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Published

2007

Conference Title

Proceedings of the 20th international congress on condition monitoring and diagnostic engineering management (COMADEM 2007)

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MODELLING COST SHARING POLICIES FOR LIFETIME WARRANTY

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ABSTRACT

In today's fiercely competitive market, the warranty period offered by the manufacturer/dealer has been progressively increasing. Estimation of warranty cost is becoming complex because of factors such as uncertainty of cost of parts, labour, downtime and ever increasing introduction of new products into the market. Manufacturer or dealers have started offering lifetime warranties for their products. Uncertainties of product failure and rectification costs are reduced by cost sharing policies. Formulation of attractive policies and estimation of costs for these warranties is important to the manufacturer/dealer for estimating future costs to build it into the sales price or contracts. This paper deals with four new cost sharing lifetime warranty policies and stochastic models for estimation of costs associated with offering these policies.

KEYWORDS:

Lifetime warranties, Cost sharing warranty policies, Cost models,

1.0 INTRODUCTION

A warranty is a contractual obligation of a manufacturer/dealer in connection with sale of a product where a manufacturer/dealer takes the responsibility to rectify defects or failures of products due to design, manufacturing and quality problems over the warranty period. The manufacturer/dealer rectifies the defects at no or a fraction of the rectification cost to buyers or refunds full or part of sale price to the buyer as per warranty terms. Warranty limits liabilities for both manufacturer and consumer in the event of premature failure. It protects manufacturer by limiting the manufacturer's liability for the product failure and at the same time it acts as a promotional banner for product quality and reliability. To the consumer it provides signals with information on the reliability and quality of the product and acts as an insurance against the early failure of the product. To the consumer, longer terms of warranty mean a peace of mind.

The warranty period offered by the manufactures or dealers, in recent years has been progressively increasing with time(Murthy and Jack 2003). In early days of the last Century, warranty period of a new

product was three/six months which became two to three years at the end of the Century. Products are being sold with long-term warranty policies in the form of lifetime warranty policies and service contracts. Some manufacturer/dealers have started offering cost sharing lifetime warranty where the rectification costs of the failed or defective items are shared by the manufacturer/dealers and the consumers. Better warranty terms have a positive promotional and protection point of view but the increased cost has a reduced probability of sale (Chattopadhyay and Murthy, 2001). McGuire (1980) showed that the servicing of warranty results additional cost of 1 to 15 percent of net sale. .

This paper is on developing cost sharing lifetime warranty policies and cost analysis for those policies.

The outline of this paper is as follows: in Section 2, details of the lifetime warranty policies are discussed. Cost sharing policies are defined in Section 3. In Section 4, cost models are developed. In Section 5, the analysis of the developed models is carried out. Finally, the contribution of this paper and some topics for further research are discussed.

2.0 LIFETIME WARRANTY POLICIES

Lifetime warranty is becoming more and more important due to its application to longer life assets and enhanced customer demand on service from a product. Despite its higher popularity and industrial potentiality, the area of lifetime warranty policy has received only a limited attention. Wells (1987) first proposed a computationally tractable expression for the total expected discounted future cost for a lifetime warranty policy. Chattopadhyay and Rahman (2004) and Rahman (2007) developed the taxonomy for lifetime warranty policies and developed cost models for free rectification lifetime policies.

The concept of lifetime (useful life) is not clear in the warranty literatures and can be a source of confusion to the consumers (The Magnuson-Moss Warranty Act, 1975). This is because it is often difficult to tell just whose life measures the period of. Rahman and Chattopadhyay (2006) first defined lifetime warranty as the manufacturer/dealer's commitment to provide free or cost sharing maintenance service such as repair or replacement of the sold product throughout the useful life of the product or the buyer's ownership period of the product. The useful lifetime is defined as the product is in the market and can be terminated in some finite, random time horizon. Outdated technology is not covered by lifetime warranty. Termination of such warranty may arise from the technological obsolescence, design modifications, change of component, or failure of a critical part that is not under coverage or change of the ownership (Rahman and Chattopadhyay, 2004). Therefore, the useful life could be:

Technical life/ Physical life – the period over which the product might be expected to last physically, to when replacement or major rehabilitation is physically required.

