Nonlinear Distributed Controller Design for Maintaining Power Balance in Islanded Microgrids

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Abstract—This paper presents a nonlinear distributed controller design for islanded operation of microgrids in order to maintain active and reactive power balance. In this paper, the microgrids are considered as inverter-dominated networks integrated with renewable energy sources (RESs) and battery energy storage systems (BESSs) where solar photovoltaic (PV) generators act as RESs and plug-in hybrid electric vehicles (PHEVs) as BESSs to supply power into the grid. The detail dynamic models of PV generators and BESSs are also developed in this paper. The proposed distributed control scheme gathers information from local and neighboring generators to achieve the desired control objectives. The proposed controller is designed by using feedback linearization and the communication between generators and control centers is developed by using the concept of graph theory. Finally, the performance of the proposed controller is demonstrated on a test microgrid and simulation results indicate the superiority under different operating conditions as compared to a linear quadratic regulator (LQR)-based controller.

Index Terms—Partial feedback linearization, distributed controller, microgrid, graph theory.

I. INTRODUCTION

Microgrid operates autonomously when it is disconnected from the main grid due to the disturbances such as faults and subsequent or preplanned switching events. The power-balance between supply and demand does not match at the event of islanding and due to this mismatch, the frequency (which is due to active power mismatches) and voltage (which occurs due to mismatch reactive power support) will fluctuate. The fluctuations in frequency and voltage will cause a blackout unless there is an appropriate power-balance between the supply and demand [1]. Since the RESs have intermittent characteristics due to their dependency on sunlight or wind speed, an energy storage system is required to supply power in case of emergency. In this paper, the batteries of plug-in hybrid vehicles (PHEVs) are considered as energy storage systems. Inverters are fast acting devices which are able to obtain the power-balance very quickly and the implementation of inverter control schemes could be meaningful during the islanded operation of microgrids [2], [3].

Most of the microgrid literature, from the control point of view, treats the voltage source behind the inverters as ideal and some of them consider the dynamics of inverters only to design linear controllers [4]–[6]. But the consideration of ideal voltage sources behind the inverters is valid when the microgrid operates at grid-connected mode as most of the system-level dynamics are dictated by the main grid due to the relatively small size of RESs [7]. However, in an islanded mode, the system dynamics are dictated by RESs and therefore it is essential to consider the dynamics of RESs for an efficient operation of microgrid. Moreover, the linear controllers are unable to provide appropriate active and reactive power with changes in atmospheric conditions and system configurations such as external disturbances and variations in loads.

Nonlinear controllers overcome the limitations of linear controllers by maintaining the power balance of microgrids over a wide range of operating conditions. A distributed nonlinear controller based on feedback linearization is proposed in [8] for secondary voltage control of microgrids where a superfluous model of microgrids has been considered rather than the exact model of RESs and BESSs. Moreover in [8], apart from the indication of distributed generators (DGs) the types of RESs have not been mentioned but the dynamics of DGs depend on the types of RESs used in microgrids. In [8], though the microgrid is partially feedback linearized, there are no clear indications about the internal dynamics of microgrids as well as there are too many assumptions and some of which are not valid. For example, it has been assumed that only one DG has the access to the reference value and the communication graph does not represent any feasible information exchange between DGs.

This paper aims to design a nonlinear distributed controller which would be capable of maintaining power balance (both active and reactive power) during the islanded operation of microgrids. The detailed dynamic model of microgrids is developed by considering solar PV generators and batteries of PHEVs as the main source of supplying power to meet the load demands during the islanding periods. Feedback linearizing controller is used to regulate voltage and frequency of microgrids for appropriate power sharing and some case studies have been presented to demonstrate the performance of the proposed scheme. The results are also compared to that of a linear quadratic regulator (LQR)-based controller.

II. MICROGRID MODELING

The microgrid considered in this paper consists of solar PV units and several PHEVs with BESSs. The detailed dynamical model of a microgrid integrated with solar PV units and PHEVs has been discussed in this section.
A. PV System Model

The PV system which supplies power into the grid consists of a PV array in which a number of cells are connected in series and parallel combinations, a voltage source converter (VSC) to convert the DC output power into the AC power, an output filter to reduce the harmonics and some connecting lines [9].

