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Author

Lu, Jian-Fei, Jeng, Dong-Sheng

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A Coupled Model for an Offshore Pile, Seabed and Seawater Interaction

Jian-Fei Lu† and Dong-Sheng Jeng‡

† Department of Civil Engineering
Jiangsu University, Zhenjiang
Jiangsu, P.R.China, 212013.
ljfdoctor@yahoo.com

‡ School of Civil Engineering
University of Sydney, Sydney
NSW 2006 Australia.
d.jeng@civil.usyd.edu.au

ABSTRACT

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A coupled model is developed to investigate the dynamic interaction between an offshore pile, poroelastic seabed and sea water. The pile and the seabed are treated as saturated porous media described by Biot's dynamic theory. The sea water is regarded as an acoustic medium which is characterised by the Helmholtz equation. Three boundary element formulations are constructed for the pile, the seabed and the sea water, respectively. The three boundary element method formulations and the continuity conditions (stress and displacement) between the pile, the seabed and the sea water are used to formulate the coupled model for the system. Airy linear wave theory is used to evaluate the external force applied on the pile and the seabed due to the incident and the scattered water waves. The dynamic response of the system is calculated by the coupled boundary element method formulation. A numerical example is used to demonstrate the capacity of the coupled model.

ADDITIONAL INDEX WORDS: *linear water waves, pile, boundary element method, porous media*

INTRODUCTION

The pile foundation is a common and important structure for offshore engineering. For example, many offshore wind farms are supported by pile foundations. Moreover, various offshore platforms use piles to transfer loads to seabed. Generally, offshore piles can be divided into three parts: the lowest part which is embedded in seabed, the middle part which interacts with seawater and the upper part which is above seawater. The possible loads associated with the three parts of the pile are as follows: seismic wave loads from the seabed, wave loads and ice loads from the water region and wind loads from the upper region, say, from a wind turbine. When performing an offshore pile foundation design, two factors must be taken into account. The first factor is the design of the pile structure itself, which makes the pile foundation fulfill strength, stiffness and stability requirement. The second one is the design of the seabed, which aims to guarantee the seabeds have enough strength to resist liquefaction and shear failure.

Numerous researches have been carried out concerning the analysis of offshore piles, seabed and water waves. For example, the effects of various waves on offshore piles have been investigated by researchers, such as (MORISON *et al.*, 1950; MACCAMY and FUCHS, 1954; CHAKARABARTI and TAM, 1975). The offshore pile capacity and stresses and strain of a pile due to wave loads were addressed by TANG (1989) and EICHER *et al.* (2003), respectively. MITWALLY and NOVAK (1987) used a linear analysis to analyse dynamic interaction between pile–soil–pile when the system is subjected to a random wave loading. Moreover, using the concept of dynamic p–y curves, the response of fixed offshore platforms to wave and current loading when taking into account soil–pile interaction was addressed by MOSTAFA and NAGGAR (2004). It is worth stressing that current researches about offshore piles, seabed and seawater mainly focussed on separate aspects of the problem rather than treated them as a coupled system. However, when subjected to dynamic loads, responses of the pile, the seabed and the seawater are coupled together. Thus, a coupled model for the offshore pile, the seabed and the seawater is crucial for a successful offshore pile foundation design. Another limitation of existing researches concerning the pile-seabed interaction is that only single phase medium model is used to describe the seabed. However, it is well-known that the seabed is a porous medium saturated by seawater. Furthermore, the evaluation of the pore pressure of the seabed around offshore piles is crucial for the estimation of the risk of liquefaction and shear failure for the seabed. Consequently, it is desirable to treat seabed as a saturated porous medium and establish a coupled model for the pile, the seabed and the seawater when the system is subjected to various dynamic loads.

In this study, a coupled model is developed to investigate the interaction between the pile, the seabed and the seawater when the system is subjected to linear water waves. The pile and the seabed are treated as saturated porous media described by Biot's dynamic theory (BIOT, 1962). The dynamic response of the sea water is described by the Helmholtz equation. Three direct boundary element formulations are established for the pile, the seabed and the seawater. The continuity conditions (stress and displacement) between the pile, the seabed and the seawater are used to couple the three BEM formulations. The dynamic response of the whole system due to water waves is calculated by the proposed coupled BEM model. To demonstrate the new model, a numerical example for the coupled system will be presented.

BEM FORMULATIONS

BEM Formulations for the Pile and the Seabed

The constitutive equations for a homogeneous porous medium have the form (BIOT, 1962)

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}e - \alpha\delta_{ij}p \quad (1)$$

$$p = -\alpha Me + M\zeta \quad (2)$$

$$e = u_{i,i}, \ \zeta = -w_{i,i} \quad (3a, b)$$

where u_i and w_i denote the average solid displacement and the infiltration displacement of the pore fluid; ε_{ij}, e are the strain tensor and the dilatation of the solid skeleton; ζ is the volume of fluid injection into a unit volume of the bulk material; σ_{ij} is the stress of the bulk porous medium; p is the excess pore pressure and δ_{ij} is the Kronecker delta. Moreover, λ , μ are Lame constants of the solid skeleton; α, M are BIOT'S parameters.

