Essays on the Time Diversification Puzzle

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Published

2012

Thesis Type

Thesis (PhD Doctorate)

School

Griffith Business School

DOI

10.25904/1912/2058

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Essays on the Time Diversification Puzzle

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Submitted in fulfillment of the requirements of the degree of

Doctor of Philosophy

in the field of Finance

July 2012

> “Funeral by funeral, theory advances.”
>
> Paul A. Samuelson

Abstract

What is the relationship between risk and investment horizon? Since Samuelson's (1969) theoretical proof that risk and time are unrelated, a half century of debate and controversy has ensued, leaving time diversification as one of the most enduring puzzles of modern finance. Over this time, the literature has developed along four identifiable streams of scholarship, with no stream demonstrating an ability to unravel the puzzle. Today, we have the same fragmented and contested debate that inspired Kritzman's (2000) lament that the time diversification had become a referendum on risk. Faced with the protagonists' entrenched positions, we offer an innovative approach to investigating the puzzle. Motivated by a critique of the literature, and an examination of the institutional setting, we provide positive insights into the relationship between risk and investment horizon by examining ten different measures on common terms. We find that the protagonists in the debate are each correct in their assessment of the puzzle on their own terms, and that resolution to the debate may hinge on more than simply the basis upon which risk is measured. Using a nested methodological approach, we advance the debate by considering whether alternative accumulation models and competing asset allocation approaches influence the relationship between risk and investment horizon. From the perspective of a long-horizon investor (e.g. a defined contribution plan member) faced with the puzzle, we find that alternative accumulation model specifications introduce the risk of the investor encountering wildly different terminal wealth outcomes. Moreover, for the investor with finite horizons, we find that asset allocation informed by a target can produce superior return and risk characteristics. We conclude that the relationship between risk and investment horizon is highly contextual, and that the time diversification puzzle is properly viewed from the perspective of a trinity of interrelated factors: risk, the accumulation model, and asset allocation strategy.

Statement of Originality

This work has not previously been submitted for a degree or diploma in any university. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

Signed: ______________________________

Adam Neil Walk

Table of Contents

List of Tables

List of Figures

Acknowledgements

Firstly, I would like to sincerely thank my thesis supervisors, Professor Michael Drew and Dr Robert Bianchi, of the Department of Accounting, Finance and Economics, Griffith Business School, Griffith University for their generous academic support, their valued friendship and their belief in this research. In particular, without Professor Drew’s guiding hand, encouragement and patience this research would not have been possible. Dr Bianchi’s technical advice made the difference on numerous occasions.

I am indebted to many colleagues, present and former, whose generosity and fellowship have helped me to this point. In particular, I would like to acknowledge Mr Steven Hackworth, Ms Sonya Sawtell-Rickson, Mr Philip Greenheld, Mr Evan Reedman and Mrs Rhona Mudford. I am also grateful for the support of Mr Adriaan Ryder who, when I combined full time employment and doctoral research, allowed me the latitude to balance my professional and academic responsibilities.

On a more personal note, I want to acknowledge my family – the Walks and the McGintys – and friends for their gracious acceptance of my apologies to all manner of social occasions. I sincerely hope they haven’t worked out I am dispensable.

To my parents, Neil and Irene, who prepared me for life in the best possible way: thank you. Your love and sacrifice will never be forgotten. Finally, and most importantly, I would like to thank my wife Megan whose kindness, patience and loving support has allowed me to pursue my intellectual passions. This man’s life would be poorer without her in it.

1 Introduction

Time diversification – the notion that extending investment horizon reduces risk – has been one of the most hotly contested ideas in finance since its first formal treatment by Samuelson (1969). Since then, nearly one hundred scholarly papers have been produced on the subject to no avail.[1] That the relationship between risk and investment horizon remains unresolved surely justifies the inclusion of time diversification as one of finance's most enduring puzzles.[2]

Conventional wisdom suggests that investment risk decreases as time horizon increases. A large number of empirical studies support this idea, finding that the standard deviation of annualised returns falls over time. Entire books have been dedicated to communicating to a popular audience this idea that risk is tamed by time: for example, Siegel's (1994) *Stocks for the Long Run: A Guide to Selecting Markets for Long-term Growth*. The widespread acceptance of the supposed inverse relationship between risk and time has led many in academe and industry to suggest that time diversification is more than just conventional wisdom, and has instead graduated to become a "stylised fact" of modern finance.[3]

So if this inverse relationship between risk and time horizon is so far beyond doubt then where is the puzzle? While many studies – including this one – confirm that the standard deviation of annualised returns decreases over time, studies also find that the standard deviation of *cumulative* returns does not diminish over time. In fact, if we frame risk in these terms, we find that dispersion actually increases over time. Figure 1.1, for example, shows these two conceptions of risk plotted against investment horizon.

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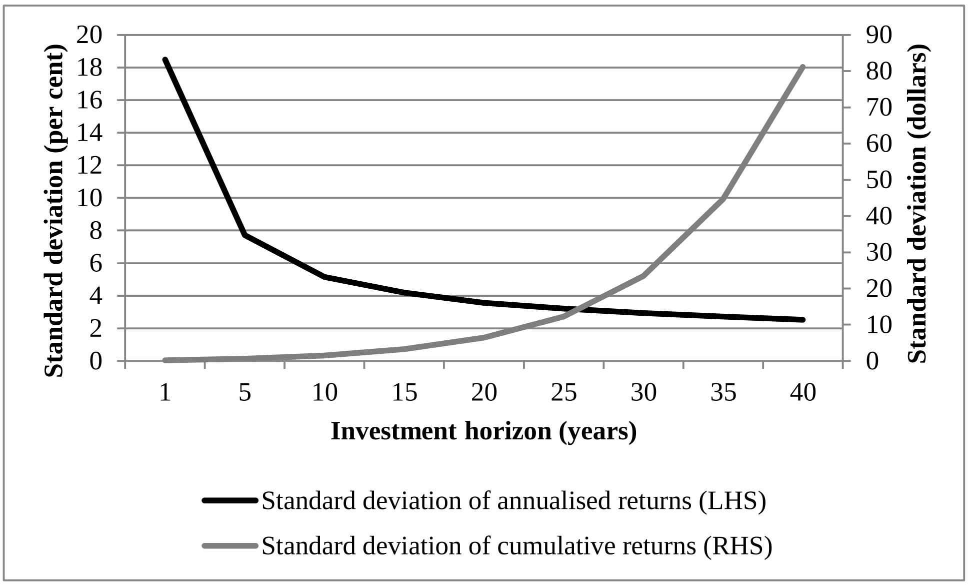
[1] The most recent of these papers is Panyagometh (2011).

[2] The Collins English Dictionary (1972) defines a puzzle as "a bewildering or perplexing question …; a conundrum." Of course, puzzles are not new to finance having exercised the minds of the field's pre-eminent scholars. In fact, one of the more recognisable names in the time diversification debate authored a book with the title "Puzzles of Finance" (Kritzman, 2000). Time diversification is the subject of Chapter 3. Alas, despite its treatment in Kritzman (2000), time diversification remains a puzzle.

[3] It is thought that the concept of a "stylised fact" was first originated by Kaldor (1961) to describe "stable patterns", "characteristic features" or "broad tendencies" which, while debatable in terms of degree, summarise a relationship between variables in empirical data (Heine et al., 2005). Kaldor (1961), investigating the statistical properties of long-term economic growth, pointed out six stylised facts which later proved to be robust across economies.

Figure 1.1: Contradictory evidence

This figure plots the standard deviation of annualised returns, in percentage terms, on the left vertical axis and the standard deviation of cumulative returns, in dollar terms, on the right vertical axis. Both series are plotted against investment horizon.[4]



This contradiction strikes at the heart of the time diversification puzzle. How is it that two versions of the one statistical measure of dispersion – standard deviation – give us opposite answers regarding the relationship between risk and time? Is risk and time negatively related – the received position – or positively related? What causes this seemingly contradictory outcome? This thesis is motivated by this key contradiction at the heart of the time diversification puzzle, a contradiction for which the academy has no satisfactory answer as yet.

By their nature, puzzles provide open space for scholars who wish to propose new ideas and innovative approaches for consideration. As will be explored in the literature review and empirical analysis sections of this thesis, the fragmented nature of the time diversification puzzle requires the arguments of the competing schools of thought to be considered on a like-for-like basis in order to move the debate forward. This thesis aims to synthesise the competing arguments that form the time diversification puzzle in the hope that there can be some satisfactory resolution to the debate. This study is not a treatise designed to induce the antagonists to recant their beliefs. Rather, in philosophical terms, this thesis is more consistent with the work of Kuhn (1970) and Lakatos (1978).

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[4] Each of these series is the product of 10,000 trials for each of nine investment horizons using a stationary bootstrap simulation method (outlined in further detail in Chapter Four). The standard deviation of annualised returns is calculated for each horizon by computing the mean monthly return for each of the 10,000 return paths, annualising each monthly mean, then taking a standard deviation of the 10,000 annualised means. The standard deviation of cumulative returns is calculated by applying the simulated return path for a given horizon to a starting value of $1. This is repeated for each of the 10,000 paths. The calculation is completed by taking the standard deviation of the 10,000 cumulative returns. Kritzman (1994), in his Figure A (p. 14), shows a 95 per cent confidence interval for annualised returns. This is the confidence interval equivalent of our standard deviation of annualised returns series.

Accordingly, a collection of puzzles will facilitate the emergence of a new paradigm, not by forcing antagonists to recant, but by offering additional empirical evidence that allows the next generation of financial economists to accept a new paradigm. In this sense, the old paradigm is not suppressed, it is lost to time through the accumulation of conflicting evidence.

Gruber (1996) notes in his seminal work on the puzzle of the growth in actively managed mutual funds, “the more time I spent thinking about mutual funds, the more I was troubled by a question: why do investors buy actively managed funds? (p.783).” In the spirit of Gruber (1996), this thesis is troubled by ongoing debate of the last half century and asks a simple question: does investment risk fall or rise over time? On surveying the literature, we find that many, many issues remain open. This thesis attempts to provide positive insights into the time diversification puzzle through the implementation of a nested methodological approach, finding that the puzzle is properly a referendum on a trinity of interrelated factors: risk, the accumulation model, and the asset allocation decision.[5]

Essentially what we have seen in the time diversification literature is a quest for *the* measure of risk that properly isolates the relationship between risk and time horizon. At the turn of the century, Kritzman (2000) remarked wistfully that “for many the time diversification debate has degenerated into a referendum on the meaning of risk (p. 50).” We agree that the debate has become, and remains, a referendum on risk, and that, in Kritzman’s (2000) words, such a referendum is “… futile (p. 50).” We argue in this thesis that the referendum on risk only addresses part of the puzzle, and that in fact the debate has been conducted on the wrong terms. Furthermore, we will show that a more robust framework for testing the relationship between risk and horizon incorporates not only competing definitions of risk, but also the way in which assets are accumulated through time (the accumulation model) and decisions regarding portfolio selection (asset allocation).

1.1 Institutional setting

When the time diversification debate began (Samuelson, 1969), an individual’s retirement income was generally received from some combination of the following sources: publicly-funded entitlements, distributions from a defined benefit (DB) pension plan, or private (non-pension) savings. We argue that this setting allowed the time diversification literature to evolve into an abstract debate amongst experts about the relationship between risk and investment horizon.

Since then, the institutional setting has evolved appreciably. Corporations decided that DB plans represented too much risk to their balance sheets and therefore replaced them with defined contribution (DC) plans, where the plan member (consciously or not) takes responsibility for their

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[5] “Accumulation model” is how we describe the evolution of wealth through time. Virtually the entire time diversification literature studies the initial endowment model, where wealth is only a function of returns. The initial endowment is often assumed to be $1, so the initial endowment model in practice collapses to a return-only model.

retirement savings. The risk that was once pooled within DB plans, is now personalised amongst DC plan members. Adequacy on an individual level thus becomes the objective of retirement savings. Governments see the merits of individual adequacy as an objective, and support the trend both rhetorically and with policy measures. In western developed economies, private retirement savings are seen as something of an imperative as adverse demographic trends challenge public finances.[6] In Australia, for example, in an effort to ensure adequacy, it is a legal requirement that employers contribute nine per cent annum of gross salary to a superannuation fund.[7]

This trend towards adequacy has focused the attention of DC plan sponsors on the investment options they offer. Of investment options, much is expected. A single option design – a default option, for example – is expected to assist a heterogeneous membership – where a range of factors are unique to the individual e.g. the combination of expectations, and contribution and salary profiles – in achieving adequacy, and avoiding periodic bouts of market turmoil. Depending on the jurisdiction, investment options may also be required to perform well on a short-term peer-relative basis as well. Given the present state-of-the-art, this represents a tall order.

But by reading the time diversification literature, none of this contextual richness would be apparent. Instead, we read about plan members who start with an initial endowment and generate terminal wealth only through returns. There is virtually no suggestion that there are other variables worthy of consideration or analysis. While there are relatively frequent references to more complete frameworks, very few time diversification studies actually analyse the impact of contributions or asset allocation on the relationship between risk and investment horizon. And those few that partly address the trinity of factors we discuss herein, fail to contrast their findings with the typical approach in the literature.

It is the institutional setting that has partly motivated a critique of the time diversification literature, and provided the inspiration for the novel elements of this thesis: the consideration of alternative accumulation models; and, an investigation of the role of asset allocation in the relationship between risk and investment horizon. After considering the institutional setting, we argue our trinity of related factors is qualitatively more persuasive. In the remainder of the thesis, we hope to make an equally persuasive quantitative case in favour of our threefold framework.

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[6] Relevant demographic trends include rising life expectancies, aging populations and lower fertility rates which together conspire to increase dependency ratios.

[7] “Superannuation” is the term used in Australia to describe retirement savings. Because of the overwhelming dominance of DC plans in Australia, many Australians would view “superannuation” and “defined contribution retirement savings” as equivalent concepts. The statutory minimum contribution rate is set to rise to twelve per cent by July 2019.

1.2 A referendum on risk

The time diversification debate began in the 1960s with the work of one of the most prolific economists of all time, Paul A. Samuelson.[8] Samuelson (1969), no doubt inspired by Markowitz's (1952) seminal work on portfolio choice under uncertainty, sought to unravel the essential relationship between investment risk and time. Samuelson's (1969) seminal work, conducted within an expected utility framework, rested on three assumptions: (1) that the investor exhibits constant relative risk aversion, (2) returns follow a random walk, and (3) wealth is only a function of returns. Maximising expected utility, he found that the allocation to risky assets is independent of time, and dependent only on risk tolerance. As he did in other fields of economics, Samuelson (1969) defined the debate. By taking this approach, and providing us these three assumptions to critique, Samuelson (1969) set out the battleground upon which most of this debate has been fought. In particular, it is the first two assumptions – those which have the most obvious bearing on risk – which have motivated much of the subsequent time diversification literature.

In the first study of this thesis we replicate the classical referendum on risk *on common terms*. We take the initial endowment accumulation model popular in the literature and, following an applied approach, we select ten separate measures of risk that are used in the literature to identify the presence of time diversification, and observe their behaviour over nine investment horizons ranging from one to 40 years. These measures range from the most common statistical measure of dispersion – standard deviation – to measures proposed specifically for the purpose of studying this puzzle – for example, the time diversification index of Fabozzi, Focardi, and Kolm (2006).

To account for the variety of asset return process assumptions adopted in the literature – most commonly, random walk and mean reversion – and the array of modeling techniques implemented, we employ four complementary quantitative methods – Monte Carlo simulation, Efron (1979) bootstrap simulation, stationary bootstrap simulation, and empirical block bootstrap simulation. The first two simulation methods are similar in two ways: firstly, each implicitly assumes that asset returns follow a random walk; and, secondly, both are common approaches in the time diversification literature. For the purposes of this first study, these two features are important because they are true to the time diversification debate thus far. A random walk assumption is actually one of the Samuelson (1969) assumptions which has attracted critique and is thus an ideal baseline assumption to adopt when attempting to synthesise the debate. Similarly, the fact that each of these methods is commonly used in earlier studies allows us to conduct the referendum on time diversification on the same terms as the literature.

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[8] Paul A. Samuelson, for "the scientific work through which he has developed static and dynamic economic theory and actively contributed to raising the level of analysis in economic science," earned him the 1970 *Sveriges Riksbank* Prize in Economic Sciences in Memory of Alfred Nobel (The Nobel Foundation, 2012).

Our referendum on risk results in two conclusions. Firstly, based on our ten measures of risk, we find contradictory evidence of time diversification, consistent with the body of literature. In fact, the verdict regarding time diversification changes depending on how risk is defined. In this sense, the main studies in the time diversification debate are correct on their own terms. And, secondly, we see the emergence of a dichotomy which might hint at a resolution to the puzzle. By observing the measurement basis of each of our ten measures – that is, whether the measure is return- or wealth-based – we see a trend emerge. Return-based measures tend to suggest the presence of time diversification, whereas wealth-based measures point in the opposite direction.

While our dichotomy suggests that time diversification depends on how one views risk – return-based or wealth-based – we argue that, from the perspective of the long horizon investor, the wealth-based perspective is paramount. In fact, our contention that wealth is the appropriate measurement basis points us to the second study of this thesis where we consider alternative accumulation models, where wealth is a function of a range of variables in addition to return.

So, after synthesising the literature on common terms, and confirming that these findings are robust to a variety of asset returns processes, we find that risk rises, and falls, with horizon depending on how risk is defined. In this way, the time diversification debate is a referendum on risk, and everyone is right on their own terms. We must therefore conclude that the time diversification puzzle is not one that can be solved objectively. Rather, it is subjective, based on the individual's conception of risk. However, we argue that the debate has been conducted in an incomplete framework. With a few exceptions, no scholar has considered what impact alternative accumulation models, or asset allocation strategies, has on the relationship between risk and investment horizon. It is these points that are explored in the remaining studies of this thesis.

1.3 A referendum on the accumulation model

After the referendum on risk, it is almost possible to have forgotten that Samuelson's (1969) work in fact hinged on a third assumption: that wealth is only a function of returns. With very few exceptions (Jagannathan and Kocherlakota, 1996; Hickman, Hunter, Byrd, Beck and Terpening, 2001; Mukerji, 2008; Panyagometh, 2011), the entire time diversification debate has been conducted in the returns-only paradigm.[9] Furthermore, those few studies that do consider intervening cash flows, fail to compare and contrast their findings to the standard initial endowment model. Rather, they propose an accumulation model, study it, but generally choose not to use their findings as a critique of the bulk of the literature which focuses on the initial endowment model. So, based on the literature to date, we are not in a position to say whether intervening cash flows affect the relationship between risk and investment horizon in any way. It is this point we take up in this second study.

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[9] Pástor and Stambaugh (2012) also consider periodic contributions but their study is probably more about financial econometrics, than time diversification.

The approach in our second study is an evolution of the one employed in the first study in two ways. Firstly, we extend the initial endowment model to two further accumulation models using our nested methodology. The first assumes that the investor contributes at the rate of nine per cent per annum from a salary that remains constant through time (“the constant contribution model”). As such contributions are constant in nominal dollar terms consistent with the literature (Jagannathan and Kocherlakota, 1996; Hickman et al., 2001; Mukerji, 2008; Panyagometh, 2011). For the final accumulation model the investor again contributes at a constant percentage rate of nine per cent per annum, but salary rises with time at a rate of three per cent per annum (“the constant percentage contribution model”). Contributions therefore increase with time.

The second innovation in this second study is the introduction of a new basis for performance evaluation: the retirement wealth ratio (*RWR*). Research has suggested that defined contribution plans should be judged by their ability to produce adequate retirement income (Baker, Logue and Rader, 2005). Building on this insight, Basu and Drew (2010) noted that expectations regarding retirement income are likely to be closely related to pre-retirement income. So, because we have introduced an accumulation model – the constant percentage contribution model – that yields a different (higher) final income, we are unable to compare terminal wealth in dollar terms. To adjust for different levels of final income, we create a ratio of terminal wealth to final income to normalise performance amongst models, and allow comparison.

To maintain consistency, we estimated the ten measures examined in the first empirical study for the 72 new models introduced in this second study.[10] When we compare the estimates for these ten measures across the three models we find that, in most cases, there are no material differences. The only measures which do differ – and they do so materially – are those measures expressed in terms of terminal wealth, and certain downside risk measures. We find this because many of the measures proposed in the literature as being indicators of the presence, or absence, of time diversification are a function of returns only. Therefore, varying the accumulation model makes no difference. We thus see some support for the dichotomous classification of risk measures we proposed in our first study: return-based on one hand; and, wealth-based on the other.

Estimates of terminal wealth and downside risk measures expressed in dollar terms are significantly different between accumulation models. Accumulation models which incorporate contributions yield significantly improved measures of terminal wealth and downside risk. One decisive illustration of the material differences between models, sees the 95% value-at-risk threshold for the constant contributions model *exceed*, in dollar terms, the *median* terminal wealth path for the equivalent initial endowment model. Thus, by adding contributions, we see a fairly extreme downside risk scenario

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[10] Recall that there are 36 models for each accumulation model set-up (four simulation methods multiplied by nine investment horizons). The second study therefore analyses the results of 108 models (36 models multiplied by three accumulation models).

outperform what many would see as measure of expected performance. We therefore conclude that the time diversification debate has been conducted in an incomplete framework, and using measures that have little relevance for a long-horizon investor.

This thesis proceeds to estimate these terminal wealth and downside risk measures – that varied between accumulation model – expressed as terminal retirement wealth ratios (RWR_T) in order to adjust for the different final income for the constant percentage contribution model. We find that our results are generally consistent no matter whether the performance is expressed in dollar or RWR_T terms. For example, the accumulation models that incorporate contributions – the constant contribution and the constant percentage contribution models – yield significantly different results compared to the initial endowment model when expressed in both dollar and RWR_T terms. The only difference is the ordering of various accumulation models: in dollar terms the constant percentage contribution model delivers the greatest terminal wealth. However, in RWR_T terms, the estimates for the constant percentage contribution model rank behind the corresponding constant contribution model estimate because its higher wealth has been adjusted for the higher final income in calculating the retirement wealth ratio.

In the first study, we synthesised the time diversification literature and found that Kritzman (2000) was in fact correct: the debate was a referendum on the meaning of risk. Furthermore we showed that, because of differences in their approaches, the assessment of all researchers were right *on their own terms*. In this study, we conducted our own referendum on the accumulation model and found the time diversification literature lacking. We show that by largely confining itself to the initial endowment model, researchers have overlooked significant factors in the dynamics of retirement savings, and perpetuated a debate about a phenomenon that is devoid of context.

1.4 A referendum on asset allocation

Until this point, asset allocation has been a constant in our analysis, to allow us to isolate the variation introduced by other factors (e.g. the introduction of alternative accumulation models in the second study). In the third and final study, we investigate whether asset allocation strategy has implications for our findings regarding time diversification. Using our nested methodological approach, we analyse a range of extant investment portfolio designs, in addition to introducing a basic rule-based dynamic strategy. By doing this, we address the paucity of asset allocation studies in the time diversification literature, and make a contribution to the literature by considering the relationship between risk and investment horizon for a selection of asset allocation strategies in the presence of contributions and salary growth (Pástor and Stambaugh, 2012[11]).

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[11] Pástor and Stambaugh (2012) note that "one must be cautious in drawing conclusions about the desirability of stocks for long-horizon investors in settings with additional risky assets such as nominal bonds, additional life

Where asset allocation is considered in the time diversification literature, it is usually as a way of addressing the time diversification literature’s underlying question: Are stocks more or less risky as investment horizon lengthens? Instead of measuring risk directly as we do in this thesis, many scholars imply the relationship between risk and investment horizon from the optimal allocation to risky assets. Thus, if the optimal allocation to stocks is higher at long horizons than at short horizons, then the implication is that the risk of stocks falls as investment horizon lengthens. The allocation to risk assets for a given horizon is seen as a proxy for risk for that horizon.

Rarely in the literature do scholars consider the performance of competing asset allocation strategies with investment horizon. Perhaps the most relevant example to this research is Hickman et al. (2001), in which the authors consider the comparative performance of four different asset allocation strategies comprised of six different assets. Hickman et al. (2001) compute a range of simple performance measures in terminal wealth terms, as well as the relative performance of each asset allocation strategy against a stock-only portfolio over investment horizons up to and including forty years. The Hickman et al. (2001) study is particularly relevant to us because it is also one of the few in the time diversification literature that incorporates contributions into the accumulation model. Reichenstein and Dorsett’s (1995) is another study that provides a benchmark for this work. Assuming a number of asset return processes, they compare the performance – distribution percentiles and downside risk – of eight different portfolios for holding periods of between one and 30 years. A drawback of their work is that it is solely focused on the initial endowment model.

The purpose of this study is to investigate whether asset allocation strategies of different types – constant allocations, non-constant allocations, and dynamic allocations – influence investment performance when compared to the stock only portfolio analysed in the earlier two studies in this thesis. We consider this question by computing the *RWR*-denominated performance measures, introduced in the second study, for the three accumulation models – the initial endowment model, the constant contribution model and the constant percentage contribution model – using four simulation methods and nine investment horizons. The five asset allocation strategies we consider are: a traditional balanced fund comprised of 60 per cent equities and 40 per cent cash; a representative target date fund incorporating a downward sloping glidepath; a simple dynamic target-driven strategy; the all-stock strategy (considered in earlier studies); and, an all-cash portfolio.

The target return employed in this thesis is a nominal return of seven per cent per annum. The setting of such a target for this study is a matter for judgment based on at least four factors: the target has to be a realistic average outcome over all horizons; it must be consistent with the proposed asset

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cycle considerations such as intermediate consumption, and optimal dynamic saving and investment decisions. Investigating asset allocation decisions in such settings while allowing higher long-run stock volatility to enter the problem is beyond the scope of this study but offers interesting directions for future research (p. 471).” We use this point as motivation for our consideration of asset allocation in our third empirical study.

allocation; for a dynamic strategy, it is critical that the investment objective not be set too high, or too low to ensure the strategy remains dynamic; and, in order to make fair comparisons, we need to select an investment target that is reasonably achievable by all strategies under consideration. We argue that a target of seven per cent per annum fulfills these criteria for all growth-oriented asset allocation strategies under consideration. While this target is used to compute a target *RWR* for the dynamic strategy, it also used to compute target-relative performance measures for all asset allocation strategies.

Our findings for this last study fall into three categories. Firstly, we find that the relative differences between the results for competing accumulation models persist for all asset allocation strategies examined, and all measures considered, with the magnitude of the difference being a function of the horizon length and the allocation to risky assets. We therefore conclude that contributions remain an important variable in retirement savings no matter what the asset allocation strategy under consideration. Secondly, our analysis is supportive of the findings of those few studies which analyse a range of asset allocation strategies. While our studies have different objectives, we confirm many of the general findings of Reichenstein and Dorsett (1995). In a similar way, whilst our approaches are not identical, we confirm many of the major findings of Hickman et al. (2001). For example, we agree that the cost of adopting an asset allocation strategy other than the all-stock portfolio is high in opportunity cost terms, without any significant downside benefits. We also find that the distributions of terminal wealth for all models over all holding periods exhibit positive skewness.

Third and finally, because of our unique approach, we make some modest contributions to the time diversification in relation to target-relative performance measures. We find that, in target-relative terms, our dynamic strategy combines the lowest probability of shortfall over all horizons with the lowest expected shortfall over all horizons greater than five years in length when compared to two strategies – the balanced and target-date strategies – that take less overall risk. The dynamic strategy also achieves the second highest target-relative Sortino ratio behind the all-stock strategy.

We contend that this surprising combination of performance attributes suggests that a dynamic target-driven strategy design may be able to influence the relationship between risk and investment horizon, and hence has implications to the time diversification debate. We must therefore ask whether it is possible to make general statements about the relationship between risk and time if we can present evidence to show that an asset allocation strategy with a higher allocation to stocks produces superior risk estimates. Thus, by providing a simple example, we show that the time diversification debate may be incomplete. We also provide practitioners evidence that dynamic target-driven investment strategies may represent a superior alternative to traditional deterministic approaches to plan design (e.g. target-risk and target-date fund designs).

1.5 Summary

This thesis makes four principal contributions to the finance literature. Firstly, we have contributed to the time diversification puzzle by unifying and synthesising a debate which, until now, has been largely disparate and fragmented. We endeavoured to replicate as much of the debate – the measures, the techniques, and the assumptions – as is practicable, so that we are in a position to critique the debate *on its own terms*. In this way, we are positioned to provide some resolution regarding the referendum on risk which has characterised the debate thus far. We show that there is no objective resolution to the puzzle, and that many of the important findings regarding time diversification are simultaneously true, as far as they go. We also find that many of the measures proposed in the literature are, by construction, designed to measure risk from a return-only perspective. The initial endowment model central to the literature has thus been hard-coded into the measures selected to study the time diversification phenomenon. We argue that wealth-related measures are preferable on two counts: (1) they are more consistent with the loss aversion ascribed to plan members; and, (2) they can be applied equally to any accumulation model.

Secondly, this thesis advances the debate by finding that insights into the time diversification puzzle require a trinity of interconnected factors to be considered, of which risk is the decisive one. Risk is the decisive factor because it lies at the heart of the time diversification question: the relationship between risk and horizon. The fact that risk can be measured in a variety of ways is the principal contribution of the literature to date. This thesis makes an original contribution by demonstrating that risk relies, in part, on two other factors until now largely overlooked in the literature. These other two factors are the accumulation model – which is the process by which wealth evolves – and asset allocation strategy – which is the principal driver of returns through time. As we have stated above, the time diversification debate has been conducted in an initial endowment framework and with the assumption of a fixed asset allocation. As such, contributions have until now been essentially irrelevant to the debate. In a similar way, a plan member's asset allocation strategy has largely been assumed away as a constant. The evidence presented here shows that both factors – the accumulation model and asset allocation strategy – have a bearing on the relationship between risk and investment horizon, particularly at longer horizons. For example, by ignoring these two factors we find that, when risk is measured by expected tail loss, risk deteriorates with investment horizon. However, when real world features such as multiple contributions and asset allocation are included, expected tail loss estimates improve with investment horizon. Our research is lent credibility by the fact that the accumulation models and asset allocation strategies studied in in this thesis are more consistent with the existing institutional setting than any other previous study in the literature.

Thirdly, we have made modest contributions to the debate by incorporating contemporary approaches to empirical finance. For example, we employ simulation techniques – the stationary bootstrap and empirical block bootstrap methods – that have not been used before in the time diversification debate. We also consider a basic form of dynamic strategy. So, while this thesis is positioned squarely in the existing debate, we also attempt to move the debate forward by incorporating state-of-the-art analytical approaches.

Finally, and perhaps most importantly, we make a number of contributions which should be of interest to plan sponsors. We provide evidence that plan sponsors need to consider our trinity of factors in plan design. Risk should be measured and managed according to the plan member's definition of risk – that is, in wealth terms – and both the accumulation model and asset allocation require attention. This will take plan sponsors in a slightly different direction to the one typically taken: an approach where asset allocation and investment manager-oriented measures of risk are the prime foci. Furthermore, we provide tentative evidence that dynamic target-driven strategies may represent an asset allocation approach worthy of further investigation.

The remainder of the thesis is organised as follows: Chapter Two reviews the literature, Chapter Three considers the institutional setting, Chapter Four outlines the data and methodology, Chapter Five through Seven contain the empirical analysis of the three referenda, and Chapter 8 summarises the findings, and discusses areas of further research.[12]

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[12] Appendices A through C contain output relating to Chapter Seven.

2 Literature review

2.1 Introduction

This thesis is concerned with the relationship between investment risk and investment horizon. Having settled on an agenda for research, we must decide how best to frame the question. When thinking about investment risk, we anchor our work in the Nobel Prize winning research of Markowitz (1952), who considered the problem of portfolio choice under uncertainty.[13] As with much ground-breaking research, Markowitz's (1952) modern portfolio theory was a caricature of a more complex problem. Most relevant to this thesis is that fact that Markowitz (1952) considered portfolio choice in a single period setting. This allowed him to remove time-variation from the problem of portfolio selection.

Naturally, the financial economics literature evolved and scholars began to consider the portfolio selection problem in a multi-period setting like that encountered with practical investment problems. Chief among these scholars was another Nobel Prize winner – Paul A. Samuelson – who considered the problem in a multi-period setting using expected utility theory. Samuelson's (1969) work is of particular interest to us for two reasons. Firstly, he was amongst the first to bring the genius of Markowitz's (1952) work into a multi-period setting which by itself is remarkable.[14] Secondly, and particularly germane to this thesis, Samuelson (1969) initiates the time diversification debate by considering whether the concept of diversification works with time, in the same way as it does amongst assets or securities (cf. Markowitz, 1952). In order to study the existence of time diversification, Samuelson (1969) selects the classical expected utility theory as his framework of choice. Expected utility theory is thus the point of departure for this debate, and all other competing streams or schools of thought tend to emerge at least in part as a reaction to Samuelson's (1969) work.

A thesis on time diversification without a discussion about expected utility theory, or any of the other major competing schools of thought, would (and should) be regarded as lacking. In order to advance the time diversification debate, we must conduct a critical survey of the literature to inform the research questions under consideration in this thesis. Fortunately, without too much effort, a process of taxonomy results in a number of distinct streams or schools of thought. Each of these competing schools tends to coalesce around an alternative theory to, or a common critique of, expected utility theory. The battle of ideas between these schools of thought has led to time diversification becoming one of finance's most enduring puzzles. As we proceed through the literature review, our critique of the major schools of thought in the time diversification debate lead us logically to an empirical approach inspired by a rather diverse and loose collection of research defined by Booth (2004) as

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[13] Markowitz's contribution earned him a share in the 1990 *Sveriges Riksbank* Prize in Economic Sciences in Memory of Alfred Nobel, which is commonly referred to as "the Nobel Prize in Economics."
[14] Others include Tobin (1965) and Merton (1969).

"applied" in nature. So, while all streams in the time diversification debate inform the questions under consideration, we operationalise this research according to an approach most sympathetic to the applied stream of the literature.

Following the literature more or less chronologically, we commence the formal review of the literature with the expected utility theory stream beginning with Samuelson (1969). Samuelson (1969) isolates the relationship between risk and time by observing the optimal allocation to risk assets with horizon, based on three assumptions. While a number of proponents confirm the mathematical certainty of his findings, even more scholars – including some who are otherwise advocates of expected utility theory – call into question Samuelson's (1969) assumptions. In fact, it is Samuelson's (1969) three assumptions –which will be scrutinised throughout the thesis – that provide later scholars the oxygen to keep the time diversification debate burning.

From the initial stream of work led by Samuelson (1969), the rise of the Black-Scholes-Merton Option Pricing Theory – another Nobel Prize winning idea – offered a convenient basis upon which Bodie (1991, 1995) could observe risk.[15] Bodie (1995) concluded that, because the price of a put option increased with investment horizon, so did risk. The option pricing theory approach of Bodie (1991, 1995) apparently emerged because of this unrelated breakthrough in economics, not as a result of a specific critique of Samuelson's (1969) work. Only later, did others highlight that Bodie's (1995) approach appeared to offer an objective measure of risk, in contrast to Samuelson's (1969) normative treatment of risk.[16] After Bodie's (1991, 1995) early contribution, the option pricing theory school of thought tended to degenerate into semantic debates about whether Bodie (1995) was correctly identifying the price of insurance, and relating it properly to the investment horizon. After a burst of scholarship in the late 1990s, this stream in the time diversification literature has to some degree faded away. Perhaps the most substantial critique of Bodie's (1995) work was that it was conducted in a risk-only framework. In a sense, this is understandable given the relationship of interest in the time diversification debate is that between risk and investment horizon. On the other hand, opponents questioned whether it is appropriate to separate risk from return, thereby overlooking one of investment's key trade-offs.

It is a generalisation that behavioural economists inevitably end up becoming amongst the most vocal opponents of any framework that tends to see economics as (hard) science, as opposed to social science. These two visions of economics mix like oil and water. Behavioural economists introduce the richness of humanity to economic problems, often in qualitative terms, whereas "scientists", of whom

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[15] Robert C. Merton and Myron S. Scholes shared the 1997 *Sveriges Riksbank* Prize in Economic Sciences in Memory of Alfred Nobel. Fischer Black died of cancer in 1995. Because the Prize in Economic Sciences is not awarded posthumously, Black is not formally recognised as a laureate although his contribution is beyond question.

[16] Recall, that to study risk by estimating the optimal allocation to risk assets, Samuelson (1969) first had to assume the investor's form of risk aversion.

Samuelson (1969) was most definitely one, prefer to take approaches characterised by theoretical formality and the rigour of mathematical reasoning, even if it means making simplistic assumptions about human behaviour. In these few sentences, we have briefly outlined both the behaviouralists' principal critique of Samuelson (1969) – the inappropriateness of his underlying assumptions – and our critique of the behavioural stream of literature – the lack of framework, and negative approach to the problem. From the author's observation of the literature, the behaviouralists tend to avoid the formal frameworks of economics in analysing the question at hand. By way of analogy, behaviouralists could be described as the insurgents of the time diversification debate. They appear, they challenge the entrenched position with the insights of psychology, only to retreat without leaving a tangible alternative framework. From the literature review conducted in this chapter, we will see that the influence of the behaviouralists is thus limited to providing critiques of the other streams of the literature. Notwithstanding its limitations, the behavioural stream in the literature does provide some compelling insights that are used in the thesis, particularly relating to the selection of risk measures for examination.

The final stream in the literature – what Booth (2004) describes as the "applied" stream – is defined more by what it's not, than what it is. While the applied stream is a somewhat nebulous confection of studies, there is the faint semblance of a unifying theme. Scholars who pursue this path tend to approach the problem of time diversification empirically, and without resting on a theoretical edifice as the more established streams tend to. Simulation techniques are also a common methodological choice as Booth (2004) suggests. Parallel to the time diversification, has emerged a rich literature on risk measures. Leaning on this literature, applied scholars tend to define risk in a certain way – for example, value-at-risk – and then proceed to estimate their selected risk measure over a number of horizons of different lengths. Scholars then draw conclusions about the presence or otherwise of time diversification by applying reasoning to these estimates. Naturally, it is possible to define risk in many ways and so the applied stream has tended to grow as new conceptions of risk emerge. As will become evident in this chapter, some scholars have even developed measures purely for the purposes of analysing the time diversification question.

So, almost fifty years after the debate began, we continue to see studies emerge which seek to resolve the debate. As noted, several recent studies have even introduced particular measures whose sole purpose is to shed light on this debate. And yet the puzzle remains unresolved. It would thus be fair to say that there are a number of entrenched camps who are unable to agree on a common basis from which to the advance the argument. This thesis does not seek to induce the debate's antagonists to recant. Rather, by leveraging off aspects of each stream in the literature, and attempting to synthesise the debate, we set out to offer an alternative view of the relationship between risk and investment horizon. Thus, as has happened to the option pricing theory stream to some extent, sides in the debate are lost to time. This Kuhnian (1970) process inspired the choice of the quotation in the front matter of this thesis which, ironically enough, is attributed to Samuelson: “Funeral by funeral, theory advances (Wilson, 1998, p.52).”

Having now briefly sketched out the literature, we turn to its relevance to this thesis. Samuelson’s (1969) three assumptions map to the three empirical studies (Chapters Five, Six and Seven) of this thesis. As is the case in all research, this thesis cannot marshal enough original critique to connect Samuelson’s (1969) assumptions directly to these three studies. Rather, this thesis leans on the whole body of scholarly literature to provide the logical and methodological motivation for the approach we pursue. The literature review thus has the purpose of showing the logical alignment between our ultimate theoretical anchor – Markowitz’s (1952) portfolio choice under uncertainty – and the research questions of this thesis.

As noted previously, this thesis is about the relationship between investment risk and investment horizon, not about the objective merits of expected utility theory, or any of the other schools of thought we discuss. We accept there are other economic problems where any one of these frameworks might represent the superior framework within which to approach the given problem. Our interest here is whether the given framework – for example, expected utility theory – is the best approach to gleaning positive insights into the puzzle of time diversification. We posit that our critique should therefore be viewed narrowly through the lens of the purpose of this thesis. Before we begin our detailed examination of the literature, we will first summarise the evolution of ideas that led to the time diversification debate as necessary context and then, in anticipation of the literature review proper, position the research questions within the literature. The literature review commences with a discussion of the work of Markowitz (1952).

2.2 The time diversification puzzle

2.2.1 Markowitzian origins

The principle concern of this thesis is the relationship between investment risk and the investor’s horizon. But before we explore the subject in detail, we must first anchor the work in the field of finance. Fortunately, when it comes to investment – or, in formal terms, portfolio choice – we can rely on one of the most famous theories in all of finance: the pioneering work of Markowitz (1952).

Markowitz’s (1952) seminal modern portfolio theory (MPT) was the first formal treatment of the benefits of portfolio diversification. The theory showed that, by constructing a portfolio of imperfectly correlated assets, it was possible to reduce portfolio risk for a given level of expected return. Formally, a portfolio, , is said to be mean-variance efficient, or a superior portfolio, if it produces greater return for a given level of risk than any other portfolio, , of the same assets, that is, where [2.1]

or less risk for a given level of return,

where [2.2]

assuming the same assets. The two parameters for portfolio, , of assets are derived as follows:

[2.3]

[2.4]

where and are, respectively, the row and column vectors of the portfolio weights of assets, is the vector of expected returns for assets, and Σ is the matrix of variances and covariances between the assets. The parameters for portfolio, , are defined analogously.

While the MPT of Markowitz (1952) remains one of the iconic theories of finance, it is founded on a number of assumptions which have been critiqued extensively in the literature. Of these assumptions, the most relevant to this thesis is its single-period character. MPT’s single-period assumption implies that the optimisation procedure need only be performed once because it is based on single, static estimates of the optimisation’s inputs – the variables in equations [2.3] and [2.4]. The single-period assumption implicitly relies on two important points: firstly, that the investor sees the life of the portfolio as one period; and, secondly, that the investor has in mind estimates of return and risk that characterise the performance of each asset for the entire period in question, along with a fixed covariance (or correlation) matrix that captures the co-movement between assets for the period.

In reality, investors view portfolio decisions as iterative. Realised returns and revised expectations necessitate a review of portfolio objectives, or the estimates used in constructing the portfolio (e.g. Grinold and Kahn, 1999). For example, further portfolio decisions may be required for the following reasons: the portfolio has failed to achieve the investment objective(s) set for it; it has achieved its objective(s) with too much risk; the investment objectives have changed; or, perhaps most importantly, expectations about the future performance of the underlying financial assets have changed. These kinds of examples give us a realistic idea of the context in which portfolio choice decisions takes place. As will become apparent, this thesis is partly motivated by a critique that the literature pays too little cognisance to the institutional setting in which long horizon portfolio choice takes place. In the second and third empirical chapters (Chapters Six and Seven) we set out to correct this deficiency.

It is thus apparent that MPT’s single-period assumption is seriously challenged by the realities of investing, with today’s institutional setting being vastly more complex than that assumed by the basic

MPT model, or by the time diversification debate. The critique of the single-period assumption is that it is too simplistic, and incompatible with the context in which real portfolio decisions are made. Multi-period approaches to the portfolio problem also have a relatively long history in the finance literature, and it is in a multi-period setting that this thesis is squarely located (Tobin, 1965). Moreover, not only does this thesis consider a multi-period problem, it is essentially about the relationship between multiple investment periods of different lengths – the "investment horizon" – and risk. Furthermore, the multi-period nature of the problem at the heart of the thesis also invites us to consider how best to model the asset return processes in the data we select.

Beginning with Samuelson (1969), financial theorists considered whether it was possible to reduce portfolio risk by investing over successively longer periods.[17] Is it possible to reduce portfolio risk by spreading risk across time periods, rather than assets? Over time, the answering of this question has become the time diversification puzzle, a debate that has been nourished by the focus on long-horizon investing born of the growth of private retirement savings which will be discussed in Chapter Three of this thesis.

Time diversification was first examined in an expected utility framework by Samuelson (1969) who found that the allocation to risky assets is independent of time, and only determined by risk tolerance. These conclusions were based on three assumptions:

1) the investor exhibits constant relative risk aversion;
2) returns follow a random walk; and
3) wealth is a function only of returns.

Much of the subsequent research within the expected utility framework has considered variations to these assumptions, and the competing streams of research use these assumptions as a critique of the

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[17] Some might be tempted to date the birth of the time diversification debate to Samuelson (1963). In this paper, Samuelson (1963) clarifies a mistaken interpretation of Bernoulli's Law of Large Numbers where it is thought that "an insurance company reduces its risk by increasing the number of ships it insures (p. 50)." In correcting this interpretation, Samuelson (1963) states that risk is not reduced by adding new risks (as in additional gambles, time periods, or ships insured), it is reduced by subdividing risks (as insurance companies and portfolio managers do). In clarifying one mistaken interpretation he appears to unwittingly introduce a dichotomy where we are choosing between adding a risk or subdividing a risk. While the problem Samuelson (1963) addressed was no doubt of interest, it has on occasion distracted scholars from the real dichotomy at the heart of the time diversification debate. For example, Oldenkamp and Vorst (1997), in an option pricing framework, compare the performance of a ten-year strategy to a one-year strategy repeated ten times. In Samuelson's (1963) terms, this is an example of subdividing risk. Oldenkamp and Vorst (1997) confidently state: "Thus, there are many scenarios in which the repeated one-year strategy outperforms the long-horizon strategy (p. 58)." We believe that Merrill and Thorley (1997) dispelled this misguided notion once and for all when, in critiquing Oldenkamp and Vorst (1997) directly, they note: "The objective in the time diversification debate is to compare risk at *different* (original emphasis) time horizons, not the same horizon (p. 62)." In the interests of complete clarity, we concur with Merrill and Thorley (1997) and see time diversification as a debate about the relative risk of horizons of different length. The comparative performance of strategies with the same horizon, but different reinvestment frequencies, would appear to be more relevant to something like the term structure literature.

framework itself. Without overstating our case, these assumptions are absolutely central to any debate relating to time diversification. In the next section, we propose to demonstrate a clear link between these three assumptions and the research questions that are the subject of this thesis.

Of Samuelson's three assumptions, the first and second have provided the motivation for much of the subsequent literature. As foreshadowed above, the time diversification literature can be neatly classified into four streams or schools of thought: (1) the expected utility theory stream; (2) the option pricing theory stream; (3) a behavioural finance stream; and, (4) an applied stream. The balance of this chapter will outline, and discuss in detail, these separate streams in the time diversification literature which tend to dwell on Samuelson's (1969) first and second assumptions. This thesis then discusses the limited literature relating to Samuelson's (1969) remaining assumption. But before we proceed to the literature review proper we will briefly discuss how Samuelson's (1969) three assumptions – the battle ground of this debate – relate to the research questions of this thesis.

2.2.2 Samuelson's first and second assumptions

As this thesis will show, much of the time diversification debate has revolved around the Samuelson's (1969) first two assumptions – that the investor exhibits constant relative risk aversion, and returns follow a random walk – with the debate taking place both within, and between, the four streams in the literature.

The classical approach requires the researcher to outline how they propose to isolate the relationship between risk and time, motivated by the literature. Predictably, the approach taken corresponds to the researcher's preferred paradigm, or their theoretical priors. For example, a proponent of the expected utility theory would typically estimate the allocation to risky assets that maximises expected utility, given a set of assumptions, and relate their results to investment horizon. Another approach might adopt a different framework, a different set of assumptions, and/or a different methodology. Studies therefore vary on a number of dimensions meaning it is difficult to isolate the essential relationship between risk and investment horizon because each difference in dimension introduces its own variation. We can therefore say that researchers have expended much more effort on adding to the debate – by considering a slightly different definition of risk, or a different set-up – than on synthesising or distilling the debate in search of generality. This willingness to re-visit the debate on different terms, and potentially arrive at a different conclusion, led Kritzman (2000) to make his memorable remark about the time diversification debate becoming a futile referendum on risk. Thus, what is needed is a study that distills the existing literature in search of durable truths. Armed with these truths – if, in fact, such truths exist – we are better positioned to bring them to bear on the essential economic activity motivating this thesis: portfolio choice.

The first research question in this thesis attempts to address this deficiency by synthesising or distilling the debate. The review of the literature will lead us to the applied stream as our vehicle for re-prosecuting the time diversification debate *on common terms*. By carefully selecting the methodology to cover all common approaches, and holding it constant throughout the thesis, we remove methodology as a source of variation, thus enabling us to make inferences about the relationship between risk and investment horizon. This research design results in some general principles that apply to the relationship between risk and investment horizon.

Formally, the first research question proposed in this thesis is:

H_0: That a relationship between risk and investment horizon exists.

H_1: That H_0 is false.

2.2.3 Samuelson's final assumption

Samuelson's (1969) final assumption – that wealth is a function only of returns – is the subject of remarkably little investigation in the time diversification literature. Most scholars are comfortable making Samuelson's (1969) final assumption theirs too. Why this is the case can only be conjectured, but it is thought to relate to the fact that an initial endowment model allows the researcher to ignore factors which both complicate their analysis, and are difficult to generalise in establishing a hypothetical investor. Unfortunately, in accepting this assumption scholars effectively divorce their research from the institutional setting in which long-horizon portfolio choice takes place. We are thus left with studies that are devoid of context. The discussion of this context takes place in Chapter Three.

Where scholars do pursue alternatives to the initial endowment accumulation model, they tend to consider their results in isolation. Thus, we see the results these models produce, but without the benchmark of the initial endowment model, we are none the wiser about how varying the accumulation model affects the relationship between risk and investment horizon. That the time diversification debate remains unresolved, and the marginal effect of alternative contributions is under-studied, is remarkable given that as this thesis is being written, and read, there are trillions of dollars of retirement savings being invested.[18] Without answers to these questions, we are left to wonder upon what basis these investment decisions are being made.

Because Samuelson's (1969) final assumption is relatively untouched by the literature, it offers a fruitful line of inquiry for this thesis. The second research question of this thesis, motivated by a critique of the literature, and an examination of the institutional setting, seeks to study whether

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[18] Pensions & Investments (2012) estimates the total assets of the world's largest one thousand retirement plans at $US6.7 trillion, as at 30 September 2011.

alternative accumulation models impact the relationship between risk and investment horizon. Essentially, by replacing a constant – the initial endowment model – with a variable, we seek to isolate the impact of realistic alternative accumulation models on the essence of the time diversification debate: the relationship between risk and time. Thus, the second research question of this thesis is:

H_0: That alternative accumulation models have no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

As we know, Samuelson's (1969) final assumption holds that wealth is only a function of returns. We also know that returns are, in turn, a function of the portfolio of assets held. Throughout much of the literature, asset allocation is a constant with stocks generally dominating portfolio composition. Given the central role of stocks to investing, this is not surprising. Furthermore, if we are to observe the essential relationship between risk and time, we would seek to limit additional sources of variation in the results and might therefore hold portfolio composition constant. However, since the time diversification debate began, approaches to asset allocation have evolved appreciably. The balanced fund design has been replaced by, or supplemented with, target date funds and a newer generation of dynamic fund designs. The review of the literature for these approaches is provided in Chapter Seven. These newer fund designs – given they all have non-constant asset allocations – implicitly seek to expose the investor to risk at the appropriate time. These designs also implicitly acknowledge that there is an important difference between return- and wealth-based conceptions of performance. So, according to modern portfolio design principles, the *timing*, as well as the magnitude of returns, influences investment performance. Our discussion of the richness of the institutional setting is the subject of Chapter Three.

To explore this point empirically, the third and final study, introduces asset allocation as a source of variation in its consideration of the time diversification phenomenon. At the end of the study, we will be able to observe how a range of competing asset allocation approaches impacts the relationship between risk and investment horizon. Thus, the third and final research question is:

H_0: That asset allocation strategy has no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

Having briefly outlined and located the research questions, we now undertake a detailed review of the literature to further inform the research agenda under consideration in this thesis.

2.3 The time diversification literature

Samuelson's (1969) seminal work on time diversification was founded on three assumptions: (1) the investor exhibits constant relative risk aversion, (2) returns follow a random walk, and (3) wealth is a function only of returns. Of these three assumptions, the first and second have motivated almost all the subsequent time diversification literature. As such, the first empirical study (Chapter Five) – which seeks to synthesise the debate thus far, and potentially offer a resolution – will be motivated by, and designed to address, the literature on these two assumptions. We also consider these two assumptions together in one section because many previous studies in the literature discuss and vary both assumptions simultaneously.

In order to make the review of the literature easier to navigate we divide the literature into a number of streams which have emerged based on common critiques, or consistent theoretical or methodological foundations. Each stream is introduced in chronological order by the date of publication of the stream's foundational work. The exception to this rule is the most diverse stream in the literature – the applied stream – which is introduced last because it leans on elements of each of the other streams, and adopts diverse and idiosyncratic approaches to studying the time diversification phenomenon.

Our discussion of Samuelson's (1969) final assumption, which motivates this thesis's second and third studies (Chapters Six and Seven), will follow separately.

2.3.1 Samuelson's first and second assumptions

2.3.1.1 Expected utility theory

The expected utility theory stream of the literature tends to observe risk indirectly. Rather than defining risk a particular way (e.g. standard deviation), then measuring it over various investment horizons, scholars within the expected utility framework typically solve for the optimal allocation to stocks for a given assumption set. In this way, it is possible to impute what happens to risk as horizon changes by observing the optimal allocation to risky assets for a variety of horizons. But in order to analyse risk in this way, Samuelson (1969), and his successors in this stream of literature, assign a risk aversion specification to their hypothetical investor via the selection of a utility function. The direction of the logic thus begins by defining the investor's risk tolerance and then works toward what this means for the relationship between risk and investment horizon. Such an approach therefore tends to be normative because it generalises how investors perceive risk first, then observes the relationship of interest. This thesis will show that the normativity of the expected utility framework, as well as its sensitivity to the specification of risk aversion, represent its key weaknesses.

As we will see, the direction of the logic in this thesis runs the opposite way. We begin by defining and measuring risk in a variety of objective ways, and then ask what these diverse measures mean for the time diversification debate. We do this for two reasons. Firstly, the professed aims of this thesis are to synthesise the fragmented time diversification debate in search of resolution, and then to extend the debate to consider alternative accumulation models and study the effect of asset allocation. We must therefore replicate much of the debate as it exists. It will be apparent once we have completed the review of the literature that much of the applied stream of literature uses the same logic that is employed in this thesis. Secondly, we contend that the population of investors is highly heterogeneous, and not as amenable to generalisation as advocates of the expected utility framework would have us believe. The very existence of the field of behavioural finance is testament to the fact that much of classical economics assumes away the human aspects of what remains a social science. This critique is not new, and has been used in the literature to motivate both the option pricing framework and behavioural streams in the literature.

Samuelson (1969), in his multi-period generalisation of Markowitz (1952), found, assuming the investor exhibits constant relative risk aversion (CRRA), that "... the optimal portfolio decision is independent of wealth at each stage and independent of all consumption-saving decisions leading to a constant [risky asset weight] w^* (p. 244)." A page later, Merton (1969), in continuous time, confirmed Samuelson's constant weight finding in the presence of CRRA, and extended his analysis to consider a constant absolute risk aversion (CARA) assumption.[19] Merton (1969) found that, assuming CARA, the dollar value of wealth invested in the risky asset remains constant, so as wealth rises the proportion falls. While Merton (1969) admits this form of utility function is behaviourally less plausible, we see that very soon after the time diversification debate began we have evidence that Samuelson's (1969) findings are sensitive to his framework's assumptions.

Kritzman and Rich (1998) clearly show in their Exhibit 2 – reproduced herein as Figure 2.1 – that the allocation to risky assets is sensitive to how each of Samuelson's (1969) first two assumptions are specified. For example, for each of three asset return processes Kritzman and Rich (1998) consider, we can see that it is possible that the allocation to risky assets may be constant, increase with time, or decrease with time depending on the utility function specification. While we can't completely discard the expected utility framework as a means of analysing the time diversification debate, we are beginning to question how it is possible to rely on conclusions so sensitive to their underlying assumptions.

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[19] Merton's (1969) paper literally begins on the next page of the same edition of the *Review of Economics and Statistics*. Merton (1969) notes that Samuelson's (1969) work is more general with respect to the probability distributions it can handle.

Figure 2.1: The impact of preferences and return characteristics on time diversification

This figure reproduces Exhibit 2 (p. 68) from Kritzman and Rich (1998) that shows how the allocation to risky assets varies with utility function, risk aversion and the asset return process.

			Impact of Time on Equity Allocation		
Utility Specification	**Absolute Risk Aversion**	**Relative Risk Aversion**	**Random Walk**	**Mean Reversion**	**Mean Aversion**
Log Utility = ln(Wealth)	Decreasing	Constant	Hold Constant	Hold Constant	Hold Constant
Square Root Utility = (Wealth)$^{\frac{1}{2}}$	Decreasing	Constant	Hold Constant	Decrease	Increase
Power Utility = -1/Wealth	Decreasing	Constant	Hold Constant	Increase	Decrease
Quadratic Utility = 25 x Wealth + 0.1 x Wealth^2	Increasing	Increasing	Decrease	Decrease	Decrease
Combination Utility = 1/Wealth + ln(Wealth)	Decreasing	Decreasing	Increase	Decrease	Increase

Even the most prolific scholars within this stream of the literature – like Samuelson (1963, 1969, 1971, 1989, 1990, 1994) and Kritzman (1994, 2000) – concede that their general findings against time diversification may not hold with alternative utility specifications.[20] Samuleson (1989), for example, states that if the logarithm of wealth less some subsistence level of consumption is the expected value to be maximised, then a lower allocation to stocks with age can be justified. Samuelson (1989) states:

> "Suppose, though, human nature is such that we are each most anxious *not to fall below a 'subsistence' level of terminal wealth* [original emphasis] - so that log(W - S) and not log W is the utility whose Expected Value we seek to maximize. In that case [the] contention is correct that older people will put less into risky stocks when they have fewer years to go before the terminal date of retiring or bequeathing (p. 11)."

Similarly, Kritzman (1994) conditions his findings in favour of Samuelson's (1969) original conclusion by highlighting five "valid reasons why you might still condition your risk posture on your

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[20] Note that both authors also published other *joint* work in the time diversification literature, for example, Merton and Samuelson (1974) and Kritzman and Rich (1998), respectively.

investment horizon (p. 17).” One of these reasons is the potential for an investor to have a discontinuous utility function.[21] Kritzman (1994) explains:

> “Consider, for example, a situation in which you require a minimum level of wealth to maintain a certain standard of living. Your lifestyle might change drastically if you penetrate this threshold, but further reductions in wealth are less meaningful. You might be more likely to penetrate the threshold given a risky investment over a short horizon than you would be if you invested in the same risky asset over the long run (p. 17).”

Milevsky (1999) tests just such a discontinuous utility function motivated by the fact that it “has been extolled as conforming to observed investor behaviour (p. 271).” Milevsky (1999) supports Samuelson’s (1969) results, finding that the optimal allocation to risky asset is independent of time. He also finds that, notwithstanding a constant allocation to risky assets, risk – defined as the probability of earning a cumulative rate of return less than that of the risk-free asset – declines exponentially with investment horizon. Milevsky (1999), thus, differentiates between the risky asset allocation and risk in a way not generally seen in this stream of the literature.

Apart from the aforementioned studies, the expected utility stream of the literature contains numerous other studies which attempt to study time diversification by estimating the optimal allocation to risky assets that maximises expected utility, given some set of assumptions (e.g. Bodie et al., 1992; Levy and Spector, 1996; Levy, 1996; Jagannathan and Kocherlakota, 1996; Van Eaton and Conover, 1998; Hansson and Persson, 2000; Strong and Taylor, 2001; Gollier, 2002; Karlsson, 2006).[22] What we have shown here is that the central players in this stream in the literature admit that the time diversification debate hinges on the form of risk aversion exhibited by the investor. If the specification changes, as Kritzman and Rich (1998) show us, then so does the relationship between risk and time.

Other streams in the time diversification literature generally motivate their resort to an alternative framework with a critique of the Samuelson (1969) approach. The option pricing protagonists contrast their framework with Samuelson (1969) by highlighting its objectivity and independence from particular models of risk aversion or utility. Fisher and Statman (1999), as representatives of the behavioural stream of the literature, critique the assumptions underlying expected utility theory noting that Samuelson (1969) implies that investors accurately assess the probability of loss, a fact not supported by the behavioural literature. Critiques of expected utility theory even pre-date the time diversification debate itself. For example, in criticising results from a similar framework to that of

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[21] Kritzman and Rich (1998) reiterate this point regarding a discontinuous utility function and give three practical situations consistent with a sudden drop in utility if a threshold is penetrated: “A decline in the value of pension assets will cause a net pension liability to appear on a company’s balance sheet; A covenant on a loan agreement will be breached if assets fall below a specified value; Your spouse will abandon you if your net worth falls by a certain amount (p. 70).”

[22] Other studies examine optimal allocations from a purely mean-variance perspective including Levy and Gunthorpe (1993), Gunthorpe and Levy (1994), Booth (2004), Mukherji (2008) and Panyagometh (2011).

Samuelson (1969), Roy (1952) comments that “in calling in a utility function to our aid, an appearance of generality is achieved at a cost of a loss of practical significance and applicability in our results. A man who seeks advice about his actions will not be grateful for the suggestion that he maximize expected utility (p. 433).”

As shown in the earlier discussion of Kritzman and Rich (1998), it is common in the expected utility theory literature for authors to vary both of Samuelson’s (1969) first and second assumptions simultaneously. This is the principal reason why they are considered together in this section of the literature review. For example, Kritzman and Rich (1998) in a matrix, reproduced herein as Figure 2.1, outline what happens to the optimal allocation to stocks for fifteen separate combinations of utility function – log utility, square root utility, power utility, quadratic utility, and combination utility – and asset return process – random walk, mean reversion, and mean aversion. Once again we see that the verdict on time diversification, from within its foundational paradigm, is highly sensitive to the model specification. Take, for example, the power utility function assumption from Figure 2.1. Depending on one’s view of the asset return process, there are three possible relationships between time and risk.

Another relevant study from the expected utility theory stream of the literature is that of Strangeland and Turtle (1999). It is relevant to this thesis because it buttresses the case against pursuing an expected utility approach, using many of the critiques pursued herein. Furthermore, it does so *from within* the expected utility framework. It thus represents another example – like Samuelson (1989) and Kritzman (1994) – where the proponents of expected utility realise that the time diversification puzzle may require alternative, or complementary, approaches to be resolved in any general way.[23] Strangeland and Turtle (1999) state that the presence of time diversification “depends critically on a number of important and highly context-dependent factors (p. 1).” They go on to cite relative risk aversion and the risky asset return process as two, out of a total of six, factors that may affect portfolio choice.[24] This context-dependency leads Strangeland and Turtle (1999) to conclude that “... there is little motivation to debate the general merit of time diversification for a *typical* (original emphasis) investor (unless we have a very clear understanding of a typical investor) (p. 1).” We can see that, like us, Strangeland and Turtle (1999) detect a fundamental incompatibility between the restrictive and deterministic nature of their framework on the one hand, and the heterogeneity of the hypothetical investor, and the inconclusive evidence on the process driving asset returns, on the other. But perhaps Strangeland and Turtle’s (1999) most revealing comment puts their views beyond doubt. They state

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[23] Resolving the time diversification debate for certain specific circumstances is not beyond any of the major approaches taken in the literature. A lack of generality is why time diversification remains a puzzle.

[24] The complete set of factors Strangeland and Turtle (1999) identify are: (1) relative risk aversion; (2) risky asset return process; (3) ability to change work habits; (4) frequency of required withdrawal for the investor's portfolio; (5) existence of non-tradable assets (e.g. human capital); and, (6) changes in investment knowledge over the investor's life.

that "... the issue of time diversification *cannot* (original emphasis) be completely resolved by resorting to an expected utility framework (p. 2)."

Strong and Taylor (2001), attempting to correct for the restrictive assumptions of earlier studies, examine time diversification with what they describe as "realistic utility functions (p. 268)." Moreover, they claim to "not impose … restrictions on the process followed by risky asset returns (p. 268)." We can see in Strong and Taylor's (2001) intent an implicit recognition of the critiques we make herein: that some of the utility function specifications adopted in the literature are more convenient than representative; and, the verdict on time diversification is, in part, a function of the assumption made regarding the process driving asset returns. At face value, therefore Strong and Taylor (2001) agree with our critiques, motivate their work with these critiques, and then set out to provide a corrective for the associated inadequacies. At first it appears that Strong and Taylor's (2001) approach may be sufficiently different to allow a resolution within the expected utility framework. For example, in contrast to Samuelson (1969), Strong and Taylor (2001) "provide support for the practitioner view that equity is a long-term investment," and that there "is evidence that equity represents a (significantly) more desirable investment over a ten-year investment horizon than over a one-month investment horizon (p. 297)." Notwithstanding this finding, we see more evidence of the weaknesses of expected utility theory. Once again, optimal allocations are not robust to "various levels of risk tolerance or various utility functions (p. 298)." Therefore, even "realistic utility functions (p. 268)" don't assist us in unravelling the puzzle. We, once again, find ourselves searching for an objective measure of risk which allows us to avoid the deterministic models of risk aversion integral to the expected utility stream of the literature.

One commonality between how the expected utility literature deals with the risk aversion specification and the asset return process is that both are approached from a deterministic perspective. Studies typically choose a risk aversion and an asset return process assumptions motivated by the relevant literature, and then proceed to analyse that chosen set-up. Unfortunately however, as with utility functions, the views of scholars on the asset return processes driving financial data are mixed. Early research, for example, concluded that stock prices contain a predictable component over short horizons, contrary to Samuelson's (1969) random walk assumption (Bodie, 1976; Jaffe and Mandelker, 1976; Nelson, 1976; Fama and Schwert, 1977). Later studies reported evidence of negative serial correlation, or mean reversion, over longer horizons (Fama and French, 1988; Poterba and Summers, 1988; Lo and Mackinlay, 1988). While attempts have been made to explain mean reversion (e.g. Malliaropulos and Priestley, 1999; Poterba and Summers, 1988; DeBondt and Thaler, 1987, 1989), no decisive argument has yet emerged. And to complicate matters further, a number of scholars find evidence against mean reversion (e.g. Richardson and Stock, 1989; Kim et al., 1991; McQueen, 1992; Miller et al., 1994). Thus, we once again see that the time diversification debate is less about the question at hand – the relationship between risk and time horizon – and more about a second-order question relating to the assumptions behind the expected utility framework, in this latter case about what asset return process drives returns.

So having critiqued how the time diversification handles the asset return process assumption of Samuelson (1969), what do we propose as an alternative? Firstly, given the first study of this thesis focusses on a synthesis of the literature we are forced to replicate some of the key approaches in the literature. We must therefore model Samuelson's (1969) random walk assumption. As will become apparent, this study pursues the applied stream in the debate and is thus drawn to simulation methods rather than closed form approaches. For example, to model the random walk assumption of Samuelson (1969), we use two separate simulation methods: one parametric (Monte Carlo simulation) and the other non-parametric (the bootstrap simulation method of Efron, 1979). Secondly, rather than imposing our views about the asset return process as is done in much of the time diversification literature, we select econometric methods amenable to reproducing the time series characteristics in the data. Here we use two types of block bootstrap simulation method – the stationary bootstrap method, and what we call the empirical block bootstrap method – where the principal difference between the methods relates to procedure for estimating the appropriate block size.[25] The analysis of the time diversification debate in this thesis is therefore strictly empirical in that the asset return process assumed – whatever that might be – is an empirical feature of the data. The alternative path is where the analysis is really the end-product of a number of positions taken on other contested questions in finance: for example, what utility function or model of risk aversion "best" characterises the "average" investor? And, what time series process drives returns? We thus prefer to let the data speak for themselves via state-of-the-art quantitative methods, as an alternative to imposing what we believe to be the most appropriate asset return process assumption.

Whilst this stream of literature is perhaps the most voluminous and long-lived in the time diversification debate, there are three principal reasons why it will not be pursued further in this thesis. Firstly, there is no consensus regarding what utility function specification best represents the "average" investor, if such a generalisation were possible. The evidence from the literature in this respect is not convincing. Secondly, as Strong and Taylor (2001) suggest, estimates of the optimal allocation to risky assets are not robust to various levels of risk tolerance or various utility functions. Thus, any generality we seek to highlight can be rejected by providing a counter-example using an alternative risk aversion or utility specification (Rabin, 1952; Booth, 2004). And, finally, we contend that expected utility theory is normative in that it imposes a utility function, or model of risk aversion, then proceeds to analyse the relationship between risk and time.[26] We prefer to begin with various

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[25] As explained later in the methodology section (Chapter Four), the use of block re-sampling is the means by which serial dependence is modelled.

[26] Thorley (1995) argues that "critics invoke mathematical models of risk aversion to argue that investors should not succumb to the time diversification 'fallacy.' The premise of [Thorley's] paper is that these arguments are a misapplication of the positive economic paradigm (p. 73)."

objective measures of risk as the basis for a study of time diversification. While this thesis shows a lack of sympathy for the expected utility framework, the fragmented and contradictory findings of the literature to date certainly gives support to our thesis that the time diversification debate is in need of synthesis.

2.3.1.2 Option pricing theory

Bodie (1991) was the first study to depart from the expected utility framework. In his paper, Bodie (1991) goes beyond "the Samuelson-Merton analysis" – which finds that investment horizon should not affect the optimal asset mix – to investigate "the implications of option pricing theory for investment policy of defined benefit pension plans (p. 57)."[27] Other than a desire to test the time diversification question using a different paradigm, Bodie (1991) provides no motivation for this innovation. It is not until later – Merrill and Thorley (1996) to be precise – that advocates of option pricing theory offer it as an objective assessment of the relationship between risk and investment horizon. As shown in the last section, Samuelson (1969) and his successors, generally proxy risk by estimating the optimal allocations to risky assets over different horizons. So how does Bodie (1991) perceive risk? Using Black-Scholes-Merton Option Pricing Theory, he equates risk with the cost of insuring against shortfall risk. In so doing, Bodie (1991) makes a distinction between the probability of shortfall – which he deems a "faulty definition of risk (p. 60)" – and the cost of insurance against shortfall risk which he estimates with option pricing theory. Bodie's (1991) basic conclusion is: "If the objective of pension asset management is to minimise the cost of providing guaranteed benefits, then the longer the time horizon, the *lower* (original emphasis) the proportion of assets that should be invested in stocks (p. 57)." This finding is both at odds with the findings of Samuelson (1969) – who suggests that the allocation to risky assets is a function of risk tolerance not investment horizon – and conventional wisdom – which argues that higher allocations to risky assets can be justified at longer investment horizons.

Bodie's (1995) motivation is identical to his earlier work (see Bodie, 1991) in that he sets out to test the "familiar proposition (p. 18)" at the heart of the time diversification debate: that investing in stocks is less risky the longer the horizon. He argues that, for this proposition to be true, the cost of insuring against underperforming the risk-free rate should fall as the investment horizon lengthens. The principal difference between the two works is that Bodie (1995) tests his findings for two types of asset return process: the random walk assumption as in Samuelson (1969); and, mean reversion. Bodie (1995) confirms the findings of Bodie (1991) that risk, measured using option pricing theory, increases rather than decreases with investment horizon. Thus, having adopted a different theoretical paradigm, Bodie (1991, 1995) produces results that contradict both conventional wisdom and the

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[27] Bodie (1991) uses the general descriptor "Samuelson-Merton analysis" because of the multiple contributions each of these scholars has made to this debate. He cites in particular Merton (1971), Merton and Samuelson (1974), and Samuelson (1963, 1989) as examples.

main findings of the expected utility theory stream of the literature discussed earlier.[28] In this sense, we see that the time diversification debate is developing *between* paradigms, as well as *within* paradigms. This matters to this thesis because, if we are to advocate the importance of synthesis in relation to this puzzle, we must first show that a puzzle exists.

A further relevant contribution of Bodie (1995) is a salient reminder that the investment decision which he examines – and which is the subject of this thesis – exists within a broader lifetime planning context as discussed in Bodie, Merton and Samuelson (1992). In their paper, Bodie et al. (1992) considers whether the presence of labour flexibility affects consumption, saving, and portfolio investment decisions over the lifecycle. According to this broader context, where *total* wealth is the sum of financial capital and human capital, the investment decision is one of several interrelated factors bearing on lifetime financial planning. A worker's lifetime income profile – which, in present value terms, equals human capital – might thus bear on the investment decision.[29] Bodie (1995) – like Samuelson (1989) and Kritzman (1994) – conditions his findings regarding time diversification by referring to this more comprehensive set-up in Bodie et al. (1992). Bodie (1995) states that: "Asset allocation for individuals should be viewed in the broader context of deciding on an allocation of *total* (original emphasis) wealth between risk-free and risky assets (p. 20)." Within this broader context, Bodie (1995) finds a potential justification for a downward sloping allocation to risk assets through time (cf. Bodie et al., 1992).