Technological life – the period until technological obsolescence dictating replacement due to the development of a technologically superior alternative

Commercial life/ Economic life – – the period over which the need for the product exists, the period until economic obsolescence dictates replacement with a lower cost alternative.

Ownership life/ Social and legal life – the period until human desire or legal requirement dictates replacement or change of ownership occurs.

3.0 COST SHARING LIFETIME WARRANTY

Under cost sharing lifetime warranties, the manufacturer/dealer and the consumer shares the cost of rectification. The basis for cost sharing can vary from product to product. These policies are currently offered by the manufacturers/dealers and would be of potential interest. The policies are:

Policy 1 - Specific Parts Excluded Lifetime Warranty [SPELTW]: Under this policy, the components of the product are grouped into two disjoint sets, Set-I (for inclusion) and Set-E (for exclusion). Here, the manufacturer/dealer rectifies failed components belonging to Set-I at no cost to the buyer over the defined

lifetime of the product. The costs of rectifying the failed components belonging to Set-E are borne by the customer. (Note: The rectification of failed components belonging to set E can be carried out either by the dealer or a third party).

Policy 2 - Limit on Individual Cost Lifetime Warranty [LICLTW]: Under this policy, if the cost of a rectification on each occasion is below the limit c_I , then it is borne completely by the manufacturer/dealer and the customer pays nothing. If the cost of a rectification exceeds c_I , the buyer pays all the costs in excess of c_I (i.e cost of rectification - c_I). This continues until the termination of lifetime.

Policy 3 - Limit on Total Cost Lifetime Warranty [LTCLTW]: Under this policy the manufacturer/dealer's obligation ceases when the total repair cost over the lifetime exceeds c_T . As a result the warranty ceases at an uncertain lifetime L or earlier if the total repair cost, at any time during the lifetime, exceeds a prefixed cut off cost c_T . Here, the warranty coverage is uncertain not only for uncertainty in exceeding the total cost limit, but also for the uncertainty of lifetime.

Policy 4 - Limit on Individual and Total Cost Lifetime Warranty [LITLTW]: Under this policy, the cost to the manufacturer/dealer has an upper limit (c_I) for each rectification and the warranty ceases when the total cost to the dealer (subsequent to the sale) exceeds a cut off cost c_T or the termination of the product life due to the defined reasons, whichever occurs first. The customer pays the difference between rectification cost and the dealer's cost if the individual rectification cost exceeds cost limit c_I .

4.0 MODELLING COSTS

4.1 Assumptions:

For the purpose of model simplification, the following assumptions are made. One can relax one or more assumptions, but these result in complexity in analysis. The assumptions are:

- Item failures are statistically independent;
- Item failure, in a probabilistic sense, is only a function of its age;
- The time to carry out a rectification action by repair or replacement is negligible compared to the mean time between failures and this time can be ignored;
- An item failure results in an immediate claim and all claims are valid;

4.2 Modelling product failures

We assume the failure rate of product is an increasing function of age. Since the number of failed components rectified at each failure is very small relative to the total number of components, the failure rate after a repair is nearly the same as that just before the failure, called a minimal repair (Barlow and Hunter, 1960). Failures are modelled as the Non-Homogeneous Poisson's Process (NHPP). Let $\Lambda(t)$ be the failure intensity associated with the failure distribution for the product. Failure cumulative probability distribution function $F(t)$ associated with density function $f(t)$ is as follows:

$$F(t) = 1 - \exp(-(\lambda t)^\beta) \quad (1)$$

$$f(t) = \frac{d[F(t)]}{dt} = \lambda\beta(\lambda t)^{\beta-1} \exp(-(\lambda t)^\beta) \quad (2)$$

And

$$\Lambda(t) = \frac{f(t)}{1 - F(t)} = \frac{\lambda\beta(\lambda t)^{\beta-1} \exp(-(\lambda t)^\beta)}{1 - (1 - \exp(-(\lambda t)^\beta))} = \lambda\beta(\lambda t)^{\beta-1} \quad (3)$$

with the parameters $\beta > 1$ and $\lambda > 0$.