The equations of motion for the bulk porous medium and a unit volume pore fluid are expressed in terms of the displacements u_i and w_i as follows (BIOT, 1962)

$$\mu u_{i,jj} + (\lambda + \alpha^2 M + \mu)u_{j,ji} + \alpha M w_{j,ji} + F_i = \rho_b \ddot{u}_i + \rho_f \ddot{w}_i \quad (4a)$$

$$\alpha\, M u_{j,ji} + M w_{j,ji} + f_i = \rho_f \ddot{u}_i + m\,\ddot{w}_i + \frac{\eta}{k}\dot{w}_i \quad (4b)$$

where ρ_b, ρ_f denote the bulk density of the porous medium and the density of the pore fluid ($\rho_b = (1-\phi)\rho_s + \phi\rho_f$), ρ_s is the density of the solid skeleton and ϕ is the porosity of the porous medium; F_i and f_i are body forces of the porous medium and the pore fluid; $m = a_\infty \rho_f / \phi$ and a_∞ is tortuosity; η and k account for the viscosity of the pore fluid and the permeability of the porous medium, respectively.

According to the analysis of BONNET (1987), although two displacement vectors are used in BIOT'S theory, there are only four independent variables in the two-phase porous medium, i.e., three displacement components for the solid skeleton and one pore pressure for the pore fluid. A four-variable-based complete boundary integral equation for saturated porous media was formulated by DOMINGUEZ (1992) and ZIMMERMAN and STERN (1993). Thus, for a saturated porous medium, the following boundary integral equation can be derived

$$c_{ij}\hat{u}_j(\mathbf{x}) = \int_\Gamma [U_{ij}^G(\mathbf{x},\xi)\hat{t}_j(\xi) - T_{ij}^G(\mathbf{x},\xi)\hat{u}_j(\xi)]d\Gamma(\xi)$$

$$i, j = 1, 2, 3, 4 \quad (5)$$

where the caret over a variable denotes the Fourier transformed variable, $\hat{u}_j(\mathbf{x})$ and $\hat{t}_j(\mathbf{x})$ are the generalised displacement and traction components (ZIMMERMAN and STERN, 1993) with $\{\hat{u}_j\big|_{j=1\sim4}\} = \{\hat{u}_1, \hat{u}_2, \hat{u}_3, \hat{p}\}$, $\{\hat{t}_j\big|_{j=1\sim4}\} = \{\hat{t}_1, \hat{t}_2, \hat{t}_3, \hat{w}_n\}$; c_{ij} depends only upon the local geometry at the $\mathbf{x}$ and reduces to a generalised delta function δ_{ij} for $\mathbf{X}$ inside Γ and to $\delta_{ij}/2$ for $\mathbf{X}$ on a smooth portion of the boundary surface; the frequency domain Green's function U_{ij}^G and T_{ij}^G can be derived as in Norris (1985) and ZIMMERMAN and STERN (1993). Moreover, $\hat{w}_n$ is the infiltration displacement of the pore fluid along the outward normal of the surface Γ and $\{\hat{t}_i\big|_{i=1\sim3}\} = \{\hat{\sigma}_{ij}n_j\big|_{i,j=1\sim3}\}$ with n_j being the direction cosine of Γ. Also, the index $i, j = 1 \sim 3$ correspond to coordinate x, y and z, respectively.

Implementation of the boundary integral equation (5) to a specific domain involves the discretisation of the boundary surface by suitable number of discrete elements, the interpolation of the field variable within an element by the node values of the element and the integration of shape function kernel product over each discrete element.

As stated above, the offshore pile is also treated as a porous medium. Thus, applying boundary integral equation (5) on the pile boundary: Γ_1, Γ_2 and Γ_3 (Figure 1) and following the procedure outlined above, the following equation is obtained

$$\mathbf{G}^{(P)} \cdot \hat{\mathbf{t}}^{(P)} = \mathbf{H}^{(P)} \cdot \hat{\mathbf{u}}^{(P)} \quad (6)$$

where the superscript P denotes the pile; $\mathbf{G}^{(P)}$ and $\mathbf{H}^{(P)}$ in (6) are the coefficient matrix obtained by integrating shape function kernel products over all the boundary elements of the pile; $\hat{\mathbf{u}}^{(P)}$ and $\hat{\mathbf{t}}^{(P)}$ are the generalised displacement and traction vectors for all the pile nodes and have the expressions

$$\hat{\mathbf{u}}^{(P)} = \{\hat{u}_1^{(P_1)}, \hat{u}_1^{(P_2)}, ...\hat{u}_1^{(P_{Np})}; \hat{u}_2^{(P_1)}, ..., \hat{u}_2^{(P_{Np})}; \hat{u}_3^{(P_1)}, ..., \hat{u}_3^{(P_{Np})};$$

