

Performance and prediction of marine clay treated with vacuum and surcharge consolidation at port of Brisbane

Author

Indraratna, Buddhima, Balasubramaniam, Bala, Poulos, Harry, Rujikiatkamjorn, Cholachat, Ameratunga, Jayantha

Published

2013

Journal Title

Australian Geomechanics Journal

Version

Accepted Manuscript (AM)

Rights statement

© 2013 Australian Geomechanics Society. This is the author-manuscript version of this paper. Reproduced in accordance with the copyright policy of the publisher. Please refer to the journal website for access to the definitive, published version.

Downloaded from

<http://hdl.handle.net/10072/60519>

Link to published version

<http://ro.uow.edu.au/eispapers/1846>

Griffith Research Online

<https://research-repository.griffith.edu.au>

PERFORMANCE AND PREDICTION OF MARINE CLAY TREATED WITH VACUUM AND SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

Buddhima Indraratna¹, A.S. Balasubramaniam², Harry Poulos³, Cholachat Rujikiatkamjorn⁴ and Jay Ameratunga⁵

¹Professor, Centre for Geomechanics and Railway Engineering, University of Wollongong, Wollongong, NSW 2522, Australia, ARC Centre of Excellence for Geotechnical Science and Engineering, Australia (corresponding author). E-mail: indra@uow.edu.au

²Adjunct Professor, School of Civil Engineering Griffith University, Gold Coast, Queensland, Australia

³Senior Principal, Coffey Geotechnics, Citadel Tower B, 799 Pacific Highway, Chatswood NSW 2067 Australia

⁴Senior Lecturer, Centre for Geomechanics and Railway Engineering, University of Wollongong, Wollongong, NSW 2522, Australia, ARC Centre of Excellence for Geotechnical Science and Engineering, Australia.

⁵Senior Principal, Coffey Geotechnics, 47 Doggett Street Newstead QLD 4006 Australia

ABSTRACT

Over the past decade the application of vacuum preloading for stabilising soft offshore-coastal clay in shallow water and other low-lying estuarine soils has become popular in Australia. Its cost effectiveness is a major factor in most projects in view of the significantly reduced time for achieving a relatively high degree of consolidation. Due to an increase in trade activities at the Port of Brisbane, new facilities on Fisherman Islands at the mouth of the Brisbane River will be constructed on the new outer area (235ha) adjacent to the existing port facilities via land reclamation. A vacuum assisted surcharge load and conventional surcharge scheme, in conjunction with prefabricated vertical drains, was selected to reduce the required consolidation time through the deeper layers of subsoil. The performance of a combined vacuum and surcharge fill system and construction of the embankment are described in this paper. A comparison of the performance of a combined vacuum and surcharge loading system with a standard surcharge fill highlights the clear benefits of vacuum consolidation. Field monitoring data are presented to demonstrate how the embankment performed during construction. This paper also evaluates the relative performance of the two contrasting preloading systems (i.e. vacuum and non-vacuum system). An analytical solution for radial consolidation that considers both time-dependent surcharge loading and vacuum pressure is proposed to predict the settlement and associated excess pore pressures of the soft Holocene clay deposits.

1 INTRODUCTION

Offshore and coastal regions of Australia contain soft clays with unfavourable geotechnical properties such as low shear strength and high compressibility. In the absence of suitable ground improvement, excessive differential settlement and lateral movement unfavourably affects the stability of buildings and port infrastructure built on these soft grounds (Holtz et al. 1991, Indraratna and Redana 2000, Ghandeharioon et al. 2010). A system of vertical drains with a combined vacuum and surcharge preloading is an effective method for promoting radial flow, which accelerates soil consolidation. The behaviour of soft clay stabilised with vertical drains and vacuum pressure can now be predicted with acceptable accuracy due to the significant progress made over the past decade through rigorous analytical and numerical analysis. Mohamedelhassan and Shang (2002) proposed an analytical solution for one-dimensional consolidation with a vacuum application. Indraratna et al. (2005) extended the unit cell radial consolidation theory for vacuum application with instantaneous loading that considers the loss of vacuum along the length of the drain.

The Port of Brisbane is Australia's third largest container port, and it is located between the mouth of the Brisbane River and Fisherman Islands (Indraratna et al. 2011). With rapid growth in trading activities, a new outer area (235ha) adjacent to the current port facilities is being reclaimed to maximise the available land, and to provide additional berths that are suitable for cargo and container handling. In this area the soil profile consists of a highly compressible layer of clay over 30m thick, with an undrained shear strength less than 15 kPa near the surface. The dredged mud used for reclamation has a much lower shear strength, depending on the time of placement and the time during which the capping material (surcharge) has been in place. Without surcharge preloading, it is estimated that full consolidation would take more than 50 years, with vertical settlements of 2.5-4.0m expected under the required service loadings. It was therefore suggested that vacuum consolidation with prefabricated vertical drains (PVDs) would speed up the consolidation process and limit horizontal deformation at the site located immediately adjacent to the Moreton Bay Marine Park (Austress Menard 2008).

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

Chu et al. (2000) and Chai et al. (2005) discussed the application of vacuum preloading combined with PVDs. In this method the suction can propagate to a greater depth of the subsoil using the PVD system and thus minimise the lengthy consolidation time due to stage construction (Indraratna et al. 2005). Indeed the height of surcharge fill may be lowered by several metres if a vacuum pressure of at least 70% the atmospheric pressure is sustained (Rujikiatkamjorn et al. 2008), and the rate at which the embankment is constructed can be increased by reducing the number of construction stages (Yan and Chu 2003). Once the soil increases its stiffness and shear strength due to consolidation, post-construction settlement will be reduced, thereby lowering the risk of differential settlement (Shang et al. 1998). The ground improvement provided by PVDs combined with a vacuum pressure may be an economically attractive alternative in deep soft clay sites, but to date there is no comprehensively reported case history where both conventional surcharge preloading and vacuum technique have been applied in the same area with different types of drains and spacing.

In this paper the performance of the vacuum and non-vacuum areas is described and compared, based on the measured vertical deformations, excess pore pressures, and horizontal displacements. The effects of the types and spacing of drains, and the technique for improvement, are described based on the observed degree of consolidation, and analytical solutions for radial consolidation that consider both time dependent surcharge loading and the vacuum pressure to predict settlement and associated excess pore pressure are proposed.

2 ANALYTICAL SOLUTIONS FOR VACUUM PRELOADING SYSTEMS

A. VACUUM PRELOADING SYSTEMS

There are two main types of vacuum preloading systems currently used in the field (Rujikiatkamjorn and Indraratna 2009, Geng et al. 2011):

A. Membrane system: After the PVDs have been installed and a sand blanket is placed with horizontal perforated pipes, a membrane is laid over the top and its edges are submerged under a trench containing bentonite slurry (Fig. 1a). Vacuum pumps are then attached to the discharge system. A major advantage of this system is that the vacuum can be distributed within the sand platform, along the surface of the soil, and down the PVDs. However, the obvious drawback of this system is that it depends entirely on the ability of the airtight system to prevent any air leaks over a significant period of time.

B. Membraneless system: When an area has to be sub-divided and progressed individually, vacuum preloading can only be conducted in one section at a time, so a membrane system may not be an economical solution. To avoid this problem, the vacuum pipes are connected directly to each individual PVD using a tubing system (Fig. 1b), such that unlike the membrane system where any air leak can affect the entire system, each drain acts independently. However this system requires enough tubing for hundreds of drains, which can obviously affect time and cost of installation.

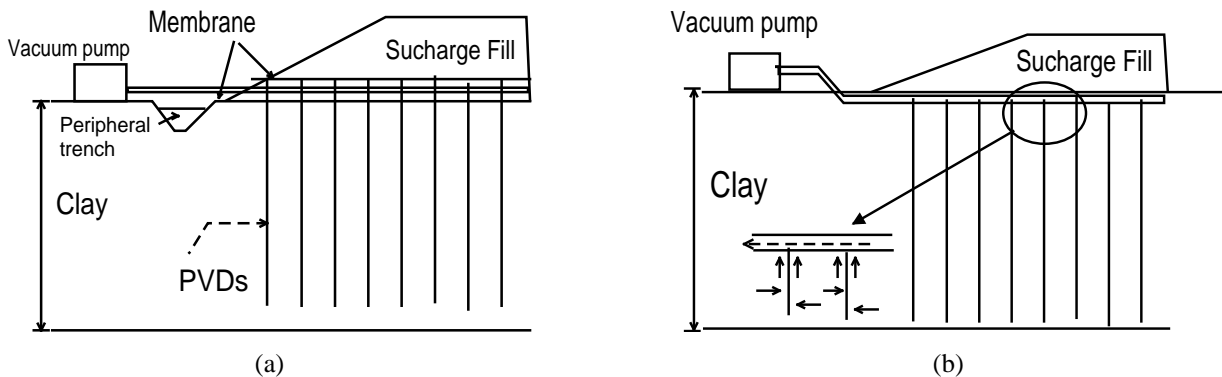


Figure 1 Types of vacuum preloading systems (a) Membrane system and (b) Membraneless system

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

2.1 GOVERNING EQUATIONS

A. Membrane system

In a membrane system the vacuum propagates from the horizontal drain through the layer of sand, PVDs, and layer of clay (Fig. 2a). This three dimensional flow in the sand blanket beneath the membrane ($0 \leq z \leq L_w$) can be expressed as:

$$\frac{\partial \varepsilon_{v1}}{\partial t} = -m_{v1} \left(\frac{\partial \bar{u}_1}{\partial t} - \frac{dq}{dt} \right) \quad (1)$$

$$-\frac{k_{h1}}{\gamma_w} \left(\frac{1}{r} \frac{\partial u_1}{\partial r} + \frac{\partial^2 u_1}{\partial r^2} \right) - \frac{k_{v1}}{\gamma_w} \frac{\partial^2 \bar{u}_1}{\partial z^2} = \frac{\partial \varepsilon_{v1}}{\partial t} \quad r_w \leq r \leq r_e \quad (2)$$

$$\frac{\partial^2 u_{w1}}{\partial z^2} = -\frac{2k_{h1}}{r_w k_{v1}} \frac{\partial u_1}{\partial r} \Big|_{r=r_w} \quad (3)$$

$$\bar{u}_1 = \frac{1}{\pi(r_e^2 - r_w^2)} \int_{r_w}^{r_e} 2\pi r u_1 dr \quad (4)$$