4.3 Modelling Uncertainties of Lifetime

The upper limit of the coverage is uncertain since the termination of life is random and unknown (see Figure 1). Conditioned on the coverage $L = a$, one can capture this uncertainty by binding lower limit l and upper limit u . One can model this as a random variable with a distribution function $H(a)$ with $H(l) = P(a \leq l) = 0$ and $H(u) = P(a \leq u) = 1$

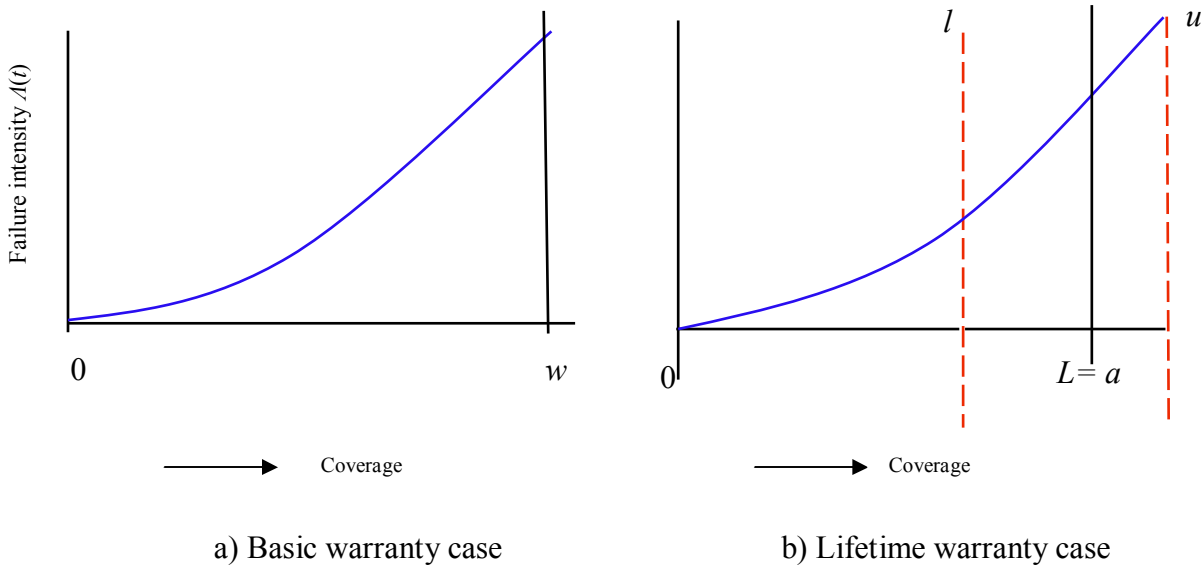


Figure 1: Failure intensity over the non lifetime and lifetime warranty coverage period

$h(a)$ is the probability density function of coverage period a associated with $H(a)$ and

$$h(a) = \frac{dH(a)}{da} \tag{4}$$

One form of $H(a)$, truncated exponential distribution (see Chattopadhyay and Murthy, 2000):

$$H(a) \text{ is } \frac{e^{-\rho l} - e^{-\rho a}}{e^{-\rho l} - e^{-\rho u}} \text{ which gives a } h(a) = \frac{\rho e^{-\rho a}}{e^{-\rho l} - e^{-\rho u}} \tag{5}$$

The mean value of useful life of the sold product can be expressed by

$$\mu_L = E(a) = \frac{(le^{-\rho l} - ue^{-\rho u}) + (e^{-\rho l} - e^{-\rho u})/\rho}{e^{-\rho l} - e^{-\rho u}} \tag{6}$$

4.4 Modelling uncertainty of rectification cost

For exponential distribution case with cost parameter γ , distribution function $G(c)$ can be expressed by

$$G(c) = 1 - e^{-\gamma c} \tag{7}$$

Then the expected cost of each rectification with cost parameter γ is given by

$$\bar{c} = [1/\gamma] \tag{8}$$

Moreover the cost can vary subsequently in the longer uncertain period of a contract and a negotiation clause can possibly be included. For a very long product life, a cost trend and discounting factor can be included in the cost model.