$$\hat{p}^{(P_1)}, \hat{p}^{(P_2)}, ..., \hat{p}^{(P_{Np})}\}^T, \ \hat{\mathbf{t}}^{(P)} = \{\hat{t}_1^{(P_1)}, \hat{t}_1^{(P_2)}, ..., \hat{t}_1^{(P_{Np})};$$

$$\hat{t}_2^{(P_1)}, ..., \hat{t}_2^{(P_{Np})}; \hat{t}_3^{(P_1)}, ..., \hat{t}_3^{(P_{Np})}; \hat{w}_n^{(P_1)}, \hat{w}_n^{(P_2)}, ..., \hat{w}_n^{(P_{Np})}\}^T \quad (7)$$

where N_P is the total node number for the pile boundary, the subscript 1, 2 , … , N_P for P denoting the node number. Multiplying both side of equation (6) by $\mathbf{G}^{(P)^{-1}}$ and letting $\mathbf{E}^{(P)} = \mathbf{G}^{(P)^{-1}} \cdot \mathbf{H}^{(P)}$, leads to

$$\hat{\mathbf{t}}^{(P)} = \mathbf{E}^{(P)} \cdot \hat{\mathbf{u}}^{(P)} \quad (8)$$

Dividing $\hat{\mathbf{u}}^{(P)}$ and $\hat{\mathbf{t}}^{(P)}$ into three parts corresponding to Γ_1, Γ_2 and Γ_3 (Figure 1), respectively and rearranging the coefficient matrix $\mathbf{E}^{(P)}$ in (8), then, one has the following partitioned matrix equation

$$\begin{Bmatrix} \hat{\mathbf{t}}^{(P\Gamma_1)} \\ \hat{\mathbf{t}}^{(P\Gamma_2)} \\ \hat{\mathbf{t}}^{(P\Gamma_3)} \end{Bmatrix} = \begin{bmatrix} \mathbf{E}_{11}^{(P)} & \mathbf{E}_{12}^{(P)} & \mathbf{E}_{13}^{(P)} \\ \mathbf{E}_{21}^{(P)} & \mathbf{E}_{22}^{(P)} & \mathbf{E}_{23}^{(P)} \\ \mathbf{E}_{31}^{(P)} & \mathbf{E}_{32}^{(P)} & \mathbf{E}_{33}^{(P)} \end{bmatrix} \cdot \begin{Bmatrix} \hat{\mathbf{u}}^{(P\Gamma_1)} \\ \hat{\mathbf{u}}^{(P\Gamma_2)} \\ \hat{\mathbf{u}}^{(P\Gamma_3)} \end{Bmatrix} \quad (9)$$

where the superscript $\Gamma_i, i = 1,2,3$ for P denote the boundary surface $\Gamma_i, i = 1,2,3$, $\hat{\mathbf{u}}^{(P\Gamma_i)}$, $\hat{\mathbf{t}}^{(P\Gamma_i)}, i = 1,2,3$ denote the generalised pile displacement and pile traction vectors for the boundary Γ_1 , Γ_2 and Γ_3 , respectively and have the following expressions

$$\hat{\mathbf{u}}^{(P\Gamma i)} = \{\hat{u}_1^{(P_1^{\Gamma i})}, \hat{u}_1^{(P_2^{\Gamma i})}, ..., \hat{u}_1^{(P_{Npi}^{\Gamma i})}; \hat{u}_2^{(P_1^{\Gamma i})}, ..., \hat{u}_2^{(P_{Npi}^{\Gamma i})}; \hat{u}_3^{(P_1^{\Gamma i})}, ..., \hat{u}_3^{(P_{Npi}^{\Gamma i})};$$

$$\hat{p}^{(P_1^{\Gamma i})}, \hat{p}^{(P_2^{\Gamma i})}, ..., \hat{p}^{(P_{Npi}^{\Gamma i})}\}^T, \ \hat{\mathbf{t}}^{(P\Gamma i)} = \{\hat{t}_1^{(P_1^{\Gamma i})}, \hat{t}_1^{(P_2^{\Gamma i})}, ..., \hat{t}_1^{(P_{Npi}^{\Gamma i})};$$

$$\hat{t}_2^{(P_1^{\Gamma i})}, ..., \hat{t}_2^{(P_{Npi}^{\Gamma i})}; \hat{t}_3^{(P_1^{\Gamma i})}, ..., \hat{t}_3^{(P_{Npi}^{\Gamma i})}; \hat{w}_n^{(P_1^{\Gamma i})}, \hat{w}_n^{(P_2^{\Gamma i})}, ..., \hat{w}_n^{(P_{Npi}^{\Gamma i})}\}^T,$$

$$i = 1, 2, 3 \quad (10)$$

where N_{Pi} is the total node number for the surface Γ_i . It is worth pointing out that to round off the corners of the seabed and the seawater, the surfaces Γ_1 , Γ_2 and Γ_3 of the pile are separated by a short segment of pile surface, respectively.