Merrill and Thorley (1996) favor Bodie's (1995) approach because they view it as an "objective way to evaluate the arguments for and against time diversification" that is "independent of any specific model of investor utility or risk aversion (p. 13)." We thus see the first sign of a formal critique of the expected utility framework motivating work within the option pricing stream of the literature. Despite their agreement with Bodie (1995) about option pricing theory's objective evaluation of risk, Merrill and Thorley (1996) use the same option pricing theory to consider two types of financially-engineered products – Protected Equity Notes and Self-Funding Market Collars – and find "that longer time horizons reduce the cost of risk elimination, and by implication, risk itself (p. 13)."[30] Once again, we

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[28] Bodie (1996) again examines the relationship between risk and investment horizon, this time in a defined benefit framework. Referring to Harlow (1991), he reiterates his critique of using the probability of shortfall as a risk measure noting that "it completely ignores how large the potential shortfall might be (p. 90)."

[29] This point – that lifetime income bears on the investment decision – and reliance on the work of Bodie et al. (1992) is common in the time diversification literature.

[30] Dempsey et al. (1996), Zou (1997) and Oldenkamp and Vorst (1997) provide critiques of Merrill and Thorley (1996). Dempsey et al. (1996) highlight what they believe to be a false analogy in Merrill and Thorley's (1996) interpretation of Bodie (1995). Zou (1997) suggests that Merrill and Thorley's (1996) findings may say more about their methodological approach, than it does about time diversification. Oldenkamp and Vorst (1997) claim to "show that Merrill and Thorley's (1996) conclusions are not as obvious as they claim" and "that their arguments do not resolve the time diversification debate (p. 57)." We note that Merrill and Thorley (1997) provide a qualitative response to Oldenkamp and Vorst's (1997) critique of their work. They state, and we agree, that "the objective in the time diversification debate is to compare risk at *different* (original emphasis) time horizons, not the same horizon. The fact remains that it costs an investor less to insure against underperforming

see an example of a study that yields polar-opposite results to another study within the same theoretical paradigm.

While Merrill and Thorley (1996) muddies the findings of the option pricing theory stream of the literature, they make at least three other critical points about the debate. Firstly, in viewing the time diversification literature, Merrill and Thorley (1996) make a distinction between “practitioner-oriented empirical research” and the work of “financial theorists” (p. 13). The work of these financial theorists is in fact the literature that we are critiquing in this literature review. The critique of the financial theoretical literature is that it tends to degenerate into a debate about the theoretical paradigm – e.g. what it the most defensible utility function – which necessarily leads to less focus on the essential relationship between risk and time horizon. Furthermore, many of Merrill and Thorley’s (1996) financial theorists, having outlined the “incontrovertible truth (Kritzman, 1994, p. 17)” and the “mathematical truth (Kritzman and Rich, 1998, p. 71)” gleaned from their theories, go on to provide a litany of reasons why their findings – and, this thesis would argue, their theory – might prove unreliable. How are we to be convinced by an argument when the theoretical edifice upon which the argument is built is undermined by the theory’s principal proponents? At least, one would argue, the “practitioner-oriented empirical research” seeks to free itself as much as possible from the false comfort of theory. It is this “practitioner-oriented empirical” approach – or what Booth (2004) describes as the “applied” stream in the literature – that will be adopted as the *modus operandi* throughout this thesis.

Secondly, Merrill and Thorley (1996) rightly point out that differences of opinion are “often rooted in semantic issues about the meaning of risk (p. 13).” This statement is both a premonition of Kritzman’s (2000) comment about the time diversification debate being “a referendum on the meaning of risk (p. 50)”, as well as one of the motivations for the focus of the first empirical study of this thesis. If we are to have a fair referendum on risk are we not obliged to conduct it on common terms? This thesis argues that it is, by comparing a comprehensive array of risk measures from the literature using a consistent methodology in Chapter Five of this thesis.

Third, and finally, Merrill and Thorley (1996) hint at an important insight into the basis upon which performance ought to be evaluated. They indicate that returns-based measures of performance may be the wrong basis for evaluation when they state: “Some critics of time diversification object to the use of annualised returns in measuring risk and return across different time horizons and suggest that dollar figures are more revealing ... Thus, the argument for time diversification is even stronger when stated, perhaps mistakenly, in simple dollar terms (p. 17).” We argue that, far from being mistaken,

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the risk-free rate at a long horizon compared to a short horizon (p. 62).” We make this point about the confusion between investment horizon and rebalancing frequency earlier when discussing the birth of the time diversification debate and Samuelson’s (1963) earlier work. Please refer to footnote 17.

wealth-relative terms is the *only* way to evaluate risk and return, particularly when we depart from the initial endowment model so popular in the literature.

As discussed earlier, Bodie (1995) found that "the cost of the insurance rises with [investment horizon] *T* (p. 20)" and, therefore, so does risk, suggesting that time diversification, as conventional wisdom conceives it, does not exist. Numerous scholars have lined up to critique Bodie's (1991, 1995, 1996) findings, many doing so in a qualitative fashion. Ferguson and Leistikow (1996) question Bodie's (1995) singular focus on risk instead noting that "if appropriate allocation proportions depend on reward in relation to risk, not just risk, then Bodie's message for individuals is irrelevant (p. 68)." Taylor and Brown (1996) challenge Bodie's (1995) analysis on three fronts. Firstly, they suggest that constant relative risk aversion might be a valid assumption over short horizons but may not be over long horizons. As an aside, this critique would apply equally to Samuelson's (1969) expected utility framework. Secondly, Taylor and Brown (1996) argues that Bodie (1995) sets-up a "straw man that he knocks down with unrealistic assumptions (p. 69)." For example, they suggest that the worst-case scenario Bodie (1995) uses is extremely unlikely and is thus unrealistic. And finally, they highlight that Bodie's (1995) argument fails when his assumption of a constant standard deviation in his application of the Black-Scholes-Merton option pricing model is replaced with a non-constant standard deviation. As we will show, very few measures of risk are constant with investment horizon.

Cohen, de Fontenay, Gould, Sirera and Bodie (1996) is a collection of four letters to the editor in response to Bodie (1995). Therefore, other than each responding to Bodie's (1995) work, there is no common theme between the letters. Rather, they represent a heterogeneous selection of critiques. Cohen, for example, points out that "the reasonable cost of insurance declines as the horizon is extended and/or return expectations are increased (p. 72)." This assertion is supported by Dempsey et al. (1996). Gould, who is clearly a practitioner, hints at two points that are of particular interest in this thesis: firstly, that "dollar savings (p. 73)" are the most important measure in pension finance problems; and, secondly, that achieving a retirement goal – which Gould expresses in dollar terms – might be a relevant way of conceiving a practical pension finance problem.[31] Dollar-denominated conceptions of performance are pursued throughout this thesis, and we explore retirement goals in the design of the dynamic asset allocation strategy in the third empirical study (Chapter Seven).

Dempsey et al. (1996) attempt to synthesise the option pricing literature and clarify the debate. On the one hand, they see, on the basis of option pricing theory, an argument that risk rises with time (e.g. Bodie, 1995) and, on the other, "a conventional insurance premium (p. 57)" argument where risk falls. Dempsey et al. (1996) find that Bodie's (1995) put option prices are correct, but that the price of a put option is a valid measure of riskiness "only in the case in which one may assume that the potential returns for holding the stock in relation to risk do not improve with the investment time horizon (p.

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[31] Bierman (1997) and Booth (2004) also suggest that the achievement of investment goals might be important.

60).” Dempsey et al. (1996) conclude that the put option prices of Bodie (1995) “cannot be taken as representing a measure of market risk. The simple reason is that the price of a put option is indicative of *two* (original emphasis) features of the market: risk and the market's reward for risk that an insurance writer on a stock can expect to achieve (p. 61).”

In conclusion, as Merrill and Thorley (1996) argue, a chief advantage of Bodie’s (1991, 1995) option pricing theory approach to investigating time diversification is that it measures risk objectively, in contrast to the more normative and contested expected utility theory stream of literature. Notwithstanding this advantage, the option pricing stream has been subjected to three specific critiques from both within the paradigm, and in competing paradigms. Firstly, Ferguson and Leistikow (1996) and Bierman (1997) argue that Bodie’s (1995) option pricing approach implicitly ignores reward-for-risk calculations in favour of risk-only ones. Bierman (1997) notes that: “We need to consider an interpretation of risk that includes good outcomes as well as the bad outcomes (p. 52).” In one sense, Bodie’s (1995) risk-only approach is defensible given that time diversification has always been about the relationship between risk and investment horizon. Bierman’s (1997) argument, on the other hand, has merit: if it wasn’t for the returns stocks offer, they wouldn’t be the financial asset *du jour*. The second critique might be appropriately described as a technical argument. As Taylor and Brown (1996) argue, when the constant standard deviation assumption of Bodie (1995) is relaxed, his findings collapse. Thus, in essence, the constant standard deviation assumption is to the Black-Scholes-Merton Option Pricing Theory, as the constant relative risk aversion assumption is to Samuelson’s (1969) expected utility theory. Each is a dubious assumption which proves lethal to its parent theory, and to our attempts to find a general relationship between risk and investment horizon. Third, and perhaps most important to this thesis, is the claim by Kritzman and Rich (1998) that, “[u]nfortunately, the option angle of time diversification has resurrected a misguided discussion about the meaning of risk (p. 71).” So not only do we see the debate taking place *between* paradigms, as well as *within* paradigms, we see the recurring theme regarding the meaning of risk. Perhaps, rather than defining risk a single way and arguing about time diversification from that point of view, we should define a dataset and methodological approach, and then replicate multiple conceptions of risk in order to formally conduct Kritzman’s (2000) referendum on risk.

So, based on the above critiques, this thesis elects not to pursue the option pricing theory approach to considering time diversification. Furthermore, when we consider the empirical chapters of this thesis (Chapters Five through Seven), returns are critical to the measures estimated, and to the overall argument of this thesis. This comment is especially true when we consider alternative accumulation models and asset allocation strategies in Chapters Six and Seven respectively. Bierman’s (1997) critique is thus decisive in the decision to discontinue consideration of option pricing theory.

2.3.1.3 Behavioural finance

Until now we have reviewed the two most enduring streams in the time diversification literature: the expected utility theory stream and the option pricing theory stream. Both streams are characterised by strong theoretical foundations, rigorous analytical approaches, and numerous studies. With this third stream – the behavioural finance stream – a different sort of literature is presented. Apart from being relatively new, the behavioural finance literature has neither strong theoretical foundations (of the economic kind), nor any particular analytical approach. There are also few studies in the behavioural finance stream of the literature. Instead, the behavioural finance research applies the insights of psychology to financial decisions in order to better define how to study problems and interpret findings. In this sense, it offers no alternative analytical framework to compete with the two approaches highlighted thus far. Rather it tends to focus on identifying and enunciating deficiencies in the earlier literature. For example, behavioural economists have critiqued the time diversification literature for not framing risk properly.

Olsen (1997) is one study that discusses risk and how it should be framed. In particular, Olsen (1997) points out that risk in pension funds management ought to be considered from the perspective of the plan member, the beneficiary, whose "risk might be related to the loss of a large amount of wealth (p. 62)" versus a manager whose "risk might be associated with a portfolio return below that of one's colleagues (p. 62)."[32] We again see here a distinction between wealth-denominated measures of risk – which Olsen (1997) sees as relevant for pension funds – and return-dominated measures of risk which are arguably more relevant to investment managers. This distinction is a persistent theme in this thesis, and the research of scholars like Olsen (1997) provides motivation for our consideration of wealth-denominated measures of performance in our empirical studies.

Olsen (1997) also describes risk as a "multiattribute phenomenon (p. 65)" where the principal risk attributes appear to be "the potential for a below-target return, the potential for a large loss, the investor's feeling of control, and the level of knowledge about an investment (p. 65)." The essentially human dimension of these latter two attributes highlights both the contribution of behavioural economists, and the difficulties presented by the qualitative nature of behavioural finance research. On this basis, we will overlook these attributes. The former two attributes are of particular interest in this thesis because they buttress the case for the research design we pursue. The first attribute – the potential for a below-target return - confirms the importance of targets to pension finance problems. We take this point up in the design of our dynamic asset allocation strategy, and the reporting of

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[32] In practice, both perspectives are – or at least should be – of interest to pension fund trustees. The plan member perspective is the appropriate terms upon which to consider whether the plan is meeting its commitments to plan members. The latter perspective – the investment manager perspective – is the lens through which evaluation of investment managers should take place. Based on the author's professional experience, the distinction between these two perspectives is not always appreciated by plan trustees, management or academia.

comparative performance, in Chapter Seven. The second attribute – the potential for a large loss – suggests once again that the magnitude of risk is important, which in turn focuses our attention on wealth-denominated measures of risk.[33] For example, it is a truism that two minus 25 per cent returns are equivalent *in percentage terms* no matter when they occur during a plan member's accumulation phase. If however we compared the impact of equivalent negative returns at two different points in the accumulation phase – say, at age 30 and age 50 – the differences could be materially different *in wealth terms*. Because this thesis is in the field of pension finance we take seriously the perspective of plan members, and thus we focus in part on wealth-denominated performance measures.

Lastly, Olsen (1997) presents evidence that the relative importance of these attributes is a function of "idiosyncratic investor and asset characteristics (p. 65)." In one sense, these findings are not surprising. In another way, the idiosyncratic nature of the relative importance of the attributes represents a telling critique of the standard expected utility theory assumption of constant relative risk aversion. Olsen's (1997) finding suggests that any attempt to generalise risk tolerance may be fraught, notwithstanding its convenience. Olsen (1997) thus lends support to our choice of using a number of objective measures of risk over the normative frameworks like Samuelson's (1969).

Olsen and Khaki (1998), in discussing how risk is treated in the time diversification literature, make three important points that support the approach taken in this thesis. Firstly, Olsen and Khaki (1998) dismiss expected utility theory as the framework that can resolve the time diversification debate. They note that "the dismissal of time diversification on positive grounds cannot be justified by appeal to the traditional discounted-SEU model (p. 58)." Olsen and Khaki (1998) are indirectly confirming our critique of expected utility theory as normative, because it seeks to impose a model of risk aversion on the hypothetical investor. Our decision to pursue a range of objective measures of risk as an alternative is again supported by the literature. A further specific critique Olsen and Khaki (1998) make of expected utility theory – or what they describe as "the traditional SEU models" – is that "decision makers do not treat probabilities and outcomes in the multiplicative fashion assumed by the traditional SEU models ... decision makers use an additive model of risk (p. 60)." We see here another example of the strengths of the behavioural stream in the literature: a willingness to look beneath the surface of the classical economic models and attempt to reconcile their assumptions with the behaviour of investors. Without this correspondence between the behaviour of investors and economic theory, we risk economic theory becoming an elegant, but not altogether informative, caricature of reality.

Secondly, Olsen and Khaki (1998) continue to provide a range of behavioural insights which support the way we design this thesis. Olsen and Khaki (1998) go further than Olsen (1997) and contend that the magnitude of loss is not only an important aspect of risk, it is of paramount importance in

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[33] Rabin (2001) also argues that investors are "loss-averse" rather than risk-averse.

understanding risk from the investor’s perspective. For example, they note “... investors consider risk a positive function of probability and size of loss, with considerably greater weight being given to the size of the loss than to the possibility of loss (p. 60).” Olsen and Khaki (1998) go on to provide a cautionary note about how the importance of loss relates to probability, emphasising that “... the tendency to ignore low-probability negative outcomes could lead to financial catastrophe. Thus, potential outcomes should not be dismissed as a matter of course when a large portion of one's wealth is at risk (p. 61).” In the context of this thesis – which studies long horizon investing and a range of accumulation models including ones that incorporate contributions – this insight of Olsen and Khaki’s (1998) is of critical importance. Although large negative returns might be rare, when the portfolio size effect of Basu and Drew (2009a) sees a rapid rise in portfolio wealth as retirement nears, even a small negative return can result in large impacts on terminal wealth. Insights like this one, encourage us to consider the full distribution of terminal wealth outcomes, as well as a number of downside risk measures. Olsen and Khaki (1998) even go as far as to question whether time diversification is indeed compatible with a behavioural conception of risk. They note that: “questions remains, however, of whether the concept of time diversification is generally consistent with the concept of risk as it has been documented in other studies of investment behaviour (p. 59).”

Olsen and Khaki’s (1998) third, and final, important point is their clear recognition that time diversification is a contested idea, and might therefore reasonably be described as a puzzle. Even at the point in time when their article was published they acknowledged that “the lack of closure on this topic stems from the [economics] profession’s failure to accept a common definition of risk (p. 58).” As will become apparent, when we review the applied stream in the literature, there are many more definitions of risk proposed in the literature. Some scholars – in particular those advocating expected utility theory – impose a theoretical framework and, based on a number of assumptions, reach a conclusion regarding the presence of time diversification. We, on the other hand, prefer to estimate risk as objectively, and as empirically, as is possible. This thesis minimises its reliance on theory, and selects a range of accepted risk measures for comparison in order to generate positive insights into the time diversification puzzle. In this sense, we agree with Olsen and Khaki (1998) that risk is “in the eye of the beholder (p. 59)”, so the intention of this thesis is to conduct an open and fair referendum on risk on common terms.

Fisher and Statman’s (1999) study has three goals. Firstly, it sets out to explore time diversification’s assumptions. In doing so, they confirm many of the earlier critiques of Samuelson’s (1969) expected utility theory approach. Fisher and Statman (1999) state plainly: “Samuelson's [1969] mathematics are right, but his assumptions are wrong (p. 90).” This statement is a neat summary of the motivation for this thesis. Fisher and Statman (1999), in the tradition of this stream of the literature, also contrast the received theory with the behavioural realities. For example, they note in relation to Samuelson (1994) that: “An unstated assumption under the mathematical truth, however, is that investors correctly assess the probabilities of losses. They do not (p. 91).” In a similar way, Fisher and Statman’s (1999) second goal is to introduce a “wide range of factors that affect investment choices,” beyond risk and return. While we agree that many such factors exist, this thesis sets out to consider the time diversification puzzle on its own terms. If one was to take into account factors beyond the investment decision, there are many studies that would assist us (e.g. Bodie et al. 1992; Vanini and Vignola, 2002; Gollier, 2002; Cocco et al., 2005). The approach taken in this thesis is that it remains focused on the relationship between risk and time, firstly within the confines of the extant debate. In the latter two studies we consider two variables – the accumulation model and asset allocation – which represent the second and third of a trinity of related factors which properly characterise the time diversification debate. The third, and final, goal of Fisher and Statman (1999) is to “explore the prudence of the time diversification prescription (p. 88).” It is almost as if the authors have grown frustrated with the time diversification debate itself, and instead resort to questioning whether time diversification’s classical generalisation – that stocks are less risky over longer horizons – is sensible given the multitude of factors that bear on investment choice. Given its subject, this thesis is not in a position to ignore the substance of the time diversification debate. In fact, the aim of this thesis is to re-prosecute the time diversification debate, using an applied approach, in order to synthesise the literature and offer positive insights into the puzzle, before reconsidering the debate in light of two understudied factors. Fisher and Statman (1999) conclude on a wistful and resigned note: “The time diversification debate teaches us little about the relationship between risk and investment horizon, but it teaches us much about the many factors that affect financial choices (p. 96).” This thesis seeks to demonstrate that the first half of this statement false.

Whilst, this behavioural stream in the literature provides useful critiques of particular research – in particular, expected utility theory – it fails to provide any comprehensive or coherent alternative framework for addressing the time diversification puzzle. Instead it provides some useful points that will be considered later in this thesis: (1) risk should be measured from the perspective of the plan member; (2) risk should be measured relative to the investor’s current status not on an absolute level (Booth, 2004); and, (3) that both the magnitude and the probability of loss are relevant considerations in understanding risk. So, while we remain cognisant of the valuable insights of the behavioural literature, this thesis does not pursue a behavioural approach. This is in large part because, as outlined above, the behaviouralists tend to focus on critiques of the existing literature rather than the introduction of new approaches that can be replicated and extended.

2.3.1.4 Applied approaches

Fourth and finally, there exists what Booth (2004) has described as an “applied” stream of literature that dwells less on theoretical paradigms and more on empirical approaches to addressing the time diversification debate. Booth (2004) states: “In contrast to the theoretical literature, an applied literature has developed based on … simulation [techniques] (p. 3).” These studies generally define risk in a certain way and then turn to measuring that risk over various time horizons in order to identify whether time diversification exists or not. Numerous studies have been conducted and, with time, later authors have sought to synthesise the previous literature often before introducing yet another measure that is claimed to settle the debate (see, for example, Kritzman and Rich, 1998). This led Kritzman (2000) to conclude that “… the time diversification debate, for many, has degenerated into a referendum on the meaning of risk, which is futile (p. 50).” Until now, however, no one study has examined all these measures using consistent data and methodologies. This lack of consistency means that it is difficult to determine whether the conflicting evidence of time diversification is truly conflicting, or whether it results from a different set-up, or a different range of measures. Before turning to Samuelson’s (1969) remaining assumption, this thesis must first discuss the important studies in this applied stream of literature, which generally revolve around particular measures of risk that will be analysed herein.

2.3.1.4.1 Standard deviation and variations

Consistent with Markowitz’s (1952) framework, the time diversification literature first defined risk as the standard deviation of annualised returns and found that, as investment horizon lengthens, risk falls (Bernstein, 1976; Garrone and Solnik, 1976; Lloyd and Haney, 1980; Lloyd and Modani, 1983; McEnally, 1985; Lee, 1990). This finding has been periodically confirmed by later studies that revisit, and attempt to synthesise, the time diversification literature (e.g. Kritzman and Rich, 1998; Kochman and Goodwin, 2002; Guo and Darnell, 2005). McEnally (1985) disagreed that this measure was an appropriate measure of risk instead arguing that “unpleasant surprises in *total* (original emphasis) returns on terminal values - the values to which the annual rates of return would compound - not surprises in the average *annualised* (original emphasis) rates of return themselves (p. 24).” Using this measure, McEnally (1985) and later authors (e.g. Bernstein, 1985; Leibowitz and Krasker, 1988; Lee, 1990; Peavy and Vaughn-Rauscher, 1994; Kochman and Goodwin, 2001; Hickman et al., 2001; Kochman and Goodwin, 2002; Gollier, 2002) found that, when measured this way, risk rises as investment horizon lengthens.

2.3.1.4.2 Distribution of outcomes

McEnally’s (1985) work is important for this thesis because it steered the debate away from parametric measures of risk and considered two additional classes of measures: firstly, measures that examine the range of outcomes, and secondly, downside measures of risk. In particular, McEnally

(1985) looked at the range of annualised and total returns. Later studies followed McEnally (1985) but examined a range of other measures over various investment horizons. Leibowitz and Krasker (1988) considered the 5th, 25th, 50th, 75th and 95th percentiles of returns. Reichenstein and Dorsett (1995) looked at ending real wealth percentiles, in particular the 5th percentile and the median (or 50th percentile). Thorley (1995) looked at the mean, 10th and 90th percentile of portfolio wealth over five different horizons. Hickman et al. (2001) consider the median, in addition to the mean and standard deviation of ending wealth. Mukherji (2008) looks at the median, the minimum, the maximum, and range of terminal wealth where $1 is invested each month over the investment horizon.

2.3.1.4.3 Downside risk measures

In studying downside risk, McEnally's (1985) estimated semi-standard deviations (below the mean) for both average annualised and total returns and found that each measure behaved similarly to their standard deviation counterparts. Mukherji (2002, 2008) found similar results using semi-standard deviation, although he describes it as downside deviation. The next downside risk measure introduced into the time diversification literature was shortfall risk, which is defined as the probability of falling short of some threshold return (Leibowitz and Krasker, 1988). In most of the literature, the threshold is the return from T-bills, which is generally regarded as the risk-free asset. Leibowitz and Krasker (1988), and later scholars (e.g. Leibowitz and Langetieg, 1989; Butler and Domian, 1991; Leibowitz and Kogelman, 1991; Reichenstein and Dorsett, 1995; Cohen et al., 1996), found that shortfall risk reduces with horizon implying that stocks are less risky over longer horizons. Other authors test these findings using different models. Reichenstein and Dorsett (1995), for example, find that estimates of shortfall risk behave in similar ways for both random walk and mean reversion models. Bierman (1997) uses a binomial model to explore the risks of hypothetical gambles, expressed in shortfall risk terms, as time horizon increases. Thorley (1995) extends the shortfall risk literature by considering the conditional risky option mean (that is, the mean return when it underperforms the risk-free option), as well as the probability of underperforming the risk-free value.

Critics of shortfall risk, like Olsen and Khaki (1998), have argued however that what needs to be taken into account is not only the probability of loss but the *magnitude* of the loss. Leaning on other work of other authors (Diamond 1988; Joag and Mowen 1990; Kaplan and Garrich 1981; Lopes 1995), Olsen and Khaki (1998) argue that not only is the magnitude of loss important, it is given greater weight by investors than the probability of loss. This finding is particularly relevant for later parts of this thesis when we discuss accumulation models which affect portfolio size, for example, where wealth is a function of contributions as well as returns. A downside risk measure that does have the ability to capture the magnitude of a loss is value-at-risk (VaR), which is proposed in the time diversification literature by Panyagometh (2011). In examining VaR and relative VaR he finds that "the risk of loss becomes lower with the increase in the length of the investment period (p. 96)."

2.3.1.4.4 Risk-adjusted measures of performance

As noted earlier, Bierman (1997), in critiquing Bodie's (1995) option pricing framework as narrowly focused on risk, made the case that reward-for-risk calculations are relevant in understanding portfolio choice problems. That expected risk and reward are positively related is, after all, one of the more durable truths of finance. The time diversification literature includes a number of different measures which seek to understand the reward-for-risk trade-off over various investment horizons. Levy (1972) found that as investment horizon lengthened the estimated Sharpe ratio increased, suggesting a better risk-return trade-off and the presence of time diversification. Levy's (1972) findings were generally confirmed by later authors (e.g. Lloyd and Modani,1983; Levy, 1984), although Hodges, Taylor and Yoder (1997) find evidence of a hump-shaped profile noting that "...the Sharpe ratio for each portfolio first increases and then decreases as the holding period is extended (p. 77)." Levy (1984) uses the Treynor ratio, a reward-for-systematic-risk measure, and finds that the risk-reward trade-off also improves with horizon for three separate groups of stocks (aggressive, defensive and neutral).[34] Using the Sortino ratio, Sinha and Sun (2005) find that the reward for downside-risk also improves with time horizon. Mukherji (2002, 2008) reaches similar conclusions using the coefficient of downside deviation, which is essentially the reciprocal of the Sortino ratio. While each of these measures allows us to consider both the return and the risk aspects of the time diversification puzzle, they each have a weakness in common with shortfall risk. Neither these reward-for-risk ratios nor shortfall risk take into account the increase in the potential magnitude of loss that results from multi-period compounding over a long horizon of, say, 40 years. This deficiency will be of even greater consequence when we leave the initial endowment paradigm and begin to incorporate contributions and salary growth, in the second and third empirical chapters of this thesis (Chapters Six and Seven).

2.3.1.4.5 Novel measures of time diversification

Until now, this literature review has only discussed measures that are well known in the broader finance literature. A small number of scholars have developed their own measures in order to shed light on the time diversification puzzle. One measure, T^*, introduced by Guo and Darnell (2005), is defined as "the investment horizon such that the total stock return over this holding period will not become negative at [a given] confidence level (p. 69)." Put another way, using the T^* measure we are, say, 95 per cent confident that we will not have a negative total stock return if the investment horizon is lengthened to T^* years or longer. While this measure is mentioned in the context of the time diversification debate, it doesn't do much to resolve the question of whether time diversification exists or not. Rather, it allows us to compare the risk of alternative asset return processes. Guo and Darnell (2005), for example, find that a mean-reverting process has a lower T^* than a random walk process. This would be the expected result because a random walk process would be expected to have paths

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[34] Systematic risk in this context, and in the Levy (1984) paper, refers to the capital asset pricing model (CAPM) beta coefficient (β).

which diverge from the mean for longer meaning that, for a given level of confidence, the T^* for a random walk process would be higher (or longer). As will become apparent, this thesis does not explicitly model a mean reverting process. Instead, we use simulation methods – in particular, a number of block bootstrap techniques – in an attempt to simulate the time series characteristics of empirical returns. We calculate T^* for each of the four simulation techniques employed in this thesis to validate the modeling techniques employed, and to compare the estimates from this thesis with those of Guo and Darnell (2005).

Another novel measure, and the last to be considered here, is the time diversification index (TDI) of Fabozzi et al. (2006). The TDI is a ratio of normalised risk measures for investment horizons of different length. Fabozzi et al. (2006) argue that the strengths of TDI as a measure include that it does not require any specific assumption regarding the risk profile of agents, and it can be computed for any model and any risk measure. For example, if we were to assume that risk is measured by standard deviation, the TDI is essentially calculated by dividing the reciprocal of the Sharpe ratio for the longer horizon by that of the shorter horizon. According to Fabozzi et al.'s (2006) rule, time diversification exists where the TDI is less than unity. Using a range of risk measures in calculating the TDI, Fabozzi et al. (2006) find little evidence of time diversification.

Having now summarised the risk measures used throughout the applied strand of the literature, it is important to re-emphasise the purpose of this thesis. The first study sets out to synthesise the findings of the applied stream of the time diversification literature by re-examining the popular measures *on the same terms*. The original contribution of the first study is to clarify the time diversification puzzle, and provide positive insights into the important task of long horizon portfolio choice. Once we have clarified the debate as it currently exists, this thesis challenges the terms of the debate by considering two further factors – the accumulation model and asset allocation strategy – that have a bearing on the relationship between risk and investment horizon. This thesis sets out to show that a complete assessment of time diversification requires the consideration of a trinity of factors.

2.3.2 Samuelson's final assumption

Before turning to the core question of this study, it is necessary to address the third and final assumption underlying Samuelson's (1969) expected utility theory which was raised at the outset of the review of the literature: that wealth is only a function of returns.

In relation to this third and final assumption, it is important to note that almost the entire time diversification literature takes place within an "initial endowment" framework. Typically, one of two approaches is taken. Firstly, many studies do not mention wealth at all and instead confine themselves to the analysis of accumulation models which are only a function of returns (e.g. Lloyd and Haney, 1980; Lloyd and Modani, 1983; Leibowitz and Langetieg, 1989; Lee, 1990). Alternatively, the study is set-up with some focus on wealth that is a function of only returns and an explicit level of initial wealth (e.g. Marshall, 1994; Reichenstein and Dorsett, 1995; Thorley, 1995; Levy, 1996).

So despite a number of papers making passing references to more realistic and complete models (e.g. Bodie et al., 1992; Kritzman and Rich, 1998), only a very small number of relatively recent studies actually incorporate periodic cash inflows, or contributions, in any way (e.g. Jagannathan and Kocherlakota, 1996; Hickman et al., 2001; Mukherji, 2008; Panyagometh, 2011; Pástor and Stambaugh, 2012). In addition, a handful of studies analyse cash outflows (or withdrawals from wealth) as a way of studying the interplay between consumption and retirement investing (e.g. Samuelson, 1969; Merton, 1969; Merton and Samuelson, 1974). Thus, the time diversification literature is overwhelmingly dominated by studies where returns are the only determinant of terminal wealth. As will be argued in Chapter Three, the institutional setting can't support such an assumption. In reality, terminal wealth is a function of not only returns, but of contributions (which in turn are partly a function of salary growth) and asset allocation. The issue of the influence of contributions and asset allocation is taken up in the second and third empirical chapters of this thesis.

2.4 Summary

Each of the competing streams in the time diversification literature has been the subject of specific criticism. The assumptions underlying Samuelson's (1969) expected utility theory approach have been comprehensively challenged by proponents and opponents alike. Of these critiques, two present themselves as being particularly convincing. Firstly, the requirement to assign a risk aversion specification to our hypothetical investor is normative and, as the behavioural literature shows us, the risk preferences of investors are by no means uniform. This normativity tends to result more in debates about risk preferences, than in discussions about the substance of time diversification: the relationship between risk and investment horizon. And, secondly, in addition to there being little consensus about what risk aversion specification best represents the "average" investor, the conclusions from the expected utility theory framework are not robust to alternative specifications. Furthermore, as Kritzman and Rich (1998) make clear, this sensitivity to specification holds for the asset return process as well. Thus, the verdict on time diversification can change dramatically by merely changing the risk aversion and/or asset return process specification, and any semblance of generality in results is lost. While we are robust in our critique of the expected utility framework, the fragmented and contradictory findings of this stream of literature certainly gives support to the notion that the time diversification debate is in need of synthesis.

Option pricing theory has been subjected to three specific critiques from both within the paradigm, and in competing paradigms. Firstly, some scholars argue that Bodie's (1995) option pricing approach implicitly ignores reward-for-risk calculations in favour of risk-only ones. Whilst this focus might be defensible given the substance of the time diversification debate, stocks are a popular investment because with the risk comes (expected) return. Risk/ return calculations should therefore at least be considered. Secondly, as Taylor and Brown (1996) argue, when Bodie's (1995) constant standard deviation assumption is relaxed, his findings fail. The option pricing framework is thus similar to the expected utility theory stream in that both appear sensitive to the major variable in their specification. Finally, as Kritzman and Rich (1998) suggest, Bodie's (1995) "option angle (p. 71)" perpetuates the debate over the meaning of risk. Thus, we see this recurring theme regarding the meaning of risk taking place between paradigms, as well as within paradigms.

The strength of the behavioural finance literature is that it provides timely critiques of the broader literature by reminding scholars that flesh-and-blood investors don't necessarily correspond to the hypothetical investor represented in much of the theory. Its principal drawback is that it fails to provide any comprehensive or coherent framework for addressing the time diversification puzzle. Instead it offers some important points to consider when we evaluate the risk measures estimated in the body of this thesis, for example, that both the magnitude and the probability of loss are relevant considerations in understanding risk.

These deficiencies motivate this thesis, and lead us to pursue an applied approach to addressing the three research questions. The three research questions in this thesis, which correspond to the three empirical chapters (Chapters Five through Seven), are:

RESEARCH QUESTION ONE

H_0: That a relationship between risk and investment horizon exists.

H_1: That H_0 is false.

RESEARCH QUESTION TWO

H_0: That alternative accumulation models have no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

RESEARCH QUESTION THREE

H_0: That asset allocation strategy has no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

The applied approach pursued in this thesis is characterised by a modest reliance on economic theory, and an empirical approach to the methodology. It also attempts to approach the research questions from within the institutional setting, in contrast to the vast body of time diversification literature which appears detached from, or indifferent to, it. Our survey of the literature has provided the rationale to position this thesis in the applied stream of the debate. By pursuing this applied research, this thesis will provide positive insights into the time diversification puzzle by conducting its own referendum on risk on common terms, before considering other factors relevant to the time diversification puzzle. This thesis will show that a complete understanding of the time diversification puzzle relies on the consideration of a trinity of inter-related factors: risk, accumulation model, and asset allocation.

Having reviewed literature, this thesis proceeds to the institutional setting where we introduce the time diversification debate to the world of long-horizon defined contribution investing.

3 Institutional setting

3.1 Introduction

This chapter outlines the institutional context in which defined contribution pension investing takes place, with a particular focus on the elements that relate directly to the second and third empirical studies. We do not pretend that this chapter represents a comprehensive study of the evolution of the retirement savings problem in general, or of all defined contribution related issues in particular. The intention of this chapter is to provide a brief account of the relevance of defined contribution investing to this research, and how it bears on the final two research questions of this thesis.

The literature review (Chapter Two) provided a critique of the time diversification literature which called into question the basis upon which the puzzle has been examined to date. In particular, the critique of Samuelson’s (1969) first two assumptions has motivated the thesis to take an applied approach to conducting a referendum on risk in the first empirical chapter (Chapter Five) to provide positive insights into the relationship between risk and investment horizon. The second and third empirical studies – which consider alternative accumulation models (Chapter Six) and asset allocation strategies (Chapter Seven) – are motivated by the fact that two of the salient features of the institutional setting (multiple cash flows and asset allocation) have been largely ignored (or assumed away) by scholars since the publication of Samuelson’s (1969) landmark work.

In simplified terms, the classical approach to studying time diversification sees the researcher reduce the problem to two dimensions: a stream of returns; and, a series of investment horizons. By analysing the risk of these returns – however risk is conceived – with investment horizon it is possible to make inferences about the existence (or otherwise) of time diversification. The central critique presented in this chapter relates to the literature’s narrow focus on this stream of returns. The received position in the literature sees wealth as a function of only returns (the initial endowment model). This thesis will show that other factors (e.g. contributions) are worthy of examination in understanding the relationship between risk and investment horizon. Furthermore, this stream of returns is usually assumed to be a function of a given, constant asset allocation. Asset allocation is thus assumed away as a constant. We will show that treating asset allocation as a constant is becoming an increasingly untenable argument to make.

This thesis finds the initial endowment model – where terminal wealth is only a function of initial wealth and returns – so central to the time diversification debate to be unpersuasive as a framework for examining defined contribution pension problems. Similarly, when one considers the range of asset allocation strategies available in the investment marketplace, we cannot justify asset allocation being marginalised as a factor unworthy of consideration. By bringing the institutional setting into the time diversification debate this thesis sets out to provide positive insights into the relationship between risk and investment horizon, and to move the debate forward. Finally, like the first empirical study, we follow the tradition of the applied stream, by framing the time diversification puzzle in light of "real world" features in the second and third empirical studies of this thesis.

3.2 Context

Because the time diversification debate looks at the relationship between investment risk and horizon, it has the potential to contribute to any application that considers long-horizon investing. Chief among these applications is retirement savings, which looks at the process by which we defer consumption to finance the retirement phase of life. In commencing our discussion on the institutional setting, we are called to refine this idea of retirement savings and relate it directly to this thesis. We start by sketching out a framework within which to consider retirement savings, based on the policy advice of the World Bank.

The World Bank (1994) – in publishing its policy recommendations regarding old age income security – envisages retirement savings as being comprised of three pillars:

- *Pillar 1: Mandatory publicly-managed pillar* – Pillar 1 provides a means-tested, minimum guaranteed pension funded by a nation's taxation base. It is thus redistributive and is designed to be an "anti-poverty (Willmore, 2000, p. 1)" pillar;
- *Pillar 2: Mandatory privately-managed pillar* – Pillar 2 is comprised of a regulated, fully-funded mandatory personal savings or occupational plan. In some jurisdictions, the government may provide taxation incentives to promote this sort of saving; and
- *Pillar 3: Voluntary pillar* – Pillar 3 is voluntary, fully-funded personal savings or occupational plan.

This thesis outlines the World Bank's view on the structure of old age income security systems, not because the author necessarily endorses their views, but because it provides an exhaustive framework that helps us locate this thesis. The institutional setting discussed in this chapter is concerned solely with Pillar 2 retirement savings.[35]

Pillar 2 retirement savings are classically delivered to a plan member by either a defined benefit (DB) scheme, or a defined contribution (DC) scheme. The essential distinction between the two types of scheme is the party that bears the risk. DB pension plans promise the employee a specific dollar payment per period in retirement, or a defined benefit. In this way, the plan sponsor – for example, an employer or government – bears the investment risk. Because the benefit is paid for the beneficiary's

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[35] In practice, Pillar 1 is more the subject of public economics, where public policy meets economics, and involves calculations regarding the public interest. Pillar 3, which considers most other forms of personal wealth, would typically be covered by the financial planning literature.

lifetime, the plan sponsor usually also bears the longevity risk.[36] From the sponsor's perspective, the most important features of a DB plan are the benefit formula, the replacement rate, and funding (Logue and Rader, 1998).

A DC plan on the hand has many of the characteristics of a bank savings account. The lump sum at retirement – from which an income stream may or may not be bought – is a function of the amount contributed by the plan sponsor and plan member, as well as the impact of market returns.[37] A DC plan member thus bears the investment risk of their Pillar 2 retirement savings, as well as their own longevity risk (Logue and Rader, 1998).

This section has briefly provided some background information with which the author builds the case that DC investing is both a growing phenomenon, and a matter of particular interest to this thesis.

3.3 Importance of defined contribution plans

The history of the private DB plans in the United States (US) stretches back to the American Express Company, a railroad freight forwarder, who in 1875 introduced the very first US private sector pension plan in an effort to promote a stable, career-oriented workforce (Williamson, 1992; Sass, 1997). Up until the latter part of the twentieth century, government- and employer-provided DB pension plans were the primary vehicle for Pillar 2 retirement savings globally (Poterba, Venti and Wise, 2006). Therefore, when the time diversification debate commenced with Samuelson (1969), an individual's retirement income was generally received from some combination of three sources: a Pillar 1 pension, benefits from a DB pension plan (Pillar 2), and perhaps some private (Pillar 3) savings. Together these sources meant that the bearing of risk in the US economy was essentially institutionalised – shared between governments and plan sponsors – thus allowing the time diversification literature to commence as an abstract debate amongst experts (academics and practitioners) about the relationship between risk and investment horizon. Since then, as this thesis will show, the institutional setting has evolved materially.

The mid-1970s marked the commencement of the secular decline in DB plans globally. For instance, in the US, the absolute number of DB plans shrunk from some 128,000 plans in 1978, to 114,000 in 1985, and around 26,000 plans in 2010 (Employment Benefit Research Institute, 2010). The precipitous decline in DB plans around the world may be attributed to pressures on plan sponsors to control costs, and the ongoing challenges of managing funding volatility and increased regulatory

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[36] The International Monetary Fund (2012) defines longevity risk as "the financial consequences associated with the risk that people live longer than expected (p. 2)." Longevity risk, from the perspective of a DB plan, is the risk that the beneficiary will live beyond the life expectancy assumed by the plan actuary. Practically, the plan actuary is interested in plan level trends, not the outcomes for individuals (herself excluded of course).

[37] The contributions made by plan sponsors on behalf of plan members are usually set via contract or statute depending on the employment arrangement and the jurisdiction under consideration.

burdens.[38, 39] Traditionally, DB plan sponsors focused exclusively on the long term, and only needed to concern themselves with the balance of plan assets and liabilities over time. However, since the 1970s, heightening regulatory demands and accounting changes have resulted in DB plans focusing more on short-term funded status, with the cash impact of plan shortfalls being felt more immediately.

Estreicher and Gold (2007) argue that the confluence of three developments also contributed to the decline in DB plans in the US:

- The exodus from the DB sector of two-thirds of all sponsors of smaller plans (those with less than 100 participants) that began in the mid-1980s in an apparent reaction to the steep increases in the costs resulting from legislative changes;
- The economic decline of the manufacturing industry, and the deregulation of the transportation industry, both of which saw many of the prime sponsors of larger DB plans close their plans; and
- The closing or freezing of their DB plans by a growing number of larger financially-sound plans for the stated purposes of bringing their compensation costs in line with those of their competitors without an active DB plan, and of controlling the volatile year-to-year increases in the cost of funding a maturing plan.

Whatever the precise set of causes, the decline of DB plans and the rise of DC plans continues unabated. Towers Watson (2012), an investment consulting firm, reports that, in the US, DC assets now comprise 57 per cent of all pension assets.[40] This makes the US, along with Australia and Switzerland, one of the three most DC-denominated jurisdictions in the world in 2011. DC plans are also of increasing importance in terms of global pension assets, with their proportion of the total rising.[41] So why is the rise of DC-style pension plans such an important trend to society, and so critical to this thesis?

The principal reason is that this trend has heralded a fundamental shift in risk from institutions, with resources and access to expertise, to individuals who are largely unprepared for the task foisted upon them. The risk that was once pooled by DB plans, is now personalised amongst individual DC plan

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[38] Funding volatility is the standard deviation of the difference between scheme asset and liabilities. It is a function of the plan's funded level, the volatility of the assets and the liabilities, and the correlation between the assets and liabilities. A typical pension plan has a funding volatility between 10 per cent and 20 per cent (Barschdorff, 2009).

[39] McCourt (2006) notes that while pension plans had become much more institutionalised in the US between 1945 and 1970, several well publicised failures (e.g. Studebaker) resulted in increased pension legislation during the period, culminating in the adoption of the *Employee Retirement Income Security Act* (ERISA) in 1974.

[40] In Australia, the imbalance in favour of DC assets is much more pronounced with DC assets comprising 81 per cent of total assets (Towers Watson, 2012).

[41] DC plans narrowly remain a minority of global pension assets with 43.1 per cent of total pension assets. That DB plans remain the majority of global pension assets is due to jurisdictions like Japan, which is large in terms of assets and almost exclusively defined benefit in nature.

members. Risks that were once implicitly shared amongst the members of a DB plan – for example, short-term market risk – are, in DC plans, the individual's to bear.

A contemporaneous trend has conspired to magnify the potential implications of this shift in risk. The move from DB plans to DC plans has occurred at the same time as the largest improvement in life expectancy in human history. This later trend has both personal and public policy consequences. On a personal level, where the individual bears longevity risk, adequacy becomes an even greater challenge with a longer retirement phase to fund.[42] On a public policy level, when combined with generally falling fertility rates, rising life expectancies introduce adverse demographic dynamic where there are fewer tax payers to support a retired population that is living longer (Collinge and Lu, 2011).[43] Together, these trends have seen private retirement savings become something of an imperative in western developed economies. In Australia, for example, successive governments, in search of retirement adequacy, have obliged employers to contribute nine per cent per annum of gross salary to a superannuation fund.[44] The Australian government thus appears to believe that Pillar 2 retirement savings, to use the World Bank's vernacular, is the best vehicle with which to fund old age.

The critical point in terms of jurisdictions dominated by DC plans is that adequacy *on an individual level* becomes the objective of retirement savings. This reality shines a bright light on the time diversification debate, and the prescriptions it has for retirement investing. We need to take a debate, which we described as an "abstract debate amongst experts" about the relationship between risk and investment horizon, and make it relevant to today's DC investors. Without robust portfolio design founded on solid principles, it is hard to see how DC investing will live up to the expectations individuals and governments have of it.

In this thesis we are focused on extracting from the time diversification debate the solid principles which we argue have been elusive. This will be achieved by firstly synthesising the debate on its own terms (Chapter Five). We then extend the debate, using our nested methodology, by incorporating two particular elements of the institutional setting we outline here: alternative accumulation models and asset allocation strategy. We begin by summarising the caricature that is the time diversification debate, before turning to a more realistic model of retirement savings. It is this latter model that motivates our consideration of alternative accumulation models and asset allocation in Chapters Six and Seven, respectively.

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[42] Retirement ages have remained remarkably steady over time given the contemporaneous increase in life expectancy.

[43] In technical terms, this is the equivalent of saying that dependency ratios are rising.

[44] This is set to rise to 12 per cent by July 2019.

3.3.1 The time diversification caricature of retirement savings

3.3.1.1 Accumulation models

Samuelson's (1969) seminal work in time diversification set the terms for the debate thereafter. Of each of Samuelson's (1969) three assumptions – discussed in detail in Chapter Two – this is particularly true for his third assumption: wealth is a function only of returns.

As discussed in more detail in Chapter Six, there are generally two approaches to dealing with the determinants of wealth. Firstly, some studies do not mention wealth in any way and focus solely on returns (e.g. Lloyd and Haney, 1980; Lloyd and Modani, 1983; Leibowitz and Langetieg, 1989; Lee, 1990). Alternatively, the scholar assumes some specific level of initial wealth to which they apply a time series of returns (e.g. Marshall, 1994; Reichenstein and Dorsett, 1995; Thorley, 1995; Levy, 1996). While this latter school touches on wealth-based conceptions of performance, there is no apparent underlying motivation to relate the accumulation model back to the prevailing institutional setting. A third approach, which is a variation of the first, is to acknowledge that other accumulation models exist, but then limit the focus to the initial endowment model (Bodie et al.,1992; Kritzman, 1994; Reichenstein and Dorsett, 1995; Kritzman and Rich, 1998; Strangeland and Turtle; 1999).

In Chapter Six, this thesis shows that a small handful of recent studies introduce more complex accumulation models that include periodic cash flows like, for example, contributions (Jagannathan and Kocherlakota, 1996; Hickman et al., 2001; Mukherji, 2008; Panyagometh; 2011; Pástor and Stambaugh, 2012). Pástor and Stambaugh (2012) provide the best explicit link to the institutional setting when, in arguing in favour of their constant savings rate, they assert that it "is consistent with the fact that the predominant use of target-date funds is in employer-sponsored retirement plans, where the employer and employee contributions are both typically pre-determined fractions of income (p. 27)." Whilst the consideration of alternative accumulation models is not unprecedented in the literature as we show, there is not a strong tradition of elucidating what these alternative models mean for the time diversification debate by relating the results back to an initial endowment equivalent. As well as looking at two alternative accumulation models inspired by the institutional setting, this thesis rectifies this deficiency and draws out what these accumulation models mean for the time diversification puzzle.

3.3.1.2 Asset allocation

As suggested in Chapter Seven, asset allocation is usually dealt with in one of two ways in the time diversification literature. The first approach sees asset allocation assumed away as a constant, as is the case in the first two empirical studies in this thesis.[45] By fixing asset allocation, and then observing how investment horizon affects risk (or in some cases, risk-adjusted performance), this thesis can

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[45] In Chapters Five and Six, one assumes that our hypothetical investor invests in an all-stock portfolio.

draw conclusions about the existence of time diversification. Where scholars take this approach, their work tends to vary on or more of the following five dimensions: the asset return process; the modeling method; the selection of investment horizons; the accumulation model; and, the measures to be used for evaluation (Leibowitz and Langetieg, 1989; Hodges, Taylor and Yoder, 1997). These five dimensions are discussed in more detail in the data and methodology section of this thesis (Chapter Four).

Where variable asset allocation is considered in the time diversification literature, it is usually as a method of indirectly measuring risk. Rather than taking a direct measurement of risk as in the first approach, many scholars compute the optimal allocation to risky assets for different investment horizons and use these optimal allocations as meta-risk measures (Samuelson, 1969; Van Eaton and Conover, 1998). Thus, if the optimal allocation to stocks is higher at long horizons than at short horizons, then the risk of stocks falls as investment horizon lengthens.

While the literature has touched on asset allocation, we can see that there are few examples where the performance of competing asset allocation strategies are compared (Reichenstein and Dorsett, 1995; Hickman et al., 2001), and none where the impact of asset allocation on the core question of time diversification is considered. This thesis fills this gap in the literature by investigating the relative performance of competing asset allocation strategies with investment horizon, in order to understand whether asset allocation has the ability to alter the balance of investment risk and thus have a bearing on the time diversification debate.

3.3.2 The reality of retirement savings

The previous section has shown that the time diversification literature typically chooses a convenient caricature of the institutional setting. In this section, elements of the institutional setting are emphasised to justify the research design adopted herein. Consistent with earlier remarks regarding the location of this thesis within the broader retirement savings context, we focus on relevant aspects of the DC investing in the United States.

The Employment Benefits Research Institute (2011) in outlining recent survey data for 401(k) plans notes that:

> "*In any given year, the change in a participant's account balance is the sum of three factors:*
>
> - *New contributions by the participant or employer, or both;*
> - *Total investment return on account balances, which depends on the performance of financial markets and on the allocation of assets in an individual's account; and*
> - *Withdrawals, borrowing, and loan repayments* (p. 10)."

This statement neatly summarises a number of issues which will be drawn out in our brief discussion of the prevailing institutional setting. Firstly, their focus on "account balance" reminds us that DC investing is principally concerned with the generation of wealth to finance retirement. This contrasts with the time diversification literature, which might lead one to conclude that returns are the sole important variable. And, secondly, it highlights the two factors that are considered both below and in the second and third empirical chapters: "contributions" in Chapter Six; and, the "allocation of assets" in Chapter Seven.[46]

3.3.2.1 Accumulation models

Defined contribution investing, as previously stated, is about providing for retirement over the course of a working life of around 40 years. If we were to believe the time diversification debate, we would assume that returns alone generate terminal wealth. This thesis will show that this is a vast simplification.

In reality, as the Employment Benefits Research Institute (2011) suggests, wealth is usually also a function of contributions. Contributions are typically made by both the employer, on the employee's behalf, and the employee. The Plan Sponsor Council of America's (PSCA) 54th Annual Survey of Profit Sharing and 401(k) Plans, which is based on 2010 data, surveys the experience of 820 companies with 10.5 million participants and $691 billion in plan assets.[47] The survey found that the level of company contributions varied between plan type with profit sharing plans offering the most generous contributions, averaging 6.8 per cent of pay (Plan Sponsor Council of America, 2012). Furthermore, company contributions in 401(k) plans averaged 2.3 per cent of pay, and in combination plans it averaged 4.6 per cent of pay. PSCA also finds that the most common default deferral – or employee contribution – is three per cent of pay (Plan Sponsor Council of America, 2012). It is clear that contributions are a feature of the US defined contribution plans and, on this basis, we incorporate contributions in each of our two alternative accumulation models.[48]

A particularly important point to note about the contribution data reported here is that all numbers are expressed as a percentage of income. Thus any change in income feeds through to plan wealth via increased contributions. In one of our accumulation models, we therefore include salary growth as a variable.

In the data and methodology section of this thesis (Chapter Four), we will outline our nested methodological approach in which the analysis from later studies builds on and subsumes the analysis

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[46] Because this thesis is concerned with DC investing, we ignore "withdrawals" from wealth.

[47] The Plan Sponsor Council of America (PSCA) is a non-profit association that provides services, best practice information, and advocacy to defined contribution plan sponsors. Membership includes 1,200 companies ranging in size from Fortune 100 firms to small, entrepreneurial businesses.

[48] There are a range of factors that ensure that DC plan enrolments will continue to remain strong into the future (e.g. auto enrolment), and that contributions will continue to rise with time (e.g. auto-escalation).

from earlier studies. For example, the second empirical chapter (Chapter Six) extends the analysis from the first empirical chapter (Chapter Five) to a further two accumulation models. Our accumulation models are similarly designed. We begin with the initial endowment model from our first empirical chapter. The marginal impact of contributions (the constant contribution model) is then considered, before examining the marginal impact of salary growth (the constant percentage contribution model). Table 3.1, a preview of Table of 4.4 from the methodology chapter, shows the various permutations of contribution rate and salary growth rate in the three accumulation models in this thesis.

Table 3.1: Accumulation models

This table presents the differences between the accumulation models studied in this thesis. From left to right the table shows the name of the accumulation model and how each of the main variables vary between models. More discussion of the methodology, including our assumptions, can be found in Chapter Four.

Accumulation model	**Contribution rate ( )**	**Salary growth rate ( )**
Initial endowment model	Zero	Zero
Constant contribution model	9% per annum	Zero
Constant percentage contribution model	9% per annum	3% per annum

By examining two accumulation models inspired by the institutional setting outlined herein – in addition to the baseline initial endowment model – we are in a position to address the substance of the second research question as follows:

RESEARCH QUESTION TWO

H_0: That alternative accumulation models have no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

We now turn to consider the institutional setting as it relates to asset allocation.

3.3.2.2 Asset allocation

While this chapter takes issue with Samuelson’s (1969) third assumption – that wealth is a function only of returns – we concede that it is at least partially true. Returns are *one* of the determinants of terminal wealth. Returns are, in turn, a function of asset allocation. With a few exceptions (for example, Hickman et al., 2001, and Reichenstein and Dorsett, 1995), the time diversification literature overlooks the relationship between returns and asset allocation, and we are left to wonder whether asset allocation has an influence on the findings of the time diversification debate.

This gap in the literature is all the more curious given our earlier discussion about the expectations placed on DC investing. The need for retirement adequacy on an individual level has led to much more scrutiny about plan design. We therefore see, in contrast to the attention it receives in the time diversification debate, an explosion in novel asset allocation strategies aimed at better serving DC plan members. Far from being uniform, DC plan members invest in a variety of ways. Based on 2010 data, the Plan Sponsor Council of America (2012) estimates that plans most frequently invest in actively managed domestic equity funds (25.1 per cent of assets), target-date funds (13.0 per cent), stable value funds (9.9 per cent), indexed domestic equity funds (8.8 per cent) and actively managed international equity funds (8.4 per cent). These numbers appear set to evolve further. The Employment Benefits Research Institute (2011) estimates that younger plan members are much more attracted to target-date funds than their older counterparts.

Macqueen and Milevsky (2009) have shown that the order in which returns occur – the so-called sequence of returns – can also have a significant impact on terminal wealth. These findings have shifted the asset allocation debate to being as much about the timing of risk as about the degree of risk to assume. As a natural consequence, innovations in asset allocation are focusing more and more on ways of managing the risk posed by an adverse sequence of returns. So, in addition to target-risk and target date funds, we are seeing very early signs of dynamic strategies which seek to achieve retirement adequacy whilst managing risk.

While we defer our discussion about the asset allocation strategies we investigate to Chapter Seven, in principle we seek to preserve the richness of the institutional setting. We are therefore drawn to consider a range of extant approaches – like target-risk and target-date designs – and relate the findings of this thesis to the time diversification debate. Furthermore, as a contribution to the literature, this thesis considers a simple dynamic investment strategy.

Through the examination of the institutional setting, we have substantiated a critique of the time diversification literature whereby we argue that asset allocation has been overlooked as a potential factor in the relationship between risk and time. Therefore, using a range of alternative asset allocations (which are discussed in detail in Chapter Seven), this thesis sets out to answer our third research question as follows:

RESEARCH QUESTION THREE

H_0: That asset allocation strategy has no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

3.4 Summary

From a reading of the time diversification literature, none of the foregoing contextual richness would be apparent. Instead the literature describes plan members who start with an initial endowment – often assumed to be one dollar – and generate wealth only through returns. There is virtually no suggestion that there are other variables worthy of consideration or analysis. While there are some references to more complete frameworks, very few time diversification studies actually bring these frameworks to life.

In this chapter, we have critiqued the initial endowment model and have shown that the prevailing institutional setting is considerably more complex than the literature would have us believe. It is this institutional setting that has provided the inspiration for the consideration of alternative accumulation models in Chapter Six, and competing approaches to asset allocation in Chapter Seven. In conducting this analysis, this thesis sheds light on the role of each in the relationship between risk and investment horizon, and contributes an element of reality to the time diversification debate.

After considering the institutional setting, this thesis argues our trinity of related factors is, at face value, persuasive. The remainder of the thesis, sets out to substantiate this claim.

4 Data and methodology

4.1 Introduction

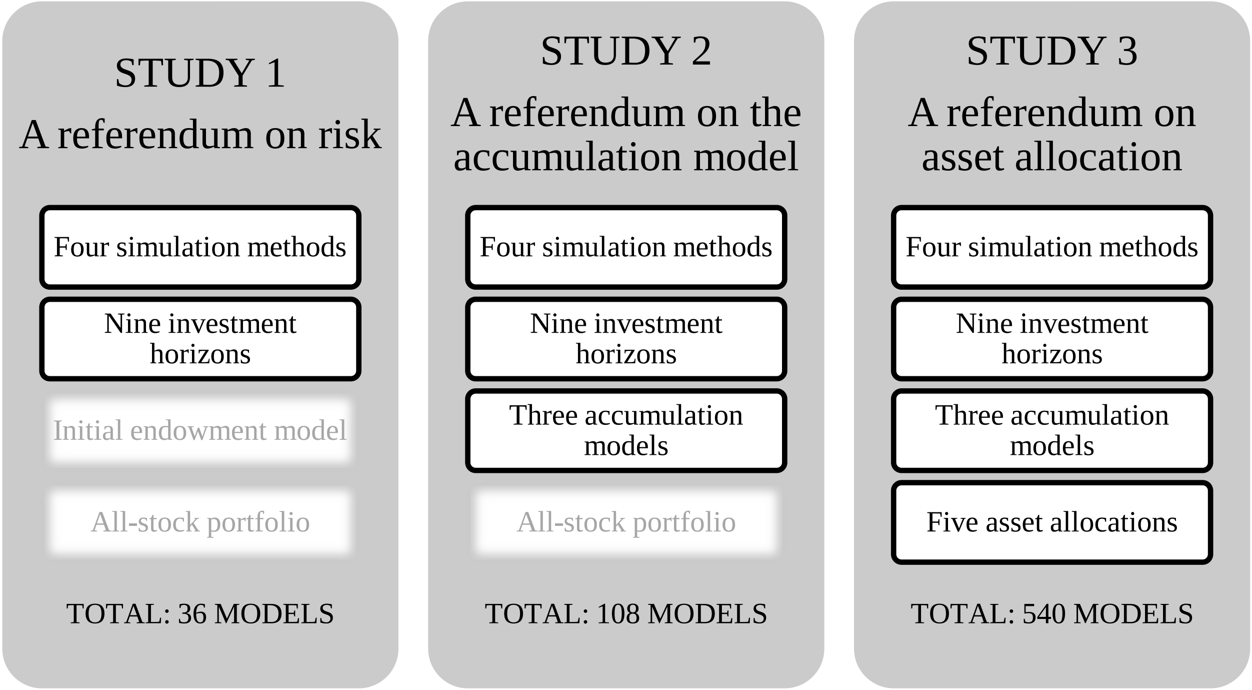
Motivated by a critique of the literature from Chapter Two, and the realities of the institutional setting discussed in Chapter Three, this thesis seeks to contribute to the enduring puzzle of time diversification by taking an applied approach.

This thesis has the professed objective of synthesising the time diversification literature (in the first study), before extending the analysis to investigate further accumulation models (in the second study), and alternative asset allocation strategies (in the third and final study). To complete these three studies, and be able to locate the analysis within the time diversification literature discussed in the Chapter Two, we need to fulfill a number of objectives in relation to data and methodology. Firstly, in the initial study – the one that seeks to synthesise the extant literature – we must replicate the key data and methodological approaches from the literature. Without sufficient coverage and consistency, the thesis would be ill-equipped to unify what is a fragmented debate. Adequate coverage is, for example, one of the driving motivations behind the selection of the ten measures that are estimated and compared in the first empirical chapter. Secondly, the methodology that is established in the first study must be easily extended to incorporate the additional accumulation models introduced in the second study, and the range of asset allocation strategies introduced in the third study. In addition to practical efficiency, this ability to easily extend the methodology allows us to relate our findings between studies. Figure 4.1 attempts to summarise the nested nature of the methodology that is employed in this thesis.

Figure 4.1: Nested methodological approach

Panel A shows how the later studies build on earlier studies by relaxing a constant. In Study 1, for our four simulation methods – Monte Carlo simulation, Efron (1979) bootstrap simulation, stationary bootstrap simulation, and empirical block bootstrap simulation – and nine horizons – 1, 5, 10, 15, 20, 25, 30, 35 and 40 years – we model a fixed accumulation model (initial endowment model) and asset allocation (all-stock portfolio) for a total of 36 models (4 x 9 x 1 x 1 = 36 models). In Study 2, the set-up is identical to Study 1 except that the accumulation model assumption is relaxed with the addition of two further models (the constant contributions model and the constant percentage contributions model) for a total of three accumulation models (4 x 9 x 3 x 1 = 108 models). In the third and final study, the remaining dimension – the asset allocation – is relaxed and we examine four additional asset allocations (a cash portfolio, a static balanced fund, a target-date fund, and a dynamic target-driven fund) for a total of five asset allocations (4 x 9 x 3 x 5 = 540 models). Panel B shows, in Venn diagram form, the nested nature of the methodological approach pursued in this thesis.

PANEL A – Empirical studies build on each other



PANEL B – Venn diagram

The ultimate objective of this thesis is to make a contribution to the literature by conducting our referendum on risk on common terms, as well as employing state-of-the-art techniques that have not before been applied to this debate.

4.2 Data

Most studies in the time diversification literature consider stocks and T-bills at a minimum, with a number considering government and/or corporate bonds (e.g. Trainer, Yawitz and Marshall, 1979; Lloyd and Modani, 1983; Harlow, 1991; Reichenstein and Dorsett, 1995; Guo and Darnell, 2005), and a smaller number considering more exotic asset classes like commodities (Milevsky, 1999; Panyagometh, 2011). The literature is almost exclusively US-focused; the exceptions being Harlow (1991) who considers hedged equity and fixed income returns from eleven developed countries (including the US), Karlsson (2006) who considers Swedish data, and Alles (2008) who employs data from four south Asian markets (India, Pakistan, Sri Lanka and Bangladesh). Studies tend to examine either monthly or annual return data.

4.2.1 Description of the dataset

The data used in this study are the well-known, and commonly used, monthly stock and Treasury bill (T-bills) returns maintained by French (2012). This thesis justifies its focus on stocks with the logic that, irrespective of the actual asset allocations of a typical long horizon investor, portfolios with a material allocation to stocks are driven first and foremost by the performance of stocks, especially in down markets where the correlations between assets tend to unity. It is therefore important to understand the performance of stocks above all others. T-bills represent a safe asset that can be used to moderate the risk of stocks.

Other than its wide acceptance, the particular data studied here have the added benefit of capturing a broad capitalisation spectrum. That is, the dataset incorporates most sub-categories of stocks, like small capitalisation stocks, and all important sectors. Above all, the reason why these data are being analysed is because the purpose of this study is to critique an existing literature which, as described above, is overwhelmingly focused on US stock returns. It is therefore necessary to retain some consistency with the data used in the literature in order to provide a basis for any critique. For this reason, all returns represent the return on US assets expressed in US dollar terms.

The excess return on the market (R_m-R_f) maintained by French (2012) is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks, from the Center for Research into Security Prices (CRSP), minus the one-month Treasury bill rate, obtained from Ibbotson Associates. To arrive at nominal total stock returns, we add back the one-month Treasury bill rate.

4.2.2 Rationale for the use of nominal returns

Before embarking on a discussion about the rationale for using nominal returns, it is appropriate to note that their use has precedent in the time diversification literature (Hickman et al., 2001; Guo and Darnell, 2005), and in the broader pension finance literature (Basu and Drew, 2009a; Basu, Byrne and Drew, 2011). It is also of some consolation that each of these cited studies approach their research question in the same applied way as in this thesis.

The use of nominal total stock returns can be justified on a number of grounds. Firstly, returns are earned in nominal terms. Because we are considering the risk of stock returns over retirement investing horizons of up to 40 years, in addition to considering questions of terminal wealth, nominal returns appear an appropriate choice. In addition, the periodic yield from each of the main asset classes – for example, the dividends from stocks or the coupons from bonds – are paid in nominal terms. Taxes are paid in nominal terms meaning that inflation adjustments should properly be made after-tax not before tax. But perhaps the most convincing argument in the context of this thesis, is that the accounts of plan members are credited with nominal returns, and members' retirement account balances are expressed in nominal dollars. We therefore conclude that on a number of counts, nominal returns are important to practical financial questions seen from the perspective of the plan member.[49]

Other arguments in favour of nominal returns could reasonably be described as technical in nature. These arguments tend to point to the complexities of making the step from nominal returns, which are observable, to real returns, which must be estimated. For example, the nominal return for any investor

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[49] We concede that academic economists may be interested in real returns. We would however argue that their motivations are different too.

for any period of time can be computed using a simple return formula familiar to any finance undergraduate. Real returns on the other hand need an appropriate inflation rate with which to adjust the nominal return. The question then becomes: what is the “best” inflation rate? Does this rate measure general inflation in the economy, or is it a specific measure of the increase in the cost of living in retirement? The former rate would certainly be more observable – via a consumer price index or a nominal GDP deflator – however the latter rate would more accurately reflect the liability for which retirement assets are being accumulated.

Our on-balance decision to use nominal returns is thus based on two arguments. Firstly, the use of nominal returns is a feature of the time diversification literature (Hickman et al., 2001; Guo and Darnell, 2005), and of the pension finance literature more broadly (Basu and Drew, 2009a; Basu, Byrne and Drew, 2011). We can therefore rely on published research as a precedent. And, secondly, this thesis argues that nominal returns are both more relevant to the problem we are examining, and more observable in practice.

4.2.3 Descriptive statistics

The descriptive statistics for monthly stock and T-bill returns are shown in Table 4.1.

Table 4.1: Descriptive statistics – Monthly returns

This table presents the summary statistics of the monthly returns of the US stock market (R_m) and of 1-month US government Treasury bills (R_f) for the period July 1926 to December 2011 (n = 1,026). Mean, standard deviation, median, maximum and minimum are each expressed in decimal terms. The remaining measures are pure numbers (i.e. have no units of measurement).

	Stocks	T-bills
Mean	0.0091	0.0029
Standard deviation	0.0544	0.0025
Skewness	0.1335	1.0324
Kurtosis	7.4145	1.2625
Median	0.0130	0.0026
Maximum	0.3837	0.0135
Minimum	-0.2901	-0.0006
Jarque-Bera statistic	836.13	311.31
Jarque-Bera p-value	0.0000	0.0000

The descriptive statistics shown in Table 4.1 generally conform to expectations. The mean, standard deviation and range of returns (i.e. the maximum minus the minimum) for stocks are significantly larger than those for T-bills, consistent with finance theory that tells us we should expect more reward for bearing greater risk. The highly statistically significant Jarque-Bera statistics tell us that both time series are significantly non-normal, with the distribution of stock returns being highly leptokurtic. In the case of stocks, the mean of 0.0091, being lower than the median 0.0130, suggests the influence of one or more significant left-tail returns. The reverse is true of the bill returns: the mean is higher than the median consistent with positive skewness and a heavier-than-normal right tail. Perhaps the only surprising aspect to this data is the marginally positive skewness for the stock data.

The descriptive statistics for annual returns are shown in Table 4.2 purely for information purposes. Only monthly returns are used in the modeling within this thesis.

Table 4.2: Descriptive statistics – Annual returns

This table presents the summary statistics of the annual returns of the US stock market (R_m) and of 1-month US government Treasury bills (R_f) for the years 1927-2011 (n = 85). Mean, standard deviation, median, maximum and minimum are each expressed in decimal terms. The remaining measures are pure numbers (i.e. have no units of measurement).

	Stocks	**T-bills**
Mean	0.1157	0.0363
Standard deviation	0.2055	0.0312
Skewness	-0.4192	0.9516
Kurtosis	2.9480	3.7627
Median	0.1439	0.0313
Maximum	0.5750	0.1472
Minimum	-0.4435	-0.0004
Jarque-Bera statistic	2.4996	15.8690
Jarque-Bera p-value	0.2866	0.0004

The descriptive statistics for annual returns shown in Table 4.2 convey an almost identical story to those statistics shown in Table 4.1. The first difference is the negative skewness for stocks which, while different to that of Table 4.1, is more consistent with our expectations regarding risky asset returns. The second and final difference is the relatively low kurtosis estimate for stocks which results in a low Jarque-Bera statistic, and an inability on our part to reject the null hypothesis that the data is drawn from the normal (or Gaussian) distribution. This finding, in particular, is neither consistent with expectations, nor with the literature relating to the statistical and distributional properties of stock returns (Mandelbrot, 1963, 1967; Fama, 1963, 1965).

4.2.4 Autocorrelation in the data

Because Samuelson's (1969) second assumption related to the nature of the asset return process, the time series characteristics of returns are the topic of some examination in the time diversification literature. The autocorrelation functions for the stock and T-bill time series are shown in Figures 4.2 and 4.3, respectively.

Figure 4.2: Autocorrelation function plot - Stocks

This plot presents the autocorrelation of monthly returns of the US stock market (R_m) up to and including lag 30. The horizontal dashed lines represent the 95% confidence interval.

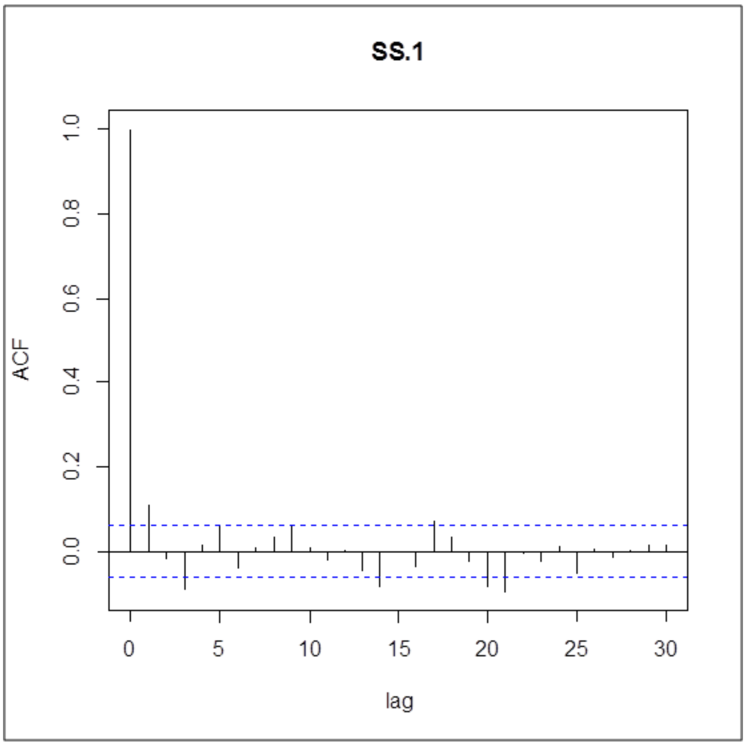
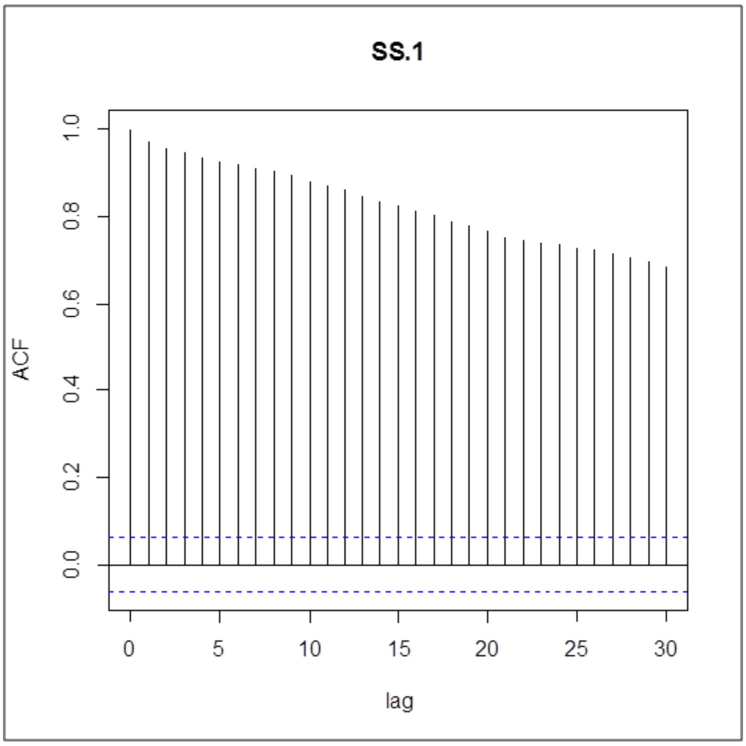


Figure 4.2 provides scant evidence of autocorrelation in the stock series, autocorrelations being slightly positive or negative from lags one to 30. Only in a small number of cases are these autocorrelations statistically significant, and these cases taken together provide little evidence of any particular sort of time series process (like, say, mean reversion). These ambiguous findings provide support for our decision to use empirical methods to model the data, rather than explicitly model a type of asset return process for which there is no strong evidence.