The governing equations for the underlying soil ($L_w \leq z \leq H$), may be expressed as:

$$\frac{\partial \varepsilon_{v2}}{\partial t} = -m_{v2} \left(\frac{\partial \bar{u}_2}{\partial t} - \frac{dq}{dt} \right) \quad (5)$$

$$-\frac{k_{s2}}{\gamma_w} \left(\frac{1}{r} \frac{\partial u_{s2}}{\partial r} + \frac{\partial^2 u_{s2}}{\partial r^2} \right) - \frac{k_{v2}}{\gamma_w} \frac{\partial^2 \bar{u}_2}{\partial z^2} = \frac{\partial \varepsilon_{v2}}{\partial t} \quad r_w \leq r \leq r_s \quad (6)$$

$$-\frac{k_{h2}}{\gamma_w} \left(\frac{1}{r} \frac{\partial u_{n2}}{\partial r} + \frac{\partial^2 u_{n2}}{\partial r^2} \right) - \frac{k_{v2}}{\gamma_w} \frac{\partial^2 \bar{u}_2}{\partial z^2} = \frac{\partial \varepsilon_{v2}}{\partial t} \quad r_s \leq r \leq r_e \quad (7)$$

$$\frac{\partial^2 u_{w2}}{\partial z^2} = -\frac{2k_{s2}}{r_w k_w} \frac{\partial u_{s2}}{\partial r} \Big|_{r=r_w} \quad (8)$$

$$\bar{u}_2 = \frac{1}{\pi(r_e^2 - r_w^2)} \left(\int_{r_w}^{r_s} 2\pi r u_{s2} dr + \int_{r_s}^{r_e} 2\pi r u_{n2} dr \right) \quad (9)$$

The boundary conditions for both the radial and vertical directions are as follows:

$$r = r_e : \frac{\partial u_{n2}}{\partial r} = 0, \quad \frac{\partial u_1}{\partial r} = 0 \quad (10a)$$

$$r = r_s : k_{s2} \frac{\partial u_{s2}}{\partial r} = k_{h2} \frac{\partial u_{n2}}{\partial r} \quad (10b)$$

$$r = r_s : u_{s2} = u_{n2} \quad (10c)$$

$$r = r_w : u_{s2} = u_{w2}, \quad \bar{u}_1 = u_{w1} \quad (10d)$$

$$z = 0 : u_{w1} = p, \quad \bar{u}_1 = p \quad (10e)$$

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

$$z = H : \frac{\partial u_{w2}}{\partial z} = 0, \frac{\partial \bar{u}_2}{\partial z} = 0 \quad (10f)$$

Continuity at the interface between the sand blanket and underlying layer of soil ($z = L_w$) may be then written as:

$$z = L_w : u_{w1} = u_{w2} \quad (10g)$$

$$z = L_w : \bar{u}_1 = \bar{u}_2 \quad (10h)$$

$$z = L_w : k_{v1} \frac{\partial u_{w1}}{\partial z} = k_w \frac{\partial u_{w2}}{\partial z} \quad (10i)$$

$$z = L_w : k_{v1} \frac{\partial \bar{u}_1}{\partial z} = k_{v2} \frac{\partial \bar{u}_2}{\partial z} \quad (10j)$$

The initial condition is:

$$\text{At } t = 0, \bar{u}_1 = \bar{u}_2 = u_0(z) = q_0 \quad (10k)$$

where i is the index number of arbitrary layer, ($i = 1, 2$), r_{si} is the radius of smear zone, r_e is the radius of influence zone, r is the radial coordinate, z is the vertical coordinate, t is the time, ε_{vi} is the vertical strain, m_{vi} is the coefficient of volume compressibility of soil, k_{hi} is the horizontal coefficient of permeability of soil, k_{vi} is the vertical coefficient of permeability of the soil, k_w is the coefficient of permeability of the vertical drain, \bar{u}_i is the average pore pressure, u_{si} is the pore pressure at any point in the smear zone, u_{ni} is the pore pressure at any point in the natural soil zone, u_{wi} is the excess pore water pressure within the vertical drain, q is the time-dependent surcharge preloading, q_0 is the initial value of preloading, L_w is the thickness of the sand layer, H is the thickness of the whole layer (i.e., for the membrane system, (both sand blanket and clay layer, and for the membraneless system, only the clay layer), p is the vacuum pressure.

B. Membraneless system

The main difference between a membrane system and a membraneless system are the boundary conditions. In a membraneless system a vacuum pump is connected directly to individual PVD's through a system of horizontal pipes (Fig. 2b). The governing equations and initial conditions of underlying soil improved by PVD's are the same as for the membrane system (Eq. (10a)-Eq. (10d) and Eq. (10k)). In order to study the loss of vacuum, the vacuum pressure along the boundary of the drain was considered to vary linearly from p at the top of the drain to ηp at the bottom, where η is a ratio between the vacuum at the top and bottom of the drain. The value of η varies between 0 and 1, but if there is no loss of vacuum at the bottom of the PVDs then $\eta = 1$, but if the vacuum pressure is 0 at the bottom of the drain then $\eta = 0$.

The boundary conditions for a membraneless system are:

$$z = 0 : u_w = p, \frac{\partial \bar{u}}{\partial z} = 0 \quad (10m)$$

$$z = H : \frac{\partial u_w}{\partial z} = \frac{\eta - 1}{H} p, \frac{\partial \bar{u}}{\partial z} = 0 \quad (10n)$$

The analytical solutions based on the above governing equations and boundary conditions are given in Appendix 1 for both membrane and membraneless systems.

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

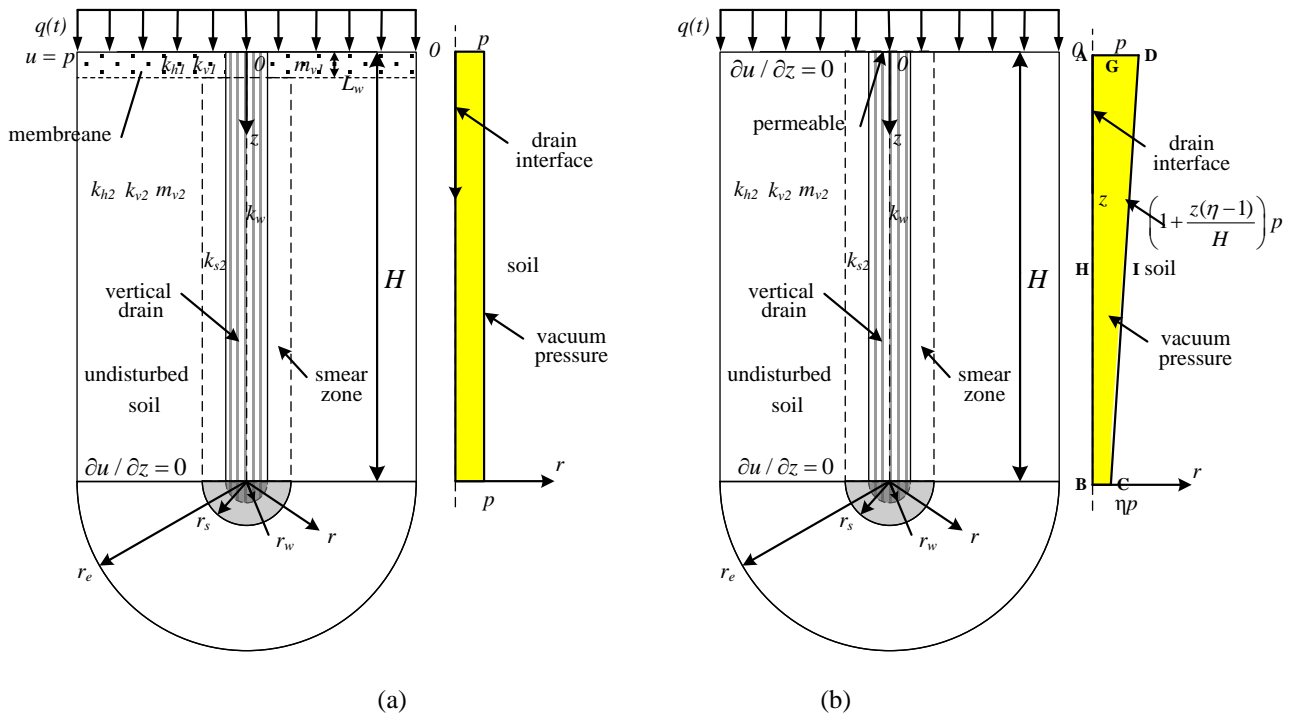


Fig. 2. Analysis schemes of unit cell with vertical drain: (a) membrane system and (b) membraneless system (Geng et al. 2011)

3 GENERAL DESCRIPTION OF THE EMBANKMENT CHARACTERISTICS AND SITE CONDITIONS

In 2003, the Port of Brisbane Pty Ltd (previously Port of Brisbane Corporation) started to reclaim a sub-tidal land area of 235ha at Fisherman Islands near the mouth of the Brisbane River (Fig. 3). The reclaimed land is expected to provide additional berths and associated infrastructure to accommodate the future growth of the Port (Port of Brisbane Corporation 2009). To compare the performance of a vacuum system with a non-vacuum system (PVD and surcharge load), a trial area was selected as shown in Fig. 4. Three contractors were selected to carry out the trials, with each being assigned an area of about 3ha for the trial. The aim was to compare their performance based on their design and construction work.

Contractor A had 8 trial areas in Area S3a, designated as WD1, WD2, WD3, WD4, WD5a, WD5b, VC 1, and VC 2. Areas WD1 to WD5a and WD5b had surcharge only, while VC1 and VC2 had surcharge and vacuum consolidation with a membrane system.

Contractor B: Their trial area T11 consisted of seven areas. Five had surcharge and different drain types. Two had surcharge in conjunction with a membraneless system.

Contractor C: Their trial area in T11 was sub-divided into Areas 4, 5, and 6. Areas 4 and 5 had a surcharge period of a year, whereas Area 6 had a surcharge period of 0.5 years. Vertical drains with 1.4m spacing were used in Areas 4 and 5 while Area 6 used vertical drains at a spacing of 1m.

The upper Holocene sand underneath the dredged mud was about 2-3m thick, and overlaid a 6-25m thick layer of Holocene clay. The highly compressible Holocene clay had low shear strength and was generally referred to as PoB clay (Ameratunga et al. 2010). The Holocene layer overlies a Pleistocene deposit comprised of highly over-consolidated clay. Site investigations including cone penetration/piezcone tests, dissipation tests, boreholes, field vane shear tests, and oedometer tests that were carried out to assess the consolidation and stability design parameters. The profile of the soil and

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

its corresponding properties are shown in Fig. 5, where the groundwater level is at +3.5m RL. The water content of the soil layers was at or beyond their liquid limits. The field vane tests indicated that the undrained shear strength of the dredged mud and Holocene clays varied from 5 to 60 kPa. The compression index (C_c) varied from 0.1-1.0. The coefficient of consolidation in a vertical direction (c_v) was approximately the same as in a horizontal direction (c_h) for the totally remoulded dredged layer of mud, while c_v/c_h was about 2 for the layer of Holocene clay.