Likewise, using the same integral equation (5) on the seabed and following the similar procedure, a partitioned matrix equation can also be established for the seabed

$$\begin{Bmatrix} \hat{\mathbf{t}}^{(S\Gamma_1)} \\ \hat{\mathbf{t}}^{(S\Gamma_4)} \end{Bmatrix} = \begin{bmatrix} \mathbf{E}_{11}^{(S)} & \mathbf{E}_{14}^{(S)} \\ \mathbf{E}_{41}^{(S)} & \mathbf{E}_{44}^{(S)} \end{bmatrix} \cdot \begin{Bmatrix} \hat{\mathbf{u}}^{(S\Gamma_1)} \\ \hat{\mathbf{u}}^{(S\Gamma_4)} \end{Bmatrix} \quad (11)$$

where the superscript S denotes the seabed, and $\hat{\mathbf{u}}^{(S^{\Gamma i})}$, $\hat{\mathbf{t}}^{(S^{\Gamma i})}, i = 1,4$ denote the generalised seabed displacement and seabed traction vectors at the boundary Γ_1 and Γ_4 (Figure 1), which have similar expressions as (10).

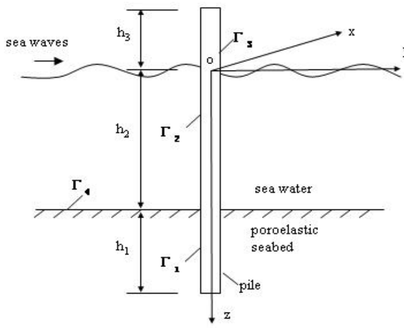

Figure 1 An offshore pile embedded in a poroelastic seabed and subjected to sea waves

BEM Formulation for the Seawater

As mentioned previously, the pile, the seabed and the seawater here are treated as a coupled forced-vibration system subjected to external force due to water waves. In view of this, the seawater here can be considered as an acoustic medium characterised by the following Helmholtz equation

$$\nabla^2 \hat{p} + k^2 \hat{p} = 0 \qquad (12)$$

where $\hat{p}$ is the pressure of the seawater and k is the wave-number ($k = \omega / v$), ω is the frequency of the wave motion and v is the acoustic velocity of the seawater. The displacement of the seawater has the form

$$\hat{\mathbf{u}}^{(W)} = \frac{1}{\rho_w \omega^2} \nabla \hat{p} \qquad (13)$$

where ρ_w is the density of the seawater. For the seawater above the seabed, due to the presence of the infinite free surface of the water layer, thus, to save computational time a boundary integral equation which can avoid the discretisation of the free surface is preferable. Consequently, the boundary integral equation and Green's function due TO SEYBERT and WU (1989) are used in this research

$$c\ \hat{p}(\mathbf{x}) = \int_{\Gamma} [\, p^G(\mathbf{x},\xi)\hat{q}(\xi) - q^G(\mathbf{x},\xi)\hat{p}(\xi)]\, d\Gamma(\xi) \quad (14)$$

where p^G and q^G are the Green's function for the half space, c depends upon the local geometry at the point $\mathbf{x}$ and $\hat{q}(\xi) = \partial \hat{p}(\xi) / \partial n(\xi)$, $n(\xi)$ is the outward normal for the boundary Γ .

Using the boundary integral equation (14) to the seawater, the following partitioned matrix equation is obtained

$$\begin{Bmatrix} \hat{\mathbf{u}}^{(W^{\Gamma 2})} \\ \hat{\mathbf{u}}^{(W^{\Gamma 4})} \end{Bmatrix} = \begin{bmatrix} \mathbf{E}_{22}^{(W)} & \mathbf{E}_{24}^{(W)} \\ \mathbf{E}_{42}^{(W)} & \mathbf{E}_{44}^{(W)} \end{bmatrix} \cdot \begin{Bmatrix} \hat{\mathbf{p}}^{(W^{\Gamma 2})} \\ \hat{\mathbf{p}}^{(W^{\Gamma 4})} \end{Bmatrix} \qquad (15)$$

where the superscript W denotes the seawater and $\hat{\mathbf{u}}^{(W^{\Gamma i})}$, $\hat{\mathbf{p}}^{(W^{\Gamma i})}$, $i = 2,4$ denote the normal seawater displacement and the seawater pressure vectors at the boundary Γ_2 and Γ_4 and have the following form

$$\hat{\mathbf{u}}^{(W^{\Gamma i})} = \{\hat{u}_n^{(W_1^{\Gamma i})}, \hat{u}_n^{(W_2^{\Gamma i})}, \ldots, \hat{u}_n^{(W_{Npi}^{\Gamma i})}\},\ \hat{\mathbf{p}}^{(W^{\Gamma i})} = \{\hat{p}^{(W_1^{\Gamma i})}, \hat{p}^{(W_2^{\Gamma i})}, \ldots, \hat{p}^{(W_{Npi}^{\Gamma i})}\} \ (16)$$

where $i = 2,4$ and $\hat{u}_n^{(W_j^{\Gamma i})}$, $\hat{p}^{(W_j^{\Gamma i})}$ denote the seawater displacement along the normal direction at j-th point of the surface Γ_i and the seawater pressure at the same point.