Figure 4.3: Autocorrelation function plot - T-bills

This plot presents the autocorrelation of monthly returns of 1-month US government Treasury bills (R_f) up to and including lag 30. The horizontal dashed lines represent the 95% confidence interval.



Consistent with our expectations, T-bill returns exhibit strong positive autocorrelation. Recall, from Tables 4.1 and 4.2, that T-bill returns have a positive mean with low volatility and a positively skewed distribution. This combination of statistical properties is certainly consistent with the autocorrelations shown in Figure 4.3. Also, from a more practical perspective, cash returns (of which T-bills are one proxy) are consistently positive with minimal volatility. We would therefore expect the sort of autocorrelation function shown in Figure 4.3. The main question that arises from the autocorrelation functions in Figures 4.2 and 4.3 is what they mean for the methodology to be adopted. We return to this question in our later discussion regarding the modeling method.

Having completed this discussion regarding the data, we now turn to the methodological approach taken to address the three research questions in this thesis.

4.3 Methodology

This section outlines the methodology employed to answer the research questions of this thesis. The structure of this methodology section will mirror the structure of the thesis. The first section will consider the methodological approach to replicating the time diversification literature on five dimensions. The second and third sections will introduce the additional aspects of methodology that are required to extend the analysis to alternative accumulation models, and variable asset allocation models, respectively.

4.3.1 Replicating the time diversification literature

As previously argued, studies within the time diversification literature are difficult to compare directly because they typically examine different measures of time diversification using a unique combination of data and methodology. As a result, it is possible to arrive at opposite conclusions about the existence of time diversification depending on how the problem is defined and measured. The fact that a debate that first emerged in academic circles with Samuelson (1969) and which continues to rage, suggests that no definitive examination of the topic has yet emerged. The principal purpose of the methodology employed in this study is to facilitate the clarification and synthesis of the time diversification debate by examining, using common data and methodology, the various measures introduced by scholars as indicators of the time diversification phenomenon. It is with this motivation that we pursue the following methodology.

Chapter Two identified a number of streams or schools of thought in the time diversification literature, of which the applied literature is of particular interest here. In the applied literature, approaches typically differ on five dimensions depending on the scholar's theoretical priors. These five dimensions are: the asset return process; the modeling method; the selection of investment horizons; the accumulation model; and, the measures to be used for evaluation.[50]

4.3.1.1 The asset return process

Scholars typically take one of two approaches in dealing with the asset return process. The first approach is to model a range of asset return processes, rather than assuming just one. Reichenstein and Dorsett (1995), for example, use historical returns to "estimate probability distributions of excess returns, real returns, and ending wealth, based on random walk and mean-reversion models, for each of eight portfolios for holding periods of one year through 30 years (p. 6)." Kritzman and Rich (1998) study five different utility specifications (log, square root, power, quadratic and combination), and consider the implications for the optimal equity allocation over time assuming three different asset return processes (random walk, mean reversion and mean aversion). Many other studies follow similar approaches (Kritzman, 2000; Strong and Taylor, 2001; Guo and Darnell, 2005; Fabozzi et al., 2006). So, rather than identifying the asset return process in the data in question and attempting to replicate it directly, advocates of this approach instead directly model a number of processes that have been identified in the finance literature.

The alternative approach is to employ one or more simulation techniques that are capable of capturing the time series characteristics of the data being studied. In the time diversification literature, fewer

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[50] Note that these dimensions are usually selected to be consistent with one another. For example, if the researcher believes that a given time series follows a random walk she would be drawn to a Monte Carlo or an Efron (1979) bootstrap simulation technique, because these techniques assume no serial correlation by construction.

authors take this approach, presumably because the quantitative techniques, and the computational power to apply them readily, emerged only in the 1990s. While simulation techniques first appeared in the time diversification literature as early as Lloyd and Haney (1980), virtually all studies employed either Monte Carlo (e.g. Leibowitz and Langetieg, 1989; Oldenkamp and Vorst, 1997; Fabozzi et al., 2006) or Efron (1979) style bootstrap methods (e.g. Butler and Domian, 1991; Hodges, Taylor and Yoder, 1997; Hickman et al., 2001; Howe and Mistic, 2003; Sinha and Sun, 2005; Alles, 2008; Panyagometh, 2011), which implicitly assume that the asset return process follows a random walk. It was not until Hansson and Persson (2000) that researchers began employing block bootstrap methods, techniques that have the ability to capture the time series characteristics of returns. Since Hansson and Persson (2000), only Mukherji (2008) has used block bootstrap methods.

This thesis will take this latter approach and use a range of simulation methods, including approaches that, as yet, have not been considered in the time diversification literature, as a way of modeling the time series characteristics in the data. How we deal with the asset return process is thus intrinsically linked to the modeling method, the subject to which we now turn our attention.

4.3.1.2 The modeling method

Having decided to address the asset return process using simulation methods, we now turn to the second methodological dimension of this study which is the selection of appropriate modeling methods. As stated earlier, one of the principal contributions of this research is to synthesise 50 years of time diversification literature in order to provide positive insights regarding the time diversification puzzle. At a minimum, in order to achieve this, the major approaches in the literature must first be replicated. The simulation methods have thus been chosen to replicate the methods used in the literature, and to propose two further methods, not yet used in the literature, which may assist in better capturing the time series characteristics in the data.

4.3.1.2.1 Monte Carlo method

Monte Carlo simulation is possibly the most commonly used method in the finance literature because of its simplicity and ease of implementation. In its most basic form, Monte Carlo simulation involves generating a vector of returns of desired length by resampling from a normal distribution with a mean and standard deviation equal to that of the sample dataset in question.[51] In this sense it is a parametric method. While Monte Carlo simulation as typically employed is relatively simple to implement, it brings with it some well documented deficiencies, most notable of which is that it implicitly assumes that returns conform to the normal distribution. This makes it a poor choice for risk management purposes, where the left tail of the distribution is of interest, and where the real distribution of returns

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[51] Technically speaking, Monte Carlo simulation can accommodate a wide variety of distributional assumptions including fat tails and mean reversion. In the author’s experience, Monte Carlo is most commonly implemented using the assumption of normality.

is important to the researcher. Notwithstanding its limitations, Monte Carlo methods have been used in the time diversification literature (Leibowitz and Langetieg, 1989; Oldenkamp and Vorst, 1997; Fabozzi et al., 2006) and will be used in this study as the baseline method.

4.3.1.2.2 Efron (1979) bootstrap method

The central problem with parametric methods like Monte Carlo simulation is that the richness of the sample data is lost once the parameters are estimated. We are therefore drawn to the class of non-parametric methods, like bootstrap simulation, to assist us in generating a large number of realistic synthetic time series as a precursor to computing a range of measures represented in the time diversification literature. Ruis and Pascual (2002) summarise the benefits of bootstrap simulation thus: “Bootstrap-based methods lead to prediction intervals that incorporate the uncertainty due to parameter estimation without distributional assumptions on the sequence of innovations (p. 21).”

Bootstrap simulation methods first appear in the literature when Lloyd and Haney (1980) used simulation *without* replacement in arguing for a “dual diversification concept,” where it's possible to make trade-offs between portfolio diversification and time diversification. The approach was similar to that developed by Efron (1979), a technique which involves resampling row vectors (i.e. a block size of one) *with* replacement to generate synthetic time series. By resampling row vectors, the technique maintains the correlations between the *n* assets whose returns are being simulated. Resampling with replacement means the technique is capable of generating a wide range of outcomes including many scenarios that, while unlikely (e.g. several 1987 crashes in succession), are at least possible in theory. This means that the technique is better able to capture the tail characteristics of the data than, say, a non-parametric method like Monte Carlo simulation.

The major drawback of Efron’s (1979) approach is that it implicitly assumes that asset returns follow a random walk. That is, because single row vectors are resampled, it doesn’t preserve the time series characteristics of the data, and by construction assumes there is no relationship between successive returns in the sample data.[52] Notwithstanding this limitation, Efron’s (1979) technique has been used in a number of studies to investigate the time diversification puzzle (Butler and Domian, 1991; Hodges et al., 1997; Hickman et al., 2001; Howe and Mistic, 2003; Sinha and Sun, 2005; Alles, 2008; Panyagometh, 2011). It has also seen use in other relevant pension finance literature, including the lifecycle investing literature (see for example Basu and Drew, 2009a). For these reasons, the Efron (1979) bootstrap method will be used in this study.

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[52] Research has shown that, contrary to the random walk assumption, stock prices contain a predictable component over short horizons. See for example Bodie (1976), Jaffe and Mandelker (1976), Nelson (1976), Fama and Schwert (1977). Later studies have reported evidence of mean reversion over longer horizons e.g. Fama and French (1988), Poterba and Summers (1988), Lo and Mackinlay (1988).

4.3.1.2.3 Stationary bootstrap method

The third and final class of methods to make an appearance in the time diversification literature is the block bootstrap which began with the work of Kunsch (1989). Whereas the Efron (1979) bootstrap selects a single row vector at random, the stationary bootstrap resamples “blocks” of successive data from the sample. By doing this, the method seeks to retain some of the serial dependence in the data while still achieving the underlying purpose of non-parametric simulation methods, which is the production of a number of synthetic time series. Each method - both Efron’s (1979) and Kunsch’s (1989) - retain the correlation structure between the assets in the dataset. Ruis and Pascual (2002) point to the block bootstrap as a method that is designed to deal with the “dependent observations (p. 28)” common in financial data. The point here is that we are not making any claims about the time series properties of the data, we are merely selecting a range of quantitative techniques that have been shown to handle the characteristics of financial data.

The central question in considering this last class of quantitative techniques is: What is the optimal block size? And, how is it determined? Where Kunsch’s (1989) block bootstrap is employed in the time diversification literature the block size is specified by the authors. Hansson and Persson (2000), for example, select a block size of 60 months arguing that “it is probably long enough to pick up most forms of possible time dependencies (p. 57)” in the data. Mukherji (2008) follows Hansson and Persson (2000) and also selects a block size of 60 months. Politis and White (2004) however point out that “… the optimal block size is never known in practice, and—more often than not—the block size used is suboptimal (p. 57).”

This statement leads us to question whether there is a better way of selecting the block size than to arbitrarily select a block length that is “probably long enough.” Building on Kunsch’s (1989) work, Politis and Romano (1994) developed the stationary bootstrap, in which the block size is a geometrically-distributed random variable. Politis and Romano (1994) show that the stationary bootstrap, where the block is random, is less sensitive to block size misspecification when compared to competing methods, for example, Politis and Romano’s (1992) circular bootstrap or the moving block bootstrap. It is for this reason, and to make a contribution to the time diversification debate, that we employ the stationary bootstrap in this study.

4.3.1.2.4 Empirical block bootstrap method

For the final simulation method, we use the routine outlined in Politis and White (2004) to determine an optimal block size based on the data. In this study we describe this method as the “empirical block bootstrap” because the block size has been estimated from the data, and in this sense is “empirical.” As we consider the question of optimal block length we must be aware of a potential complicating issue. Given that we are resampling from two datasets – representing stocks and T-bills – what happens if the estimated optimal block sizes are different?

Let us first consider the optimal block sizes which emerge from Politis and White's (2004) routine. To estimate the optimal block lengths using Politis and White's (2004) routine we use Patton's (2012) Matlab code. For the two times series in this thesis, Patton's (2012) code estimates an optimal block size of 2.7577 months for stocks and 33.0000 months for T-bills. Before considering how we deal with the different optimal block lengths, it is worth reviewing the autocorrelation function of each series at Figures 4.2 and 4.3 to validate these estimated block lengths. From cursory inspection, each estimated optimal block size appears consistent with the corresponding autocorrelation function. For stocks, Figure 4.2 shows little strong evidence of serial dependence, and where there is evidence it is up to around lag three which appears consistent with an optimal block size of 2.7577. Figure 4.3 suggests strong serial dependence in the bill time series, and so it is not surprising that the estimated optimal block size for T-bills is as high as 33 months.

We now consider how to reconcile the differing block lengths for the two time series, as only one block length may be used in the estimation process. If we were to choose the shorter of the two suggested block lengths (2.7577 months for stocks) we would miss the serial dependence in the bill series thereby undermining our reason for employing block bootstrap methods in the first place. We are therefore prompted to consider the longer of the two estimated block lengths. Conveniently, the longer block size (33 months) is a multiple of the rounded value of the shorter block size (2.7577 months rounded to three months).[53] By selecting the 33 month block size to conduct the simulation, we are effectively resampling eleven successive three month blocks thereby indirectly retaining the optimal block size for stocks within the larger block length. Therefore, throughout this study we use a 33 month block size when estimating the empirical block bootstrap.[54]

As with Politis and Romano's (1994) extension to Kunsch's (1989) work outlined above, this fourth technique, the empirical block bootstrap, has not yet been used in the time diversification literature and thus represents an original contribution to the body of knowledge on time diversification.

4.3.1.3 Investment horizons

Having addressed how we propose to deal with the asset return process, and outlined the modeling methods to be employed, we now turn to the third dimension of this study: the investment horizons to be studied.

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[53] Rounding the optimal block size for stocks to three appears to be both a logical and reasonable exercise given it is hard to imagine a block of size 2.7577 months being selected from the data.

[54] Where the investment horizon is less than 33 months in length (e.g. 12 months), we replace the optimal block length with the investment horizon. For example, for a one-year horizon all simulated data will be twelve month blocks drawn from the original data.

Early studies in the time diversification literature tended to look at relatively short investment horizons. For example, McEnally (1985) and Lee (1990) considered investment horizons up to 10 years in length, whereas Butler and Domian (1991) examined longer holding periods of up to 25 years. More recent work however has recognised that retirement saving takes place over an entire working life, with a number of studies examining investment horizons of 30 years (Reichenstein and Dorsett 1995; Hodges, Taylor and Yoder, 1997; Hickman et al., 2001) or 40 years (Mukherji, 2002).

The institutional setting discussed earlier in this thesis (Chapter Three) supports the consideration of investment horizons up to and including 40 years. For example, a typical university graduate in any developed country might, in approximate terms, enter the workforce in their early twenties and retire in their early sixties. This implies an investment horizon of around 40 years. Furthermore, recent work on lifecycle investing (e.g. Basu and Drew, 2009a; Basu, Byrne and Drew, 2011) routinely considers investment horizons of up to 40 years in length.

Thus, there are at least three reasons to look at a range of horizons up to and including 40 years: (1) the existing time diversification literature examines horizons of this length; (2) such horizons are supported by the institutional setting outlined earlier in this thesis; and, (3) related pension finance literature regularly considers investment problems over these sorts of horizons. In this study we will examine the following nine investment horizons for each combination of modeling method and accumulation model: 1, 5, 10, 15, 20, 25, 30, 35 and 40 years.

4.3.1.4 The accumulation model

The fourth dimension of this study is the accumulation model, by which we mean the process by which wealth evolves in determining terminal wealth. With very few exceptions (e.g. Jagannathan and Kocherlakota, 1996; Mukherji, 2008; Panyagometh, 2011), the entire time diversification debate is conducted within an initial endowment framework. This means that at time $t = 0$, the investor begins their retirement savings with an initial endowment – often assumed to be $1 – and from that point the only determinant of terminal wealth is returns.

It is important to emphasise that we are not examining the initial endowment framework in this study because we believe it is a fair representation of reality. Our treatment of the institutional setting (Chapter Three) confirms that the initial endowment accumulation model is a gross simplification. Rather, we consider the initial endowment model here because the purpose of this study is to synthesise the time diversification literature and, if we are to do this, we must approach the task from within the same paradigm. In Chapter Six, we will examine what we argue are more realistic (and thus complex) accumulation models, including ones that incorporate regular contributions.

4.3.1.4.1 Setting the initial endowment

A factor that needs to be considered in examining the accumulation model over a range of different investment horizons is what initial wealth should be. Essentially we are left with two options. Firstly, we can assume that all hypothetical investors, no matter what their investment horizon, begin with $1 and generate terminal wealth based only on returns. The problem with this approach is that a person with, say, a 20-year investment horizon will not usually be starting out with $1 in their account. Instead, they could be aged 45 with a non-trivial retirement account balance. Using a $1 initial endowment therefore limits the inferences that can be made about terminal wealth expressed in dollar terms.

The alternative is to assume what might be considered a realistic initial endowment for a person of a given age. According to this view, we would commence the analysis with initial wealth assumptions that correspond to a plan member of an age consistent with the particular horizon. Let us consider again the 20-year investment horizon, and how we might generate a realistic assumption for initial wealth. One approach – the approach pursued herein – is to equate a 20-year investment horizon with a person aged 45 years (45 years plus an investment horizon of 20 years gives us a realistic retirement age of 65 years). Then, using an appropriate publicly-available data source, we determine a “typical” retirement account balance for a 45 year old plan member. This balance assumption would thus represent initial wealth for a hypothetical investor with a 20-year horizon. The detailed process by which we determine our initial wealth assumption is outlined in more detail below.

This thesis effectively deals with both conceptions of initial endowment. The first, an initial endowment of $1, is effectively measured by calculating cumulative returns. For example, we calculate the standard deviation of cumulative returns as our second measure of risk in Chapter Five. This initial wealth assumption allows us to make very general comparisons regarding the effect of time horizon in isolation, because initial wealth is identical in each case.

In the latter case, where the initial endowment is some realistic dollar wealth for the particular age, we obscure the effect of investment horizon, but introduce a measure of reality regarding terminal wealth expressed in nominal dollar terms. With a realistic terminal wealth expressed in dollars we are in a position to compare the baseline accumulation model with other accumulation models (e.g. ones including contributions and/or salary growth) for which cumulative returns is not a relevant measure. Estimates of terminal wealth also allow us to consider other measures – for example, the retirement wealth ratio (the ratio of terminal wealth to final salary) – which are arguably more intuitive for retirement investors.

In this study, we use median weekly earnings data from the United States Bureau of Labor Statistics (2009) to provide earnings benchmarks for US workers of ages that correspond to the nine investment horizons used in this study. We use this median earnings data by age as the basis for identifying median account balances from the Employment Benefit Research Institute (2009). We cross-check the EBRI data against United States Census Bureau (2012) data to validate our assumptions. Table 4.3 summarises the data.

Table 4.3: Earnings and account balance data

This table presents earnings and related account balance data in order to approximate initial wealth (W_0) for various horizons. Row one shows the investment horizon. Row two shows the assumed investor age that corresponds to the investment horizon. Row three shows Bureau of Labour Statistics (BLS) (2009) median earnings data for the fourth quarter of 2008 (annualised, rounded). Row four shows raw Employment Benefit Research Institute (EBRI) (2009) median account balance data that corresponds to the annualised BLS earnings data in row three (Only includes 401(k) accounts. Previous employer accounts, and IRAs are excluded). Row five shows the EBRI data rounded to the nearest thousand dollars. The rounded data is used as initial wealth (W_0) in the analysis in this thesis. Row six shows data that was sourced to validate the account balance data shown in rows four (in raw form) and five (in rounded form). The data was obtained from the US Census Bureau (2012) and represents the median value of retirement accounts by age (including IRAs, Keogh accounts, 401(k), 403(b)). Note that there are two major differences between the data in rows five and six: (1) row six data is more recent by around two years allowing the sampled population to accumulate more assets; and, (2) row six data includes a more complete variety of account types. These two differences would lead us to expect the row six data to be greater, an expectation that is born out in the numbers. Given these reconcilable differences, we suggest that the US Census bureau data provides a reasonable cross check for the EBRI data. Investment horizon and assumed age are expressed in years. All other data are expressed in dollars.

Investment horizon (years)	**40**	**35**	**30**	**25**	**20**	**15**	**10**	**5**	**1**
Assumed age	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**64**
Median earnings data	25,000	35,000	39,000	42,000	42,000	43,000	43,000	43,000	33,000
Raw median account bal.	4,757	10,108	15,458	34,176	52,893	62,242	71,591	72,713	73,834
Median account balance	5,000	10,000	15,000	34,000	53,000	62,000	72,000	73,000	74,000
Validating account bal.	N/A	10,000	23,000	36,000	51,500	67,000	82,500	98,000	77,000

We focus on median earnings data, and the account balances corresponding to median wage earners, as the most relevant for a study of pension finance because it is these individuals that are most likely to have to rely on their retirement accounts to fund their retirement. Lower income earners, for example, are more likely to be receiving some sort of social security or supplementary assistance, and high income earners will likely have at least some significant assets outside their retirement accounts.

We acknowledge that the data are not perfect. For example, time lags exist between the datasets, and the EBRI data don't include certain types of account that are included in the Census data. We would however argue that our overarching objective should be to present believable raw data as the basis for analysis. By their very nature both earnings and wealth data are not well represented by averages or medians. There is enormous cross sectional variation in both earnings and wealth as each are the product of the idiosyncratic nature of individual lives and career paths. We therefore only pretend to provide reasonable and believable starting points as part of an analysis that involves many more important variables, not least of all the sequence of returns.

4.3.1.4.2 Accumulation model specification

We employ a simple, but flexible, accumulation model which drives the evolution of terminal wealth. Terminal wealth is given by:

[4.1]

where = terminal wealth at time T (i.e. at retirement)

= wealth at time $t = 0$ (i.e. initial wealth)

= change in wealth over one time period

= contribution rate

= employment state in month t

= monthly salary in month t

= rate of investment return earned in month t

= number of months in the plan before retirement

To estimate , we need to model the contribution cash flows and the investment returns for each period and apply these to initial wealth, . Contributions depend on salary, contribution rate, and the probability of unemployment in any period. The salary in any period, in turn, depends on the starting salary, salary growth rate, and the number of periods elapsed since commencing employment. This is given by:

[4.2]

where = starting salary of the plan member

= salary growth rate

Investment returns are dependent on the returns on individual asset classes included in the portfolio and the weights assigned to them. The weights are determined by the asset allocation strategy of the plan. Mathematically:

[4.3]

where = the weight assigned to the asset in year

= is the nominal return on the asset in year

We base all our analysis on nine hypothetical workers – corresponding to each of our nine investment horizons – who each retire at age 65 years. The starting salary ( ) profile of our nine hypothetical workers is shown in row three of Table 4.3, and pictorially in Figure 4.4.[55]

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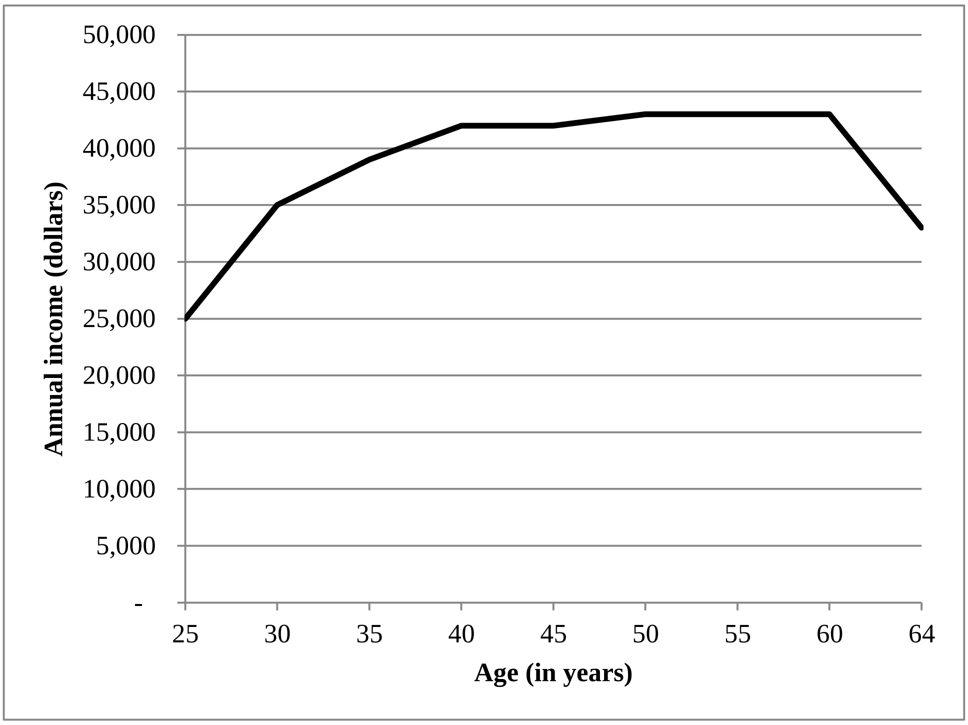
[55] This “humped” wage profile reflects the UK experience (see Byrne et al. 2006).

Figure 4.4: Starting salary by age

This figure shows the starting salary ( ) assumptions plotted against the age of our hypothetical investor. Age is calculated by the deducting the investment horizon from the assumed retirement age of 65 years (e.g. the salary assumption for an investor age 25 years corresponds to the salary assumed for a 40-year investment horizon). This is the same Bureau of Labour Statistics (2009) median earnings data shown in row three of Table 4.3.



While the flexibility exists to model the employment state as a binary variable, we assume that our hypothetical plan members remain employed at all times.[56] Further, we assume that contributions are credited to the plan at the end of each month, and that portfolios are rebalanced at the end of each month to maintain the target asset allocation ( ). We ignore any taxes payable on investment returns and transaction costs that may be incurred in managing the investments. To generate our nominal returns, , we employ the four quantitative methods outlined in section 4.3.1.2.

In this thesis, we consider three accumulation models, with each model corresponding to a unique combination of assumed inputs for equations [4.1] to [4.3]. The first empirical chapter (Chapter Five) confines its analysis to the initial endowment accumulation model. The second (Chapter Six) and third (Chapter Seven) empirical chapters add a further two accumulation models – the constant contribution model and the constant percentage contribution model. The *differences* between these models are summarised in Table 4.4. All other inputs to equations [4.1]-[4.3] are consistent across accumulation models for any combination of simulation method and investment horizon.

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[56] This has been assumed for two reasons: (1) for simplicity, and (2) because employment status is not relevant to our research questions. For an interesting study on the effects of career breaks see Basu and Drew (2009b).

Table 4.4: Accumulation models

This table presents the differences between the accumulation models studied in this thesis. From left to right the table shows the name of the accumulation model and how each of the variables from equations [4.1]-[4.3] vary between models. While the percentage rates are quoted in per annum terms, these annual rates are applied on a monthly basis in the modeling.

Accumulation model	**Contribution rate ( )**	**Salary growth rate ( )**
Initial endowment model	Zero	Zero
Constant contribution model	9% per annum	Zero
Constant percentage contribution model	9% per annum	3% per annum

The only variable in the accumulation model that remains undiscussed are the asset weights, . In the first and second empirical chapters (Chapters Five and Six), asset allocation is constant with the hypothetical investor allocating their wealth to an all-stock portfolio (i.e. = 100% for all *t*). It is only in the final empirical chapter (Chapter Seven), that asset allocation becomes a variable, and we consider the comparative performance of a further four strategies in addition to our baseline all stock portfolio.

4.3.1.5 Measures to be used for evaluation

4.3.1.5.1 Survey of risk measures in the time diversification literature

Having described the accumulation model, we now turn to the fifth and final dimension of this study which is the measures to be used for evaluation. But before specifying the formulae, we provide a survey of the evaluation measures that appear in the time diversification literature.

Once again, this study commences with a synthesis of the time diversification literature following the applied stream. In order to achieve this synthesis, this thesis must reproduce the important measures covered by that literature. Popularity and novelty in the literature are thus the main criteria for inclusion in this study. We defer the literature-based review of the ten measures we analyse to the first empirical chapter where such a review more naturally fits.

Therefore, here we limit ourselves to listing the ten measures that will be estimated in the first empirical study, after which we proceed with a technical specification of each measure in the following section. The measures we consider are:

1. Standard deviation of annualised returns;
2. Standard deviation of cumulative returns;
3. Distribution of terminal wealth;
4. Shortfall risk;
5. Value-at-risk;
6. Expected tail loss;
7. Sharpe ratio;
8. Sortino ratio;
9. Guo and Darnell's *T**; and
10. Fabozzi et al.'s time diversification index (TDI).

4.3.1.5.2 Specification of the measures used for evaluation

In the first study, assuming an initial endowment accumulation model, we estimate 10,000 return paths for each of the Monte Carlo, Efron (1979) bootstrap, stationary bootstrap, and empirical block bootstrap models for each of the nine horizons (1, 5, 10, 15, 20, 25, 30, 35, 40 years), for a total of 36 models. For each combination of simulation method and investment horizon, we have 10,000 simulated return paths.

Before we compute our ten measures, we estimate the four statistical moments – mean, standard deviation, skewness and kurtosis – for each of the 36 models. We calculate these moments, and discuss them briefly, in order to validate the raw output from our simulations. For example, for Monte Carlo simulations we would expect to see skewness of zero and kurtosis of three in line with the technique's implicit assumption of normality.

This thesis uses the following formulae to compute the statistical moments for each of the 10,000 paths for a given model (say, a 480 month Monte Carlo simulation) (Rachev et al., 2008):

Mean $-$ [4.4]

Standard deviation $-$ [4.5]

Skewness $\frac{-}{\quad}$ [4.6]

Kurtosis $\frac{-}{\quad}$ [4.7]

where = is the arithmetic return at time *t*

= is mean of arithmetic returns

= is the number of observations

= is the standard deviation of returns

We now have 10,000 means, standard deviations, and estimates of skewness and kurtosis. In order to bring these to a single number, we take the average of the 10,000 means, the average of the standard deviations, and so on. For the mean and standard deviation, we annualise these monthly means before reporting them. For skewness and kurtosis, we report the raw mean.

We now turn to the specification of the ten measures which will be used in this thesis.

4.3.1.5.2.1 Standard deviation of annualised returns

In the previous section, we calculated separate standard deviations for each of our 10,000 simulated return paths. We then took an average of these 10,000 standard deviations in order to estimate the expected volatility across the total number of trials. We then annualised the standard deviation in order to make the estimate more intuitive.

In estimating the first of our ten measures – the standard deviation of annualised returns – we take a slightly different approach by changing the order of the operations. Instead of taking a standard deviation, averaging and then annualising as we did in the last section, we average our monthly returns for each path, we then annualise, and finally we compute a standard deviation of the annualised returns. We are left with a total of 36 standard deviations of annual returns.

4.3.1.5.2.2 Standard deviation of cumulative returns

To measure what McEnally (1985) describes as “the values to which the annual rates of return would compound (p. 24),” we apply the returns for each of the 10,000 paths to a notional initial investment of one dollar to yield 10,000 cumulative returns. We then compute the mean and standard deviation of the 10,000 cumulative returns for each model. Cumulative returns are calculated as follows (Mukherji, 2002):

Cumulative returns [4.8]

4.3.1.5.2.3 Distribution of terminal wealth

In order to estimate what value returns might grow to in nominal dollar terms, we take the initial wealth levels from Table 4.3, apply the returns for the corresponding investment horizons, in order to generate 10,000 simulated estimates for terminal wealth per model. This allows us to build a distribution of terminal wealth. To bring to life the distribution of terminal wealth, we report measures of central tendency (i.e. mean and median paths), measures of dispersion (i.e. standard deviation and range), and percentiles both above (i.e. 75th and 95th percentiles and the maximum) and below (i.e. the minimum, and the 5th and 25th percentiles) the median.

4.3.1.5.2.4 Downside risk measures

Now we consider a number of downside risk measures that have been used in the time diversification debate. We report all four measures but we only count three – shortfall risk, value-at-risk, and expected tail loss – in our final list of ten measures.

Firstly we calculate the target semi-standard deviation, or downside deviation, which summarises the negative component of standard deviation against a target, which in our case is the return from T-bills. The downside deviation is the equivalent of the square root of the lower partial moment of degree 2, or .[57]

Downside deviation [4.9]

– [4.10]

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[57] Lower partial moments ( ), of which we consider a number of variations, were developed by Bawa (1975) and Fishburn (1977). The variable λ is the order of the lower partial moment.

Shortfall risk, or the probability of shortfall, is related to downside deviation in that it is also a lower partial moment, but in this case of order zero. Using a simulation approach, shortfall risk measures the proportion of paths where the stock portfolio underperforms the T-bill portfolio for the given horizon.

Shortfall risk – [4.11]

To analyse the worst outcomes, we use two common measures of estimating tail risk – value-at-risk (VaR) and expected tail loss (ETL). Rachev et al. (2008) define value-at-risk as "the minimum level of loss at a given, sufficiently high, confidence level for a predefined time horizon (p. 182)." A confidence level, , implies that we are interested in the th quantile of the distribution of terminal wealth, where . If is the th quantile of the distribution of terminal wealth then VaR at the confidence level is:

Value-at-risk [4.12]

In this study we consider the 95% confidence level as per the recommendation of Rachev et al. (2008). We also measure expected tail loss (ETL), which is often proposed as a better candidate than VaR because it is a coherent risk measure (Artzner, Delbaen, Eber, and Heath, 1999; Rachev et al., 2008).[58] ETL gives the probability weighted average of terminal wealth estimates that fall below VaR. In our case, if is the th outcome of terminal wealth , and is the probability of the th outcome, then:

Expected tail loss — [4.13]

Since we have specified at 95%, and each of our 10,000 simulated returns paths are equally probable, ETL is actually the average of the worst 5% of terminal wealth outcomes.

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[58] According to Rachev et al. (2008), a coherent risk measures has the following properties:

- *Monotonicity* – Monotonicity states that "if investment *A* has random return *Y* that is not less than the return *X* of investment *B* at a given horizon in all states of the world, then the risk of *A* is not greater than the risk of *B* (p. 195)";
- *Positive homogeneity* – Positive homogeneity refers to the property where "scaling the return of the portfolio by a positive factor scales the risk by the same factor (p. 195)." For example, if a position size is doubled, so is its risk;
- *Subadditivity* – If *X* and *Y* are random variables, then "the subadditivity property states that the risk of the portfolio is not greater than the sum of the risks of the random [returns] (p. 196)." The subadditivity and positive homogeneity properties together imply a convex functional. This convexity is a manifestation of the diversification effect expected from the combination of random variables; and
- *Invariance* – The invariance property suggests that adding cash to a portfolio reduces its risk by an amount proportional to the portfolio weight of the cash added.

4.3.1.5.2.5 Reward-to-variability ratios

Reward-to-variability ratios, or risk-adjusted measures of performance, are the next class of measures to be considered. For each model (i.e. for a given technique and investment horizon) we generate 10,000 return paths for each asset class (stocks and T-bills), and one arithmetic mean and standard deviation per path. For each asset class, we average the 10,000 arithmetic means and standard deviations to obtain the “average” mean and standard deviation of the 10,000 paths. For the Sortino ratio and the coefficient of downside deviation, we use the downside deviation formula from equation [4.10], which is denoted as in equations [4.15] and [4.16] below. We use these component calculations to compute the following measures:

Sharpe ratio [4.14]

where is the average over 10,000 simulated paths of standard deviation, , computed using equation [4.5] (Sharpe, 1966).

Sortino ratio [4.15]

where − (Sortino and Price, 1994).

Coefficient of downside deviation [4.16]

where the average return on T-bills over the 10,000 simulated paths, , represents the target value (Mukherji, 2008).

We report all three reward-to-variability ratios, but only count the Sharpe and Sortino ratios amongst the ten measures which are of interest in this thesis.

4.3.1.5.2.6 Guo and Darnell's $T*$

Guo and Darnell (2005) introduced a new measure to the time diversification, christening it $T*$. Guo and Darnell (2005) define $T*$ as "the investment horizon such that the total stock return over this holding period will not become negative at [a given] confidence level (p. 69)." We calculated $T*$ for a given time horizon and modeling technique as follows:

1. Simulate 10,000 return paths of length T. For example, for a horizon of 40 years, each simulated return path is 480 months long.
2. For each return path we calculate cumulative returns for each period of the investment horizon from a starting value of \$1 at time $t = 0$ until time T. In the 40 year example, we have a matrix of 10,000 x 480 month long cumulative return paths. Cumulative returns are calculated as follows:

   Cumulative returns [4.17]

3. For the 95% confidence level as used in this study, track the fifth percentile (1 minus the 95% confidence level) cumulative return for all periods in the investment horizon.
4. Beginning at time $t = 0$ search the fifth percentile cumulative returns until it exceeds unity.
5. Find the time t that corresponds to the period when the fifth percentile cumulative return exceeds unity. This is $T*$.

$T*$ is reported in units of months.

4.3.1.5.2.7 Fabozzi et al.'s (2006) time diversification index

Fabozzi et al. (2006) created the time diversification index (TDI) as a flexible metric that "can be computed theoretically for any model and any risk measure (p. 10)." The TDI is a ratio of normalised risk measures for two investment horizons, and is calculated thus:

TDI [4.18]

where and are time horizons and , and are the expected returns at the two time horizons and and are any two risk measures of the risk at the two time horizons. Fabozzi et al. (2006) suggest that a TDI less than unity represents evidence of time diversification.

In summary, for the baseline accumulation model we estimate Monte Carlo, Efron (1979) bootstrap, stationary bootstrap and empirical block bootstrap models for each of nine investment horizons (1, 5, 10, 15, 20, 25, 30, 35, 40 years). Thus, for any one accumulation model, we simulate a total of 36 models (four techniques multiplied by nine investment horizons) and from the simulated data we compute our ten measures.[59] With these metrics, we will have replicated the important measures in the applied stream of the time diversification literature.

Based on the estimates of these ten measures, we will be in a position to attempt to identify if there is a satisfactory solution to the time diversification puzzle consistent with the first research question of this thesis, that is:

RESEARCH QUESTION ONE

H_0: That a relationship between risk and investment horizon exists.

H_1: That H_0 is false.

We now turn to the particular methodological issues relating to our second and third studies.

4.3.2 Extending the analysis to alternative accumulation models

The methodology to be employed in the second empirical study (Chapter Six) follows that of the first study, with the addition of two further aspects which we now discuss.

4.3.2.1 New accumulations models defined

In our review of the institutional setting (Chapter Three), we critiqued the initial endowment model, where terminal wealth is a function of only returns and the size of the initial endowment. The first of two additional aspects to the methodology employed in the second study, is the consideration of two further accumulation models.

Firstly, we examine the *constant contribution accumulation model* where contributions are fixed at nine per cent of salary (credited monthly), and salary remains constant in nominal terms over the investment horizon. And, secondly, we consider a *constant percentage contribution model* where contributions are again fixed at nine per cent of salary (credited monthly), but salary increases at a constant rate of three per cent per annum (applied on monthly basis). These accumulation models were previously summarised in Table 4.4, in terms of the accumulation model variables.

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[59] In the later chapters of this thesis we will consider other, more realistic, accumulation models which introduce contributions and/ or salary growth.

4.3.2.2 The retirement wealth ratio - A new basis for evaluation

The second additional methodological aspect pursued in the second empirical study (Chapter Six), is the introduction of a further basis upon which to judge terminal wealth outcomes. The challenge with return or dollar-based measures of performance is that neither is particularly informative for the investor in terms of what performance means to their spending power in retirement. Baker, Logue, and Rader (2005) argue that defined contribution plans should be measured in terms of their ability to generate sufficient retirement income. What we need therefore is a measure that sizes terminal wealth against some relevant benchmark.

One such measure is the retirement wealth ratio (RWR_T) of Basu and Drew (2010), which is calculated by dividing terminal wealth (W_T) by salary at time T. The rationale for the introduction of the retirement wealth ratio into the pension finance literature was because "it is very likely that the participant's post-retirement income expectations are closely linked to their immediate income before retirement (Basu and Drew, 2010, p. 292)." For example, terminal wealth of one million dollars will appear much more attractive to an individual whose final salary is $50,000 per annum when compared to another individual whose final salary in $200,000 per annum. If we were to judge each scenario based on terminal wealth alone, performance would be equivalent with each individual retiring with one million dollars. Expressed in retirement wealth ratio terms, the worker on the lower income would retire with a RWR_T of 20 times ($1,000,000 dollars divided by $50,000) versus an RWR_T of five times ($1,000,000 divided by $200,000) for the individual on the higher income.

The RWR_T therefore allows us to compare accumulation models where incomes at retirement (time T) are not equivalent. In the second and third empirical studies (Chapters Six and Seven), we are able to compare terminal wealth for the initial endowment and constant contribution models because, in each case, salary is constant in nominal dollar terms over the investment horizon. But by introducing the constant percentage contributions model, where contributions rise due to the effect of salary growth, we have a model with a different final income. We must therefore evaluate performance in RWR_T terms so as to avoid over-estimating the performance of the constant percentage contribution model because we have ignored the higher final salary, and hence higher post-retirement income expectations. Throughout the thesis, salary is expressed in gross terms.

In the second empirical study (Chapter Six), by examining the marginal impact of contributions and salary growth, we hope to provide positive insights into the importance of the accumulation model in our assessment of the existence of the time diversification phenomenon, consistent with our second research question:

RESEARCH QUESTION TWO

H_0: That alternative accumulation models have no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

Furthermore, we offer our insights into the importance of the accumulation model as a novel contribution to the time diversification literature.

4.3.3 Asset allocation as a variable

The new element in the methodology employed in the third empirical study (Chapter Seven) is the introduction of a range of asset allocation strategies which will be compared to our baseline all stock portfolio. In deciding the asset allocation strategies to examine, we are seeking to fulfill a number of objectives. Firstly, in order to locate this study in the time diversification debate, and relate the findings of this thesis back to the debate, we must replicate some of the important strategies represented in those few papers in the time diversification literature that examine asset allocation strategies.

A significant motivation behind the introduction of the second and third accumulation models in Chapter Six was the incompatibility of the time diversification literature – which focuses almost exclusively on the initial endowment model – with the prevailing institutional setting outlined in Chapter Three. In selecting a range of representative asset allocation strategies for our final empirical study, we must also keep the institutional setting in mind. In the context of our third study, the institutional setting can be thought of from two perspectives. Firstly, because this thesis takes a US focus, we must cover the spectrum of asset allocation approaches observed in that market. And, apart from being the focus for the time diversification debate and this thesis, the United States also happens to be a large, mature retirement savings system where DC plans are widespread, and where innovative asset allocation strategy is a feature. And, secondly, given the provenance of the author, we hope to represent the major approaches of superannuation funds in the Australian market. We would argue that, in covering the approaches taken in these two markets, we are replicating the popular asset allocation approaches taken in DC plans globally.

The final objective influencing the asset allocation strategies to be considered in this study, is the need to showcase state-of-the-art asset allocation strategies. By analysing such strategies here, we are contributing to both the pension finance literature, and the body of evidence which will underpin more widespread adoption of these approaches in industry.

We must therefore balance the achievement of these objectives, against a desire to limit the number of strategies under consideration to the minimum number so as to avoid making the analysis and reporting of results unwieldy. We leave the in-depth discussion of the particular asset allocation strategies employed to Chapter Seven.

4.3.3.1 Setting a target return

Chapter Seven, the final empirical study, will investigate the role of asset allocation in the relationship between risk and investment horizon. In doing so, we will compare the performance of a number of competing asset allocation strategies. The principal challenge in making comparisons between strategies with different design principles, and different asset allocations at any point in time, is to find a basis on which to conduct a comparison.

A target return – or investment objective – is the primary criterion in designing an asset allocation strategy, and generally seeks to balance upside potential against downside risk. Target returns are thus a matter of judgment based on a number of considerations. The third empirical study of this study introduces a target return which is used to estimate a target retirement wealth ratio for each permutation of accumulation model and investment horizon. While we highlight the importance of target returns here as a particular methodological point, we defer the detailed discussion of target returns for Chapter Seven.

The next valid next question is: how are target returns important to the methodology in Chapter Seven? As briefly mentioned, our common target return is used to estimate unique target retirement wealth ratios (RWR_{target}) for each combination of accumulation model and investment horizon. As argued earlier, when we first introduced the retirement wealth ratio (RWR_{T}), such ratios fulfill a number of purposes. Firstly, as Baker, Logue, and Rader (2005) suggested, the retirement wealth ratio focuses performance measurement on the ability of the plan to generate sufficient retirement income. Furthermore, the retirement wealth ratio expresses performance in terms of final salary, which Basu and Drew (2010) assert is an important anchor for plan members.

So how do we propose to use the retirement wealth ratio in the final empirical chapter? Firstly, as in the second empirical chapter (Chapter Six), performance will be reported in retirement wealth ratio terms because, in comparing different accumulation models, we still have the problem of evaluating performance in the presence of different final salary levels. Thus, RWR_{T} remains the principal reporting basis in the final study.

Furthermore, the unique target retirement wealth ratios (RWR_{target}) we estimate for each combination of accumulation model and investment horizon will serve two purposes. Firstly, these target retirement wealth ratios will be used as the performance benchmark for three target-relative variations of the performance measures which are reported in this thesis. We specify these three measures in the next section of this chapter, but defer the literature-based review of the measures to Chapter Seven proper. Secondly, the series of RWR_{target} will be used as the target in one of the five asset allocation strategies we examine: the dynamic target-driven strategy. In this strategy, the relevant target retirement wealth ratio acts as the benchmark against which the algorithm makes rule-based switching decisions. We also discuss our dynamic strategy design in detail in Chapter Seven.

4.3.3.2 Specification of target-relative measures

In order to evaluate target-relative performance we introduce variations to three relatively common performance measures. But before introducing these measures, we reiterate that in this study there are 27 different target retirement wealth ratios, corresponding to the 27 different combinations of accumulation model and investment horizon. The reason for these different targets is that we need to be able to compare performance across accumulation models. For example, if a plan member is making contributions to their retirement plan, she would target a higher retirement wealth ratio than an equivalent plan member that does not contribute *ceteris paribus*. The common factor in the setting of all target RWRs is the nominal return assumption – seven per cent per annum – used in their calculation. In this sense we have used a common nominal return target to adjust target *RWR*s to account for the underlying accumulation model and investment horizon thereby making performance comparable.

We defer the literature-based review of these three target-relative measures to the third empirical chapter (Chapter Seven) where such a review more naturally fits. Suffice it to say that the specification of each is virtually identical to the specifications in section 4.3.1.5.2, except for two variations: (1) the terms of the component measures are changed from return-based measures to wealth-based RWR_T measures; and, (2) the threshold or benchmark against which each measure is computed has changed from the *variable* risk-free return to the *constant* target *RWR*.

The first of the three target-relative measures to be considered here is the probability of shortfall (Bawa, 1975; Fishburn, 1977):

Shortfall risk $-$ [4.19]

where = the target outcome

= is the outcome for the th observation

= is the number of observed *RWR* outcomes

= is the maximisation function that selects the larger of the two quantities

= the degree of the lower partial moment. In this case, $\lambda = 0$

The second measure, expected shortfall, is something of a companion to the probability of shortfall (Bawa, 1975; Fishburn, 1977). Where the probability of shortfall tells us about the likelihood of falling short of the target RWR, expected shortfall tells us about the average magnitude of a shortfall when they occur. In technical terms, shortfall risk is the lower partial moment of order zero ($\lambda = 0$), and expected shortfall is the lower partial moment of order one ($\lambda = 1$). Notice that each of these target-relative measures focus exclusively on risk.

Expected shortfall $-$ [4.20]

where = the target outcome

= is the outcome for the th observation

= is the number of observed *RWR* outcomes

= is the maximisation function that selects the larger of the two quantities

= the degree of the lower partial moment. In this case, $\lambda = 1$

In the final target-relative measure to be considered here – the Sortino ratio (Sortino and Price, 1994) – we introduce the return dimension to our evaluation of the performance of the five asset allocation strategies.

Sortino ratio [4.21]

where $-$

and = the target outcome

= is the mean of *n RWR* outcomes

= is the number of observed *RWR* outcomes

= is the maximisation function that selects the larger of the two quantities

In this section of the methodology, we have highlighted a number of additional methodological issues prompted by our last study's particular focus on asset allocation. In most cases, we have merely introduced the issue, ahead of a more detailed treatment in Chapter Seven. Therefore, in order to gain a complete picture of the methodology to be employed in the last empirical study, this chapter and Chapter Seven should be read together.

In the third empirical chapter (Chapter Seven), we extend the analysis from Chapters Five and Six to consider whether asset allocation influences the relationship between risk and investment horizon consistent with our final research question:

RESEARCH QUESTION THREE

H_0: That asset allocation strategy has no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

4.4 Summary

This chapter commenced by outlining the three objectives which had to be fulfilled by our approach to data and methodology in order achieve the purpose of this thesis: to provide positive insights into the time diversification puzzle by first synthesising a fragmented and contradictory time diversification literature (in the first study), before extending the analysis to investigate the influence of alternative accumulation models (in the second study), and competing asset allocation strategies (in the third and final study).

Firstly, in our initial study (Chapter Five) – which seeks to synthesise the extant literature – we must replicate the key data and methodological approaches of the applied stream of the literature. By considering five separate methodological dimensions – the asset return process, the modeling method, the selection of investment horizons, the accumulation model, and the measures to be used for evaluation – we have outlined how this thesis replicates the existing debate. By employing four separate simulation methods, we reproduce the main techniques employed in the literature and allow for the presence of a range of potential asset return processes. We select nine investment horizons ranging in length from one to 40 years. In the first study, we confine our analysis to the initial endowment model which we showed in Chapter Two was the dominant assumption in the literature. And last, and most importantly, we have selected ten popular measures from the literature as the basis for our referendum on risk in the first empirical chapter (Chapter Five).

Secondly, the methodology we employ must be easily extended to investigate alternative accumulation models, and competing approaches to asset allocation. In this way, we are in the position to relate the findings of our second and third empirical chapters to our first study, and to the extant time diversification literature. To reiterate this point about the need for a nested methodological approach, we reproduce Figure 4.1, Panel B here as Figure 4.5.

Figure 4.5: Nested methodological approach

This figure shows, in Venn diagram form, the nested nature of the methodological approach pursued in this thesis. In Study 1, for our four simulation methods – Monte Carlo simulation, Efron (1979) bootstrap simulation, stationary bootstrap simulation, and empirical block bootstrap simulation – and nine horizons – 1, 5, 10, 15, 20, 25, 30, 35 and 40 years – we model a fixed accumulation model (initial endowment model) and asset allocation (all-stock portfolio) for a total of 36 models (4 x 9 x 1 x 1 = 36 models). In Study 2, the set-up is identical to Study 1 except that the accumulation model assumption is relaxed with the addition of two further models (the constant contributions model and the constant percentage contributions model) for a total of three accumulation models (4 x 9 x 3 x 1 = 108 models). In the third and final study, the remaining dimension – the asset allocation – is relaxed and we examine a four additional asset allocations (a cash portfolio, a static balanced fund, a target-date fund, and a dynamic target-driven fund) for a total of five asset allocations (4 x 9 x 3 x 5 = 540 models).

The final objective of this thesis is to make a contribution to the literature by employing state-of-the-art techniques not before applied to the time diversification debate. In relation to the first empirical chapter, we have made modest contributions to move the debate forward by, for example, employing two simulation methods not employed before in the literature. Furthermore, we are the first study to consider such a wide range of measures on the same terms. We thus conduct the referendum on risk that Kritzman (2000) foreshadowed. The final two chapters (Chapters Six and Seven) represent relatively significant contributions to the time diversification literature. We consider two alternatives to the initial endowment accumulation model, each of which is more consistent with the institutional setting than any other model from the literature. Furthermore, we relate the results from these alternative accumulation models back to the time diversification debate, which is something not done in the literature. And, in the final study (Chapter Seven), we consider a total of five asset allocation strategies, including a dynamic target-driven strategy which has no precedent in the literature.

We now turn to the empirical chapters of this thesis, where we will operationalise the data and methodology we have just discussed.

5 A referendum on risk

5.1 Introduction

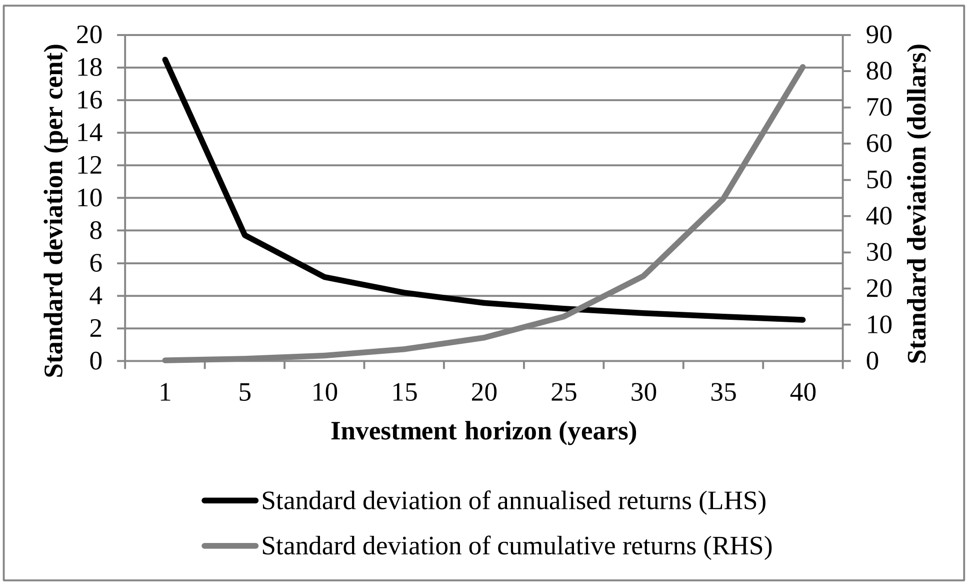
The time diversification puzzle is concerned with the relationship between risk and investment horizon. In this first empirical study, this thesis re-prosecutes the time diversification debate *on common terms* by pursuing a research design inspired by the applied stream in the literature. By doing so, we hope to synthesise the range of applied studies in the literature, and to provide positive insights (and potentially some resolution) to the enduring time diversification puzzle.

In the review of the literature in Chapter Two, this thesis posited that the main streams in the time diversification literature – expected utility theory, option pricing theory, and the behavioural stream – have yet to resolve this puzzle for a variety of reasons. We were thus drawn to the emergent field of applied literature as the source of an objective framework to facilitate a re-examination of the time diversification puzzle. At the time of writing, we find no scholar(s) that have attempted to revisit the debate by examining such a wide range of measures using the same data and methodology.

Over many decades, the time diversification literature has reported some truly contradictory evidence about the relationship between risk and investment horizon. Reconsider for a moment Figure 5.1, the same figure that was presented in the introduction to this thesis.

Figure 5.1: Contradictory evidence

This figure plots the standard deviation of annualised returns, in percentage terms, on the left vertical axis and the standard deviation of cumulative returns, in dollar terms, on the right vertical axis. Both series are plotted against investment horizon. This figure is a reproduction of Figure 1.1. For details on how these series were estimated, refer to footnote 4, page 12 of this thesis.



We see that competing measures of risk yield *opposite* conclusions about the presence of time diversification. The natural question is: How is this possible? How can two versions of a widely-accepted statistical measure of dispersion – standard deviation – give us polar-opposite answers regarding the relationship between risk and time? This first empirical study is motivated by the need to resolve this contradiction at the heart of the time diversification puzzle.

Figure 5.1 is merely one example of conflicting evidence from the literature. In truth, there are many more examples of risk measures that yield different conclusions about the presence of time diversification. This body of contradictory evidence prompted Kritzman's (2000) famous remark that the debate had "degenerated into a referendum on the meaning of risk (p. 50)." This study proposes to conduct just such a referendum.

To operationalise our referendum, we select ten popular measures of "risk" from the time diversification debate.[60] Within an initial endowment accumulation model, and using a common dataset and a consistent methodological approach, this study estimates these ten measures for nine investment horizons ranging in length from one to 40 years. We then compare the ten measures on two dimensions: (1) the functional relationship between risk and time; and, (2) the measurement basis. We discover three things. Firstly, our estimates, across all four simulation methods, generally confirm the findings in the literature. Secondly, the evidence of time diversification remains contradictory even when examined using the same data and methodology. We find that four measures are suggestive of time diversification, and that six measures are not. And, finally, we show that the measurement basis might offer some resolution to the puzzle. Return-based measures – where the measure is a function of average returns – tend to be supportive of time diversification, whereas wealth-based measures – where the measure is a function of a chain of realised returns – tend to see risk rise with time.

Thus, by considering ten measures from the extant applied literature, this study finds that every scholar is right about the time diversification debate *on their own terms*. In a sense then, the puzzle is enduring. We also provide a potential resolution to the debate by considering measurement basis. By changing the measurement basis, the answer regarding time diversification appears to change. This finding also appears to offer some resolution to the contradictory evidence shown in Figure 5.1: the standard deviation of annualised returns is a return-based measure, whereas the standard deviation of cumulative returns is a wealth-based measure.

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[60] In fact, not all these measures are, strictly speaking, measures of risk. Some, for example, are measures of risk-adjusted performance. We use the term "risk" loosely here because the time diversification debate is principally about the relationship between risk and investment horizon. We argue that the term risk is used loosely in the debate because many scholars conflate the essential question of time diversification – the relationship between risk and investment horizon – and next natural question – what does this implied allocation to stocks over the lifecycle mean for performance.

Finally, this contrast between the findings for these two measurement bases is something to revisit in the second study of this thesis (Chapter Six). In Chapter Six, we consider two alternative accumulation models – each involving contributions – which will inevitably impact wealth, but not returns. If measurement basis is an important aspect of risk, we might therefore see further evidence of this tentative dichotomy between return- and wealth-based measures.

Before turning to the substance of this study – the data, the methodology, and the empirical analysis – we consider the key literature that motivates the research in this chapter.

5.2 Literature review

Time diversification was first examined in an expected utility framework by Samuelson (1969) who found that the allocation to risky assets is independent of time. As this thesis has highlighted several times already, this conclusion was based on three assumptions: (1) the investor exhibits constant relative risk aversion, (2) returns follow a random walk, and (3) wealth is a function only of returns. In Chapter Two, this thesis showed that the time diversification literature has developed around critiques of the first two of these three assumptions.[61] In the context of these two assumptions, we will briefly recap on the literature that motivates this study.

5.2.1 Samuelson’s (1969) first assumption

5.2.1.1 An applied approach

Samuelson’s (1969) first assumption has provided the motivation for most of the subsequent literature on time diversification, which we have divided into four streams, or schools of thought. Whilst each contributes to the debate in its own way, we showed in Chapter Two that three of these streams – expected utility theory, option pricing theory, and the behavioural stream – suffer from material deficiencies. Because of these deficiencies, and their demonstrated inability to shed light on the time diversification puzzle, this thesis discontinues its consideration of these three streams in the literature. Instead, this study is drawn to an applied approach to conducting our referendum on risk.

The applied literature – which was identified and named first by Booth (2004) – tends to dwell less on theoretical paradigms, instead taking a more empirical approach to addressing the time diversification debate. Booth (2004) also noted that the applied literature is characterised by a methodological preference for employing simulation techniques. Applied studies generally begin by defining risk in a certain way – for example, the distribution of terminal wealth as in Reichenstein and Dorsett (1995) – and then measuring *their* conception of risk over various time horizons, in order to identify whether time diversification exists or not.

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[61] In fact, our critique that the third assumption is virtually ignored in the literature motivates the second and third studies of this thesis, which investigate alternative accumulation models and asset allocation respectively.

Numerous studies have been conducted in the applied tradition and, with time, later authors have sought to synthesise the previous literature, or propose another measure for consideration. This growing array of risk measures no doubt motivated Kritzman’s (2000) observation that “… the time diversification debate, for many, has degenerated into a referendum on the meaning of risk, which is futile” (p. 50). Until now, however, no one study has examined all the important measures using consistent data and methodologies. This lack of consistency means that we are unable to identify the source of conflicting evidence. Is the evidence of time diversification truly conflicting? Or does the conflicting evidence result from a different set-up, or a different measure or set of measures?

So, while this debate has created much heat, it has not produced much in the way of light. Almost 50 years after the debate began, we are left wondering about a number of questions which are critical to long-horizon investors. Is there a relationship between risk and investment horizon? Is this relationship robust? Or does it depend on one’s conception of risk? We propose to correct this deficiency in the literature by examining a range of measures using a common dataset and methodology in the applied tradition. By removing the variation due to the research design, we set out to synthesise the debate, and identify if there are any general findings regarding the relationship between risk and investment horizon.

Using a consistent methodology, we estimate ten different measures for each of four simulation methods for horizons of between one and 40 years. By observing the behaviour of these estimates with investment horizon, this thesis generalises the functional relationship between these two variables. From this functional relationship we are in a position to interpret a relationship between risk and investment horizon, and hence generate ten separate views on the existence of time diversification. We will have thus conducted our own referendum on risk – to use Kritzman’s (2000) turn of phrase – on common terms.

We also consider the measurement basis of each of these ten measures in order to see if there is any common functional relationship for measures of the same basis. By measurement basis, we mean the way in which the measures are calculated. Return-based measures – for example, the standard deviation of annualised returns – are the function of returns, and are usually expressed in percentage terms, or as a ratio. By nature, return-based measures fail to make the connection between returns and wealth. The alternative is to calculate wealth-based measures, which are usually expressed in dollar terms. Wealth-based measures are a function of a series of realised returns, not a chain of average returns as return-based measures would suggest. Wealth-based measures acknowledge that plan members invest to earn returns in order to generate wealth to finance their retirement. Retirees, after all, spend dollars not returns. By tracking this measurement basis alongside the functional relationship between risk and investment horizon implied from each of the ten risk measures, this thesis hopes to identify whether the measurement basis offers any insight into the time diversification debate.

The ten measures that will be considered in this study are as follows:

1. Standard deviation of annualised returns;
2. Standard deviation of cumulative returns;
3. Distribution of terminal wealth;
4. Shortfall risk;
5. Value-at-risk;
6. Expected tail loss;
7. Sharpe ratio;
8. Sortino ratio;
9. Guo and Darnell's *T**; and
10. Fabozzi et al.'s time diversification index (TDI).

Before addressing the study proper, we will now briefly outline the pedigree of these ten measures in the time diversification literature.

5.2.1.2 Ten measures of risk

In conducting our referendum on risk, we must carefully select our risk measures based on a review of the literature. In grounding our selection of measures in the literature, we maintain a connection between the body of scholarship on time diversification and our particular research question, which is whether the relationship between risk and investment horizon exists.

Consistent with Markowitz's (1952) framework, the time diversification literature first defined risk as the standard deviation of annualised returns and found that as investment horizon lengthens, risk falls (Bernstein, 1976; Garrone and Solnik, 1976; Lloyd and Haney, 1980; Lloyd and Modani, 1983; McEnally, 1985; Lee, 1990). This finding has been periodically confirmed by later studies that revisit, and attempt to synthesise, the time diversification literature (e.g. Kritzman and Rich, 1998; Kochman and Goodwin, 2002; Guo and Darnell, 2005). The standard deviation of annualised returns is thus a fixture of the debate.

McEnally (1985) disagreed that an annualised measure was an appropriate measure of risk instead arguing that "what matters is unpleasant surprises in *total* (original emphasis) returns on terminal values - the values to which the annual rates of return would compound - not surprises in the average *annualised* (original emphasis) rates of return themselves (p. 24)." Using this total return measure – or what we call cumulative returns – McEnally (1985) and later authors (e.g. Bernstein, 1985; Leibowitz and Krasker, 1988; Lee, 1990; Peavy and Vaughn-Rauscher, 1994; Kochman and Goodwin, 2001; Hickman et al., 2001; Kochman and Goodwin, 2002; Gollier, 2002) found that when measured this way risk rises as investment horizon lengthens. In his statement, McEnally (1985) is drawing an important distinction. He is contrasting the two measurement bases we propose in this thesis, and emphasising that, in his opinion, the wealth basis "matters" most. Furthermore, the two measures he contrasts in his statement are those very measures plotted against each other in Figure 5.1.

McEnally's (1985) work was important because it steered the debate away from "average" measures of risk and considered two additional classes of measures: firstly, measures that examine the range of outcomes, and secondly, downside measures of risk. In particular, McEnally (1985) looked at the range of annualised and total returns. Later authors followed McEnally (1985) but examined a range of other measures over various investment horizons. Leibowitz and Krasker (1988) considered the 5th, 25th, 50th, 75th and 95th percentiles of returns. Reichenstein and Dorsett (1995) look at ending real wealth percentiles, in particular the 5th percentile and the median (or 50th percentile). Thorley (1995) chose the mean, 10th and 90th percentile of portfolio wealth over five different horizons. Hickman et al. (2001) consider the median, in addition to the mean and standard deviation of ending wealth. Mukherji (2008) looks at the median, the minimum, the maximum, and range of terminal wealth where $1 is invested each month over the investment horizon. It is therefore clear that scholars have increasingly used the distribution of terminal wealth to study the range of wealth outcomes.

In studying downside risk, McEnally's (1985) estimated semi-standard deviations (below the mean) for both average annualised and total returns and found that each measure behaved similarly to their standard deviation counterparts. Mukherji (2002, 2008) found similar results using semi-standard deviation, although he described it as downside deviation. The next downside risk measure introduced into the time diversification literature was shortfall risk, which is defined as the probability of falling short of some threshold return (Leibowitz and Krasker, 1988). In most of the literature, the threshold is the return from T-bills, which is generally regarded as the risk-free asset. Leibowitz and Krasker (1988), and later scholars (e.g. Leibowitz and Langetieg, 1989; Butler and Domian, 1991; Leibowitz and Kogelman, 1991; Reichenstein and Dorsett, 1995; Cohen et al., 1996), found that shortfall risk reduces with horizon implying that stocks are less risky over longer horizons. Other authors test these findings using different models. Reichenstein and Dorsett (1995), for example, find that estimates of shortfall risk behave in similar ways for both random walk and mean reversion models. Bierman (1997) uses a binomial model to explore the risks of hypothetical gambles, expressed in shortfall risk terms, as time horizon increases. Thorley (1995) extends the shortfall risk literature by considering the conditional risky option mean (that is, the mean return conditional on it being less than the risk-free option), as well as the probability of underperforming the risk-free value.

Critics of shortfall risk, like Olsen and Khaki (1998), have argued however that what needs to be taken into account is not only the probability of loss but the *magnitude* of the loss. Leaning on the work of other authors (Diamond 1988; Joag and Mowen 1990; Kaplan and Garrich 1981; Lopes 1995), Olsen and Khaki (1998) argue that not only is the magnitude of loss important, it is given greater weight by investors than the probability of loss. This finding is particularly relevant for later parts of this thesis when we discuss accumulation models which affect portfolio size, for example, ones where wealth is a function of contributions as well as returns. A downside risk measure that does have the ability to capture both the probability and magnitude of loss is value-at-risk (VaR), which is proposed in the time diversification literature by Panyagometh (2011). In examining VaR and relative VaR he finds that "the risk of loss becomes lower with the increase in the length of the investment period (p. 96)." In this study, we estimate VaR and its close relative, expected tail loss (ETL), in understanding downside risk in wealth terms.

As noted earlier, Bierman (1997), in critiquing Bodie's (1995) option pricing framework as narrowly focused on risk, made the case that reward-for-risk calculations are relevant in understanding portfolio choice problems. That expected risk and reward are positively related is, after all, one of the more durable truths of finance. The time diversification literature includes a number of different measures which seek to understand the reward-for-risk trade-off over various investment horizons.[62] Levy (1972) found that as investment horizon lengthened the estimated Sharpe ratio increased, suggesting a better risk-return trade-off and the presence of time diversification. Levy's (1972) findings were generally confirmed by later authors (e.g. Lloyd and Modani,1983; Levy, 1984), although Hodges et al. (1997) find evidence of a hump-shaped profile noting that "...the Sharpe ratio for each portfolio first increases and then decreases as the holding period is extended (p. 77)." Using the Sortino ratio, Sinha and Sun (2005) find that the reward for downside-risk trade-off also improves with time horizon. Mukherji (2002, 2008) reaches similar conclusions using the coefficient of downside deviation, which is essentially the reciprocal of the Sortino ratio. While these measures allow us to consider both the return and the risk aspects of the time diversification puzzle, they each have a weakness in common with shortfall risk. As is true for all return-based measures, neither these reward-for-risk ratios nor shortfall risk take into account the increase in the potential magnitude of loss that results from multi-period compounding over a long horizon of, say, 40 years. This deficiency will be of even greater consequence when we leave the initial endowment paradigm and begin to incorporate contributions and salary growth, as we will see later in this thesis.

Until now we have only discussed measures that are well known in the finance literature. A small number of scholars have developed their own measures in order to shed light on the time diversification puzzle. One measure, *T**, introduced by Guo and Darnell (2005), is defined as "the investment horizon such that the total stock return over this holding period will not become negative

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[62] In addition to the reward-for-risk measures estimated herein, others are considered in the literature. Levy (1984), for example, uses the Treynor ratio, a reward-for-systematic-risk measure, and finds that the risk-reward trade-off also improves with horizon for three separate groups of stocks (aggressive, defensive and neutral). Systematic risk in this context, and in the Levy (1984) paper, refers to the capital asset pricing model (CAPM) beta coefficient (β).

at [a given] confidence level (p. 69).” Put another way, using the *T** measure we are, say, 95 per cent confident that we will not have a negative return if the investment horizon is lengthened to *T** years or longer. While this measure is mentioned in the context of the time diversification debate it doesn’t do much to resolve the question of whether time diversification exists or not. Rather it allows us to compare the risk of different asset return processes. Guo and Darnell (2005), for example, find that a mean-reverting process has a lower *T** than a random walk process. This would be the expected result because a random walk process would be expected to have paths which diverge from the mean for longer meaning that for a given level of confidence, the *T** for a random walk process would be higher (or longer). As outlined in Chapter Four, we do not explicitly model a mean reverting process in this research. Instead this thesis employs simulation methods – in particular, a number of block bootstrap techniques – in an attempt to simulate the time series characteristics of empirical returns. We calculate *T** for each of the four simulation techniques employed in this thesis, for completeness and to validate the modeling techniques employed.

Another novel measure, and the last to be considered here, is the time diversification index (TDI) of Fabozzi et al. (2006). The TDI is a ratio of normalised risk measures for investment horizons of different length. Fabozzi et al. (2006) argue that the strengths of TDI as a measure include that it does not require any specific assumption regarding the risk profile of agents, and it can be computed for any model and any risk measure. For example, if we were to assume that risk is measured by standard deviation, the TDI is essentially calculated by dividing the reciprocal of the Sharpe ratio for the longer horizon by that of the shorter horizon. According to Fabozzi et al.’s (2006) rule, time diversification exists where TDI is less than unity. Using a range of risk measures in calculating the TDI, Fabozzi et al. (2006) find little evidence of time diversification.

With Fabozzi et al.’s (2006) time diversification index, we complete our brief synopsis of the ten measures we examined in this thesis. From the synopsis, it should be apparent that this study is firmly grounded in the applied stream of the time diversification literature.

Based on the literature, we have now resolved our selection of ten measures which will form the basis of our referendum on risk. This thesis has thus moved beyond Kritzman’s (2000) rhetoric, and is now prepared to let these risk measures cast their vote for or against time diversification.

But, before turning to the research design and the empirical evidence, we first briefly discuss Samuelson’s (1969) second assumption and its relevance to this study.

5.2.2 Samuelson’s (1969) second assumption

In his pioneering work, Samuelson (1969) assumed that returns followed a random walk. Since then, random walk and other asset return processes have been covered extensively in the time diversification literature.

As discussed in Chapter Four, the evidence regarding asset return processes is ambiguous. This ambiguity has motivated many authors to study a range of serial dependence relationships. The dominant approach in the literature sees the researcher specify and model the asset return process directly.