Figure 3. Map of the proposed extension area at the Port of Brisbane (adopted from Port of Brisbane Corporation 2009)

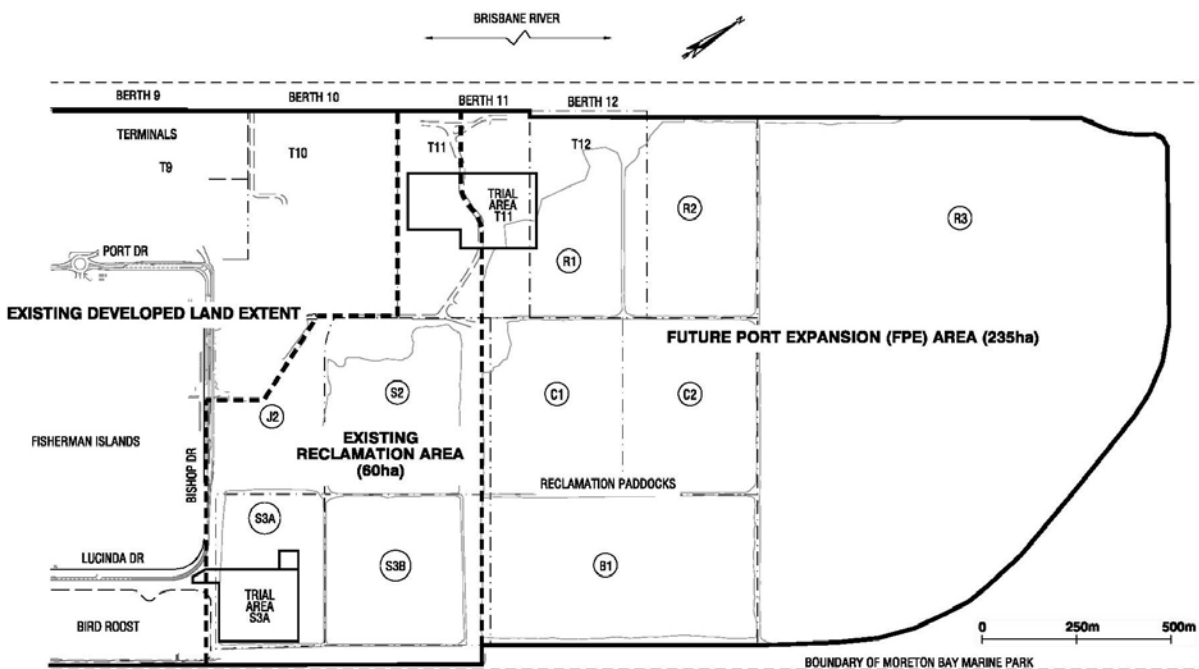


Figure 4. General site layout

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

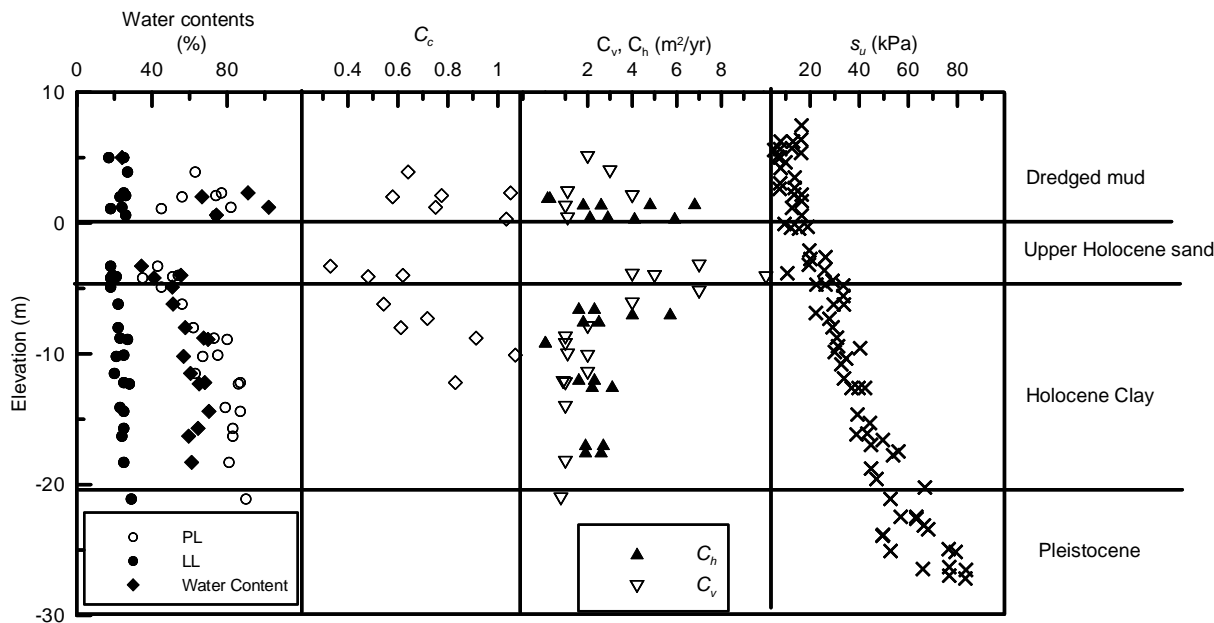


Figure 5. Soil properties and profile at S3A, Port of Brisbane (Indraratna et al. 2011)

Because the layer of Holocene clay is quite thick, two approaches to preloading were used to minimise long term settlement, they included a conventional surcharge preloading system and a vacuum consolidation system, both applied to PVDs. Rigorous design specifications were considered for the design and construction of fill embankments and vacuum application over the deposits of soft Holocene: (a) Service load of 15-25 kPa, (b) maximum residual settlement of not more than 250 mm over 20 years after the application of a service load.

4 ASSESSMENT OF RELATIVE EFFECTIVENESS OF THE TRIAL SCHEMES

A. DEGREE OF CONSOLIDATION (DOC) WITH TIME

The degree of consolidation DOC versus time plots, derived from the measured data, are shown in Figure 6 for an array of locations, and they all show a very similar behaviour, irrespective of the treatment site (S3A and T11) and the type of improvement (vacuum vs. surcharge only). In all these settlement plate areas, a relatively high DOC was achieved after one year, and all plots converged to $DOC > 80\%$. In order to separate the ‘clustering,’ especially towards the one year period, the DOC is divided by the dimensionless parameter β -factor.

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

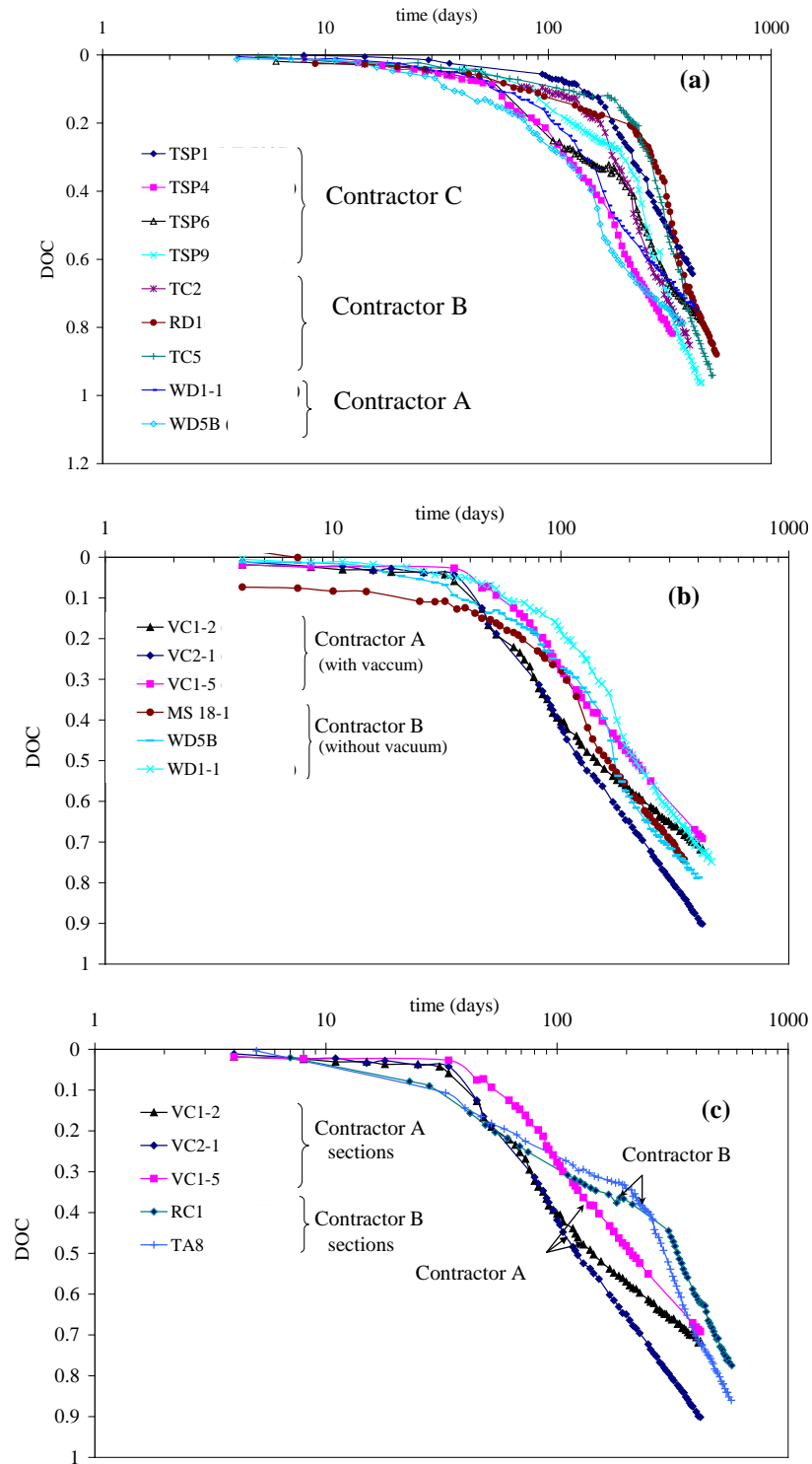


Figure 6. Analytically computed DOC with time for (a) non-vacuum in S3A and T11, (b) treatment in S3A only and (c) vacuum areas in S3A and T11

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

In this respect, this dimensionless parameter β can be defined as:

$$\beta = (l_d/\alpha s_d) \times (H/h_c) \quad (11)$$

The β -factor is totally independent of the soil properties and is designed to capture the loading conditions of the drain and site; it comprises the favourable effects of: (i) increasing the drain length (l_d), (ii) decreasing the drain spacing (s_d) and its pattern ($\alpha = 1.05$ for triangular and 1.13 for square spacing), and (iii) increasing the surcharge load height (H) to consolidate the given clay thickness (h_c), represented by the ratio (H/h_c).