COUPLED BEM MODEL FOR THE PILE, THE SEABED AND THE SEAWATER

In above section, three discrete boundary formulations are established for the pile, the seabed and the seawater. In this section, the three discrete boundary formulations and the interface conditions between the pile, the seabed and the seawater are used to construct the coupled boundary element formulation for the system.

Continuity conditions between the three domains

The continuity conditions at the common boundary Γ_1 between the pile and the seabed (Figure 1) have the following form (DERESIEWICZ & SKALAK, 1963)

$$\hat{\mathbf{u}}^{(P^{\Gamma 1})} = \hat{\mathbf{u}}^{(S^{\Gamma 1})},\ \ \hat{\mathbf{t}}^{(P^{\Gamma 1})} = -\hat{\mathbf{t}}^{(S^{\Gamma 1})} \qquad (17)$$

The continuity conditions between the pile and the seawater at their common boundary Γ_2 (Figure 1) have the following form (STOLL and KAN, 1981)

$$\hat{\mathbf{p}}^{(P^{\Gamma 2})} = \hat{\mathbf{p}}^{(W^{\Gamma 2})},\ \ \hat{\mathbf{u}}_n^{(P^{\Gamma 2})} + \hat{\mathbf{w}}_n^{(P^{\Gamma 2})} = -\hat{\mathbf{u}}^{(W^{\Gamma 2})},\ \hat{\mathbf{t}}_x^{(P^{\Gamma 2})} = \mathbf{n}_x^{(W^{\Gamma 2})} \cdot \hat{\mathbf{p}}^{(W^{\Gamma 2})} + \hat{\bar{\mathbf{t}}}_x^{(P^{\Gamma 2})},$$

$$\hat{\mathbf{t}}_y^{(P^{\Gamma 2})} = \mathbf{n}_y^{(W^{\Gamma 2})} \cdot \hat{\mathbf{p}}^{(W^{\Gamma 2})} + \hat{\bar{\mathbf{t}}}_y^{(P^{\Gamma 2})},\ \hat{\mathbf{t}}_z^{(P^{\Gamma 2})} = \mathbf{0} \qquad (18)$$

where $\hat{\mathbf{p}}^{(P^{\Gamma 2})}$, $\hat{\mathbf{u}}_n^{(P^{\Gamma 2})}$s and $\hat{\mathbf{w}}_n^{(P^{\Gamma 2})}$ are the pile pore pressure, the normal pile displacement of the solid frame and the normal infiltration displacement of the pile pore fluid at Γ_2 ; $\hat{\mathbf{t}}_x^{(P^{\Gamma 2})}$, $\hat{\mathbf{t}}_y^{(P^{\Gamma 2})}$, $\hat{\mathbf{t}}_z^{(P^{\Gamma 2})}$ are the traction for the x, y and z direction at the pile surface Γ_2 ; $\mathbf{n}_x^{(W^{\Gamma 2})}$, $\mathbf{n}_y^{(W^{\Gamma 2})}$ are the direction cosine matrix along the x and y direction for the seawater surface Γ_2 ; $\hat{\bar{\mathbf{t}}}_x^{(P^{\Gamma 2})}$, $\hat{\bar{\mathbf{t}}}_y^{(P^{\Gamma 2})}$ are the tractions along x , y direction applied on the pile surface Γ_2 due to the incident wave and the scattered wave. Using $\hat{\boldsymbol{\chi}}^{(P^{\Gamma 2})}$ to represent the vectors $\hat{\mathbf{p}}^{(P^{\Gamma 2})}$, $\hat{\mathbf{u}}_n^{(P^{\Gamma 2})}$, $\hat{\mathbf{w}}_n^{(P^{\Gamma 2})}$, $\hat{\mathbf{t}}_x^{(P^{\Gamma 2})}$, $\hat{\mathbf{t}}_y^{(P^{\Gamma 2})}$, $\hat{\mathbf{t}}_z^{(P^{\Gamma 2})}$, $\hat{\bar{\mathbf{t}}}_x^{(P^{\Gamma 2})}$ and $\hat{\bar{\mathbf{t}}}_y^{(P^{\Gamma 2})}$, then, the above vectors have the following uniform expression

$$\hat{\boldsymbol{\chi}}^{(P^{\Gamma 2})} = \{\hat{\chi}^{(P_1^{\Gamma 2})}, \hat{\chi}^{(P_2^{\Gamma 2})}, \ldots, \hat{\chi}^{(P_{Np2}^{\Gamma 2})}\} \qquad (19)$$