This thesis takes a different approach. Rather than modeling particular asset return processes directly as many authors have done, in this study we prefer to deal with the data's time series characteristics as an empirical issue. This leads us to choose a range of simulation techniques that are designed to preserve the time series characteristics present in the data. In this sense, we avoid expressing a view about the nature of the data, instead we let the data speak for themselves. The techniques we employ, and their relationship to the various asset return process assumptions, are covered in more detail in Chapter Four. We now consider the data and methodology we use to generate our risk estimates, before turning to discuss the empirical findings.

5.3 Data

As outlined in Chapter Four, the data used in this study are the well-known, and commonly used, monthly stock and T-bill returns maintained by French (2012).

The excess return on the market (R_m-R_f) maintained by French (2012) is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks, from the Center for Research into Security Prices (CRSP), minus the one-month US government Treasury bill rate, obtained from Ibbotson Associates. To arrive at nominal total stock returns, we add back the one-month Treasury bill rate. We consider nominal total stock returns for the reasons outlined in Chapter Four.

5.4 Methodology

Studies within the time diversification literature are difficult to compare directly because they typically examine different measures using a unique combination of data and methodology. As a result, it is possible to arrive at opposite conclusions about the existence of time diversification depending on how the problem is defined and measured. The principal purpose of the methodology employed in this study is to clarify and synthesise the time diversification debate by examining, using common data and methodology, the various important measures introduced by scholars as indicators of the time diversification phenomenon.

As outlined in Chapter Four, the studies in the applied stream of the literature typically differ on five dimensions depending on the scholar's theoretical priors. These five dimensions are: the asset return process; the modeling method; the selection of investment horizons; the accumulation model; and, the measures to be used for evaluation. To achieve our goal of synthesising the debate we must design a methodology that covers the major approaches to each of these five dimensions. For convenience, we briefly recap on our approach to these five dimensions.

5.4.1 The asset return process

Samuelson’s (1969) second assumption – that returns follow a random walk – is the baseline assumption in the time diversification debate, and a common assumption in finance. At a minimum, we must therefore replicate this assumption. In departing from Samuelson’s (1969) assumption we have two choices: (1) directly model an alternative process as is commonly done in the literature; or, (2) select a modeling approach that reproduces the serial dependence in the data.

We must therefore choose between the deterministic on the one hand, or the empirical on the other. Unsurprisingly given the foregoing discussion, this thesis takes the empirical approach. Thus, how we deal with the first two dimensions of this study – the asset return process and the modeling method – are intrinsically linked.

5.4.2 The modeling method

Our selection of four different simulation methods is a direct response to Samuelson’s (1969) second assumption: that returns follow a random walk. We use two methods – one parametric, the other non-parametric – that implicitly assume a random walk. These methods – Monte Carlo simulation and the Efron (1979) bootstrap – serve two purposes. Firstly, they allow us to replicate Samuelson’s (1969) assumption as a baseline. And, secondly, they enable a comparison between our findings and the other studies in the literature that assume returns follow a random walk.

The selection of our remaining two methods – the stationary bootstrap and the empirical block bootstrap – also serves two purposes. Firstly, their use acknowledges that other potential return processes exist, in addition to Samuelson’s (1969) random walk assumption. The literature studies a number of these alternatives, in particular mean reversion. Secondly, it further evidences our preference for empirical methods. Rather than directly modeling a particular asset return process (e.g. mean reversion) as is commonly done in the literature, we select methods that have the demonstrated ability to model the serial dependence properties of the data. In this way, the asset return process is an empirical matter, not an assumption imposed on the analysis by the modeler. This approach should also serve as a reminder that this thesis approaches the puzzle from the perspective of the literature’s applied stream, which Booth (2004) identifies as being characterised by the use of simulation methods.

5.4.3 Investment horizons

Consistent with the literature, we select a nine investment horizons ranging from one year to 40 years.

5.4.4 The accumulation model

Because this study attempts to approach the time diversification debate on its own terms, we assume that the accumulation model conforms to the initial endowment thinking so common to the literature. Without locating our study in the literature we would be unable to succeed in our ambition to synthesise the debate, and to identify general principles behind the relationship between risk and investment horizon.

According to the initial endowment model, wealth is a function of the initial endowment – which is often assumed to be $1 – and returns. No other variables figure in the calculation of terminal wealth. As has been foreshadowed already, we study the impact of alternative accumulation models on the time diversification debate in the second empirical study (Chapter Six).

5.4.5 Measures to be used for evaluation

The purpose of this study is to investigate the relationship between risk and investment horizon. To observe risk, we estimate ten different measures from the literature over our nine investment horizons ranging from one to 40 years in length. By considering a number of risk measures, we seek to develop an understanding of how risk relates to horizon without prejudging the investor's risk attitude. For example, expected tail loss might give us an idea about how an investor, who defines risk in terms of the worst downside outcomes, sees the relationship between risk and investment horizon. Alternatively, standard deviation or the distribution of terminal wealth, might be more appropriate measures for an investor who sees risk as a more symmetric concept.

Furthermore, by considering measures proposed in the literature this thesis anchors its analysis in the time diversification debate, and provides the groundwork for a synthesis of existing studies. And, perhaps most importantly, the consideration of this array of risk measures seeks to address the critique of Samuelson's (1969) first assumption, which focuses on the investor's risk attitude. The difference between Samuelson's (1969) approach and ours is that we leave open a range of potential attitudes to risk, whereas Samuelson (1969) specifies the investor's attitude by imposing a particular model of risk aversion.

In this study, assuming an initial endowment model, we estimate the following measure for each of four simulation techniques and nine investment horizons:

1. Standard deviation of annualised returns;
2. Standard deviation of cumulative returns;
3. Distribution of terminal wealth;
4. Shortfall risk;
5. Value-at-risk;
6. Expected tail loss;
7. Sharpe ratio;
8. Sortino ratio;
9. Guo and Darnell's *T**; and
10. Fabozzi et al.'s time diversification index (TDI).

The technical specifications of these measures are outlined in Chapter Four.

5.5 Empirical evidence

5.5.1 Moments of the simulated return paths

Using the base accumulation model (an initial endowment framework), we estimate Monte Carlo, Efron (1979) bootstrap, stationary bootstrap, and empirical block bootstrap models for each of the nine horizons (1, 5, 10, 15, 20, 25, 30, 35, 40 years), for a total of 36 models. In this section, we report the results of the modeling, and compare our findings with the time diversification literature.

Table 5.1 reports the annualised moments of returns calculated as outlined in the methodology section. We report these results, and discuss them briefly, in order to validate the raw output from our simulations – 10,000 returns paths per simulation method per investment horizon – and to compare the results with our expectations of each method.

Table 5.1: Annualised moments of returns

This table presents the four moments of annualised returns for the four modeling techniques for each of the nine investment horizons. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison, the results are grouped by measure and read from left to right as follows:

Arithmetic mean: $-$. Standard deviation: $-$. Skewness: $\frac{-}{\quad}$. Kurtosis: $\frac{-}{\quad}$.

Estimates for mean and standard deviation are expressed in percentage terms, and estimates of skewness and kurtosis are pure numbers (i.e. have no units of measurement).

Investment	**Mean**				**Standard deviation**				**Skewness**				**Kurtosis**			
horizon	**mc (%)**	**b (%)**	**bs (%)**	**bb (%)**	**mc (%)**	**b (%)**	**bs (%)**	**bb (%)**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	10.87	10.92	10.89	10.90	18.82	18.78	18.53	18.55	0.00	0.06	-0.11	-0.08	3.01	9.80	7.98	8.30
35	10.90	10.90	10.85	10.88	18.81	18.77	18.49	18.53	0.00	0.06	-0.13	-0.11	3.01	9.76	7.72	8.02
30	10.85	10.95	10.90	10.94	18.81	18.79	18.40	18.44	0.00	0.07	-0.17	-0.14	3.01	9.73	7.44	7.74
25	10.92	10.87	10.86	10.89	18.79	18.72	18.38	18.39	0.00	0.04	-0.19	-0.17	3.01	9.53	7.11	7.44
20	10.86	10.91	10.89	10.83	18.76	18.67	18.26	18.25	0.00	0.02	-0.23	-0.21	3.01	9.33	6.73	7.05
15	10.83	10.86	10.85	10.90	18.75	18.65	18.14	18.08	0.01	-0.01	-0.28	-0.26	3.02	9.04	6.21	6.47
10	10.80	10.81	10.86	10.78	18.65	18.52	17.72	17.86	0.00	-0.06	-0.34	-0.32	3.02	8.50	5.49	5.71
5	10.80	10.81	10.86	10.77	18.47	18.21	17.19	17.22	0.01	-0.16	-0.32	-0.33	3.04	7.19	4.57	4.66
1	10.15	10.21	10.16	10.44	16.98	16.21	15.04	14.95	0.05	-0.22	-0.15	-0.14	3.20	4.39	3.47	3.42

The first noteworthy feature about the results reported in Table 5.1 is that the estimates for each moment are broadly similar for any given horizon and, where they do differ, the differences are consistent with expectations. The means and standard deviations are similar for each of the four models, and the standard deviations are relatively constant with time horizon as Kritzman and Rich (1998) find. If there is some noticeable difference between the standard deviation measures, it is that the estimates for the two block bootstrap models – the stationary bootstrap (bs) and empirical block bootstrap (bb) models – are slightly lower than those of the Monte Carlo and Efron (1979) bootstrap models. We argue that this is because the outer bounds of the simulated return paths – particularly the positive paths – are wider for the latter two methods, than the former two. This hypothesis is supported to some extend by the high kurtosis estimates for the Efron (1979) bootstrap method reported in Table 5.1, and the larger estimates for the terminal wealth range (Panel A) and maximum terminal wealth (Panel C) reported later in Table 5.5.

It is when we start to examine the skewness and kurtosis estimates that noticeable differences between the simulation methods begin to emerge. Skewness estimates for the Monte Carlo simulations (column headed "mc") are zero in almost all cases consistent with the method's implicit assumption of normality. The kurtosis estimates for the Monte Carlo simulations are approximately three in all cases for the same reason. The Efron (1979) bootstrap – where single row vectors are resampled with replacement – will generally produce a wider distribution of outcomes because the technique can produce long runs of either positive or negative returns. This can produce either positive or negative skewness (depending on whether the long runs of returns are positive or negative), and significant leptokurtosis. Table 5.1 shows both of these features with skewness ranging from -0.22 to 0.07, and the highest kurtosis of any model.

The stationary (bs) and empirical block (bb) bootstrap models produce the classical distributional characteristics of stock returns: negative skewness and leptokurtosis. This combination of characteristics implies a strongly non-normal distribution with a long left, or negative, tail. The kurtosis estimates for these two models are lower than that of the Efron (1979) bootstrap because each method involves resampling blocks of returns. As a result, we would expect these methods to (at least) partially capture the time series characteristics of returns, including phenomena like mean reversion (Poterba and Summers, 1988). The classical view of mean reversion sees the chances of either strongly positive or strongly negative paths as less likely, because the underlying return process will induce returns to revert to the mean after a sustained run of either positive or negative returns.

5.5.2 Ten measures of risk

In this section, we study the estimates for the ten measures of "risk" identified as popular in the literature. We are well aware that some of these measures – for example, the Sharpe ratio – are not strictly speaking measures of risk. To be precise, the Sharpe ratio is a measure of risk-adjusted return.

While we recognise this distinction, we also note that each of these measures is proposed in the literature as a way of potentially studying the presence, or otherwise, of time diversification.

5.5.2.1 Standard deviation of annualised returns

One of the more common findings of the time diversification literature is that the standard deviation of annualised returns falls as investment horizon lengthens (Bernstein, 1976; Garrone and Solnik, 1976; Lloyd and Haney, 1980; Lloyd and Modani, 1983; McEnally, 1985; Lee, 1990; Kritzman and Rich, 1998; Kochman and Goodwin, 2002; Guo and Darnell, 2005). Table 5.2 shows the standard deviation of annualised returns for each of the four modeling methods. We observe a similar pattern to that observed in the literature. At the one-year horizon, standard deviation estimates are of a similar order of magnitude to the annualised moments reported in Table 5.1. As horizon is lengthened, the standard deviation falls to low single figures.

Table 5.2: Standard deviation of annualised returns

This table presents the standard deviation of annualised returns for the four modeling techniques for each of the nine investment horizons. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). Estimates are expressed in percentage terms.

Investment	Standard deviation			
horizon	mc (%)	b (%)	bs (%)	bb (%)
40	2.98	2.96	2.52	2.98
35	3.15	3.15	2.72	3.21
30	3.45	3.43	2.94	3.40
25	3.76	3.80	3.21	3.75
20	4.20	4.16	3.56	4.19
15	4.87	4.88	4.19	4.90
10	5.87	5.81	5.15	5.99
5	8.28	8.25	7.72	8.36
1	17.60	17.37	18.49	18.93

Kritzman and Rich (1998) in exploring the question “What is Risk?” examine five different conceptions of risk.[63] In Table 5.3, we replicate Kritzman and Rich’s (1998) analysis, using the same shortcut, for our simulated data. Table 5.3 is thus a parametric counterpart of Table 5.2.[64]

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[63] These measures are shown in Kritzman and Rich (1998), Exhibit 1 (p. 67.)

[64] A comparison of Tables 5.2 and 5.3 provides some interesting insights into the different distributions produced by the four simulation methods employed herein. Because Monte Carlo simulation is a parametric method, the estimates in Table 5.2 are predictably close to those of Table 5.3. The Efron (1979) bootstrap also produces similar estimates but for a different reason: the technique tends to produce relatively symmetric distributions. With the block bootstrap methods – the stationary and empirical block bootstrap methods – we begin to see larger differences between the corresponding estimates in each table. These result from the information lost during the parameterisation process.

Table 5.3: Standard deviation of annualised returns – Parametric shortcut

This table presents the standard deviation of annualised returns for the four modeling techniques for each of the nine investment horizons using the shortcut employed in Kritzman and Rich (1998): ‾ (see column 4, Exhibit 1, p. 67). The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). Estimates are expressed in percentage terms.

Investment	Standard deviation			
horizon	mc (%)	b (%)	bs (%)	bb (%)
40	2.98	2.97	2.93	2.93
35	3.18	3.17	3.13	3.13
30	3.43	3.43	3.36	3.37
25	3.76	3.74	3.68	3.68
20	4.19	4.18	4.08	4.08
15	4.84	4.81	4.68	4.67
10	5.90	5.86	5.60	5.65
5	8.26	8.14	7.69	7.70
1	16.98	16.21	15.04	14.95

Using their short-cut, Kritzman and Rich (1998) see a reduction in the standard deviation of annualised returns from 20 per cent at the one-year horizon to 4.47 per cent at the 20-year horizon. Table 5.2 sees a standard deviation of 17-19 per cent at the one-year horizon fall to approximately four per cent at the 20-year horizon (depending on the model). Table 5.3 presents a similar trend with standard deviation falling from between 15-17 per cent to around four per cent.

A qualitative interpretation of this phenomenon – the inverse relationship between the standard deviation of annualised return and horizon – is that, over a long horizon, we have greater relative certainty about the expected return we would receive from investing in stocks. Over a one year horizon on the contrary, we have relatively wide range of potential outcomes. We might, for example, have a strong year of, say, 30 per cent or a particularly poor year of minus 30 per cent. Measured in standard deviation terms, our one-year prospects are relatively uncertain. Over a longer horizon, with a greater number of observations, we would expect to have less uncertainty about our expected return.

Kritzman (1994) observes this same phenomenon in a different way. Rather than parameterising the dispersion via a standard deviation calculation as we have done above, Kritzman (1994) instead shows a 95 per cent confidence interval for annualised returns against investment horizon.[65] He shows that, as the investment horizons lengthens, the confidence interval narrows. Thus, we see uncertainty or "risk" – this time expressed as a confidence interval – fall with investment horizon.

For the first of our first ten measures, we have shown two things: we find an inverse relationship between the standard deviation of annualised returns and investment horizon consistent with the

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[65] Kritzman (1994), Figure A, p. 14.

literature; and, in approximate terms, our results are consistent with the numerical estimates of other studies.

5.5.2.2 Standard deviation of cumulative returns

McEnally (1985) challenged the focus on annualised return measures, instead arguing that *total*, or cumulative, return measures are more relevant to understanding time diversification. In suggesting this focus, he begins to move the time diversification debate away from annual average conceptions of risk, to conceptions of risk that recognise investing as being a function of a series of realised returns. These two camps represent the two measurement bases we discuss in this study: the returns basis and the wealth basis respectively. Table 5.4 presents mean cumulative returns – which is the average value to which one dollar would grow over the particular given horizon – as well as the standard deviation around the mean value.

Table 5.4: Mean and standard deviation of cumulative returns

This table presents the mean and standard deviation for the four modeling techniques for each of the nine investment horizons. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). The results are grouped by measure and read from left to right: mean and standard deviation. Results are expressed in dollars.

Investment	**Mean**				**Standard deviation**			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	77.64	77.37	66.85	76.93	134.73	133.18	81.14	108.79
35	44.39	44.28	39.50	44.94	67.77	66.05	44.72	57.63
30	26.04	26.33	23.64	26.13	35.66	34.09	23.43	28.82
25	15.26	15.25	13.82	15.13	18.14	17.73	12.22	14.76
20	8.76	8.78	8.23	8.72	8.72	8.50	6.45	7.54
15	5.09	5.14	4.87	5.18	4.22	4.47	3.22	3.73
10	2.95	2.94	2.89	2.96	1.89	1.87	1.52	1.69
5	1.73	1.73	1.72	1.73	0.75	0.75	0.64	0.66
1	1.12	1.12	1.12	1.12	0.21	0.21	0.21	0.22

The results in Table 5.4 confirm the findings of McEnally (1985) and later authors (e.g. Bernstein, 1985; Leibowitz and Krasker, 1988; Lee, 1990; Peavy and Vaughn-Rauscher, 1994; Kochman and Goodwin, 2001; Hickman et al., 2001; Kochman and Goodwin, 2002; Gollier, 2002) who found that, when measured on a cumulative basis, risk rises as investment horizon lengthens. So, when we turn from an average return basis to a cumulative return basis, we observe a dramatic widening in the range of outcomes as horizon lengthens. We thus see that the investment experience of a plan member is highly path-dependent, and not merely a chain of average returns as our last measure would have us believe. This thesis will return to this theme later in this chapter when it considers other wealth-based measures, and in the next chapter (Chapter Six) when it investigates alternative accumulation models.

These results also hint at an important feature of long horizon investing that will be explored in more detail later in this thesis. Basu and Drew (2009a) identified a rapid increase in wealth in the later years of a typical 40 year investment horizon, describing it as the "portfolio size effect." In these results, we see an almost threefold increase in mean cumulative returns when comparing the 30- and 40-year horizons, providing tentative confirmatory evidence of such an effect. Evidence of this phenomenon in these results is particularly noteworthy because, at this point in the thesis, we are considering the initial endowment model where outcomes are only a function of returns. When this thesis considers alternative accumulation models in Chapter Six, we would expect to see an even stronger portfolio size effect as contributions (in particular) intensify the compounding effect at longer horizons.

In comparing the results produced by the various simulation methods, we once again find results that are consistent with our expectations. The two methods that implicitly assume that stock returns follow a random walk – Monte Carlo simulation and the Efron (1979) bootstrap method – have noticeably higher dispersion, as measured by standard deviations, particularly at the longer horizons. Those techniques that better capture the time series characteristics of the data – especially the stationary bootstrap – have lower cumulative returns for every horizon, as well as less dispersion.

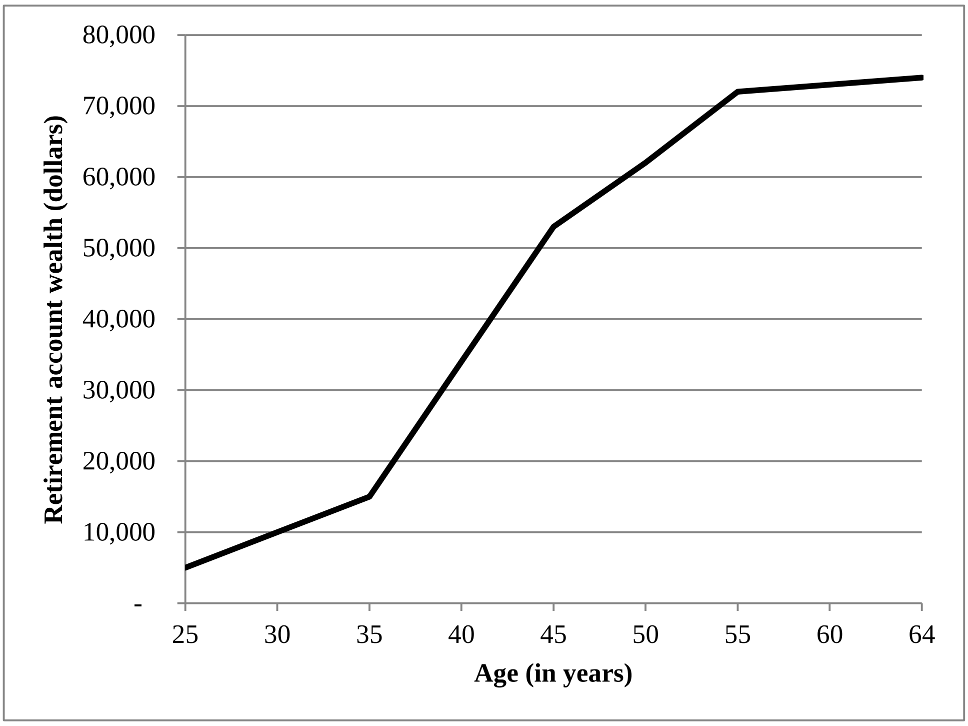
Two measures into our study of ten, we see early evidence of two things. Firstly, this empirical analysis has confirmed the findings of the relevant literature thus far. And, secondly, we see the emergence of conflicting evidence regarding time diversification. Therefore, at this point in the study, the time diversification puzzle remains very much alive.

5.5.2.3 Distribution of terminal wealth

From observing the effect of returns on one dollar as we have in Table 5.4, we now apply returns to the realistic initial wealth (W_0) levels discussed in Chapter Four (and shown in Table 4.3). For convenience, we plot the assumed initial wealth against the assumed age of our hypothetical investor in Figure 5.2.

Figure 5.2: Initial wealth by age

This figure shows our initial wealth assumptions plotted against the age of our hypothetical investor. Age is calculated by the deducting the investment horizon from our assumed retirement age of 65 years (e.g. the initial wealth assumed for an investor age 25 years corresponds to the wealth assumed for a 40-year investment horizon). Discussion of the data, and the reasoning behind its use, can be found in Chapter Four (section 4.3.1.4).



Estimates of the distributions of terminal wealth for each simulation method are reported in Table 5.5.

Table 5.5: Distribution of terminal wealth

This table presents the distribution of terminal wealth for the four modeling techniques for each of the nine investment horizons. Initial wealth (W_0) is as per row five of Table 4.3, which is reproduced in graphical form as Figure 5.2. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into three panels. Panel A summarises, from left to right, measures of central tendency (mean and median) and dispersion (standard deviation and range). Panel B summarises the lower half of the distribution (percentiles from the minimum to the median) with the median repeated for ease of reference. Panel C summarises the upper half of the distribution (percentiles from the median to the maximum) with the median again repeated for ease of reference. Results are presented in thousands of dollars.

PANEL A – Measures of central tendency and dispersion

Investment	Mean				Median				Standard deviation				Range			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	385.57	382.96	331.60	381.55	183.85	191.45	202.62	211.05	678.34	656.99	401.65	538.56	16,508.53	17,604.27	5,074.34	12,115.05
35	440.32	439.12	391.32	445.73	238.38	244.25	250.64	263.62	670.02	651.49	441.55	570.42	20,655.57	16,113.09	6,528.95	10,112.98
30	386.84	391.64	351.54	388.83	224.61	235.06	244.24	255.04	525.30	507.24	348.01	428.13	9,213.28	10,003.73	3,910.91	5,151.72
25	513.47	514.25	465.94	509.63	334.86	328.77	350.29	361.61	603.51	598.81	411.64	497.24	14,174.63	7,746.01	5,121.88	5,359.70
20	460.22	460.84	431.81	458.19	326.17	331.54	345.55	352.22	456.00	445.28	337.35	394.44	5,926.59	8,580.06	3,824.32	4,965.63
15	312.86	315.96	299.49	318.04	242.26	241.77	254.09	267.58	257.75	273.77	197.49	228.32	5,410.81	6,234.49	2,617.68	2,173.98
10	210.20	210.18	206.42	211.66	176.96	177.43	188.23	193.86	133.51	133.24	107.87	120.39	1,800.49	2,055.81	975.11	1,208.56
5	125.06	124.99	124.38	124.90	114.70	115.24	122.39	122.89	54.19	53.98	46.04	47.24	499.70	551.94	370.76	358.33
1	81.80	81.91	82.03	82.35	80.50	81.06	83.27	83.52	14.89	14.67	14.99	15.17	124.63	143.91	141.84	141.84

PANEL B – Lower half of the distribution of terminal wealth

Investment	**Minimum**				**5th percentile**				**25th percentile**				**Median**			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	2.95	2.02	0.27	0.06	26.97	26.91	26.53	19.38	84.92	86.57	91.77	85.05	183.85	191.45	202.62	211.05
35	3.60	2.83	1.36	0.63	38.59	39.32	37.17	26.84	115.58	115.27	122.02	115.51	238.38	244.25	250.64	263.62
30	6.09	2.83	2.17	0.80	43.18	41.40	41.90	33.35	112.33	116.88	122.95	117.78	224.61	235.06	244.24	255.04
25	10.72	9.86	6.63	2.55	69.67	70.01	67.62	53.65	177.08	172.02	186.15	179.31	334.86	328.77	350.29	361.61
20	14.65	12.81	8.60	2.19	81.85	81.39	82.56	64.63	183.95	186.60	199.08	189.91	326.17	331.54	345.55	352.22
15	11.90	11.33	7.63	1.48	70.57	72.87	68.96	57.38	146.64	147.08	156.94	156.51	242.26	241.77	254.09	267.58
10	24.62	16.91	5.73	7.07	66.91	67.25	63.43	53.37	119.69	120.45	127.64	125.78	176.96	177.43	188.23	193.86
5	19.91	25.13	6.31	4.10	57.84	57.68	51.52	49.33	86.95	87.69	91.55	92.58	114.70	115.24	122.39	122.89
1	36.33	35.56	25.41	25.41	59.44	59.45	57.45	57.36	71.35	71.93	72.80	73.29	80.50	81.06	83.27	83.52

PANEL C – Upper half of the distribution of terminal wealth

Investment	**Median**				**75th percentile**				**95th percentile**				**Maximum**			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	183.85	191.45	202.62	211.05	414.03	424.12	415.48	472.00	1,401.84	1,330.25	1,066.88	1,279.57	16,511.48	17,606.29	5,074.61	12,115.10
35	238.38	244.25	250.64	263.62	511.13	510.96	493.91	551.71	1,491.76	1,442.99	1,216.17	1,472.72	20,659.17	16,115.92	6,530.32	10,113.61
30	224.61	235.06	244.24	255.04	452.41	464.35	456.99	508.83	1,257.29	1,242.67	1,031.52	1,181.23	9,219.38	10,006.57	3,913.08	5,152.52
25	334.86	328.77	350.29	361.61	630.36	620.90	617.92	663.71	1,530.15	1,566.08	1,261.83	1,495.54	14,185.35	7,755.87	5,128.51	5,362.25
20	326.17	331.54	345.55	352.22	570.77	574.77	566.41	607.74	1,296.00	1,285.11	1,083.36	1,220.41	5,941.24	8,592.88	3,832.92	4,967.83
15	242.26	241.77	254.09	267.58	390.54	395.23	395.55	418.24	797.04	800.05	675.99	755.87	5,422.72	6,245.83	2,625.30	2,175.46
10	176.96	177.43	188.23	193.86	263.06	262.56	267.20	276.46	464.50	455.14	406.13	435.29	1,825.11	2,072.72	980.83	1,215.63
5	114.70	115.24	122.39	122.89	151.43	152.26	152.85	155.05	228.28	224.38	200.22	206.26	519.62	577.07	377.08	362.43
1	80.50	81.06	83.27	83.52	90.94	90.83	91.11	91.55	108.36	107.35	103.92	104.29	160.96	179.47	167.25	167.25

The first observation from the results in Table 5.5 is that contrary to the mean measures reported earlier, mean terminal wealth does not rise monotonically with time horizon. This is obviously due to the obscuring factor of the different initial wealth levels shown in Figure 5.2. While this might obscure mean terminal wealth to some degree, what can be said, however, is that initial wealth is a relevant variable when comparing the results from different time horizons. When we compare the means of the different models for the same horizon, estimates of terminal wealth are generally more consistent although there remains some variation in results due to model specification. Take, for example, the 35-year horizon where the mean terminal wealth for the stationary bootstrap (bs) model is around 10 per cent lower than that of the nearest estimate of terminal wealth (391.32 versus 439.12 for the Efron (1979) bootstrap model).

Median terminal wealth is significantly lower than the mean especially at long horizons suggesting that strongly positive return paths – the so-called "right tail" of the distribution – are inflating the mean for all models. This finding is consistent with Leibowitz and Krasker (1988) who find that the gap between the mean and median tends to rise with investment horizon. We can observe these strongly positive outcomes in the right half of Panel C where terminal wealth over a 40-year horizon for the maximum path is around 90 times higher than that of the median path (17,606.29 divided by 191.45) for the Efron (1979) bootstrap model. Perhaps the most striking evidence of the influence of the right tail on the mean and standard deviation measures reported in Table 5.5, are the substantial differences between the 95th percentile and maximum values for each model for investment horizons of 10 years or longer. For example, for the Efron (1979) bootstrap model, the estimated maximum at the 40-year horizon is around thirteen times larger than the corresponding 95th percentile estimate (17,606.29 divided by 1,330.25). Leibowitz and Krasker (1988) report a similar finding although their analysis highlighted the divergence between the 75th and 95th percentiles which while significant, is not as material as the divergence between the 95th percentile and the maximum.

In order to validate the results reported in Table 5.5, we re-examined the significant work of Reichenstein and Dorsett (1995) who computed, *inter alia*, ending real wealth percentiles.[66] We compared the ratio of the 95th percentile to the median, and the ratio of the median to the 5th percentile in their work with that of ours for 1, 5, 10, 20 and 30 year horizons. The purpose of this comparison was to test whether the distribution of terminal wealth was similar despite a number of differences between our study and theirs. For example, Reichenstein and Dorsett (1995) studied the S&P 500 (amongst other datasets) from 1926-1993 whereas this thesis considers a broader definition of stocks for a longer period (1926-2011). Furthermore, Reichenstein and Dorsett (1995) consider terminal wealth in real terms whereas we consider wealth in nominal terms. Table 5.6 reports these ratios.

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[66] Reichenstein and Dorsett's (1995) work is significant in at least two respects. Firstly, it sets out to synthesise the time diversification debate as it stood when their work was published. In this sense, it is a forerunner to this study. Secondly, their study runs to 89 pages.

Table 5.6: Comparison of terminal wealth distributions

This table presents a simple comparison of terminal wealth distributions. The ratio of the 95th percentile to the median (95/median), and the ratio of the median to the 5th percentile (Median/5) are reported. Because Reichenstein and Dorsett (1995) considered a random walk model in Table 11 (p. 32) of their study, we report the ratios for the two models from this study that assume a random walk by construction: the Monte Carlo (mc) simulation and the Efron (1979) bootstrap (b) simulation. From left to right we report ratios for: Monte Carlo simulation (mc), Efron (1979) bootstrap (b) and the Reichenstein and Dorsett (1995) results (R&D95). Because the results are ratios they are pure numbers (i.e. have no units of measurement).

Investment horizon	mc		b		R&D95	
	95/median	Median/5	95/median	Median/5	95/median	Median/5
30	5.60	5.20	5.29	5.68	6.29	6.29
20	3.97	3.98	3.88	4.07	4.48	4.48
10	2.62	2.64	2.57	2.64	2.89	2.90
5	1.99	1.98	1.95	2.00	2.11	2.11
1	1.35	1.35	1.32	1.36	1.39	1.39

The first thing that emerges from the ratios in Table 5.6 is the nature of each distribution. The Monte Carlo simulation (mc) results suggest a relatively symmetrical distribution, as do the ratios computed for Reichenstein and Dorsett's (1995) work. In both cases, the upside (95/median) and downside (Median/5) relativities are nearly identical (with the exception of those for the Monte Carlo simulation at the 30-year horizon). For the Efron (1979) bootstrap, the downside relativities are larger in magnitude than their upside equivalents, consistent with a technique that captures the tail characteristics of the empirical distribution.

Apart from these technical findings, it is noteworthy that despite differences in data and methodology all results are of similar orders of magnitude. We can therefore conclude that this study has replicated the dynamics of this earlier time diversification study to some degree. Furthermore, whilst direct comparisons are difficult because of a range of different assumptions, the findings of this study are also generally consistent with those of other studies that consider terminal wealth in one form or another (e.g. Thorley, 1995; Hickman et al., 2001; Mukherji, 2008).

5.5.2.4 Downside risk measures

Our consideration of the distribution of terminal wealth highlighted the range of outcomes that are possible for long term investors. Following the time diversification literature, we report first a downside risk measure that ignores the magnitude of loss, shortfall risk, followed by others that capture the size of a loss in different ways, such as downside deviation, value-at-risk and expected tail loss. These downside risk measures are reported in Table 5.7.[67]

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[67] Note that the 95% VaR estimates in Table 5.7 correspond to the 5th percentile terminal wealth estimates in Panel B of Table 5.5.

Table 5.7: Downside risk measures

This table presents four downside risk measures for the four modeling techniques employed for each of the nine investment horizons. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). The measures are: downside deviation, shortfall risk, 95% value-at-risk (VaR) and 95% expected tail loss. All results are expressed in thousands of dollars, except for shortfall risk which is expressed in percentage terms.

Investment horizon	Shortfall risk				Downside deviation				Value-at-risk				Expected tail loss			
	mc (%)	b (%)	bs (%)	bb (%)	mc ($)	b ($)	bs ($)	bb ($)	mc ($)	b ($)	bs ($)	bb ($)	mc ($)	b ($)	bs ($)	bb ($)
40	3.05	3.00	3.01	5.37	1.35	1.45	1.70	2.53	26.97	26.91	26.53	19.38	17.84	17.53	16.49	11.14
35	3.97	3.80	4.17	6.88	2.74	2.73	3.12	4.72	38.59	39.32	37.17	26.84	25.78	25.85	23.61	15.76
30	5.03	5.43	5.02	6.99	3.60	3.82	4.48	5.74	43.18	41.40	41.90	33.35	29.79	28.24	27.42	20.29
25	7.04	7.06	6.37	9.18	7.90	8.24	9.00	12.05	69.67	70.01	67.62	53.65	49.98	48.30	44.03	32.92
20	9.30	9.13	8.55	11.22	11.56	11.62	12.13	16.98	81.85	81.39	82.56	64.63	60.06	59.07	56.61	41.67
15	12.36	12.80	11.82	13.58	12.79	12.64	13.54	17.51	70.57	72.87	68.96	57.38	53.83	54.90	48.89	36.16
10	17.23	17.16	15.37	17.38	12.93	12.93	13.93	17.66	66.91	67.25	63.43	53.37	53.95	53.71	46.38	36.18
5	24.82	24.32	22.38	20.83	10.95	11.11	12.35	13.77	57.84	57.68	51.52	49.33	49.17	48.69	37.79	33.92
1	38.40	36.76	32.02	31.05	6.82	6.85	7.97	8.04	59.44	59.45	57.45	57.36	55.30	54.40	48.59	47.82

5.5.2.4.1 Shortfall risk

Risk is typically viewed as being a function of the consequences and likelihood of an event occurring.[68] The first downside risk measure we consider, shortfall risk – or the probability of falling short of some threshold return, which we define as the return from T-bills – focuses on only one these elements of risk: likelihood. In the next section, we will turn to alternative downside measures which capture both these two elements in different ways.

The estimates for shortfall risk in Table 5.7 are consistent with the time diversification literature which finds that, as investment horizon lengthens, shortfall risk reduces (Leibowitz and Krasker, 1988; Leibowitz and Langetieg, 1989; Butler and Domian, 1991; Leibowitz and Kogelman, 1991; Reichenstein and Dorsett, 1995; Cohen et al., 1996). The shortfall risk estimates of this study also confirm that this trend holds equally for each of the four modeling techniques. Whilst we do not make claims about the nature of the asset return process in the data used in this study, it is argued that the four techniques employed herein provide us some insights into random walk (Monte Carlo and Efron (1979) bootstrap) and serially-correlated (stationary and empirical block bootstraps) asset return processes. In this sense, these findings show that that the fall in shortfall risk as investment horizon lengthens is somewhat robust to the asset return process specification, consistent with the findings of Reichenstein and Dorsett (1995) who explicitly examine random walk and mean reversion models and reach the same conclusion.

Having now considered four of our ten selected measures, this study continues to see conflicting evidence of time diversification. On the one hand, two measures – the standard deviation of annualised returns and shortfall risk – suggest that risk falls as investment horizon lengthens. On the other, the remaining two measures – the standard deviation of cumulative returns and the distribution of terminal wealth – suggest that the range of outcomes, and hence risk, increases with investment horizon. In each case, these findings are consistent with the literature. At this point in the analysis we seem destined to conclude that the time diversification debate remains a puzzle, depending on how one frames risk.

5.5.2.4.2 Wealth-denominated downside risk measures

The behavioural literature (e.g. Olsen and Khaki, 1998) has argued that, in considering risk, investors weight the magnitude of loss over its probability. We therefore turn to several measures of risk that capture the magnitude of loss in different ways. Firstly, we consider semi-standard deviation which is a parametric measure of downside risk. Because it is a parametric measure, it provides a *summary*

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[68] For generic guidance on risk management, international standards like AS/NZS ISO 31000:2009 (or its predecessors like AS/NZS ISO 4360:2004) are useful. In defining risk, AS/NZS ISO 31000:2009 notes that "risk is often expressed in terms of a combination of the *consequences* of an event … and the associated *likelihood* of occurrence (p. 1)" (emphasis added) (Standards Australia, 2009).

measure of the variation below a given threshold. This study then considers two non-parametric measures of risk expressed in dollar terms: value-at-risk; and, expected tail loss.[69]

McEnally (1985) found that the estimated semi-standard deviation (below the mean) for annualised average returns falls as investment horizon increases in the same way that the standard deviation of annualised returns reported in Table 5.2 falls. He also found that the semi-standard deviation of *total* returns generally increases with time up to the 10-year horizon, which was the longest horizon he examined. In Table 5.7, we report semi-standard deviation, or downside deviation, expressed in dollar terms. These figures are the counterparts to the results reported in Table 5.5, and are calculated using the same initial wealth estimates. This means that the trend identified by McEnally (1985), and confirmed by Mukherji (2002, 2008), are obscured somewhat by different starting wealth levels. Another difference that obscures these results is the level against which the downside deviation is measured. McEnally (1985) measures semi-standard deviation against the mean of the series, whereas Mukherji (2008) against a number of arbitrary real return targets (e.g. two, five and eight per cent). Here we measure downside deviation against the return from T-bills, which represents the zero risk opportunity cost for our hypothetical investor.

We now turn to the final measure of downside risk that appears in the literature – value-at-risk (VaR) – and a more robust variant – expected tail loss (ETL) – which we add as a minor contribution to the literature. ETL measures the average loss conditional on the loss exceeding the VaR threshold at a given confidence level. In this way, ETL gives us more information about the nature of tail risk than VaR, which only provides us the threshold at which tail risk begins. In line with Olsen and Khaki's (1998) claims about the importance to investors of the potential *magnitude* of loss, VaR and ETL each incorporate the concept of probability via the confidence level, as well as the potential magnitude of the loss should a loss occur.

Before we consider the VaR and ETL estimates in Table 5.7 in detail, recall that the values are expressed in dollar terms and have been computed using different initial wealth levels for each horizon. While different levels of initial wealth again obscure the results to some degree, this approach does have the benefit of providing us a realistic idea of the terminal wealth in nominal dollar terms. Despite this complicating factor, it is possible to observe an underlying trend in the results especially for horizons of 20 years and longer. From the 20-year to the 40-year horizon, we observe a monotonic reduction in both VaR and ETL for all models, suggesting that risk increases with investment horizon. Between the one-year and the 15-year horizons the results are less clear.

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[69] Expected tail loss (ETL) is known by other names. Rachev et al. (2008) highlight that this measure, for a discrete distribution function, is variously known as average value-at-risk, conditional value-at-risk, or expected shortfall. Rachev et al. (2008) define ETL as the equivalent measure for a continuous distribution function. We prefer ETL because it is, to us, the most descriptive term.

We are unable to compare these results to the findings of Panyagometh (2011) – the only other time diversification study to investigate VaR – who studies an accumulation model that involves monthly contributions. This thesis considers similar accumulation models in Chapter Six.

Finally, from a technical perspective, these results highlight the limitations of VaR used in isolation. If we look at the VaR results by themselves, we might form certain expectations of the magnitude of downside risk. Take, for example, the VaR estimates for the 40-year horizon (Table 5.7), where they range from $19,380 for the empirical block bootstrap model (bb) to $26,970 for the Monte Carlo model (mc). When we consider the corresponding ETL estimates – the average terminal wealth conditional on a VaR event occurring – we see that outcomes can indeed be much worse than VaR suggests, with estimates ranging from $11,140 (bb) to $17,840 (mc).

After completing our consideration of downside risk measures, we now review a number of reward-to-variability ratios observed in the time diversification literature.

5.5.2.5 Reward-to-variability ratios

Until now we have looked at measures incorporating returns, and measures of risk, but we haven't combined these into what were first known as reward-to-variability ratios (Sharpe, 1966), and which are now commonly described as risk-adjusted measures of performance.[70] In this study, consistent with the time diversification literature, we estimate Sharpe ratios, Sortino ratios and coefficients of downside deviation for each of the four models for all nine investment horizons. The results are reported in Table 5.8.

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[70] Mercifully, the unwieldy "reward-to-variability ratio" became the "Sharpe ratio" as a tribute to Sharpe's seminal work on mutual fund performance.

Table 5.8: Reward-to-variability ratios

This table presents reward-to-variability ratios for the four modeling techniques for each of the nine investment horizons. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). The measures are: the Sharpe ratio, the Sortino ratio and the coefficient of downside deviation. The results are ratios so are therefore pure numbers (i.e. have no units of measurement).

Investment	**Sharpe ratio**				**Sortino ratio**				**Coefficient of downside deviation**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	2.47	2.50	2.93	2.47	79.70	65.68	96.69	31.28	0.03	0.03	0.02	0.07
35	2.34	2.34	2.71	2.29	46.95	45.11	71.05	24.83	0.04	0.05	0.03	0.08
30	2.13	2.16	2.52	2.19	36.71	39.86	38.54	20.27	0.06	0.05	0.05	0.10
25	1.97	1.94	2.29	1.97	25.32	24.86	27.69	14.76	0.08	0.08	0.08	0.14
20	1.75	1.78	2.08	1.75	15.63	16.46	21.24	9.82	0.13	0.13	0.10	0.21
15	1.50	1.51	1.75	1.51	9.87	10.21	12.30	6.25	0.21	0.21	0.17	0.34
10	1.25	1.26	1.43	1.22	6.12	6.26	6.16	3.86	0.34	0.33	0.34	0.54
5	0.89	0.89	0.96	0.87	2.85	2.91	2.56	1.93	0.74	0.73	0.83	1.09
1	0.39	0.40	0.37	0.38	0.78	0.79	0.63	0.63	2.76	2.69	3.37	3.48

5.5.2.5.1 Sharpe ratio

The risk-adjusted performance measure estimates reported in Table 5.8 are generally consistent with other studies in the time diversification literature that estimate the same measures. We find that estimated Sharpe ratios rise as time horizon lengthens confirming the findings of a number of other studies beginning with Levy (1972) (see also Lloyd and Modani,1983; Levy, 1984). We also observe that this rising trend holds for all four model types suggesting that our finding is robust to a range of asset return processes. Comparing the Sharpe ratios produced by each of the four models, we see that for any given horizon they are of a similar magnitude.

Our Sharpe ratio findings, however, contradict those of Hodges et al. (1997) who find that Sharpe ratios first rise with investment horizon, then fall, forming a hump-shaped profile. It is not clear how their study differs to this one however we suspect it is based on different methodological approaches.

By considering the Sharpe ratio, this thesis has considered the relationship between risk-adjusted performance and investment horizon showing a positive relationship consistent with the literature. With conflicting evidence continuing to mount, it is becoming apparent that the time diversification puzzle – as it is currently defined – remains.

5.5.2.5.2 Sortino ratio

The Sortino ratio estimates we report in Table 5.8 confirm the findings of Sinha and Sun (2005) who show that Sortino ratios rise with investment horizon. While the reported Sortino ratios do rise with investment horizon, there is a difference in magnitude between the ratios produced by each simulation method for a given horizon, and the differences between models become larger with time.

This divergence in results is an expected consequence of the way each technique models the distribution of returns. For example, the 40-year horizon Sortino ratio for the Monte Carlo (mc) technique is more than twice as high as that of the empirical block bootstrap (bb) simulation technique (79.70 divided by 31.28). Because the empirical block bootstrap (bb) technique better captures the distributional characteristics of the stock returns, we would expect a higher downside deviation estimate when compared to the Monte Carlo method. Indicative evidence of this can be found in Table 5.7, where the downside deviation estimate (in slightly different terms) for the empirical block bootstrap is around twice as high as that of the corresponding Monte Carlo estimate (2.53 versus 1.35). Therefore, with the denominator almost twice as high, we would expect, *ceteris paribus*, a Sortino ratio of around half the magnitude. This expectation is born out in Table 5.8. The growing difference between the estimates for each of the methods through time is simply the result of the compounding effect of these methodological differences through time.

Finally, we consider the coefficient of downside deviation (CDD) together with the Sortino ratio, because it is essentially its reciprocal. If the CDD estimates are to be consistent with the findings for both the Sharpe and Sortino ratios, they should fall as time horizon lengthens. Table 5.8 reports that for each model the CDD monotonically falls with horizon, confirming the findings of Mukherji (2002, 2008) who also considered this measure.

So this study has shown that each of the Sharpe ratio, the Sortino ratio and the CDD tell the same story: the reward-for-risk calculation improves with investment horizon. While this statement isn't strictly about the relationship between risk and time – the core question of the time diversification debate – it does lead to the same prescription provided by advocates of the concept: stocks should be more attractive to those with longer investment horizons. It is also noteworthy that each of these measures is a return-based measure, and exhibits the same functional relationship with time that we have identified for other return-based measures discussed thus far.

*5.5.2.6 Guo and Darnell's (2005) T**

We now turn to two measures which were specifically developed to shed light on a particular element of the time diversification debate. In this sense, these measures are unique to this literature and, to our knowledge, neither measure has been re-examined in the subsequent time diversification literature. Firstly, in Table 5.9, we consider estimates of Guo and Darnell's (2005) T^* measure which they define as "the investment horizon such that the total stock return over this holding period will not become negative at the [selected] confidence level (p. 69)."

Table 5.9: Guo and Darnell's T^*

This table presents Guo and Darnell's (2005) T^* for the four modeling techniques for each of the nine investment horizons. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). T^* is defined as "the investment horizon such that the total stock return over this holding period will not become negative at the [95%] confidence level (Guo and Darnell, 2005, p. 69)." Results are expressed in units of months. Highlighted cells represent occasions where T^* is greater than investment horizon for which it is being calculated.

Investment horizon	T^* mc	b	bs	bb
40	168	170	166	220
35	170	172	178	233
30	164	171	166	209
25	164	175	174	223
20	168	169	162	208
15	166	171	168	>180
10	>120	>120	>120	>120
5	>60	>60	>60	>60
1	>12	>12	>12	>12

Before embarking on a discussion about the results in Table 5.9, it is worth reiterating that while the *T** measure is introduced by Guo and Darnell (2005) within the context of the time diversification debate, it apparently wasn't designed as a measure of time diversification *per se*. Instead Guo and Darnell (2005) developed *T** to compare the relative riskiness of different asset return processes – random walk, mean reverting, and positively autocorrelated processes – using a measure expressed in terms of time. By observing different *T** estimates for each of the three processes they conclude that "optimal asset allocation policy should … depend upon the length of the investment horizon (p. 69)."

The results in Table 5.9 show a mixed picture. For the Monte Carlo (mc), Efron (1979) bootstrap (b) and the stationary bootstrap (bs) models, all estimates for *T** are approximately the same, falling in the range 162 to 178 months. If there is some detectable contrast between these three models it is that the Efron (1979) bootstrap (b) has slightly higher *T** estimates than the Monte Carlo (mc) simulation.

The most obvious finding in Table 5.9 is the difference between the empirical block bootstrap (bb) results and those of the other three models, with *T** estimates being around 30 months higher. In practical terms, these higher *T** estimates mean that it takes more time for the compounding of returns to counteract the heavier left tail produced by the empirical block bootstrap technique. Our suspicions about the empirical block bootstrap method producing a heavier left tail are validated to some degree by the downside risk measures reported in Table 5.7. Note, for example, that all downside risk estimates for the empirical bootstrap method are worse that the other three methods over all horizons.

It is notable that in our discussion of the results relating to *T** we have spent significantly more time contrasting the simulation methods we employ, than on relating risk to investment horizon. This is because Guo and Darnell's (2005) is less interested in the substance of the time diversification debate, and more interested in what the asset return process means for long horizon investing. In this sense, it is another way of analysing various conceptions of Samuelson's (1969) second assumption. Therefore, while Guo and Darnell's (2005) *T** measure doesn't help us resolve the time diversification debate, it does buttress the case for horizon-dependent asset allocation policy, a line of inquiry this thesis touches on in Chapter Seven.

5.5.2.7 Fabozzi et al.'s (2006) time diversification index

The final measure under consideration in this study is the time diversification index (TDI) of Fabozzi et al. (2006). The TDI is a ratio of normalised risk measures for two investment horizons. The estimates we report in Table 5.10 compare the normalised risk measure for a horizon of, say, 25 years against the same risk measure for the 40-year horizon. All TDIs are computed against a 40-year horizon.

Table 5.10: Time diversification index

This table presents Fabozzi et al.'s (2006) time diversification index (TDI) for the four modeling techniques for each of the nine investment horizons. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). The time diversification index is a ratio of normalised risk measures for two horizons. The calculated TDIs reported in this table are the ratio of the normalised risk measures for each investment horizon against the 40 year investment horizon. As such, there is no 40 year measure. According to Fabozzi et al.'s (2006) rule, a TDI of less than unity suggests the presence of time diversification. The results are ratios so are therefore pure numbers (i.e. have no units of measurement).

Investment horizon	Time diversification index			
	mc	b	bs	bb
40				
35	1.0030	0.9984	0.9989	0.9994
30	0.9984	1.0015	1.0086	1.0096
25	1.0063	0.9984	1.0056	1.0083
20	1.0022	1.0042	1.0147	1.0097
15	1.0002	1.0015	1.0180	1.0267
10	1.0025	1.0035	1.0432	1.0280
5	1.0124	1.0206	1.0751	1.0651
1	1.0351	1.0824	1.1495	1.1883

By calculating Fabozzi et al.'s (2006) TDI measure, and applying their decision rule – a TDI of less than unity is suggestive of time diversification – we find very little evidence of time diversification. For each simulation method, virtually all estimates exceed unity. The higher TDIs are recorded for shorter horizons, and the estimates fall in value as horizon lengthens.

Where the TDI estimates do fall below unity, it is generally at longer horizons. For example, for three of the four models – the Efron (1979) bootstrap, the stationary bootstrap and the empirical block bootstrap – the time diversification index is less than unity at the 35-year horizon. This means that the normalised risk measure for the longer of the two horizons in the calculation (40 years) is lower than that of the shorter horizon (35 years). One interpretation of a sub-unity TDI in qualitative terms might go as follows: for the longer horizon, in this case 40 years, we have less risk per unit of return than at the shorter horizon of 35 years. Alternatively, we could say that there is an improvement in return per unit of risk as we lengthen the horizon, in this example from 35 years to 40 years. Regardless of the interpretation, we expect that this feature of the results – sub-unity TDIs at longer horizons – is a manifestation of the rapid rise in portfolio size at the end of plan member's working life. According to this view, expected portfolio size grows faster than the additional risk assumed. We submit that this different dynamic in the last decade or so of a typical investment horizon may provide further evidence of the portfolio size effect identified by Basu and Drew (2009a).

Overall, however, we find no clear evidence of time diversification, in line with the findings of Fabozzi et al. (2006).

5.6 Concluding remarks

In this first empirical study, motivated by a critique of the literature, we have taken an applied approach to reconsidering the essential relationship between risk and investment horizon. In particular, we have selected ten measures used in the literature to study the time diversification debate. Using four simulation methods, we estimated these ten measures for nine investment horizons from one to 40 years long. In so doing, this thesis has removed from the study any variation due to data or methodology, thereby allowing a comparison of these ten measures *on the same terms*. We reproduce our findings in summary form in Table 5.11.

Table 5.11: Summary findings

This table presents, in summary form, the findings of this study. From left to right, we report the: (1) the measure of interest; (2) the functional relationship between risk and investment horizon suggested by the measure; (3) whether this relationship is indicative of time diversification ("Yes" or "No"); and, (4) whether the measure is a returns-only ("Returns") or wealth-based ("Wealth") measure.[71]

Measure	Functional relationship	Time diversification?	Measurement basis
Standard deviation of annualised returns	Decreasing	Yes	Returns
Standard deviation of cumulative returns	Increasing	No	Wealth
Distribution of terminal wealth	Increasing	No	Wealth
Shortfall risk	Decreasing	Yes	Wealth
Value-at-risk	Increasing	No	Wealth
Expected tail loss	Increasing	No	Wealth
Sharpe ratio	Increasing	Yes	Returns
Sortino ratio	Increasing	Yes	Returns
Guo and Darnell's (2005) *T**	N/A	No	Wealth
Time diversification index	Unclear	No	Returns

The empirical evidence presented in this study, and summarised in Table 5.11, leads us to two possible conclusions. Firstly, based on an approach inspired by the applied stream of the literature, there is mixed evidence of time diversification depending on one's preferences regarding how risk should be measured. These findings hold for a range of quantitative techniques that implicitly model a variety of asset return processes. Thus, whether or not equities are riskier over long investment horizons is really not a question that can be answered objectively. Rather, it is a subjective matter

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[71] A wealth-based measure in this context includes a measure of cumulative returns where returns are applied to a notional starting value of $1.

depending on how one frames risk. Table 5.11 effectively provides us ten different frames in which to consider risk. In this sense, Kritzman's (2000) statement that the time diversification has become a "referendum on the meaning of risk" is insightful. Thus, we may conclude that the time diversification puzzle lives on, and that we can accept our null hypothesis: that a relationship between risk and investment horizon exists. It is the ambiguity of this relationship between risk and investment horizon that leads us to another potential conclusion.

The second potential conclusion is that, in Table 5.11, we have seen the emergence of a dichotomy. Rather than ten different risk frames, maybe we really have the choice between two measurement bases: a returns-basis or a wealth-basis. Let us consider this point in more detail. Firstly let us exclude those measures where there is no clear functional relationship between risk and investment horizon. This removes the T^* and TDI measures from consideration, leaving eight measures remaining. For the remaining eight measures, let us compare columns three and four of Table 5.11. When we do this, we see that for each return-based measure there is a suggestion of the presence of time diversification. A further measure, shortfall risk, provides evidence of time diversification despite being wealth-based measure. As the literature tells us however, shortfall risk does have one significant drawback. As a probability measure, it ignores the magnitude dimension of risk.

On the other hand, we see that wealth-based measures in every case are not consistent with time diversification. This is particularly telling, because wealth-based measures we argue better represent the experience of long-horizon investors. Terminal wealth is, after all, a function of a series of realised returns applied to an accumulation model (in this case the initial endowment model). Terminal wealth is not a function of a series of annualised averages as a return-based view of the world implies. So, while the findings of this thesis suggest that the presence of time diversification might be an outworking of the investor's risk frame, it might also be a function of the basis upon which risk is measured.

If the one accepts the prescription that wealth-denominated measures are paramount, then time diversification does not exist in the initial endowment accumulation model. But what about other accumulation models? If our preferred measures are wealth-based, then the accumulation model – the variables that drive the evolution of wealth – most definitely matters. This last point is particularly critical as we move onto the second and third empirical chapters (Chapters Six and Seven). Whilst this "referendum on the meaning of risk" may be appropriate in focusing the minds of scholars and practitioners on appropriate ways of measuring risk, the debate we would argue has been conducted in an incomplete framework. With very few exceptions, none of the literature considers the institutional setting *as it currently exists*. For example, virtually no scholar takes seriously the importance of contributions to the question of how risk changes with investment horizon. Furthermore, few scholars in the time diversification literature consider whether asset allocation strategies – like target date strategies or dynamic strategies – affect the risk calculation. It is these gaps in the literature that we take up in the next two chapters.

6 A referendum on the accumulation model

6.1 Introduction

In our first empirical study, we re-prosecuted the time diversification debate within an initial endowment framework and concluded that the time diversification puzzle remains a puzzle because each scholar approaches the problem with a unique set of assumptions and measures. Thus, all researchers are right *on their own terms*. We also suggested that some resolution to the puzzle may lie in the risk measure's basis – whether the measure is return-based or wealth-based – but we deferred judgment until this study is completed because the consideration of alternative accumulation models may provide us additional evidence to consider.

In this study, we use a critique of the time diversification literature to motivate our consideration of two additional accumulation models: the constant contribution model, where terminal wealth is a function of initial endowment, returns and constant contributions; and, the constant percentage contribution model where terminal wealth is a function of the initial endowment, returns and contributions which grow as a result of salary growth. We replicate the same battery of measures for these two alternative accumulation models and compare the results with those that were calculated for the initial endowment model so central to the time diversification literature. Our nested methodological approach therefore allows us to conduct a difference referendum: a referendum on the accumulation model.

The critique of the time diversification debate that motivates this study related to the lack of consistency between what the literature assumes, and the realities of the institutional setting we outlined in Chapter Three. In particular, the literature largely ignores at least one variable – contributions – that significantly impacts terminal wealth. In Chapter Three we showed that contributions are a central feature of the retirement savings system which this thesis considers – the United States. The inconsistency between the literature and the institutional setting, and the dearth of studies that compare alternative accumulation models to the typical initial endowment approach, motivated us to extend our research design from the first empirical study to two alternative types of accumulation model. In this chapter, the accumulation model goes from being a constant, to being a variable. In doing so, we consider the results from a further 72 models as we show in Figure 6.1 (two additional accumulation models times our base set-up of 36 models).

Figure 6.1: Nested methodological approach

This figure shows, in Venn diagram form, the nested nature of the methodological approach pursued in this thesis. In this second study, the set-up is identical to Study 1 except that the accumulation model assumption is freed with two further models being added (for a total of three) (3 x 1 x 4 x 9 = 108 models).

By releasing the accumulation model as a variable, we set out to investigate whether the accumulation model has any influence on the relationship between risk and investment horizon consistent with our second research question:

RESEARCH QUESTION TWO

H_0: That alternative accumulation models have no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

In this second study, we extend the approach from Chapter Five to a further two accumulation models – the constant contribution model and the constant percentage contribution model – so we can contrast the results. We find that, with the exception of measures expressed in terms of terminal wealth, all measures are virtually identical giving us the evidence to critique the time diversification literature. By ignoring contributions, and measuring risk the way they have, scholars have been debating a phenomenon in a context devoid of reality. If the time diversification debate really is about long term investing, then it should be reframed to allow realistic accumulation models to be examined, after which conclusions about the relationship between risk and investment horizon can be reached.

Based on the analysis in this study, we assert that the time diversification debate cannot be resolved by considering risk and investment horizon in isolation. The accumulation model is a factor that must be considered before complete understanding of the relationship between risk and horizon can be resolved. In the final empirical study (Chapter Seven), we will see whether asset allocation is a similarly important factor.

We now commence the study proper by briefly reviewing the literature.

6.2 Literature review

Samuelson (1969) initiated the time diversification debate by observing how time horizon affected the optimal allocation to risky assets. Using an expected utility framework, he concluded that the allocation to risky assets was independent of time, and only a function risk tolerance. Samuelson’s (1969) conclusions were based on three assumptions: (1) the investor exhibits constant relative risk aversion, (2) returns follow a random walk, and (3) wealth is a function only of returns. Much of the subsequent research within the expected utility framework has considered variations to these assumptions (Kritzman, 1994; Milevsky, 1999; Gollier, 2002), and many of the competing streams of research use these assumptions as a critique of the framework itself (Fisher and Statman, 1999; Booth, 2004). We pursue a competing approach to that of Samuelson’s (1969) and used critiques of his first two assumptions – that investors exhibit constant relative risk aversion, and returns follow a random walk – as the basis for the first study in this thesis. In this second study, we use a critique of the last of Samuelson’s (1969) assumptions – that wealth is a function only of returns – and of the time diversification literature in general, to motivate an examination of the impact of contributions on the question at the root of the time diversification debate: Are risky assets more or less risky over longer horizons?

6.2.1 Contributions in the time diversification literature

When considered in the light of the institutional setting outlined earlier in this thesis – that is, DC investing involves regular cash contributions over the working life – it is astonishing that almost the entire time diversification literature takes place within an “initial endowment” framework. In the extant literature one of the following two approaches is generally taken. Firstly, many studies do not mention wealth at all and instead confine themselves to references to returns only (e.g. Lloyd and Haney, 1980; Lloyd and Modani, 1983; Leibowitz and Langetieg, 1989; Lee, 1990). Predictably this stream of the literature tends to focus on returns-based performance measures as the primary way for judging the presence of time diversification. The alternative approach is for the author(s) to assume some explicit level of initial wealth and apply a series of returns to generate some range of terminal wealth estimates (e.g. Marshall, 1994; Reichenstein and Dorsett, 1995; Thorley, 1995; Levy, 1996).

These studies generally consider wealth-based measures of performance to supplement, or as a substitute for, return-based measures (e.g. McEnally, 1985).

While very few studies in the time diversification literature address accumulation models beyond the initial endowment model, a number do at least acknowledge that other models exist. Bodie et al., (1992), for example, hint at more complete models when they state that: "At any time *t*, … current wealth *F*(t) is determined by *past saving and investment decisions* (p. 428)" (emphasis added). Kritzman and Rich (1998) acknowledge "three sources of wealth appreciation: investment returns, wages, and the fraction of wages saved (p. 70)." [Other studies that hint at contributions include Kritzman (1994) and Strangeland and Turtle (1999).[72]]

Some studies almost appear to cite the work of Bodie et al. (1992) as a way of avoiding the need to consider contributions in their accumulation model. According to Bodie et al. (1992) wealth is comprised of financial capital and human capital, and an individual can compensate for changes in financial capital by deploying human capital differently. Consumption, investment, and human capital allocation are thus a function of past decisions and return realisations. Kritzman and Rich (1998), for example, in dealing with this assumption of Samuelson (1969) appear to imply that human capital is relevant only in compensating for changes in financial wealth. There appears to be no recognition, or acceptance, on their part that individuals are *systematically* translating their human capital (the present value of future earnings) into financial capital via regular contributions to their retirement account as their human capital is realised in monetary terms through the receipt of salary income. As we highlight in the institutional setting of this thesis, regular contributions to retirement accounts are increasingly common in a number of jurisdictions around the world as defined contribution plans grow in popularity.[73]

6.2.2 Contributions considered

Of the entire time diversification literature, only a small number of relatively recent studies actually incorporate periodic cash inflows, or contributions, in any way.[74] Jagannathan and Kocherlakota (1996), using an expected utility approach similar to that of Samuelson (1969), determine the optimal risky asset weights over various time horizons for varying degrees of constant relative risk aversion

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[72] For example, Strangeland and Turtle (1999) state: "As in much of the time diversification literature, we do not consider the more general case of intertemporal consumption and income flows. This [Strangeland and Turtle's (1999)] framework is consistent with the notion that time diversification is typically posited as advice that is dependent solely on an investor's age and time until retirement, with little concern for future cash flows. (p. 12)."

[73] In some jurisdictions, contributions are not only regular they are required by law. For example, in Australia, all employers are obligated to contribute a minimum of nine per cent of gross salary to a retirement account. This percentage is set to rise to twelve per cent by July 2019.

[74] A number of studies analyse cash outflows (or withdrawals from wealth) as a way of studying the interplay between consumption and retirement investing (e.g. Samuelson, 1969; Merton, 1969; Merton and Samuelson, 1974).

for “a household that has $20,000 per year available for investment from its salary income (p. 16).” Jagannathan and Kocherlakota (1996) show that the median allocation to stocks for a median household with a risk aversion coefficient of five is a decreasing function of time horizon: allocations fall non-linearly from approximately 80 per cent at age 35 to around 40 per cent at age 65.

Mukherji (2008), using a block bootstrap technique to simulate returns, estimates optimal portfolios from six asset classes for a number of horizons with monthly investments, where the optimality criterion is the minimisation of downside relative risk relative to a range of different target returns. Apart from finding that optimal allocations to risky assets fall as investment horizon shortened, he confirmed that for the same horizon a higher target return may result in significantly different portfolio composition than a lower target return. Mukherji’s (2008) use of the block bootstrap technique, and his argument that it preserves the time series properties of returns, provided the motivation for our use of this technique throughout this thesis. Mukherji’s (2008) insights regarding return targets will be revisited later in this thesis when considering the target return for a dynamic lifecycle strategy.

Panyagometh (2011) follows a similar approach to that of Mukherji (2008) with some minor variations. Instead of downside deviation, Panyagometh (2011) computes the optimal asset allocation by minimising portfolio standard deviation. He also studies value-at-risk and relative value-at-risk as alternative measures of risk. Panyagometh’s (2011) study differs from Mukherji (2008) in that it considers different levels of post retirement spending needs, employs a different form of bootstrap simulation technique, and considers the problem from within a Thai institutional setting. Panyagometh (2011) finds that the optimal allocation to risk assets falls as investment horizon increases and/or the post-retirement spending needs decrease.

Whereas Mukherji (2008) and Panyagometh (2011) are firmly located in the time diversification literature, recent work by Pástor and Stambaugh (2012) in financial econometrics analyses the relationship between “predictive variance” and time horizon within a Bayesian empirical framework.[75] Pástor and Stambaugh (2012) make a distinction between predictive variance and true variance and find that the relationship between risk and time horizon may be even more complicated than the time diversification literature would have one believe. For example, they suggest that: “Investors might well infer from the data that the true variance is lower at long horizons while at the same time assessing the predictive variance to be higher at long horizons (p. 1).” But most relevant to this study is that Pástor and Stambaugh (2012) assume a constant savings rate because it “is consistent with the fact that the predominant use of target-date funds is in employer-sponsored retirement plans,

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[75] In this paper, the authors introduce the measure “predictive variance” and define it as the variance of the k-period return conditional on D_T, the data available at time T.

where the employer and employee contributions are both typically pre-determined fractions of income (p. 27).”

But perhaps the most relevant authority for this study is the work of Hickman et al. (2001). Hickman et al. (2001) use six different asset classes and compare performance over various investment horizons in order to determine which assets offer the dominant investment strategy under a variety of holding-period and risk aversion assumptions. The authors then turn to considering the relative performance of four types of lifecycle asset allocation strategy where $100 monthly contributions are made throughout the investment horizon. Hickman et al. (2001) find evidence supporting greater risk allocations for those investors with longer horizons, and a shift toward lower risk holdings as retirement approaches. Whilst our approach differs in certain important ways, the approach of Hickman et al. (2001) is probably the one with which we have most sympathy.

Where Hickman et al. (2001) study a regular contribution accumulation model in isolation, we will compare the results for such a model against the initial endowment accumulation model that is so popular in the time diversification literature, as well as a further model where contributions are a constant percentage of salary that grows at a constant rate with time. We will also consider a more comprehensive set of performance measures, as well as testing the robustness of all three accumulation models to different asset return processes by employing the same range of simulation techniques used in the first study of this thesis.

6.2.3 Our approach

Having now introduced the few studies that go beyond the initial endowment model, we turn to the purpose of this study. This study will take the same framework that we employed to synthesise the literature in Chapter Five of this thesis, and extend it to two further accumulation models: (1) a model where terminal wealth is a function of contributions in addition to returns and the initial endowment; and, (2) a model where terminal wealth is the function of contributions, salary growth, returns and initial endowment.

We also introduce a new basis upon which to assess and compare performance between the accumulation models. By introducing salary growth in the third and final accumulation model, we introduce a difficulty: terminal income will be higher for the third model as a result of salary growth when compared to the initial endowment and constant contributions models where income is constant throughout the investment horizon. If one accepts that terminal wealth expectations are somehow anchored to terminal income (as argued in this thesis), then we need a way of adjusting terminal wealth to account for differing levels of terminal income. This new basis for performance evaluation – the retirement wealth ratio (RWR) – will be discussed in greater detail later in the methodology section of this study.

By comparing the results of these two additional models to those of the first study set out to provide insights into the importance of contributions to terminal wealth, and consider whether the addition of contributions to the accumulation model has any implications for the relationship between risk and investment horizon.

At the end of this study we will present two findings: (1) the literature overlooks at least one variable – contributions – that significantly impacts terminal wealth; and, (2) the measures proposed by the literature as ways of measuring time diversification, with few exceptions, ignore the influence of contributions. This study will conclude that time diversification cannot be properly understood in the absence of realistic accumulation models.

6.3 Data

The data used in this study are the well-known, and commonly used, monthly stock and T-bills returns maintained by French (2012). The excess return on the market (R_m-R_f) maintained by French (2012) is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks, from the Center for Research into Security Prices (CRSP), minus the one-month Treasury bill rate, obtained from Ibbotson Associates. To arrive at nominal total stock returns, we add back the one-month Treasury bill rate.

We consider nominal total stock returns for the reasons outlined in Chapter Four (section 4.2.2).

6.4 Methodology

The methodology to be employed in this study follows that of the first study, with the addition of two further aspects: two new accumulation models; and, the introduction of the retirement wealth ratio as a new metric for evaluation.

We take stock and T-bill data and, using four separate simulation methods, we simulate 10,000 synthetic returns paths for each of nine different investment horizons. The four simulation methods have been chosen for several reasons. Firstly, in order to contrast our findings with those of the time diversification literature we must replicate methods used in the literature, namely a parametric Monte Carlo method and the non-parametric bootstrap simulation method of Efron (1979). Secondly, we make a contribution to the time diversification literature by adding newer, and arguably more defensible, non-parametric block bootstrap techniques. These block bootstrap techniques also fulfill another purpose in this study: they are recognised as being better able to capture the time series characteristics of financial returns (Pascual and Ruiz, 2002; Mukherji, 2008). The nine investment horizons to be considered are 1, 5, 10, 15, 20, 25, 30, 35 and 40 years.

6.4.1 New accumulation models

Earlier in this thesis, we considered and critiqued the initial endowment model, where terminal wealth is a function of only returns and the magnitude of the initial endowment. The first of the two additional aspects to the methodology employed in this study, is the consideration of two further accumulation models: the constant contribution model; and, the constant percentage contribution model. Each model is summarised in Table 6.1.

Table 6.1: **Accumulation models**

This table presents the differences between the accumulation models studied in this thesis. From left to right the table shows the name of the accumulation model and how each of the variables from equations [4.1]-[4.3] in Chapter Four vary between models. While the percentage rates are quoted in per annum terms, these annual rates are applied on a monthly basis in the modeling.

Accumulation model	**Contribution rate ( )**	**Salary growth rate ( )**
Initial endowment model	Zero	Zero
Constant contribution model	9% per annum	Zero
Constant percentage contribution model	9% per annum	3% per annum

Firstly, we will examine a constant contribution model where contributions are fixed at nine per cent of salary (credited monthly), and salary remains constant in nominal terms over the investment horizon. Secondly, we consider a constant percentage contribution model where contributions are again fixed at nine per cent of salary (credited monthly), but salary increases at a constant rate of three per cent per annum (applied on monthly basis). By examining the marginal impact of contributions and salary growth, we hope to provide insights into the relative importance of each variable. Furthermore, we hope to provide positive insights into the importance of contributions as a novel contribution to the time diversification literature. By incorporating multiple cash flows as we observe in DC investing, we also introduce the real world to the time diversification debate.

6.4.2 Earnings and account balance data

In this study, we use median weekly earnings data from the US Bureau of Labor Statistics (BLS) to provide benchmark income levels for US workers of ages that correspond to the nine investment horizons used in this study. Recall, that this income data was used in Chapter 4 (reported in Table 4.3), as the basis for identifying median account balances from the Employment Benefit Research Institute (EBRI) for use as initial wealth in the simulation of terminal wealth paths. For convenience we reproduce this data in Table 6.2, where row three reports the starting income for each investment horizon (row one) and the corresponding age (row two).