Based on the magnitude of β determined at the location of each settlement plate for S3A and T11, conditions at the drain and site at the 3 trial paddocks can be differentiated as follows:

- (i) Low β impact: 2-6 (for S3A area under Contractor A),
- (ii) Moderate β impact: 8-12 (for T11 area under Contractor B), and
- (iii) High β impact: 12 -18 (for T11 area under Contractor C).

Although the value of β has no specific relationship to the converging target of DOC that is intended to be attained at the date the contractors remove all the fill, it can act as a 'filter' in distinguishing the relative performance in S3A and T11, by dividing the DOC by β . Figure 7 shows the variation of DOC divided by the β -factor (U/β) plotted against time. This results in a separation between the vacuum and non-vacuum areas, and also separates the vacuum consolidation effects of Contractors A and B. When all 3 sets of plots (Figs. 7a-c) are considered, the relative consolidation in Contractor A treatment areas where vacuum consolidation has been used, are superior to all the other locations in S3A and T11.

B. DISSIPATION OF EXCESS PORE WATER PRESSURE

Figure 8 shows that VC2 in S3A had the largest reduction in pore water pressure with time, closely followed by VWP3 in T11 (Contractor C), but because the fill heights and thickness of clay varied in the S3A and T11 paddocks, these plots cannot be compared directly since most of them were clustered together during the first 3 months and showed little difference. Figure 9a indicates the rate of change of excess pore water pressure for the same locations; with VC2, VC1, and WD1 indicating the highest rate of change of excess pore pressure at the start, whereas VC1 maintained a steady state over a long period of time. The membraneless systems do not seem to indicate a high rate of excess pore pressure dissipation compared to the VC1 and VC2 areas. When these plots are normalised by the β -factor (Figure 9b), it is clear that VC1 and VC2 have the best treatment in view of excess pore pressure dissipation, compared to all the other areas. While the fill height has been reduced in the VC areas of S3A and thereby involves less mucking operations, the applied suction (-70 kPa) more than compensates for the accelerated excess pore pressure dissipation rates, and thus confirms the effective performance of the membrane-type vacuum consolidation technique.

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

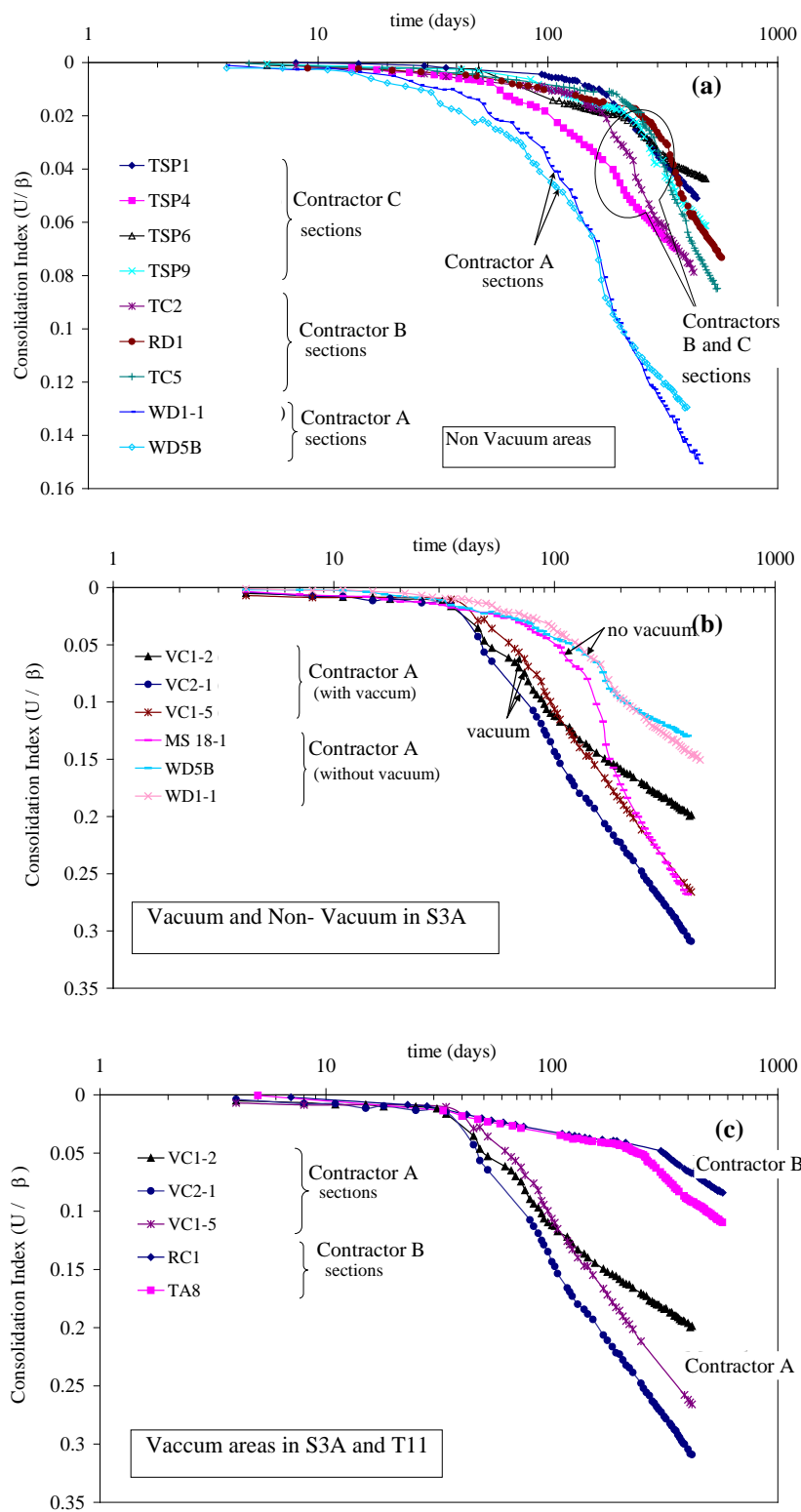


Figure 7. Computed DOC/β with time for (a) non-vacuum in S3A and T11, (b) treatment in S3A only and (c) vacuum areas in S3A and T11

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

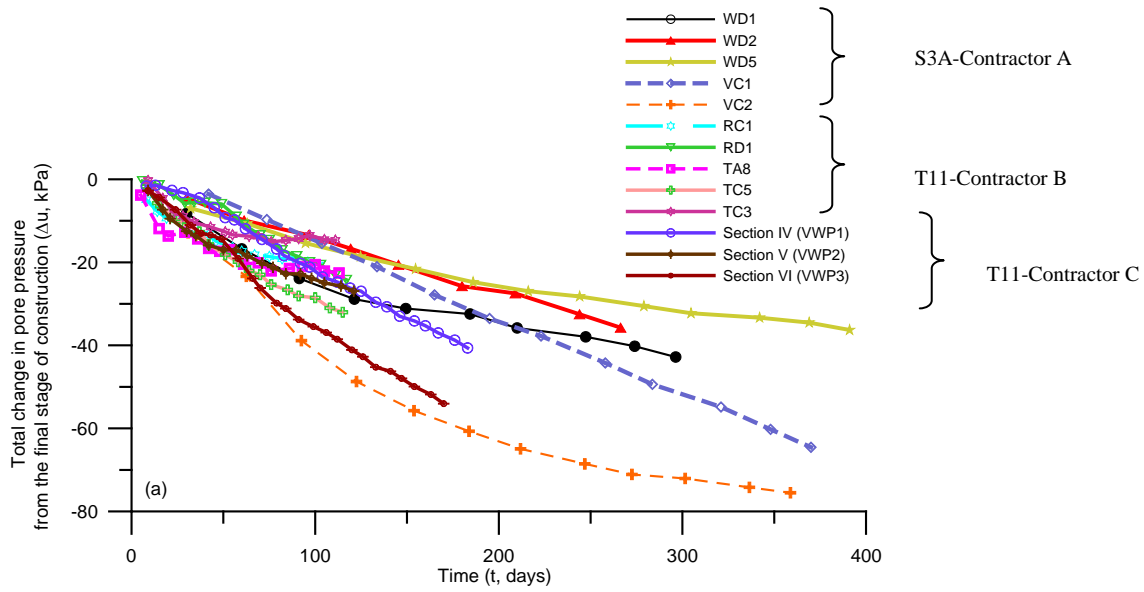


Figure 8. Reduction in Excess Pore Water Pressure with Time in S3A and T11 areas

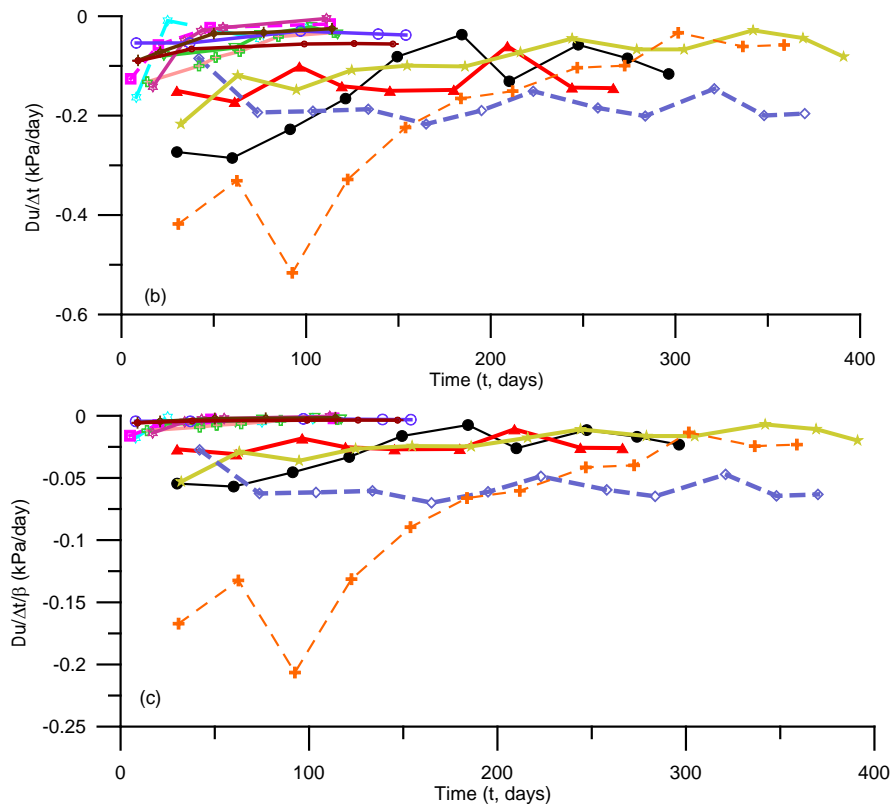


Figure 9. Comparison of excess pore pressure dissipation between S3A and T11 (a) Rate of dissipation of Excess pore pressure, (b) Excess pore pressure dissipation rate normalised by β

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

C. CONTROLLING LATERAL DISPLACEMENT

It is quite well known that vertical drains effectively reduce the lateral yield in soil and that a vacuum pressure controls lateral movement even more. Indeed in some cases it may even force any lateral movement inwards rather than outwards (Indraratna et al 1997, Indraratna et al. 2005, Indraratna et al. 2008). The use of a vacuum pressure to precisely control lateral displacement will be very important in sensitive areas near marine parks. In this particular POB site, only very limited field data has been available from a few inclinometers, but nevertheless, to compare the lateral movement of selected vacuum and non-vacuum areas with very different soil profiles and surcharge load conditions, lateral displacement can be divided by the applied effective stress at the same depth.