Note that the vector $\hat{\mathbf{u}}_n^{(P^{\Gamma 2})}$ can be calculated by the pile surface displacement along Γ_2 and the corresponding direction cosine. The direction cosine vectors $\mathbf{n}_x^{(W^{\Gamma 2})}$, $\mathbf{n}_y^{(W^{\Gamma 2})}$ for the Γ_2 of the seawater have the expression

$$\mathbf{n}_j^{(W^{\Gamma 2})} = \{n_j^{(W_1^{\Gamma 2})}, n_j^{(W_2^{\Gamma 2})}, \ldots, n_j^{(W_{Np2}^{\Gamma 2})}\},\quad j = x,\ y \qquad (20)$$

The continuity conditions between the seabed and the seawater at the common boundary Γ_4 (Figure 1) have the following form (STOLL and KAN, 1981)

$$\hat{\mathbf{p}}^{(S^{\Gamma 4})} = \hat{\mathbf{p}}^{(W^{\Gamma 4})},\ \hat{\mathbf{u}}_z^{(S^{\Gamma 4})} + \hat{\mathbf{w}}_n^{(S^{\Gamma 4})} = -\hat{\mathbf{u}}^{(W^{\Gamma 4})},\ \hat{\mathbf{t}}_x^{(S^{\Gamma 4})} = \mathbf{0}$$

$$\hat{\mathbf{t}}_y^{(S^{\Gamma 4})} = 0,\ \hat{\mathbf{t}}_z^{(S^{\Gamma 4})} = \hat{\mathbf{p}}^{(W^{\Gamma 4})} + \hat{\bar{\mathbf{t}}}_z^{(S^{\Gamma 4})} \qquad (21)$$

where $\hat{\bar{\mathbf{t}}}_z^{(S^{\Gamma 4})}$ are the z direction traction on Γ_4 of the seabed due to the incident and the scattered water wave.

Coupled BEM Formulation for the System

Using the continuity conditions (17) between the pile and the seabed on Γ_1 and equations (9) and (11), the following equation for Γ_1 is obtained

$$(\mathbf{E}_{11}^{(P)}+\mathbf{E}_{11}^{(S)})\cdot\hat{\mathbf{u}}^{(P^{\Gamma_1})}+\mathbf{E}_{12}^{(P)}\cdot\hat{\mathbf{u}}^{(P^{\Gamma_2})}+\mathbf{E}_{13}^{(P)}\cdot\hat{\mathbf{u}}^{(P^{\Gamma_3})}+\mathbf{E}_{14}^{(S)}\cdot\hat{\mathbf{u}}^{(S^{\Gamma_4})}=\mathbf{0} \quad (22)$$

Using the continuity conditions (18) and equations (9) and (15), the following equation is obtained for the boundary Γ_2

$$[\mathbf{E}_{21}^{(P)},\mathbf{E}_{22}^{(P)},\mathbf{E}_{23}^{(P)}]\cdot\begin{Bmatrix}\hat{\mathbf{u}}^{(P^{\Gamma_1})}\\ \hat{\mathbf{u}}^{(P^{\Gamma_2})}\\ \hat{\mathbf{u}}^{(P^{\Gamma_3})}\end{Bmatrix}+\begin{Bmatrix}-\mathbf{n}_x^{(W^{\Gamma_2})}\cdot\hat{\mathbf{p}}^{(W^{\Gamma_2})}\\ -\mathbf{n}_y^{(W^{\Gamma_2})}\cdot\hat{\mathbf{p}}^{(W^{\Gamma_2})}\\ \mathbf{0}\\ -\mathbf{n}_x^{(W^{\Gamma_2})}\cdot\hat{\mathbf{u}}_x^{(P^{\Gamma_2})}-\mathbf{n}_y^{(W^{\Gamma_2})}\cdot\hat{\mathbf{u}}_y^{(P^{\Gamma_2})}\end{Bmatrix}$$

$$-[\mathbf{0},\mathbf{0},\mathbf{0},\mathbf{E}_{22}^{(w)}\cdot\hat{\mathbf{p}}^{(P^{\Gamma_2})}+\mathbf{E}_{24}^{(w)}\cdot\hat{\mathbf{p}}^{(S^{\Gamma_4})}]^T=[\hat{\hat{\mathbf{t}}}_x^{(P^{\Gamma_2})},\hat{\hat{\mathbf{t}}}_y^{(P^{\Gamma_2})},\mathbf{0},\mathbf{0}]^T \quad (23)$$

where $\hat{\mathbf{u}}_x^{(P^{\Gamma_2})}$, $\hat{\mathbf{u}}_y^{(P^{\Gamma_2})}$ have the expression

$$\hat{\mathbf{u}}_j^{(P^{\Gamma_2})}=\{\hat{u}_j^{(P_1^{\Gamma_2})},\hat{u}_j^{(P_2^{\Gamma_2})},\ldots,\hat{u}_j^{(P_{Np2}^{\Gamma_2})}\},\ j=x,y \quad (24)$$

