# Modified Vogel's Approximation Method for Solving Transportation Problems


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# Modified Vogel's Approximation Method For Solving Transportation Problems

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## Abstract

In this research, we have Modified Vogel's Approximation Method (MVAM) to find an initial basic feasible solution for the transportation problem whenever VAM was developed in 1958. Three methods North West Corner Method (NWCM), least Cost Method (LCM) and Vogel's Approximation Method (VAM) have been used to find initial basic feasible solution for the transportation model. We have taken same transportation models and used MVAM to find its initial basic feasible solution and compared its result with above three methods, but MVAM gives minimum transportation cost and also optimal and in some problems the result of MVAM is same as VAM but better than NWCM and LCM.

**Keywords:** Transportation problem, Vogel's Approximation Method (VAM), Maximum Penalty of largest numbers of each Row, Minimum Penalty of smallest numbers of each column.

## 1. Introduction

One of the earliest and most important applications of linear programming has been the formulation and solution of the transportation problem as a linear programming problem. In this problem we determine optimal shipping schedule of a single commodity between sources and destinations. The objective is to determine the number of units to be shipped from the source i to the destination j, so that the total demand at the destinations is completely satisfied and the cost of transportation is minimum.

Let $x_{ij} \geq 0$ be the quantity shipped from the source i to the destination j. The mathematical formulation of the problem is as follows:

$$\text{Minimize } \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}\, x_{ij} \quad (\textit{Total transportation cost})$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} = a_i \quad (\textit{Supply from sources})$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad (\textit{Demand from destinations})$$

$$x_{ij} \geq 0, \ \textit{for all } i \textit{ and } j.$$

where Z : Total transportation cost to be minimized.

$C_{ij}$ : Unit transportation cost of the commodity from each source i to destination j.

$x_{ij}$ : Number of units of commodity sent from source i to destination j.

$a_i$ : Level of supply at each source i.

$b_j$ : Level of demand at each destination j.

**NOTE:** Transportation model is balanced if Supply $\left(\sum_{i=1}^{m} a_i\right)$ = Demand $\left(\sum_{i=1}^{m} b_i\right)$.

Otherwise unbalanced if *Supply* $\left(\sum_{i=1}^{m} a_i\right) \neq$ *Demand* $\left(\sum_{i=1}^{m} b_i\right)$.

The total number of variables is *mn*. The total number of constraints is *m+n*, while the total number of allocations (*m+n–1*) should be in feasible solution. Here the letter *m* denotes the number of rows and *n* denotes the number of columns.

## General Computational Procedure for Transportation Model

The basic steps for solving transportation model are:

**Step 1 -** Determine a starting basic feasible solution. We use any one method NWCM, LCM, or VAM, to find initial basic feasible solution.

**Step 2**-Optimality condition. If solution is optimal then stop the iterations otherwise go to step 3.

**Step 3 -** Improve the solution**.** We use any one optimal method MODI or Stepping Stone method.

**Table 1:** Transportation array

**DESTINATIONS**

| | | D1 | | D2 | | …….. | Dn | | Supply $a_i$ |
|---|---|---|---|---|---|---|---|---|---|
| Sources | $S_1$ | $C_{11}$ | $x_{11}$ | $C_{12}$ | $x_{12}$ | …….. | $C_{1n}$ | $x_{1n}$ | $a_1$ |
| | $S_2$ | $C_{21}$ | $x_{21}$ | $C_{22}$ | $x_{22}$ | …….. | $C_{2n}$ | $x_{2n}$ | $a_2$ |
| | . . . . | …….. | | …….. | | …….. | …….. | | …….. |
| | $S_m$ | $C_{m1}$ | $x_{m1}$ | $C_{m2}$ | $x_{m2}$ | …….. | $C_{mn}$ | $x_{mn}$ | $a_m$ |
| Demand $b_j$ | | $b_1$ | | $b_2$ | | …….. | $b_n$ | | Balanced model $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ |

## 2. Methodology

The following methods are always used to find initial basic feasible solution for the transportation problems and are available in every text book of Operations Research [1].

**Initial Basic Feasible Solutions Methods**

(i) Column Minimum Method (CMM)
(ii) Row Minimum Method (RMM)
(iii) North West-Corner Method (NWCM)
(iv) Least Cost Method (LCM)
(v) Vogel's Approximation Method (VAM)

**Optimal Methods**

(i) Modified Distribution (MODI) Method or u-v Method
(ii) Stepping Stone method.