The profiles of normalised lateral displacement with depth for the limited data sections are shown in Figure 10. These four plots clearly indicate that while vacuum consolidation definitely helped control lateral movement, the Contractor A vacuum system with 70 kPa suction demonstrated the most significant reduction in normalised lateral displacement (compare VC1-MS28 with WD3-MS27). In a Membraneless vacuum system with 50 kPa suction, while lateral movement was definitely reduced (compare MS24 with MS34), the reduction was not as large as the Membrane system. The shape of the lateral displacement curves suggest that in all VC areas the suction head propagated significantly with depth such that both the Lower Holocene Clay (LHC) and Upper Holocene Clay (UHC) layers were favourably influenced.

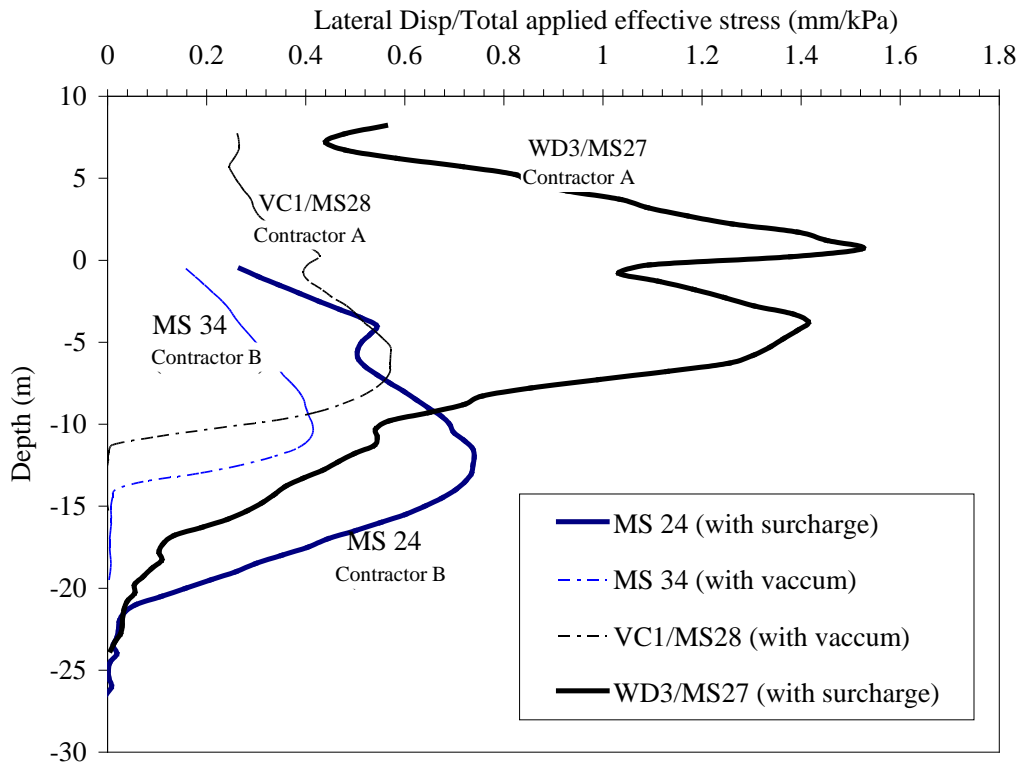


Figure 10. Role of Vacuum Consolidation on Lateral Displacement

D. RESIDUAL SETTLEMENTS (RS)

All the contractors had to aim for a residual settlement (RS) that would be less than 150mm or 250mm, depending on the thickness of the clay and the anticipated service loads in the respective areas. In Figure 11, the estimated values of RS for both S3A and T11 paddocks were calculated and are plotted against the β -factor, based on methods incorporating soil creep provided by Terzaghi et al., (1996) and Yin and Graham, (1994). This figure suggests that the critical RS occurs in the range $4 < \beta < 16$. In this critical zone, which includes locations from all 3 contractors from both S3A and T11 paddocks, the

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

RS is close to the permissible limits. At low values where $\beta < 4$, the residual settlements were much smaller because of vacuum consolidation, but at very high values where $\beta > 16$ (T11), the RS tended to decrease because the fill surcharge levels were high compared to the thickness of the clay (i.e. relatively high H/h_c ratio).

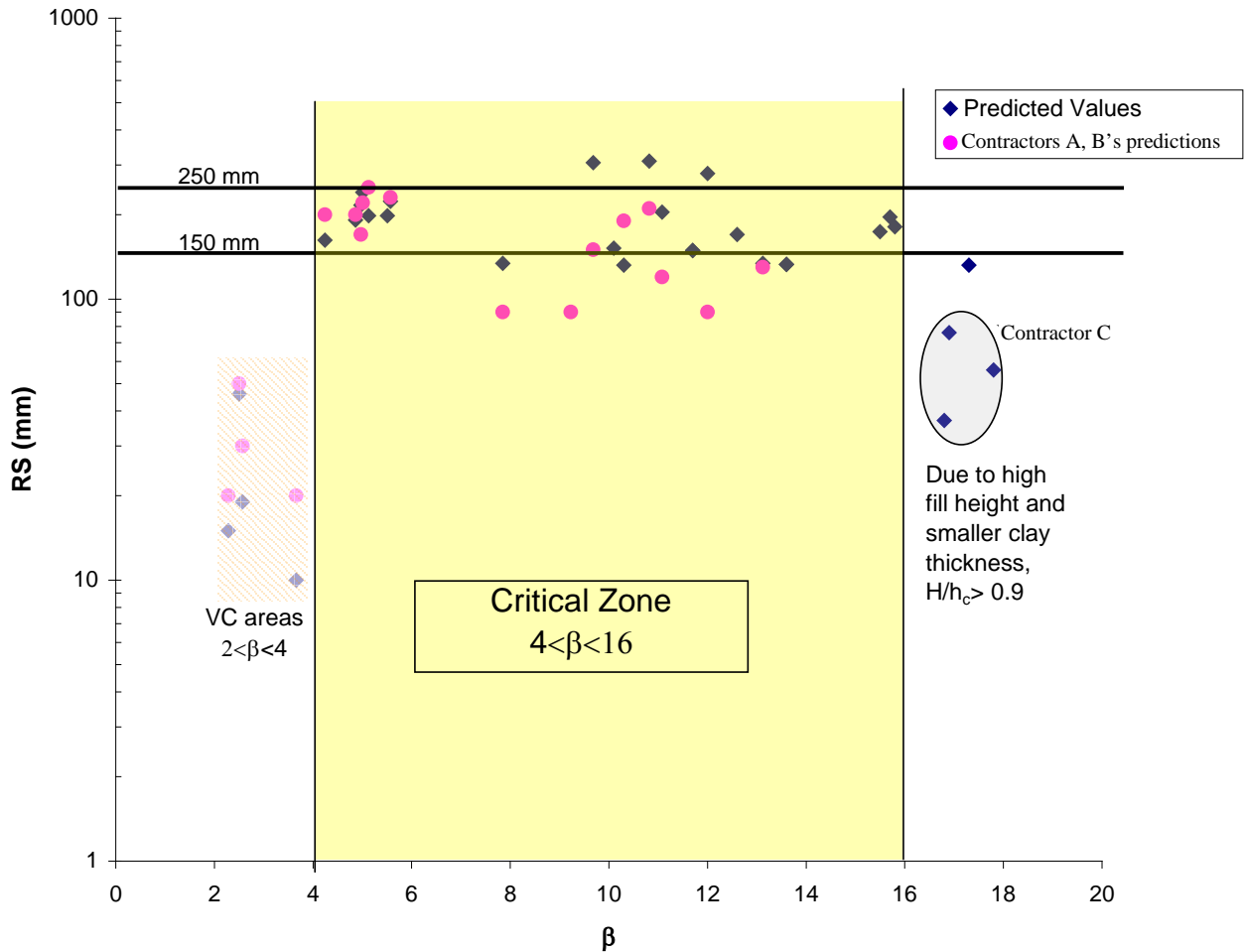


Figure 11 Critical β values for permissible Residual Settlement in S3A and T11

Figure 12 provides approximately linear relationships between the RS and clay thickness for a range of OCR from 1.1 to 1.4, for a DOC exceeding 80%. As expected, when the OCR increased the RS decreased substantially, but in general, as the Holocene clay became thicker, the RS also increased and the corresponding regression lines and best-fit equations are also shown in Figure 12. The vacuum consolidation locations of S3A (VC1-2, VC2-2 and VC2-3) in particular, show a considerably reduced RS, which is well below the permissible limit, at an OCR approaching 1.4. At an OCR of approximately 1.3, the residual settlements associated with membraneless consolidation (TA8,) and VC1-5 (S3A) were also small.