Table 6.2: Earnings and account balance data

This table presents earnings and related account balance data in order to approximate initial wealth (W_0) for various horizons. Row one shows the investment horizon. Row two shows the assumed investor age that corresponds to the investment horizon. Row three shows Bureau of Labour Statistics (BLS) (2009) median earnings data for the fourth quarter of 2008 (annualised, rounded). Row four shows raw Employment Benefit Research Institute (EBRI) (2009) median account balance data that corresponds to the annualised BLS earnings data in row three (Only includes 401(k) accounts. Previous employer accounts, and IRAs are excluded). Row five shows the EBRI data rounded to the nearest thousand dollars. The rounded data is used as initial wealth (W_0) in the analysis in this thesis. Row six shows data that was sourced to validate the account balance data shown in rows four (in raw form) and five (in rounded form). The data was obtained from the US Census Bureau (2012) and represents the median value of retirement accounts by age (including IRAs, Keogh accounts, 401(k), 403(b)). Note that there are two major differences between the data in rows five and six: (1) row six data is more recent by around two years allowing the sampled population to accumulate more assets; and, (2) row six data includes a more complete variety of account types. These two differences would lead us to expect the row six data to be greater, an expectation that is born out in the numbers. Given these reconcilable differences, we suggest that the US Census bureau data provides a reasonable cross check for the EBRI data. Investment horizon and assumed age are expressed in years. All other data are expressed in dollars.

Investment horizon (years)	**40**	**35**	**30**	**25**	**20**	**15**	**10**	**5**	**1**
Assumed age	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**64**
Median earnings data	25,000	35,000	39,000	42,000	42,000	43,000	43,000	43,000	33,000
Raw median account bal.	4,757	10,108	15,458	34,176	52,893	62,242	71,591	72,713	73,834
Median account balance	5,000	10,000	15,000	34,000	53,000	62,000	72,000	73,000	74,000
Validating account bal.	N/A	10,000	23,000	36,000	51,500	67,000	82,500	98,000	77,000

To reiterate a point made earlier, in this thesis we focus on median earnings data, and the account balances corresponding to median wage earners, as the most relevant for a study of pension finance because it these individuals that are most likely to have to rely on their retirement accounts to fund their consumption in retirement. Lower income earners, for example, are more likely to be on some sort of social security or supplementary assistance and high income earners will likely be able to rely on assets over and above their retirement accounts. In using these income levels, we seek to employ reasonable, representative income levels as the basis for calculating contributions, and as reasonable starting points to which we apply salary growth.

Throughout this thesis, we assume that the income levels that correspond to the nine investment horizons (row three, Table 6.2) grow at a constant rate of three per cent per annum for the length of the investment horizon, applied on the same monthly basis as contributions are calculated and added to wealth at time *t*. As for the problem of determining an appropriate income level for a given age, arriving at a single representative "average" level of salary growth is problematic for a number of reasons. Firstly, income growth over a working life is rarely uniform or constant in nature. Income profiles are affected by both macroeconomic trends, and by factors idiosyncratic to the individual like gender, occupation, education level, employer and industry. Scholars have shown that a typical income profile is "humped" in nature, rising to its zenith in the early-to-mid fifties after which it falls as the individual transitions from full time work to semi-retirement (Byrne et al., 2006). Whilst we concede that our assumption could be subject to criticism, we take consolation from the fact that the purpose of this thesis does not hinge on the accuracy of the salary growth assumption (if such an assumption exists). Rather, we investigate an accumulation model incorporating salary growth (the constant percentage contribution model) to understand the marginal impact of salary growth when compared to the constant contribution and initial endowment models.

In summary, the three accumulation models evolve as follows: the initial endowment model begins with initial wealth and is affected only by returns; the constant contribution model sees terminal wealth as a function of initial wealth, returns, and constant nominal dollar contributions; and, the constant percentage contribution model generates terminal wealth from the interplay of initial wealth, returns and contributions that rise with income growth. These accumulation models are summarised above in Table 6.1.

6.4.3 Evaluating outcomes using the retirement wealth ratio

The second additional methodological aspect in this study, is the introduction of a further basis upon which to judge terminal wealth outcomes. The challenge with return- or dollar-based terminal wealth measures of performance is that neither is particularly informative for the investor in terms of what performance means to their spending power in retirement. Baker, Logue, and Rader (2005), for example, argue that defined contribution plans should be measured in terms of their ability to generate sufficient retirement income. What we need therefore is a measure that sizes terminal wealth against some relevant benchmark (Booth, 1997; Clarkson, 1989; Booth and Yakoubov, 2000).

One such measure is the retirement wealth ratio (RWR_T) of Basu and Drew (2010), which is calculated by dividing terminal wealth (W_T) by gross income at time T.[76] Basu and Drew's (2010) rationale for the introduction of the retirement wealth ratio into the pension finance literature was because "it is very likely that the participant's post-retirement income expectations are closely linked to their immediate income before retirement (p. 292)." For example, terminal wealth of one million dollars will appear much more attractive to an individual whose final salary is $50,000 per annum when compared to another individual whose final salary in $200,000 per annum. If we were to judge each scenario based on terminal wealth alone, performance would be equivalent with each individual retiring with one million dollars. Expressed in retirement wealth ratio terms, the worker on the lower income would retire with a RWR_T of 20 times ($1,000,000 divided by $50,000) versus an RWR_T of five times ($1,000,000 divided by $200,000) for the individual on the higher income.

The RWR_T therefore allows us to compare accumulation models where incomes at time T are not equivalent. In this study, we are able to compare terminal wealth for the initial endowment and constant contribution models because in each case salary is constant in nominal dollar terms over the investment horizon. By introducing the constant percentage contributions model, where contributions rise due to the effect of salary growth, we have a model with a different and higher final income. We therefore evaluate performance in RWR_T terms so as to avoid over-estimating the performance of the constant percentage contribution model because we have ignored the higher final salary, and hence higher post-retirement income expectations.

6.5 Empirical evidence

For second and third accumulation models – constant contributions and constant percentage contribution – we estimate Monte Carlo, Efron (1979) bootstrap, stationary bootstrap, and empirical block bootstrap models for each of the nine horizons (1, 5, 10, 15, 20, 25, 30, 35, 40 years), for a total of 72 models. For ease of comparison, we repeat the results for the 36 models (four techniques multiplied by nine investment horizons) estimated for the initial endowment accumulation model in Chapter Five of this thesis. Consequently, in the below tables we report 108 versions of the same measure in order to compare the results on three dimensions: (1) the accumulation model, which looks at different sets of determinants of terminal wealth; (2) the modeling technique, which allows for different conceptions of the asset return process (e.g. random walk); and, (3) investment horizon, which is the main subject of the time diversification literature.

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[76] A similar measure – the "expected accumulation" – is used by Booth and Yakoubov (2000).

6.5.1 Extending the analysis to include contributions and salary growth

In this section we extend our analysis from the first study of this thesis to two further accumulation models using our nested approach to methodology. By doing this we set out to compare three different accumulation models *on the same basis*, in order to substantiate a critique of the time diversification literature, and to make a contribution to the debate.

6.5.1.1 Moments of annualised returns

We report the moments of annualised returns in Table 6.3, not because they should differ between accumulation models, but because we wish to show that they don't differ materially. For all modeling techniques, and for all accumulations models, we simulate 10,000 synthetic return paths of a length equal to the investment horizon times twelve months (e.g. 480 for a horizon of 40 years) *from a common dataset*. Therefore, for all 108 models each of the four distributional moments should be approximately the same, with some tolerance for the effect of the modeling technique and for differences in horizon. For example, because the Monte Carlo simulation is a parametric method that assumes normality, we expect to see estimates of skewness and kurtosis of approximately zero and three respectively for all Monte Carlo models. We find this in the estimates reported in Table 6.3. Similarly, we would expect to see a combination of negative skewness and leptokurtosis for those methods most capable of modeling the statistical features of the data. For both the stationary bootstrap and the empirical block bootstrap, we see both these statistical features reproduced faithfully for each accumulation model and investment horizon with minimal variation. To reiterate, the results in Table 6.3 do not provide any insights regarding the question at hand. Rather, the results show that in the process of conducting the simulations, we replicate the properties of the data, and remove the modeling process as a source of variation when we compare the results of the three accumulation model later in this study.

Table 6.3: Annualised moments of returns

This table presents the four moments of annualised returns for the four modeling techniques for each of the nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). The accumulation models are: initial endowment, constant contributions, and constant percentage contributions. For ease of comparison the results are grouped by measure: Panel A reports mean, Panel B reports standard deviations, Panel C reports skewness and Panel D reports kurtosis. Estimates for mean and standard deviation are expressed in percentage terms, and estimates of skewness and kurtosis are pure numbers (i.e. have no units of measurement).

PANEL A – Mean

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contribuions**			
	mc (%)	**b (%)**	**bs (%)**	**bb (%)**	**mc (%)**	**b (%)**	**bs (%)**	**bb (%)**	**mc (%)**	**b (%)**	**bs (%)**	**bb (%)**
40	10.87	10.92	10.89	10.90	10.89	10.94	10.94	10.93	10.93	10.89	10.93	10.89
35	10.90	10.90	10.85	10.88	10.92	10.91	10.91	10.89	10.86	10.89	10.88	10.89
30	10.85	10.95	10.90	10.94	10.96	10.87	10.95	10.91	10.92	10.90	10.94	10.93
25	10.92	10.87	10.86	10.89	10.86	10.98	10.91	10.94	10.96	10.93	10.85	10.90
20	10.86	10.91	10.89	10.83	10.87	10.88	10.87	10.88	10.93	10.85	10.93	10.88
15	10.83	10.86	10.85	10.90	10.87	10.80	10.92	10.90	10.86	10.92	10.90	10.81
10	10.80	10.81	10.86	10.78	10.77	10.99	10.77	10.81	10.86	10.91	10.84	10.89
5	10.80	10.81	10.86	10.77	10.75	10.72	10.67	10.76	10.63	10.78	10.85	10.76
1	10.15	10.21	10.16	10.44	10.16	10.07	9.92	10.17	10.35	10.06	9.91	10.32

PANEL B – Standard deviation

Investment horizon	Initial endowment				Constant contributions				Constant percentage contribuions			
	mc (%)	b (%)	bs (%)	bb (%)	mc (%)	b (%)	bs (%)	bb (%)	mc (%)	b (%)	bs (%)	bb (%)
40	18.82	18.78	18.53	18.55	18.82	18.78	18.48	18.54	18.84	18.77	18.51	18.53
35	18.81	18.77	18.49	18.53	18.83	18.76	18.43	18.48	18.81	18.77	18.51	18.46
30	18.81	18.79	18.40	18.44	18.79	18.72	18.36	18.39	18.81	18.76	18.41	18.43
25	18.79	18.72	18.38	18.39	18.78	18.75	18.25	18.36	18.79	18.74	18.39	18.41
20	18.76	18.67	18.26	18.25	18.76	18.70	18.20	18.29	18.77	18.70	18.25	18.37
15	18.75	18.65	18.14	18.08	18.76	18.65	17.97	18.16	18.73	18.65	18.06	18.13
10	18.65	18.52	17.72	17.86	18.68	18.57	17.76	17.95	18.67	18.58	17.76	17.88
5	18.47	18.21	17.19	17.22	18.49	18.27	17.13	17.27	18.47	18.24	17.05	17.23
1	16.98	16.21	15.04	14.95	17.01	16.29	15.16	15.04	17.05	16.18	15.01	15.11

PANEL C – Skewness

Investment horizon	Initial endowment				Constant contributions				Constant percentage contribuions			
	mc	b	bs	bb	mc	b	bs	bb	mc	b	bs	bb
40	0.00	0.06	-0.11	-0.08	0.00	0.07	-0.11	-0.08	0.00	0.07	-0.12	-0.09
35	0.00	0.06	-0.13	-0.11	0.00	0.06	-0.14	-0.12	0.00	0.07	-0.13	-0.13
30	0.00	0.07	-0.17	-0.14	0.00	0.03	-0.17	-0.16	0.00	0.06	-0.16	-0.14
25	0.00	0.04	-0.19	-0.17	0.00	0.04	-0.21	-0.16	0.00	0.05	-0.19	-0.17
20	0.00	0.02	-0.23	-0.21	0.00	0.01	-0.24	-0.20	0.00	0.03	-0.23	-0.20
15	0.01	-0.01	-0.28	-0.26	0.00	-0.01	-0.29	-0.27	0.00	0.00	-0.28	-0.26
10	0.00	-0.06	-0.34	-0.32	0.00	-0.03	-0.33	-0.30	0.01	-0.06	-0.33	-0.30
5	0.01	-0.16	-0.32	-0.33	0.01	-0.17	-0.35	-0.32	0.01	-0.18	-0.33	-0.33
1	0.05	-0.22	-0.15	-0.14	0.05	-0.23	-0.14	-0.15	0.04	-0.22	-0.15	-0.13

PANEL D – Kurtosis

Investment horizon	Initial endowment				Constant contributions				Constant percentage contribuions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	3.01	9.80	7.98	8.30	3.01	9.87	7.98	8.30	3.01	9.81	7.98	8.30
35	3.01	9.76	7.72	8.02	3.01	9.75	7.73	8.03	3.00	9.79	7.76	8.04
30	3.01	9.73	7.44	7.74	3.01	9.57	7.42	7.76	3.01	9.74	7.42	7.78
25	3.01	9.53	7.11	7.44	3.01	9.59	7.07	7.44	3.01	9.58	7.10	7.45
20	3.01	9.33	6.73	7.05	3.02	9.30	6.65	7.05	3.02	9.33	6.70	7.09
15	3.02	9.04	6.21	6.47	3.02	9.06	6.13	6.49	3.01	9.07	6.14	6.49
10	3.02	8.50	5.49	5.71	3.02	8.56	5.48	5.72	3.02	8.53	5.53	5.74
5	3.04	7.19	4.57	4.66	3.04	7.22	4.55	4.63	3.06	7.24	4.53	4.65
1	3.20	4.39	3.47	3.42	3.19	4.39	3.48	3.44	3.19	4.36	3.48	3.42

As stated in the first study of this thesis, one of the more durable findings in the time diversification literature is that the standard deviation of annualised returns falls as investment horizon lengthens (Bernstein, 1976; Garrone and Solnik, 1976; Lloyd and Haney, 1980; Lloyd and Modani, 1983; McEnally, 1985; Lee, 1990; Kritzman and Rich, 1998; Kochman and Goodwin, 2002; Guo and Darnell, 2005). Table 6.4 shows the standard deviation of annualised returns for each of the four modeling methods across each accumulation model. For each combination of accumulation model and simulation method, we observe a monotonic fall in the standard deviation of annualised returns. We thus report a similar pattern to that observed in the literature, irrespective of the accumulation model we consider.

Table 6.4: Standard deviation of annualised returns

This table presents the standard deviation of annualised for the four modeling techniques for each of the nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). The accumulation models are: initial endowment, constant contributions, and constant percentage contributions. Estimates are expressed in percentage terms.

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contribuions**			
	mc (%)	**b (%)**	**bs (%)**	**bb (%)**	**mc (%)**	**b (%)**	**bs (%)**	**bb (%)**	**mc (%)**	**b (%)**	**bs (%)**	**bb (%)**
40	2.98	2.96	2.52	2.98	2.96	2.97	2.52	2.98	3.01	2.97	2.52	2.97
35	3.15	3.15	2.72	3.21	3.21	3.17	2.68	3.15	3.21	3.17	2.68	3.20
30	3.45	3.43	2.94	3.40	3.44	3.42	2.87	3.43	3.44	3.46	2.93	3.40
25	3.76	3.80	3.21	3.75	3.74	3.76	3.16	3.78	3.76	3.78	3.17	3.74
20	4.20	4.16	3.56	4.19	4.22	4.21	3.58	4.19	4.22	4.18	3.61	4.12
15	4.87	4.88	4.19	4.90	4.84	4.85	4.10	4.75	4.83	4.89	4.14	4.86
10	5.87	5.81	5.15	5.99	5.89	5.97	5.18	5.99	5.91	6.01	5.22	5.93
5	8.28	8.25	7.72	8.36	8.39	8.42	7.75	8.32	8.35	8.29	7.68	8.17
1	17.60	17.37	18.49	18.93	17.65	17.55	18.65	18.78	17.32	17.17	18.59	18.84

6.5.1.2 Cumulative returns

Another way of observing the variation due to returns between accumulation models is by considering cumulative returns. Cumulative returns are the values to which $1 would grow over the horizon in question, so any variation between models is only the result of difference in the simulated returns. Table 6.5 reports the mean and standard deviation of cumulative returns.

Table 6.5: Mean and standard deviation of cumulative returns

This table presents the mean and standard deviation of cumulative returns for the four modeling techniques for each of the nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). The accumulation models are: initial endowment, constant contributions, and constant percentage contributions. For ease of comparison the results are grouped by measure: Panel A reports means and Panel B reports standard deviations. Results are expressed in dollars.

PANEL A – Mean

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contribuions**			
	mc ($)	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	77.64	77.37	66.85	76.93	76.32	78.86	68.27	78.61	78.95	77.18	67.96	76.93
35	44.39	44.28	39.50	44.94	45.62	44.88	39.97	44.50	45.12	44.54	38.93	45.43
30	26.04	26.33	23.64	26.13	26.66	25.77	23.67	26.25	26.28	26.69	24.01	26.21
25	15.26	15.25	13.82	15.13	15.01	15.44	13.90	15.46	15.56	15.40	13.70	15.08
20	8.76	8.78	8.23	8.72	8.84	8.82	8.23	8.78	8.85	8.70	8.32	8.69
15	5.09	5.14	4.87	5.18	5.11	5.06	4.89	5.08	5.11	5.19	4.89	5.06
10	2.95	2.94	2.89	2.96	2.95	3.02	2.87	2.97	2.97	3.01	2.89	2.99
5	1.73	1.73	1.72	1.73	1.73	1.72	1.71	1.72	1.71	1.72	1.72	1.72
1	1.12	1.12	1.12	1.12	1.12	1.11	1.12	1.12	1.12	1.11	1.12	1.12

PANEL B – Standard deviation

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contribuions**			
	mc ($)	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	134.73	133.18	81.14	108.79	126.42	151.92	82.42	112.74	136.17	133.62	83.25	106.93
35	67.77	66.05	44.72	57.63	70.45	65.20	45.11	56.19	73.06	67.83	42.44	58.53
30	35.66	34.09	23.43	28.82	34.72	33.60	23.07	29.76	36.05	39.99	24.18	29.94
25	18.14	17.73	12.22	14.76	17.59	17.78	12.14	15.66	18.84	17.96	12.16	14.53
20	8.72	8.50	6.45	7.54	9.03	8.96	6.43	7.42	8.48	8.63	6.46	7.39
15	4.22	4.47	3.22	3.73	4.26	4.17	3.14	3.53	4.26	4.62	3.18	3.57
10	1.89	1.87	1.52	1.69	1.92	1.95	1.50	1.71	1.92	2.00	1.53	1.69
5	0.75	0.75	0.64	0.66	0.76	0.77	0.64	0.66	0.75	0.74	0.64	0.65
1	0.21	0.21	0.21	0.22	0.21	0.21	0.22	0.22	0.21	0.21	0.21	0.22

The estimated means and standard deviations of cumulative returns reported in Table 6.5 conform to our expectations. In general, estimates are consistent across the three accumulations models, except those for the stationary bootstrap (bs) model which are noticeably lower than those of the other three techniques, especially at longer investment horizons. A proportion of the differences between the four model types is due to the compounding of small, but expected, variations in returns resulting from different simulation processes. Note, for example, that the variation between all models increases as time horizon lengthens.

But the magnitudes of the differences between the stationary bootstrap model and the other model types suggest that other effects are at play. The principal reason for selecting the stationary bootstrap and the empirical block bootstrap as methods to be studied in this thesis is because we believe, and the literature agrees, that they are each more likely than either Monte Carlo or Efron (1979) bootstrap methods to capture the moments of the return series, as well as their time series characteristics. These *a priori* expectations regarding the performance of the stationary and empirical block bootstrap models are to some extent born out in the results we report in Table 6.3 with both models methods producing negative skewness and leptokurtosis.

The question however remains: What is causing the significantly different results between the stationary bootstrap model and the other three types estimates? We believe there are two possible answers. Firstly, it is likely that the estimated kurtosis for the stationary bootstrap is more left tailed in nature, and that the effect of this heavier left tail is compounded as horizon lengthens giving lower cumulative returns. Other evidence – like having the most negative skewness of all four methods (Panel C, Table 6.3) – supports this argument. Secondly, because of their random walk assumption, the Monte Carlo and Efron (1979) bootstrap techniques are capable of producing long series of positive returns which may cause the right tails of their simulated distributions to be heavier than that of the stationary or empirical block bootstrap techniques. In the absence of these strongly positive returns paths, and the associated compounding effect, cumulative returns, and the standard deviation of cumulative returns, will necessarily be lower for the stationary and empirical block bootstrap.

6.5.1.3 Terminal wealth

Until now we have considered performance measures which look at the relationship between investment horizon and returns in isolation. Firstly, in Tables 6.3 and 6.4 we looked at return-based measures, and secondly in Table 6.5 we looked at how cumulative returns evolve from a common one dollar initial wealth. We observed patterns consistent with those found in Chapter Five, regardless of which accumulation model we consider.

We will now consider how returns interact with the realistic initial wealth (W_0) levels shown in Table 6.2 and compare the results across accumulation models first in dollar terms, and later in retirement wealth ratio terms. Recall the three accumulation models: (1) the initial endowment model begins with initial wealth and grows only with returns; (2) the constant contribution model begins with initial wealth and evolves as a result of the interaction between returns and monthly contributions at the rate of nine per cent per annum; and, (3) the constant percentage contribution model adds a further variable to returns and contributions which is that of salary growth. Thus the difference between the constant contribution model and the constant percentage contribution model is that the former is a constant percentage contribution from a constant salary whereas the latter is a constant percentage contribution from a salary which grows monthly at a rate of three per cent per annum.

Table 6.6 reports measures of central tendency and dispersion expressed in terminal wealth terms, as well as the distribution of terminal wealth.

Table 6.6: Distribution of terminal wealth

This table presents the distribution of terminal wealth for the four modeling techniques over nine investment horizons for the three accumulation models. Initial wealth (W_0) is as per row five of Table 6.1. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. Performance measures are group by measure type with measures of central tendency reported in Panel A (mean terminal wealth) and Panel B (median, or 50th percentile terminal wealth). Measures of dispersion are reported in Panels C (standard deviation of terminal wealth) and D (terminal wealth range). Panels E through G summarise the lower half of the distribution of terminal wealth (minimum terminal wealth, 5th percentile terminal wealth and 25th percentile terminal wealth respectively). Panels H through J summarise the upper half of the distribution of terminal wealth (75th percentile terminal wealth, 95th percentile terminal wealth and maximum terminal wealth respectively). Results are presented in thousands of dollars.

PANEL A – Mean terminal wealth

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	385.57	382.96	331.60	381.55	1,940.89	2,003.28	1,784.04	1,992.89	2,534.31	2,499.37	2,300.79	2,493.83
35	440.32	439.12	391.32	445.73	1,741.68	1,733.40	1,573.94	1,722.18	2,141.86	2,121.24	1,936.52	2,160.48
30	386.84	391.64	351.54	388.83	1,217.70	1,190.13	1,117.64	1,210.18	1,453.04	1,466.98	1,360.60	1,452.86
25	513.47	514.25	465.94	509.63	997.55	1,022.83	938.80	1,023.73	1,160.99	1,155.29	1,054.04	1,133.06
20	460.22	460.84	431.81	458.19	737.74	737.96	695.97	733.63	802.62	792.45	764.60	791.87
15	312.86	315.96	299.49	318.04	461.93	457.38	444.55	459.45	488.73	494.97	472.43	486.09
10	210.20	210.18	206.42	211.66	280.77	286.93	274.30	282.96	292.32	295.65	285.93	293.84
5	125.06	124.99	124.38	124.90	150.98	150.75	149.57	150.98	151.79	152.81	152.62	152.56
1	81.80	81.91	82.03	82.35	85.21	85.16	85.19	85.45	85.37	85.18	85.30	85.58

PANEL B – Median (50th percentile) terminal wealth

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc ($)	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	183.85	191.45	202.62	211.05	1,169.91	1,197.53	1,251.13	1,285.09	1,595.48	1,575.48	1,694.23	1,704.72
35	238.38	244.25	250.64	263.62	1,119.10	1,120.59	1,161.35	1,202.82	1,419.76	1,422.37	1,488.07	1,568.16
30	224.61	235.06	244.24	255.04	829.22	822.54	880.51	901.02	1,030.47	1,029.72	1,084.43	1,123.29
25	334.86	328.77	350.29	361.61	713.03	740.29	760.40	786.10	841.66	843.34	868.27	906.58
20	326.17	331.54	345.55	352.22	550.44	552.46	586.48	601.79	624.63	608.42	646.98	653.46
15	242.26	241.77	254.09	267.58	373.50	373.16	394.98	404.09	395.84	399.70	420.47	428.12
10	176.96	177.43	188.23	193.86	242.21	246.77	255.11	259.15	253.41	254.83	266.49	273.31
5	114.70	115.24	122.39	122.89	139.64	139.64	146.66	149.19	141.64	142.63	149.33	150.02
1	80.50	81.06	83.27	83.52	84.24	84.27	86.64	86.76	84.33	84.25	86.81	86.81

PANEL C – Standard deviation of terminal wealth

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc ($)	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	678.34	656.99	401.65	538.56	2,491.82	2,920.93	1,729.55	2,253.57	3,116.45	3,210.65	2,106.80	2,574.04
35	670.02	651.49	441.55	570.42	2,122.76	2,013.02	1,413.94	1,754.30	2,493.56	2,373.89	1,587.78	2,080.87
30	525.30	507.24	348.01	428.13	1,268.63	1,239.10	882.06	1,101.93	1,512.96	1,569.10	1,020.83	1,240.43
25	603.51	598.81	411.64	497.24	977.40	987.78	687.66	871.02	1,101.40	1,066.64	740.05	866.43
20	456.00	445.28	337.35	394.44	647.67	650.27	468.92	534.78	643.47	651.96	492.23	558.69
15	257.75	273.77	197.49	228.32	332.55	327.40	248.66	281.01	343.84	369.24	260.52	292.75
10	133.51	133.24	107.87	120.39	162.54	164.66	127.38	145.59	165.27	172.29	132.12	146.81
5	54.19	53.98	46.04	47.24	60.84	61.38	51.45	53.08	60.10	59.71	51.96	52.76
1	14.89	14.67	14.99	15.17	15.18	15.18	15.44	15.59	14.97	14.86	15.29	15.46

PANEL D – Terminal wealth range

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	16,508.53	17,604.27	5,074.34	12,115.05	58,514.94	113,083.28	17,984.94	37,395.37	60,837.23	97,517.20	31,322.49	37,622.73
35	20,655.57	16,113.09	6,528.95	10,112.98	46,969.86	34,965.61	25,004.79	25,885.51	54,228.63	53,121.45	22,918.17	30,332.42
30	9,213.28	10,003.73	3,910.91	5,151.72	16,494.23	29,003.89	9,818.57	15,248.28	44,873.38	50,571.85	10,584.77	19,820.38
25	14,174.63	7,746.01	5,121.88	5,359.70	22,413.79	21,217.72	7,563.23	11,576.91	22,166.41	15,897.77	8,701.30	9,640.34
20	5,926.59	8,580.06	3,824.32	4,965.63	8,767.03	14,826.23	4,574.81	4,573.65	7,356.80	10,164.55	5,344.59	7,397.77
15	5,410.81	6,234.49	2,617.68	2,173.98	4,949.02	5,036.05	2,621.46	2,387.51	4,698.53	11,630.51	2,444.07	2,794.37
10	1,800.49	2,055.81	975.11	1,208.56	2,000.53	1,667.37	1,056.04	1,619.32	2,098.95	2,258.11	1,040.21	1,261.13
5	499.70	551.94	370.76	358.33	541.24	865.74	512.13	368.28	586.41	538.93	465.00	429.25
1	124.63	143.91	141.84	141.84	113.00	161.11	161.25	145.09	125.12	154.19	145.13	145.13

PANEL E – Minimum terminal wealth

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	2.95	2.02	0.27	0.06	50.68	44.99	18.29	20.93	105.34	107.57	58.98	53.26
35	3.60	2.83	1.36	0.63	55.22	63.66	37.07	24.53	103.01	130.48	68.40	44.33
30	6.09	2.83	2.17	0.80	44.16	57.08	36.21	19.06	94.71	92.61	49.84	48.14
25	10.72	9.86	6.63	2.55	51.51	44.66	38.36	15.15	88.95	94.14	47.95	40.29
20	14.65	12.81	8.60	2.19	38.23	45.79	31.99	18.49	75.15	28.52	43.42	23.14
15	11.90	11.33	7.63	1.48	39.05	41.90	20.46	10.50	45.65	43.14	29.52	20.03
10	24.62	16.91	5.73	7.07	44.63	34.79	18.64	13.01	46.44	43.45	33.47	12.77
5	19.91	25.13	6.31	4.10	33.01	35.01	15.83	9.80	29.63	23.37	14.68	9.88
1	36.33	35.56	25.41	25.41	40.55	36.65	27.19	27.19	40.32	40.21	27.22	27.22

PANEL F – 5th percentile terminal wealth

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	26.97	26.91	26.53	19.38	267.78	260.75	272.58	214.94	393.09	399.40	412.43	343.76
35	38.59	39.32	37.17	26.84	266.55	271.71	277.97	229.23	391.17	387.10	408.33	336.07
30	43.18	41.40	41.90	33.35	232.47	222.46	233.32	189.69	311.11	307.66	308.37	277.43
25	69.67	70.01	67.62	53.65	211.64	210.59	217.25	182.98	278.64	267.71	263.16	234.00
20	81.85	81.39	82.56	64.63	177.37	175.58	180.37	153.02	209.15	208.20	211.85	187.41
15	70.57	72.87	68.96	57.38	136.25	134.76	138.68	120.86	154.84	152.79	152.96	130.83
10	66.91	67.25	63.43	53.37	105.95	105.10	100.35	89.80	109.86	110.30	106.69	95.53
5	57.84	57.68	51.52	49.33	74.85	73.51	68.98	65.50	75.88	75.82	72.98	69.24
1	59.44	59.45	57.45	57.36	62.20	61.96	59.75	60.05	63.08	62.45	59.79	59.79

PANEL G – 25th percentile terminal wealth

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	84.92	86.57	91.77	85.05	616.15	629.42	684.29	640.71	884.08	859.83	949.52	905.77
35	115.58	115.27	122.02	115.51	612.96	613.17	659.24	630.22	809.59	827.54	888.10	860.54
30	112.33	116.88	122.95	117.78	475.35	475.11	516.86	500.19	612.08	614.13	658.04	644.54
25	177.08	172.02	186.15	179.31	423.13	437.64	458.83	453.95	516.82	518.30	542.77	535.40
20	183.95	186.60	199.08	189.91	342.01	347.21	368.54	362.72	392.31	389.85	414.08	405.01
15	146.64	147.08	156.94	156.51	244.76	241.00	263.91	260.03	264.88	267.65	283.00	277.01
10	119.69	120.45	127.64	125.78	171.03	173.01	180.12	178.85	181.24	179.99	189.18	187.50
5	86.95	87.69	91.55	92.58	107.86	107.07	113.08	114.83	108.56	110.49	116.14	116.47
1	71.35	71.93	72.80	73.29	74.73	75.03	75.42	75.74	74.77	75.34	75.54	76.38

PANEL H – 75th percentile terminal wealth

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc ($)	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	414.03	424.12	415.48	472.00	2,280.90	2,330.69	2,256.30	2,515.73	2,987.56	2,958.56	2,922.59	3,156.09
35	511.13	510.96	493.91	551.71	2,059.70	2,060.26	2,037.77	2,176.63	2,551.02	2,551.46	2,480.06	2,727.85
30	452.41	464.35	456.99	508.83	1,481.26	1,438.73	1,453.37	1,552.22	1,778.19	1,750.13	1,752.50	1,853.81
25	630.36	620.90	617.92	663.71	1,218.68	1,252.41	1,214.54	1,315.65	1,389.07	1,422.10	1,351.98	1,472.20
20	570.77	574.77	566.41	607.74	906.64	914.37	891.05	959.20	997.87	977.88	993.86	1,023.59
15	390.54	395.23	395.55	418.24	575.46	570.16	570.59	590.60	606.63	612.20	601.35	629.10
10	263.06	262.56	267.20	276.46	345.08	359.00	351.01	363.12	360.34	362.76	361.88	374.45
5	151.43	152.26	152.85	155.05	181.46	181.46	181.87	184.51	181.75	183.75	183.91	184.52
1	90.94	90.83	91.11	91.55	94.52	94.10	94.52	94.96	94.82	94.05	94.81	95.01

PANEL I – 95th percentile terminal wealth

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc ($)	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	1,401.84	1,330.25	1,066.88	1,279.57	6,174.53	6,240.48	5,201.72	6,094.95	7,613.23	7,619.62	6,269.83	7,145.31
35	1,491.76	1,442.99	1,216.17	1,472.72	5,214.54	5,207.53	4,178.77	5,016.62	6,062.82	6,002.48	4,955.39	6,045.65
30	1,257.29	1,242.67	1,031.52	1,181.23	3,545.07	3,382.69	2,794.38	3,257.86	3,925.80	4,045.02	3,338.04	3,735.03
25	1,530.15	1,566.08	1,261.83	1,495.54	2,747.30	2,781.86	2,255.95	2,667.92	3,126.32	3,046.09	2,465.11	2,824.33
20	1,296.00	1,285.11	1,083.36	1,220.41	1,932.91	1,895.18	1,609.85	1,771.12	1,992.19	1,960.49	1,701.95	1,862.15
15	797.04	800.05	675.99	755.87	1,081.91	1,060.08	916.40	986.95	1,122.19	1,130.30	972.28	1,034.26
10	464.50	455.14	406.13	435.29	585.73	602.26	505.46	550.86	600.63	610.17	523.61	565.15
5	228.28	224.38	200.22	206.26	265.67	265.71	236.20	242.11	264.85	265.30	240.49	245.07
1	108.36	107.35	103.92	104.29	111.74	111.18	107.62	107.84	111.73	111.01	107.67	108.06

PANEL J – Maximum terminal wealth

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc ($)	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	16,511.48	17,606.29	5,074.61	12,115.10	58,565.61	113,128.26	18,003.22	37,416.30	60,942.57	97,624.77	31,381.47	37,675.99
35	20,659.17	16,115.92	6,530.32	10,113.61	47,025.08	35,029.27	25,041.86	25,910.04	54,331.65	53,251.94	22,986.57	30,376.75
30	9,219.38	10,006.57	3,913.08	5,152.52	16,538.39	29,060.96	9,854.78	15,267.34	44,968.08	50,664.46	10,634.61	19,868.53
25	14,185.35	7,755.87	5,128.51	5,362.25	22,465.30	21,262.38	7,601.59	11,592.06	22,255.37	15,991.91	8,749.26	9,680.63
20	5,941.24	8,592.88	3,832.92	4,967.83	8,805.26	14,872.02	4,606.80	4,592.15	7,431.95	10,193.07	5,388.02	7,420.92
15	5,422.72	6,245.83	2,625.30	2,175.46	4,988.07	5,077.95	2,641.92	2,398.01	4,744.18	11,673.64	2,473.60	2,814.39
10	1,825.11	2,072.72	980.83	1,215.63	2,045.16	1,702.16	1,074.68	1,632.33	2,145.39	2,301.55	1,073.69	1,273.90
5	519.62	577.07	377.08	362.43	574.25	900.76	527.95	378.08	616.04	562.29	479.68	439.12
1	160.96	179.47	167.25	167.25	153.55	197.76	188.44	172.28	165.44	194.40	172.35	172.35

The most striking (and perhaps unsurprising) finding from the results reported in Table 6.6 is the material impact that contributions have on all reported measures of terminal wealth, be they measures of central tendency, measures of dispersion, or distribution percentiles. Over a full investing life of forty years, contributing at the rate of nine per cent per annum can yield the investor median terminal wealth of around six times that of an equivalent initial endowment accumulation model. Even over a relatively short horizon of only 15 years, a contributing investor will be around 50 per cent better off than a non-contributing investor in nominal dollar terms. Where contributions rise due to the effect of salary growth (the constant percentage contribution model), we are better off still with median terminal wealth being around 30 per cent higher over a 40-year horizon than the accumulation model where contributions are constant over the investment horizon (the constant contributions model). These results show clearly that making regular contributions to a retirement account, and any factor which can induce contributions to rise with time (e.g. salary growth in cases where contributions are a percentage of salary, or auto-escalation initiatives), have the potential to deliver significant improvements in terminal wealth for investors. The public policy implications are clear: plan members should be encouraged to contribute to their defined contribution plan in order to ensure retirement adequacy.

Whilst the magnitude of the results varies materially as we compare competing accumulation models, the relativities within each of the accumulation models generally do not. If we rank by simulation method the results for a given measure and horizon, these rankings generally remain consistent across accumulation models. Where the rankings do differ, they do so only marginally. One noticeable, and predictable, difference between the results for the three accumulation models is the different strength of the compounding effect. As we add contributions, then salary growth, to the accumulation model we notice an increasingly strong compounding effect as investment horizon lengthens. This phenomenon can be observed by comparing for each accumulation model the absolute differences in terminal wealth between successive horizons for a given simulation method. For example, consider median terminal wealth in Panel B of Table 6.6 for the stationary bootstrap (bs) model over a 40-year horizon, and compare it to the corresponding value for 35 years. For each accumulation model in turn, the differences between the 40- and 35-year median terminal wealth are: initial endowment model -$48,020 ($202,620 minus $250,640); constant contribution model $89,780 ($1,251,130 minus $1,161,350); and, constant percentage contribution model $206,160 ($1,694,230 minus $1,488,070). [Note that for the initial endowment model reported first, the 40-year terminal wealth is *lower* than the 35-year terminal resulting in a negative difference. This has resulted because the additional five years of returns are insufficient to compensate for the lower initial wealth level for the 40-year horizon.]

Where risk is measured as the standard deviation of terminal wealth, Table 6.6 shows us that risk increases with investment horizon consistent with the literature (Reichenstein and Dorsett, 1995; Hickman et al., 2001). At the 40-year horizon, the constant contribution and constant percentage contribution models are approximately four times (between 3.7 and 4.4 times depending on the simulation method) and five times (between 4.6 and 5.2 times depending on the simulation method) more risky than for the initial endowment model, respectively. Expressed another way, the standard deviation of terminal wealth for the constant contribution model at the 20-year horizon is approximately the same as the risk of the initial endowment model at the 40-year horizon (between 400 and 700 depending on the simulation method). A similar risk equivalence holds when comparing the constant percentage contribution model to the initial endowment model.

Review of the terminal wealth range estimates in Table 6.6 confirms earlier findings about the rapid increase in dispersion of outcomes in the final decade of a 40-year investment horizon. The results also confirm the significant variation between the results for the four different methods with the Monte Carlo and Efron (1979) bootstrap methods having (predictably) wider ranges because of their tendency to produce long series of negative and positive returns which, with the associated compounding effects, produce more extreme tail wealth paths. Because contributions and salary growth introduce wealth to the constant contribution and constant percentage contribution models throughout the investment horizon, the terminal wealth range for these two models is significantly wider than for the initial endowment model, at around 3.4-6.5 times and 3.1-6.2 times larger respectively at the 40-year horizon.

Earlier, in the discussion of the results reported in Table 6.5, we were seeking explanations as to why the results for the stationary bootstrap method were so different to that of the other three methods, but particularly the Monte Carlo and Efron (1979) bootstrap methods. We suggested that the estimated kurtosis was perhaps more left tailed in nature, consistent with the stationary bootstrap method having the most negative estimated skewness. In Panel E of Table 6.6 we see results that confirm our earlier hypothesis, with the stationary bootstrap (bs) minimum terminal wealth being significantly lower than the Monte Carlo (mc) and Efron (1979) bootstrap models over all horizons including the shorter horizons. For example, for the initial endowment model, minimum terminal wealth for the stationary bootstrap is around 8-10 times smaller than either the Monte Carlo or the Efron (1979) bootstrap methods at the 40-year horizon, with the difference narrowing as investment horizon shortens. While the same differences exist for the other accumulation models, the size of the difference is less pronounced.

While this finding was consistent with our expectations, one surprising finding from the results in Panel E of Table 6.6 is that the minimum terminal wealth estimates for the empirical block bootstrap (bb) are even worse than that of the stationary bootstrap (bs). In the face of the evidence in Table 6.5, where the means and standard deviations of cumulative returns for empirical block bootstrap return sit between those of the higher Monte Carlo and Efron (1979) bootstrap models and the lower stationary bootstrap model, we would not necessarily expect this result. This leads us to our second hypothesis for explaining the lower results shown in Table 6.5 for the stationary bootstrap: the absence of strongly positive return paths.

By switching our attention to Panel J of Table 6.6, we find the key to reconciling the results of Tables 6.5 and 6.6. As discussed above, the stationary bootstrap model produces a heavy left tail consistent with the negative skewness and leptokurtosis observed in Table 6.3, but with the absence of strongly positive return paths as can be seen by it producing the lowest maximum terminal wealth paths for almost all investment horizons and accumulation models. In some cases its maximum terminal wealth path is significantly lower than that of the empirical block bootstrap, the nearest of any method. For example, for the initial endowment model over the 40-year horizon, the maximum terminal wealth path for the stationary bootstrap is less than half that of the empirical block bootstrap, and around 70 per cent lower than that of the other two methods.

At the same time we find the explanation for why the means and standard deviations of cumulative returns shown in Table 6.5 for the empirical block bootstrap are higher than that of the stationary bootstrap. While the empirical block bootstrap has the worst of all terminal wealth paths (cf. Panel E, Table 6.6) it produces better upside wealth paths than the stationary bootstrap (cf. Panels H-J, Table 6.6). These positive paths counteract the impact of the strongly negative paths, resulting in higher mean (cf. Panel A, Table 6.6) and median (cf. Panel B, Table 6.6) terminal wealth paths. Expressed another way, the empirical block bootstrap method produces a more leptokurtic empirical distribution (cf. Panel D, Table 6.3), but with a more positive skew (cf. Panel C, Table 6.3).

In our first look at wealth-based measures, we begin to observe significant differences between the results from our three accumulation models. We thus see early evidence that regular contributions – which is a feature of the institutional setting – have a significant marginal impact where performance is expressed in wealth terms.

6.5.1.4 Downside risk

While the distribution of terminal wealth gives us a picture of the range of possible outcomes, research has shown that investors are concerned with downside risk. Downside risk can be measured in probability terms – which measures likelihood of a risk event – or in terms of magnitude – which gives the analyst an idea of the potential impact of a loss. The ideal measure usually combines these two dimensions of risk.

The behavioural literature has shown that investors generally care more about the magnitude of a loss, than its probability (e.g. Olsen and Khaki, 1998). In Table 6.7 we report downside risk measures that capture the probability of loss, like shortfall risk, as well as others that size the magnitude of a loss in different ways, such as downside deviation, value-at-risk and expected tail loss.

Table 6.7: Downside risk measures

This table presents four downside risk measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures are: downside deviation is reported in Panel A, shortfall risk is reported in Panel B, 95% value-at-risk (VaR) is reported in Panel C, and 95% expected tail loss is reported in Panel D. All results are expressed in thousands of dollars, except for shortfall risk which is expressed in percentage terms.

PANEL A – Downside deviation

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	1.35	1.45	1.70	2.53	12.86	12.50	13.96	21.04	19.81	19.47	22.14	31.82
35	2.74	2.73	3.12	4.72	15.82	15.58	17.38	25.95	23.04	21.93	26.32	36.53
30	3.60	3.82	4.48	5.74	14.54	16.19	17.17	24.39	20.19	21.60	23.66	32.02
25	7.90	8.24	9.00	12.05	17.69	17.93	19.57	26.80	19.57	21.60	23.59	31.82
20	11.56	11.62	12.13	16.98	18.98	20.14	20.28	27.45	21.23	21.73	22.28	28.83
15	12.79	12.64	13.54	17.51	17.85	18.82	18.72	23.58	18.23	19.22	19.98	26.12
10	12.93	12.93	13.93	17.66	16.09	16.44	18.04	21.51	16.95	17.38	18.56	22.16
5	10.95	11.11	12.35	13.77	12.79	13.17	14.32	15.77	13.01	13.03	13.77	15.14
1	6.82	6.85	7.97	8.04	7.10	7.14	8.43	8.42	6.82	7.03	8.40	8.35

PANEL B – Shortfall risk

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc (%)**	**b (%)**	**bs (%)**	**bb (%)**	**mc (%)**	**b (%)**	**bs (%)**	**bb (%)**	**mc (%)**	**b (%)**	**bs (%)**	**bb (%)**
40	3.05	3.00	3.01	5.37	3.18	3.16	2.91	5.05	3.79	3.42	3.23	5.23
35	3.97	3.80	4.17	6.88	4.43	4.09	3.71	5.97	4.52	4.77	4.18	6.15
30	5.03	5.43	5.02	6.99	4.99	5.46	4.52	7.03	5.82	5.76	5.00	6.84
25	7.04	7.06	6.37	9.18	6.51	6.60	5.89	7.88	6.20	6.87	6.16	8.60
20	9.30	9.13	8.55	11.22	8.87	8.91	7.94	10.06	9.15	8.93	8.05	10.06
15	12.36	12.80	11.82	13.58	12.10	12.49	10.15	12.36	12.40	12.07	10.33	13.05
10	17.23	17.16	15.37	17.38	17.28	16.90	15.91	16.81	16.49	17.55	14.81	16.91
5	24.82	24.32	22.38	20.83	25.47	25.91	22.41	21.41	25.81	24.59	21.19	21.36
1	38.40	36.76	32.02	31.05	37.54	37.36	32.05	31.64	38.06	36.74	32.53	31.36

PANEL C – 95% value-at-risk

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	26.97	26.91	26.53	19.38	267.78	260.75	272.58	214.94	393.09	399.40	412.43	343.76
35	38.59	39.32	37.17	26.84	266.55	271.71	277.97	229.23	391.17	387.10	408.33	336.07
30	43.18	41.40	41.90	33.35	232.47	222.46	233.32	189.69	311.11	307.66	308.37	277.43
25	69.67	70.01	67.62	53.65	211.64	210.59	217.25	182.98	278.64	267.71	263.16	234.00
20	81.85	81.39	82.56	64.63	177.37	175.58	180.37	153.02	209.15	208.20	211.85	187.41
15	70.57	72.87	68.96	57.38	136.25	134.76	138.68	120.86	154.84	152.79	152.96	130.83
10	66.91	67.25	63.43	53.37	105.95	105.10	100.35	89.80	109.86	110.30	106.69	95.53
5	57.84	57.68	51.52	49.33	74.85	73.51	68.98	65.50	75.88	75.82	72.98	69.24
1	59.44	59.45	57.45	57.36	62.20	61.96	59.75	60.05	63.08	62.45	59.79	59.79

PANEL D – 95% expected tail loss

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc ($)	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	17.84	17.53	16.49	11.14	190.50	190.70	190.86	143.62	294.72	302.52	294.84	237.27
35	25.78	25.85	23.61	15.76	199.90	203.17	198.35	153.94	303.13	302.84	296.12	237.66
30	29.79	28.24	27.42	20.29	177.78	169.64	169.76	131.01	247.81	241.25	232.92	197.85
25	49.98	48.30	44.03	32.92	163.95	161.87	156.46	127.46	219.62	210.28	201.92	167.52
20	60.06	59.07	56.61	41.67	139.09	135.04	132.45	108.29	167.04	164.46	160.69	135.56
15	53.83	54.90	48.89	36.16	110.17	107.25	105.47	87.55	128.09	123.03	118.31	95.84
10	53.95	53.71	46.38	36.18	88.38	87.34	76.98	66.99	91.94	91.13	82.49	72.40
5	49.17	48.69	37.79	33.92	64.25	62.93	52.47	48.62	66.16	64.72	55.16	51.61
1	55.30	54.40	48.59	47.82	57.70	56.80	50.43	50.45	58.64	56.97	50.44	50.30

In Table 6.7, we once again see our earlier findings confirmed. For the initial endowment model we see no real trend in the estimates for downside deviation because of the obscuring influence of different levels of initial wealth.[77] For the constant contribution and constant percentage contribution accumulation models, we begin to see a faint pattern of an increase in downside deviation with investment horizon as identified by McEnally (1985), although the evidence is by no means emphatic. In comparing accumulation models, as we incorporate contributions and then salary growth, we see a rise in measured downside deviation for all corresponding models and investment horizons consistent with our other findings where performance is measured in wealth terms.

With shortfall risk (Panel B, Table 6.7) we return briefly to measures that are a function of returns only. We would thus expect to see virtually identical results for each simulation method across the accumulation models because the only possible source of difference in the results is the (expected) variation between the return paths simulated from the data. The results reported confirm our expectations with results for a given simulation method being of similar order of magnitude across the three accumulation models. Furthermore, the relative results amongst simulation methods within a given accumulation model are consistent for all accumulation models: the empirical block bootstrap method yields the highest estimates for shortfall risk for the longer horizons for all accumulation models. Finally, the results for all twelve models confirm the overwhelming finding of the time diversification literature which is that shortfall risk reduces as investment horizon lengthens (Leibowitz and Krasker, 1988; Leibowitz and Langetieg, 1989; Butler and Domian, 1991; Leibowitz and Kogelman, 1991; Reichenstein and Dorsett, 1995; Cohen et al., 1996).

We now turn to our final two measures of downside risk, value-at-risk (VaR), and its more robust variant, expected tail loss (ETL) (Rachev et al., 2008). Earlier in this thesis, obscured somewhat by the complicating factor of different levels of initial wealth, we observed a monotonic reduction in both VaR and ETL for all models from the 20-year to the 40-year horizon suggesting that risk increases with investment horizon. The picture for horizons between the one-year and the 15-year were less clear. When we consider the results for the constant contribution and constant percentage contribution models, two particular results stand out: firstly, that VaR and ETL estimates generally rise with investment horizon, and, secondly, that VaR and ETL estimates are, in absolute terms, substantially larger as contributions, and then salary growth, are incorporated into the accumulation model.

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[77] Recall also that the downside deviation figures we report here are downside deviation against the return from T-bills, representing the zero risk opportunity cost. McEnally (1985) measures downside deviation against the mean, whereas Mukherji (2008) measures downside deviation against a number of arbitrary and fixed real return targets.

This first finding is consistent with that of Panyagometh (2011), the only other work in the time diversification literature that analyses the behaviour of VaR with investment horizon in the presence of monthly contributions. Panyagometh (2011) finds that as horizon lengthens, VaR estimates across a number of asset classes rise (i.e. improve). Our second finding provides us more evidence that contributions and salary growth are more important factors in long horizon portfolio choice than the time diversification literature would have us believe.

6.5.1.5 Risk-adjusted performance

Until now we have reviewed performance from either a return or a risk perspective, without explicitly combining the two concepts. This begs the question: How does risk-adjusted performance change with horizon when comparing the three accumulation models? We report risk-adjusted performance measures in Table 6.8.

Table 6.8: Risk-adjusted measures

This table presents risk-adjusted measures of performance for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). The measures are: the Sharpe ratio is reported in Panel A, the Sortino ratio is reported in Panel B, and the coefficient of downside deviation is reported in Panel C. The results are ratios so are therefore pure numbers (i.e. have no units of measurement).

PANEL A – Sharpe ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	2.47	2.50	2.93	2.47	2.49	2.49	2.95	2.49	2.46	2.48	2.94	2.48
35	2.34	2.34	2.71	2.29	2.30	2.33	2.76	2.34	2.29	2.33	2.74	2.30
30	2.13	2.16	2.52	2.19	2.16	2.15	2.58	2.16	2.15	2.14	2.53	2.18
25	1.97	1.94	2.29	1.97	1.96	1.99	2.34	1.96	1.98	1.96	2.32	1.97
20	1.75	1.78	2.08	1.75	1.75	1.75	2.06	1.75	1.76	1.76	2.05	1.78
15	1.50	1.51	1.75	1.51	1.52	1.50	1.81	1.55	1.52	1.52	1.79	1.50
10	1.25	1.26	1.43	1.22	1.24	1.25	1.40	1.22	1.25	1.23	1.41	1.25
5	0.89	0.89	0.96	0.87	0.87	0.86	0.93	0.87	0.86	0.88	0.96	0.89
1	0.39	0.40	0.37	0.38	0.39	0.39	0.36	0.37	0.41	0.40	0.36	0.38

PANEL B – Sortino ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	79.70	65.68	96.69	31.28	61.01	57.96	91.22	35.03	72.98	58.38	114.55	33.96
35	46.95	45.11	71.05	24.83	43.91	50.24	60.95	25.29	49.59	54.44	59.77	23.81
30	36.71	39.86	38.54	20.27	37.50	32.41	51.06	19.15	35.82	35.37	46.07	21.44
25	25.32	24.86	27.69	14.76	24.36	26.75	30.46	13.28	27.26	25.12	31.50	13.75
20	15.63	16.46	21.24	9.82	16.53	15.13	18.29	9.36	15.78	15.04	17.84	10.41
15	9.87	10.21	12.30	6.25	10.35	9.68	11.81	6.90	10.93	9.96	11.95	6.25
10	6.12	6.26	6.16	3.86	6.15	6.31	6.04	3.93	5.80	6.02	6.10	4.12
5	2.85	2.91	2.56	1.93	2.73	2.73	2.44	1.96	2.70	2.72	2.60	2.05
1	0.78	0.79	0.63	0.63	0.76	0.76	0.59	0.62	0.82	0.78	0.59	0.63

PANEL C – Coefficient of downside deviation

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.03	0.03	0.02	0.07	0.03	0.04	0.02	0.06	0.03	0.04	0.02	0.06
35	0.04	0.05	0.03	0.08	0.05	0.04	0.03	0.08	0.04	0.04	0.04	0.09
30	0.06	0.05	0.05	0.10	0.06	0.06	0.04	0.11	0.06	0.06	0.05	0.10
25	0.08	0.08	0.08	0.14	0.09	0.08	0.07	0.16	0.08	0.08	0.07	0.15
20	0.13	0.13	0.10	0.21	0.13	0.14	0.11	0.22	0.13	0.14	0.12	0.20
15	0.21	0.21	0.17	0.34	0.20	0.22	0.18	0.30	0.19	0.21	0.18	0.33
10	0.34	0.33	0.34	0.54	0.34	0.34	0.34	0.53	0.36	0.35	0.34	0.51
5	0.74	0.73	0.83	1.09	0.77	0.76	0.85	1.06	0.77	0.77	0.83	1.03
1	2.76	2.69	3.37	3.48	2.77	2.75	3.45	3.51	2.66	2.69	3.45	3.52

As with our earlier discussion of shortfall risk, we would expect to see little variation between the results for each of the accumulation models because the three risk-adjusted measures we consider only account for returns. The various effects of initial endowment, contributions and salary growth are ignored in their calculation, so any variation between estimates results from the random differences in the return path simulation procedure.

Confirming our earlier findings for the initial endowment model (re-reported in Table 6.8 for convenience), estimated Sharpe and Sortino ratios for all models rise with investment horizon, and estimated coefficients of downside deviation fall for all models. These findings are generally consistent with the time diversification literature on the Sharpe ratio (Levy, 1972; Lloyd and Modani,1983; Levy, 1984), the Sortino ratio (Sinha and Sun, 2005), and the coefficient of downside deviation (Mukherji, 2002, 2008).[78]

The relative results for each simulation method, when comparing results across accumulation models and measures, are also consistent. For example, in each accumulation model, and for each of the three measures, the stationary bootstrap produces the superior reward-for-risk tradeoff. The overall implication of these results is that the reward-for-risk trade-off improves with investment horizon, if one assumes that return is measured by the arithmetic mean and risk is measured by a variant of standard deviation.[79]

6.5.1.6 Measures specific to the time diversification literature

The remaining two measures to be considered – Guo and Darnell's (2005) *T** and the time diversification index (TDI) of Fabozzi et al. (2006) – bring with them the same drawbacks as shortfall risk and the risk-adjusted measures we consider: they each ignore the influence of any other variable other than returns. For this reason, we expect to observe broadly consistent estimates across the three accumulation models we examine. Table 6.9 reports Guo and Darnell's (2005) *T**, and Table 6.10 reports the time diversification index.

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[78] Earlier in this thesis we noted a finding regarding the behaviour of Sharpe ratios with investment horizon that is at odds with both the time diversification literature and the evidence reported here. Hodges, Taylor and Yoder (1997) find that Sharpe ratios first rise with investment horizon, then fall, forming a hump-shaped profile.

[79] Standard deviation in the case of the Sharpe ratio, and downside semi-standard deviation in the case of the Sortino ratio and the coefficient of downside deviation.

Table 6.9: Guo and Darnell’s *T**

This table presents Guo and Darnell’s (2005) *T** for the four modeling techniques for each of the nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). *T** is defined as “the investment horizon such that the total stock return over this holding period will not become negative at the [95%] confidence level” (Guo and Darnell, 2005, p. 69). Results are expressed in units of months. Highlighted cells represent occasions where the *T** is greater than for investment horizon for which it is being calculated.

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	168	170	166	220	165	180	164	215	166	177	175	226
35	170	172	178	233	173	184	171	221	172	173	174	211
30	164	171	166	209	165	172	168	222	165	165	160	209
25	164	175	174	223	169	171	168	224	162	169	173	211
20	168	169	162	208	164	172	166	216	167	177	172	215
15	166	171	168	>180	169	176	156	>180	166	167	163	>180
10	>120	>120	>120	>120	>120	>120	>120	>120	>120	>120	>120	>120
5	>60	>60	>60	>60	>60	>60	>60	>60	>60	>60	>60	>60
1	>12	>12	>12	>12	>12	>12	>12	>12	>12	>12	>12	>12

It is first worth reiterating here that while Guo and Darnell (2005) introduced their *T** measure in the context of the time diversification debate, it was apparently proposed as a way of comparing the relative riskiness of different asset return processes using a measure expressed in units of time. By observing different *T** estimates for each of the three processes they conclude that "optimal asset allocation policy should … depend upon the length of the investment horizon" (p. 69).

The results in Table 6.9 continue the mixed picture observed earlier in this thesis. For the Monte Carlo, Efron (1979) bootstrap and stationary bootstrap models, all estimates for *T** are approximately the same, falling in the range 160 to 184 months with no discernible pattern. Perhaps the only possible variation amongst these three methods are the results for the Efron (1979) bootstrap, which are marginally higher than *T** estimates for the Monte Carlo and stationary bootstrap methods.

The most obvious feature of the results in Table 6.9 is the material difference between those of the empirical block bootstrap when compared to the other models, with *T** estimates around 30 months longer, which we argue is the result of the heavier left tail produced by this simulation method (cf. Panels E and F, Table 6.6). The addition of two further accumulation models to the initial endowment model studied earlier in this thesis (and re-reported in this study for ease of comparison) brings us no closer to resolving the time diversification debate one way or the other, but it does reinforce the case that Guo and Darnell (2005) made in favour investment horizon dependent asset allocation policy. The matter of asset allocation policy will be considered in the third and final study of this thesis.

The time diversification index (TDI) estimates reported in Table 6.10 are similar to the *T** estimates reported in Table 6.9 in one way: there is little in the way of discernible patterns in either set of results. Of the 96 separate TDI estimates, only 17 fall below the unity threshold proposed by Fabozzi et al. (2006) as being evidence of time diversification. Furthermore, where the TDI is less than unity there are no compelling trends by type of simulation method or accumulation model. If there is the faint outline of a trend, it is that sub-unity TDIs tend to occur at the longer horizons, with ten of 17 being at the longest two horizons (30- or 35-years) and, of those, eight are at a horizon of 35 years.

Earlier, we suggested that this evidence of time diversification at longer horizons might be a manifestation of the portfolio size effect of Basu and Drew (2009a): by extending the investment horizon to 40 years, one is able to access the rapid jump in portfolio size between 35- and 40-year investment horizons with comparatively little additional risk. For example, consider the significant difference between mean cumulative returns for the 35- and 40-year horizon in Panel A of Table 6.5, and the estimated improvement in risk-adjusted performance suggested by the Sharpe and Sortino ratios shown in Panels A and B of Table 6.8. Overall, however, we find no evidence of time diversification, confirming the conclusions of Fabozzi et al. (2006).

Table 6.10: Time diversification index

This table presents Fabozzi et al.’s (2006) time diversification index (TDI) for the four modeling techniques for each of the nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). The time diversification index is a ratio of normalised risk measures for two horizons. The calculated TDIs reported in this table are the ratio of the normalised risk measures for each investment horizon against the 40 year investment horizon. As such there is no 40 year measure. According to Fabozzi et al.’s (2006) rule, a TDI of less than unity suggests the presence of time diversification. The results are ratios so are therefore pure numbers (i.e. have no units of measurement).

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40												
35	1.0030	0.9984	0.9989	0.9994	1.0018	0.9993	0.9999	0.9994	0.9956	1.0006	0.9951	1.0040
30	0.9984	1.0015	1.0086	1.0096	1.0074	0.9971	1.0069	1.0061	1.0009	1.0025	1.0069	1.0084
25	1.0063	0.9984	1.0056	1.0083	0.9990	1.0056	1.0093	1.0113	1.0058	1.0059	0.9993	1.0073
20	1.0022	1.0042	1.0147	1.0097	1.0014	0.9989	1.0091	1.0092	1.0041	1.0004	1.0141	1.0072
15	1.0002	1.0015	1.0180	1.0267	1.0014	0.9946	1.0266	1.0183	1.0000	1.0101	1.0230	1.0138
10	1.0025	1.0035	1.0432	1.0280	0.9967	1.0159	1.0246	1.0217	1.0033	1.0129	1.0342	1.0362
5	1.0124	1.0206	1.0751	1.0651	1.0044	1.0070	1.0521	1.0574	0.9920	1.0189	1.0780	1.0627
1	1.0351	1.0824	1.1495	1.1883	1.0326	1.0612	1.1053	1.1473	1.0468	1.0720	1.1178	1.1612

6.5.1.7 Summary

In this study, we use a critique of the time diversification literature to motivate the consideration of two additional accumulation models: the constant contribution model, where terminal wealth is a function of initial endowment, returns and constant contributions; and, the constant percentage contribution model where terminal wealth is a function of the initial endowment, returns and contributions which grow as a result of salary growth. We replicated the same battery of measures for these two accumulation models and compared the results with those we calculated for the initial endowment model so central to the time diversification literature. We found that none of these "measures" of time diversification differed materially across accumulation models, with the exception of measures of terminal wealth and certain downside risk measures. The measures of terminal wealth – measures of central tendency, measures of dispersion and percentiles – and downside risk are significantly different for accumulation models that incorporate contributions versus the one that does not. It is therefore possible to conclude that the time diversification debate to date has been conducted within a relatively narrow framework, and with a focus on measures that are not of key concern to long horizon investors.

Perhaps in framing the time diversification problem scholars have failed to take account of the purpose of retirement savings. In the time diversification literature, risk is measured from within an initial endowment framework using a range of well-known risk measures. Only a limited number of studies consider terminal wealth and downside risk, and none considers whether this terminal wealth is sufficient to fund the plan member's retirement. Thus a debate which is ostensibly about long horizon investing seems to ignore one of the major risks facing long horizon investors, which is the adequacy of terminal wealth.

If we are to manage the risk of adequacy then, based on these results, one of the largest risks will be not contributing enough. Compare for example, median terminal wealth for all models in the initial endowment framework (Panel B, Table 6.6) with the 95% VaR for the two accumulation models which incorporate contributions. We highlight that all figures are expressed in thousands of dollars and are therefore comparable to some degree. Terminal wealth estimates for the median path for the initial endowment model range from $183,000 and $212,000 depending on the simulation method. The 95% value-at-risk estimates range from $214,940 to $272,580 for the constant contributions model, and $343,760 to $412,430 for the constant percentage contributions model. Here we see that by adding contributions, outcomes are improved to such a great extent that the middle path in the initial endowment model results in less terminal wealth than a relatively extreme downside risk scenario for either of the other two accumulation models. Expressed another way, more than 95 per cent of all terminal wealth scenarios for the constant contribution and constant percentage contribution models will exceed the median path for the initial endowment model. This suggests two important findings: (1) contributions are an extremely important variable when considering pension finance problems; and, (2) the time diversification literature comprehensively fails to identify its importance, or study its effects.

Before we make too many claims about the superiority of accumulation models that incorporate contributions, we do concede that contributing to one's retirement does come at a cost. Contributions are made usually by the plan member, their employer, or often some combination of the two. Where the worker makes contributions to their retirement account they are deferring current consumption. All other things equal, for every dollar of retirement account contribution there is one dollar of current consumption foregone.[80] In this sense, any amount of additional terminal wealth resulting from contributions, and the returns earned on them, is reward for deferring consumption. Similarly, when an employer makes contributions to an employee's defined contribution plan on their behalf it is an employment cost that is incorporated in the present price of goods, or it is compensation that the employee would otherwise have received as current income. Expressed another way, any increase in employer contributions has an economic impact via higher prices or a reduction in the worker's current income (*ceteris paribus*).

6.5.2 Comparing performance in retirement wealth ratio terms

6.5.2.1 Introducing the retirement wealth ratio

Until now, where we have examined terminal wealth, we have done so in absolute nominal dollar terms. Without an appropriate yardstick we are neither able to properly size wealth measures, nor fairly compare the terminal wealth levels yielded by each of the accumulation models. For example, when assessing the terminal wealth estimates produced by the constant contribution and constant percentage contribution models, we are unable to directly compare performance on terminal wealth grounds alone because of differing final salary levels. Assuming that investors' expectations regarding their retirement income are somehow a function of their final salary as suggested in Basu and Drew (2010), then a performance measure expressed in terms of final salary would be an appropriate measure for the purposes of comparison.

As outlined in the methodology section of this study, one such measure is the retirement wealth ratio (RWR_T) of Basu and Drew (2010). It is calculated by dividing terminal wealth (W_T) by income at time T. In the methodology section, we used the following example to draw out the ability of RWR_T to provide a basis for comparison where there are two dimension being compared. Terminal wealth of one million dollars will appear more adequate to an individual whose final salary is $50,000 per

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[80] In general, when contributions are made it does not involve a full dollar of consumption foregone *by design*. In Australia, for example, the taxation system benefits superannuation investors. Thus, for every dollar of voluntary contributions to superannuation, the worker forgoes an amount of current consumption equivalent to one dollar less the worker's marginal rate of taxation.

annum when compared to another individual whose final salary in $200,000 per annum. If we were to judge each scenario based on terminal wealth alone, performance would be equivalent with each individual retiring with one million dollars. Expressed in retirement wealth ratio terms, the worker on the lower income would retire with a RWR_T of 20 times ($1,000,000 divided by $50,000) versus an RWR_T of five times ($1,000,000 divided by $200,000) for the individual on the higher income.

The RWR_T therefore allows us to compare accumulation models where both terminal wealth and income at time T are not equivalent. Had we restricted our analysis to the initial endowment and constant contribution accumulation models, we would be able to compare performance on a wealth basis because final salaries for the two models are equal. But by introducing the constant percentage contributions model, where contributions rise due to the effect of salary growth, we have a model with a different final income thereby adding a variable that must be controlled for. For the balance of this study we review performance in RWR_T terms.

6.5.2.2 Distribution of retirement wealth ratios

The first part of the analysis in this study took a range of measures from the time diversification literature and reconsidered them for two additional, arguably more realistic, accumulation models that incorporate some combination of contributions and salary growth. By doing this, we wished to achieve two closely related goals. Firstly, we wished to contrast the results of the latter two accumulation models with those of the initial endowment model so popular in the time diversification literature as the basis for a critique. And, secondly, we sought to understand whether the measures proposed in the literature as the “best” measure of time diversification yielded different results for the two accumulation models introduced in this study. We found that, when measured in terms of terminal wealth, the constant contribution and constant percentage contribution models produced significantly different terminal wealth and downside risk measures. Furthermore we found that wealth was the only measurement basis which varied materially between the three models.

The commonality between the results for many measures (e.g. risk-adjusted measures) across accumulation models results from the nature of these measures themselves. With the exception of wealth-based measures, none of these measures incorporates the effect of any variable other than returns. This leads us to conclude that many of the measures proposed in the literature are appropriate only for accumulation models, like the initial endowment model, where returns are the only determinant of performance (other than initial endowment). Because we have shown that most of the measures estimated above do not vary according with the accumulation model under investigation, it is not necessary to recalculate most of the measures from the literature because they have already been calculated earlier in this thesis.

We now report those measures reported earlier that were expressed in terms of wealth. As such, Tables 6.11 and 6.12 are the RWR_T counterparts to tables 6.6 and 6.7.

Table 6.11: Distribution of retirement wealth ratios

This table presents the distribution of terminal wealth for the four modeling techniques over nine investment horizons for the three accumulation models. Initial wealth (W_0) is as per row five of Table 6.1. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. Performance measures are group by measure type with measures of central tendency reported in Panel A (mean retirement wealth ratio) and Panel B (median, or 50th percentile retirement wealth ratio). Measures of dispersion are reported in Panels C (standard deviation in retirement wealth ratio terms) and D (retirement wealth ratio range). Panels E through G summarise the lower half of the distribution of terminal wealth (minimum, 5th percentile and 25th percentile retirement wealth ratio respectively). Panels H through J summarise the upper half of the distribution of terminal wealth (75th percentile, 95th percentile and maximum retirement wealth ratio respectively). Results are presented as multiples (i.e. in unit of "times" as in "*x* times final salary").