## 3. Initial Basic Feasible Solution Methods and Optimal Methods

There are several initial basic feasible solution methods and optimal methods for solving transportation problems satisfying supply and demand.

### *Initial Basic Feasible Solution Methods*

We have used following four methods to find initial basic feasible solution of the transportation problem:

 

- North West-Corner Method (NWCM)
- Least Cost Method (LCM)
- Vogel's Approximation Method (VAM)
- Modified Vogel's Approximation Method (MVAM)

### *Optimal Methods*

For optimal solution we have used the Modified Distribution (MODI) Method.

## 4. ALGORITHMS OF INITIAL BASIC FEASIBLE SOLUTION METHODS

### 1. North-West Corner Method (NWCM)

**Algorithm**

**Step 1.** Select the North-West (upper left-hand) corner cell of the transportation table and allocate units according to the supply and demand.

**Step 2.** If the demand for the first cell is satisfied, then move horizontally to the next cell in the second column.

**Step 3.** If the supply for the first row is exhausted, then move down to the first cell in the second row.

**Step 4.** Continue the process until all supply and demand values are exhausted.

### 2. Least Cost Method (LCM)

**Algorithm**

**Step 1.** First examine the cost matrix and choose the cell with **minimum cost** and then allocate there as much as possible. If such a cell is not unique, select arbitrary any one of these cells.

**Step 2.** Cross out the satisfied row or a column. If either a column or a row is satisfied simultaneously, only one may be crossed out.

**Step 3.**Write the reduced transportation table and repeat the process from step 1 to step 2, until one row or one column is left out.

### 3. Vogel's Approximation Method (VAM)

**Algorithm**

**Step 1.** Compute penalty of each row and a column. The penalty will be equal to the difference between the two smallest shipping costs in the row or column.

**Step 2.** Identify the row or column with the largest penalty and assign highest possible value to the variable having smallest shipping cost in that row or column.

**Step 3.** **C**ross out the satisfied row or column.

**Step 4.** Compute new penalties with same procedure until one row or column is left out.

**Note:** Penalty means the difference between two smallest numbers in a row or a column.

## 4. Modified Vogel's Approximation Method (MVAM)

## Algorithm

**Step 1.** Compute penalty of each row and a column. The penalty of each row will be equal to the difference between the two largest shipping costs but the penalty of each column is equal to the difference between smallest costs.

**Step 2.** Identify the row or column with the largest penalty and assign minimum possible value to the variable having smallest shipping cost in that row or column.

**Step 3.** **C**ross out the satisfied row or column.

**Step 4.** Compute new penalties with same procedure until one row or column is left out.

## Optimal Method

### *Modified Distribution (MODI) Method*

This method always gives the total minimum transportation cost to transport the goods from sources to the destinations.

## Algorithm

1. If the problem is unbalanced, balance it. Setup the transportation tableau
2. Find a basic feasible solution.
3. Set $u_1 = 0$ and determine $u_i$'s and $v_j$'s such that $u_i + v_j = c_{ij}$ for all basic variables.
4. If the reduced cost $c_{ij} - u_i - v_j \geq 0$ for all non-basic variables (minimization problem), then the current BFS is optimal. Stop! Else, enter variable with most negative reduced cost and find leaving variable by looping.
5. Using the new BFS, repeat steps 3 and 4.

## 5 The Numerical Problems

**Example 1:** Consider the following transportation problem:

**Table 2: Cost Matrix for the Numerical example**

| | | D | E | F | G | Supply |
|---|---|---|---|---|---|---|
| | | Destination | | | | |
| Factory | A | 20 | 22 | 17 | 4 | 120 |
| Factory | B | 24 | 37 | 9 | 7 | 70 |
| Factory | C | 32 | 37 | 20 | 15 | 50 |
| Demand | | 60 | 40 | 30 | 110 | 240 |

Applying the algorithm of the **Row Minimum Method (RMM)**, we obtain the following allocations:

**Table 3: Initial Basic Feasible Solution using Row Minimum Method**

| | D | E | F | G | Supply |
|---|---|---|---|---|---|
| A | 20 | 22 | 17 (10) | 4 (10) | 120 |
| B | 24 (50) | 37 | 9 (20) | 7 | 70 |
| C | 32 (10) | 37 (40) | 20 | 15 | 50 |
| Demand | 60 | 40 | 30 | 110 | 240 |

$$z = 17\times10 + 4\times10 + 24\times50 + 9\times20 + 32\times10 + 37\times40 = 3790$$