Based on Figure 12, a lower bound and upper bound for RS in terms of clay thickness (h_c) can be obtained as follows for the entire range of over-consolidation upon fill removal:

Lower Bound: $RS = 3.8 h_c - 27$ (vacuum consolidation in S3A at OCR = 1.4)

Upper Bound: $RS = 14.3 h_c + 34$ (surcharge only sites at OCR = 1.1)

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

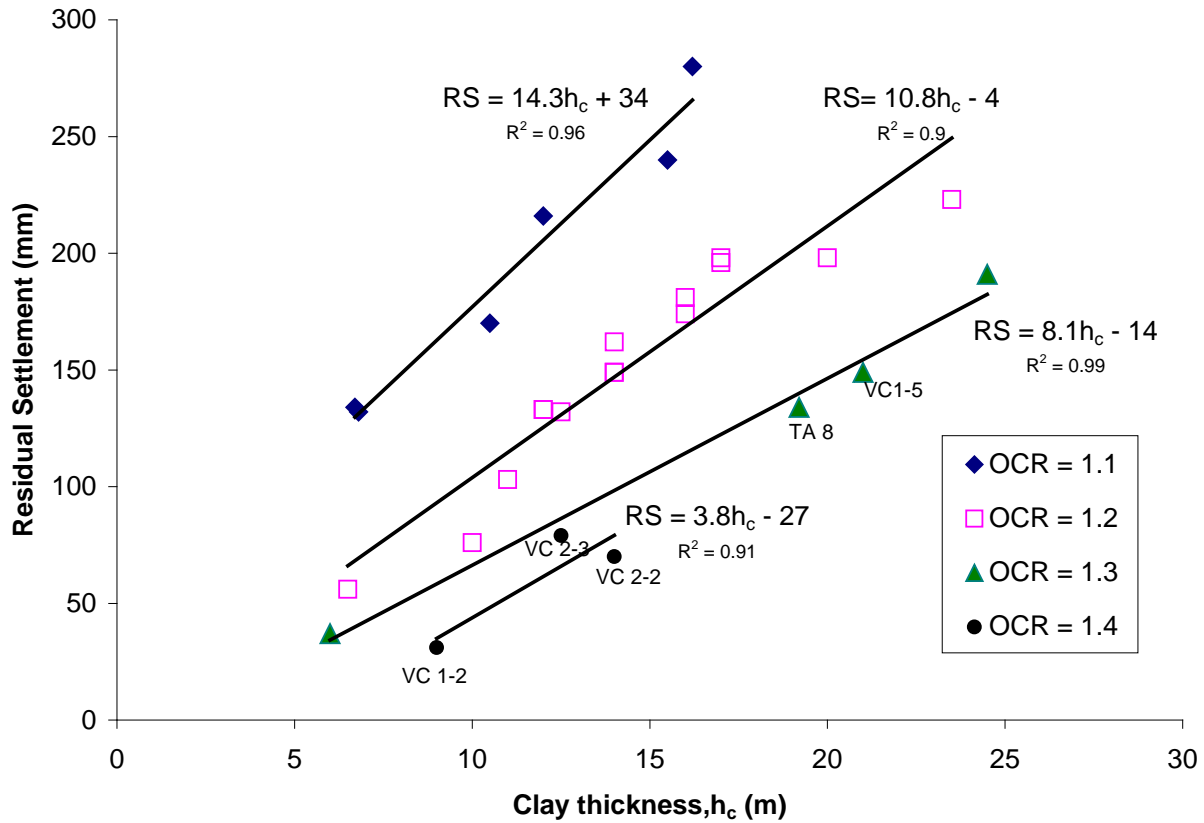


Figure 12 Effect of OCR and Clay Thickness on Residual Settlement

To predict excess pore pressures and associated settlements, Equations (1)-(11) were used in conjunction with Tables 5 and 6 because they respectively summarise the properties for each layer and thickness of soil for each section. In the analysis, the soil compression index (C_c) obtained from the oedometer test is related to the actual stress state within a given region of the foundation. The vertical and horizontal coefficients of consolidation were measured using oedometer and Rowe cells. For the completely remoulded dredged mud that was reclaimed from the seabed, and the Upper Holocene Sand, the ratio k_h/k_s was assumed to be unity. For the upper and lower Holocene clay, the ratios of k_h/k_s and d_s/d_w were assumed to be 2 and 3, respectively, in accordance with the laboratory tests conducted by Indraratna and Redana (2000).

Table 5. Soil profiles, equivalent drain diameter and drain influence zone diameter used for prediction (Indraratna et al. 2011)

Area	Layer Thickness (m)				Drain influence zone diameter (m)	Equivalent drain diameter (m)
	Dredged mud	Upper Holocene sand	Upper Holocene Clay	Lower Holocene Clay		
WD1	2	1	4	11.5	1.23	0.034
WD2	2	1.5	2	19	1.57	0.034
WD3	2	1	2	8	1.24	0.05
WD4	2	1.5	2	21	1.47	0.05
WD5A	0	1	2	8	1.36	0.05
WD5B	2.5	1	2	7	1.24	0.05
VC1	2.5	2.5	2	5	1.36	0.034
VC2	0.5	3	2.5	16	1.36	0.034

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

Table 6. Soil properties for each layer (Indraratna et al. 2011)

Soil layer	Soil type	γ_t (kN/m ³)	$Cc/(1+e_0)$	c_v (m ² /yr)	c_h (m ² /yr)	k_h/k_s	$s=d_s/d_w$
1	Dredged Mud	14	0.235	1	1	1	1
2	Upper Holocene Sand	19	0.01	5	5	1	1
3	Upper Holocene Clay	16	0.18	1	2	2	3
4	Lower Holocene Clay	16	0.2	0.8	1.9	2	3

The embankment load was simulated according to a staged construction (with a compacted unit weight of 20 kN/m³). Settlement and associated excess pore pressure predictions were conducted at the centreline of the embankment using the proposed analytical model. As the computation of consolidation settlement and excess pore pressure at the centreline (zero lateral displacements) is straightforward and followed basic 1-D consolidation theory, the use of a MATLAB spreadsheet formulation was most convenient. It is noted that at the beginning of each subsequent stage, the initial in-situ effective stress was calculated based on the final degree of consolidation at the previous stage. In vacuum areas a suction pressure of 65 kPa was used to compute the settlement and excess pore pressure.

Figures 13 and 14 shows the calculated settlements and associated excess pore pressures with the data measured in Areas WD4 and VC1. Overall, the comparisons between prediction and field observation show that settlement and the associated pore water pressure can be predicted very well. In vacuum areas the degree of consolidation exceeded 90% at 400 days, whereas in the non-vacuum area it was less than 85% at the same time. This confirms that at a given time, the combined vacuum preloading would accelerate consolidation faster than surcharge preloading alone because in non-vacuum areas, the embankment had to be constructed gradually to avoid potential undrained failure in the remoulded dredged layer.

5 CONCLUSIONS

A system of vertical drains with vacuum preloading is an effective method for speeding up soil consolidation. The performance of ground treatment operations at the Port of Brisbane was analysed and discussed. The land was reclaimed using mud dredged from the seabed of shipping channels and berths. Several trial areas were chosen to study the behaviour of surcharge and vacuum consolidation. Purely on the basis of settlements of Degree of Consolidation (DOC), it is not possible to compare the relative treatments applied in the two paddocks S3A and T11 because they all achieved a relatively high target DOC irrespective of the type of drains and their installation pattern, the nature of surcharge loading (with or without vacuum), and the thickness of the clay. In order to clarify the differences between the locations of the trial a drain and site representation factor totally independent of soil consolidation properties, defined as the β -factor, was adopted to capture the drain and site loading conditions. It comprises the favourable effects of: (i) increasing the drain length (l_d), (ii) decreasing the drain spacing (S_d) and its pattern ($\alpha = 1.05$ for triangular and 1.13 for square spacing), and (iii) increasing the surcharge load height (H) in relation to a given clay thickness (h_c).

Dividing the DOC, settlement and lateral displacement/settlement ratio by this β -factor, provided a performance indicator that represented the returns per unit value of β . In such a comparison, the vacuum consolidation applied by Contractor A in S3A seemed to be the most beneficial. The application of a Membraneless vacuum system was also effective at controlling lateral displacement, but data from the field inclinometer was too limited to make firm conclusions. However, it is clear that effective control of lateral displacement in sensitive areas such as marine parks can benefit immensely by the application of vacuum pressure and thereby decrease the fill heights required on the surface.

While a distinct relationship between the DOC and RS is difficult to determine for the given conditions, there is no doubt that the residual settlement RS decreased almost linearly with the increase in the over-consolidation ratio, and also RS tended to move closer to the prescribed 150mm limit for the critical range $4 < \beta < 16$. The minimum RS was attained in the vacuum consolidation sites in S3A when the OCR exceeded 1.3. The RS tended to become critical when the OCR is close to or less than 1.1, and this situation mainly occurs for surcharge-only sites with very thick clay where the treatment is not as effective as when a vacuum pressure is applied. This verifies the fact that, without a vacuum pressure, a large surcharge fill height becomes necessary to keep the RS less than the prescribed limit, and the need to remove a large amount of fill in order to achieve a significant OCR can be a cumbersome process in the field. The higher the service load, the greater will be the advantage of applying a vacuum to reduce the need for excessive fill heights and control lateral displacement. In

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

view of the stringent residual settlement and lateral displacement criteria, the application of a sufficiently high vacuum pressure in conjunction with some surcharge fill to achieve a relatively high DOC (i.e. > 85%) and subsequent unloading for attaining an OCR > 1.3 would be the optimum choice for the site characteristics and loading conditions encountered here.

A unit cell theory that considers a time-dependent surcharge load and vacuum application was developed to predict the settlement and associated excess pore pressure, and the analysis results were shown to be in good agreement with the field measurements. At 400 days, the degree of consolidation in the vacuum areas was much greater than in the non-vacuum areas for the same total stress applied at the surface. The system of PVDs subjected to vacuum combined surcharge preloading is a useful method for accelerating radial consolidation and controlling lateral displacement. While the analytical model discussed here is a useful tool to predict the performance of soft clay stabilised by PVDs, to model pressure preloading accurately requires field observations to examine the correct distribution of vacuum pressure because the fluctuation of suction with time and depth has been a not-uncommon problem in numerous case studies.

6 ACKNOWLEDGEMENTS

The writers acknowledge the support of the Port of Brisbane Pty Ltd, Coffey Geotechnics and Austress Menard. The research funding from the Australia Research Council is acknowledged. The assistance of Guy Schweitzer of Port of Brisbane Pty Ltd, Daniel Berthier of Austress Menard Bachy, Cynthia de Bok, and Dr Chamari Bamunawita of Coffey Geotechnics and Dr Xue-yu Geng and Dr Jayan Vinod at the University of Wollongong is appreciated. More elaborate details of the contents discussed in the paper can also be found in previous publications of the first Author, co-workers and his research students in ASCE and Canadian Geotechnical Journals, since the mid 1990's. Valuable comments from A/Prof Hadi Khabbaz of UTS are greatly appreciated.