PANEL A – Mean retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	15.42	15.32	13.26	15.26	77.64	80.13	71.36	79.72	30.53	30.11	27.72	30.05
35	12.58	12.55	11.18	12.74	49.76	49.53	44.97	49.21	21.41	21.21	19.36	21.60
30	9.92	10.04	9.01	9.97	31.22	30.52	28.66	31.03	15.15	15.29	14.18	15.15
25	12.23	12.24	11.09	12.13	23.75	24.35	22.35	24.37	13.06	12.99	11.85	12.74
20	10.96	10.97	10.28	10.91	17.57	17.57	16.57	17.47	10.49	10.35	9.99	10.35
15	7.28	7.35	6.96	7.40	10.74	10.64	10.34	10.68	7.25	7.34	7.01	7.21
10	4.89	4.89	4.80	4.92	6.53	6.67	6.38	6.58	5.04	5.09	4.93	5.06
5	2.91	2.91	2.89	2.90	3.51	3.51	3.48	3.51	3.04	3.06	3.05	3.05
1	2.48	2.48	2.49	2.50	2.58	2.58	2.58	2.59	2.51	2.50	2.51	2.52

PANEL B – Median (50th percentile) retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	7.35	7.66	8.10	8.44	46.80	47.90	50.05	51.40	19.22	18.98	20.41	20.54
35	6.81	6.98	7.16	7.53	31.97	32.02	33.18	34.37	14.20	14.22	14.88	15.68
30	5.76	6.03	6.26	6.54	21.26	21.09	22.58	23.10	10.74	10.73	11.31	11.71
25	7.97	7.83	8.34	8.61	16.98	17.63	18.10	18.72	9.47	9.48	9.77	10.20
20	7.77	7.89	8.23	8.39	13.11	13.15	13.96	14.33	8.16	7.95	8.45	8.54
15	5.63	5.62	5.91	6.22	8.69	8.68	9.19	9.40	5.87	5.93	6.24	6.35
10	4.12	4.13	4.38	4.51	5.63	5.74	5.93	6.03	4.37	4.39	4.59	4.71
5	2.67	2.68	2.85	2.86	3.25	3.25	3.41	3.47	2.84	2.86	2.99	3.00
1	2.44	2.46	2.52	2.53	2.55	2.55	2.63	2.63	2.48	2.48	2.55	2.55

PANEL C – Standard deviation of retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	27.14	26.28	16.07	21.54	99.68	116.84	69.19	90.15	37.55	38.68	25.38	31.01
35	19.14	18.62	12.62	16.30	60.65	57.52	40.40	50.13	24.93	23.74	15.88	20.81
30	13.47	13.01	8.92	10.98	32.53	31.77	22.62	28.26	15.77	16.36	10.64	12.93
25	14.37	14.26	9.80	11.84	23.27	23.52	16.37	20.74	12.39	12.00	8.32	9.75
20	10.86	10.60	8.03	9.39	15.42	15.48	11.17	12.73	8.41	8.52	6.43	7.30
15	5.99	6.37	4.59	5.31	7.73	7.61	5.78	6.54	5.10	5.48	3.86	4.34
10	3.11	3.10	2.51	2.80	3.78	3.83	2.96	3.39	2.85	2.97	2.28	2.53
5	1.26	1.26	1.07	1.10	1.42	1.43	1.20	1.23	1.20	1.20	1.04	1.06
1	0.45	0.44	0.45	0.46	0.46	0.46	0.47	0.47	0.44	0.44	0.45	0.45

PANEL D – Retirement wealth ratio range

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	660.34	704.17	202.97	484.60	2,340.57	4,523.30	719.40	1,495.86	732.95	1,174.90	377.37	453.27
35	590.16	460.37	186.54	288.94	1,342.02	998.98	714.42	739.59	542.19	531.12	229.14	303.27
30	236.24	256.51	100.28	132.10	422.93	743.69	251.76	390.98	467.80	527.20	110.34	206.62
25	337.49	184.43	121.95	127.61	533.66	505.18	180.08	275.64	249.30	178.80	97.86	108.42
20	141.11	204.29	91.06	118.23	208.74	353.01	108.92	108.90	96.13	132.82	69.84	96.67
15	125.83	144.99	60.88	50.56	115.09	117.12	60.96	55.52	69.67	172.46	36.24	41.44
10	41.87	47.81	22.68	28.11	46.52	38.78	24.56	37.66	36.16	38.90	17.92	21.73
5	11.62	12.84	8.62	8.33	12.59	20.13	11.91	8.56	11.74	10.79	9.31	8.59
1	3.78	4.36	4.30	4.30	3.42	4.88	4.89	4.40	3.68	4.53	4.27	4.27

PANEL E – Minimum retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.12	0.08	0.01	0.00	2.03	1.80	0.73	0.84	1.27	1.30	0.71	0.64
35	0.10	0.08	0.04	0.02	1.58	1.82	1.06	0.70	1.03	1.30	0.68	0.44
30	0.16	0.07	0.06	0.02	1.13	1.46	0.93	0.49	0.99	0.97	0.52	0.50
25	0.26	0.23	0.16	0.06	1.23	1.06	0.91	0.36	1.00	1.06	0.54	0.45
20	0.35	0.31	0.20	0.05	0.91	1.09	0.76	0.44	0.98	0.37	0.57	0.30
15	0.28	0.26	0.18	0.03	0.91	0.97	0.48	0.24	0.68	0.64	0.44	0.30
10	0.57	0.39	0.13	0.16	1.04	0.81	0.43	0.30	0.80	0.75	0.58	0.22
5	0.46	0.58	0.15	0.10	0.77	0.81	0.37	0.23	0.59	0.47	0.29	0.20
1	1.10	1.08	0.77	0.77	1.23	1.11	0.82	0.82	1.19	1.18	0.80	0.80

PANEL F – 5th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	1.08	1.08	1.06	0.77	10.71	10.43	10.90	8.60	4.73	4.81	4.97	4.14
35	1.10	1.12	1.06	0.77	7.61	7.76	7.94	6.54	3.91	3.87	4.08	3.36
30	1.11	1.06	1.07	0.85	5.96	5.70	5.98	4.86	3.24	3.21	3.21	2.89
25	1.66	1.67	1.61	1.28	5.04	5.01	5.17	4.36	3.13	3.01	2.96	2.63
20	1.95	1.94	1.97	1.54	4.22	4.18	4.29	3.64	2.73	2.72	2.77	2.45
15	1.64	1.69	1.60	1.33	3.17	3.13	3.22	2.81	2.30	2.27	2.27	1.94
10	1.56	1.56	1.48	1.24	2.46	2.44	2.33	2.09	1.89	1.90	1.84	1.65
5	1.35	1.34	1.20	1.15	1.74	1.71	1.60	1.52	1.52	1.52	1.46	1.38
1	1.80	1.80	1.74	1.74	1.88	1.88	1.81	1.82	1.85	1.84	1.76	1.76

PANEL G – 25th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	3.40	3.46	3.67	3.40	24.64	25.17	27.37	25.63	10.65	10.36	11.44	10.91
35	3.30	3.29	3.49	3.30	17.51	17.52	18.83	18.00	8.09	8.27	8.88	8.60
30	2.88	3.00	3.15	3.02	12.19	12.18	13.25	12.83	6.38	6.40	6.86	6.72
25	4.22	4.10	4.43	4.27	10.07	10.42	10.92	10.81	5.81	5.83	6.10	6.02
20	4.38	4.44	4.74	4.52	8.14	8.27	8.77	8.64	5.13	5.09	5.41	5.29
15	3.41	3.42	3.65	3.64	5.69	5.60	6.14	6.05	3.93	3.97	4.20	4.11
10	2.78	2.80	2.97	2.93	3.98	4.02	4.19	4.16	3.12	3.10	3.26	3.23
5	2.02	2.04	2.13	2.15	2.51	2.49	2.63	2.67	2.17	2.21	2.32	2.33
1	2.16	2.18	2.21	2.22	2.26	2.27	2.29	2.30	2.20	2.22	2.22	2.25

PANEL H – 75th percentile retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	16.56	16.96	16.62	18.89	91.24	93.26	90.25	100.64	36.00	35.65	35.22	38.03
35	14.61	14.60	14.11	15.76	58.86	58.87	58.22	62.19	25.51	25.52	24.80	27.28
30	11.60	11.91	11.72	13.05	38.00	36.89	37.27	39.80	18.54	18.25	18.27	19.33
25	15.01	14.78	14.71	15.80	29.02	29.82	28.92	31.33	15.62	16.00	15.21	16.56
20	13.59	13.69	13.49	14.47	21.59	21.77	21.22	22.84	13.04	12.78	12.99	13.38
15	9.08	9.19	9.20	9.73	13.38	13.26	13.27	13.74	9.00	9.08	8.92	9.33
10	6.12	6.11	6.21	6.43	8.03	8.35	8.16	8.44	6.21	6.25	6.24	6.45
5	3.52	3.54	3.55	3.61	4.22	4.22	4.23	4.29	3.64	3.68	3.68	3.69
1	2.76	2.75	2.76	2.77	2.86	2.85	2.86	2.88	2.79	2.77	2.79	2.79

PANEL I – 95th percentile retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	56.07	53.21	42.68	51.18	246.98	249.62	208.07	243.80	91.72	91.80	75.54	86.08
35	42.62	41.23	34.75	42.08	148.99	148.79	119.39	143.33	60.62	60.01	49.55	60.45
30	32.24	31.86	26.45	30.29	90.90	86.74	71.65	83.53	40.93	42.17	34.80	38.94
25	36.43	37.29	30.04	35.61	65.41	66.23	53.71	63.52	35.16	34.26	27.72	31.76
20	30.86	30.60	25.79	29.06	46.02	45.12	38.33	42.17	26.03	25.62	22.24	24.33
15	18.54	18.61	15.72	17.58	25.16	24.65	21.31	22.95	16.64	16.76	14.42	15.34
10	10.80	10.58	9.44	10.12	13.62	14.01	11.75	12.81	10.35	10.51	9.02	9.74
5	5.31	5.22	4.66	4.80	6.18	6.18	5.49	5.63	5.30	5.31	4.81	4.91
1	3.28	3.25	3.15	3.16	3.39	3.37	3.26	3.27	3.29	3.26	3.17	3.18

PANEL J – Maximum retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	660.46	704.25	202.98	484.60	2,342.60	4,525.10	720.13	1,496.70	734.22	1,176.20	378.08	453.91
35	590.26	460.45	186.58	288.96	1,343.60	1,000.80	715.48	740.29	543.22	532.42	229.82	303.71
30	236.39	256.58	100.34	132.12	424.06	745.15	252.69	391.47	468.79	528.17	110.86	207.13
25	337.75	184.66	122.11	127.67	534.89	506.25	180.99	276.00	250.30	179.86	98.40	108.88
20	141.46	204.59	91.26	118.28	209.65	354.10	109.69	109.34	97.11	133.19	70.40	96.97
15	126.11	145.25	61.05	50.59	116.00	118.09	61.44	55.77	70.35	173.10	36.68	41.73
10	42.44	48.20	22.81	28.27	47.56	39.59	24.99	37.96	36.96	39.65	18.50	21.95
5	12.08	13.42	8.77	8.43	13.35	20.95	12.28	8.79	12.33	11.26	9.60	8.79
1	4.88	5.44	5.07	5.07	4.65	5.99	5.71	5.22	4.87	5.72	5.07	5.07

The first point to note from Table 6.11 is that the large absolute differences in the results for the constant contribution and constant percentage contribution models when compared to the initial endowment model persist when measured in RWR_T terms. This result re-emphasises the two important findings from the previous section of this study: that contributions are an extremely important variable in pension finance problems; and, secondly that the time diversification literature fails to identify the importance of contributions, or study their effect.

All measures reported in Table 6.11 for the constant contribution and constant percentage contribution models increase in magnitude with investment horizon. For some measures, like the standard deviation of RWR_T reported in Panel C, we see evidence that supports the time diversification literature. Hickman et al. (2001), for example, also found that the standard deviation of terminal wealth increased with time horizon. The range of RWR_T estimates increases in investment horizon confirming the results of McEnally (1985) and others (e.g. Mukherji, 2008).

Other measures appear to yield opposite results when we move beyond the initial endowment model. Compare, for example, the minimum RWR_T estimates (Panel E, Table 6.11) for initial endowment model with those of the other two accumulation models. For the initial endowment model, estimates tend to fall (i.e. worsen) as investment horizon lengthens. For the two accumulation models that incorporate contributions, minimum RWR_T estimates tend to rise (i.e. improve) with investment horizon. Whilst this evidence suggests that the near universal absence of contributions from the time diversification literature may be a significant deficiency, caution is required for two reasons. Firstly, because of the low absolute value of wealth for the minimum paths, the differences in initial wealth for the various horizons obscure the results somewhat. And, secondly, while different trends appear to exist for different accumulation models, in economic terms, all outcomes are equally poor.

The noticeable (and expected) difference in the RWR_T estimates for the constant percentage contribution model reported in Table 6.11 is that they are now lower than the estimates for the constant contribution model. When expressed in terminal wealth terms as in Table 6.6 the reverse is true. This is because, in the former of the two models, the denominator in the calculation of RWR_T (i.e. final salary) is also growing with time hence the ratio of terminal wealth to final salary will be lower. Whilst this might at first suggest an inferior terminal wealth outcome for the plan member, this is not the case for two reasons: (1) terminal wealth remains higher in dollar terms as shown in Table 6.6; and (2) the hypothetical plan member has also earned higher incomes throughout their working life because of salary growth. Thus, whilst terminal wealth isn't lower in absolute terms, the terminal wealth relative to the plan member's expectations is lower, where expectations are assumed to be related to final salary.

Table 6.12: Downside risk measures

This table presents selected downside risk measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures are: 95% value-at-risk (VaR) is reported in Panel A, and 95% expected tail loss is reported in Panel B. All results are expressed in thousands of dollars.

PANEL A – 95% value-at-risk

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	1.08	1.08	1.06	0.78	10.71	10.43	10.90	8.60	4.74	4.81	4.97	4.14
35	1.10	1.12	1.06	0.77	7.62	7.76	7.94	6.55	3.91	3.87	4.08	3.36
30	1.11	1.06	1.07	0.86	5.96	5.70	5.98	4.86	3.24	3.21	3.21	2.89
25	1.66	1.67	1.61	1.28	5.04	5.01	5.17	4.36	3.13	3.01	2.96	2.63
20	1.95	1.94	1.97	1.54	4.22	4.18	4.30	3.64	2.73	2.72	2.79	2.45
15	1.64	1.69	1.60	1.33	3.17	3.13	3.23	2.81	2.30	2.27	2.27	1.94
10	1.56	1.56	1.48	1.24	2.46	2.44	2.33	2.09	1.89	1.90	1.84	1.65
5	1.35	1.34	1.20	1.15	1.74	1.71	1.60	1.52	1.52	1.52	1.46	1.39
1	1.80	1.80	1.74	1.74	1.89	1.88	1.81	1.82	1.86	1.84	1.76	1.76

PANEL B – 95% expected tail loss

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc ($)	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	0.71	0.70	0.66	0.45	7.63	7.63	7.64	5.75	3.55	3.65	3.55	2.86
35	0.74	0.74	0.68	0.45	5.72	5.81	5.67	4.40	3.03	3.03	2.96	2.38
30	0.76	0.72	0.70	0.52	4.56	4.35	4.36	3.36	2.58	2.52	2.43	2.06
25	1.19	1.15	1.05	0.78	3.91	3.86	3.73	3.04	2.47	2.37	2.27	1.89
20	1.43	1.41	1.35	0.99	3.31	3.22	3.16	2.58	2.18	2.15	2.10	1.77
15	1.25	1.28	1.14	0.84	2.56	2.50	3.45	2.04	1.90	1.83	1.76	1.42
10	1.26	1.25	1.08	0.84	2.06	2.03	1.79	1.56	1.58	1.57	1.42	1.25
5	1.14	1.13	0.88	0.79	1.49	1.46	1.22	1.13	1.32	1.30	1.10	1.03
1	1.68	1.65	1.47	1.45	1.75	1.72	1.53	1.53	1.72	1.68	1.48	1.48

As we observed with minimum RWR_T estimates reported in Panel E of Table 6.11, the estimates for value-at-risk and expected tail loss for the constant contribution and the constant percentage contribution models have a positive relationship with time, in contrast with the inverse relationship observed for the initial endowment model. For the initial endowment model, downside risk increases (i.e. RWR_Ts fall) with time. For the two models introduced in this study, downside risk decreases (i.e. RWR_Ts rise) with time due to the positive effect of ongoing contributions and, for the constant percentage contributions model, of salary growth. The effect of contributions and salary growth can be observed by comparing expected tail loss estimates (in RWR_T terms) through time for the initial endowment and constant percentage contribution models.

Figure 6.2: Expected tail loss with investment horizon

Using the empirical block bootstrap (bb) simulation method, this figure presents estimates of the 95% expected tail loss in RWR_T terms for the initial endowment (heavy line) and constant percentage contribution (dashed line) models against investment horizon.

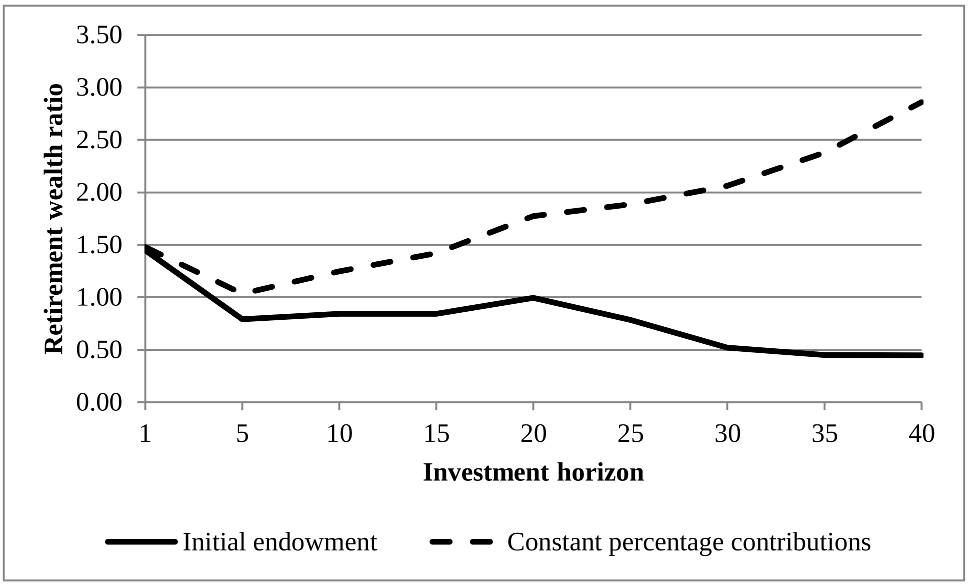

Figure 6.2 shows that over short horizons of one to five years contributions make little difference with ETL estimates for the two accumulation models differing by only a small amount. As horizon lengthens, however, the combined effects of contributions, salary growth, and returns (and associated compounding) leads to significantly different outcomes with the constant percentage contribution model yielding a 40-year RWR_T over six times larger than the initial endowment model equivalent. Thus, we see the difference that contributions and salary growth make even for the average of the five per cent worst portfolio outcomes.

6.6 Concluding remarks

From this study, we can therefore conclude that the time diversification literature suffers from two related flaws. Firstly, the literature largely ignores at least one variable – contributions – that significantly impacts terminal wealth. As we explain earlier, contributions are a well-documented feature of most developed systems of retirement savings. We therefore have both empirical and practical reasons to reject our null hypothesis: That alternative accumulation models have no bearing on the relationship between risk and investment horizon. And secondly, the measures proposed by the literature as ways of measuring time diversification, with few exceptions, ignore the influence of contributions. These exceptions are where performance is expressed in terms of terminal wealth. This distinction between wealth-based measures and return-based measures provides further support for our finding from Chapter Five which highlighted that resolution of the time diversification debate might be more related to the risk measure's measurement basis (return- or wealth-based), rather than the risk measure itself.

Because of these two flaws the time diversification literature has, to paraphrase Kritzman (2000), become a "referendum on the meaning of risk" for a mis-specified problem. From the perspective of retirement savings, whether risk rises or falls with investment horizon in the absence of contributions is not a particularly interesting question because it is devoid of important context. We can see this by comparing, say, the 95% value-at-risk estimates for the initial endowment and constant percentage contribution models for the 40-year horizon in Table 6.12. In retirement wealth ratio terms, the risk for the constant percentage contribution model – arguably the most realistic under consideration – is more than four times larger than that of the initial endowment model suggesting better downside performance. But this model also generally brings with it a wider range of outcomes depending on one's view of the asset return process.

If the time diversification debate really is about long term investing, then it should be reframed to allow realistic accumulation models to be examined, after which conclusions about the relationship between risk and investment horizon can be reached. Recent research by Basu and Drew (2009a) has shown that, as a plan member approaches retirement, portfolio size rises rapidly – the portfolio size effect – raising the potential impact of a large negative return. Macqueen and Milevsky (2009) describe this as sequencing risk, or the risk of experiencing a poor sequence of returns. Risk is thus highly path-dependent and virtually unique to the individual. The key implication of the work of Basu and Drew (2009a) and Macqueen and Milevsky (2009) is that, because of the effect of the compounding of returns, contributions and salary growth, a negative 25 per cent return five years into one's working life is not the same as an equivalent return five years prior to retirement. Another closely related way of perceiving risk was proposed by Fabozzi et al. (2006) who suggested risk be thought of as episodic, rather than as a durable trend with time. So rather than either rising or falling with time as is claimed in the time diversification literature, risk according to this view manifests itself suddenly, persists for a period, then disappears only to return later.

In study one we reviewed the time diversification literature within the initial endowment framework and concluded that the time diversification puzzle remains a puzzle because each scholar approaches the problem with a unique set of assumptions. In this sense all researchers are right *on their own terms*. In the second study we added a further two accumulation models – the constant contribution model and the constant percentage contribution model – and replicated the measures from study one in order to contrast the results. We found that, with the exception of measures expressed in terms of terminal wealth, all measures were virtually identical giving us the evidence to critique the time diversification literature. By ignoring contributions, and measuring risk the way they have, scholars have been debating a phenomenon in a context devoid of reality. Without this reality, and in the face of emerging research, the generality of such findings is limited.

Regardless of one's conceptualisation of risk, plan members are certainly subject to risk over their investing life. The question remains: Can plan sponsors do anything to manage risk over long investment horizons? Until now we have considered the performance of an all-stock portfolio in absolute terms, and in relative terms against an all-cash portfolio. The third and final study in this thesis will seek to answer this question by analysing the performance of a number of competing investment strategies in order to study whether asset allocation techniques are able to assist in managing investment risk over long investment horizons, and generate improved terminal outcomes.

7 A referendum on asset allocation

7.1 Introduction

Samuelson (1969) initiated the time diversification debate by observing how time horizon affected the optimal allocation to risky assets. Using an expected utility framework, he concluded that the allocation to risky assets was independent of time, only being a function risk tolerance. Samuelson's (1969) conclusions were based on three assumptions: (1) the investor exhibits constant relative risk aversion, (2) returns follow a random walk, and (3) wealth is a function only of returns. Much of the subsequent research within the expected utility framework has considered variations to these assumptions (Kritzman, 1994; Milevsky, 1999; Gollier, 2002), and many of the competing streams of research use these assumptions as a critique of the framework itself (Fisher and Statman, 1999; Booth, 2004).

In earlier studies in this thesis, we have pursued a competing approach to that of Samuelson (1969), and used critiques of his first two assumptions as the basis for our first study, in which we synthesise the time diversification literature on common terms. In the second study, we use a critique of the last of Samuelson's (1969) three assumptions, and of the time diversification literature in general, to motivate an examination of the impact of contributions on the question of whether stocks are more or less risky over longer horizons.

Samuelson's (1969) third assumption – that wealth is a function only of returns – despite being critiqued in the second study is at least partially true. Returns are *one* of the determinants of terminal wealth. Research has also shown that the order in which returns occur – the so-called sequence of returns – can also have a significant impact on terminal wealth (Macqueen and Milevsky, 2009). But what Samuelson (1969) does not explicitly say, and what the time diversification literature rarely studies, is that returns themselves are in turn a function of asset allocation. Does asset allocation, however conceived, have an influence on the findings of the time diversification debate?

Furthermore, the question at the heart of the time diversification debate – whether risk and time horizon are related – is not an end in itself. It is merely an initial question in a broader question about how to invest one's retirement savings to achieve one's financial goals.[81] For example, if we conclude that risk falls with investment horizon, we might incorporate this insight into the design of our pension plan by setting a target-date style investment strategy where risk is reduced according to some policy as retirement approaches.

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[81] Of course, this micro-economic question is in the realm of pension finance, itself a sub-field of financial economics. Its macro-economic counterpart relates to how we, as a society, best fund our society's retirement system. This macroeconomic question – in the realm of public economics – is a pressing public policy concern as Western developed economies face a number of significant demographic headwinds (e.g. rising dependency ratios as the Baby Boomer generation retires, and generally falling fertility rates). Some of these issues are touched upon in Chapter Three.

In this third and final study, we investigate whether the implementation of different asset allocation strategies has implications for our findings regarding time diversification. Using our nested methodological approach, we consider a range of extant investment portfolio designs, as well as introducing a basic rule-based dynamic strategy from the pension finance literature. By doing this we make a contribution to the time diversification literature by considering the relationship between risk and investment horizon for a range of asset allocation strategies in the presence of contributions and salary growth.

7.2 Literature review

7.2.1 Time diversification literature

In the literature we typically see one of two approaches in analysing time diversification. According to the first approach, scholars take a given asset allocation and then observe how investment horizon affects risk (or in some cases, risk-adjusted performance). Thus, by keeping the asset allocation constant it is possible to see how investment horizon affects risk, by observing risk measures directly. The extant literature that takes this approach tends to vary the following five dimensions: the asset return process; the modeling method; the selection of investment horizons; the accumulation model; and, the measures to be used for evaluation.

For example, in one of the earlier studies, Leibowitz and Langetieg (1989) used a Monte Carlo simulation method to generate synthetic time series in order to study shortfall risk over investment horizons of one, five, ten and 20 years. Leibowitz and Langetieg (1989) define their five dimensions as follows: there is an implicit assumption that the asset return process follows a random walk; Monte Carlo simulation is employed; investment horizons of one, five, ten and 20 years are examined; they assume an initial endowment model similar to the first accumulation model considered in this thesis; and, shortfall risk is the primary measure used for evaluation. Leibowitz and Langetieg (1989) contend that their "... analysis indicates that risk persists at surprisingly high levels, even over investment horizons as long as twenty years (p. 61)" but they fall short of expressing an opinion about the time diversification debate *per se*.[82]

Hodges et al. (1997), on the other hand, take a different approach regarding three of the five dimensions. Like Leibowitz and Langetieg (1989), they implicitly assume that asset returns evolve according to a random walk but instead employ an Efron (1979) style bootstrap method to simulate returns, as we also have in this thesis. Instead of shortfall risk, their study considers how two types of Sharpe ratio – single-period, and multi-period, Sharpe ratios – change as investment horizon lengthens. The investment horizons examined by Hodges et al. (1997) extend to 30 years for single-

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[82] On this point, the literature is not necessarily in agreement. Thorley (1995), for example, suggests that the Leibowitz and Langetieg (1989) study "assumes the validity of time diversification and concerns itself with measuring its economic significance" (p. 68).

period Sharpe ratios and 60 years for multi-period Sharpe ratios. Hodges et al.’s (1997) “results show that the Sharpe ratio cannot be meaningfully used independently of the planned investment horizon (p. 75).” Furthermore, they find that the Sharpe ratio is found to vary substantially with holding period, as do rankings among portfolios.

This first approach is the one we have pursued in the first two studies of this thesis. A variation of this approach – where a variety of competing asset allocation strategies is examined – will be employed in this study.

Where asset allocation is considered in the time diversification literature it is usually as a way of accessing the question at the core of the time diversification debate: Are stocks more or less risky as investment horizon lengthens? Rather than directly measuring risk as in the first approach, many scholars indirectly estimate risk by computing the optimal allocation to risky assets for different investment horizons. According to this approach, if the optimal allocation to stocks is higher at long horizons than at short horizons, then the risk of stocks falls as investment horizon lengthens. Thus, the allocation to risk assets for a given horizon is seen as a proxy for risk for that horizon. The higher the allocation to stocks (as the typical proxy for growth assets), the higher is the risk.

Samuelson (1969) in his seminal work in the time diversification literature, for example, found that the allocation to risky assets was constant with time, with the level of allocation being a function of risk tolerance. Van Eaton and Conover (1998) begin by directly critiquing Samuelson’s (1969) and Kritzman’s (1984) work in an expected utility framework, before turning to the work of Thorley (1995) and calling into question its generality by providing counter-examples. Van Eaton and Conover (1998) conclude that “although we have shown that rational investors may rationally refer larger risky-asset allocations at longer periods, we have not shown that any investor *should* (original emphasis) prefer such increased allocations (p. 58).” We will not pursue this second approach for two reasons. Firstly, critiques of a number of the assumptions underlying this approach motivated much of this thesis. Relying on this approach at this stage would thus be inappropriate. And, secondly, it is more challenging to study the range of accumulation models we examine is this thesis, especially those involving non-constant contributions.[83]

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[83] While Jagannathan and Kocherlakota (1996), for example, estimate the optimal allocations to risk assets in the presence of contributions, thereby demonstrating it is possible, we argue that the former of the two approaches is a more flexible way to compare the three accumulation models studied herein. It also avoids us making many of the assumptions underlying expected utility theory, which is the theoretical paradigm adopted in Jagannathan and Kocherlakota (1996), and the one critiqued herein in the guise of Samuelson’s (1969) paper.

Earlier in this thesis, we took the first of these approaches in investigating the time diversification debate. In the first study, we synthesised the time diversification literature, and replicated the important authorities in the empirical stream of the literature using a nested methodological approach. Our objective was to see whether there was a solution to the time diversification puzzle. We found that there was a solution. Scholars were generally correct in their assessment, with their different findings being a function of the way the researcher specified the five dimensions outlined above. In the second study, we used a critique Samuelson's (1969) third assumption – that wealth is only a function of returns – as the motivation for the introduction of two, arguably more realistic, accumulation models: the constant contribution model, where wealth is also a function of contributions, and the constant percentage contribution model, where a further variable, salary growth, is introduced to the accumulation model. While the results varied according to the accumulation model and simulation method employed, the conclusion of the study was clear. Contributions are a critical variable in pension finance problems, and the fact the time diversification literature largely ignores contributions represents a significant deficiency.

Throughout the first two studies asset allocation was a constant. By keeping asset allocation constant, we could observe how a variety of risk measures changed with investment horizon without any additional source of variation. Because our asset allocation was 100% stocks, we were retaining the time diversification literature's focus on the relationship between the investment risk of stocks (or related betas) and horizon. In very few cases in the literature do scholars consider the performance of competing asset allocation strategies with investment horizon, effectively testing whether asset allocation has the ability to alter the balance of investment risk. The best example in the extant literature of such a study is Hickman et al. (2001), in which the authors consider the comparative performance of four different asset allocation strategies comprised of six different assets. Hickman et al. (2001) compute mean and median terminal wealth, the standard deviation of terminal wealth, and the underperformance of each asset allocation strategy against a stock-only portfolio over investment horizons up to and including forty years. Hickman et al. (2001) has further strengths in that it is one of the few in the time diversification literature to incorporate contributions into the accumulation model.[84]

A further example with which we have some affinity is the work of Reichenstein and Dorsett (1995) who employ "... historical returns to estimate probability distributions of excess returns, real returns, and ending wealth, based on random walk and mean-reversion models, for each of eight portfolios for holding periods of 1 year through 30 years (p. 6)." They measure performance by observing distribution percentiles and shortfall risk, assuming two different return processes for a range of different portfolios over a variety of investment horizons. In relation to the last of the remaining

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[84] Furthermore, Hickman et al.'s (2001) use of constant nominal dollar contributions is very similar in nature to the constant contribution model of this study.

dimensions – the accumulation model – whilst contributions are referred to in their work (top of page 30), Reichenstein and Dorsett (1995) limit their analysis to an initial endowment model for their eight different portfolios.

We now turn to our approach which is similar in spirit to Hickman et al. (2001) and Reichenstein and Dorsett (1995).

7.3 Our approach

The purpose of this study is to investigate whether asset allocation strategies of different types – constant allocations, non-constant allocations, and dynamic allocations – influence the relationship between risk and investment horizon consistent with our research question:

RESEARCH QUESTION THREE

H_0: That asset allocation strategy has no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

Using our nested methodological approach, we will be able to contrast our results with that of the stock only portfolio analysed in the earlier two studies in this thesis. We will consider this question by computing the measures analysed earlier for the three accumulation models – the initial endowment model, the constant contribution model and the constant percentage contribution model – using four simulation methods for nine investment horizons.

7.4 Data

Once again, the data used in this study are the well-known, and commonly used, monthly stock and T-bills returns maintained by French (2012). The excess return on the market (R_m-R_f) maintained by French (2012) is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks, from the Center for Research into Security Prices (CRSP), minus the one-month Treasury bill rate, obtained from Ibbotson Associates. To arrive at nominal total stock returns, we add back the one-month Treasury bill rate. The one-month Treasury bill is used as a proxy for the low risk asset in the asset allocation strategies examined in this study.

We consider nominal total stock returns for the reasons outlined in Chapter Four (section 4.2.2).

7.5 Methodology

7.5.1 Consistent base methodology

The nested methodological approach from the first and second empirical studies, is again employed here as shown in Figure 7.1. Accordingly, we extend the analysis to a further four asset allocation strategies beyond our baseline all-stock strategy.

Figure 7.1: Nested methodological approach

This figure shows, in Venn diagram form, the nested nature of the methodological approach pursued in this thesis. In this third and final study, the set-up is identical to Study 2 except that the remaining dimension – the asset allocation – is freed and we examine a further four asset allocations for a total of five different asset allocations (3 x 5 x 4 x 9 = 540 models).

We take stock and T-bill data and, using four separate simulation methods, we simulate 10,000 synthetic returns paths for each of nine different investment horizons. The four simulation methods have been chosen for several reasons. Firstly, in order to contrast our findings with those of the time diversification literature we must replicate methods used in the literature, namely the parametric Monte Carlo method and the non-parametric bootstrap simulation method of Efron (1979). Secondly, we make a contribution to the time diversification literature by adding newer, and arguably more defensible, non-parametric block bootstrap techniques. These block bootstrap techniques also fulfill another purpose in this study: they are recognised as being better able to capture the time series characteristics of financial returns (Pascual and Ruiz, 2002; Mukherji, 2008). The nine investment horizons to be considered are one, five, 10, 15, 20, 25, 30, 35 and 40 years.

To retain consistency throughout this thesis we assume the same earnings and account balance data introduced in studies one and two. Earnings data are used as estimates of income at time $t = 0$ for the ages that correspond to the nine investment horizons examined. For example, for the 30-year investment horizon we assume that the plan member is aged 35 – retirement age of 65 years less the investment horizon of 30 years – and we use median earnings data appropriate to that age. The median account balance data represent our assumptions for initial wealth based on the corresponding median income assumptions.

The three accumulation models defined in the second study remain unchanged in this study. For the initial endowment model, as the name suggests, terminal wealth is a function of only the initial endowment (or wealth at time $t = 0$) and returns. For the constant contribution model, we introduce constant nominal dollar contributions at the rate of nine per cent per annum of income credited on a monthly basis. In the final accumulation model, the constant percentage contribution model, we introduce salary growth at the rate of three per cent per annum (applied monthly), resulting in rising nominal dollar contributions as well as a higher terminal income when compared to the other accumulation models.

7.5.2 Return-based performance measures excluded

In the first two studies in this thesis we selected and examined a number of measures that were used by scholars to judge whether time diversification existed or not. By considering these measures on the same terms we concluded that the findings of all the major studies were simultaneously correct. So, what looked like a genuine puzzle, was in fact shown to be a debate conducted at cross-purposes.

Motivated by a general critique of the literature regarding its indifference to contributions, we replicated the measures used in study one for a further two accumulation models, both of which included contributions. We found that of all the measures, only those expressed in wealth terms differed in any significant way. Many of the measures proposed in the literature were thus shown to systematically ignore the influence of contributions, and were thus set aside. To account for the higher final salary of the constant percentage contribution model, we considered the retirement wealth ratio (RWR_T) counterparts of the terminal wealth denominated measures proposed in the literature.

Because we will continue to consider all three accumulation models in this study – and for the reasons outlined in the Chapter Three on the institutional setting – we will not report any of the measures reported in the first study of this thesis. We will confine the analysis in this study to key measures of central tendency and dispersion, distribution percentiles, and downside measures of risk, all measured in retirement wealth ratio (RWR_T) terms.

7.5.3 Evaluating outcomes using the retirement wealth ratio

Because in this study we are again comparing the results of the three accumulation models, in addition to comparing the performance of different asset allocation strategies, we are forced to consider performance in RWR_T terms for the same reasons outlined in Chapter Six. We briefly reiterate our earlier point for convenience. The challenge with return or dollar-based terminal wealth measures of performance is that neither is particularly informative for the investor in terms of what performance means to their spending power in retirement. Baker, Logue, and Rader (2005), for example, argue that defined contribution plans should be measured in terms of their ability to generate sufficient retirement income, and Basu and Drew (2010) contend that a plan member's expectations will somehow be related to their salary immediately prior to their retirement. We therefore adopt Basu and Drew's (2010) retirement wealth ratio (RWR_T), which is calculated by dividing terminal wealth (W_T) by income at time T. The RWR_T provides as a way of relating terminal wealth to some benchmark for the plan member's post-retirement expectations.

In addition to providing an arguably more intuitive and informative measure of terminal wealth, RWR_T fulfills another critical role in this thesis. If we were to be evaluating terminal wealth alone, comparing alternative accumulation model would be relatively simple. But when we introduce an accumulation model incorporating salary growth, we are adding a model that has different final salary to the other two accumulation models under consideration. If we assume that final salary somehow forms expectations regarding the adequacy of terminal wealth, we therefore need to utilise a measure that has the ability to balance what is now two relevant variables: terminal wealth and final salary. RWR_T does precisely this. By creating a ratio of terminal wealth to final salary we create a measure that is both anchored to expectations, and allows comparison. Therefore, in this final study we compare performance solely in RWR_T terms.

7.5.4 Asset allocation strategies

In selecting asset allocation strategies for this study, we are seeking to fulfill a number of objectives. Firstly, in order to locate this study in the time diversification debate, and relate our findings back to the debate, we must replicate some of the important strategies represented in those few papers that look at asset allocation strategies.

A significant motivation behind the introduction of the second and third accumulation models in Chapter Six, was the incompatibility of the time diversification literature – which focuses almost exclusively on the initial endowment model – with the prevailing institutional setting outlined in Chapter Three. Therefore, when we consider the selection of representative asset allocation strategies, we must also keep the institutional setting in mind. When we speak of the institutional setting we think of at least two market perspectives. Firstly, given much of the time diversification literature has a United States focus, we seek to fairly represent the spectrum of asset allocation approaches adopted there. Apart from being the focus for the time diversification debate, the United States also happens to be a large retirement savings system where defined contribution plans are well established, and where asset allocation strategy is characterised by innovation. And, secondly, given the provenance of the author, we hope to represent the major approaches of superannuation funds in the Australian market. It is our view that, in covering the approaches taken in these two markets, we are replicating the popular asset allocation approaches taken in DC plans globally.[85]

The final objective influencing the asset allocation strategies to be considered in this study, is the need to include approaches that are at the very forefront of innovation. By analysing such strategies here we are contributing to both the pension finance literature, and the body of evidence which will underpin acceptance of these approaches in practice.

We must therefore balance the achievement of these objectives, against a desire to limit the number of strategies under consideration to the minimum number so as to avoid making the analysis and reporting of results unwieldy.

7.5.4.1 All-stock portfolio

As we argue earlier in this thesis we justify our focus on stocks in three ways. Firstly, stocks are examined in every study in the time diversification literature. We must therefore retain a focus on stocks in order to be in a position to lean on, and critique, the literature. The dataset we use also has the benefit of incorporating many of the sub-categories of stocks that are analysed in the time diversification literature, for example, small capitalisation stocks as in Hickman et al. (2001). Secondly, stocks represent the single largest risk exposure of any long-horizon defined contribution pension plan, whatever the actual stock allocation might be. Focusing on stocks is thus consistent with the institutional setting in which this thesis is located. Finally, stocks provide us with one of the longest and most reliable sources of financial data. This justification is of particular relevance to us given our methodological predisposition to data-intensive non-parametric simulation methods.

An additional reason for considering an all-stock portfolio in a study which compares the performance of various asset allocation strategies is the desire to have an immediate answer to the common refrain: How does that compare to the performance of 100% stocks? The all-stock portfolio in this context is a benchmark for a wealth maximising long horizon investment approach advocated by scholars such as Siegel (1994). By modeling the all-stock portfolio beside more common pension plan portfolio designs we can show approximations for the upper and lower limits of performance for unleveraged portfolios.

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[85] We acknowledge that there are other jurisdictions in which asset allocation approaches are characterised by innovation, e.g. the United Kingdom and the Netherlands. We would however argue that these jurisdictions are, historically at least, dominated by defined benefit plans.

7.5.4.2 Cash portfolio

In the same way that the all stock-portfolio provides the outer limits of performance for an investment portfolio, the cash portfolio gives an indication of the performance of a zero-risk portfolio over the investment horizon in question. In this sense we provide a benchmark for all other non-cash portfolios, and introduce a basis for performance evaluation. For example, having such a benchmark allows us to balance the upside benefits of growth-oriented portfolios against the opportunity cost of the same portfolio in poor equity market conditions.

7.5.4.3 Static balanced fund

Target risk funds are designed to expose the plan member to a particular static level of investment risk over the planned investment horizon. The level of risk is typically determined based on the investment objective of the fund. A higher objective would imply a greater allocation to risk assets (typically stocks), than a portfolio with a lower objective *ceteris paribus*. From a practical perspective, while the risk exposure for a target risk fund is constant by design, the underlying investments may change with time (McMurdy, 2009).[86]

Target risk funds are widespread in jurisdictions where defined contribution plans are predominant, for example, in the United States, and in Australia where they remain the cornerstone of superannuation fund offerings.[87] Target risk funds are typically labeled in such a way that indicates to the plan member the level of risk which the fund targets. For example, funds may be labeled as "conservative", "balanced" or "aggressive" in nature. Between jurisdictions, the risk exposure for a given descriptive label varies. In the United States, for example, a typical "balanced" fund would be comprised of 60 per cent growth assets and 40 per cent defensive assets; a so-called "60/40 fund" (Fidelity Investments, 2012; The Vanguard Group, 2012).[88] In contrast, Australian funds have traditionally set growth allocations at a slightly higher level than in the United States at around 70 per cent; funds are thus known as "70/30 funds." For example, SuperRatings, an information provider to the Australian superannuation industry, defines their balanced fund universe as funds with allocations to growth assets of between 60 per cent and 76 per cent (SuperRatings Pty Limited, 2012).

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[86] In the vernacular, asset allocation is constant, whereas asset selection may vary.

[87] "Superannuation" is the generic name given to retirement savings in Australia. Because the Australian system is overwhelmingly defined contribution in nature most Australians would see "superannuation funds" and "defined contribution pension plans" as equivalent concepts.

[88] According to the P&I/Towers Watson World 500 survey (Pensions & Investments, 2011), Fidelity Investments and The Vanguard Group are the fourth and fifth largest investment managers in the world (as at 31 December 2010) when ranked by total assets under management. Together these investment firms account for approximately $3.5 trillion in assets under management. Even when investment managers eschew the balanced fund label in favour of more sophisticated sounding labels a balanced-style portfolio is often used as a performance benchmark. For example, BlackRock's actively-managed Global Allocation Fund uses a 60/40 portfolio as a "reference benchmark" (BlackRock, Inc., 2012, p. 4). By adopting such a reference benchmark, this investment manager is signalling that it considers balanced funds as its competitor universe. From the perspective of a plan member, for BlackRock's approach to be of value it must outperform a 60/40 benchmark (after fees) or the (rational) plan member would consider switching to a lower cost passive balanced option.

Target risk funds, implicitly at least, accept Samuelson’s (1969) prescription that the optimal asset allocation to stocks is independent of time, and only a function of risk tolerance. A person with low risk tolerance would thus select a “conservative” target risk fund and remain invested in this fund until retirement. Where the investor disagreed with Samuelson (1969), and instead believed in the existence of time diversification, the plan member would commence their retirement savings life in an aggressive target risk fund and progressively switch to lower risk funds as retirement approaches. An assumption of plans that pursue target risk style investing is that plan members are both engaged and informed enough to make optimal decisions throughout their investment horizon in order to meet their retirement objectives. Although beyond the scope of this thesis, research has shown that this assumption may be described as heroic, and that the vast majority of plan members are neither well-prepared nor inclined to manage their retirement savings (Baldwin, 2008; Bor and Masters, 2008).

If we are to analyse the performance of a target risk fund, what level of risk should we assume and why? Because most of the time diversification research has taken place with a focus on US data, we are compelled to consider a US-focused institutional setting in order to be able to relate our findings back to the literature. In this thesis we will examine the performance of a typical US-style “balanced” fund as an approximation of a static fund design that targets a moderate risk exposure. We select this design for two reasons.

Firstly, balanced-style funds are common default options in pension plans. This means that the trustees of these funds have decided, explicitly or implicitly, that such a level of risk is appropriate for a typical inactive plan member over their accumulation phase. Therefore, if a plan member “defaults” into this fund, and remains inactive throughout their working life, they will be invested in the same balanced fund when they reach retirement. The inactive plan member is obviously an important focus for scholars, plan sponsors and pension fund managers alike because it is these individuals that must be served adequately by plan design. Active plan members are by definition more willing to make use of a range of tools in an attempt to improve their retirement outcomes. These tools might include obtaining financial advice, making additional voluntary contributions and actively selecting their investment fund option to meet their own investment objective.

The second, related reason for considering a balanced-style design is because it represents an appropriate comparator for the other plan designs to be considered in this thesis. The next design to be considered – the target date fund – has a non-constant allocation to risk assets that is determined by a pre-set glidepath. Typical extant glidepaths start with higher allocations to risk assets than a balanced fund and, as retirement approaches, this allocation falls to a level below that of a balanced fund. In approximate terms therefore, a balanced fund may be thought of as a constant allocation equivalent of a target date fund.

In our two asset world, we assume that this balanced fund has a constant allocation to stocks of 60 per cent and an allocation to cash of 40 per cent.

7.5.4.4 Target-date fund

Target-date funds are a type of age-based investing in which investors allocate a greater portion of their retirement savings to stocks, or other risky assets, when they are young and have a relatively long retirement horizon, and gradually shift this allocation towards less risky assets as they approach retirement (Yoon, 2009). The rule that determines the asset allocation at any point in time is known as the "glidepath", a metaphorical reference to the trajectory of an aircraft approaching a runway.

The rationale behind target-date funds varies but there appears to be some recognition, albeit implicit, of the time diversification debate. The conventional wisdom argues that, as assets accumulate and the target date approaches, capital protection should take precedence over performance enhancement since even small percentage losses can mean very large dollar losses in retirement funds. Moreover, when retirement (the target date) draws near, the probability that mean reversion in stock returns will restore any substantial losses is greatly reduced. Naturally, therefore, as the retirement date draws closer investors want to reduce portfolio risk in order to preserve their retirement funds. When the target date is reached, TDFs are typically folded into a balanced-style fund that keeps its target asset allocation constant, or follows a more conservative glide path (Yoon, 2009). These sorts of arguments are reminiscent of those in the time diversification literature that argue that stocks are less risky over longer horizons.

Whilst there can be significant differences between individual glidepaths, designs tend to conform to a number of general principles. In explaining these principles we refer to the target-date fund glidepath (shaded grey) shown in Figure 7.2. Target-date funds tend to have higher allocations to risk assets than comparable target risk funds over (at least) the first half of a typical 40-year investment horizon. Stock allocations typically range between 80 and 100 per cent depending on the provider. From about 20 years before the retirement date, allocations to risk assets tend to fall linearly to levels of around 60 per cent or less. In the post-retirement, or decumulation, phase risky asset allocations generally level off somewhere between 40 and 60 per cent.[89] Bennyhoff (2009), for example, shows the glidepaths of nine well-known US investment firms. At age 25 (40 years from an assumed retirement age of 65 years), stock allocations range from 80 to 100 per cent and remain relatively constant until around age 40, or around 25 years from retirement. At this point in time, the majority of glidepaths reduce stock

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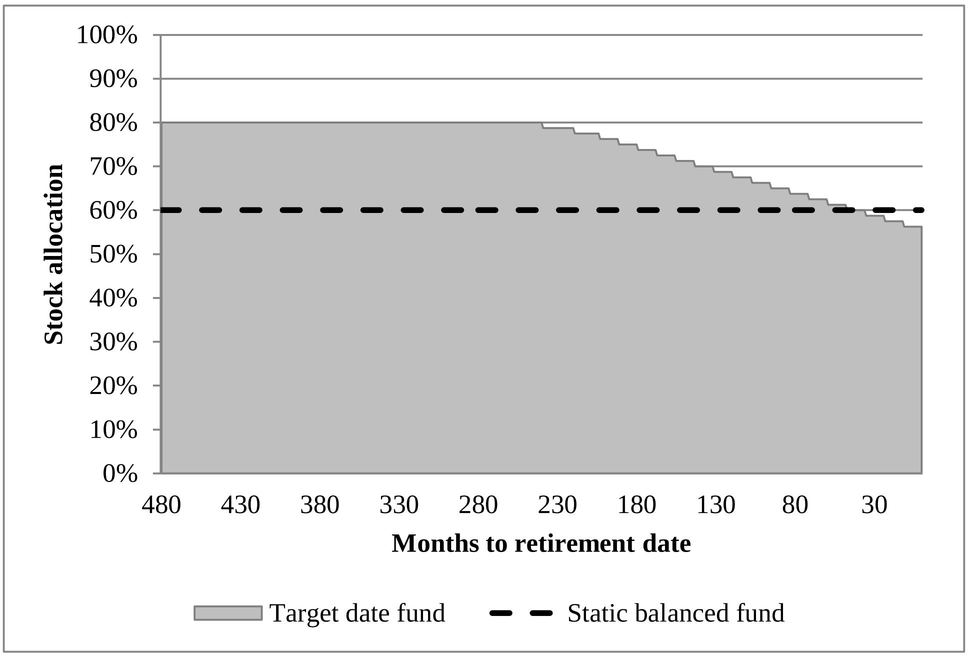
[89] Since the onset of the so-called global financial crisis (GFC), the glidepaths of target date funds have been the subject of scrutiny and review. For example, the US Department of Labor and the Securities and Exchange Commission conducted a joint hearing into the performance of target-date funds in June 2009. Testimony presented before the hearing highlighted that the "… returns of 2010 target date funds in 2008 [ranged] from minus 3.6 per cent to minus 41 per cent (Securities and Exchange Commission, 2009, p. 14)" suggesting two things: (1) designs varied widely between target-date funds; and, (2) that risk levels may have been, in hindsight, too high.

allocations linearly to levels of around 45-65 per cent at age 65 years. In a minority of cases, the glidepaths shown by Bennyhoff (2009) evolve according to a more complex algorithm e.g. curvilinear, smooth step-like function.

Figure 7.2: Glidepath design of the target-date fund

This figure shows the glidepath design of the target date fund to be considered in this thesis (shaded area) against the allocation to risk assets for the static balanced fund considered herein (dashed line).



By examining an investment strategy that implicitly assumes that risk increases as the retirement date nears we are in a position to test whether investment performance is superior as a consequence. By comparing performance of the target date fund to the static balanced fund we are able to contrast competing perspectives on the time diversification debate.

The target-date fund that will be considered in this thesis (shown in Figure 7.2) has the following design. For the first 20 years (from 480 months to 240 months to retirement), the glidepath has a constant allocation to stocks of 80 per cent. From year 20 (240 months to retirement), the allocation to stocks falls linearly *on an annual basis* from 80 per cent to 56.25 per cent at retirement. During the “descent” in the glidepath, the allocation changes once per annum by minus 1.25 per cent and this allocation is maintained for the balance of that year. When viewed on a monthly basis, as in Figure 7.2, the glidepath appears as a step function where each step is one year long. The annual basis equivalent of Figure 7.2 would have a smooth downward-sloping from 80 per cent to 56.25 per cent.

7.5.4.5 Dynamic, target-driven fund

Perhaps the most obvious similarity between target risk and target-date funds is their deterministic nature. No matter what the experience of the investor over the course of the accumulation phase, the allocation to risk assets for each of these deterministic strategies at any single point in time is known *with certainty*. For a target risk design – like a 60/40 balanced fund – at every point in the accumulation phase the allocation to stocks will be 60 per cent. Similarly, with the target-date fund, one can determine the allocation to risky assets at any point in the accumulation phase by cursory inspection of the glidepath algorithm. In theory, each of these fund strategies has been set in order to achieve a given investment objective. But once a plan member invests in one of these strategies, the investment objective and the investment strategy cease to correspond. For example, if at any point in time the investment strategy is failing to achieve the fund's objective, no corrective action is taken. The strategy remains as originally designed and is effectively ignorant of the objective.[90] As a consequence of this apparent deficiency, the following question arises: Is there a fund design that remains focused on the investment objective throughout the accumulation phase?

A dynamic target-driven fund is one possible answer we consider in this study. Before discussing a possible design we must first consider the target for the fund.

7.5.4.5.1 Setting a desired target return

The desired target return (DTR) is defined by Sortino (2010) as the annualised rate of return required from investments to be able to support expenditures during retirement. This rate of return depends on many factors, including life expectancy, contributions in retirement, and retirement income from sources other than investments. As many of these factors are not known with certainty, this rate can only be approximated (Sortino, 2010). Furthermore, because we are considering three different accumulation models it is not a simple task to set a common target return because terminal wealth in each model is a function of a different set of variables. We discuss the way in which we incorporate a common target return in competing accumulation models below.

The academic foundations of target return funds are attributed to Fishburn (1977).[91] At their core, strategies that incorporate a desired target return seek to balance upside potential against downside risk but remain primarily focused on the target. This has real practical implications for pension fund management, and for plan members. For example, when at any point in time performance is exceeding the investment target, risk will be reduced. In doing this the fund design is implicitly acknowledging it is appropriate to forego upside in order to reduce risk. Conversely, if performance

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[90] Generally, investment strategies are periodically reviewed by boards of trustees to ensure that they are achieving the goals set for them.

[91] Mukherji (2008) gives us a practical example from the time diversification literature about how target returns might figure in performance evaluation.

has been poor and the portfolio is below target, risk will be added even in cases where recent equity returns have been poor.

The target we use in this thesis is a nominal return of seven per cent per annum. The setting of such a target for this study is a matter for judgment based on at least four considerations. Firstly, the target has to be a realistically achievable average outcome over the horizon being examined. Secondly, the target must be consistent with the proposed asset allocation. Whilst the preferred approach to portfolio design sees the asset allocation as a function of the investment objective, in practice the process is not purely sequential. Typically there is feedback between the investment objective and the asset allocation, based on the risk tolerance of the investor. For example, an investor with low risk tolerance should not set an investment objective of, say, ten per cent per annum because they would be unable to tolerate the equity risk required to meet such a target.

Thirdly, for a dynamic strategy, it is critical that the investment objective not be set too high, or too low. If we were to again set a target of, say, 10 per cent per annum we raise the probability that we are below target and hence increase the probability that we are invested in the high risk portfolio. If we raise the objective far enough, we will always be below target and therefore always in the high risk portfolio. In this circumstance, the dynamic portfolio would not be dynamic at all and would tend to perform like a target-risk portfolio. The same holds for an objective that is too low. If we are always above our target then we will never need to increase our allocation to risk assets to make up for any shortfall. A moderate target therefore allows us to better ascertain the potential of a dynamic strategy of the sort examined herein.

And, finally, in order to make fair comparisons we need to select an investment target that is reasonably achievable by all strategies under consideration. We argue that a target of seven per cent per annum fulfills this criterion for all the asset allocation strategies under consideration. The exceptions would be the all-stock portfolio, which should achieve this objective relatively comfortably but with greater risk, and the cash portfolio, which is extremely unlikely to be able to achieve this objective. Recall, however, that these strategies are being included for quite specific reasons: the all-stock portfolio to show the outer limits of performance and risk for unlevered portfolios, and as an informal benchmark for all other growth-oriented portfolios; and, the cash portfolio to provide an indication of the performance of a zero-risk portfolio.

In this study, we will extend again the nested methodological approach employed in the first two studies in this thesis. In particular, we will extend the analysis of the all-stock portfolio in the second study to the four additional investment strategies introduced in this study. We will thus be considering the same three accumulation models, and will consequently need to compare performance in retirement wealth ratio terms because final salary will be higher for the constant percentage contribution model due to the effects of salary growth.

Earlier in this thesis we cited the work of Baker, Logue and Rader (2005) who have argued that defined contribution plans should be measured in terms of their ability to generate sufficient retirement income. This argument motivated our use of the retirement wealth ratio (RWR_T) of Basu and Drew (2010). How do we incorporate our target return for three different accumulation models and make retirement wealth ratio the principal mechanism for the dynamic strategy, and the terms in which performance is reported? The modeling process tracks the retirement wealth ratio throughout the investment horizon, RWR_t, and compares it to the target RWR, RWR_{target}. If RWR_t is greater than RWR_{target}, wealth at time t, W_t, is invested in the low risk portfolio for one month after which RWR_{t+1} is compared to RWR_{target} and the next portfolio decision is made. Conversely, where RWR_t is less than RWR_{target}, portfolio wealth W_t is invested in the high risk portfolio in attempt to recover the deficit versus the target.

A practical difficulty with this approach is that there is no universal RWR_{target}. Rather, there is a unique RWR_{target} for each combination of investment horizon and accumulation model meaning we have a total of 27 different target retirement wealth ratios. For each combination of investment horizon and accumulation model, we replicate a single wealth path assuming our nominal return target of seven per cent per annum. We are thus simulating the experience of a plan member that earns the investment objective for every year of the given investment horizon. We then calculate the terminal retirement wealth ratio (RWR_T) by dividing terminal wealth by final salary. The calculated RWR_{target} for a given accumulation model and investment horizon is used as the target upon which the dynamic strategy makes it periodic asset allocation decisions. It must be emphasised though that while the RWR_{target} differs for each of the 27 different combinations of investment horizon and accumulation models, the return used to generate the target RWRs is seven per cent per annum in each case. With a common return, we can isolate the differences in performance due to the accumulation model for a given investment horizon and simulation method.

However, while the issue of a desired target is vital, the issue of managing the glidepath in a dynamic, or non-deterministic, manner remains. One potential solution, which we analyse in this thesis, is the dynamic, target-driven strategy.

7.5.4.5.2 Dynamic strategy design

The dynamic strategy studied herein will be of the same nature to that studied by Basu, Byrne and Drew (2011). The strategy begins with a constant allocation of 100 per cent stocks for the first 30 years of our assumed 40-year accumulation phase. The rationale for this high allocation to risk assets is that with a minimum investment horizon of 10 years the objective of the plan member over the first 30 years of his investing life will be the maximisation of wealth. Conventional wisdom would also have it that over a 10-year horizon the plan member will have sufficient time to recover wealth lost due to a large drawdown. As we lead into the last 10 years – the time during which the portfolio size effect becomes manifest (Basu and Drew, 2009a) – we seek to reduce risk when we are on target to achieve the fund objective. So, for the last 10 years of the accumulation phase, there is a binary choice between a high risk portfolio when cumulative performance is below target, or a low risk portfolio when performance is above target. In this way, when we are short of our target we increase risk to recover ground, rather than maintaining our asset allocation based on an arbitrary pre-determined rule as happens in target risk and target-date fund designs. Similarly, when we are exceeding our target return we maintain a lower level of risk for as long as we remain above this target. The strategy is truly target-driven in that progress against the target is monitored and used to adjust the asset allocation accordingly.

The last ten years of the 40-year accumulation phase is divided into two separate five year periods with slightly different asset allocation rules. For both periods, the below-target portfolio is 100 per cent stocks. The rationale for this allocation is based on the importance of the target. If we are below the target we increase risk in an effort to recover ground such that wealth exceeds the target at time t. Recall that our ultimate objective is that wealth at time T meets or exceeds our target. Between years 30 and 35 (from 10 to 5 years from the retirement date), when wealth exceeds the target we allocate 80 per cent to stocks and 20 per cent to cash. In the final five years of the accumulation phase (between years 35 and 40 of the accumulation phase), the above-target allocation to stocks is 60 per cent, with the remaining 40 per cent invested in cash. In this way, the maximum risk allocation when we are above target falls as we approach the investment horizon. Conversely, when we are below target we will always assume the maximum amount risk in order to ensure we meet our target. We have therefore essentially constructed a dynamic counterpart to the target date fund.

The reason why we consider this simple dynamic strategy is to contrast its performance to comparators that are representative of extant portfolio designs. By doing this we set out to study whether it is possible to alter the balance of risk using a simple rule-based asset allocation. In reality, there are a number of aspects of this design which would make it unlikely to be implemented in practice. Firstly, such large changes in asset allocation are unlikely to be approved by boards of trustees on risk management grounds alone. Furthermore, such large swings in asset allocation would also impact returns through significant transaction costs, particularly in the foreseeable circumstance where cumulative performance is near the target and these large swings in allocations could occur quite frequently as performance switches above and below target in successive periods. Thirdly, from a behavioural perspective, plan members may have significant difficulty accepting a portfolio rule which reduces risk amidst a bull market when recent performance has been strong, or adds risk during a bear market when immediate past performance has been poor. Practically a dynamic investment strategy would be more nuanced and would take account of financial market conditions in addition to progress against an investment target.

This should not however distract us from the objective of this study: which is to compare a small number of competing approaches to asset allocation to observe whether our findings regarding the time diversification debate can be influenced through plan design.

7.6 Empirical evidence

In the first two studies in this thesis we examined the performance of an all-stock portfolio for each of three accumulation models – the initial endowment, the constant contribution and the constant percentage contribution models. For each of these accumulation models, we estimated Monte Carlo, Efron (1979) bootstrap, stationary bootstrap, and empirical block bootstrap models for each of nine horizons (1, 5, 10, 15, 20, 25, 30, 35, 40 years), for a total of 108 models. Although asset allocation strategies other than an all-stock portfolio haven’t been mentioned until this study, when we estimated the initial 108 models, we simultaneously estimated models for each of the asset allocation strategies which we outline and study here. This means that for each of the 108 model permutations we have five sets of results corresponding to the number of asset allocation strategies we consider below. This way of conducting the modeling task has one particular real benefit: the performance of all five asset allocation strategies are estimated from an *identical* set of 10,000 simulated return paths. Thus, any differences in performance amongst the asset allocation strategies for any one of the 108 models are *entirely* the result of differences in strategy design. There is no minor variation to results introduced by conducting separate simulation procedures for each of the asset allocation strategies.

For ease of comparison, we repeat the results for the 108 models (three accumulation models, times four techniques, times nine investment horizons) estimated for the all-stock portfolios in the first two studies of this thesis. Therefore, in the below tables we report 108 versions of the same measure for each of five asset allocation strategies. By comparing the various asset allocation strategies against the baseline all-stock portfolio for all 108 model permutations we are able to isolate the influence of asset allocation strategy upon performance.

7.6.1 Extending the analysis to consider asset allocation strategies

In this section we extend the analysis in the first two studies of this thesis to consider how different asset allocations affect our earlier findings. Continuing the theme, this study may be considered a referendum on asset allocation as it relates to the time diversification debate. We will compare five different asset allocation strategies *on the same basis*, in order to provide insights as to the importance of asset allocation to the relationship between investment risk and investment horizon.

This study introduces a further four asset allocation strategies to the all-stock strategy considered until this point. As a result we have five times the results to report and discuss. We will therefore be selective and confine our discussion to those results which illustrate the findings of this thesis. In the interests of convenience, and brevity in the thesis proper, we show in the body of this study only those results referred to in the text. For completeness, all output for this study (including that illustrated in this chapter) is reported for each asset allocation strategy in Appendix A, for the distributions of retirement wealth ratios, in Appendix B, for downside risk measures, and in Appendix C, for target-relative measures.

7.6.1.1 Consistency with the findings of earlier time diversification studies

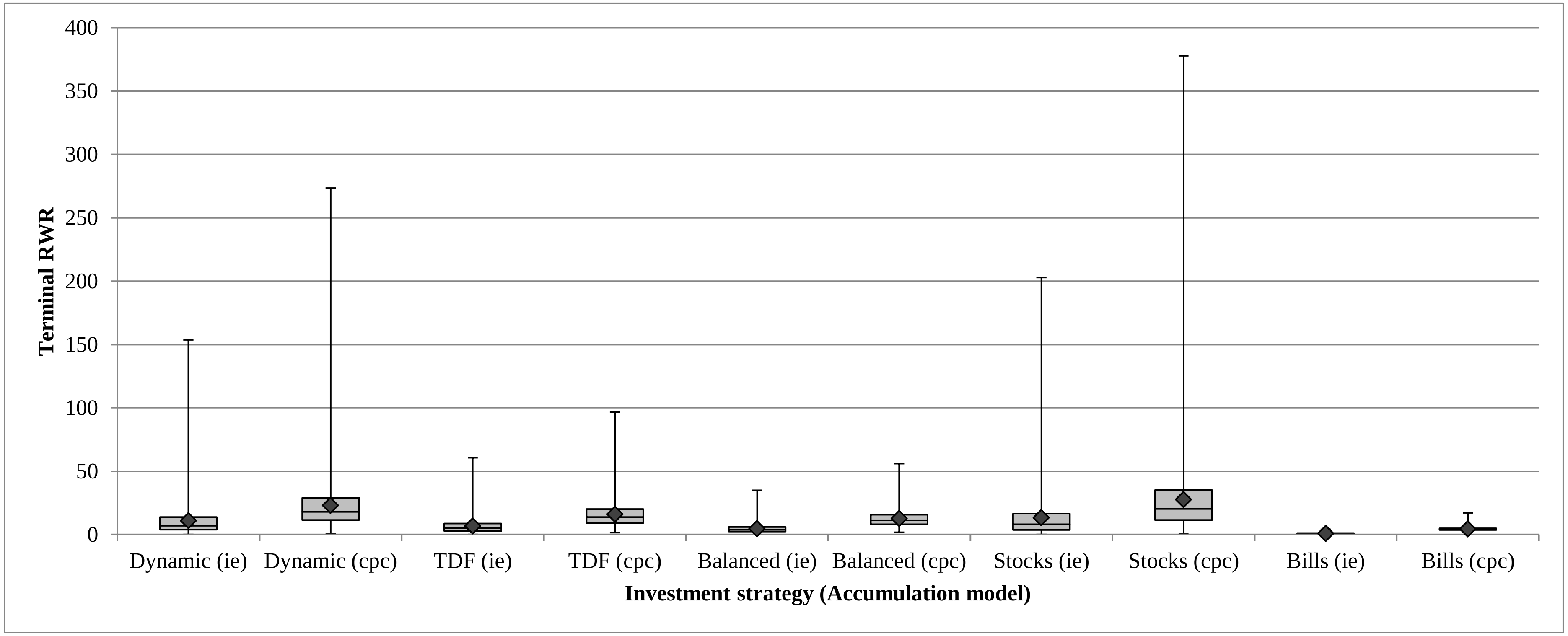
In the second study of this thesis, we introduced the retirement wealth ratio at retirement date T, RWR_T, as a way of comparing the performance of competing accumulation models for an all-stock asset allocation. Recall that a measure like the retirement wealth ratio was found to be necessary because it allows us to adjust terminal wealth for differing levels of final salary, which has been argued in the literature to be an anchor for retirement expectations. For the all-stock portfolio, we found that there were large absolute differences between the results for the three accumulation models which confirmed two findings from this thesis: (1) that contributions are a critical variable in realistic pension finance problems; and, (2) that the extant time diversification literature largely overlooks their importance.

The large absolute differences between the results for the accumulation models persist irrespective of the asset allocation strategy examined, or the measure under consideration, although the magnitude of the difference is predictably larger at longer horizons, and for higher risk asset allocations. Figure 7.3, for example, compares terminal retirement wealth ratios for the initial endowment (ie) and constant percentage contribution (cpc) models for each of the five asset allocation strategies outlined in section 7.5.4. In each case, we are considering the estimates computed using the stationary bootstrap (bs) simulation method over a 40-year horizon. Therefore, the only difference between each of the pairs of results for each strategy in Figure 7.3 is the accumulation model. Thus we are essentially considering the difference that contributions and salary growth makes to terminal wealth when expressed in retirement wealth ratio terms. This comparison can also be thought of as a direct contrast between the accumulation model favoured in the time diversification literature – the initial endowment model – and the accumulation model observed in many retirement systems globally. The dramatic differences between estimates for each strategy, and the consistency of these differences amongst a wide variety of investment strategies, allows us to conclude that contributions remain an important variable no matter what the investment strategy, and re-emphasises that the time diversification literature omits a significant variable in the determination of terminal wealth.

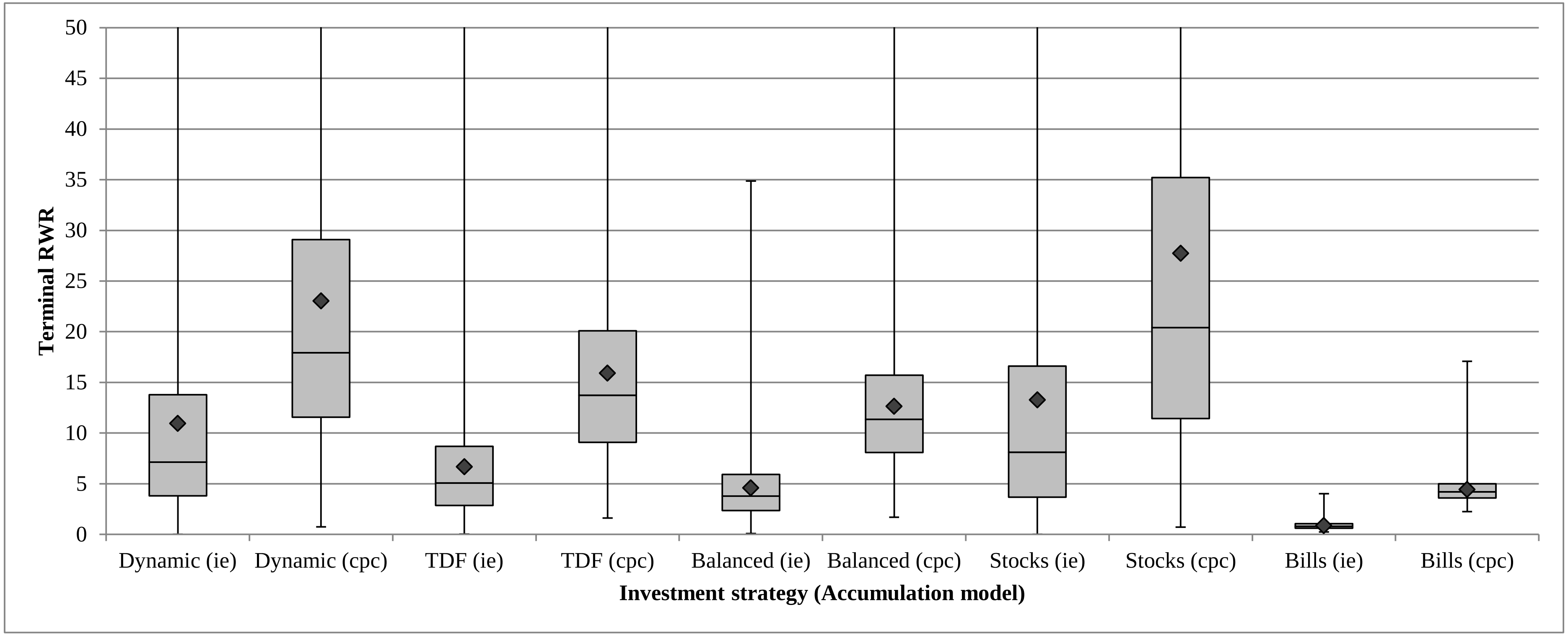
Figure 7.3: Comparison of accumulation models by asset allocation strategy

Using the stationary bootstrap (bs) simulation method, this figure presents comparative box-and-whisker plots for the initial endowment (ie) and the constant percentage contribution (cpc) accumulation models for each of the five asset allocation strategies over a 40-year investment horizon. The horizontal lines for each box represent, from the top, the 75th percentile, median and 25th percentile retirement wealth ratios (RWR_T). The upper and lower error bars are the maximum and minimum retirement wealth ratios respectively. The diamond-shaped marker represents the mean RWR_T for each model. Panel A shows the complete box-and-whisker plot, whereas Panel B focuses on retirement wealth ratios less than or equal to 50 times (y-axis) in the interests of viewability.

PANEL A – Complete distribution



PANEL B – RWR_Ts less than or equal 50 times



In the last chapter we also showed for the all-stock strategy, that nearly all retirement wealth ratio estimates for the constant contribution and constant percentage contribution models increase in magnitude with investment horizon. We also showed that this increase in the magnitude of a measure with horizon generally held for the minimum retirement wealth ratio path, contrary to our findings relating to minimum RWRs for the initial endowment model which tend to reduce in magnitude (i.e. worsen) with horizon. For each of the asset allocation strategies examined in this study both of these generalisations are again supported by the evidence.

Where a measure does not increase monotonically with horizon, it is usually for one or both of the following two reasons: (1) the asset allocation strategy under consideration is lower risk in nature; and/ or, (2) the measure under consideration represents the poorer end of all outcomes e.g. the minimum retirement wealth ratio. To illustrate the first exception we consider the range of retirement wealth ratio estimates for Cash strategy in Table 7.1. In particular we consider the initial endowment model – which we studied in detail in Chapter Five – and the constant percentage contribution model – the model which we argue best represents the prevailing institutional setting. Notice firstly that the estimates for the initial endowment model form a hump-shaped profile with investment horizon, mirroring the hump-shaped profile of the initial wealth (W_0) discussed earlier. Because we have a low risk investment strategy, we can conclude that returns are not of a sufficient magnitude to outweigh the influence of initial wealth, even when we are considering a measure such as the range of outcomes.[92]

When we turn the constant percentage contributions model in the right half of Table 7.1 we see that for one of the simulation methods – Monte Carlo simulation (mc) – the addition of both contributions and salary growth, and the compounding effects of each, have been sufficient to cause the estimates to rise monotonically with time. However for the remaining three simulation methods – the Efron (1979) bootstrap, the stationary bootstrap (bs) and the empirical block bootstrap (bb) – low cash returns even when combined with contributions, salary growth and compounding are insufficient to obscure the influence of initial wealth. This suggests that initial wealth can have a significant impact on terminal wealth even over long horizons, and even when combined with accumulation models that incorporate contributions. These mixed results for the constant percentage contribution model also suggest that the nature of the asset return process may influence our estimates.

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[92] Recall that a number of studies in the time diversification literature established that the range of RWR_T estimates generally increases with investment horizon (e.g. McEnally, 1985; Mukherji, 2008).

Another feature of the results reported in Table 7.1 is the large differences in magnitude between corresponding estimates for each of the two accumulation models *through time*. Table 7.1 in this way gives us a slightly different perspective of the influence of contributions to that provided in Figure 7.3. Rather than comparing the magnitudes of estimates between accumulation models for a given horizon and simulation method as in Figure 7.3, Table 7.1 allows us to compare the magnitudes of RWR_T estimates between accumulation models for a single strategy with investment horizon.

Table 7.1: Retirement wealth ratio range – Cash strategy

This table presents the retirement wealth ratio range for the four modeling techniques over nine investment horizons for the two of the three accumulation models – the initial endowment model and the constant percentage contributions model. Initial wealth (W_0) is as per row five of Table 4.3. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). Results are presented as multiples (i.e. in unit of "times" as in "*x* times final salary"). This is an excerpt from Panel D, Table A.5 in Appendix A.