Applying the algorithm of the **Column Minimum Method (CMM)**, we obtain the following allocations:

**Table 4: Initial Basic Feasible Solution using Column Minimum Method**

| | D | E | F | G | Supply |
|---|---|---|---|---|---|
| A | 20 / 60 | 22 / 40 | 17 | 4 / 20 | 120 |
| B | 24 | 37 | 9 / 30 | 7 / 40 | 70 |
| C | 32 | 37 | 20 | 15 / 50 | 50 |
| Demand | 60 | 40 | 30 | 110 | 240 |

$$z = 20\times60+22\times40+4\times20+9\times30+7\times40+15\times50=3370$$

Applying the algorithm of the **North West Corner Method (NWCM)**, we obtain the following allocations:

**Table 5: Initial Basic Feasible Solution using North West Corner Method**

| | D | E | F | G | Supply |
|---|---|---|---|---|---|
| A | 20 / 60 | 22 / 40 | 17 / 20 | 4 | 120 |
| B | 24 | 37 | 9 / 10 | 7 / 60 | 70 |
| C | 32 | 37 | 20 | 15 / 50 | 50 |
| Demand | 60 | 40 | 30 | 110 | 240 |

$$z = 20\times60+22\times40+17\times20+9\times10+7\times60+15\times50=3680$$

Applying the algorithm of the **Least Cost Method (LCM)**, we obtain the following allocations:

**Table 6: Initial Basic Feasible Solution using Least Cost Method**

| | D | E | F | G | Supply |
|---|---|---|---|---|---|
| A | 20 / 10 | 22 | 17 | 4 / 110 | 120 |
| B | 24 / 40 | 37 | 9 / 30 | 7 | 70 |
| C | 32 / 10 | 37 / 40 | 20 | 15 | 50 |
| Demand | 60 | 40 | 30 | 110 | 240 |

$$z = 20\times10+4\times110+24\times40+9\times30+32\times10+37\times400=3580$$

Applying the algorithm of the **Vogel's Approximation Method (VAM)**, we obtain the following allocations:

**Table 7: Initial Basic Feasible Solution using Vogel's Approximation Method**

| | D | E | F | G | Supply | Row Penalty | | | |
|---|---|---|---|---|---|---|---|---|---|
| A | 20 | 22 / 40 | 17 | 4 / 80 | 120 | (13) | (13) | -- | -- |
| B | 24 / 10 | 37 | 9 / 30 | 7 / 30 | 70 | (2) | (2) | (2) | 17 |
| C | 32 / 50 | 37 | 20 | 15 | 50 | (5) | (5) | (5) | 17 |
| Demand | 60 | 40 | 50 | 110 | 240 | | | | |
| Column Penalty | (4) | (15) | (8) | (3) | | | | | |
| | (4) | -- | (8) | (3) | | | | | |
| | (8) | -- | (11) | (8) | | | | | |
| | (8) | | | (8) | | | | | |

$$z = 22\times40+4\times80+24\times10+9\times30+7\times30+32\times50+=3520$$

Applying the algorithm of the **Modified Vogel's Approximation Method (VAM)**, we obtain the following allocations:

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**Table 8: Initial Basic Feasible Solution using Modified Vogel's Approximation Method**

| | D | E | F | G | Supply | Row Penalty | | | |
|---|---|---|---|---|---|---|---|---|---|
| A | 20 / 60 | 22 / 40 | 17 | 4 / 20 | 120 | (2) | (13) | (16) | 16 |
| B | 24 | 37 | 9 / 30 | 7 / 20 | 70 | (13) | (15) | (17) | -- |
| C | 32 | 37 | 20 | 15 / 50 | 50 | (5) | (12) | (17) | 17 |
| Demand | 60 | 40 | 50 | 110 | 240 | | | | |
| Column Penalty | (4) | (15) | (8) | (3) | | | | | |
| | (4) | -- | (8) | (3) | | | | | |
| | (4) | -- | -- | (3) | | | | | |
| | (4) | | | (3) | | | | | |

$$z = 20\times 60 + 22\times 40 + 4\times 20 + 9\times 30 + 7\times 20 + 15\times 50 + = 3460$$