7 REFERENCES

- Ameratunga, J., Boyle, P., De Bok, C. and Bamunawita, C. 2010. Port of Brisbane (POB) clay characteristics and use of wick drains to improve deep soft clay deposits. Proceedings 17th Asian Geotechnical Conference, Taipei, Vol I, pp 116-119.
- Austress Menard 2008. Personal communication internal report and confidential, 79p.
- Berthier, D., Boyle, P., Ameratunga, J., De Bok, C. and Vincent, P. 2009. A successful trial of vacuum consolidation at in the Port of Brisbane. PIANC 2009, Christchurch, New Zealand (in CD).
- Chai, J. C., Carter, J.P., and Hayashi, S. 2005. Ground deformation induced by vacuum consolidation. *Journal of Geotechnical and Geoenvironmental Engineering*, 131(12):1552-1561.
- Chu, J. Yan, S.W., and Yang, H. 2000. Soil improvement by the vacuum preloading method for an oil storage station. *Geotechnique*, 50(6): 625-632.
- Geng, X. Y., Indraratna, B. and Rujikiatkamjorn, C. 2011. Analytical solutions for a single vertical drain with vacuum and time-dependent surcharge preloading in membrane and membraneless systems. *International Journal of Geomechanics*, ASCE, (Accepted December 2011).
- Ghandeharioon, A., Indraratna, B., and Rujikiatkamjorn, C. 2010. Analysis of soil disturbance associated with mandrel-driven prefabricated vertical drains using an elliptical cavity expansion theory. *International Journal of Geomechanics*, ASCE. 10(2), 53-64.
- Holtz, R. D., Jamiolkowski, M., Lancellotta, R. and Pedroni, S. 1991. Prefabricated vertical drains: design and performance. CIRIA ground engineering report: ground improvement. Butterworth-Heinemann Ltd, UK, 131.
- Indraratna, B. and Redana, I. W. 2000. Numerical modeling of vertical drains with smear and well resistance installed in soft clay. *Canadian Geotechnical Journal*, 37: 132-145.
- Indraratna, B., Balasubramaniam, A.S. and Sivanesarwan 1997. Analysis of Settlements and Lateral Deformation of Soft Clay Foundation beneath Two Embankments. *International Journal of Numerical and Analytical Methods in Geomechanics*, 31(9): 599-618.
- Indraratna, B., Sathanathan, I., Rujikiatkamjorn C. and Balasubramaniam, A. S. 2005. Analytical and numerical modelling of soft soil stabilized by PVD incorporating vacuum preloading. *International Journal of Geomechanics*, 5(2). 114-124.
- Indraratna, B., Aljorany, A., and Rujikiatkamjorn C., 2008. Analytical and numerical modelling of consolidation by sand drains beneath a circular embankment. *International Journal of Geomechanics*, ASCE. 8(3), 199-206.

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

- Indraratna, B., Rujikiatkamjorn, C., Ameratunga, J., and Boyle, P. 2011. Performance and prediction of vacuum combined surcharge consolidation at Port of Brisbane. *J. of Geotechnical & Geoenvironmental Engineering*, ASCE 137 (11), 1009-1018.
- Mohamedelhassan, E., and Shang, J.Q. 2002. Vacuum and surcharge combined one-dimensional consolidation of clay soils. *Can. Geotech. J.* 39: 1126-1138.
- Port of Brisbane Corporation 2009. Annual report 2008/2009, Port of Brisbane Corporation, Brisbane, Queensland, 92p.
- Rujikiatkamjorn C. and Indraratna, B. 2009. Design procedure for vertical drains considering a linear variation of lateral permeability within the smear zone. *Canadian Geotechnical Journal* 46(3), 270-280.
- Rujikiatkamjorn, C., Indraratna, B. and Chu, J. 2008. 2D and 3D numerical modeling of combined surcharge and vacuum preloading with vertical drains. *International Journal of Geomechanics*, 8(2): 144-156.
- Shang, J.Q., Tang, M., and Miao, Z. 1998. Vacuum preloading consolidation of reclaimed land: a case study. *Canadian Geotechnical Journal*, 35: 740-749.
- Terzaghi, K., Peck, R. B. and Mesri, G., 1996 *Soil Mechanics in Engineering Practice*, 3rd Ed. Wiley-Interscience.
- Yan, S.W. and Chu, J. 2003. Soil improvement for a road using a vacuum preloading method. *Ground Improvement*, 7(4): 165-172.
- Yin, J.-H., and Graham, J. 1994. Equivalent times and one-dimensional elastic visco-plastic modelling of time-dependent stress-strain behaviour of clays. *Canadian Geotechnical Journal*, 31: 42-52.

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

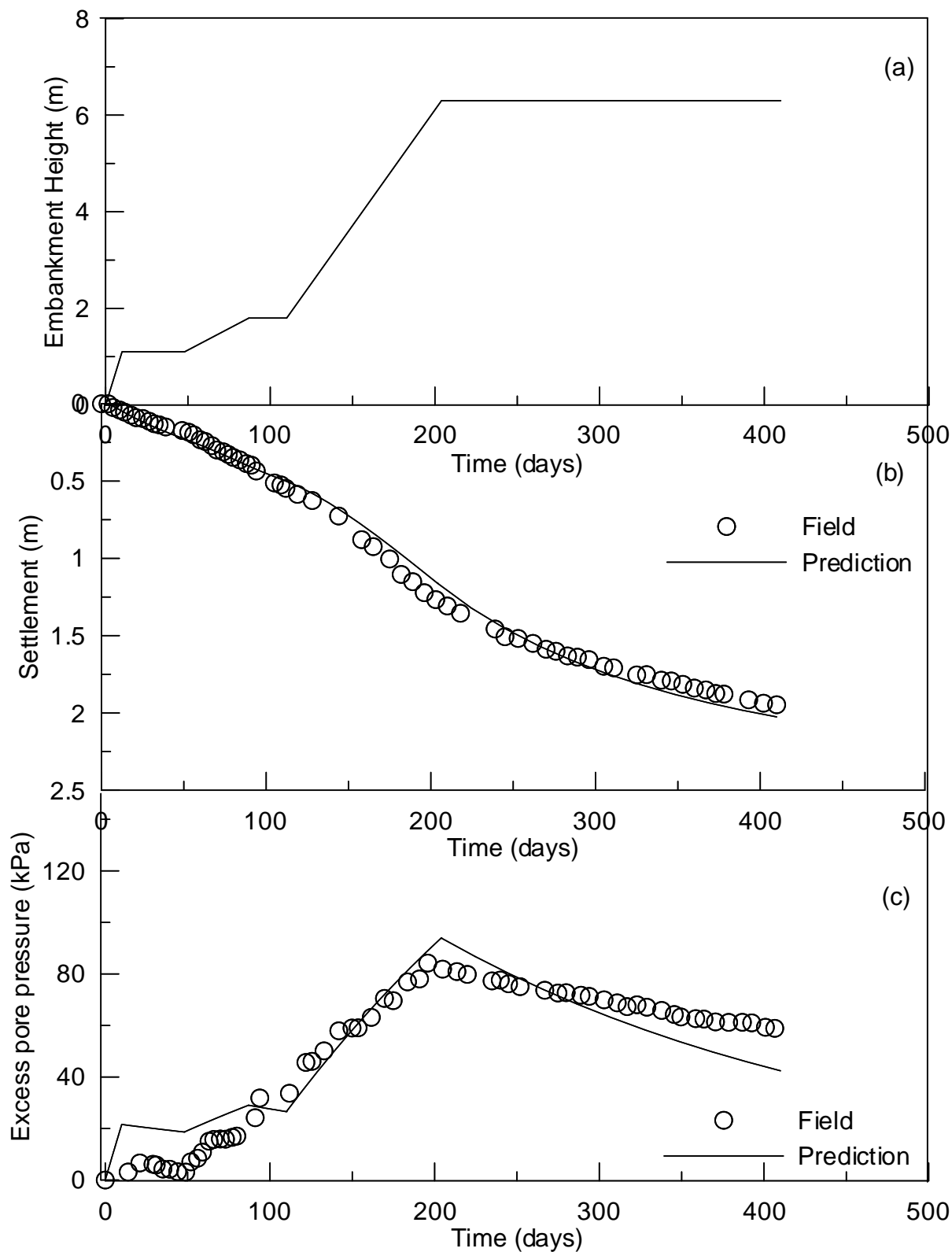


Figure 13. WD4 area: (a) stages of loading, (b) surface settlements under the centreline of the embankment and (c) excess pore pressures

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

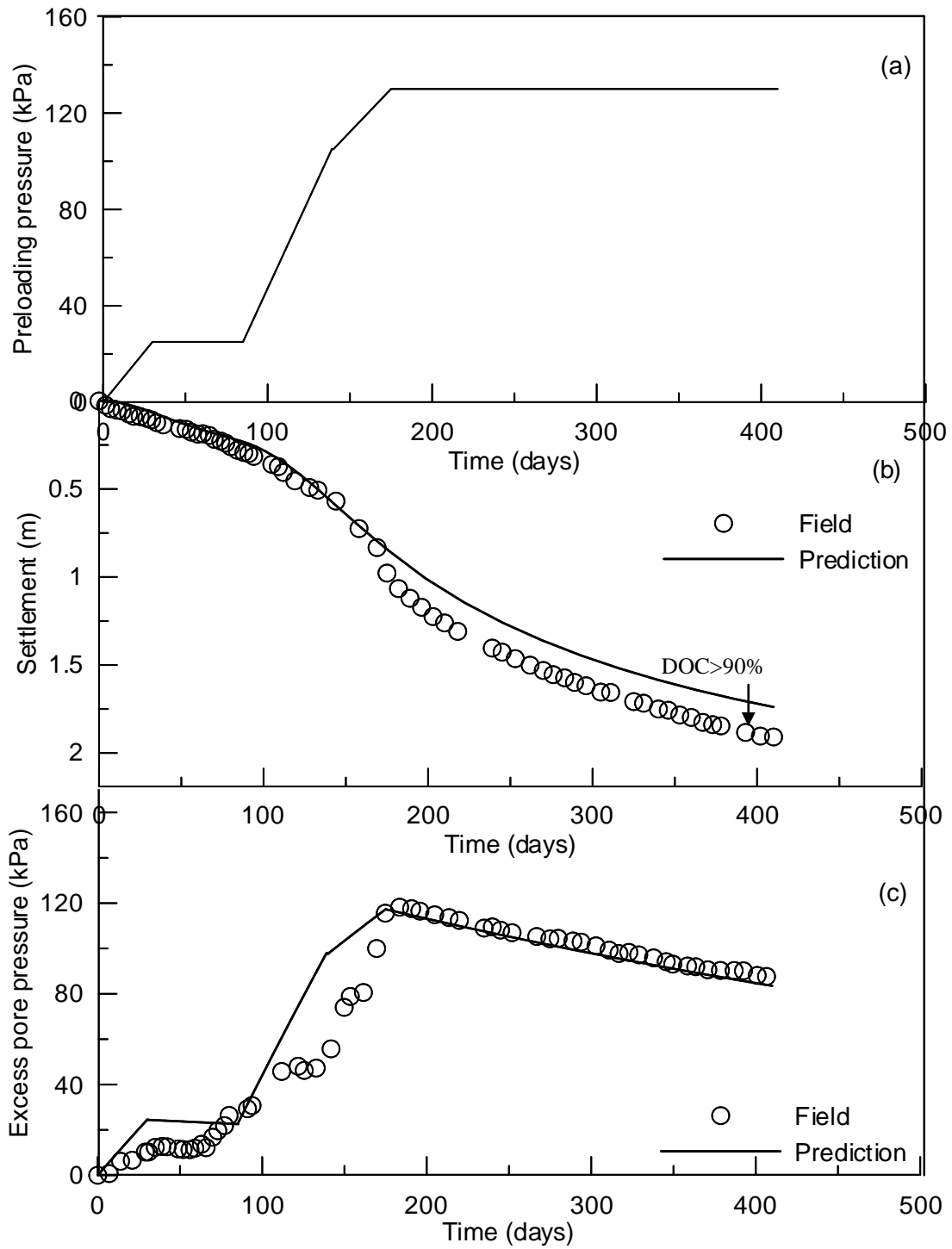


Figure 14. VC1 area: (a) stages of loading, (b) surface settlements under the centreline of the embankment and (c) excess pore pressures