4.5 Model 1: Specified Parts Exclusive Lifetime Warranty (Policy 1):

The components of the item are grouped into two disjointed sets; I and E. Failures of components belonging to Set I are covered by lifetime free replacement warranty and those belonging to Set E are not covered under warranty. Let $N_I(l, u)$ and $N_E(l, u)$ denote the number of failures over the lifetime warranty period. One can model the failures during warranty using two different approaches. We assume that the total costs are not affected by the inflation or the discounting of money.

Approach 1

Here the failures are modelled by considering point process with intensity function $\Lambda(t)$. However, with each point (corresponding to a failure) there is a mark which indicates whether the item failure is covered under warranty or not. This mark is modelled as a binary random variable Y with $Y = 1$ indicating that the failure is covered and $Y = 0$ indicating that the failure is not covered under warranty.

Let $P\{Y = 0\} = p$ and $P\{Y = 1\} = 1 - p = q$

In this case, the expected number of failures of the components of the sold product that are covered with free replacement warranty over the lifetime is given by

$$E[N_I(l, u)] = \int_l^u \left\{ q \int_0^a \Lambda(t) dt \right\} h(a) da \quad (9)$$

where, l and u are the lower and upper limits of the useful life distribution and a is the lifetime coverage of the product.

Similarly, the expected number of failures of components that are not covered with free replacement under this warranty policy is given by

$$E[N_E(l, u)] = \int_l^u \left\{ p \int_0^a \Lambda(t) dt \right\} h(a) da \quad (10)$$

Approach 2

Here, the component failures belonging to Set I are modelled by an intensity function $\Lambda_I(t)$ and those belonging to Set E are modelled by another intensity function $\Lambda_E(t)$. Both of these are increasing functions of time t . Then $N_I(L)$ and $N_E(L)$ are distributed according to non-stationary Poisson processes with intensity functions $\Lambda_I(t)$ and $\Lambda_E(t)$ respectively. In this case, the expected number of failures over the useful life of the products can be expressed as

$$E[N_I(l, u)] = \int_l^u \left\{ \int_0^a \Lambda_I(t) dt \right\} h(a) da \quad (11)$$

$$E[N_E(l, u)] = \int_l^u \left\{ \int_0^a \Lambda_E(t) dt \right\} h(a) da \quad (12)$$

Model 2: Limit on Individual Cost Lifetime Warranty (Policy 2)

Here the cost of individual claims to the dealer is limited to fixed cost c_l whereby the manufacturer/dealer carries out all rectification action free of cost to the customer if the cost of rectification is below a limit c_l . If the cost of rectification exceeds c_l , then the customer pays the difference between the cost of

rectification and c_I . That is the manufacturer/dealer pays all costs up to c_I and the customer pays an amount $(C-c_I)$, where C is the total rectification costs of an individual claim.

Then for the j^{th} failure,

$$M_j = \min\{C_j, c_I\} \tag{13}$$

$$B_j = \min\{0, (C_j - c_I)\} \tag{14}$$

Where M_j and B_j represent the manufacturer's cost and customer's costs for j th failure respectively.

Let the individual cost of rectification, C_j be given by a distribution function $G(c)$ with density function $g(c)$.

