omega t)}\rfloor$, where $\kappa_{jt} = [(is_2\omega/T_2) - (L/T_2)]^{1/2}$ (the subscript $jt$ indicates that the solution is that of *Jiao and Tang* [1999]). One can show that $\kappa_2$ is just the first two terms of an expansion of $\kappa_{jt}$. The solution can thus be improved by replacing $\kappa_2$ in (12b) with $\kappa_{tj}$.

The second fluctuating term (i.e., $Re\{[(AL)/(\kappa_1^2 T_2 - is_2\omega)][e^{i(\kappa_1 x-\omega t)} - e^{i(\kappa_{20} x-\omega t)}]\}$) accounts for the dynamic effects of the overlying phreatic aquifer. Such effects were neglected by *Jiao and Tang* [1999]. Since the amplitude damping is dominated by $\mathrm{Im}(\kappa_{20})$ (a small number typically and $\ll \mathrm{Im}(\kappa_1)$), the dynamic effects can influence the head fluctuations over a large distance. In Figure 2, we show head fluctuations at $x$ = 100 and 250 m as predicted by the present solution and that of *Jiao and Tang* [1999]. The predicted amplitudes of the local head fluctuations, varying with $x$, are also compared (Figures 2c and 2d). The dynamic effects are clearly evident, affecting the amplitude and phase of the head fluctuations. In particular, the dynamic effects lead to reduced amplitude damping. The parameter values used in the calculation are $s_1$ = 0.25, $s_2$ = 0.001, $T_1$ = 0.2 m$^2$ s$^{-1}$, $T_2$ = 0.02 m$^2$ s$^{-1}$, $L$ = 0.02 d$^{-1}$, $A$ = 1 m, and $\omega$ = 0.2168 rad h$^{-1}$ (diurnal tide).

To derive conditions under which dynamic effects are important, we will focus on the magnitude of $(AL)/(\kappa_1^2 T_2 - is_2\omega)$ (denoted as $A_d$), i.e.,

$$A_d = \frac{A}{\sqrt{a_1^2 + \left(\frac{s_1 a_1}{\varepsilon} - \frac{a_1\varepsilon}{4s_1} - \frac{s_1}{\varepsilon a_2}\right)^2}}, \tag{13a}$$

$$a_1 = T_2/T_1, \tag{13b}$$

$$a_2 = s_1/s_2. \tag{13c}$$

Typically, we have $s_1 \gg \varepsilon \gg s_2$ [*Jiao and Tang*, 2000]; thus (13a) can be simplified to the approximation

$$A_d = (A\varepsilon)/(s_1 a_1). \tag{14}$$

Equation (14) suggests that the dynamic effects cannot be neglected where leakage is large and the transmissivity of the confined aquifer is smaller than or comparable to that of the phreatic aquifer. On the basis of (14), the following condition for $L$ may be derived, under which the dynamic effects are important:

$$L \geq 0.01 s_1 a_1 \omega, \tag{15}$$

where the critical value for $A_d/A$ has been assumed to be 0.01.

## 4. Conclusions

We have derived an analytical solution for tidal head fluctuations in a leaky confined aquifer, including effects of the overlying fluctuating phreatic aquifer. These effects, neglected by previous studies, are important under a relatively large leakage and phreatic aquifer's transmissivity. Obviously, the solution presented here can also be applied to two interacting confined aquifers, in which case fluctuations in both aquifers (i.e., the dynamic effects) cannot be neglected.

## References

Bear, J., and A. Verruijt, *Modelling Groundwater Flow and Pollution*, D. Reidel, Norwell, Mass., 1987.

Jiao, J. J., and Z.-H. Tang, An analytical solution of groundwater response to tidal fluctuations in a leaky confined aquifer, *Water Resour. Res.*, *35*, 747–751, 1999.

Jiao, J. J., and Z.-H. Tang, Reply to comment by R. E. Volker and Q. Zhang on "An analytical solution of groundwater response to tidal fluctuations in a leaky confined aquifer," *Water Resour. Res.*, *37*, 187–188, 2000.

Li, L., D. A. Barry, F. Stagnitti, J.-Y. Parlange, and D.-S. Jeng, Beach water table fluctuations due to spring-neap tides: Moving boundary effects, *Adv. Water Resour.*, *23*, 817–824, 2000.

Li, L., D. A. Barry, F. Stagnitti, and J.-Y. Parlange, Beach water table fluctuations due to spring-neap tides: Seepage face effects, *Adv. Water Resour.*, in press, 2001.

Liu, P. L. F., and J.-G. Wen, Nonlinear diffusive surface waves in porous media, *J. Fluid Mech.*, *347*, 119–139, 1997.

Parlange, J.-Y., F. Stagnitti, J. L. Starr, and R. D. Braddock, Free-surface flow in porous media and periodic solution of the shallow-flow approximation, *J. Hydrol.*, *70*, 251–263, 1984.

Walker, G., Solution of a class of coupled linear partial differential equations, *IMA J. Appl. Math.*, *38*, 35–48, 1987.

D. A. Barry and L. Li, School of Civil and Environmental Engineering and Contaminated Land Assessment and Remediation Research Centre, University of Edinburgh, The King's Buildings, Edinburgh EH9 3JN, Scotland, United Kingdom. (A.Barry@ed.ac.uk; ling.li@ed.ac.uk)

D.-S. Jeng, School of Engineering, Griffith University, Gold Coast, Queensland 9726, Australia. (D.Jeng@mailbox.gu.edu.au)

(Received July 13, 2000; revised December 11, 2000; accepted December 12, 2000.)