Besides, for the boundary Γ_3, it is assumed that $\hat{\mathbf{t}}^{(P^{\Gamma_3})}$ is known a prioir and thus using (9), one has the following equation

$$[\mathbf{E}_{31}^{(P)},\mathbf{E}_{32}^{(P)},\mathbf{E}_{33}^{(P)}]\cdot[\hat{\mathbf{u}}^{(P^{\Gamma_1})},\hat{\mathbf{u}}^{(P^{\Gamma_2})},\hat{\mathbf{u}}^{(P^{\Gamma_3})}]^T=\hat{\mathbf{t}}^{(P^{\Gamma_3})} \quad (25)$$

For the boundary Γ_4, using the continuity condition (21) and equations (11) and (15), the following matrix equation is obtained

$$[\mathbf{E}_{41}^{(S)},\mathbf{E}_{44}^{(S)}]\cdot\begin{Bmatrix}\hat{\mathbf{u}}^{(P^{\Gamma_1})}\\ \hat{\mathbf{u}}^{(S^{\Gamma_4})}\end{Bmatrix}+[\mathbf{0},\mathbf{0},-\hat{\mathbf{p}}^{(S^{\Gamma_4})},\hat{\mathbf{u}}_z^{(S^{\Gamma_4})}]^T$$

$$-[\mathbf{0},\mathbf{0},\mathbf{0},\mathbf{E}_{42}^{(W)}\cdot\hat{\mathbf{p}}^{(P^{\Gamma_2})}+\mathbf{E}_{44}^{(w)}\cdot\hat{\mathbf{p}}^{(S^{\Gamma_4})}]^T=[\mathbf{0},\mathbf{0},\hat{\hat{\mathbf{t}}}_z^{(S^{\Gamma_4})},\mathbf{0}]^T \quad (26)$$

Combination of equations (22)–(23) and (25)–(26) yields all the equations for the unknowns $\hat{\mathbf{u}}^{(P^{\Gamma_1})}$, $\hat{\mathbf{u}}^{(P^{\Gamma_2})}$, $\hat{\mathbf{u}}^{(P^{\Gamma_3})}$ and $\hat{\mathbf{u}}^{(S^{\Gamma_4})}$. Once $\hat{\mathbf{u}}^{(P^{\Gamma_1})}$, $\hat{\mathbf{u}}^{(P^{\Gamma_2})}$, $\hat{\mathbf{u}}^{(P^{\Gamma_3})}$ and $\hat{\mathbf{u}}^{(S^{\Gamma_4})}$ are determined, unknown tractions along the surface of the boundary can be evaluated by (9) and (11).

It is worth emphasising that when calculating the external loads due to the incident wave and the scattered wave, the pile and the seabed are assumed to be fixed, thus, $\hat{\hat{\mathbf{t}}}_x^{(P^{\Gamma_2})}$, $\hat{\hat{\mathbf{t}}}_y^{(P^{\Gamma_2})}$ and $\hat{\hat{\mathbf{t}}}_z^{(S^{\Gamma_4})}$ can be evaluated uniquely by the pressure of the incident and the scattered water wave. Also, compared with $\hat{\hat{\mathbf{t}}}_x^{(P^{\Gamma_2})}$ and $\hat{\hat{\mathbf{t}}}_y^{(P^{\Gamma_2})}$, the influence of $\hat{\hat{\mathbf{t}}}_z^{(S^{\Gamma_4})}$ is negligible, so it is neglected in this study.

NUMERICAL EXAMPLES

In this example, the embedded depth of the pile inside the seabed is $h_1=15$ m and the depth of the water is $h_2=15$ m. The length of the pile which is above the water is $h_3=1$ m and the diameter of the pile is $d=0.6$ m (Figure 1). The $\hat{\mathbf{t}}^{(P^{\Gamma_3})}$ along the Γ_3 of the pile is supposed to vanish. The water wave is a linear harmonic wave with angular frequency being 1 1/s and 3 1/s, respectively and the wave height is 2 m. The parameters for the seabed and the corresponding fluid assume the following values: $\mu=1.0\times10^7$ Pa, $\lambda=2.0\times10^7$ Pa, $\rho_s=2.0\times10^3$ kg/m^2, $\rho_f=1.0\times10^3$ kg/m^3, $\phi=0.3$, $\eta=1.0\times10^{-3}$ Pa.s, $k=1.0\times10^{-12}$ m^2, $\alpha=0.85$, $M=1.0\times10^7$ Pa, $a_\infty=3$. For the pile, $\mu=1.0\times10^8$ Pa, $\lambda=2.0\times10^8$ Pa, $\rho_s=2.3\times10^3$ kg/m^3, $\phi=0.25$, $k=1.0\times10^{-13}$ m^2. The other parameters for the pile and its saturating fluid assume the same values as those for the seabed. The density and the acoustic velocity of the seawater are equal to 1.0×10^3 kg/m^3 and 1400 m/s, respectively.

The shear force and the horizontal displacement of the pile for the two cases with $\omega=1$ and 3 1/s are plotted in Figures 2 and 3, respectively. The pore pressure of the seabed along the pile side with the coordinates $x=0, y=-0.3$ m and $z=15.0\sim30.0$ m is shown in Figure 4.