Therefore, the expected cost of each rectification to the manufacturer/dealer is given by

$$\bar{c}_m = \int_0^{c_I} c g(c) dc + c_I \bar{G}(c_I) \tag{15}$$

and that to the buyer is given by

$$\bar{c}_b = \int_{c_I}^{\infty} (C - c_I) g(c) dc \tag{16}$$

The expected number of failures is given by

$$E[N(l, u)] = \int_l^u \left\{ \int_0^a \Lambda(t) dt \right\} h(a) da \tag{17}$$

The total expected warranty cost to the manufacturer/dealer, $E[C_m(l, u)]$, is given by

$$E[C_m(l, u)] = \bar{c}_m \left[\int_l^u \left\{ \int_0^a \Lambda(t) dt \right\} h(a) da \right] \tag{18}$$

and the total expected cost to the customer over the lifetime, $E[C_b(L)]$, is given by

$$E[C_b(l, u)] = \bar{c}_b \left[\int_l^u \left\{ \int_0^a \Lambda(t) dt \right\} h(a) da \right] \tag{19}$$

Model 3: Limit on Total Cost Lifetime Warranty (Policy 3)

Under the conditions of this policy, the warranty ceases either at the point of termination of the useful life or the total warranty cost reached at a prefixed cost limit c_T , whichever occurs first.

Let TC_j denote the cost of rectifying the first j failures subsequent to the sale. It is given by

$$TC_j = \sum_{i=1}^j C_i \quad (j = 1, 2, \dots) \tag{20}$$

where C_i is the cost of rectification of the i^{th} failure.

Since the total cost (of claims) to the manufacturer/dealer over the life period is limited to c_T , we have for the i^{th} failure, the cost to the manufacturer/dealer is

$$M_i = \min\{C_i, (c_T - TC_{(i-1)})\} \tag{21}$$

and the cost to the customer is

$$B_i = \max\{0, (TC_i - c_T)\} \tag{22}$$

TM_j denote the cost to manufacturer/dealer associated with the first j numbers of failures and this can be expressed by

$$TM_j = \sum_{i=1}^j M_i \quad (j=1,2,\dots) \tag{23}$$

TB_j denote the cost to buyer associated with the first j failures. It is given by

$$TB_j = \sum_{i=1}^j B_i \quad (j=1,2,\dots) \tag{24}$$

$$TM_0 = TB_0 = 0.$$

The warranty can cease either at lifetime a with number of failures $N(t)$ or earlier at the j^{th} failure if $TM_{(j-1)} < c_T \leq TM_j$ and $j < N(t)$ (Figure 2). The total cost to the manufacturer/dealer is as follows:

Let C_L be the cost of rectifications of all the failures and $V(c)$ be the distribution function for C_L . Then using conditional argument

$$V(c) = \sum_{r=0}^{\infty} \Pr ob. \{C_L \leq c | N(t) = r\} \Pr ob. \{N(t) = r\} \tag{25}$$

Note that C_L , conditional on $N(t) = r$, is the sum of r independent and identically distributed random variables with distribution $G(c)$.

$$V(c) = \sum_{r=0}^{\infty} \left[G^{(r)}(c) \left\{ \frac{\left\{ \int_0^u \left[\int_0^a \Lambda(t) dt \right] h(a) da \right\}^r \exp\left(-\int_0^u \left[\int_0^a \Lambda(t) dt \right] h(a) da\right)}{r!} \right\} \right] \tag{26}$$

where, $G^{(r)}(c)$ is the r -fold convolution of $G(c)$ with itself.