The dynamical model of a grid-connected PV system in dq-frame using the angular frequency $\omega$ of the grid and can be written as [9]

\[
\begin{align*}
\dot{I}_d &= -\frac{R}{L} I_d + \omega I_q - \frac{E_d}{L} + \frac{v_{pv}}{L} K_d \\
\dot{I}_q &= -\omega I_d - \frac{R}{L} I_q - \frac{E_q}{L} + \frac{v_{pv}}{L} K_q \\
\dot{v}_{pv} &= \frac{1}{C} i_{pv} - \frac{1}{C} I_d K_d - \frac{1}{C} I_q K_q
\end{align*}
\]

where $R$ is the resistance of the connecting lines, $L$ is the inductance of the filter and connecting lines, $C$ is the DC-link capacitance, $v_{pv}$ is the voltage across the DC-link capacitor, $I$ is the output current of the inverter, $i_{pv}$ is the output current of PV array, $K_d$ and $K_q$ are the binary input switching signals, respectively. The subscripts $d$ and $q$ stand for direct- and quadrature-axes components, respectively.

B. BESS Model

The BESS of PHEVs consists of a battery, a VSC, an output filter and some connecting lines. When the batteries of PHEVs are used to supply power into the grid, the system is also known as vehicle-to-grid (V2G) systems. The most commonly used battery model is proposed in [10] in which the charge stored in the battery, is the integral of only a part $I_m$ of the total current $I_d$ entering the battery. The detailed battery elements such as resistors ($R_0$ and $R_1$), capacitor ($C_1$), and internal voltage ($E_{in}$) can be seen in [10]. Thus, BESSs of PHEVs can be modeled in a similar way to that of a grid-connected PV system as discussed above.

In dq-frame the dynamical model of a BESS as shown in Fig.?? can be written as

\[
\begin{align*}
\dot{I}_d &= \frac{1}{\tau_1} (M_d I_{bd} + M_q I_{bq} - I_d) \\
\dot{I}_{bd} &= -\frac{R}{L} I_{bd} + \omega I_{bq} - \frac{E_d}{L} + \frac{v_{dc}}{L} M_d \\
\dot{I}_{bq} &= -\omega I_{bd} - \frac{R}{L} I_{bq} - \frac{E_q}{L} + \frac{v_{dc}}{L} M_q
\end{align*}
\]

where $M_d$ and $M_q$ are the switching functions in d and q frame respectively, $I_{bd}$ and $I_{bq}$ are the currents in d and q frame respectively, and $E_d$ and $E_q$ are the grid voltages in d and q frame respectively.

The mathematical models of PV systems and BESSs as discussed in this section are valid for both single- and three-phase systems [11]. The only difference between single- and three-phase systems is the power flow. The active and reactive power delivered by both PV system and BESSs can be written as follows by assuming that $E_d = 0$ [12].

For a three-phase system,

\[
\begin{align*}
P &= \frac{3}{2} E_q I_q \\
Q &= \frac{3}{2} E_q I_d
\end{align*}
\]

and for a single-phase system,

\[
\begin{align*}
P &= E_q I_q \\
Q &= E_q I_d
\end{align*}
\]

The mathematical model developed in this section is used to design the proposed distributed controller for microgrid. However before designing a distributed feedback linearizing controller, it is essential formulate the problem which is known in the following section.

III. CONTROL PROBLEM FORMULATION

The main target of the proposed distributed controller is to deliver active and reactive power from PV units and BESSs. From the power equations as described by equations (3) and (4), it can be seen that the active power ($P$) can be regulated by controlling the current $I_d$ and reactive power ($Q$) regulation can be obtained by controlling $I_q$ as there is nothing to do with the microgrid voltage $E_q$.

The design of feedback linearizing controller depends on the output functions, i.e., the control objectives of the system. The feedback linearizing controller as proposed in [9] was for achieving the maximum power point tracking (MPPT) as well as delivering maximum active power into the grid. For this reason, in [9] the control objectives were $v_{pv}$ and $I_q$. But to serve the purpose of this paper, the active and reactive current components $I_d$ and $I_q$ need to be chosen as control objectives. In this case, $v_{pv}$ as a control objective can be neglected because if there are enough BESSs, it is not essential to use MPPT system. Similarly in case of BESSs, $I_d$ and $I_q$ can be selected as control objectives.