Figure 2 shows that the shear force reaches its maximum at the interface between the seawater and the seabed. Also, for the upper part of the pile the shear force for the case of $\omega=3$ 1/s is larger than that for the case of $\omega=1$ 1/s, while for the lower part of the pile, opposite tendency occurs. For the horizontal displacement, the displacement for the case of $\omega=1$ 1/s is larger than that of the case with $\omega=3$ 1/s. Moreover, the $\omega=1$ 1/s case achieves its maximum displacement at the upper part of the pile, while for the $\omega=3$ 1/s case, the maximum displacement occurs around $z/(h_1+h_2)=0.37$.

Figure 4 indicates that the pore pressure for the case of $\omega=3$ 1/s is slightly larger than that for the case of $\omega=1$ 1/s. Moreover, the pore pressure decreases sharply with increasing depth of the seabed. Thus, the upper part of the seabed has a high risk of liquefaction and shear failure due to the larger value of the pore pressure.

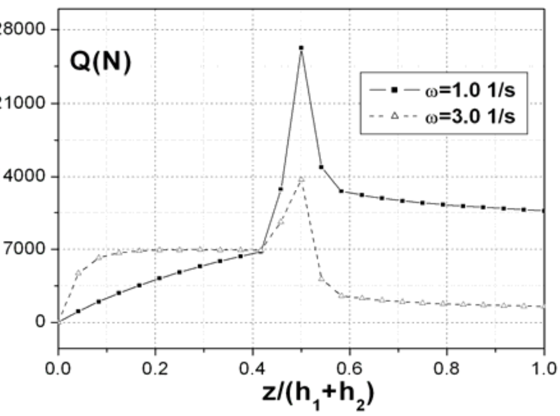

Figure 2. The shear force of the pile subjected to linear water waves with wave height being 2 m and $\omega=1,\ 3$ s^{-1}.

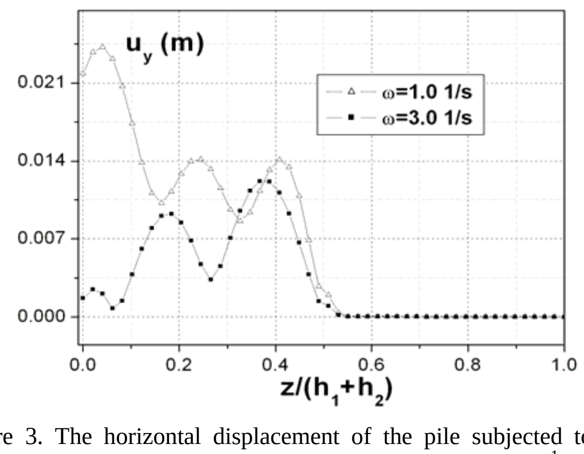

Figure 3. The horizontal displacement of the pile subjected to linear water waves with wave height being 2 m and $\omega=1,\ 3$ s^{-1}.

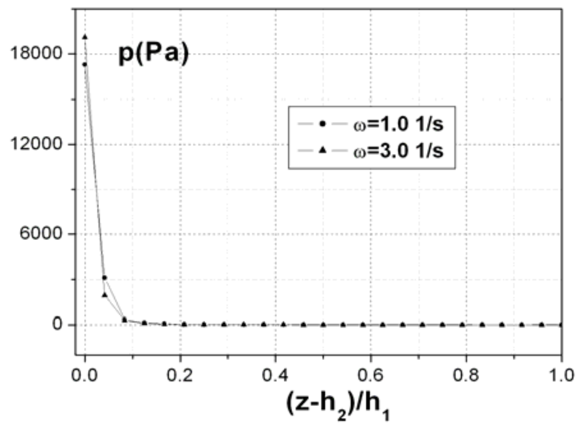

Figure 4 The pore pressure of the seabed along the pile side with the coordinates $x = 0, y = -0.3$ m and $z = 15.0 \sim 30.0$ m when the pile is subjected to linear water waves with wave height being 2 m and $\omega = 1, 3$ s^{-1}.

CONCLUSION

A frequency domain coupled model accounting for the dynamic interaction between the pile, the seabed and the seawater has been developed in this paper. The coupled model is based on the boundary element method for the porous medium and the acoustic medium as well as the continuity conditions at the interface between the three domains. Although only the linear harmonic wave in the frequency domain is considered in the paper, using the Fourier transform method, random waves in the time domain can also be taken into account by our model. Moreover, the new model can also be used to investigate the dynamic response of the system to nonlinear water waves, seismic waves, wind loads or ice loads. Also, the current model can be extended to solve the problem of the interaction between seabed, pile groups and seawaves.

A numerical example demonstrating the response of the coupled system to linear sea waves is given in the paper, which indicates that wave loads can generate considerable pore pressure at some parts of the seabed. Clearly, the obtained pore pressure will facilitate the liquefaction and the shear failure analysis for the seabed.

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