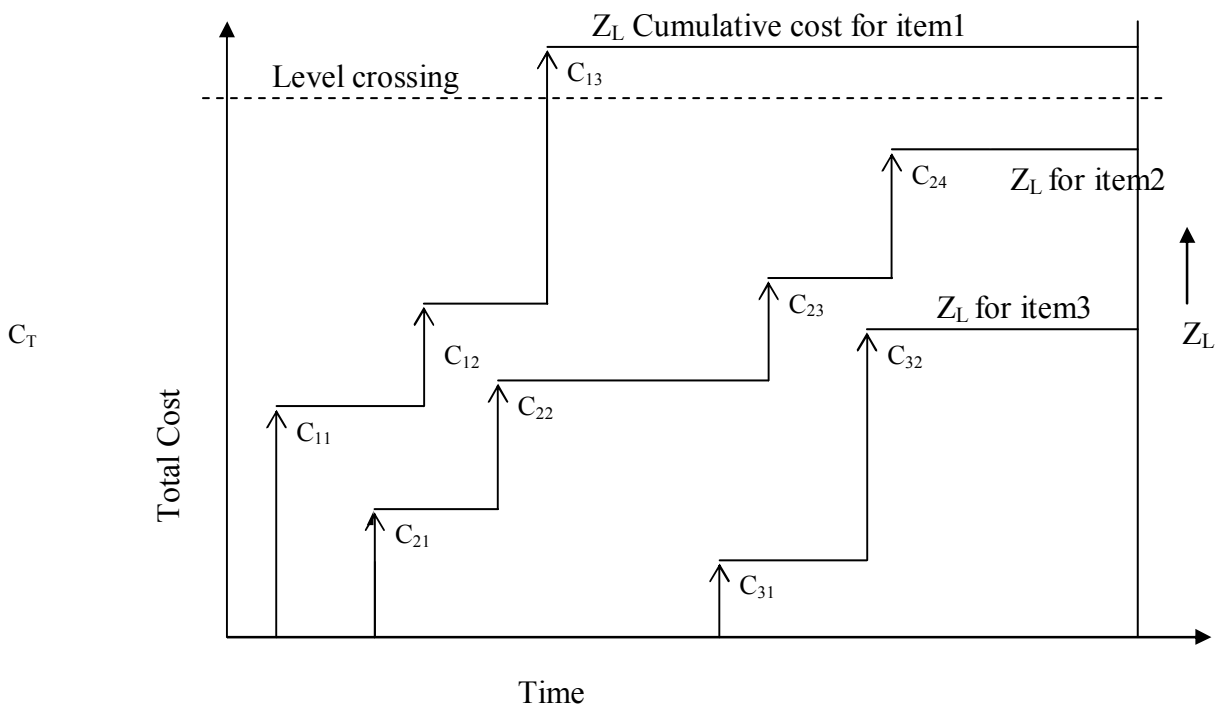


Figure 2: Limit on Total Cost Lifetime Warranty

The cost to the manufacturer/dealer equals C_L if $C_L \leq c_T$, and equals c_T if $C_L > c_T$. In the former case, the warranty ceases at a which is a random variable and in the later case it ceases before a . $E[C_m(l,u)]$ denote this expected cost to the manufacturer and it is given by

$$E[C_m(l,u)] = \int_0^{c_T} cv(c)dc + c_T \bar{V}(c_T) \tag{27}$$

where $v(c)$ is the density function ($= dV(c)/dc$) associated with the distribution function $V(c)$.

The cost to the buyer over the period $[0, a]$ is $\max \{0, C_L - c_T\}$. Therefore the expected cost to the buyer can be expressed by

$$E[C_b(l,u)] = \int_{c_T}^{\infty} (c - c_T)v(c)dc \tag{28}$$

When $G(c)$ is exponentially distributed and $\Lambda(t) = \lambda$ (ie, occurring according to stationary Poisson process) then $v(c)$ can be obtained analytically (Cox, 1962). For a general $\Lambda(t)$, it is not possible to obtain $v(c)$ analytically and simulation can be used.

Model 4: Limit on Individual and Total Cost Lifetime Warranty (Policy 4)

Under this policy, the cost to the manufacturer/dealer has an upper limit (c_I) for each rectification and the warranty ceases when the total cost to the manufacturer/dealer (subsequent to the sale) exceeds c_T or at the termination of useful life a , whichever occurs first. The customer pays the difference between rectification cost and the manufacturer/dealer’s limit cost. That is the total cost of claims to the manufacturer/dealer is limited to c_T and the cost of an individual claim is limited by c_I , then for the j^{th} failure

$$M_j = \begin{cases} \min\{C_j, c_I, (c_T - TM_{(j-1)})\} & \text{for } TM_{(j-1)} \leq c_T \\ 0 & \text{for } TM_{(j-1)} > c_T \end{cases} \tag{29}$$

and

$$B_j = \begin{cases} \max\{0, (C_j - c_I), (TM_j - c_T)\} & \text{for } TM_{(j-1)} \leq c_T \\ C_j & \text{for } TM_{(j-1)} > c_T \end{cases} \tag{30}$$

where TM_j denotes the sum of the costs incurred by the manufacturer/dealer with the servicing of the first j failures under warranty.