Feedback linearization technique may transform the PV system and BESS into a fully or partially linear one and when the orders of the transformed system equal to the orders of original system, the controller design is straightforward. But if the transformed system is a reduced-order linear system, some additional works need to be done to check the stability of untransformed dynamics (which are also known as internal dynamics) before designing the controller. To have a clear understanding about the requirement of an appropriate feedback linearizing scheme, an overview on the feedback linearizability of microgrids has been provided in the following section.

IV. FEEDBACK LINEARIZABILITY OF MICROGRIDS

The feedback linearizability of a nonlinear system determines the approach of feedback linearization that needs to be used in order to design the controller. The PV system and BESS model as described in Section II can be written in the following form of a generalized nonlinear systems

\[
\begin{align*}
\dot{x} &= f(x) + g_1(x) u_1 + g_2(x) u_2 \\
y_1 &= h_1(x) \\
y_2 &= h_2(x)
\end{align*}
\]
Since the microgrid comprises two power sources (solar PV and BESSs) to supply the load center, the feedback linearizability of these sources can be obtained by calculating the total relative degree of the system which is shown in the following two subsections [9]:

A. Feedback Linearizability of PV Systems

The PV system represented by equation (1) can written in the form of equation (5) where

\[ x = [I_d \ I_q \ \psi_{pv}]^T \]

\[ f(x) = \begin{bmatrix} \frac{-R}{L}I_d + \omega I_q - \frac{E_d}{L} \\ -\omega I_d - \frac{R}{L}I_q - \frac{E_q}{L} \end{bmatrix}, \quad g(x) = \begin{bmatrix} \frac{\psi_{pv}}{L} \\ 0 \end{bmatrix} \]

\[ u = [K_d \ K_q]^T \quad \text{and} \quad y = [I_d \ I_q]^T \]

The relative degree corresponding to \( h_1 = I_d \) can be calculated as

\[ L_d h_1 = L_d L_f^{-1} I_d = \frac{\psi_{pv}}{L} \]

where \( r_1 = 1 \). Similarly, the relative degree corresponding to \( h_2 = I_q \) can be calculated as follows:

\[ L_d h_2 = L_d L_f^{-1} I_q = \frac{\psi_{pv}}{L} \]

which indicates \( r_2 = 1 \). Therefore, \( r_1 + r_2 = 2 \) which means that \( (r_1 + r_2) < n \) as \( n = 3 \). Therefore, the PV system used in microgrids is partially linearizable for active and reactive power sharing which can be written as

\[ \begin{align*}
\dot{\bar{z}}_1 &= -\frac{R}{L}I_d + \omega I_q - \frac{E_d}{L} + \frac{\psi_{pv}}{L} K_d \bar{v}_1 \\
\dot{\bar{z}}_2 &= -\omega I_d - \frac{R}{L}I_q - \frac{E_q}{L} + \frac{\psi_{pv}}{L} K_q \bar{v}_2
\end{align*} \]  

B. Feedback Linearizability of BESSs

Similarly the BESS as described by equation (2) can be written as equation (5) in which

\[ x = [I_b \ \bar{I}_{bd} \ \bar{I}_{bq}]^T \]

\[ f(x) = \begin{bmatrix} -\frac{\bar{I}_{bd}}{L} \\ -\frac{\bar{I}_{bq}}{L} \end{bmatrix}, \quad g(x) = \begin{bmatrix} \frac{I_{bd}}{L} \\ \frac{I_{bq}}{L} \end{bmatrix} \]

\[ u = [M_d \ M_q]^T \quad \text{and} \quad y = [I_{bd} \ I_{bq}]^T \]