APPENDIX 1: ANALYTICAL SOLUTIONS

A. Membrane system

The pore water pressure within a vertical drain and the average pore water pressure for a membrane system, which can be solved by considering the applicable boundary conditions and loading pattern (detailed derivations can be found in Appendix A), in the Laplace frequency domain are:

$$\hat{u}_{w1}(Z, S) = X_1 e^{a_1 Z} + X_2 e^{-a_1 Z} + X_3 e^{a_2 Z} + X_4 e^{-a_2 Z} + \hat{Q}(S) \quad (11)$$

$$\hat{u}_{w2}(Z, S) = Y_1 e^{b_1 Z} + Y_2 e^{-b_1 Z} + Y_3 e^{b_2 Z} + Y_4 e^{-b_2 Z} + \hat{Q}(S) \quad (12)$$

$$\hat{u}_1(Z, S) = X_1 \left(1 - \frac{a_1^2}{B_2}\right) e^{a_1 Z} + X_2 \left(1 - \frac{a_1^2}{B_2}\right) e^{-a_1 Z} + X_3 \left(1 - \frac{a_2^2}{B_2}\right) e^{a_2 Z} + X_4 \left(1 - \frac{a_2^2}{B_2}\right) e^{-a_2 Z} + \hat{Q}(S) \quad (13)$$

$$\hat{u}_2(Z, S) = Y_1 \left(1 - \frac{b_1^2}{B_4}\right) e^{b_1 Z} + Y_2 \left(1 - \frac{b_1^2}{B_4}\right) e^{-b_1 Z} + Y_3 \left(1 - \frac{b_2^2}{B_4}\right) e^{b_2 Z} + Y_4 \left(1 - \frac{b_2^2}{B_4}\right) e^{-b_2 Z} + \hat{Q}(S) \quad (14)$$

where,

$$a_1 = \sqrt{\frac{\left(\frac{\Theta}{C} + B_1 + B_2\right) + \sqrt{\left(\frac{\Theta}{C} + B_1 + B_2\right)^2 - 4\frac{\Theta}{C}B_2}}{2}}$$

$$a_2 = \sqrt{\frac{\left(\frac{\Theta}{C} + B_1 + B_2\right) - \sqrt{\left(\frac{\Theta}{C} + B_1 + B_2\right)^2 - 4\frac{\Theta}{C}B_2}}{2}}$$

$$b_1 = \sqrt{\frac{\left(\Theta + B_3 + B_4\right) + \sqrt{\left(\Theta + B_3 + B_4\right)^2 - 4\Theta B_4}}{2}}$$

$$b_2 = \sqrt{\frac{\left(\Theta + B_3 + B_4\right) - \sqrt{\left(\Theta + B_3 + B_4\right)^2 - 4\Theta B_4}}{2}}$$

$$\Theta = \frac{SK_3 h_2^2}{n^2}$$

By considering the boundary conditions (Eq.10a-10f), the continuity conditions at the interface between the underlying soil and sand blanket (Eq. 10g-10j), and the initial condition (Eq. 10k), the following matrix can be obtained to get X_i and Y_i ($i = 1, 2, 3, 4$):

$$\xi_{8 \times 8} \psi^T = P^T \quad (15)$$

where $\xi_{8 \times 8}$ is shown in Appendix A as,

$$\psi = [X_1 \ X_2 \ X_3 \ X_4 \ Y_1 \ Y_2 \ Y_3 \ Y_4], \quad P = [\hat{P} - \hat{Q}, \ 0, \ 0, \ 0, \ 0, \ 0, \ 0, \ 0]$$

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

$$B_1 = \frac{8K_1 h_2^2}{F_{a1} n^2}, \quad B_2 = \frac{8h_2^2 (n^2 - 1) K_2}{F_{a1} n^2}, \quad B_3 = \frac{8K_3 h_2^2}{F_{a2} n^2}, \quad B_4 = \frac{8h_2^2 (n^2 - 1) K_4}{F_{a2} n^2},$$

$$F_{a1} = \left(\ln n - \frac{3}{4}\right) \frac{n^2}{n^2 - 1} + \frac{1}{n^2 - 1} \left(1 - \frac{1}{4n^2}\right),$$

$$F_{a2} = \left(\ln \frac{n}{m} + K_5 \ln m - \frac{3}{4}\right) \frac{n^2}{n^2 - 1} + \frac{m^2}{n^2 - 1} (1 - K_5) \left(1 - \frac{m^2}{4n^2}\right) + K_5 \frac{1}{n^2 - 1} \left(1 - \frac{1}{4n^2}\right),$$

$$n = \frac{r_e}{r_w}, \quad m = \frac{r_s}{r_w}, \quad s = r_s / r_w, \quad c_{vi} = \frac{k_{vi}}{m_{vi} \gamma_w}, \quad c_{hi} = \frac{k_{hi}}{m_{vi} \gamma_w}, \quad K_1 = \frac{k_{h1}}{k_{v1}}, \quad K_2 = \frac{k_{h1}}{k_w}, \quad K_3 = \frac{k_{h2}}{k_{v2}}, \quad K_4 = \frac{k_{h2}}{k_w},$$

$$K_5 = \frac{k_{h2}}{k_{s2}}, \quad h_2 = \frac{H}{d_w}, \quad T_{h2} = \frac{c_{h2} \cdot t}{de^2}, \quad C = \frac{c_{v1}}{c_{v2}}, \quad Z = \frac{z}{H}.$$

And $\hat{u}_{w1}(Z, S)$, $\hat{u}_{w2}(Z, S)$, $\hat{u}_1(Z, S)$, $\hat{u}_2(Z, S)$, $\hat{Q}(S)$, S is the Laplace transform of $u_{w1}(Z, T_{h2})$, $u_{w2}(Z, T_{h2})$, $\bar{u}_1(Z, T_{v1})$, $\bar{u}_2(Z, T_{v1})$, $q(T_{h1})$, T_{h2} .

The solutions to the excess pore water pressure u_{wi} and average pore water pressure \bar{u}_i ($i = 1, 2$) were obtained using the inverse Laplace transform of Eqs. (11) - (14), hence:

$$u_{wi}(Z, S) = \frac{1}{2\pi I} \int_{a-I\infty}^{a+I\infty} \hat{u}_{wi}(Z, S) e^{ST} dS \quad (i = 1, 2) \quad (16)$$

$$\bar{u}_i(Z, S) = \frac{1}{2\pi I} \int_{a-I\infty}^{a+I\infty} \hat{\bar{u}}_i(Z, S) e^{ST} dS \quad (i = 1, 2) \quad (17)$$

where $I = \sqrt{-1}$. The analytical solutions for Eqs. (16) and (17) were obtained using the numerical inversion of Laplace transform.

B. Membraneless system

As with the membrane system, the pore water pressure within a vertical drain and the average pore water pressure for a membraneless system, which can be solved by considering the applicable boundary conditions and loading pattern (Appendix A), in the Laplace frequency domain are:

$$\hat{u}_w(Z, S) = \chi_1 e^{b_1 Z} + \chi_2 e^{-b_1 Z} + \chi_3 e^{b_2 Z} + \chi_4 e^{-b_2 Z} + \hat{Q}(S) \quad (18)$$

$$\hat{\bar{u}}(Z, S) = \chi_1 \left(1 - \frac{b_1^2}{B_6}\right) e^{b_1 Z} + \chi_2 \left(1 - \frac{b_1^2}{B_6}\right) e^{-b_1 Z} + \chi_3 \left(1 - \frac{b_2^2}{B_6}\right) e^{b_2 Z} + \chi_4 \left(1 - \frac{b_2^2}{B_6}\right) e^{-b_2 Z} + \hat{Q}(S) \quad (19)$$

The matrix from the Eqs. (18) and (19) are determined by:

$$\xi'_{4 \times 4} \psi^T = P^T \quad (20)$$

where

ξ' is shown in Appendix A as,

$$\psi' = [\chi_1 \quad \chi_2 \quad \chi_3 \quad \chi_4], \quad \mathbf{P}' = [\hat{P} - \hat{Q}, \quad 0, \quad (\eta - 1)P, \quad 0]$$

PERFORMANCE AND PREDICTION OF VACUUM COMBINED SURCHARGE CONSOLIDATION AT PORT OF BRISBANE

$$F_a = \left(\ln \frac{n}{m} + K_5 \ln m - \frac{3}{4} \right) \frac{n^2}{n^2 - 1} + \frac{m^2}{n^2 - 1} (1 - K_5) \left(1 - \frac{m^2}{4n^2} \right) + K_5 \frac{1}{n^2 - 1} \left(1 - \frac{1}{4n^2} \right)$$

$$B_5 = \frac{8K_3 h_2^2}{n^2 F_a}, \quad B_6 = \frac{8h_2^2 (n^2 - 1) K_4}{F_a n^2}.$$

Using the inverse Laplace transform, the excess pore water pressure u_w and average pore water pressure \bar{u} can be obtained.

The settlement of the soil is given by:

$$s(t) = \int_{L_w}^H \varepsilon_2 dz \quad (21)$$

Theoretically, the average degree of consolidation may be defined either in terms of strain or pore pressure. While the former shows the rate of settlement, the latter indicates the dissipation rate of excess pore water pressure.

The average degree of consolidation in terms of settlement can be expressed as:

$$\bar{U}_s = 1 - \frac{m_{v1} \int_0^\rho L^{-1}(\hat{u}_1) dZ + m_{v2} \int_\rho^1 L^{-1}(\bar{u}_2) dZ}{m_{v1} \int_0^\rho (q_u - p) dZ + m_{v1} \int_\rho^1 (q_u - p) dZ} \quad (22)$$

The average degree of consolidation may be defined in terms of effective stress (i.e. dissipation of excess pore water pressure) as:

$$\bar{U}_p = 1 - \frac{\int_0^\rho L^{-1}(\hat{u}_1) dZ + \int_\rho^1 L^{-1}(\bar{u}_2) dZ}{\int_0^\rho (q_u - p) dZ + \int_\rho^1 (q_u - p) dZ} \quad (23)$$