Warranty expires before a if the total cost to the manufacturer/dealer exceeds c_T . T_b denote the time at which the warranty expires. It is given by the first passage of a process C_t which represents the total cost to the manufacturer/dealer by time t . It is seen that

$$T_b > t \text{ if } C_t < c_T \tag{31}$$

Let $V_m(c; t)$ denote the distribution function for the total cost to the manufacturer/dealer by time t . Then $V_m(c; t)$ is given by

$$V_m(c; t) = \sum_{r=0}^{\infty} G_m^r(c) [\text{Pr ob}\{N(t) = r\}] \tag{32}$$

where $G_m(c)$ is given by

$$G_m(c) = \begin{cases} G(c) & \text{for } c < c_I \\ 1 & \text{for } c \geq c_I \end{cases} \quad (33)$$

Since, the cost of each single rectification action to the manufacture/dealer is constrained to be less than c_I .

$$Prob\{N(t) = r\} = \left[\int_0^u \left\{ \int_0^a \Lambda(t) dt \right\} h(a) da \right]^r \left[e^{-\int_0^u \int_0^a \Lambda(t) dt h(a) da} \right] / r! \quad (34)$$

Let $Q(t; c_T)$ denotes the distribution and $q(t; c_T)$ is the density function for T_b . Then, we have

$$\int_t^\infty q(t; c_T) dx = \int_0^{c_T} v_m(t; c) dc \quad (35)$$

The expected warranty cost to the manufacture/dealer can be obtained by using a level crossing of the above cumulative process. Let $v_m(c)$ be the density function associated with distribution function $V_m(c)$. Then the expected total cost to the manufacturer/dealer is given by

$$E[C_m(l, u)] = \int_0^{c_T} c v_m(c) dc + c_T \bar{V}_m(c_T) \quad (36)$$

The expected cost to the customer/buyer until warranty expires, when C_t first time crosses c_T at T_b , is given by

$$E[C_b(l, u)] = E[N(T_b)] \int_{c_I}^\infty (c - c_I) g(c) dc \quad (37)$$

where expected number of failures over $[0, T_b)$ is given by

$$E[N(t_b)] = \int_0^a \int_0^t \Lambda(x) dx q(t; c_T) dt \quad (38)$$

It is not possible to evaluate the costs analytically and one needs to use a simulation approach to obtain these costs.

5.0 ANALYSIS OF THE MODELS

Analysis of Specific Parts Exclusive Lifetime Warranty [SPELTW]:

Let, $\beta = 2$ and $\lambda = 0.443$ per year. Let the lifetime distribution parameter $\rho = 0.4$. This implies that the mean time to first failure, $\mu = 2$ years. Let expected cost of rectifications for warranty included and warranty excluded components be $\bar{c}_I = \$100$ and $\bar{c}_E = \$70$ respectively and let the probability of warranted excluded component $p = 0.4$ (40%), and probability of warranty covered component $q = 1 - p = 0.6$.

Table 1: Warranty cost (\$) to the Manufacturer for lifetime SPELTW: Approach 1

u	l							
	0.5	1	1.5	2	2.5	3	3.5	4
2	18.43	26.36	35.96	-	-	-	-	-
4	51.42	65.58	82.11	100.66	120.9	142.49	165.09	-
4.5	60.41	76.2	94.51	115.02	137.4	161.23	186.26	212.04
5	69.38	86.77	106.81	129.33	153.8	179.98	207.45	235.9
5.5	78.2	97.16	119.01	143.42	170.7	198.5	228.46	259.57
6	86.78	107.23	130.83	157.14	185.86	216.62	249.07	282.85
6.5	94.98	117.05	142.2	170.4	201.16	234.2	269.11	305.54
7	102.8	126.2	153.5	183.02	215.81	251.06	288.04	327.48