The relative degree for the first output function \( h_1(x) = I_{bd} \) can be calculated as

\[ L_d h_1 = L_d L_f^{-1} I_d = \frac{\psi_{dc}}{L} \]

where \( r_1 = 1 \) and that of for the second output function \( h_2(x) = I_{bq} \) can be calculated as follows:

\[ L_d h_2 = L_d L_f^{-1} I_q = \frac{\psi_{dc}}{L} \]

which indicate the total relative degree is 2 for a third-order BESS. Therefore, the BESS is also partially linearized for sharing power into a microgrid. The partially linearized BESSs can be written as

\[ \begin{align*}
\dot{\bar{z}}_{1b} &= -\frac{R}{L}I_{bd} + \omega I_{bq} - \frac{E_d}{L} + \frac{\psi_{dc}}{L} M_d = v_{1b} \\
\dot{\bar{z}}_{2b} &= -\omega I_{bd} - \frac{R}{L}I_{bq} - \frac{E_q}{L} + \frac{\psi_{dc}}{L} M_q = v_{2b}
\end{align*} \]  

(11)

The design of a nonlinear distributed controller based on the partial feedback linearization is shown in the following section.

V. DISTRIBUTED CONTROLLER DESIGN FOR MICROGRID

In this section, the distributed controller is designed for the feedback linearized microgrid which can be written as

\[ \dot{\bar{z}}_i = \tilde{A}_i \bar{z}_i + \tilde{B}_i \bar{v}_i \]  

(12)

where \( \tilde{A}_i \) is the system matrix, \( \tilde{B}_i \) is the input matrix, and \( \bar{v}_i \) is the new linear control input for \( i^{th} \) partially linearized system.

Since the feedback linearization technique decouples the microgrid into several subsystems depending on the number of distributed generators (DGs), the distributed controller can be designed by considering the communication among different subsystems. The communication among different subsystems can represented by using a directed graph (digraph) and in this paper, the digraph is represented through bidirectional communication among subsystems.

If a microgrid is represented by using a digraph \( G \) and \( A_G \) represents the adjacency matrix, the Laplacian matrix can be defined as

\[ L = \Delta - A_G \]  

(13)

where \( \Delta = \text{diag}\{d_i\} \) is the degree matrix with \( d_i = \sum_{j \in N_i} a_{ij} \). The adjacency and Laplacian matrices are used to incorporate the communication in the distributed controller among different subsystems.

To achieve the control objectives by incorporating communication among different subsystems, e.g., between different nodes \( i \) and \( j \), the control objectives for each subsystem need to be set in a cooperative manner which can be expressed as

\[ e_i = \sum_{j \in N_i} a_{ij} (Y_i - Y_j) + g_i (Y_i - Y_{net}) \]  

(14)

where \( e \) is the tracking error in terms of the local neighborhood, \( g \) is the pinning gain, \( Y \) represents the output vectors, and the subscript \( i \) represents the subsystem. If a solar PV unit is connected at the node \( i \), then \( Y_i \) can be written as

\[ Y_i = [I_{di} \ I_{qi}]^T \]  

(15)

and similarly, if a BESS is connected at the node \( j \), then

\[ Y_j = [I_{bdj} \ I_{bqj}]^T \]  

(16)

Now it is essential to obtain the distributed control law for each subsystem which can be determined by using any linear control technique. But before obtaining the control law, it is essential to analyze the stability of internal dynamics of each subsystem which can be done in a similar way as presented in [9].
If a proportional integral controller is used to achieve $e \to 0$, then the linear control law for the $i^{th}$ partially linearized subsystem can be written as

$$\tilde{v}_i = c \left( K_p + \frac{K_i}{s} \right) e_i$$  \hspace{1cm} (17)

where $K_p$ and $K_i$ are proportional and integral gain, respectively and these gain can be chosen in a similar way as presented in [9]. Since for each subsystem there are two control objectives, the boldface represents a vector to define two control laws. The coupling factor $c$ is a quantity obtained from the the graph theory which can be written as [8]

$$c \geq \frac{1}{\lambda_{\text{min}}}$$  \hspace{1cm} (18)

where $\lambda_{\text{min}} = \min_{i \in N} Re(\lambda_i)$ and $\lambda_i$ is the eigenvalue of $L+G$ with $G = \text{diag}\{g_i\}$.