Investment	**Initial endowment**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.33	0.34	3.77	2.46	1.19	1.15	14.81	6.59
35	0.40	0.40	3.64	2.54	0.92	0.93	8.05	6.03
30	0.39	0.42	3.74	3.34	0.81	0.78	5.78	4.13
25	0.62	0.62	4.99	3.79	0.78	0.71	6.10	4.80
20	0.77	0.79	6.93	4.31	0.73	0.78	6.41	4.75
15	0.65	0.63	4.69	3.80	0.58	0.62	4.44	3.20
10	0.51	0.50	3.02	2.90	0.45	0.55	2.87	2.55
5	0.29	0.29	1.24	1.37	0.27	0.28	1.34	1.28
1	0.15	0.15	0.32	0.32	0.14	0.15	0.31	0.31

For the constant percentage contribution model, RWR_T range estimates tend to rise with investment horizon and the absolute difference at longer horizons is large. Take, for example, the relativities between the 20- and 40-year range estimates for the empirical block bootstrap model. At the 20-year horizon, the range estimate for the constant percentage contribution model is only 1.1 times larger than the corresponding estimate for the initial endowment model (4.75 divided by 4.31). When we reach the 40-year horizon, the estimate for the initial endowment model is less than half that of the constant percentage contributions model. This provides yet more evidence that the near universal absence of contributions from the time diversification literature is a significant deficiency. This rapid rise in range in the latter half of the investment horizon also provides evidentiary support for the portfolio size effect identified by Basu and Drew (2009a).

But, in terms of retirement outcomes, the important question is what proportion of this increase in the range or terminal retirement wealth ratios is positive versus negative? Unlike returns, contributions are a purely positive contributor to portfolio wealth throughout the accumulation phase so we would expect that the increase in range observed for accumulation models involving contributions would be skewed to the positive. By returning to Figure 7.3 we can see our expectations confirmed. Contributions, salary growth and the associated compounding effects through time sees the range of positive outcomes – the spread of outcomes between the maximum and median retirement wealth ratios – grow faster and to a greater magnitude than do the negative outcomes – the spread between the median and minimum retirement wealth ratios. Over the 40-year horizon shown in Figure 7.3 the positive half of the distribution of terminal retirement wealth ratios is so large that we are forced to include Panel B to allow us to see the detail of the each distribution.

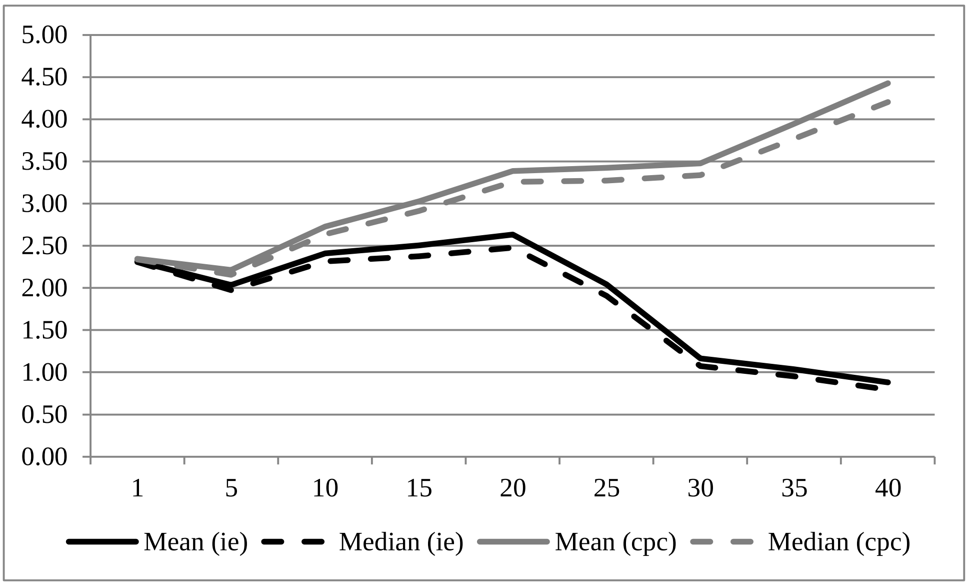
Another way of highlighting this particular phenomenon is by comparing the means and medians of the initial endowment and constant percentage contribution models through time for a given combination of strategy and simulation method. In Figure 7.4 we compare these two accumulation models for the cash strategy modeled using the stationary bootstrap method. In comparing the means and medians for these two models, we should see two results. Firstly, we would expect to see different profiles for each of the two accumulation models as a result of their different set-ups. For, the constant percentage contribution model we should see the mean and median both rise with investment horizon due to the addition of contributions. For the initial endowment model, we would expect to see signs of the profile of the initial endowment assumption because of the absence of contributions, and the relatively low returns provided by the cash strategy. Each of these expectations are born out in Panel A of Figure 7.4: the initial endowment model series are hump-shaped like the profile of the initial endowment assumptions used in the modeling, and the mean and median of the constant percentage contribution model series rise consistently with investment horizon. The difference in distance between the grey series (e.g. the mean) and its black counterpart may be interpreted as the impact of contributions and salary growth on terminal wealth. Unsurprisingly, the gap between the models rises with investment horizon. We would expect to see similar relativities between the means and medians of these accumulation models for other investment strategies. The gradients would however differ with steeper gradients for investment strategies with higher allocations to risk assets.

Secondly, we should see a positive distance between the mean and median through time reflecting the impact of strongly positive paths inflating the mean. We see this positive difference for each accumulation model shown in Panel B. Panel B also shows that the initial endowment series is hump-shaped, consistent with the profile of the initial endowment assumptions of the model. This suggests that the comparatively low returns from the cash strategy are insufficient to outweigh the importance of the initial endowment, and that it is the prime variable in the evolution of terminal wealth for the cash strategy. Conversely, consider the constant percentage contribution model series which rises as horizon lengthens. This rising profile suggests that, as horizon lengthens, strong positive paths resulting from the combination of returns, contributions, salary growth and the associated compounding effects cause the mean to rise faster than the median.

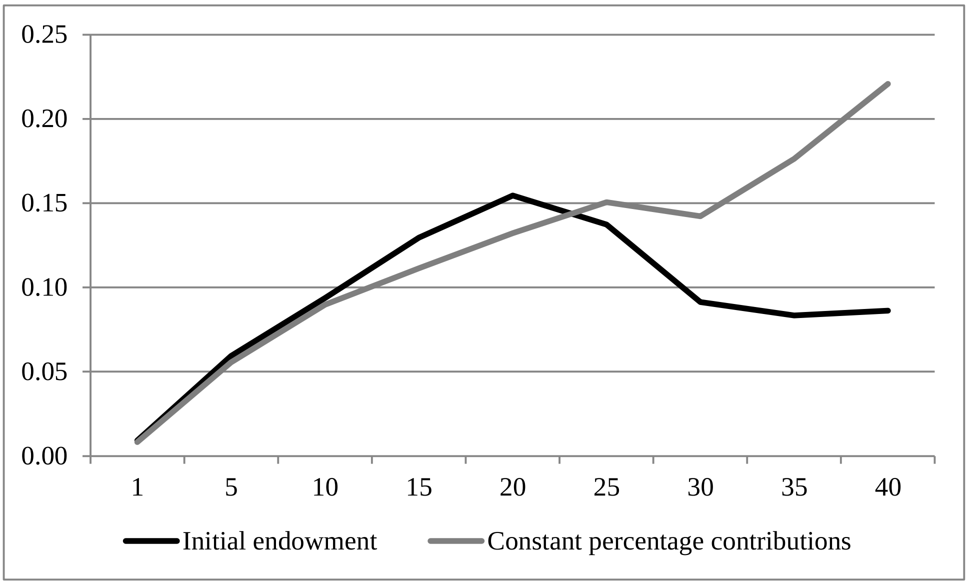
Figure 7.4: Comparison of means and medians with time horizon – Cash strategy

Using the stationary bootstrap (bs) simulation method, Panel A presents the mean and the median RWR_T for the cash strategy for two accumulation models – the initial endowment model (ie) in black and the constant percentage contribution model (cpc) in grey – for each investment horizon. Means are shown as full lines, and medians are shown as dashed lines. Panel B shows the difference between the mean and median for each of the two accumulation models.

PANEL A – Means and medians by accumulation model



PANEL B – Differences by accumulation model



For the latter exception to the generally positive relationship between investment horizon and the measure being considered, consider Table 7.2 which shows the minimum retirement wealth ratio path for the target-date strategy for the initial endowment and constant percentage contribution models. For all simulation methods employed for the initial endowment model, we observe an almost perfect *inverse* relationship between investment horizon and minimum retirement wealth. Two influences conspire to cause this effect. Firstly, the absence of contributions from the initial endowment models means there are no periodic inflows to the portfolio to support the retirement wealth ratio. And, secondly, the adverse return outcomes for this minimum RWR path are sufficiently poor to erase any sign of the hump-shaped profile of initial wealth. When we consider the constant percentage contribution model, we see the difference that growing contributions make through time. The absolute values for all RWR estimates for the constant percentage contributions model are significantly higher than the corresponding initial endowment estimate, with the magnitude of the difference rising with time as growing contributions compound. While the estimates generally rise with time, for each of the simulation methods we see that estimates do not always rise over successive investment horizons. For example, for the empirical block bootstrap method (bb), our hypothetical investor earns less over a 35-year horizon (0.96 times final salary) when compared to a 30-year horizon (1.08 times final salary). This result, and the poor absolute results shown in in Table 7.2, reminds us that even with contributions, returns are an important variable. This and earlier findings motivate our comparison of asset allocation strategies later in this study.

Table 7.2: Minimum retirement wealth ratio – Target-date strategy

This table presents the minimum retirement wealth ratio for the four modeling techniques over nine investment horizons for the two of the three accumulation models – the initial endowment model and the constant percentage contributions model. Initial wealth (W_0) is as per row five of Table 4.3. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). Results are presented as multiples (i.e. in unit of "times" as in "*x* times final salary"). This is an excerpt from Panel E, Table A.3 in Appendix A.

Investment	Initial endowment				Constant percentage contributions			
horizon	mc	b	bs	bb	mc	b	bs	bb
40	0.21	0.16	0.04	0.01	1.72	2.00	1.62	1.28
35	0.19	0.18	0.09	0.08	1.88	1.90	1.32	0.96
30	0.21	0.18	0.11	0.06	1.54	1.47	1.04	1.08
25	0.43	0.44	0.31	0.20	1.41	1.56	1.12	0.88
20	0.73	0.68	0.47	0.22	1.43	0.93	1.18	0.68
15	0.78	0.63	0.50	0.20	1.14	1.08	1.08	0.78
10	0.88	0.86	0.43	0.51	1.15	1.28	1.00	0.55
5	0.88	1.03	0.48	0.36	1.15	0.91	0.66	0.53
1	1.66	1.55	1.28	1.28	1.66	1.62	1.31	1.31

As we observed for the all-stock strategy in Chapter Six, the estimates for value-at-risk and expected tail loss for the constant contribution and the constant percentage contribution models have a positive relationship with time for all four new strategies, in contrast with the mixed relationship observed for the initial endowment model (see Appendix B). For the initial endowment model, downside risk appears to have less of a relationship with time than with initial endowment. Because in downside risk scenarios returns are poor by construction, other variables comprise a greater part of terminal wealth over any given horizon. We are therefore not surprised to see that we observe a hump-shaped trend amongst downside risk measures mirroring the same profile for initial wealth. When we move to the models that include contributions we see that the profile of downside risk changes due to the positive impact of contributions, salary growth and the associated compounding effects.

7.6.1.2 Consistent with relevant asset allocation studies

Reichenstein and Dorsett (1995) analyses many of the variables we consider in this thesis. For example, they estimate the probability distributions of terminal wealth for eight portfolios for investment horizons ranging from one through 30 years. They also consider two different asset return processes (random walk and mean reversion), which we envisage in selecting the simulation methods used in this study. The Monte Carlo and Efron (1979) bootstrap methods explicitly assume a random walk, whereas the block bootstrap methods employed herein are consciously chosen to capture the time series properties in the data, which would include a mean reversion.

While Reichenstein and Dorsett (1995) do examine a range of portfolio designs, the goal of their study was not to examine the interplay of portfolio choice and the time diversification debate. Rather, their study's goals included: "to assess the investment implications of time diversification" (p. 46); and, "to provide investors with a picture of the risk-return trade-off of several portfolios for holding periods of 1 through 30 years" (p. 47). So while much of their analysis is of interest to us, they did not seek to link these goals and see whether the implications of time diversification varied with asset allocation. Therefore, while Reichenstein and Dorsett's (1995) study is not of direct relevance as far as its findings go, the design of their study and some general results are certainly of interest.

Of the few papers from the time diversification literature that compare asset allocations, Hickman et al. (2001) is probably the most relevant for a variety of reasons. Most pertinent to this study is the fact that the authors compare four different asset allocation strategies: Malkiel, 100 – Age, Implied and S&P. Their strategies while not precisely the same as the five examined in this study cover similar ground. The Malkiel, 100 – Age and Implied strategies each have non-constant allocations to risk assets and so are similar to the target-date strategy that we model. The S&P strategy of Hickman et al. (2001) is very similar to our all-stock strategy, although our stock universe is broader. We make a contribution to the time diversification literature by also considering a cash strategy, a constant allocation balanced strategy and a simple dynamic target-driven strategy. Hickman et al. (2001) assumes a basic accumulation model which includes constant nominal dollar contributions making it roughly equivalent to our second accumulation model, although we argue that our analysis is more meaningful in dollar terms because we attempt to incorporate some reality into our assumptions regarding initial wealth, contribution rates, and salary for a given age. This thesis is methodologically similar to Hickman et al. (2001) in that we both use simulation methods with replacement. Because Hickman et al. (2001) only analyse one accumulation model, they are able to compare performance between strategies in terminal wealth terms, whereas we need to consider performance in retirement wealth ratio terms because we are comparing outcomes across three accumulation models where terminal wealth isn't directly comparable because of different final salary. While we use *RWR*, in essence we agree with Hickman et al. (2001) that terminal wealth is the correct terms in which to evaluate the performance.

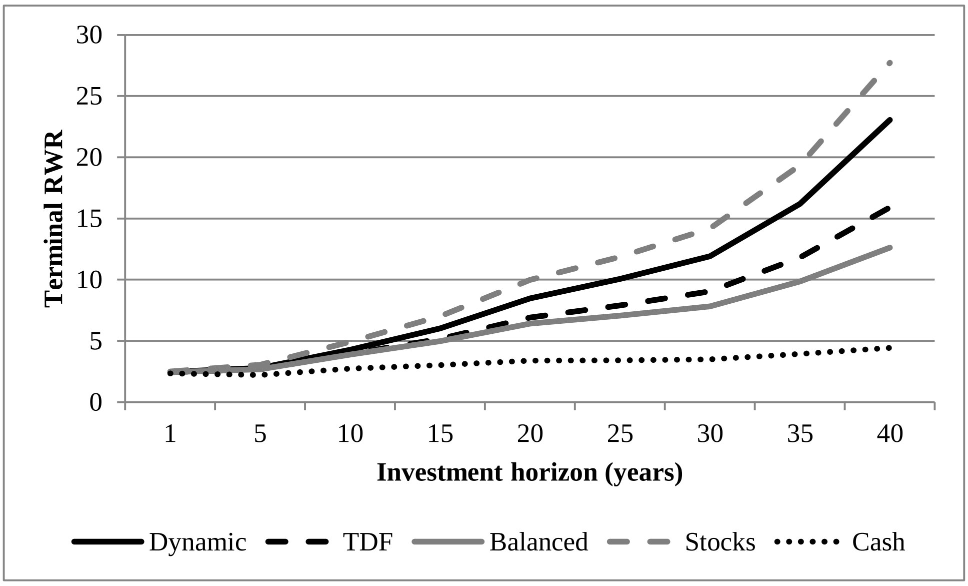
In order to simplify the comparisons we make with Hickman et al. (2001) we will focus on one accumulation model – the constant percentage contribution (cpc) model – and the results from one simulation method – the stationary bootstrap method. We select these because we argue they are the most realistic of all those studied. The constant percentage accumulation model is consistent with the institutional setting as discussed in Chapter Three of this thesis. The stationary bootstrap method is designed to model the time series characteristics of the data, and has the demonstrated ability to generate simulated distributions with similar statistical characteristics as the sample dataset, namely negative skewness and leptokurtosis.

One of the major findings of Hickman et al. (2001) is that, over a 40 year horizon, the cost of adopting an asset allocation strategy other than the all-stock portfolio "is high in terms of foregone wealth" (p. 110) and the benefits in terms of underperformance avoided "appear insignificant" (p. 110). Furthermore, they find that over shorter horizons the benefits of adopting asset allocation strategies increase, suggesting that all-stock portfolios might not be as superior over short horizons. In comparing our analysis to that of Hickman et al. (2001) we point to Figure 7.5, where we show two different types of expectations – expected return, or mean, and expected tail loss – that will allow us to compare our results to the findings of Hickman et al. (2001).

Figure 7.5: Comparison of investment strategies – Absolute performance

Using the stationary bootstrap (bs) simulation method, Panel A presents the mean RWR_T for each strategy for the constant percentage contribution model (cpc) with investment horizon. Panel B shows the 95% expected tail loss, in RWR_T terms, for each strategy for the constant percentage contribution model (cpc) with investment horizon.

PANEL A – Mean RWR_T



PANEL B – 95% Expected tail loss in RWR_T terms

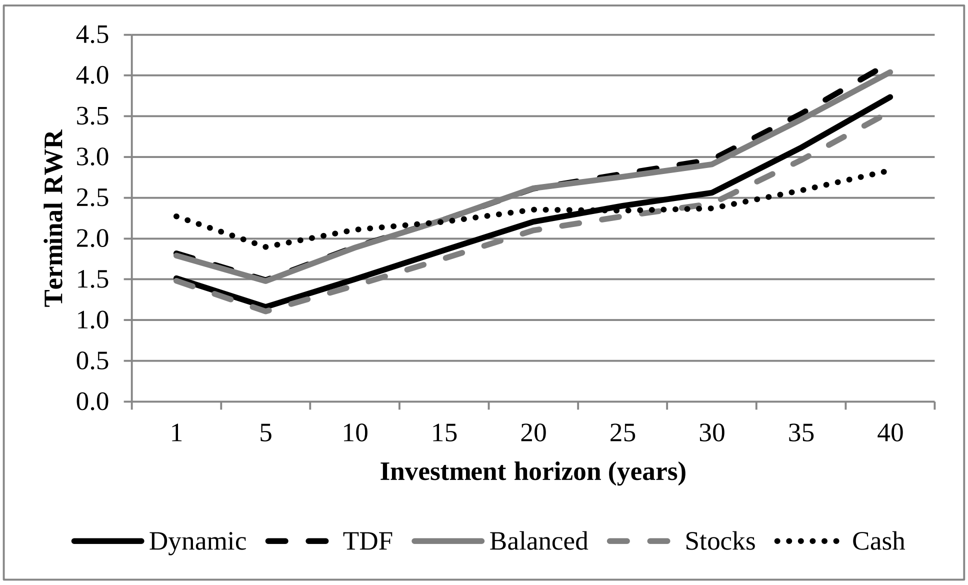


Figure 7.5 confirms the findings of Hickman et al. (2001) that, by adopting an asset allocation strategy other than the all-stock portfolio, one gives up significant upside with little in the way of downside benefits. Compare, for example, the absolute differences between each series in Panel A versus Panel B.

Turning to the distribution of terminal wealth, Hickman et al. (2001) find that all models over all holding periods exhibit positive skewness which they identify as occasions where "mean exceeds median" (p. 110). Our analysis strongly supports this finding and we show estimates of means and medians for the cash strategy over our nine horizons in Figure 7.4 in the previous section of this study.[93] We argue that this positive skewness results from the combination of two things: contributions and compounding. The effects of both of these variables – but especially contributions – are persistently positive contributors to wealth even during poor market conditions.

7.6.1.3 Asset allocation findings

In comparing the results of this study to the work of Hickman et al. (2001), we found that growth-oriented asset allocation strategies, when compared to an all-stock strategy, produced significantly less upside potential but without material improvements in downside performance. Over shorter horizons the evidence is more mixed. We are therefore left to wonder whether it is indeed worth the effort to attempt to design an asset allocation strategy to outperform an all-stock portfolio. Are we not just better off to take Siegel's (1994) prescription and invest in "stocks for the long run?"

On two counts we answer in the negative. Firstly, when one invests in an all-stock portfolio one doesn't access the whole distribution of potential outcomes – which we have shown is quite attractive – one gets an unknown *single* path of returns. We must therefore seek to design a strategy which is robust to a range of potential futures including those rare but catastrophic ones. And, secondly, when investing for retirement we are generally seeking to generate enough terminal wealth to fund an adequate income stream, where "adequate" is in practice defined by the board of trustees on behalf of individual plan members. In this sense we aren't interested in the pure maximisation of wealth. Perhaps then we are willing to forego upside in order to create some certainty around a particular level of terminal wealth.

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[93] Recall, in Figure 7.2 we compare means and medians for two accumulation models – the initial endowment (ie) and constant percentage contribution (cpc) models – over nine investment horizons modelled using the stationary bootstrap simulation method (bs). The strategy we consider in Figure 7.2 is the cash strategy. If we were to show higher risk strategies, the contrast between the means and medians would be even larger.

This brings us back to the very objective behind the dynamic strategy we consider in this study: to maximise our chances of achieving our return target. The time diversification literature is silent on such strategies and where asset allocation strategy is considered – as in Hickman et al. (2001) – the implicit and overriding objective is terminal wealth maximisation versus a range of comparators. Where shortfall risk is a consideration, the threshold is typically the risk-free asset or, in the case of Hickman et al. (2001), the all-stock portfolio. Expressed in simple terms, the time diversification literature suggests that more is better. In this study, by examining a dynamic target-driven strategy, we seek to investigate whether a particular level of wealth *with more certainty* is indeed possible.

In order to evaluate target-relative performance we introduce variations to three relatively common performance measures. But before introducing these measures, we reiterate that in this study there are 27 different target retirement wealth ratios, corresponding to the 27 different combinations of accumulation model and investment horizon. The reason for these different targets is that we need to be able to compare performance across accumulation models. For example, if a plan member is making contributions to their retirement plan, she would target a higher retirement wealth ratio than an equivalent plan member that does not contribute. The common factor in the setting of all target RWRs is the nominal return assumption – seven per cent per annum – used their calculation. In this sense we have used a common nominal return target to adjust target RWRs to account for the underlying accumulation models thereby making performance comparable.

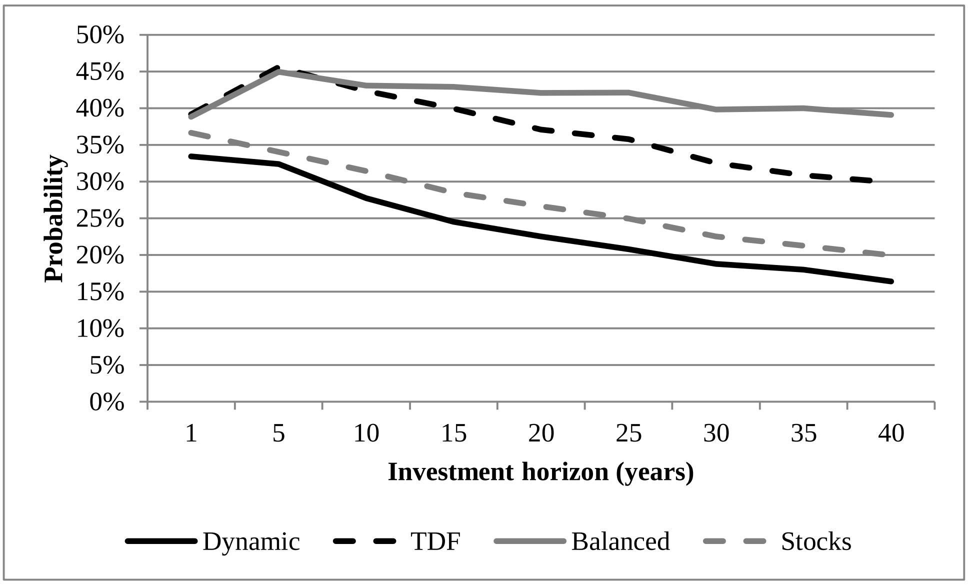
The first of the three target-relative measures to be considered here is the probability of shortfall. The probability of shortfall is a relatively common measure in finance where it is usually measured against an absolute target or some competing investment strategy. Hickman et al. (2001) for example measure the probability of shortfall for three strategies against an all-stock alternative. Probability of shortfall has been used elsewhere in the time diversification literature (e.g. Leibowitz and Krasker, 1988; Leibowitz and Langetieg, 1989; Butler and Domian, 1991; Bierman, 1997; Milevsky, 1999). The second measure, expected shortfall, is something of a companion to the probability of shortfall. Where the probability of shortfall tells about the likelihood of falling short of the target RWR, expected shortfall tells us about the average magnitude of a shortfall when they occur. Notice that each of these target-relative measures focus exclusively on risk. In the final target-relative measure to be considered here – the Sortino ratio – we introduce the return dimension to our evaluation of the performance of the five asset allocation strategies. Here we calculate the Sortino ratio as the difference between the mean RWR_T and the target RWR, divided by the below-target semi-deviation. We are therefore considering the expected reward above the target for the below-target volatility assumed, thereby trying to balance the upside potential against downside risk.

Each of these three target-relative measures, for four of our five asset allocation strategies, are plotted against investment horizon in Figure 7.6. We exclude the cash strategy from Figure 7.6 because its addition makes the results for the other strategies difficult to compare. Furthermore, the estimates for these three measures for the cash strategy conform to our expectations. For example, the probability of shortfall is very close to unity over all horizons suggesting that over the horizons considered cash will struggle to outperform the nominal target return of seven per cent per annum. This is no surprise. To peruse the results for all strategies, including the cash strategy, please refer to Appendix C.

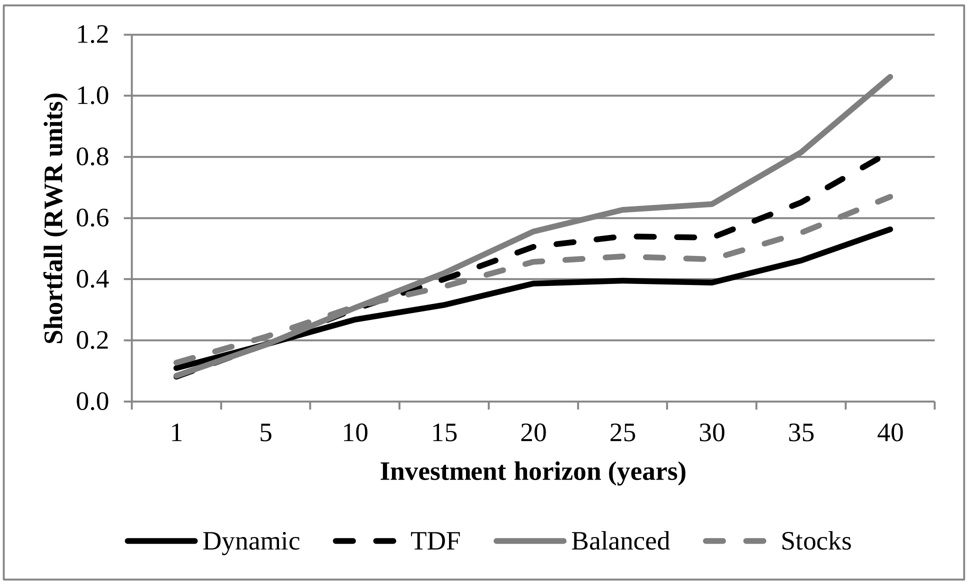
Figure 7.6: Comparison of investment strategies – Target-related measures

Using the stationary bootstrap (bs) simulation method, and assuming the constant percentage contribution model, we report three target-relative measures. Panel A presents the probability of shortfall expressed in percentage terms, Panel B shows the expected shortfall in RWR_T terms, and Panel C presents the Sortino ratio as a ratio of above-target reward to below-target variation.

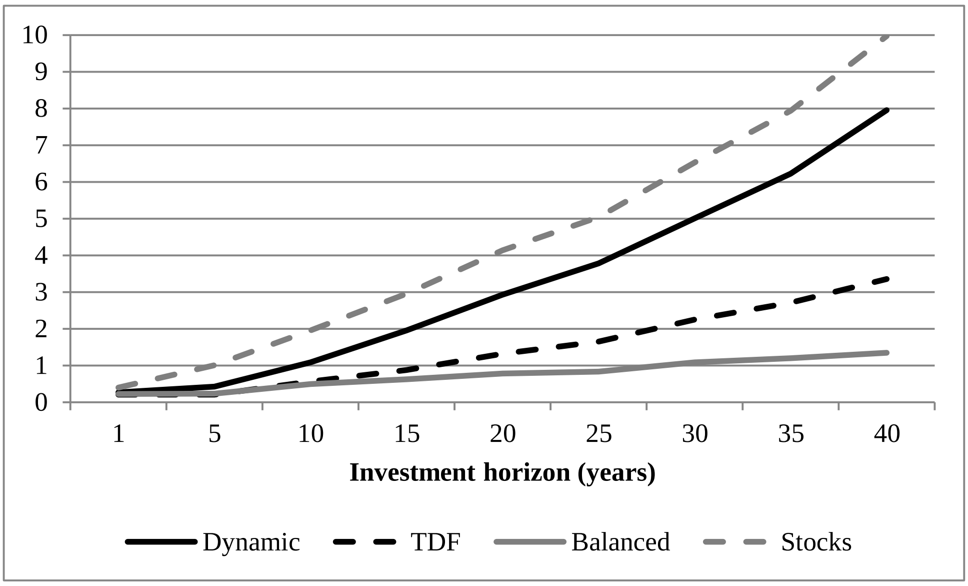
PANEL A – Probability of shortfall



PANEL B – Expected shortfall



PANEL C – Sortino ratio



Upon reviewing the probability of shortfall estimates shown in Panel A of Table 7.6 two themes emerge. Firstly, we see a relationship between the allocation to risk assets and the absolute level of shortfall probability. Generally speaking, the higher the allocation to risk assets the lower is the absolute level of probability over all horizons. Expressed another way, reading the chart from top to bottom we see series representing increasing allocations to risk assets. Secondly, as the allocation to risk assets increases, the more the probability falls with horizon. That is, the gradient of the series is generally steeper for those strategies with higher risk allocations. Compare, for example, the gradients of the target-date (TDF) and stock series: the target-date series falls from 40 per cent at a one-year horizon to around 30 per cent at a 40-year horizon, whereas the stock series falls from around 37 per cent at a one-year horizon to 20 per cent at a 40-year horizon.

Two further features are noticeable from Panel A. Firstly, the probability of shortfall for the balanced strategy is relatively constant for all investment horizons, ranging between 40 and 45 per cent. This likely says something about the relationship between the target and the expected return of the investment strategy. Secondly, note that the dynamic strategy produces lower estimates for the probability of shortfall for all horizons when compared to the all-stock strategy. As noted earlier, increased allocations to risk assets generally result in lower estimated probabilities. However with the dynamic strategy we are achieving lower shortfall probabilities with lower average allocations to risk assets. Recall that the stocks strategy has a constant allocation to stocks of 100 per cent, whereas the dynamic strategy has an allocation to stocks of 100 per cent for the first 30 years, followed by two five-year periods where the allocation varies between 100 per cent stocks and progressively lower allocations to risk assets with time depending on progress versus the target. Given the relative simplicity of the strategies considered here it is too early to tell, but these results might suggest that dynamic target-driven strategies may be effective alternatives for DC pension plans that seek to achieve a particular target.

When we turn to Panel B, we see that the average shortfall rises with investment horizon for all strategies, given our assumption of constant percentage contribution (cpc) accumulation model. This result is consistent with our intuition. As returns, contributions and salary growth compound through time, the range of outcomes widens, and the average shortfall when we are below the target RWR increases. The surprising trend in Panel B is perhaps the ordering of the strategies. For a higher allocation to risk assets, one expects larger potential drawdowns, and, one might think, a larger average shortfall. Panel B suggests otherwise, and this again relates to the relationship between the target and the asset allocation strategy. Take the all-stock strategy for example. An all-stock strategy would be expected to achieve an annual return greater than the nominal target of seven per cent more often than, say, the balanced fund. As a result the stock strategy would be in shortfall less often than the balanced fund, and when it is, it will be by a lesser amount because of the cumulative effect of the return premium over the target rate of return. This accounts for the higher risk strategies achieving comparatively smaller expected shortfall estimates.

Once again, the dynamic strategy outperforms all other strategies, including the all-stock strategy, when we consider expected shortfall. Expected shortfall is the lowest of all strategies over all horizons. And, furthermore, the dynamic strategy complicates our ability to make generalisations about the relationship between the allocation to risk assets and risk. Given it has a lower overall allocation to risk assets than the all-stock strategy, we are left to conclude that the dynamic strategy may achieve better target-relative performance by taking risk at the right times based on performance versus the target.[94]

Before turning to the Sortino ratios in Panel C, we take this opportunity to re-emphasise the findings of our second study from a slightly different perspective. In this section, we have looked at the constant percentage contributions model for two reasons: to simplify the reporting of our results, and to focus on the model which we argue is most consistent with the institutional setting. However when we compare the target-relative expected shortfall across accumulation models we see more evidence of the deficiencies of the initial endowment assumption so popular in the time diversification literature. The expected shortfall estimates in Panel B rise consistently as investment horizon lengthens. This finding holds no matter which simulation method is employed. But if we consider the results for the initial endowment model – see Appendix C for all the results – we see a different profile. Once again see the hump-shape profile of our initial wealth assumptions. These different findings suggest that the time diversification debate may be of limited relevance for those who design and manage DC plans because by using a simple counter example – that is, changing the accumulation model – we yield different answers to the question: does risk increase or decrease with investment horizon?

And, finally, let us consider the Sortino ratio estimates in Panel C. In this case, our estimates accord more closely with our expectations, and with the findings of other studies in the time diversification literature. Firstly, the Sortino ratios for all strategies rise monotonically with investment horizon. We expect these results because, as shown earlier in this study, positive outcomes grow at a greater rate than negative outcomes as horizon lengthens. Described another way, the numerator in the Sortino ratio calculation grows quicker than its denominator. This positive relationship between Sortino ratio and horizon is also consistent with the only other study in the time diversification that considers the measure, namely Sinha and Sun (2005). But perhaps the most noticeable difference between Panel C, and the results in Panels A and B, is that the ranking of the strategies by Sortino ratio neatly matches the ranking of strategies by allocation to risk assets. This finding is not at all surprising. Because the mean RWR_T is used in

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[94] In fact, the dynamic strategy and the stock strategy will have equivalent allocations to stocks in the very unlikely event that the dynamic strategy is at all times below target. In any other case, the dynamic strategy will have lower allocations to risk assets.

the calculation of the Sortino ratio, it is impossible for the stock strategy to produce anything other than the superior Sortino ratio. This is because the extremely positive paths for stock strategy bias the mean higher.

Hickman et al. (2001), when evaluating performance in absolute terminal wealth terms, found that growth-oriented asset allocation strategies, when compared to an all-stock strategy, produced significantly less upside potential but without any downside performance pay-off. They also found that over shorter horizons the evidence was not as clear cut. This analysis, in which we evaluate performance against a target, shows that we agree with Hickman et al. (2001) on some points, but disagree on others. Our estimated Sortino ratios suggest the stock strategy has far superior upside potential even against strategies with non-trivial allocations to risk assets. We also found that the results were clearer for longer horizons: the estimates for horizons between one and ten years were harder to differentiate. The point upon which we disagree with Hickman et al. (2001) relates to risk. In a target-relative paradigm, we find that pursuing a dynamic strategy results in us forgoing upside in exchange for materially altering the downside risk characteristics of a portfolio when compared to a static alternative.

7.7 Concluding remarks

In this final study, extending our nested methodological approach, we examine five different asset allocation strategies in order to investigate whether asset allocation has the potential to influence the relationship between risk and investment horizon. In addition to a selection of extant asset allocation strategies, we introduce a basic rule-based dynamic strategy from the pension finance literature.

Our findings for this last study are threefold. Firstly, we find that the relative differences between the results for competing accumulation models persist for all asset allocation strategies examined, and all measures considered, with the magnitude of the difference being a function of the horizon length and the allocation to risky assets. We conclude that contributions remain an important variable in retirement savings no matter what the asset allocation strategy under consideration. Secondly, our analysis supports those few studies which analyse a range of asset allocation strategies (Reichenstein and Dorsett, 1995; Hickman et al. 2001). For example, we show that little is gained in the way of downside benefits by adopting a strategy other than the all-stock strategy.

Third and finally, because of our novel approach, we make some modest contributions to the time diversification literature in relation to target-relative performance measures. We find that, in target-relative terms, our dynamic strategy combines the lowest probability of shortfall over all horizons with the lowest expected shortfall over all horizons greater than five years in length when compared to two strategies – the balanced and target-date strategies – that take less overall risk. The dynamic strategy also achieves the second highest target-relative Sortino ratio behind the all-stock strategy.

We posit that this combination of performance attributes indicates that our dynamic target-driven strategy design may be able to influence the relationship between risk and investment horizon, and hence has implications for the time diversification debate and our third research question. We question whether it is possible to make general statements about the relationship between risk and time if we can present evidence to show that a portfolio design with a higher overall equity allocation can produce more attractive risk estimates. This allows us to reject our null hypothesis: That asset allocation strategy has no bearing on the relationship between risk and investment horizon. We also provide practitioners evidence that dynamic target-driven investment strategies may represent a superior alternative to traditional deterministic approaches to plan design (e.g. target risk and target date fund designs).

Thus, we show that the time diversification debate as it is currently constituted is incomplete. For a complete understanding of the relationship between risk and investment horizon, we must consider a trinity of related factors: risk; the accumulation model; and, asset allocation.

8 Conclusion

Having concluded the empirical chapters of this thesis, we first summarise the results of our three referenda, before drawing out the implications of our findings for long-horizon investors. We close by briefly outlining the limitations of this research, and suggesting areas for future research.

8.1 A referendum on risk

The time diversification puzzle is concerned with the relationship between risk and investment horizon. In our first empirical study, using a nested methodological approach, we re-prosecuted the time diversification debate *on common terms* by pursuing a research design inspired by the applied stream in the literature. Our objective was to synthesise a fragmented and contested debate, and glean positive insights into our research question:

RESEARCH QUESTION ONE

H_0: That a relationship between risk and investment horizon exists.

H_1: That H_0 is false.

In the literature review, we identify three schools of thought – expected utility theory, option pricing theory, and the behavioural stream – which have each failed to resolve this puzzle for a variety of reasons. Attracted by its empirical approach, and its consistency with the institutional setting outlined in Chapter Three, we were attracted to a fourth stream, the applied literature, as the source of an objective framework for re-examining the time diversification puzzle. At the time of writing, we are aware of no study that has attempted to resolve this enduring puzzle by employing such a range of measures using a unifying approach to data and methodology.

Over nearly 50 years, the time diversification literature has produced contradictory evidence about the relationship between risk and investment horizon. If we believe Samuelson (1969), there is no relationship between these variables, because risk is a function of risk tolerance not horizon. Some scholars see risk – for example, the standard deviation of annualised returns – falling with investment horizon. And, finally, another group of scholars see risk – for example, the standard deviation of cumulative returns – rise with investment horizon. How can these statements simultaneously be true? Our first study, by selecting ten important measures from the literature, re-visits this conflicting evidence. Where Kritzman (2000) talks about "a referendum on the meaning of risk (p. 50)", we conduct just such a referendum. Using a common methodological approach, we metaphorically invite these risk measures into the public square and ask them by way of referendum: what is the relationship between risk and investment horizon?

To bring this referendum to life, we selected ten popular measures of "risk" from the time diversification debate. Within an initial endowment accumulation model, and using the nested methodological approach shown in Figure 8.1, we estimate these ten measures for nine investment horizons ranging in length from one to 40 years.

Figure 8.1: Nested methodological approach

This figure shows, in Venn diagram form, the nested nature of the methodological approach pursued in this thesis. In study one, the accumulation model (initial endowment model) and asset allocation (all-stock portfolio) assumptions are fixed as we consider four simulation methods over nine horizons (1 x 1 x 4 x 9 = 36 models). In study two, the set-up is identical to Study 1 except that the accumulation model assumption is freed with two further models being added (for a total of three) (3 x 1 x 4 x 9 = 108 models). In the third and final study, the remaining dimension – the asset allocation – is freed and we examine a further four asset allocations for a total of five different asset allocations (3 x 5 x 4 x 9 = 540 models).

We then compare the ten measures on two dimensions: (1) the functional relationship between risk and time; and, (2) the measurement basis. Our results are summarised in Table 8.1.

Table 8.1: Summary findings

This table presents, in summary form, the findings of this study. From left to right, we report the: (1) the measure of interest; (2) the functional relationship between risk and investment horizon suggested by the measure; (3) whether this relationship is indicative of time diversification ("Yes" or "No"); and, (4) whether the measure is a returns-only ("Returns") or wealth-based ("Wealth") measure.

Measure	Functional relationship	Time diversification?	Measurement basis
Standard deviation of annualised returns	Decreasing	Yes	Returns
Standard deviation of cumulative returns	Increasing	No	Wealth
Distribution of terminal wealth	Increasing	No	Wealth
Shortfall risk	Decreasing	Yes	Wealth
Value-at-risk	Increasing	No	Wealth
Expected tail loss	Increasing	No	Wealth
Sharpe ratio	Increasing	Yes	Returns
Sortino ratio	Increasing	Yes	Returns
Guo and Darnell's (2005) T^*	N/A	No	Wealth
Time diversification index	Unclear	No	Returns

We find three things. Firstly, our estimates, across all four simulation methods, generally confirm the findings in the literature. We can therefore conclude that the puzzle doesn't owe its existence to faulty analysis. Secondly, the evidence of time diversification remains contradictory even when examined using the same data and methodology. We find that four measures are suggestive of time diversification, and that six measures are not (cf. Table 8.1). Thus, whether or not equities are riskier over long investment horizons is not a question that can be answered objectively. The presence of time diversification becomes a function of how one frames risk. In our first study, we have essentially provided ten different ways in which risk may be framed. An emphatic resolution to the puzzle therefore appears out of reach.

And, finally, we show that the measurement basis might offer some resolution to the puzzle. By comparing columns three and four of Table 8.1 (and ignoring the two measures where the functional relationship between risk and investment horizon is unclear), we see that return-based measures – where the measure in question is a function of average returns – tend to be supportive of time diversification, whereas wealth-based measures – where the measure is a function of a chain of realised returns – tend to see risk rise with time. Viewed according to this dichotomy, the time diversification puzzle appears somewhat clearer.

In our first study, by considering ten measures from the extant applied literature, we find that everyone is right about the time diversification debate *on their own terms*. In this sense the time diversification puzzle is enduring. In our measurement basis dichotomy, we offer a tentative resolution to the puzzle. By changing the measurement basis, our findings regarding the presence of time diversification appear to change. This contrast between the findings for these two measurement bases is something to revisit in the second study of this thesis. In Chapter Six, we considered two alternative accumulation models – each involving contributions – which will inevitably impacted wealth, but not returns. If measurement basis is an important aspect of risk, we will see further evidence of this tentative dichotomy between return- and wealth-based measures.

We now turn our second empirical study where we extend our nested methodology to consider alternative accumulation models, inspired by a critique of the time diversification literature, and its treatment of the prevailing institutional setting.

8.2 A referendum on the accumulation model

In our first empirical study, we re-prosecuted the time diversification debate within an initial endowment framework and concluded that the time diversification puzzle remains a puzzle because each scholar approaches the problem with their own views on risk, and with a particular set of assumptions. Thus, all researchers are right *on their own terms*. We also suggested that some resolution to the puzzle may lie in the risk measure's basis – whether the measure is return-based or wealth-based – but we deferred judgment until this study is completed because alternative accumulation models should provide us additional evidence to consider.

In this study, we use a critique of the time diversification literature to motivate our consideration of two additional accumulation models: the constant contribution model; and, the constant percentage contribution model. The differences between these new accumulation models and the initial endowment model employed in our first are shown in Table 8.2.

Table 8.2: Accumulation models

This table presents the differences between the accumulation models studied in this thesis. From left to right the table shows the name of the accumulation model and how each of the variables from equations [4.1]-[4.3] vary between models. While the percentage rates are quoted in per annum terms, these annual rates are applied on a monthly basis in the modeling.

Accumulation model	**Contribution rate ( )**	**Salary growth rate ( )**
Initial endowment model	Zero	Zero
Constant contribution model	9% per annum	Zero
Constant percentage contribution model	9% per annum	3% per annum

We replicated the same battery of ten risk measures for these two alternative accumulation models and compare the results with those that were calculated for the initial endowment model in the first study. By extending the analysis to alternative accumulation models, our nested methodological approach allows us to conduct a different referendum: a referendum on the accumulation model.

The critique of the time diversification debate that motivates this study related to the lack of consistency between what the literature assumes, and the realities of the institutional setting we outline in Chapter Three. In particular, the literature largely ignores at least one variable – contributions – that significantly impacts terminal wealth. In Chapter Three we showed that contributions are a central feature of the retirement savings system which this thesis considers – the United States. The inconsistency between the literature and the institutional setting, and the dearth of studies that compare alternative accumulation models to the typical initial endowment approach, motivated us to extend our research design from the first empirical study to two alternative types of accumulation model. Thus, in our second study, the accumulation model goes from being a constant, to being a variable. In doing so, we consider the results from a further 72 models as we show in Figure 8.1 (two additional models times our base set-up of 108 models).

By releasing the accumulation model as a variable, we set out to investigate whether the accumulation model has any influence on the relationship between risk and investment horizon consistent with our second research question:

RESEARCH QUESTION TWO

H_0: That alternative accumulation models have no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

In this second study, we extended our approach from Chapter Five to a further two accumulation models – the constant contribution model and the constant percentage contribution model – in order to contrast the results, and address our research question. We find that, with the exception of measures expressed in terms of terminal wealth, all measures are virtually identical giving us the evidence to critique the time diversification literature. By ignoring contributions, and measuring risk the way they have, scholars have been debating a phenomenon in a context devoid of reality. If the time diversification debate really is about long term investing, then it should be reframed to allow realistic accumulation models to be examined, after which conclusions about the relationship between risk and investment horizon can be reached.

Furthermore, our results suggest that our measurement basis dichotomy remains a possible solution to the time diversification debate. Returns-based measures do not vary with accumulation model because, by definition, they are a function of returns. Wealth-based measures are however significantly different between accumulation models as one would expect. Contributions, according to our alternative accumulation models and the institutional setting (Pástor and Stambaugh, 2012), are always positive contributors to portfolio wealth.

Because of these two flaws the time diversification literature has, to paraphrase Kritzman (2000), become a referendum on the meaning of risk for a mis-specified problem. From the perspective of retirement savings, whether risk rises or falls with investment horizon in the absence of contributions is not a particularly interesting question because it is without context. Based on the analysis in our second study, we assert that the time diversification debate cannot be resolved by considering risk and investment horizon in isolation. The accumulation model is a factor that must be considered before complete understanding of the relationship between risk and horizon can be resolved. In the final empirical study, based on a similar set of motivations, we will see if asset allocation is a similarly important factor.

Regardless of one's conceptualisation of risk, plan members are certainly subject to risk over their investing life. The question remains: Can plan sponsors do anything to manage risk over long investment horizons? For our first two studies we considered the performance of an all-stock portfolio in absolute terms, and in relative terms against an all-cash portfolio. In our last study, we seek to answer this question by analysing the performance of a number of competing investment strategies.

8.3 A referendum on asset allocation

In this thesis, we have pursued an applied approach based on a critique of each of the competing schools of thought in the time diversification debate. In particular, we used critiques of Samuelson's (1969) first two assumptions as the basis for our first study, in which we synthesise the time diversification literature on common terms. In the second study, we use a critique of the last of Samuelson's (1969) three assumptions, and of the literature's treatment of the institutional setting, to motivate an examination of the impact of contributions on the relationship between risk and investment horizon.

Samuelson's (1969) third assumption – that wealth is a function only of returns – despite being critiqued in the second study remains partially true. Returns are *one* of the determinants of terminal wealth. But what Samuelson (1969) does not explicitly say, and what the time diversification literature rarely studies, is that returns themselves are in turn a function of asset allocation. Does asset allocation, however conceived, have an influence on the findings of the time diversification debate? This question is encapsulated in our final research question:

RESEARCH QUESTION THREE

H_0: That asset allocation strategy has no bearing on the relationship between risk and investment horizon.

H_1: That H_0 is false.

In our third and final study, we investigated whether the implementation of different asset allocation strategies has implications for our findings regarding time diversification. Using our nested methodological approach, we consider a range of extant investment portfolio designs, as well as introducing a basic rule-based dynamic strategy from the pension finance literature. By doing this we hope to make a contribution to the time diversification literature by considering the relationship between risk and investment horizon for a range of asset allocation strategies in the presence of contributions and salary growth. In doing so, we conduct our own referendum on asset allocation.

Our findings for this last study are threefold. Firstly, we find that the relative differences between the results for competing accumulation models persist for all asset allocation strategies examined, and all measures considered, with the magnitude of the difference being a function of the horizon length and the allocation to risky assets. Contributions are thus an important variable in retirement savings no matter what the asset allocation strategy under consideration. Secondly, our analysis supports the findings of those few studies which analyse a range of asset allocation strategies. For example, we show that by adopting an investment strategy other than the all-stock strategy the investor gives up significant upside without noticeable improvement downside performance. In economic terms, all downside outcomes are poor.

Third and finally, because of our unique approach, we make some modest contributions to the time diversification literature in relation to target-relative performance measures. We find that, in target-relative terms, our dynamic strategy combines the lowest probability of shortfall over all horizons with the lowest expected shortfall over all horizons greater than five years in length when compared to two strategies – the balanced and target-date strategies – that take less overall risk. The dynamic strategy also achieves the second highest target-relative Sortino ratio behind the all-stock strategy.

We posit that this combination of performance attributes indicates that our dynamic target-driven strategy design may be able to influence the relationship between risk and investment horizon, and hence has implications to the time diversification debate. We therefore question whether it is possible to make general statements about the relationship between risk and horizon if we can present evidence to show that an asset allocation strategy with more equity risk produces, more attractive risk estimates. We also provide practitioners evidence that dynamic target-driven investment strategies may represent a superior alternative to traditional deterministic approaches to plan design (e.g. target risk and target date fund designs).

Thus, by providing a simple example, we have shown that the time diversification debate as it is currently constituted is incomplete. Instead, a proper understanding of the relationship between investment risk and horizon requires consideration of a trinity of related factors: risk; accumulation model; and, asset allocation.

In concluding this thesis, we note the limitations of this research and highlight some avenues for future research.

8.4 Implications for long-horizon investors

In addressing the question at the heart of this thesis – the relationship between risk and investment horizon – we have placed great importance on the institutional setting. It is therefore incumbent on us to briefly explore the practical implications of this work for DC plan members who, critically, bear the investment risk of their retirement savings:

- In our recapitulation of the time diversification debate in the first empirical study, we showed that by changing the definition of risk it was possible to arrive at opposite conclusions regarding the relationship between risk and investment horizon. Risk is thus highly subjective, and making generalisations about an individual's attitude to risk is both difficult and dangerous;
- In our second study we show, by contrasting the results of just three accumulation models, that wildly different terminal wealth outcomes are possible. Such a finding, for example, calls into question the efficacy of designing DC plans around a small number of stylised or hypothetical investors in order to achieve adequacy for all. A more granular approach to DC plan design – which takes into account individual circumstances – might therefore appear to be a sensible next step; and
- Portfolio designs in many DC plans – e.g. so-called balanced funds – tend to implicitly rely on investors having perpetual investment horizons. In this way, the investor will achieve the fund's objective over the long run. In reality, DC investors have finite investment horizons and some (perhaps ill-formed) expectations about adequacy means to them. The performance of the simple dynamic strategy from our final empirical study shows that, by remaining cognisant of both the target and investment horizon, it is possible to nudge the balance of reward and risk in the favour of the plan member. Trustees should perhaps consider incorporating these design features in the menu of investment options they offer.

The common denominator in our findings, and in the implications for DC investors, is the critical role framing plays in long-horizon investing. Pension fund trustees would do well to remember this when they turn their minds to plan design.

8.5 Limitations

We note the following limitations of this research:

- *Two asset framework* – This thesis assumes that the investment opportunity set is characterised by two assets: a risky asset (stocks); and, a riskless asset (T-bills). Practically, the investment opportunity set is much larger, extending to fixed interest securities, commodities and unlisted assets. We are drawn to this simplistic framework because our methodology calls for large reliable datasets;
- *Simplistic accumulation models* – Each of the accumulation models relies on simplistic assumptions. For example, the constant contribution and constant percentage contribution models assume smooth and constant contribution and salary growth through time. Realistically, salary growth is not smooth and contributions may be interrupted by discontinuities in employment (e.g. career breaks). Furthermore, we assume our hypothetical investor is always employed. As many can attest, this is also not always true;
- *Salary and account balance data* – We admit that our salary and account balance data could be subject to critique. Our rationale for selecting this data, was to find reasonable estimates of median salary and account balance data for workers in the United States;
- *Considers the investment problem in isolation* – This thesis, being in the field of finance, looks at the investment problems in isolation. More complex models might examine, for example, the interplay between investment, consumption and other factors (e.g. Vanini and Vignola, 2002); and
- *Assets outside of DC plan balance* – A common criticism of studies in this vein is that they narrowly equate wealth with the balance of the investor's retirement account. We accept that plan members have wealth outside of their DC plan – what the World Bank calls Pillar 3 savings – and that this wealth affects an investor's financial calculations. Such broad definitions of wealth do however make it extremely difficult to arrive at a hypothetical investor profile.

It is some consolation to note that we are not alone in making many of these assumptions. As Samuelson (1969) shows us, in order to make a problem tractable, it is often necessary to make certain assumptions.

8.6 Areas for future research

We proposed the following as obvious potential areas of future research:

- *N-asset frameworks* – Given the wide range of asset classes available in the investment marketplace, it is natural to want to revisit elements of this study in settings involving more than two assets;
- *Alternative asset allocation approaches* – In this thesis we have considered a modest number of competing asset allocation strategies. Alternative approaches to asset allocation strategy, e.g. more sophisticated dynamic strategy algorithms, are thus an obvious avenue for future research; and
- *Risk measures* – In this thesis, we review a risk measures that represent a range of perspectives on risk. It would be helpful to investigate whether there is a measure, or combination of measures, that best corresponds with investor behaviour. By identifying the “right” battery of risk measures, this sort of analysis would present greater potential for application.

Appendix A

This appendix reports the complete set of distributions of retirement wealth ratios for each of the five asset allocation strategies considered in the third empirical study (Chapter Seven).

Table A.1: Distribution of retirement wealth ratios – Stock strategy

This table presents the distribution of terminal wealth for the four modeling techniques over nine investment horizons for the three accumulation models. Initial wealth (W_0) is as per row five of Table 6.1. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. Performance measures are group by measure type with measures of central tendency reported in Panel A (mean retirement wealth ratio) and Panel B (median, or 50th percentile retirement wealth ratio). Measures of dispersion are reported in Panels C (standard deviation in retirement wealth ratio terms) and D (retirement wealth ratio range). Panels E through G summarise the lower half of the distribution of terminal wealth (minimum, 5th percentile and 25th percentile retirement wealth ratio respectively). Panels H through J summarise the upper half of the distribution of terminal wealth (75th percentile, 95th percentile and maximum retirement wealth ratio respectively). Results are presented as multiples (i.e. in unit of “times” as in “*x* times final salary”).

PANEL A – Mean retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	15.42	15.32	13.26	15.26	77.64	80.13	71.36	79.72	30.53	30.11	27.72	30.05
35	12.58	12.55	11.18	12.74	49.76	49.53	44.97	49.21	21.41	21.21	19.36	21.60
30	9.92	10.04	9.01	9.97	31.22	30.52	28.66	31.03	15.15	15.29	14.18	15.15
25	12.23	12.24	11.09	12.13	23.75	24.35	22.35	24.37	13.06	12.99	11.85	12.74
20	10.96	10.97	10.28	10.91	17.57	17.57	16.57	17.47	10.49	10.35	9.99	10.35
15	7.28	7.35	6.96	7.40	10.74	10.64	10.34	10.68	7.25	7.34	7.01	7.21
10	4.89	4.89	4.80	4.92	6.53	6.67	6.38	6.58	5.04	5.09	4.93	5.06
5	2.91	2.91	2.89	2.90	3.51	3.51	3.48	3.51	3.04	3.06	3.05	3.05
1	2.48	2.48	2.49	2.50	2.58	2.58	2.58	2.59	2.51	2.50	2.51	2.52

PANEL B – Median (50^{th} percentile) retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	7.35	7.66	8.10	8.44	46.80	47.90	50.05	51.40	19.22	18.98	20.41	20.54
35	6.81	6.98	7.16	7.53	31.97	32.02	33.18	34.37	14.20	14.22	14.88	15.68
30	5.76	6.03	6.26	6.54	21.26	21.09	22.58	23.10	10.74	10.73	11.31	11.71
25	7.97	7.83	8.34	8.61	16.98	17.63	18.10	18.72	9.47	9.48	9.77	10.20
20	7.77	7.89	8.23	8.39	13.11	13.15	13.96	14.33	8.16	7.95	8.45	8.54
15	5.63	5.62	5.91	6.22	8.69	8.68	9.19	9.40	5.87	5.93	6.24	6.35
10	4.12	4.13	4.38	4.51	5.63	5.74	5.93	6.03	4.37	4.39	4.59	4.71
5	2.67	2.68	2.85	2.86	3.25	3.25	3.41	3.47	2.84	2.86	2.99	3.00
1	2.44	2.46	2.52	2.53	2.55	2.55	2.63	2.63	2.48	2.48	2.55	2.55

PANEL C – Standard deviation of retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	27.14	26.28	16.07	21.54	99.68	116.84	69.19	90.15	37.55	38.68	25.38	31.01
35	19.14	18.62	12.62	16.30	60.65	57.52	40.40	50.13	24.93	23.74	15.88	20.81
30	13.47	13.01	8.92	10.98	32.53	31.77	22.62	28.26	15.77	16.36	10.64	12.93
25	14.37	14.26	9.80	11.84	23.27	23.52	16.37	20.74	12.39	12.00	8.32	9.75
20	10.86	10.60	8.03	9.39	15.42	15.48	11.17	12.73	8.41	8.52	6.43	7.30
15	5.99	6.37	4.59	5.31	7.73	7.61	5.78	6.54	5.10	5.48	3.86	4.34
10	3.11	3.10	2.51	2.80	3.78	3.83	2.96	3.39	2.85	2.97	2.28	2.53
5	1.26	1.26	1.07	1.10	1.42	1.43	1.20	1.23	1.20	1.20	1.04	1.06
1	0.45	0.44	0.45	0.46	0.46	0.46	0.47	0.47	0.44	0.44	0.45	0.45

PANEL D – Retirement wealth ratio range

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	660.34	704.17	202.97	484.60	2,340.57	4,523.30	719.40	1,495.86	732.95	1,174.90	377.37	453.27
35	590.16	460.37	186.54	288.94	1,342.02	998.98	714.42	739.59	542.19	531.12	229.14	303.27
30	236.24	256.51	100.28	132.10	422.93	743.69	251.76	390.98	467.80	527.20	110.34	206.62
25	337.49	184.43	121.95	127.61	533.66	505.18	180.08	275.64	249.30	178.80	97.86	108.42
20	141.11	204.29	91.06	118.23	208.74	353.01	108.92	108.90	96.13	132.82	69.84	96.67
15	125.83	144.99	60.88	50.56	115.09	117.12	60.96	55.52	69.67	172.46	36.24	41.44
10	41.87	47.81	22.68	28.11	46.52	38.78	24.56	37.66	36.16	38.90	17.92	21.73
5	11.62	12.84	8.62	8.33	12.59	20.13	11.91	8.56	11.74	10.79	9.31	8.59
1	3.78	4.36	4.30	4.30	3.42	4.88	4.89	4.40	3.68	4.53	4.27	4.27

PANEL E – Minimum retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.12	0.08	0.01	0.00	2.03	1.80	0.73	0.84	1.27	1.30	0.71	0.64
35	0.10	0.08	0.04	0.02	1.58	1.82	1.06	0.70	1.03	1.30	0.68	0.44
30	0.16	0.07	0.06	0.02	1.13	1.46	0.93	0.49	0.99	0.97	0.52	0.50
25	0.26	0.23	0.16	0.06	1.23	1.06	0.91	0.36	1.00	1.06	0.54	0.45
20	0.35	0.31	0.20	0.05	0.91	1.09	0.76	0.44	0.98	0.37	0.57	0.30
15	0.28	0.26	0.18	0.03	0.91	0.97	0.48	0.24	0.68	0.64	0.44	0.30
10	0.57	0.39	0.13	0.16	1.04	0.81	0.43	0.30	0.80	0.75	0.58	0.22
5	0.46	0.58	0.15	0.10	0.77	0.81	0.37	0.23	0.59	0.47	0.29	0.20
1	1.10	1.08	0.77	0.77	1.23	1.11	0.82	0.82	1.19	1.18	0.80	0.80

PANEL F – 5th percentile retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	1.08	1.08	1.06	0.77	10.71	10.43	10.90	8.60	4.73	4.81	4.97	4.14
35	1.10	1.12	1.06	0.77	7.61	7.76	7.94	6.54	3.91	3.87	4.08	3.36
30	1.11	1.06	1.07	0.85	5.96	5.70	5.98	4.86	3.24	3.21	3.21	2.89
25	1.66	1.67	1.61	1.28	5.04	5.01	5.17	4.36	3.13	3.01	2.96	2.63
20	1.95	1.94	1.97	1.54	4.22	4.18	4.29	3.64	2.73	2.72	2.77	2.45
15	1.64	1.69	1.60	1.33	3.17	3.13	3.22	2.81	2.30	2.27	2.27	1.94
10	1.56	1.56	1.48	1.24	2.46	2.44	2.33	2.09	1.89	1.90	1.84	1.65
5	1.35	1.34	1.20	1.15	1.74	1.71	1.60	1.52	1.52	1.52	1.46	1.38
1	1.80	1.80	1.74	1.74	1.88	1.88	1.81	1.82	1.85	1.84	1.76	1.76

PANEL G – 25th percentile retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	3.40	3.46	3.67	3.40	24.64	25.17	27.37	25.63	10.65	10.36	11.44	10.91
35	3.30	3.29	3.49	3.30	17.51	17.52	18.83	18.00	8.09	8.27	8.88	8.60
30	2.88	3.00	3.15	3.02	12.19	12.18	13.25	12.83	6.38	6.40	6.86	6.72
25	4.22	4.10	4.43	4.27	10.07	10.42	10.92	10.81	5.81	5.83	6.10	6.02
20	4.38	4.44	4.74	4.52	8.14	8.27	8.77	8.64	5.13	5.09	5.41	5.29
15	3.41	3.42	3.65	3.64	5.69	5.60	6.14	6.05	3.93	3.97	4.20	4.11
10	2.78	2.80	2.97	2.93	3.98	4.02	4.19	4.16	3.12	3.10	3.26	3.23
5	2.02	2.04	2.13	2.15	2.51	2.49	2.63	2.67	2.17	2.21	2.32	2.33
1	2.16	2.18	2.21	2.22	2.26	2.27	2.29	2.30	2.20	2.22	2.22	2.25

PANEL H – 75th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	16.56	16.96	16.62	18.89	91.24	93.26	90.25	100.64	36.00	35.65	35.22	38.03
35	14.61	14.60	14.11	15.76	58.86	58.87	58.22	62.19	25.51	25.52	24.80	27.28
30	11.60	11.91	11.72	13.05	38.00	36.89	37.27	39.80	18.54	18.25	18.27	19.33
25	15.01	14.78	14.71	15.80	29.02	29.82	28.92	31.33	15.62	16.00	15.21	16.56
20	13.59	13.69	13.49	14.47	21.59	21.77	21.22	22.84	13.04	12.78	12.99	13.38
15	9.08	9.19	9.20	9.73	13.38	13.26	13.27	13.74	9.00	9.08	8.92	9.33
10	6.12	6.11	6.21	6.43	8.03	8.35	8.16	8.44	6.21	6.25	6.24	6.45
5	3.52	3.54	3.55	3.61	4.22	4.22	4.23	4.29	3.64	3.68	3.68	3.69
1	2.76	2.75	2.76	2.77	2.86	2.85	2.86	2.88	2.79	2.77	2.79	2.79

PANEL I – 95th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	56.07	53.21	42.68	51.18	246.98	249.62	208.07	243.80	91.72	91.80	75.54	86.08
35	42.62	41.23	34.75	42.08	148.99	148.79	119.39	143.33	60.62	60.01	49.55	60.45
30	32.24	31.86	26.45	30.29	90.90	86.74	71.65	83.53	40.93	42.17	34.80	38.94
25	36.43	37.29	30.04	35.61	65.41	66.23	53.71	63.52	35.16	34.26	27.72	31.76
20	30.86	30.60	25.79	29.06	46.02	45.12	38.33	42.17	26.03	25.62	22.24	24.33
15	18.54	18.61	15.72	17.58	25.16	24.65	21.31	22.95	16.64	16.76	14.42	15.34
10	10.80	10.58	9.44	10.12	13.62	14.01	11.75	12.81	10.35	10.51	9.02	9.74
5	5.31	5.22	4.66	4.80	6.18	6.18	5.49	5.63	5.30	5.31	4.81	4.91
1	3.28	3.25	3.15	3.16	3.39	3.37	3.26	3.27	3.29	3.26	3.17	3.18

PANEL J – Maximum retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	660.46	704.25	202.98	484.60	2,342.60	4,525.10	720.13	1,496.70	734.22	1,176.20	378.08	453.91
35	590.26	460.45	186.58	288.96	1,343.60	1,000.80	715.48	740.29	543.22	532.42	229.82	303.71
30	236.39	256.58	100.34	132.12	424.06	745.15	252.69	391.47	468.79	528.17	110.86	207.13
25	337.75	184.66	122.11	127.67	534.89	506.25	180.99	276.00	250.30	179.86	98.40	108.88
20	141.46	204.59	91.26	118.28	209.65	354.10	109.69	109.34	97.11	133.19	70.40	96.97
15	126.11	145.25	61.05	50.59	116.00	118.09	61.44	55.77	70.35	173.10	36.68	41.73
10	42.44	48.20	22.81	28.27	47.56	39.59	24.99	37.96	36.96	39.65	18.50	21.95
5	12.08	13.42	8.77	8.43	13.35	20.95	12.28	8.79	12.33	11.26	9.60	8.79
1	4.88	5.44	5.07	5.07	4.65	5.99	5.71	5.22	4.87	5.72	5.07	5.07

Table A.2: Distribution of retirement wealth ratios – Dynamic strategy

This table presents the distribution of terminal wealth for the four modeling techniques over nine investment horizons for the three accumulation models. Initial wealth (W_0) is as per row five of Table 6.1. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. Performance measures are group by measure type with measures of central tendency reported in Panel A (mean retirement wealth ratio) and Panel B (median, or 50th percentile retirement wealth ratio). Measures of dispersion are reported in Panels C (standard deviation in retirement wealth ratio terms) and D (retirement wealth ratio range). Panels E through G summarise the lower half of the distribution of terminal wealth (minimum, 5th percentile and 25th percentile retirement wealth ratio respectively). Panels H through J summarise the upper half of the distribution of terminal wealth (75th percentile, 95th percentile and maximum retirement wealth ratio respectively). Results are presented as multiples (i.e. in unit of "times" as in "*x* times final salary").