The expected warranty costs to the manufacturer/dealer are shown in Table 1. Warranty cost increases with the increase of lifetime. The higher the proportion of warranty covered components and the expected rectification cost, the higher is the manufacturer's expected warranty cost. Let the parameter values for failures of components belonging to Set E be $\lambda_E = 0.241$ per year and $\beta_E = 2.31$ and let the expected cost of each rectification $\bar{c}_E = \$70$. The corresponding values for Set I are $\lambda_I = 0.443$ per year, $\beta_I = 2$ and $\bar{c}_I = \$100$. The expected warranty cost to the manufacturer/dealer $E[C_M(l, u)]$ are shown in Tables 2.

Table 2: Manufacturer's costs for Specified parts excluded policy Approach 2

u	l							
	0.5	1	1.5	2	2.5	3	3.5	4
2	28.41	29.44	52.43					
4	78.85	92.12	138.89	137.64	162.9	189.24	216.27	
4.5	83.94	105.07	128.88	154.82	182.42	211.25	240.89	270.98
5	94.78	117.38	143.55	171.65	201.59	232.9	265.19	298.05
5.5	105.24	129.96	157.71	187.94	220.19	253.99	288.92	324.57
6	115.28	141.65	171.25	203.58	238.09	274.34	311.89	350.34
6.5	124.68	152.72	184.15	218.46	255.16	293.8	333.94	375.98
7	133.54	163.12	196.27	232.5	271.31	312.27	354.93	398.86

Warranty costs increase with the increase of the statutory base of the warranty and also with the increase of the useful life of the product. Costs are influenced by the proportion warranted components and the failure intensities of the warranted and unwarranted components.

Analysis of Limit on Individual Cost (LICTW) model

For $\beta = 2$ and $\lambda = 0.443$ per year. Let the manufacturer set individual limit cost $c_l = \$125$ and we assume the cost of each rectification is not more than \$200 which implies that the customer has to pay all the costs rectification from 125 to \$200. The expected warranty costs, $E[C_m(l, u)]$ for different combinations of l and u are shown in Tables 3.

Table 3: Manufacturer's/dealers costs for policy LICLTW

u (upper limit)	l (lower limit of useful life in years)							
	0.5	1	1.5	2	2.5	3	3.5	4
4	45.97	58.64	73.41	90.01	108.11	127.41	147.62	
4.5	54.02	68.14	84.5	102.84	122.84	144.17	166.54	189.65
5	62.05	77.59	95.96	115.64	137.54	160.93	185.5	210.94
5.5	69.92	86.88	106.42	128.24	152.04	177.49	204.29	232.1
6	77.58	95.9	116.98	140.51	166.19	193.7	222.78	252.91
6.5	84.93	104.4	127.15	152.34	179.87	209.41	240.63	273.2
7	91.92	112.58	136.85	163.66	192.97	224.49	257.89	292.92

Expected warranty cost are increasing function of useful life of the products and the increases with the increase of span limits. Costs are sensitive to mean cost of rectification and the manufacturer's limit on cost per rectification.

CONCLUSIONS

Lifetime warranties are a relatively new concept. These types of warranties, especially, cost sharing lifetime warranties have received a very little attention. In this paper, cost sharing lifetime warranty policies have been formulated and stochastic models for estimation of costs associated with these policies have been developed.

There is huge scope for future research in this area. In this paper cost models were developed at a system level. One can also developed cost models at a component level by applying renewal, modified renewal or other related processes and aggregating to the system level. For model simplification purposes a number of assumptions were made. To make the model more realistic some of these assumptions can be relaxed in future research. In estimating warranty cost, it was assumed that an item failure results in an immediate claim and all claims are valid (that is the buyer always exercises his/her warranty). In practice, there are a number of instances when a warranty is not exercised, even though it is possible to do so.

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