The nonlinear feedback linearizing control laws for the subsystem with PV generators can be written from equation (8) as

$$K_d = \frac{v_{pu}}{L} \left[ \bar{v}_1 + \frac{R}{L} I_d - \omega I_q + \frac{E_d}{L} \right]$$

$$K_q = \frac{v_{pu}}{L} \left[ \bar{v}_2 + \omega I_d + \frac{R}{L} I_q + \frac{E_q}{L} \right]$$ \hspace{1cm} (19)

and that of for BESSs can be written as

$$M_d = \frac{v_{dc}}{L} \left[ \bar{v}_{1b} + \frac{R}{L} I_{db} - \omega I_{q} + \frac{E_d}{L} \right]$$

$$M_q = \frac{v_{dc}}{L} \left[ \bar{v}_{2b} + \omega I_{db} + \frac{R}{L} I_{dq} + \frac{E_q}{L} \right]$$ \hspace{1cm} (20)

where $v_1$, $v_2$, $v_{1b}$, and $v_{2b}$ can be calculated from from equation (17). Equations (19) and (20) are the final control laws for the microgrid for sharing active and reactive power. The performance of the designed controller has been evaluated in the following section.

VI. CONTROLLER PERFORMANCE EVALUATION

The performance of the designed controller is evaluated on a 6-bus islanded test microgrid which is shown in Fig. 1. In this microgrid, two PV units and PHEVs are supplying a load center with total load 25 kW and 16.10 kVAr and the inverters connecting both PV units are operated at 0.95 power factor. Since the distributed controller is designed in such a way that the controller designed for each DG unit has the capability to communicate with the controller connected to other DGs, the communication topology for the test microgrid can be represented in a manner as shown in Fig. 2.

The adjacency and Laplacian matrices are calculated based on the communication topology as shown in Fig. 2. To simulate the performance of the designed controller, the pinning gain for each subsystem is considered as 1 from which the corresponding coupling factor is obtained as 0.25. The performance of the designed controller is evaluated by applying a three-phase fault at the terminal of one PV unit and by changing the loads at the load center.

A. Controller performance during three-phase short-circuit fault

Three-phase short-circuit faults are the most severe faults in power systems. To simulate the performance of the distributed controller, a three-phase short-circuit fault is applied at bus 2 by considering the following fault sequence:

- Fault is applied at $t=1$ s
- Fault is cleared at $t=1.2$ s

During the faulted condition, the PV unit connected to bus 2 will not supply any active and reactive power. Thus, there will be a shortage of 10 KW active power and 3.3 kVAR reactive power. The controller connected to other buses will receive this information and the controllers connected BESSs of PHEVs will supply this power. At normal operating conditions the
BESS of each PHEV is supplying 3.5 kW and 5 kVAr. But During the faulted condition, the controller will regulate the active power of each BESS from 3.5 kW to 8.5 kW and 5 kVAr to 6.74 kVAr to maintain the power balance within the microgrid. This scenario is shown in Fig. 3 and Fig. 4 which represent the active and reactive power sharing in microgrid in the event of a three-phase short-circuit fault. When the fault is cleared, the controller with the PV unit at bus 2 will start working and the other controllers will receive this information to settle the system into the pre-fault steady-state. From Fig. 3 and Fig. 4, it can be seen that the proposed controller (solid line) maintains the power sharing in a better way as compared to an LQR-based controller.

**B. Controller performance with changes in loads**

In this case study, it is considered that the load at bus 2 is reduced to 10 kW, 6.32 kVAr at t=1 s and the microgrid continues to operate with this reduced load. Due to the utilization of maximum benefits from PV units, they will deliver power as of normal operating conditions. Thus, the controllers with BESSs need to be adjusted with reduced output power to maintain the power balance. In this situation, the BESS with each PHEV will deliver 1 kW active power and 1.62 kVAr reactive power rather than supplying 3.5 kW and 5 kVAr during normal operating conditions. Fig. 5 and Fig. 6 show active and reactive power sharing with permanent changes in loads in a microgrid from where it can be seen that the proposed control ensures the appropriate power balance (solid line).

**REFERENCES**


**VII. Conclusion**

A distributed controller is designed for islanded operation of microgrid to share active and reactive power and the designed controller adopts the flexibilities in communication topology could be useful in practical applications. The design of the proposed distributed controller does not require too many assumptions and the design process is straight-forward. The designed controller achieves the control objectives in a faster way as compared to that of an LQR-based controller as the control approach is capable of canceling the inherent nonlinearity within the system by maintaining stable internal dynamics. Simulation results clearly indicate that faster responses of the designed controller under different operating conditions. Future works will include the communication delays and uncertainties within the microgrids.