PANEL A – Mean retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	12.61	12.43	10.94	12.42	65.26	65.11	59.44	64.76	24.65	24.57	23.04	24.60
35	10.30	10.25	9.26	10.34	40.49	40.56	37.50	40.19	17.78	17.41	16.20	17.72
30	8.29	8.19	7.48	8.14	25.55	25.19	24.00	25.45	12.41	12.57	11.92	12.52
25	10.07	10.00	9.26	9.94	19.83	20.02	18.82	20.08	10.71	10.72	10.06	10.60
20	9.11	9.05	8.65	9.01	14.53	14.57	14.03	14.58	8.69	8.67	8.47	8.68
15	6.15	6.11	5.90	6.14	9.03	8.98	8.86	9.03	6.16	6.19	6.02	6.10
10	4.18	4.16	4.11	4.15	5.63	5.71	5.55	5.63	4.36	4.37	4.28	4.33
5	2.64	2.64	2.61	2.62	3.21	3.20	3.17	3.19	2.79	2.79	2.78	2.78
1	2.45	2.45	2.44	2.45	2.54	2.55	2.54	2.55	2.48	2.47	2.47	2.47

PANEL B – Median (50th percentile) retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	6.70	6.81	7.13	7.40	41.84	42.54	43.79	45.28	16.68	16.82	17.94	17.95
35	6.14	6.18	6.32	6.63	27.94	28.37	29.44	30.42	12.80	12.69	13.27	13.80
30	5.37	5.34	5.57	5.79	19.13	18.94	19.95	20.35	9.54	9.61	10.04	10.28
25	7.07	7.07	7.46	7.63	15.40	15.83	16.09	16.63	8.51	8.55	8.76	9.07
20	7.11	7.19	7.44	7.59	12.09	12.11	12.54	12.85	7.34	7.37	7.62	7.72
15	5.27	5.30	5.42	5.63	8.14	8.11	8.32	8.58	5.55	5.57	5.68	5.78
10	3.98	3.96	4.05	4.12	5.43	5.48	5.51	5.61	4.20	4.19	4.26	4.33
5	2.65	2.64	2.71	2.71	3.23	3.22	3.27	3.30	2.82	2.82	2.86	2.87
1	2.48	0.48	2.50	2.50	2.57	2.58	2.60	2.60	2.51	2.51	2.53	2.53

PANEL C – Standard deviation of retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	18.85	18.36	11.90	15.71	77.45	80.31	50.41	63.39	25.99	26.19	18.09	21.85
35	13.45	13.41	9.28	11.61	40.58	39.99	28.46	35.00	16.47	15.97	11.08	14.32
30	9.43	9.06	6.47	7.82	22.21	21.47	15.62	19.23	10.39	10.72	7.40	8.77
25	9.92	9.80	6.92	8.27	15.86	15.22	11.25	13.91	8.03	7.78	5.63	6.57
20	7.22	6.91	5.49	6.04	9.41	9.70	7.41	8.39	5.45	5.41	4.23	4.74
15	3.97	3.85	2.98	3.39	4.59	4.60	3.66	4.06	3.09	3.17	2.45	2.69
10	1.79	1.75	1.53	1.67	2.08	2.13	1.81	2.00	1.62	1.64	1.38	1.50
5	0.75	0.73	0.70	0.72	0.83	0.84	0.79	0.81	0.70	0.71	0.67	0.68
1	0.35	0.35	0.37	0.37	0.35	0.36	0.38	0.39	0.34	0.34	0.37	0.37

PANEL D – Retirement wealth ratio range

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	395.74	415.20	153.66	338.82	1,654.23	2,883.40	477.58	972.11	405.46	595.63	272.68	345.76
35	225.55	246.04	135.92	195.44	724.35	598.00	365.10	549.34	233.44	290.66	111.71	197.66
30	174.06	136.54	83.59	100.72	410.42	289.39	148.53	232.62	219.59	282.49	77.57	121.18
25	149.36	128.66	73.32	82.84	375.30	237.17	112.44	173.40	129.97	103.58	54.75	68.32
20	78.27	97.35	56.35	68.11	116.22	176.63	65.45	70.22	70.96	86.81	41.39	60.93
15	46.40	64.02	28.06	29.91	49.19	60.26	32.89	36.18	32.63	65.41	20.96	25.12
10	23.25	20.28	13.53	13.22	18.92	20.07	14.96	16.98	17.46	16.77	12.26	12.54
5	5.92	6.20	5.74	5.57	6.63	9.03	7.04	6.32	5.77	5.78	5.78	5.58
1	2.65	3.00	4.00	4.00	2.54	3.69	4.30	4.08	2.60	3.04	3.96	3.96

PANEL E – Minimum retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.10	0.08	0.01	0.00	2.67	1.80	0.73	0.84	0.95	1.30	0.74	0.64
35	0.10	0.08	0.04	0.02	1.45	1.82	1.08	0.70	0.83	1.30	0.68	0.44
30	0.13	0.07	0.06	0.02	1.57	1.46	0.93	0.49	0.54	0.97	0.52	0.51
25	0.19	0.23	0.16	0.06	1.42	1.06	0.93	0.36	0.98	1.06	0.54	0.45
20	0.24	0.31	0.20	0.05	0.93	1.10	0.76	0.44	0.95	0.37	0.65	0.30
15	0.47	0.26	0.18	0.03	0.98	0.97	0.48	0.25	0.83	0.64	0.45	0.35
10	0.42	0.40	0.13	0.16	0.99	0.82	0.44	0.30	0.75	0.82	0.58	0.23
5	0.57	0.62	0.17	0.10	0.84	0.81	0.38	0.24	0.76	0.47	0.29	0.21
1	1.25	1.08	0.77	0.77	1.24	1.14	0.82	0.82	1.28	1.18	0.80	0.80

PANEL F – 5th percentile retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	1.09	1.08	1.09	0.80	10.53	10.72	11.26	9.00	4.89	4.95	5.31	4.34
35	1.11	1.13	1.09	0.77	7.72	7.91	8.34	6.85	4.06	3.99	4.36	3.58
30	1.11	1.07	1.11	0.88	5.90	5.91	6.37	5.17	3.32	3.31	3.41	3.02
25	1.67	1.69	1.65	1.31	5.20	5.11	5.39	4.59	3.09	3.11	3.15	2.75
20	1.96	1.98	2.04	1.59	4.40	4.30	4.53	3.83	2.81	2.79	2.91	2.54
15	1.71	1.73	1.71	1.39	3.28	3.23	3.40	2.94	2.35	2.32	2.40	2.03
10	1.63	1.64	1.56	1.31	2.57	2.53	2.47	2.18	1.98	1.98	1.92	1.73
5	1.39	1.41	1.26	1.21	1.80	1.79	1.68	1.60	1.58	1.58	1.53	1.47
1	1.83	1.84	1.75	1.76	1.92	1.91	1.84	1.84	1.87	1.87	1.79	1.78

PANEL G – 25th percentile retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	3.60	3.61	3.81	3.65	26.03	26.08	27.53	26.62	10.98	10.92	11.55	11.30
35	3.52	3.54	3.67	3.58	18.36	18.58	19.50	19.11	8.75	8.68	9.08	9.00
30	3.26	3.27	3.39	3.30	13.27	13.08	13.80	13.60	6.88	6.87	7.18	7.10
25	4.60	4.57	4.81	4.77	11.14	11.25	11.61	11.58	6.30	6.28	6.48	6.45
20	4.87	4.96	5.19	5.09	9.12	9.06	9.42	9.42	5.53	5.59	5.80	5.74
15	3.81	3.79	4.04	4.04	6.28	6.20	6.62	6.56	4.35	4.36	4.53	4.46
10	3.08	3.08	3.20	3.19	4.34	4.40	4.49	4.48	3.34	3.38	3.47	3.48
5	2.16	2.20	2.23	2.27	2.68	2.68	2.74	2.81	2.35	2.36	2.42	2.45
1	2.23	2.24	2.27	2.27	2.32	2.34	2.36	2.36	2.26	2.27	2.29	2.30

PANEL H – 75th percentile retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	14.38	14.26	13.77	15.76	75.58	75.92	75.09	81.84	29.28	29.05	29.10	30.82
35	11.96	12.16	11.82	12.94	48.71	48.56	47.67	50.46	21.23	20.90	20.33	22.01
30	10.00	9.90	9.72	10.65	30.65	30.40	30.33	32.04	14.77	14.97	14.88	15.71
25	12.21	12.15	12.00	12.92	23.84	24.10	23.54	25.22	12.77	12.99	12.43	13.29
20	11.22	11.21	11.04	11.61	17.70	17.57	17.32	18.37	10.43	10.39	10.50	10.89
15	7.46	7.51	7.48	7.80	10.88	10.86	10.91	11.19	7.34	7.39	7.31	7.55
10	5.06	5.00	5.03	5.14	6.68	6.82	6.70	6.82	5.18	5.16	5.11	5.24
5	3.07	3.07	3.05	3.07	3.70	3.69	3.66	3.70	3.21	3.22	3.19	3.21
1	2.68	2.66	2.66	2.67	2.78	2.77	2.76	2.76	2.71	2.68	2.68	2.68

PANEL I – 95th percentile retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	41.88	40.92	33.25	39.38	194.30	189.29	159.82	181.20	68.53	68.63	57.78	64.95
35	32.78	32.15	26.88	32.10	113.50	110.16	90.42	106.59	46.71	45.42	37.68	44.82
30	24.93	24.32	20.10	22.60	64.85	64.36	53.83	61.61	30.08	30.54	26.24	29.00
25	27.71	27.16	22.70	26.10	47.10	48.18	40.34	46.29	24.89	25.04	20.87	23.24
20	22.70	22.06	19.02	21.11	32.06	32.42	28.33	30.49	18.97	18.57	16.34	17.74
15	13.42	13.14	11.44	12.45	17.46	17.36	15.52	16.45	11.88	11.81	10.54	11.01
10	7.37	7.29	6.69	6.99	9.31	9.57	8.52	8.97	7.22	7.21	6.58	6.84
5	3.89	3.85	3.60	3.64	4.57	4.60	4.27	4.35	3.96	3.98	3.74	3.78
1	2.99	2.98	2.88	2.89	3.08	3.09	3.00	3.00	3.01	2.99	2.91	2.91

PANEL J – Maximum retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	395.83	415.28	153.67	338.83	1,656.90	2,885.20	478.31	972.95	406.40	596.93	273.42	346.40
35	225.64	246.12	135.96	195.46	725.80	599.81	366.18	550.04	234.27	291.97	112.40	198.10
30	174.19	136.62	83.65	100.74	411.99	290.85	149.46	233.11	220.13	283.46	78.09	121.69
25	149.55	128.90	73.48	82.90	376.72	238.23	113.37	173.76	130.95	104.64	55.29	68.77
20	78.50	97.65	56.55	68.16	117.15	177.73	66.21	70.66	71.91	87.18	42.04	61.23
15	46.87	64.28	28.24	29.95	50.17	61.23	33.37	36.42	33.47	66.05	21.41	25.47
10	23.66	20.68	13.67	13.39	19.92	20.89	15.40	17.28	18.22	17.58	12.84	12.76
5	6.49	6.82	5.91	5.67	7.46	9.84	7.42	6.56	6.52	6.25	6.07	5.79
1	3.90	4.09	4.77	4.77	3.78	4.83	5.13	4.91	3.89	4.22	4.76	4.76

Table A.3: Distribution of retirement wealth ratios – Target-date strategy

This table presents the distribution of terminal wealth for the four modeling techniques over nine investment horizons for the three accumulation models. Initial wealth (W_0) is as per row five of Table 6.1. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. Performance measures are group by measure type with measures of central tendency reported in Panel A (mean retirement wealth ratio) and Panel B (median, or 50th percentile retirement wealth ratio). Measures of dispersion are reported in Panels C (standard deviation in retirement wealth ratio terms) and D (retirement wealth ratio range). Panels E through G summarise the lower half of the distribution of terminal wealth (minimum, 5th percentile and 25th percentile retirement wealth ratio respectively). Panels H through J summarise the upper half of the distribution of terminal wealth (75th percentile, 95th percentile and maximum retirement wealth ratio respectively). Results are presented as multiples (i.e. in unit of “times” as in “*x* times final salary”).

PANEL A – Mean retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	7.25	7.18	6.67	7.21	41.58	41.67	39.65	41.71	16.65	16.39	15.92	16.50
35	6.53	6.36	5.99	6.44	27.64	27.61	26.42	27.56	12.27	12.27	11.82	12.46
30	5.41	5.44	5.15	5.44	18.38	18.22	17.87	18.45	9.23	9.29	9.05	9.31
25	7.08	7.11	6.80	7.12	15.11	15.10	14.56	15.16	8.15	8.24	7.90	8.19
20	6.97	6.90	6.72	6.89	11.60	11.56	11.33	11.61	6.98	6.97	6.92	7.00
15	4.98	5.00	4.90	5.03	7.65	7.60	7.57	7.66	5.20	5.25	5.17	5.21
10	3.74	3.72	3.71	3.74	5.12	5.16	5.08	5.13	3.94	3.96	3.92	3.96
5	2.51	2.50	2.50	2.50	3.06	3.06	3.05	3.06	2.67	2.67	2.67	2.67
1	2.41	2.41	2.41	2.41	2.50	2.51	2.51	2.51	2.44	2.43	2.43	2.44

PANEL B – Median (50^{th} percentile) retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	4.85	4.90	5.08	5.22	31.69	32.21	33.08	33.59	13.17	13.09	13.73	13.77
35	4.65	4.64	4.69	4.92	22.14	22.27	22.72	23.45	10.10	10.17	10.45	10.79
30	4.06	4.14	4.27	4.42	15.33	15.33	15.90	16.18	7.91	7.86	8.13	8.34
25	5.66	5.68	5.89	6.04	12.81	12.98	13.17	13.53	7.12	7.13	7.23	7.46
20	5.89	5.93	6.05	6.15	10.29	10.20	10.49	10.75	6.20	6.23	6.44	6.51
15	4.45	4.47	4.57	4.74	7.01	7.02	7.22	7.36	4.76	4.82	4.94	4.99
10	3.50	3.49	3.58	3.66	4.85	4.88	4.95	5.02	3.72	3.74	3.82	3.91
5	2.43	2.43	2.51	2.52	2.98	2.98	3.05	3.09	2.60	2.61	2.66	2.68
1	2.40	2.40	2.43	2.44	2.49	2.50	2.53	2.53	2.43	2.43	2.46	2.46

PANEL C – Standard deviation of retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	7.80	7.66	5.68	6.81	33.95	34.77	26.15	30.74	12.49	12.24	9.65	10.95
35	6.30	6.03	4.73	5.58	20.16	19.91	15.81	18.19	8.03	8.10	6.38	7.61
30	4.69	4.56	3.59	4.11	12.16	11.60	9.42	10.78	5.42	5.69	4.48	5.01
25	5.16	5.34	4.18	4.75	9.18	8.82	7.12	8.24	4.39	4.54	3.61	4.03
20	4.40	4.16	3.55	3.92	5.91	6.00	4.99	5.44	3.45	3.38	2.89	3.11
15	2.50	2.55	2.14	2.33	3.23	3.19	2.72	2.94	2.11	2.19	1.83	1.95
10	1.43	1.39	1.25	1.34	1.17	1.72	1.49	1.62	1.28	1.32	1.14	1.21
5	0.63	0.62	0.58	0.59	0.69	0.71	0.65	0.66	0.60	0.60	0.56	0.56
1	0.24	0.24	0.25	0.26	0.25	0.25	0.26	0.26	0.24	0.24	0.25	0.25

PANEL D – Retirement wealth ratio range

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	123.78	130.49	60.68	96.92	419.82	766.07	228.65	351.66	230.56	204.18	95.24	124.51
35	79.58	93.60	53.27	73.29	302.10	253.40	176.39	204.49	124.87	117.33	55.26	85.13
30	89.34	53.21	35.94	45.90	150.08	152.78	82.06	108.28	67.93	109.48	37.93	56.26
25	55.72	59.05	37.41	38.10	135.71	117.64	67.63	80.54	41.51	49.22	32.46	37.08
20	45.29	52.94	35.66	34.47	77.31	85.86	42.20	43.57	43.44	41.02	24.12	33.50
15	24.76	36.43	20.96	17.95	43.70	35.55	20.70	23.46	27.73	39.23	14.45	15.32
10	14.28	14.88	9.46	10.71	14.90	14.62	10.51	13.92	13.52	12.92	8.73	10.49
5	5.42	5.27	4.73	4.50	5.22	7.50	5.69	4.74	4.57	4.91	4.62	4.30
1	1.85	2.25	2.44	2.44	1.88	2.52	2.66	2.50	2.01	2.31	2.42	2.42

PANEL E – Minimum retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.21	0.16	0.04	0.01	3.69	3.10	2.06	1.78	1.72	2.00	1.62	1.28
35	0.19	0.18	0.09	0.08	2.43	2.97	2.50	1.86	1.88	1.90	1.32	0.96
30	0.21	0.18	0.11	0.06	2.35	2.47	2.05	0.95	1.54	1.47	1.04	1.08
25	0.43	0.44	0.31	0.20	2.37	1.95	1.75	1.04	1.41	1.56	1.12	0.88
20	0.73	0.68	0.47	0.22	1.69	2.11	1.53	0.99	1.43	0.93	1.18	0.68
15	0.78	0.63	0.50	0.20	1.94	1.82	1.15	0.76	1.14	1.08	1.08	0.78
10	0.88	0.86	0.43	0.51	1.58	1.44	0.96	0.71	1.15	1.28	1.00	0.55
5	0.88	1.03	0.48	0.36	1.27	1.33	0.80	0.62	1.15	0.91	0.66	0.53
1	1.66	1.55	1.28	1.28	1.75	1.59	1.35	1.35	1.66	1.62	1.31	1.31

PANEL F – 5th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	1.17	1.13	1.14	0.94	10.97	11.11	11.30	9.79	5.12	5.13	5.24	4.61
35	1.22	1.22	1.20	0.95	8.27	8.40	8.52	7.54	4.29	4.21	4.41	3.88
30	1.22	1.19	1.21	1.03	6.36	6.32	6.56	5.75	3.57	3.55	3.62	3.27
25	1.94	1.88	1.86	1.59	5.58	5.63	5.69	5.12	3.36	3.34	3.33	3.06
20	2.29	2.29	2.31	1.98	4.79	4.80	4.90	4.38	3.10	3.08	3.13	2.88
15	2.08	2.04	1.98	1.76	3.73	3.67	3.73	3.41	2.62	2.62	2.62	2.37
10	1.90	1.92	1.87	1.67	2.87	2.90	2.84	2.62	2.28	2.25	2.21	2.07
5	1.62	1.63	1.52	1.49	2.07	2.05	1.97	1.92	1.81	1.80	1.76	1.72
1	2.03	2.02	1.99	1.99	2.12	2.11	2.07	2.07	2.06	2.06	0.01	2.01

PANEL G – 25th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	2.72	2.72	2.85	2.72	20.23	20.37	21.48	20.63	8.79	8.68	9.08	8.91
35	2.65	2.68	2.78	2.70	14.66	14.73	15.30	14.92	6.97	7.01	7.30	7.22
30	2.46	2.51	2.58	2.54	10.52	10.50	11.15	10.89	5.66	5.61	5.81	5.78
25	3.65	3.60	3.74	3.72	8.94	9.14	9.41	9.37	5.15	5.17	5.30	5.30
20	4.02	4.00	4.15	4.09	7.49	7.51	7.75	7.70	4.62	4.66	4.82	4.77
15	3.22	3.23	3.34	3.35	5.39	5.32	5.60	5.54	3.70	3.75	3.83	3.81
10	2.72	2.74	2.80	2.81	3.91	3.93	3.99	4.01	3.04	3.04	3.10	3.10
5	2.05	2.07	2.11	2.13	2.57	2.55	2.61	2.64	2.23	2.25	2.30	2.31
1	2.23	2.25	2.26	2.26	2.33	2.34	2.35	2.35	2.27	2.28	2.28	2.29

PANEL H – 75th percentile retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	8.85	8.91	8.69	9.56	51.45	51.23	50.49	53.81	20.35	20.23	20.09	21.00
35	8.19	7.96	7.77	8.37	34.29	34.02	33.61	35.17	15.06	15.08	14.78	15.64
30	6.78	6.86	6.73	7.20	22.72	22.36	22.43	23.38	11.28	11.25	11.24	11.67
25	8.87	8.90	8.85	9.33	18.51	18.49	18.17	19.05	9.88	10.05	9.74	10.27
20	8.70	8.64	8.53	8.88	14.27	14.11	13.90	14.61	8.44	8.41	8.55	8.75
15	6.11	6.15	6.16	6.35	9.21	9.15	9.22	9.42	6.22	6.27	6.23	6.37
10	4.49	4.44	4.49	4.56	6.04	6.12	6.05	6.16	4.61	4.64	4.66	4.73
5	2.87	2.86	2.90	2.90	3.47	3.47	3.50	3.51	3.03	3.02	3.05	3.04
1	2.57	2.56	2.57	2.57	2.66	2.66	2.67	2.67	2.59	2.58	2.60	2.60

PANEL I – 95th percentile retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	21.54	20.38	17.67	20.00	103.75	103.54	90.76	100.47	39.81	38.83	34.33	37.17
35	18.05	17.36	15.20	16.99	65.26	64.62	56.15	63.00	27.95	27.25	24.18	26.97
30	13.76	13.92	12.11	13.26	40.57	40.27	35.87	38.38	19.14	19.57	17.53	18.67
25	16.90	17.13	14.87	16.35	32.74	31.72	28.26	30.90	16.84	16.70	14.81	15.85
20	15.42	14.90	13.37	14.28	23.01	22.83	20.85	21.73	13.47	13.28	12.17	12.83
15	9.79	9.78	8.88	9.29	13.63	13.45	12.55	12.94	9.17	9.25	8.64	8.74
10	6.37	6.26	5.93	6.07	9.31	8.39	7.68	7.95	6.32	6.36	5.93	6.05
5	3.65	3.61	3.41	3.43	4.30	4.33	4.09	4.11	3.75	3.74	3.57	3.57
1	2.82	2.82	2.76	2.76	2.93	2.93	2.87	2.86	2.84	2.84	2.78	2.78

PANEL J – Maximum retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	123.99	130.64	60.72	96.93	423.52	769.16	230.71	353.44	232.28	206.18	96.86	125.79
35	79.77	93.78	53.36	73.37	304.53	256.37	178.89	206.35	126.76	119.24	56.58	86.09
30	89.55	53.39	36.04	45.96	152.43	155.25	84.11	109.23	69.48	110.95	38.97	57.34
25	56.15	59.49	37.72	38.30	138.08	119.58	69.38	81.58	42.92	50.77	33.58	37.96
20	46.02	53.62	36.13	34.69	79.00	87.97	43.73	44.56	44.87	41.95	25.30	34.18
15	25.54	37.06	21.46	18.15	45.64	37.37	21.85	24.21	28.87	40.31	15.52	16.10
10	15.16	15.74	9.89	11.22	16.47	16.06	11.47	14.62	14.67	14.20	9.73	11.04
5	6.30	6.30	5.21	4.86	6.49	8.83	6.49	5.36	5.72	5.82	5.28	4.82
1	3.51	3.80	3.72	3.72	3.63	4.12	4.01	3.85	3.67	3.93	3.73	3.73

Table A.4: Distribution of retirement wealth ratios – Balanced strategy

This table presents the distribution of terminal wealth for the four modeling techniques over nine investment horizons for the three accumulation models. Initial wealth (W_0) is as per row five of Table 6.1. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. Performance measures are group by measure type with measures of central tendency reported in Panel A (mean retirement wealth ratio) and Panel B (median, or 50th percentile retirement wealth ratio). Measures of dispersion are reported in Panels C (standard deviation in retirement wealth ratio terms) and D (retirement wealth ratio range). Panels E through G summarise the lower half of the distribution of terminal wealth (minimum, 5th percentile and 25th percentile retirement wealth ratio respectively). Panels H through J summarise the upper half of the distribution of terminal wealth (75th percentile, 95th percentile and maximum retirement wealth ratio respectively). Results are presented as multiples (i.e. in unit of "times" as in "*x* times final salary").

PANEL A – Mean retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	4.75	4.76	4.59	4.81	31.02	31.11	30.37	31.23	12.80	12.73	12.63	12.86
35	4.61	4.55	4.40	4.61	21.74	21.85	21.33	21.91	10.05	10.04	9.86	10.17
30	4.11	4.17	4.05	4.18	15.35	15.25	15.19	15.45	7.85	7.94	7.83	7.98
25	5.85	5.86	5.72	5.89	12.97	13.14	12.85	13.21	7.26	7.28	7.07	7.26
20	6.16	6.14	6.02	6.13	10.66	10.58	10.45	10.63	6.47	6.43	6.41	6.46
15	4.75	4.72	4.66	4.76	7.32	7.29	7.29	7.36	4.99	5.04	4.99	5.02
10	3.66	3.67	3.66	3.69	5.03	5.11	5.03	5.08	3.92	3.92	3.89	3.92
5	2.50	2.51	2.52	2.52	3.07	3.07	3.06	3.08	2.69	2.68	2.68	2.68
1	2.41	2.41	2.42	2.42	2.52	2.51	2.51	2.52	2.44	2.44	2.44	2.45

PANEL B – Median (50th percentile) retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	3.71	3.70	3.79	3.91	26.15	26.24	26.66	27.15	11.04	10.99	11.35	11.50
35	3.70	3.69	3.70	3.86	18.70	18.92	19.09	19.71	8.84	8.81	9.01	9.28
30	3.38	3.47	3.52	3.63	13.65	13.50	13.87	14.14	7.05	7.10	7.25	7.42
25	5.04	4.98	5.12	5.27	11.47	11.79	11.92	12.23	6.53	6.54	6.59	6.82
20	5.46	5.46	5.54	5.64	6.54	9.57	9.82	10.06	5.90	5.87	6.03	6.13
15	4.33	4.28	4.39	4.52	6.77	6.81	6.99	7.13	4.65	4.67	4.79	4.85
10	3.43	3.44	3.53	3.62	4.77	4.84	4.91	4.98	3.73	3.72	3.80	3.87
5	2.43	2.43	2.52	2.53	2.99	2.98	3.06	3.11	2.62	2.62	2.67	2.69
1	2.40	2.41	2.44	2.45	2.50	2.50	2.54	2.54	2.43	2.43	2.47	2.47

PANEL C – Standard deviation of retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	3.82	3.83	3.20	3.56	19.69	19.67	16.57	18.38	7.40	7.34	6.34	6.83
35	3.48	3.31	2.87	3.22	12.69	12.45	10.72	11.81	5.30	5.26	4.52	5.08
30	2.74	2.79	2.39	2.60	7.89	9.94	6.96	7.57	3.71	3.93	3.34	3.61
25	3.53	3.61	3.05	3.34	6.53	6.46	5.55	6.16	3.39	3.39	2.85	3.09
20	3.26	3.20	2.84	3.08	4.86	4.82	4.18	4.46	2.75	2.73	2.44	2.57
15	2.18	2.20	1.91	2.04	2.89	2.83	2.48	2.65	1.88	1.94	1.67	1.77
10	1.34	1.32	1.20	1.28	1.60	1.66	1.46	1.57	1.24	1.27	1.11	1.18
5	0.63	0.63	0.59	0.60	0.71	0.73	0.66	0.68	0.61	0.61	0.57	0.57
1	0.26	0.26	0.27	0.27	0.27	0.27	0.28	0.28	0.26	0.25	0.27	0.27

PANEL D – Retirement wealth ratio range

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	66.38	54.81	34.78	36.99	245.67	334.68	146.58	185.65	114.60	113.56	54.34	57.71
35	58.63	46.10	25.72	34.77	182.67	136.16	105.66	108.92	65.18	64.89	37.69	48.19
30	31.82	31.20	24.79	26.16	88.19	111.11	59.46	66.74	36.49	60.60	27.96	37.07
25	45.75	33.59	28.43	23.50	95.91	79.64	51.78	55.26	32.65	34.05	24.59	25.53
20	37.42	38.17	26.21	24.97	55.15	63.80	37.00	34.52	28.62	28.61	19.58	25.06
15	20.10	30.29	19.21	15.40	32.19	29.20	18.30	20.41	20.78	32.03	13.37	13.79
10	12.43	14.51	8.75	10.49	13.73	13.91	10.36	13.87	11.49	12.26	8.06	10.10
5	4.93	5.38	4.77	4.56	5.49	7.77	5.84	4.82	5.48	5.11	4.77	4.40
1	2.11	2.42	2.60	2.60	2.13	2.71	2.84	2.66	2.05	2.49	2.59	2.59

PANEL E – Minimum retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.19	0.24	0.08	0.03	4.31	3.57	2.24	1.95	1.94	2.14	1.70	1.39
35	0.23	0.27	0.17	0.11	3.22	3.46	2.60	1.96	1.97	2.08	1.37	1.01
30	0.42	0.26	0.22	0.11	2.60	2.67	2.10	1.01	1.33	1.59	1.10	1.09
25	0.52	0.63	0.53	0.32	1.96	2.08	1.80	1.07	1.42	1.69	1.15	0.95
20	0.86	0.82	0.61	0.30	2.57	2.12	1.59	1.06	1.47	0.91	1.17	0.70
15	0.82	0.71	0.56	0.24	1.65	1.82	1.18	0.73	1.17	1.13	1.00	0.73
10	0.95	0.87	0.47	0.52	1.38	1.45	0.97	0.72	1.24	1.30	1.02	0.54
5	0.96	1.01	0.45	0.35	1.29	1.31	0.78	0.59	1.11	0.87	0.64	0.50
1	1.59	1.51	1.23	1.23	1.72	1.55	1.30	1.30	1.56	1.58	1.26	1.26

PANEL F – 5th percentile retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	b	bs	bb	mc	b	bs	bb	mc	b	bs	bb
40	1.14	1.15	1.12	0.98	10.87	10.76	10.87	9.59	4.87	4.96	4.97	4.49
35	1.24	1.24	1.20	1.00	8.08	8.27	8.18	7.41	4.16	4.15	4.25	3.80
30	1.26	1.22	1.24	1.10	6.28	6.27	6.34	5.66	3.55	3.51	3.49	3.23
25	1.96	1.98	1.92	1.71	5.53	5.62	5.63	5.14	3.29	3.34	3.29	3.05
20	2.38	2.37	2.36	2.07	4.87	4.83	4.90	4.44	3.14	3.11	3.12	2.90
15	2.07	2.10	2.03	1.84	3.73	3.72	3.74	3.45	2.65	2.63	2.62	2.41
10	1.92	1.94	1.88	1.69	2.92	2.90	2.82	2.64	2.28	2.26	2.20	2.07
5	1.62	1.62	1.51	1.47	2.07	2.04	1.96	1.90	1.81	1.80	1.75	1.71
1	2.01	2.01	1.97	1.97	2.11	2.09	2.05	2.05	2.03	2.04	1.99	1.99

PANEL G – 25th percentile retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	b	bs	bb	mc	b	bs	bb	mc	b	bs	bb
40	2.29	2.31	2.35	2.30	17.94	18.10	18.60	18.05	7.89	7.81	8.07	7.96
35	2.36	2.36	2.40	2.36	13.19	13.37	13.73	13.47	6.47	6.46	6.65	6.63
30	2.27	2.30	2.32	2.30	9.91	9.80	10.21	10.04	5.24	5.26	5.43	5.37
25	3.40	3.38	3.50	3.47	8.47	8.65	8.83	8.84	4.93	4.93	5.01	5.03
20	3.90	3.88	3.97	3.91	7.23	7.27	7.48	7.46	4.55	4.53	4.65	4.62
15	3.21	3.19	3.28	3.29	5.29	5.25	5.49	5.45	3.65	3.70	3.78	3.75
10	2.70	2.74	2.80	2.80	3.88	3.92	3.98	3.99	3.04	3.03	3.09	3.09
5	2.05	2.07	2.12	2.13	2.57	2.55	2.62	2.64	2.25	2.25	2.31	2.31
1	2.23	2.24	2.25	2.26	2.33	2.34	2.34	2.35	2.25	2.27	2.27	2.28

PANEL H – 75th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	5.94	5.95	5.93	6.36	38.14	38.54	38.06	40.16	15.75	15.65	15.70	16.22
35	5.81	5.75	5.61	5.99	26.71	26.88	26.71	27.65	12.16	12.26	12.12	12.65
30	5.18	5.22	5.18	5.48	18.86	18.62	18.77	19.37	9.47	9.60	9.57	9.85
25	7.30	7.30	7.33	7.59	15.81	16.01	15.81	16.41	8.71	8.83	8.62	9.00
20	7.65	7.59	7.52	7.81	12.92	12.80	12.71	13.19	7.82	7.72	7.84	7.95
15	5.78	5.75	5.78	5.97	8.77	8.71	8.79	8.96	5.95	5.99	5.97	6.09
10	4.39	4.37	4.40	4.49	5.94	6.04	5.97	6.07	4.58	4.59	4.60	4.67
5	2.87	2.88	2.92	2.92	3.50	3.49	3.52	3.53	3.05	3.04	3.06	3.06
1	2.58	2.58	2.58	2.59	2.69	2.68	2.69	2.69	2.60	2.60	2.61	2.62

PANEL I – 95th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	11.82	11.81	10.80	11.57	67.85	67.90	61.93	66.51	26.40	26.58	24.66	25.74
35	10.92	10.70	9.90	10.79	45.61	45.64	41.27	44.89	20.22	19.74	18.38	19.68
30	9.33	9.45	8.62	9.20	30.16	30.43	28.37	29.20	14.94	15.29	14.10	14.74
25	12.60	12.78	11.47	12.31	25.39	25.25	23.27	24.78	13.81	13.64	12.39	12.98
20	12.26	12.29	11.34	11.85	19.95	19.65	18.26	18.85	11.53	11.55	10.82	11.21
15	8.83	8.82	8.16	8.45	12.71	12.51	11.75	12.07	8.44	8.60	8.14	8.16
10	6.14	6.07	5.79	5.92	8.01	8.20	7.57	7.80	6.21	6.25	5.85	5.93
5	3.64	3.65	3.45	3.47	4.34	4.38	4.13	4.15	3.79	3.77	3.60	3.61
1	2.85	2.85	2.79	2.79	2.98	2.96	2.90	2.90	2.89	2.87	2.81	2.82

PANEL J – Maximum retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	66.57	55.06	34.86	37.02	249.98	338.26	148.83	187.60	116.54	115.70	56.04	59.10
35	58.86	46.36	25.89	34.88	185.89	139.62	108.26	110.88	67.15	66.97	39.06	49.20
30	32.24	31.47	25.01	26.26	90.79	113.79	61.56	67.74	37.82	62.18	29.06	38.16
25	46.27	34.22	28.96	23.82	97.88	81.72	53.58	56.33	34.07	35.74	25.74	26.48
20	38.28	38.98	26.83	25.27	57.72	65.92	38.60	35.59	30.08	29.52	20.75	25.76
15	20.93	31.00	19.77	15.64	33.84	31.02	19.48	21.14	21.95	33.16	14.37	14.52
10	13.38	15.38	9.21	11.01	15.11	15.36	11.33	14.59	12.73	13.56	9.07	10.64
5	5.89	6.40	5.23	4.90	6.78	9.07	6.62	5.40	6.59	5.98	5.41	4.90
1	3.69	3.93	3.83	3.83	3.85	4.26	4.13	3.96	3.61	4.07	3.84	3.84

Table A.5: Distribution of retirement wealth ratios – Cash strategy

This table presents the distribution of terminal wealth for the four modeling techniques over nine investment horizons for the three accumulation models. Initial wealth (W_0) is as per row five of Table 6.1. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. Performance measures are group by measure type with measures of central tendency reported in Panel A (mean retirement wealth ratio) and Panel B (median, or 50th percentile retirement wealth ratio). Measures of dispersion are reported in Panels C (standard deviation in retirement wealth ratio terms) and D (retirement wealth ratio range). Panels E through G summarise the lower half of the distribution of terminal wealth (minimum, 5th percentile and 25th percentile retirement wealth ratio respectively). Panels H through J summarise the upper half of the distribution of terminal wealth (75th percentile, 95th percentile and maximum retirement wealth ratio respectively). Results are presented as multiples (i.e. in unit of "times" as in "*x* times final salary").

PANEL A – Mean retirement wealth ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.81	0.82	0.88	0.85	8.73	8.73	9.19	8.94	4.26	4.26	4.43	4.36
35	0.98	0.98	1.04	1.02	7.20	7.20	7.49	7.38	3.80	3.80	3.95	3.88
30	1.10	1.10	1.16	1.16	5.91	5.91	6.16	6.03	3.38	3.38	3.48	3.45
25	1.95	1.95	2.04	2.00	5.57	5.57	5.77	5.69	3.33	3.33	3.42	3.39
20	2.54	2.54	2.63	2.60	5.17	5.17	5.31	5.27	3.30	3.30	3.39	3.36
15	2.44	2.44	2.51	2.47	4.23	4.22	4.32	4.29	2.97	2.97	3.03	3.00
10	2.37	2.37	2.41	2.40	3.46	3.46	3.52	3.49	2.69	2.69	2.73	2.71
5	2.02	2.02	2.03	2.03	2.52	2.52	2.53	2.53	2.20	2.20	2.21	2.21
1	2.32	2.32	2.32	2.32	2.42	2.42	2.42	2.41	2.35	2.35	2.35	2.34

PANEL B – Median (50th percentile) retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.81	0.81	0.79	0.81	8.72	8.72	8.66	8.67	4.25	4.25	4.21	4.25
35	0.98	0.97	0.95	0.97	7.20	7.20	7.10	7.18	3.80	3.80	3.77	3.77
30	1.10	1.10	1.07	1.09	5.91	5.91	5.87	5.86	3.38	3.38	3.34	3.37
25	1.95	1.95	1.91	1.92	5.56	5.56	5.51	5.55	3.33	3.33	3.27	3.32
20	2.54	2.54	2.48	2.52	5.16	5.16	5.09	5.15	3.30	3.30	3.26	3.28
15	2.44	2.44	2.38	2.41	4.22	4.22	4.16	4.19	2.97	2.97	2.91	2.94
10	2.37	2.37	2.32	2.35	3.46	3.46	3.40	3.43	2.69	2.69	2.64	2.67
5	2.02	2.02	1.98	2.00	2.52	2.52	2.47	2.50	2.20	2.20	2.16	2.18
1	2.32	2.32	2.31	2.31	2.42	2.41	2.41	2.41	2.35	2.34	2.34	2.34

PANEL C – Standard deviation of retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.04	0.05	0.38	0.27	0.34	0.34	2.70	1.88	0.14	0.14	1.14	0.80
35	0.05	0.05	0.41	0.29	0.26	0.26	2.06	1.46	0.12	0.12	0.95	0.68
30	0.05	0.05	0.43	0.30	0.20	0.20	1.57	1.11	0.10	0.10	0.77	0.57
25	0.08	0.08	0.68	0.48	0.18	0.18	1.41	1.04	0.10	0.10	0.76	0.56
20	0.10	0.10	0.77	0.55	0.16	0.16	1.22	0.90	0.10	0.10	0.72	0.52
15	0.08	0.08	0.63	0.45	0.12	0.12	0.88	0.64	0.08	0.08	0.58	0.42
10	0.07	0.07	0.47	0.36	0.08	0.08	0.59	0.44	0.06	0.06	0.44	0.33
5	0.04	0.04	0.25	0.21	0.04	0.04	0.28	0.23	0.04	0.04	0.24	0.20
1	0.02	0.02	0.06	0.06	0.02	0.02	0.06	0.06	0.02	0.02	0.06	0.06

PANEL D – Retirement wealth ratio range

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.33	0.34	3.77	2.46	2.96	2.90	22.86	17.34	1.19	1.15	14.81	6.59
35	0.40	0.40	3.64	2.54	1.89	1.96	18.50	13.17	0.92	0.93	8.05	6.03
30	0.39	0.42	3.74	3.34	1.64	1.58	14.31	9.31	0.81	0.78	5.78	4.13
25	0.62	0.62	4.99	3.79	1.35	1.52	11.84	8.70	0.78	0.71	6.10	4.80
20	0.77	0.79	6.93	4.31	1.25	1.28	9.20	8.00	0.73	0.78	6.41	4.75
15	0.65	0.63	4.69	3.80	0.84	0.94	5.76	5.16	0.58	0.62	4.44	3.20
10	0.51	0.50	3.02	2.90	0.61	0.66	3.74	3.72	0.45	0.55	2.87	2.55
5	0.29	0.29	1.24	1.37	0.35	0.36	1.36	1.55	0.27	0.28	1.34	1.28
1	0.15	0.15	0.32	0.32	0.16	0.14	0.32	0.32	0.14	0.15	0.31	0.31

PANEL E – Minimum retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.67	0.65	0.25	0.30	7.38	7.53	4.29	4.76	3.77	3.77	2.26	2.68
35	0.78	0.81	0.32	0.41	6.34	6.30	3.73	4.27	3.37	3.42	2.16	2.40
30	0.93	0.94	0.45	0.52	5.18	5.18	3.27	3.57	2.99	3.07	2.04	2.23
25	1.65	1.68	0.86	0.98	4.95	4.86	3.12	3.35	2.97	3.00	2.06	2.20
20	2.19	2.20	1.30	1.41	4.57	4.59	3.14	3.25	2.95	2.91	2.09	2.21
15	2.14	2.16	1.46	1.48	3.82	3.81	2.83	2.95	2.67	2.67	2.03	2.09
10	2.14	2.14	1.68	1.69	3.17	3.20	2.60	2.60	2.48	2.46	2.04	2.02
5	1.88	1.87	1.70	1.70	2.36	2.36	2.16	2.15	2.07	2.07	1.89	1.89
1	2.24	2.26	2.24	2.24	2.34	2.36	2.34	2.34	2.28	2.29	2.27	2.27

PANEL F – 5th percentile retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.74	0.74	0.44	0.51	8.18	8.20	5.88	6.41	4.03	4.03	3.01	3.27
35	0.89	0.90	0.55	0.63	6.79	6.78	4.94	5.40	3.61	3.61	2.73	2.98
30	1.02	1.02	0.65	0.74	5.59	5.60	4.17	4.52	3.21	3.22	2.49	2.65
25	1.81	1.81	1.21	1.36	5.28	5.27	4.00	4.27	3.17	3.17	2.46	2.62
20	2.38	2.39	1.67	1.84	4.91	4.92	3.76	4.05	3.15	3.15	2.46	2.63
15	2.30	2.31	1.72	1.86	4.04	4.04	3.20	3.41	2.84	2.85	2.29	2.42
10	2.27	2.27	1.81	1.91	3.33	3.33	2.76	2.88	2.59	2.59	2.17	2.26
5	1.96	1.96	1.72	1.74	2.45	2.45	2.18	2.21	2.14	2.14	1.90	1.93
1	2.28	2.29	2.24	2.24	2.38	2.38	2.34	2.34	2.31	2.32	2.27	2.27

PANEL G – 25th percentile retirement wealth ratio

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.78	0.78	0.61	0.66	8.50	8.50	7.26	7.59	4.16	4.16	3.61	3.79
35	0.94	0.94	0.75	0.81	7.03	7.02	5.99	6.33	3.72	3.72	3.27	3.39
30	1.07	1.07	0.86	0.92	5.77	5.78	5.03	5.23	3.31	3.31	2.91	3.04
25	1.89	1.89	1.56	1.65	5.44	5.44	4.76	4.95	3.26	3.26	2.87	2.99
20	2.48	2.48	2.08	2.21	5.06	5.06	4.41	4.62	3.24	3.23	2.86	2.97
15	2.38	2.38	2.04	2.15	4.15	4.14	3.67	3.82	2.91	2.92	2.60	2.69
10	2.33	2.33	2.06	2.14	3.40	3.40	3.08	3.18	2.65	2.65	2.39	2.47
5	1.99	1.99	1.84	1.88	2.49	2.49	2.32	2.36	2.18	2.17	2.02	2.06
1	2.30	2.30	2.26	2.26	2.40	2.40	2.36	2.36	2.33	2.33	2.29	2.29

PANEL H – 75th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.84	0.85	1.06	0.99	8.95	8.95	10.55	10.00	4.35	4.35	4.99	4.79
35	1.01	1.01	1.23	1.17	7.37	7.37	8.57	8.21	3.89	3.89	4.43	4.26
30	1.14	1.14	1.37	1.30	6.04	6.04	6.96	6.66	3.45	3.45	3.89	3.78
25	2.00	2.00	2.38	2.26	5.69	5.69	6.49	6.26	3.39	3.39	3.83	3.71
20	2.61	2.61	3.02	2.91	5.27	5.27	5.97	5.78	3.37	3.36	3.77	3.66
15	2.49	2.49	2.85	2.72	4.30	4.30	4.81	4.66	3.02	3.02	3.33	3.24
10	2.42	2.42	2.67	2.60	3.52	3.51	3.83	3.74	2.73	2.73	2.97	2.91
5	2.04	2.04	2.18	2.15	2.55	2.55	2.70	2.67	2.23	2.23	2.35	2.32
1	2.33	2.33	2.35	2.35	2.43	2.43	2.45	2.45	2.36	2.36	2.38	2.38

PANEL I – 95th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.89	0.89	1.59	1.36	9.30	9.30	14.42	12.36	4.50	4.50	6.55	5.81
35	1.06	1.06	1.80	1.56	7.65	7.65	11.39	10.11	4.01	4.01	5.75	5.16
30	1.19	1.19	1.99	1.69	6.24	6.25	9.15	8.05	3.55	3.55	4.95	4.49
25	2.09	2.09	3.37	2.90	5.87	5.87	8.43	7.60	3.50	3.50	4.89	4.39
20	2.71	2.72	4.10	3.60	5.43	5.43	7.67	6.90	3.46	3.46	4.75	4.30
15	2.57	2.58	3.74	3.29	4.42	4.42	6.01	5.49	3.10	3.10	4.15	3.77
10	2.48	2.48	3.35	3.07	3.60	3.60	4.69	4.29	2.79	2.80	3.62	3.32
5	2.08	2.09	2.55	2.41	2.59	2.59	3.07	2.98	2.26	2.27	2.69	2.59
1	2.35	2.35	2.43	2.44	2.45	2.45	2.54	2.53	2.38	2.38	2.46	2.47

PANEL J – Maximum retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	1.00	0.99	4.02	2.76	10.34	10.42	27.15	22.10	4.96	4.93	17.07	9.27
35	1.18	1.21	3.96	2.94	8.24	8.25	22.23	17.44	4.29	4.35	10.21	8.43
30	1.32	1.36	4.18	3.86	6.82	6.75	17.58	12.88	3.80	3.85	7.82	6.36
25	2.28	2.30	5.85	4.77	6.30	6.38	14.96	12.05	3.75	3.71	8.16	7.00
20	2.97	2.99	8.23	5.73	5.82	5.88	12.33	11.25	3.68	3.69	8.50	6.96
15	2.79	2.79	6.15	5.28	4.66	4.75	8.59	8.12	3.25	3.29	6.47	5.30
10	2.65	2.64	4.70	4.59	3.78	3.86	6.34	6.33	2.93	3.01	4.90	4.57
5	2.16	2.16	2.94	3.07	2.71	2.73	3.51	3.70	2.34	2.35	3.23	3.16
1	2.39	2.41	2.56	2.56	2.50	2.50	2.66	2.66	2.42	2.43	2.58	2.58

Appendix B

This appendix reports the complete set of downside risk measures for each of the five asset allocation strategies considered in the third empirical study (Chapter Seven).

Table B.1: Downside risk measures – Stock strategy

This table presents selected downside risk measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures are: 95% value-at-risk (VaR) is reported in Panel A, and 95% expected tail loss is reported in Panel B. All results are expressed in *RWR* units.

PANEL A – 95% value-at-risk

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	1.08	1.08	1.06	0.78	10.71	10.43	10.90	8.60	4.74	4.81	4.97	4.14
35	1.10	1.12	1.06	0.77	7.62	7.76	7.94	6.55	3.91	3.87	4.08	3.36
30	1.11	1.06	1.07	0.86	5.96	5.70	5.98	4.86	3.24	3.21	3.21	2.89
25	1.66	1.67	1.61	1.28	5.04	5.01	5.17	4.36	3.13	3.01	2.96	2.63
20	1.95	1.94	1.97	1.54	4.22	4.18	4.30	3.64	2.73	2.72	2.79	2.45
15	1.64	1.69	1.60	1.33	3.17	3.13	3.23	2.81	2.30	2.27	2.27	1.94
10	1.56	1.56	1.48	1.24	2.46	2.44	2.33	2.09	1.89	1.90	1.84	1.65
5	1.35	1.34	1.20	1.15	1.74	1.71	1.60	1.52	1.52	1.52	1.46	1.39
1	1.80	1.80	1.74	1.74	1.89	1.88	1.81	1.82	1.86	1.84	1.76	1.76

PANEL B – 95% expected tail loss

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.71	0.70	0.66	0.45	7.63	7.63	7.64	5.75	3.55	3.65	3.55	2.86
35	0.74	0.74	0.68	0.45	5.72	5.81	5.67	4.40	3.03	3.03	2.96	2.38
30	0.76	0.72	0.70	0.52	4.56	4.35	4.36	3.36	2.58	2.52	2.43	2.06
25	1.19	1.15	1.05	0.78	3.91	3.86	3.73	3.04	2.47	2.37	2.27	1.89
20	1.43	1.41	1.35	0.99	3.31	3.22	3.16	2.58	2.18	2.15	2.10	1.77
15	1.25	1.28	1.14	0.84	2.56	2.50	3.45	2.04	1.90	1.83	1.76	1.42
10	1.26	1.25	1.08	0.84	2.06	2.03	1.79	1.56	1.58	1.57	1.42	1.25
5	1.14	1.13	0.88	0.79	1.49	1.46	1.22	1.13	1.32	1.30	1.10	1.03
1	1.68	1.65	1.47	1.45	1.75	1.72	1.53	1.53	1.72	1.68	1.48	1.48

Table B.2: Downside risk measures – Dynamic strategy

This table presents selected downside risk measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures are: 95% value-at-risk (VaR) is reported in Panel A, and 95% expected tail loss is reported in Panel B. All results are expressed in *RWR* units.

PANEL A – 95% value-at-risk

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	1.09	1.08	1.09	0.80	10.55	10.73	11.27	9.00	4.90	4.94	5.31	4.34
35	1.11	1.13	1.09	0.77	7.72	7.91	8.35	6.85	4.07	3.99	4.36	3.58
30	1.11	1.07	1.12	0.88	5.90	5.91	6.37	5.17	3.32	3.32	3.41	3.02
25	1.67	1.69	1.65	1.32	5.20	5.11	5.39	4.59	3.09	3.11	3.15	2.75
20	1.97	1.98	2.04	1.59	4.40	4.30	4.54	3.83	2.81	2.79	2.91	2.54
15	1.71	1.73	1.71	1.39	3.28	3.23	3.40	2.95	2.35	2.32	2.40	2.03
10	1.63	1.64	1.56	1.32	2.57	2.53	2.47	2.19	1.98	1.98	1.92	1.73
5	1.39	1.41	1.26	1.21	1.80	1.79	1.68	1.60	1.58	1.58	1.53	1.47
1	1.83	1.84	1.75	1.76	1.92	1.91	1.84	1.84	1.87	1.88	1.79	1.78

PANEL B – 95% expected tail loss

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.73	0.70	0.67	0.45	7.90	7.75	7.98	5.96	3.67	3.70	3.74	2.98
35	0.76	0.74	0.69	0.46	5.83	5.87	5.91	4.55	3.10	3.08	3.11	2.50
30	0.74	0.73	0.72	0.53	4.47	4.41	4.59	3.52	2.60	2.56	2.56	2.16
25	1.17	1.15	1.08	0.80	4.02	3.91	3.91	3.16	2.40	2.41	2.40	1.97
20	1.42	1.42	1.40	1.02	3.45	3.26	3.30	2.69	2.21	2.19	2.21	1.85
15	1.31	1.29	1.20	0.88	2.62	2.54	2.60	2.13	1.91	1.86	1.86	1.49
10	1.27	1.30	1.15	0.89	2.12	2.10	1.89	1.63	1.64	1.63	1.50	1.31
5	1.16	1.19	0.94	0.84	1.56	1.53	1.29	1.20	1.37	1.35	1.16	1.09
1	1.70	1.68	1.51	1.48	1.78	1.75	1.56	1.55	1.74	1.70	1.51	1.51

Table B.3: Downside risk measures – Target-date strategy

This table presents selected downside risk measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures are: 95% value-at-risk (VaR) is reported in Panel A, and 95% expected tail loss is reported in Panel B. All results are expressed in *RWR* units.

PANEL A – 95% value-at-risk

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	1.17	1.13	1.14	0.94	10.98	11.12	11.30	9.79	5.12	5.13	5.24	4.61
35	1.22	1.22	1.20	0.95	8.27	8.43	8.52	7.54	4.29	4.21	4.41	3.88
30	1.23	1.19	1.21	1.03	6.36	6.32	6.56	5.75	3.57	3.55	3.62	3.28
25	1.94	1.88	1.86	1.59	5.58	5.64	5.69	5.13	3.36	3.34	3.33	3.06
20	2.29	2.29	2.31	1.98	4.79	4.81	4.90	4.38	3.10	3.08	3.13	2.88
15	2.08	2.04	1.98	1.76	3.73	3.67	3.73	3.41	2.62	2.62	2.62	2.37
10	1.90	1.92	1.87	1.67	2.87	2.90	2.84	2.62	2.28	2.25	2.21	2.07
5	1.62	1.63	1.52	1.49	2.07	2.05	1.97	1.92	1.81	1.80	1.76	1.72
1	2.03	2.02	1.99	1.99	2.12	2.11	2.07	2.07	2.06	2.06	2.01	2.01

PANEL B – 95% expected tail loss

Investment horizon	Initial endowment				Constant contributions				Constant percentage contributions			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.85	0.81	0.80	0.60	8.76	8.77	8.95	7.30	4.19	4.21	4.16	3.58
35	0.89	0.88	0.84	0.63	6.72	6.79	6.70	5.67	3.60	3.55	3.53	3.03
30	0.94	0.90	0.89	0.71	5.28	5.21	5.26	4.43	3.00	2.99	2.97	2.63
25	1.48	1.45	1.38	1.11	4.66	4.67	4.61	3.98	2.86	2.85	2.79	2.43
20	1.84	1.82	1.79	1.44	4.08	4.02	3.97	3.47	2.69	2.63	2.61	2.34
15	1.72	1.70	1.59	1.30	3.25	3.16	3.15	2.75	2.30	2.27	2.23	1.93
10	1.65	1.67	1.54	1.31	2.54	2.58	2.40	2.17	2.03	2.00	1.90	1.73
5	1.47	1.47	1.28	1.20	1.90	1.87	1.68	1.60	1.66	1.64	1.49	1.43
1	1.95	1.93	1.81	1.79	2.03	2.01	1.88	1.87	1.98	1.95	1.82	1.82

Table B.4: Downside risk measures – Balanced strategy

This table presents selected downside risk measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures are: 95% value-at-risk (VaR) is reported in Panel A, and 95% expected tail loss is reported in Panel B. All results are expressed in *RWR* units.

PANEL A – 95% value-at-risk

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	1.14	1.15	1.12	0.98	10.79	10.77	10.87	9.60	4.87	4.97	4.97	4.49
35	1.24	1.24	1.20	1.00	8.08	8.27	8.19	7.41	4.16	4.15	4.25	3.80
30	1.26	1.22	1.24	1.10	6.28	6.27	6.34	5.66	3.55	3.51	3.49	3.24
25	1.96	1.98	1.92	1.71	5.53	5.62	5.63	5.14	3.29	3.34	3.29	3.06
20	2.38	2.37	2.37	2.07	4.88	4.83	4.90	4.44	3.14	3.11	3.12	2.90
15	2.07	2.10	2.03	1.84	3.73	3.72	3.74	3.46	2.65	2.63	2.62	2.41
10	1.92	1.94	1.88	1.69	2.92	2.90	2.82	2.64	2.28	2.26	2.20	2.07
5	1.62	1.62	1.51	1.47	2.07	2.04	1.96	1.90	1.81	1.80	1.75	1.71
1	2.01	2.01	1.97	1.97	2.11	2.09	2.05	2.05	2.03	2.04	1.99	1.99

PANEL B – 95% expected tail loss

Investment horizon	Initial endowment mc	b	bs	bb	Constant contributions mc	b	bs	bb	Constant percentage contributions mc	b	bs	bb
40	0.88	0.88	0.84	0.67	8.86	8.82	8.74	7.34	4.10	4.17	4.04	3.55
35	0.96	0.95	0.90	0.72	6.69	6.87	6.65	5.74	3.51	3.55	3.46	3.01
30	0.99	0.97	0.95	0.79	5.21	5.27	5.21	4.44	3.02	3.00	2.91	2.62
25	1.57	1.58	1.49	1.26	4.71	4.75	4.61	4.06	2.87	2.87	2.76	2.45
20	1.97	1.94	1.89	1.57	4.20	4.10	4.00	3.53	2.72	2.67	2.62	2.35
15	1.74	1.76	1.65	1.38	3.25	3.21	3.16	2.80	2.32	2.30	2.23	1.95
10	1.68	1.69	1.56	1.34	2.58	2.59	2.40	2.19	2.01	2.00	1.89	1.73
5	1.46	1.46	1.27	1.18	1.90	1.85	1.67	1.58	1.66	1.63	1.48	1.41
1	1.92	1.90	1.78	1.76	2.01	1.99	1.85	1.84	1.93	1.93	1.79	1.79

Table B.5: Downside risk measures – Cash strategy

This table presents selected downside risk measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures are: 95% value-at-risk (VaR) is reported in Panel A, and 95% expected tail loss is reported in Panel B. All results are expressed in *RWR* units.

PANEL A – 95% value-at-risk

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.74	0.74	0.44	0.51	8.18	8.20	5.88	6.41	4.03	4.03	3.01	3.27
35	0.89	0.90	0.55	0.63	6.79	6.78	4.94	5.40	3.61	3.61	2.73	2.98
30	1.02	1.02	0.65	0.74	5.59	5.60	4.17	4.52	3.21	3.22	2.49	2.65
25	1.81	1.81	1.21	1.36	5.28	5.28	4.00	4.27	3.17	3.17	2.46	2.62
20	2.38	2.39	1.67	1.84	4.91	4.92	3.76	4.05	3.15	3.15	2.46	2.63
15	2.30	2.31	1.72	1.86	4.04	4.04	3.20	3.41	2.84	2.85	2.29	2.42
10	2.27	2.27	1.81	1.91	3.33	3.33	2.76	2.88	2.59	2.59	2.17	2.26
5	1.96	1.96	1.72	1.74	2.45	2.45	2.18	2.21	2.14	2.14	1.90	1.93
1	2.28	2.29	2.24	2.24	2.38	2.38	2.34	2.34	2.31	2.32	2.27	2.27

PANEL B – 95% expected tail loss

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.73	0.73	0.39	0.46	8.05	8.07	5.45	6.03	3.97	3.97	2.84	3.10
35	0.87	0.88	0.50	0.58	6.69	6.69	4.64	5.07	3.57	3.57	2.59	2.82
30	1.00	1.00	0.59	0.68	5.51	5.52	3.95	4.29	3.17	3.18	2.37	2.53
25	1.78	1.78	1.11	1.26	5.21	5.21	3.78	4.06	3.13	3.14	2.34	2.50
20	2.34	2.35	1.57	1.73	4.85	4.85	3.57	3.84	3.11	3.11	2.35	2.53
15	2.27	2.28	1.63	1.77	3.99	3.99	3.07	3.27	2.81	2.82	2.20	2.33
10	2.24	2.25	1.76	1.84	3.30	3.30	2.69	2.79	2.57	2.57	2.11	2.19
5	1.94	1.94	1.71	1.72	2.43	2.43	2.17	2.18	2.12	2.13	1.90	1.91
1	2.28	2.28	2.24	2.24	2.38	2.38	2.34	2.34	2.31	2.31	2.27	2.27

Appendix C

This appendix reports the complete set of target-relative measures for each of the five asset allocation strategies considered in the third empirical study (Chapter Seven).

Table C.1: Target-relative measures – Stock strategy

This table presents selected target-related measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures, all calculated versus the target *RWR*, are: probability of shortfall is reported in Panel A, expected shortfall is reported in Panel B and the Sortino ratio is reported in Panel C. Probability of shortfall is expressed as percentages, expected shortfall is expressed in *RWR* units, and Sortino ratio is expressed as a ratio of return to risk.

PANEL A – Probability of shortfall

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.24	0.24	0.22	0.24	0.23	0.22	0.20	0.22	0.22	0.23	0.20	0.22
35	0.25	0.25	0.24	0.25	0.24	0.23	0.21	0.23	0.25	0.24	0.21	0.22
30	0.28	0.26	0.24	0.26	0.25	0.25	0.22	0.24	0.26	0.26	0.22	0.23
25	0.28	0.29	0.26	0.27	0.27	0.26	0.24	0.25	0.27	0.27	0.25	0.25
20	0.31	0.31	0.28	0.29	0.30	0.29	0.26	0.27	0.29	0.30	0.27	0.27
15	0.33	0.33	0.30	0.30	0.32	0.33	0.28	0.29	0.33	0.32	0.28	0.30
10	0.37	0.37	0.33	0.33	0.36	0.35	0.32	0.32	0.35	0.35	0.31	0.31
5	0.40	0.40	0.33	0.33	0.38	0.38	0.32	0.31	0.42	0.40	0.34	0.34
1	0.46	0.45	0.39	0.38	0.46	0.45	0.39	0.38	0.43	0.42	0.37	0.36

PANEL B – Expected shortfall

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.34	0.34	0.33	0.39	1.83	1.82	1.61	2.04	0.75	0.76	0.67	0.81
35	0.34	0.35	0.34	0.40	1.38	1.35	1.21	1.51	0.63	0.62	0.55	0.67
30	0.34	0.33	0.31	0.36	1.01	1.05	0.90	1.11	0.52	0.52	0.47	0.54
25	0.50	0.52	0.49	0.56	0.95	0.92	0.83	0.97	0.50	0.52	0.48	0.55
20	0.58	0.57	0.53	0.62	0.84	0.84	0.76	0.87	0.51	0.51	0.46	0.52
15	0.47	0.47	0.44	0.48	0.62	0.64	0.55	0.62	0.42	0.42	0.38	0.44
10	0.38	0.38	0.35	0.40	0.46	0.45	0.43	0.47	0.33	0.34	0.31	0.34
5	0.23	0.23	0.22	0.22	0.24	0.25	0.23	0.24	0.24	0.23	0.21	0.22
1	0.14	0.13	0.14	0.13	0.14	0.14	0.14	0.14	0.12	0.12	0.13	0.12

PANEL C – Sortino ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	15.11	14.98	12.43	13.11	12.07	12.56	11.19	11.09	11.02	10.89	9.97	9.76
35	11.68	11.44	9.74	10.25	9.84	9.95	8.86	8.75	8.94	8.82	7.94	8.18
30	9.03	9.20	8.06	8.35	8.00	7.45	7.13	6.97	7.15	7.22	6.52	6.57
25	6.87	6.80	5.84	6.06	3.10	6.45	5.71	5.90	6.11	5.89	5.04	5.16
20	4.80	4.83	4.35	4.31	4.72	4.66	4.26	4.24	4.43	4.31	4.14	4.00
15	3.32	3.40	3.00	3.14	3.28	3.12	3.10	3.02	3.16	3.25	2.95	2.81
10	2.00	2.02	1.89	1.83	2.03	2.18	1.83	1.88	2.20	2.19	1.96	1.96
5	1.18	1.17	1.06	1.04	1.29	1.25	1.14	1.16	0.97	1.03	1.00	0.96
1	0.31	0.33	0.31	0.34	0.32	0.31	0.28	0.30	0.48	0.45	0.40	0.43

Table C.2: Target-relative measures – Dynamic strategy

This table presents selected target-related measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures, all calculated versus the target *RWR*, are: probability of shortfall is reported in Panel A, expected shortfall is reported in Panel B and the Sortino ratio is reported in Panel C. Probability of shortfall is expressed as percentages, expected shortfall is expressed in *RWR* units, and Sortino ratio is expressed as a ratio of return to risk.

PANEL A – Probability of shortfall

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.21	0.21	0.20	0.22	0.19	0.19	0.16	0.19	0.19	0.19	0.16	0.18
35	0.22	0.22	0.21	0.22	0.20	0.20	0.17	0.19	0.20	0.20	0.18	0.19
30	0.23	0.23	0.21	0.23	0.21	0.22	0.18	0.20	0.21	0.21	0.19	0.20
25	0.25	0.25	0.23	0.24	0.23	0.22	0.20	0.21	0.23	0.23	0.21	0.22
20	0.27	0.26	0.24	0.25	0.24	0.25	0.22	0.23	0.26	0.25	0.23	0.23
15	0.28	0.28	0.26	0.26	0.27	0.28	0.24	0.24	0.27	0.27	0.24	0.26
10	0.31	0.31	0.29	0.29	0.31	0.30	0.28	0.28	0.30	0.29	0.28	0.27
5	0.34	0.33	0.31	0.30	0.32	0.32	0.30	0.28	0.35	0.35	0.32	0.31
1	0.41	0.40	0.36	0.34	0.41	0.40	0.36	0.35	0.37	0.37	0.33	0.32

PANEL B – Expected shortfall

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.32	0.31	0.30	0.36	1.63	1.62	1.39	1.81	0.67	0.65	0.56	0.71
35	0.33	0.32	0.31	0.37	1.25	1.18	1.04	1.32	0.52	0.54	0.46	0.58
30	0.31	0.30	0.28	0.33	0.88	0.91	0.76	0.97	0.45	0.44	0.39	0.47
25	0.47	0.47	0.44	0.51	0.80	0.80	0.71	0.85	0.44	0.45	0.39	0.47
20	0.53	0.51	0.47	0.55	0.71	0.72	0.65	0.76	0.44	0.43	0.39	0.45
15	0.41	0.41	0.38	0.42	0.53	0.54	0.46	0.53	0.35	0.35	0.32	0.37
10	0.32	0.31	0.30	0.35	0.38	0.37	0.37	0.41	0.28	0.28	0.27	0.30
5	0.19	0.18	0.19	0.19	0.20	0.20	0.20	0.20	0.19	0.19	0.19	0.19
1	0.12	0.11	0.12	0.11	0.12	0.12	0.12	0.12	0.10	0.10	0.11	0.11

PANEL C – Sortino ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	11.86	11.64	9.87	10.28	9.73	9.67	8.96	8.58	8.28	8.39	7.95	7.55
35	8.89	8.82	7.62	7.86	7.35	7.56	6.94	6.67	7.01	6.65	6.23	6.22
30	7.09	6.94	6.20	6.34	5.80	5.58	5.51	5.20	5.15	5.31	5.00	4.88
25	5.03	4.96	4.38	4.46	4.62	4.66	4.28	4.29	4.22	4.20	3.78	3.73
20	3.39	3.39	3.15	3.04	3.25	3.21	3.06	2.96	2.99	2.98	2.93	2.78
15	2.28	2.23	2.02	2.05	2.09	2.02	2.08	1.95	2.05	2.08	1.96	1.77
10	1.14	1.13	1.04	0.95	1.13	1.23	0.99	0.99	1.22	1.23	1.08	1.06
5	0.60	0.62	0.50	0.48	0.72	0.70	0.56	0.59	0.48	0.50	0.42	0.41
1	0.22	0.22	0.17	0.19	0.18	0.19	0.15	0.17	0.37	0.35	0.27	0.29

Table C.3: Target-relative measures – Target-date strategy

This table presents selected target-related measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures, all calculated versus the target *RWR*, are: probability of shortfall is reported in Panel A, expected shortfall is reported in Panel B and the Sortino ratio is reported in Panel C. Probability of shortfall is expressed as percentages, expected shortfall is expressed in *RWR* units, and Sortino ratio is expressed as a ratio of return to risk.

PANEL A – Probability of shortfall

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.33	0.33	0.31	0.32	0.31	0.31	0.28	0.30	0.32	0.33	0.30	0.30
35	0.34	0.34	0.32	0.33	0.32	0.33	0.30	0.31	0.34	0.33	0.31	0.31
30	0.36	0.35	0.33	0.33	0.35	0.35	0.31	0.32	0.35	0.35	0.32	0.32
25	0.37	0.38	0.35	0.34	0.37	0.36	0.34	0.33	0.38	0.38	0.36	0.34
20	0.40	0.40	0.38	0.38	0.39	0.39	0.37	0.36	0.41	0.40	0.37	0.37
15	0.43	0.43	0.40	0.38	0.43	0.43	0.39	0.38	0.44	0.43	0.40	0.39
10	0.47	0.47	0.44	0.42	0.46	0.45	0.43	0.42	0.45	0.45	0.42	0.40
5	0.48	0.48	0.42	0.41	0.45	0.45	0.41	0.38	0.50	0.50	0.46	0.44
1	0.51	0.50	0.44	0.43	0.51	0.50	0.44	0.44	0.46	0.45	0.39	0.38

PANEL B – Expected shortfall

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.41	0.42	0.39	0.45	2.21	2.18	1.97	2.33	0.88	0.91	0.82	0.93
35	0.42	0.41	0.40	0.45	1.61	1.58	1.45	1.67	0.71	0.72	0.65	0.74
30	0.39	0.38	0.36	0.39	1.18	1.19	1.04	1.20	0.58	0.59	0.54	0.59
25	0.56	0.58	0.55	0.59	1.07	1.02	0.94	1.04	0.58	0.58	0.54	0.58
20	0.62	0.62	0.58	0.64	0.90	0.90	0.83	0.90	0.56	0.55	0.51	0.55
15	0.48	0.48	0.46	0.48	0.63	0.65	0.57	0.62	0.44	0.43	0.40	0.44
10	0.37	0.36	0.35	0.37	0.44	0.43	0.42	0.44	0.32	0.32	0.30	0.32
5	0.19	0.19	0.18	0.19	0.19	0.20	0.19	0.19	0.20	0.20	0.19	0.19
1	0.09	0.09	0.09	0.09	0.10	0.09	0.10	0.09	0.08	0.08	0.08	0.08

PANEL C – Sortino ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	4.67	4.58	4.05	4.23	3.98	4.02	3.78	3.70	3.62	3.45	3.35	3.28
35	3.86	3.67	3.27	3.43	3.25	3.27	3.02	3.00	2.92	2.89	2.71	2.80
30	3.06	3.07	2.77	2.89	2.57	2.48	2.49	2.42	2.33	2.37	2.25	2.26
25	2.28	2.26	2.02	2.12	2.12	2.17	1.97	2.04	1.84	1.91	1.65	1.75
20	1.62	1.55	1.43	1.43	1.52	1.49	1.40	1.42	1.32	1.32	1.32	1.29
15	1.02	1.03	0.93	1.00	0.99	0.92	0.95	0.94	0.88	0.95	0.88	0.83
10	0.53	0.50	0.48	0.48	0.54	0.61	0.48	0.51	0.61	0.63	0.56	0.58
5	0.32	0.31	0.30	0.28	0.46	0.43	0.40	0.42	0.19	0.20	0.21	0.19
1	0.05	0.05	0.05	0.08	0.02	0.04	0.03	0.05	0.25	0.23	0.20	0.23

Table C.4: Target-relative measures – Balanced strategy

This table presents selected target-related measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures, all calculated versus the target *RWR*, are: probability of shortfall is reported in Panel A, expected shortfall is reported in Panel B and the Sortino ratio is reported in Panel C. Probability of shortfall is expressed as percentages, expected shortfall is expressed in *RWR* units, and Sortino ratio is expressed as a ratio of return to risk.

PANEL A – Probability of shortfall

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.44	0.43	0.42	0.41	0.41	0.40	0.39	0.39	0.41	0.42	0.39	0.39
35	0.43	0.44	0.43	0.42	0.42	0.41	0.40	0.39	0.41	0.42	0.40	0.38
30	0.44	0.43	0.41	0.40	0.42	0.43	0.39	0.38	0.43	0.43	0.40	0.39
25	0.44	0.45	0.42	0.41	0.44	0.42	0.40	0.39	0.44	0.43	0.42	0.40
20	0.45	0.45	0.43	0.43	0.44	0.44	0.41	0.40	0.45	0.45	0.42	0.41
15	0.45	0.46	0.44	0.41	0.46	0.45	0.42	0.40	0.46	0.46	0.43	0.41
10	0.49	0.48	0.45	0.43	0.48	0.46	0.44	0.43	0.46	0.46	0.43	0.41
5	0.48	0.47	0.42	0.41	0.45	0.45	0.41	0.37	0.49	0.49	0.45	0.43
1	0.50	0.49	0.44	0.43	0.50	0.49	0.43	0.43	0.46	0.45	0.39	0.38

PANEL B – Expected shortfall

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	0.52	0.52	0.51	0.54	2.78	2.74	2.58	2.88	1.13	1.15	1.06	1.14
35	0.49	0.50	0.49	0.52	2.00	1.93	1.89	1.99	0.87	0.87	0.82	0.88
30	0.45	0.44	0.43	0.45	1.37	1.39	1.27	1.39	0.69	0.69	0.65	0.68
25	0.64	0.64	0.61	0.64	1.22	1.15	1.10	1.16	0.65	0.65	0.63	0.65
20	0.65	0.66	0.63	0.68	0.97	0.97	0.91	0.96	0.58	0.59	0.56	0.59
15	0.49	0.49	0.47	0.49	0.67	0.68	0.60	0.64	0.46	0.45	0.42	0.45
10	0.38	0.36	0.35	0.37	0.45	0.43	0.42	0.44	0.32	0.32	0.31	0.32
5	0.19	0.19	0.18	0.19	0.19	0.20	0.19	0.19	0.20	0.12	0.19	0.19
1	0.10	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.09	0.08	0.08	0.08

PANEL C – Sortino ratio

Investment horizon	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
	mc	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	1.57	1.59	1.41	1.53	1.57	1.59	1.49	1.50	1.39	1.36	1.35	1.35
35	1.49	1.41	1.24	1.38	1.35	1.42	1.29	1.34	1.27	1.27	1.19	1.27
30	1.28	1.35	1.21	1.31	1.21	1.16	1.19	1.17	1.07	1.14	1.08	1.12
25	1.10	1.12	1.00	1.08	1.04	1.15	1.04	1.12	0.99	1.00	0.83	0.93
20	0.92	0.89	0.80	0.82	0.96	0.90	0.84	0.88	0.83	0.78	0.78	0.78
15	0.75	0.72	0.65	0.71	0.70	0.66	0.69	0.69	0.61	0.68	0.63	0.60
10	0.41	0.43	0.41	0.41	0.43	0.54	0.42	0.45	0.57	0.56	0.50	0.52
5	0.31	0.34	0.32	0.31	0.50	0.46	0.43	0.44	0.26	0.23	0.24	0.22
1	0.05	0.08	0.08	0.11	0.10	0.08	0.06	0.08	0.23	0.26	0.23	0.25

Table C.5: Target-relative measures – Cash strategy

This table presents selected target-related measures for the four modeling techniques over nine investment horizons for the three accumulation models. The modeling techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures, all calculated versus the target *RWR*, are: probability of shortfall is reported in Panel A, expected shortfall is reported in Panel B and the Sortino ratio is reported in Panel C. Probability of shortfall is expressed as percentages, expected shortfall is expressed in *RWR* units, and Sortino ratio is expressed as a ratio of return to risk.

PANEL A – Probability of shortfall

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00
20	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00
15	1.00	1.00	0.98	1.00	1.00	1.00	0.98	1.00	1.00	1.00	0.98	1.00
10	1.00	1.00	0.96	0.99	1.00	1.00	0.95	0.99	1.00	1.00	0.95	0.98
5	1.00	1.00	0.92	0.94	1.00	1.00	0.91	0.92	1.00	1.00	0.93	0.95
1	1.00	1.00	0.89	0.89	1.00	1.00	0.89	0.89	1.00	0.99	0.83	0.84

PANEL B – Expected shortfall

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	2.49	2.48	2.42	2.45	14.27	14.27	13.81	14.06	5.64	5.64	5.48	5.54
35	2.32	2.32	2.26	2.28	9.60	9.60	9.31	9.42	4.20	4.20	4.06	4.12
30	2.00	2.00	1.94	1.96	6.39	6.39	6.15	6.27	3.12	3.12	3.02	3.05
25	2.65	2.65	2.56	2.60	5.13	5.13	4.93	5.01	2.77	2.77	2.68	2.71
20	2.56	2.56	2.47	2.50	3.83	3.83	3.70	3.73	2.30	2.30	2.22	2.24
15	1.66	1.66	1.60	1.63	2.27	2.28	2.19	2.21	1.53	1.53	1.48	1.50
10	1.03	1.03	1.00	1.00	1.24	1.24	1.20	1.21	0.91	0.91	0.89	0.89
5	0.38	0.38	0.38	0.38	0.38	0.38	0.39	0.38	0.40	0.40	0.40	0.40
1	0.08	0.08	0.09	0.09	0.08	0.08	0.09	0.09	0.05	0.05	0.06	0.06

PANEL C – Sortino ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	-1.00	-1.00	-0.99	-0.99	-1.00	-1.00	-0.98	-0.99	-1.00	-1.00	-0.98	-0.99
35	-1.00	-1.00	-0.98	-0.99	-1.00	-1.00	-0.98	-0.99	-1.00	-1.00	-0.97	-0.99
30	-1.00	-1.00	-0.98	-0.99	-1.00	-1.00	-0.97	-0.98	-1.00	-1.00	-0.97	-0.98
25	-1.00	-1.00	-0.97	-0.98	-1.00	-1.00	-0.96	-0.98	-1.00	-1.00	-0.96	-0.98
20	-1.00	-1.00	-0.96	-0.98	-1.00	-1.00	-0.95	-0.97	-1.00	-1.00	-0.95	-0.97
15	-1.00	-1.00	-0.93	-0.96	-1.00	-1.00	-0.93	-0.96	-1.00	-1.00	-0.93	-0.96
10	-1.00	-1.00	-0.91	-0.94	-1.00	-1.00	-0.90	-0.94	-1.00	-1.00	-0.89	-0.94
5	-0.99	-0.99	-0.83	-0.88	-0.99	-0.99	-0.81	-0.85	-1.00	-1.00	-0.86	-0.89
1	-0.97	-0.97	-0.82	-0.81	-0.97	-0.97	-0.81	-0.82	-0.94	-0.94	-0.70	-0.70

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