## 5.2 Optimality Test of Example 1

Taking initial Basic Feasible Solution due to proposed method, we now proceed for optimality using Modified Distribution Method. Here we calculate $u_i$ and $v_j$ for occupied basic cells using $u_i + v_j = c_{ij}$ and assuming $u_1 = 0$:

$$u_1 + v_1 = c_{11} \quad \Rightarrow 0 + v_1 = 20 \quad \Rightarrow v_1 = 20$$
$$u_1 + v_2 = c_{12} \quad \Rightarrow 0 + v_2 = 22 \quad \Rightarrow v_2 = 22$$
$$u_1 + v_4 = c_{14} \quad \Rightarrow 0 + v_4 = 4 \quad \Rightarrow v_4 = 4$$
$$u_2 + v_3 = c_{23} \quad \Rightarrow 3 + v_3 = 9 \quad \Rightarrow v_3 = 6$$
$$u_2 + v_4 = c_{24} \quad \Rightarrow u_2 + 4 = 7 \quad \Rightarrow u_2 = 3$$
$$u_3 + v_4 = c_{34} \quad \Rightarrow u_3 + 4 = 15 \quad \Rightarrow u_3 = 11$$

Calculation of opportunity cost for non basic variable using $\Delta_{ij} = c_{ij} - (u_i + v_j)$ :

| | |
|---|---|
| (1 , 3) | $\Delta_{13} = c_{13} - (u_1 + v_3) = 17 - (0 + 6) = 11$ |
| (2 , 1) | $\Delta_{21} = c_{21} - (u_2 + v_1) = 24 - (3 + 20) = 1$ |
| (2 , 2) | $\Delta_{22} = c_{22} - (u_2 + v_2) = 37 - (3 + 22) = 12$ |
| (3 , 1) | $\Delta_{31} = c_{31} - (u_3 + v_1) = 32 - (11 + 20) = 1$ |
| (3 , 2) | $\Delta_{32} = c_{32} - (u_3 + v_2) = 37 - (11 + 22) = 4$ |
| (3 , 3) | $\Delta_{33} = c_{33} - (u_3 + v_3) = 20 - (11 + 6) = 3$ |

Since all opportunity costs are positive, the basic feasible solution obtained by the proposed method is an optimal solution.

## Table 9: A comparison of the methods

| Problems | Size of a problem | NWCM | LCM | VAM | MODI | MVAM | Remarks |
|---|---|---|---|---|---|---|---|
| 1.* | 3X4 | 3860 | 3670 | 3520 | 3460 | 3460 | Optimum |
| 2. | 3x3 | 6600 | 6460 | 5920 | 5920 | 5920 | Optimum |
| 3. | 3x3 | 730 | 555 | 555 | 555 | 555 | Optimum |
| 4. | 3x5 | 363 | 305 | 290 | 290 | 290 | Optimum |
| 5. | 3x4 | 176 | 150 | 149 | 149 | 149 | Optimum |
| 6. | 3x3 | 5925 | 4550 | 5125 | 4525 | 4550 | Near to Optimum |
| 7. | 3x4 | 7750 | 7350 | 7350 | 7350 | 7350 | Optimum |
| 8. | 3x4 | 310 | 180 | 180 | 180 | 180 | Optimum |
| 9.* | 3x4 | 670 | 665 | 650 | 610 | 610 | Optimum |
| 10. | 3x3 | 148 | 150 | 150 | 148 | 148 | Optimum |

The cost of transportation shows that the:

(i) Modified Vogel's Approximation Method (MVAM) and Vogel's approximation method (VAM), provide the same result but almost optimal but in some problems very close to optimal.

(ii) In (MVAM), all results are better than both methods NWCM and LCM.

(iii) In MVAM, we have used penalty of maximum numbers of each row but kept same penalty of minimum numbers of each column as in VAM.

(iv) In Problem No1 the result of MVAM is better than the result of VAM.

(v) In MVAM and VAM, the penalty of each row makes the problem simple, easy and takes a same time in calculation.

## 6. Conclusion

We have used here four methods North West Corner Method (NWCM), Least Cost Method (LCM), Vogel's Approximation Method (VAM) and Modified Vogel's Approximation Method (MVAM) to find an initial basic feasible solution for the transportation model. The results of MVAM and VAM are almost same optimal but better than NWCM and LCM.

In some problems the result of MVAM is better than VAM. However, it is important to note that we have used penalty of each row of maximum numbers but kept same penalty of minimum numbers of each column as in VAM.

Thus our method is also easily applied to find the initial basic feasible solution for the balanced and unbalanced transportation problems.

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